Fiscal and Monetary Policy in the Presence of Informality and the Incentive to Join a Currency Union [1]

Mahama Samir Bandaogo [2]
October 17, 2016

Abstract:

How does economic activity outside of government control –informality– affect the conduct of fiscal and monetary policy? I study this question in a New Keynesian, small open economy model. The model is assumed to feature informality in both goods and labor markets. A non-traded sector produces a non-taxed informal good. The traded sector produces a formal good and is subject to taxation, but it can hire workers using both formal and informal contracts. I show that the presence of informality decreases the optimal tax rate and increases macroeconomic volatility. Moreover, when the country cannot credibly precommit to the optimal policy, informality significantly increases the incentive to peg the currency. This result can help explain why many sub-Saharan African countries have plans to either expand existing currency unions or to form new ones.

Keywords: Informal Sector, Monetary Policy, Fiscal Policy, Currency Union.
JEL Classification No: J46, E26, E52, E62, F41, H21, H26.

[1] I would like to thank the members of my committee, Fabio Ghironi, Yu-chin Chen and Philip Brock for their guidance and support. I also thank all the participants in the International & Macroeconomics workshops for their useful comments.

[2] Northeastern University, Seattle, WA. Email: bandaogo@uw.edu

1 Introduction

This paper studies how informality affects optimal fiscal and monetary policy in a small open economy. Informality refers to economic activities outside of government regulation, such as firms not declared to the authorities and illegal employment. It is an empirical relevant phenomenon in both developing and industrial countries. Schneider (2005) estimates that the informal sector varies between 38% and 58% of GDP in a number of sub-Saharan African countries.[3] Several countries in the Euro area are similarly characterized by large informal sectors and those with the largest informality are at the center of the recent euro area crisis.[4] A common trait across these developing and industrial economies is that they have either given up monetary independence by joining the euro or they are considering taking the same action and joining a monetary union: The West African currency union of the Colonies Françaises Africaines (CFA) plans to expand from eight to fifteen member countries by 2020. All current members and those planning to join the new arrangement have large informal sectors. Since informality reduces the tax base and constrains tax revenues, it decreases the room to maneuver fiscal policy for counter-cyclical stabilization. With the loss of monetary policy independence that occurs when countries join a currency union, the constraint placed on fiscal policy by informality may have implications for the desirability of giving up monetary independence. At the same time, informality may exacerbate problem caused by time inconsistency in the conduct of policy. The purpose of this paper is to explore how the different aspects of informality affect fiscal and monetary policy incentives, including the desirability of giving up monetary independence by pegging the currency.

I set up a New Keynesian, dynamic stochastic general equilibrium (DSGE) that extends the benchmark model developed by Galí and Monacelli (2005) to study fiscal and monetary policy in small, industrial economies by introducing informality in goods and labor markets. As in Galí and Monacelli (2005), I assume a continuum of small open economies. Each country has two sectors of production: a non-traded sector and a traded sector. The Home country's non-traded sector is "informal": It is characterized by low productivity, and it is "invisible" to the government. Thus, it is unregulated and not taxed. Non-traded sectors in the rest of the world need not be unregulated. The Home country's traded sector is instead the formal one, regulated and taxed by the fiscal authority. The government collects taxes from firms operating in the formal sector and provides a productive public good that is used by firms in both sectors to produce their respective goods. I follow Loayza (1996) and assume that the informal firms only have partial access to the public good since they are not taxed and regulated by the government. The fiscal authorities have access only to distortionary taxes and can only see and tax the formal sector. When setting the optimal tax rate, the government must take into account the presence of the informal sector. On one hand, the tax decreases formal output since it increases the marginal cost of the formal firms. On the other hand, tax proceeds are used to finance provision of a public good that decreases marginal costs also in the formal sector, implying a positive effect of taxation of output through this channel. On balance, taxes levied in the formal sector only increase informal output, since the public good decreases the marginal cost of the informal firms. Formal firms do not evade taxes, but they hire a mix of formal and informal workers in order to decrease their marginal cost of

[3]Nigeria (58%), Benin (45%), Niger (42%), Mali (41%), Côte d'Ivoire (40%), Ghana (38%), Burkina-Faso (38%)

[4]Greece (29%), Italy (27%), Spain (23%), Belgium (23%), Germany (16%), France (15%)

production. Formal firms pay the informal workers a lower wage compared to the formal workers.

I use the Ramsey approach of Schmitt-Gohé and Uríbe (2004) to characterize optimal fiscal and monetary policy, and I compare optimal policy in the economy with no informality to the economy with informality. I find that the optimal tax rate in the presence of informality is significantly lower than the tax rate when there is no informality. This result is due to the fact that in the presence of informality, the government finds it optimal to decrease the tax rate to prevent the formal good from becoming relatively more expensive than the informal good. Importantly, if the only type of informality in the economy is the use of informal labor in the formal sector, then the government finds it optimal to increase the tax rate. This is because informal labor decreases the formal firms' marginal cost and allows them to increase output. In this case, an increase in the tax rate restores output to its constrained-efficient level. Long run inflation is zero under the optimal polity regardless of the presence of informality, consistent with much of the literature on optimal monetary policy in industrial countries. Informality does not alter the conclusion that zero inflation is optimal under commitment. However, in contrast to the current literature that prescribes price stability over the business cycle as optimal, I find that significant departures from price stability is optimal in response to shocks in the presence of informality. When there is no informality, the volatility of the tax rate and inflation are very low and similar across these policy instruments. Policy authorities should adopt a policy of price stability and smooth taxation over time, in line with the with standard results in the literature on optimal fiscal and monetary policy. In the presence of informality, tax smoothing remains optimal, but price stability ceases to be so, as indicated by the drastic increase in optimal inflation volatility. When faced with significant informality, the Ramsey policy maker finds tax variation more costly than fluctuations in inflation. This result stands in contrast to those in Schmitt-Gohé and Uríbe (2004), where changes in the tax rate were used more often than changes in inflation. If the country pegs its currency, then it can no longer use changes in inflation, and can only rely on fiscal policy. Historically, the CFA-zone countries adopted a common long-run tax rate of 18%. Comparing the welfare under the CFA countries' historical policy to the welfare under the optimal Ramsey policy under commitment and optimal policy under discretion, I find that informality decreases the incentive to join a currency union if the countries can credibly commit to their own optimal policy. However, if the countries do not have the technology or the credibility to commit, pegging the currency is better than pursuing discretionary policy: Under discretion, the policy maker uses both changes in inflation and taxation significantly more often, with costs that rare amplified in the presence of informality. Thus, informality strengthens the argument for currency pegs originally explored by Giavazzi and Pagano (1988).

The findings in this paper contribute to the literature on informality by focusing on the impact of informality on the conduct of fiscal and monetary policy. Very little is known about how informality affects these policies. The majority of studies on the informal sector, such as Áureo de Paula and Scheinkman (2007) and Marjit and Kar (2009), have been more concerned with its determinants and less with its impact on macroeconomic variables and policy making. Some recent papers have shown that informal labor markets have macroeconomic implications through their impact on aggregate variables, such as inflation (Castillo and Montoro (2012)), and their volatility (Shapiro (2015)). This paper contributes further by exploring the impact of informality on other macroeconomic variables. A small existing literature (Castillo and Montoro (2012) and Batini

et al. (2011)) explores the impact of informality on inflation dynamics and the conduct of monetary policy. These papers consider informality in labor and credit markets, ignoring informality in the goods market, and they conduct their analyses in the context of a closed economy. My framework differs from theirs in that it considers goods and labor informality in the context of a small open economy. Moreover, I address the question of optimal taxation, which many believe is the driver of informality, by studying its impact on the optimal tax rate.

This paper contributes also to the extensive literature on optimal fiscal and monetary policy for small economies. For instance, Galí and Monacelli (2005) and Benigno and Benigno (2003) explore the scenarios in which price stability is optimal for open economies.Schmitt-Gohé and Uríbe (2004) find that in the presence of nominal rigidity, the Ramsey planner resorts to price stability and uses fiscal policy more often. The framework laid out in this paper follows the literature on optimal monetary policy by developing a New Keynesian DSGE (NK-DSGE) model with different features of informality. Informality in the labor market allows firms to decrease their marginal cost of production by hiring cheaper labor, and perfect competition in the informal goods market – which implies flexible prices – affects the overall (CPI) inflation dynamics in the economy. Informality thus has direct implications for the conduct of monetary policy and this paper contributes to the literature by laying out the conduct of optimal policy in the presence of informality.

Finally the paper also contributes the literature on optimal exchange rate regimes for small open economies. Scholars have been giving opposing policy prescriptions on exchange rate regimes for small countries. On one hand, Aghion et al. (2009) conclude that developing countries with low financial development should adopt a less flexible exchange rate in order to enjoy faster economic growth. On the other hand, Levy-Yeyati and Sturzenegger (2003) and Galí and Monacelli (2005) suggest that countries should adopt a more flexible exchange rate regime. Except for Galí and Monacelli (2005), these studies are concerned with the impact of the exchange rate regime on economic growth. I re-evaluate the desirability of relinquishing monetary policy independence in the presence of informality, and I find that informality strengthens the classical argument for exchange rate pegs in Giavazzi and Pagano (1988): If precommitment to optimal policy is not feasible, a peg (or a currency union) is more desirable than discretion, the more so the larger the informal sector. This result can help explain why many sub-Saharan African countries have plans to either expand existing currency unions or the form new ones.

The rest of the paper is organized as follows. Section 2 discusses some characteristics of informality, which are the basis of the assumptions I make later in the model. Section 3 lays out the model. Section 4 studies the consequences of informality in a version of the model without nominal rigidity and solves numerically for the optimal tax rate in this environment. Section 5 solves for the optimal fiscal and monetary policies in the presence of price stickiness for the economy with and without informality. Section 6 explores alternative policies and performs a welfare ranking exercise. Section 7 concludes.

2 The Informal Sector

Informality can be defined in many ways, but in this paper I use informality to refer to economic activities out of the control of authorities. In the literature, it is also referred

to as the underground economy and the black market. Batini et al. (2010) provides a very good summary of informality around the world. Informality can be defined in terms of goods, credit and lending, and labor. In this paper, I cast informality in terms of consumption good and labor. In particular, I use the term informal sector to refer to the non-traded, non-regulated (not taxed) and low productivity sector of the economy. I also introduce informality in the formal sector, to refer to the phenomenon of formal firms hiring informal labor, also known as "under the table" labor. This captures the informalization of the formal sector. Employees with "under the table" contracts receive lower wages that those with formal contracts. All countries have some degree of informality, but the phenomenon is more pervasive in developing countries, especially those in Sub-Saharan Africa. Schneider (2005) estimates the size of the underground economy between 1999 and 2004 at around 41% of GPD in developing economies and about 18% in the OECD countries. In recent years, policy makers have shown a particular interest in the informal sector and its implication for different policies. Chen (2007) and Williams (2014) both discuss the shift in how the informal sector is viewed. Firstly, there is the change in how informality is defined, shifting from a narrower to a much broader definition. Secondly, there is increased recognition that informality is pervasive and here to stay as long as what is considered to be the formal sector thrives. Policy makers now talk about the accommodation, rather than the eradication, of the informal sector. Since theory predicted that as economies grow, the informal workforce moves to the formal sector and informality decreases, the informal sector was long viewed as a short term phenomenon. However, Kanbur (2015) and Williams (2014) document and discuss its pervasive and long term nature. Many reasons have been given to justify the existence of the informal sector ranging from the burden of regulation (taxation) to the shortage of employment in the formal sector. I do not explicitly model the creation of the informal sector: I take it as given and incorporate its key characteristics affecting the conduct of fiscal policy and monetary policy. I use several studies and surveys of the informal sector to support some of my assumptions. Below, I summarize some key assumptions and use a 2013 study from ANSD (2013) as reference.

Level of Education in the Informal Sector:

I take the informal sector as given and a low level of education as the main cause of its existence. Agents with high level of education are able to secure employment in the formal sector and those with less education find employment in the informal sector. This is in line with previous papers that have modeled the informal sector such as Azuma (2008), Rauch (1991) and Fiess et al. (2001). The informal sector is documented to have very low productivity (McMillan and Rodrik (2012)) and this can be justified by the low level of education of its workforce, at least in Sub-Saharan Africa. A study from Senegal (ANSD (2013)) supports the claim that the education level of agents operating in the informal sector is low. Many agents did not attend school, some attended only a few years of elementary education and many received traditional Islamic schooling (learning the Koran without any formal training). A very small fraction, less than 5%, graduated (at least) high school. The following summarizes the findings in the study.

Competition:

Another important assumption that has implications for the results of the model is the assumption of a perfectly competitive informal sector. In this way, informal prices adjust every period and monetary policy has no direct impact on real (informal) variables. Hence, as the degree of informality increases, the economy becomes more and more

Table 1: Level of Education in the Informal Sector in Senegal

Level of Education	Percentage
No School	28
Some Primary School	28.2
Some High School	18
Islamic School	16
Some High School and more	4.5

competitive and overall price level (CPI) becomes more flexible. ANSD (2013) conducted a survey of firms in the informal sector and found that competition is the biggest problem encountered by those firms, which is due to the non-differentiation of their products. This finding constitutes the basis of my assumption of a perfectly competitive informal sector.

3 The Model

The world economy consists of a continuum of small open economies (of size zero) on the unit interval with identical preferences and technology, as in Galí and Monacelli (2005). Each economy has two sectors of production, a traded sector and a non-traded sector. The representative household consists of a continuum of members with different levels of education. In the Home country, members of the household supply their labor to both the (traded) formal sector and the (non-traded) informal sector. In the formal sector, some members of the household (those with the highest level of education) receive a formal wage contract and some (with a relatively lower level of education) receive an informal ("under the table") wage contract, which pays a lower wage than the formal contract. Consumption consists of the informal good and a bundle of formal goods produced domestically and abroad. The government collects a unit tax on the production of formal goods and uses the proceeds to provide non-rival productive public services that are used by both formal and informal firms. Following Loayza (1996), formal firms are registered and known to the authorities, so they have full access to the public services provided by the government. The informal firms, on the other hand, are not declared to authorities and only have access to a fraction of the public services.

3.1 Household and Preferences

All households have identical preferences over a consumption bundle C_t and three types of labor N_t^I, N_t^F and L_t. With a discount rate β, the lifetime utility of each household is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t; N^F_{\phi_i,t}; N^I_{\phi_i,t}; L_{\phi_i,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \int_\phi \frac{\left(N^F_{\phi_i,t}\right)^{1+\psi}}{1+\psi} d\phi_i - ... \right\} \left\{ ... \int_\phi \frac{\left(N^I_{\phi_i,t}\right)^{1+\psi}}{1+\psi} d\phi_i - \int_\phi \frac{L^{1+\mu}_{\phi_i,t}}{1+\mu} d\phi_i \right\} \tag{1}$$

where ϕ_i represents individual i's level of education. $N^F_{\phi_i,t}; N^I_{\phi_i,t}$ and $L_{\phi_i,t}$ represent the amount of hours individually spent by household members with talent level ϕ_i working. The individual hours are then aggregated over levels of education for each type of contract in order to get N^F_t and N^I_t, which are the aggregate hours devoted to working in the formal sector under formal contract and under informal contract, respectively. L_t represents the aggregate hours devoted to working in the informal sector. This specification of the household's utility function assumes risk sharing within the household; all members consume the same consumption basket regardless of wage earnings. C_t is the consumption basket of the household and it is, in turn, a composite of consumption indexes defined as:

$$C_t = \left[\alpha^{\frac{1}{\eta}} C_{I,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where α is the proportion of the informal good in the consumption bundle. It can also act as a proxy for the size of the informal (goods) sector in the Home country. $C_{I,t}$ is the informal good and $C_{F,t}$ is a composite of consumption index of Home formal goods and Foreign formal goods. η is the elasticity of substitution between the informal good and the formal good. $C_{F,t}$ is defined as:

$$C_{F,t} = \left[(1-a)^{\frac{1}{\gamma}} C_{HF,t}^{\frac{\gamma-1}{\gamma}} + a^{\frac{1}{\gamma}} C_{FF,t}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{3}$$

where a is the home country's degree of openness and also a measure of home bias ($a < 0.5$). γ is the elasticity of substitution between a domestically produced formal good and a foreign formal good. $C_{HF,t}$ and $C_{FF,t}$ are both indexes of formal goods:

$$C_{HF,t} = \left(\int_0^1 C_{HF,t}(j)^{\frac{\lambda-1}{\lambda}} dj\right)^{\frac{\lambda}{\lambda-1}} \qquad C_{FF,t} = \left(\int_0^1 (C^i_{FF,t})^{\frac{\upsilon-1}{\upsilon}} di\right)^{\frac{\upsilon}{\upsilon-1}}$$

where $C^i_{FF,t}$ is an index of all imported formal goods from country i and defined by:

$$C^i_{FF,t} = \left(\int_0^1 C^i_{FF,t}(j)^{\frac{\lambda-1}{\lambda}} dj\right)^{\frac{\lambda}{\lambda-1}}$$

$j \in [0,1]$ represents the different varieties of formal goods in each country, $i \in [0,1]$ represents the different countries in the world economy, λ is the elasticity of substitution between formal goods within the same country and υ the elasticity between formal goods of two different countries. Each country is assumed to be in financial autarky, so each household maximizes (1) subject to a sequence of budget constraints as follows:

$$P_t C_t \leq \int_{\phi_i} W^F_{\phi_i,t} N^F_{\phi_i,t} d\phi_i + \int_{\phi_i} W^I_{\phi_i,t} N^I_{\phi_i,t} d\phi_i + \int_{\phi_i} \Omega_{\phi_i,t} L_{\phi_i,t} d\phi_i \tag{4}$$

where $W^F_{\phi_{i,t}}$ is the wage earned by an individual with education level ϕ_i in the formal sector under a formal wage contract, $W_{\phi^I_{i,t}}$ is wage earned in the formal sector under informal wage contract and $\omega_{\phi_{i,t}}$ is wage earned in the informal sector. Given the above specification of preferences, the optimal allocation of spending yields the following demand functions. The Household's demand for home formal good j and the price index for home formal goods are :

$$C_{HF,t}(j) = \left(\frac{P_{HF,t}(j)}{P_{HF,t}}\right)^{-\lambda} C_{HF,t} \qquad P_{HF,t} = \left(\int_0^1 P_{HF,t}(j)^{1-\lambda} dj\right)^{\frac{1}{1-\lambda}}$$

And the HH's demand for foreign formal goods is:

$$C^i_{FF,t}(j) = \left(\frac{P^i_{FF,t}(j)}{P^i_{FF,t}}\right)^{-\lambda} C^i_{FF,t} \qquad C^i_{FF,t} = \left(\frac{P^i_{FF,t}}{P_{FF,t}}\right)^{-\upsilon} C_{FF,t}$$

$C^i_{FF,t}(j)$ is the HH's demand for foreign formal good j from country i and $C^i_{FF,t}$ is the total imports of formal goods from country i. The Price index for goods from country i, $P^i_{FF,t}$, and the Price index for total imports, $P_{FF,t}$, can be expressed as:

$$P^i_{FF,t} = \left(\int_0^1 P_{FF,t}(j)^{1-\lambda} dj\right)^{\frac{1}{1-\lambda}} \qquad P_{FF,t} = \left(\int_0^1 (P^i_{FF,t})^{1-\upsilon} di\right)^{\frac{1}{1-\upsilon}}$$

The demands for (index of) Home formal goods and Foreign formal good are respectively given by:

$$C_{HF,t} = (1-a)\left(\frac{P_{HF,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \qquad C_{FF,t} = a\left(\frac{P_{FF,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t}$$

The price index for formal goods is:

$$P_{F,t} = \left[(1-a)P^{1-\gamma}_{HF,t} + aP^{1-\gamma}_{FF,t}\right]^{\frac{1}{1-\gamma}} \tag{5}$$

The demand functions for the informal good and formal goods are:

$$C_{I,t} = \alpha\left(\frac{P_{I,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = (1-\alpha)\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

The Consumer Price Index is:

$$P_t = \left[\alpha P^{1-\eta}_{I,t} + (1-\alpha)P^{1-\eta}_{F,t}\right]^{\frac{1}{1-\eta}} \tag{6}$$

The households intratemporal optimality conditions are:

$$\left(N^F_{\phi_i,t}\right)^{\psi} C^{\sigma}_t = \frac{W^F_{\phi_i,t}}{P_t} \qquad \left(N^I_{\phi_i,t}\right)^{\psi} C^{\sigma}_t = \frac{W^I_{\phi_i,t}}{P_t}$$

$$L^{\mu}_{\phi_i,t} C^{\sigma}_t = \frac{\Omega_{\phi_i,t}}{P_t} \tag{7}$$

3.2 Employment and Wage Setting

The labor market for each type of employment is perfectly competitive. Each household member takes the wage rate as given and decides how many hours to supply. The labor markets are segmented and labor is immobile. That is, members with no or very little schooling receive informal wages in the informal sector; members with high school education receive informal ("under the table") wage contracts; and members with college or advanced degrees receive formal wages contracts. Given the competitiveness of the markets, the wage is constant across each type of employment:

$$W^F_{\phi_i,t} = W^F_t \qquad W^I_{\phi_i,t} = W^I_t \qquad \Omega_{\phi_i,t} = \Omega_t$$

The household's optimality conditions can be then be written simply as:

$$\left(N_t^F\right)^{\psi} C_t^{\sigma} = \frac{W_t^F}{P_t} \qquad \left(N_t^I\right)^{\psi} C_t^{\sigma} = \frac{W_t^I}{P_t} \qquad L_t^{\mu} C_t^{\sigma} = \frac{\Omega_t}{P_t} \tag{8}$$

And the intertemporal optimality condition of the household can be written as:

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = Q_{t+1} \tag{9}$$

with Q_{t+1} being the discount factor.

3.3 Related Prices

I define the relative price of the informal good to the formal good as follows:

$$\Theta_t = \frac{P_{I,t}}{P_{F,t}} \tag{10}$$

I further define the bilateral terms of trade between the domestic country and a given country i as the price of country i's goods in terms of the home goods: $S_{i,t} = \frac{P_{i,FFt}}{P_{HF,t}}$. The effective terms of trade can then be written as:

$$S_t = \frac{P_{FF,t}}{P_{HF,t}} = \left(\int_0^1 S_{i,t}^{1-\upsilon} di\right)^{\frac{1}{1-\upsilon}}$$

The bilateral real exchange rate between the Home country and country i is defined as the price of goods from country i in terms of the price of the domestic country's goods, $Q_{i,t} = \frac{\xi_{i,t} P_{i,t}}{P_t}$, and the effective real exchange rate (the price of the goods from the rest of the world in terms of the price of the domestic goods) is $Q_t = \frac{\xi_t P_t^*}{P_t}$. I define the bilateral real exchange rate for formal goods as $Q_{i,t}^F = \frac{\xi_{i,t} P_{iF,t}}{P_{F,t}}$ and the effective real exchange rate as $Q_t^F = \frac{\xi_t P_{F,t}^*}{P_{F,t}}$. Purchasing Power Parity (PPP) is violated due to the presence of the non-traded informal good.

3.4 International Risk Sharing

I assume financial autarky, which implies the value of all imports must equal the value of all exports. The Home country cannot borrow to finance its spending. This yields the following:

$$S_t \left(\frac{P_{FF,t}}{P_{F,t}}\right)^{-\gamma} C_t = \left(\frac{P_{HF,t}}{P_{F,t}}\right)^{-\gamma} \int_0^1 \left(S_t^i S_{i,t}\right)^{\upsilon-\gamma} \left(Q_{i,t}^F\right)^{\upsilon-\eta} Q_{i,t}^{\eta} C_t^i di$$

$$S_t^{1-\gamma} C_t = \int_0^1 \left(S_t^i S_{i,t}\right)^{\upsilon-\gamma} \left(Q_{i,t}^F\right)^{\upsilon-\eta} Q_{i,t}^{\eta} C_t^i di$$

Under the Cole and Obtsfeld specification ($\upsilon = \gamma = \eta = 1$) this condition boils down to:

$$C_t = Q_t C_t^* \tag{11}$$

which is essentially equivalent to the market sharing condition under complete (market) risk sharing, where C_t^* is the rest of the world's consumption.

3.5 Firms and Production

Any given firm in the formal sector produces a differentiated traded good with two types of labor. Both types are perfectly substitutable and produce identical good j:

$$Y_{H,t}(j) = \left(A_t^F N_t^F(j) + A_t^I N_t^I(j)\right) G_t^{\chi} \tag{12}$$

where N_t^F and N_t^I are, respectively, the number of hours worked by the employees with formal wage contracts and employees with no formal wage contracts employed by firm j. This specification is similar to the one used by Castillo and Montoro (2012). A_t^F and A_t^I are their corresponding levels of productivity, for which I assume that A_t^I is a fraction κ of A_t^F. Following Loayza (1996) I introduce the flow of government spending as productivity-enhancing with an output elasticity of χ such that the formal firm production function is:

$$Y_{H,t}(j) = \left(N_t^F(j) + \kappa N_t^I(j)\right) A_t^F G_t^{\chi} \tag{13}$$

Aggregating over each type of formal good produced, the index of formal good is given by:

$$Y_{H,t} = \left(\int_0^1 Y_{H,t}\left(j\right)^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}} \tag{14}$$

This aggregate output can then be related to an aggregate level of employment and the price dispersion $\Delta_{p_{h,t}}$:

$$Y_{H,t} = \left(N_t^F + \kappa N_t^I\right) \Delta_{p_{h,t}} A_t^F G_t^{\chi} \tag{15}$$

The level of productivity follows an AR(1) process as $A_t^F = \rho_F A_{t-1}^F + \epsilon_t^F$.

The perfectly competitive informal firm combines labor and a fraction of the productive public good to produce the informal good. The production function is given by:

$$Y_{I,t} = A_{I,t} L_t \left(\delta G_t\right)^{\chi} \tag{16}$$

δ is the fraction of public services to which firms operating in the informal sector have access. Consequently, δ can also serve as an indicator of the effectiveness of the authorities' crackdown on informal firms. The harder they crack down the lower the fraction of public services informal firms have access to, hence the lower δ is. One could assume that public goods have different impacts on both sectors or that congestion might arise with rapid growth of the informal sector, but to simplify the analysis I assume that output elasticity of government spending is constant across both sectors. The level of productivity is an AR(1) process, $A_t^I = \rho_I A_{t-1}^I + \epsilon_t^I$.

3.6 The Government

The government or the fiscal authorities collect a unit tax τ_t on every formal good sold. It then uses the tax revenues to provide a non-rival public good G_t. The government runs a balanced budget in every period as follows:

$$G_t = \tau_t Y_{H,t} \tag{17}$$

From this relation, it is clear that the size of the public good directly depends on the size of the formal sector and the tax rate. Since the public good is productive, an expansion of the formal sector has a positive externality on the informal sector. The tax in itself is distortionary for the formal sector, because it decreases the amount of goods produced, but at the same time it is productivity-enhancing for both sectors through the provision of the public good.

3.7 Pricing

Since the informal sector is perfectly competitive, the price is equal to the marginal cost and is fully flexible.

$$P_{I,t} = MC_{I,t}$$
$$MC_{I,t} = \frac{\Omega_t}{A_{I,t}\left(\delta G_t\right)^{\chi}}$$

Using the labor consumption substitution rate from the household optimality, $\Omega_t / P_t = L_t^{\mu} C_t^{\sigma}$, the informal price setting relation becomes:

$$P_{I,t}^{1-\alpha} = \frac{L_t^{\mu} C_t^{\sigma}}{A_{I,t}\left(\delta G_t\right)^{\chi}} P_{F,t}^{1-\alpha} \tag{18}$$

And in terms of inflation rates:

$$\Pi_{I,t}^{1-\alpha} = \left(\frac{G_{t-1}}{G_t}\right)^{\chi} \left(\frac{C_t}{C_{t-1}}\right) \left(\frac{A_{i,t-1}}{A_{i,t}}\right) \Pi_{F,t}^{1-\alpha} \tag{19}$$

Prices are sticky in the formal sector, à la Calvo (1983). In each period, a fraction θ of all formal firms get to reset their price. The firms set the optimal price by maximizing the discounted values of all expected profits. The optimal price must satisfy the following first order condition:

$$P_{H,t}^{*} = \left(\frac{\gamma}{\gamma - 1}\right) \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[Q_{t+k} P_{H,t+k}^{\gamma+1} Y_{H,t+k} MC_{t+k}^{n}\right]}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[(1-\tau_{t+k}) Q_{t+k} P_{H,t+k}^{\gamma} Y_{H,t+k}\right]} \tag{20}$$

where $Q_{t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$. Expressed in terms of inflation rates and written recursively this equation becomes:

$$\Pi_{HF,t}^{*} = \left(\frac{\gamma}{\gamma - 1}\right) \frac{F_{1t}}{F_{2t}}$$
$$F_{1t} = Y_{H,t} MC_{F,t} + \theta\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma} F_{1,t+1}$$
$$F_{2t} = Y_{H,t}\left(1 - \tau_t\right) + \theta\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \Pi_{t+1}^{-1} \Pi_{HF,t+1}^{\gamma-1} F_{2,t+1}$$

The optimal price inflation and the overall formal sector inflation are linked by the following equation:

$$\Pi_{HF,t}^{*} = \left(\frac{1 - \theta \Pi_{HF,t}^{\gamma-1}}{1 - \theta}\right)^{\frac{1}{1-\gamma}} \tag{21}$$

The price dispersion $\Delta_{ph,t}$, due to price stickiness is expressed as:

$$\Delta_{p_{h,t}} = \theta \Pi_{HF,t}^{\gamma} \Delta_{p_h,t-1} + (1-\theta)\, \Pi_{HF,t}^{*(-\gamma)} \tag{22}$$

3.8 Equilibrium

The informal firms only have to meet domestic demand, so market clearing in the informal sector entails output being equal to domestic demand:

$$Y_{I,t} = \alpha \left(\frac{P_{I,t}}{P_t} \right)^{-\eta} C_t \tag{23}$$

The formal good is consumed both domestically and abroad, so market clearing in the formal sector entails output being equal to domestic demand plus foreign demand (exports):

$$Y_{H,t} = (1-\alpha) \left(\frac{P_{HF,t}}{P_{F,t}} \right)^{-\gamma} \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} \left[(1-a) C_t + a \int_0^1 \left(S_t^i S_{i,t} \right)^{\upsilon-\gamma} \left(Q_{i,t}^F \right)^{\upsilon-\eta} Q_{i,t}^{\eta} C_t^i di \right] \tag{24}$$

where $Q_{i,t}^F$ is the formal good real exchange rate. And assuming the Cole and Obstfeld specification ($\sigma = \eta = \upsilon = \gamma = 1$) these two market clearing conditions reduce to:

$$Y_{I,t} = \alpha \Theta_t^{\alpha-1} C_t \tag{25}$$

$$Y_{H,t} = (1-\alpha) S_t^a \Theta_t^{\alpha} \left[(1-a) C_t + a Q_t C_t^* \right] \tag{26}$$

Using the risk sharing condition under financial autarky ($C_t = Q_t C_t^*$), the formal sector market-clearing equilibrium can be written as:

$$Y_{H,t} = (1-\alpha) S_t^a \Theta_t^{\alpha} C_t \tag{27}$$

When there is no good informality $\alpha = 0$ and $Y_{I,t} = 0$. The formal sector market-clearing then becomes $Y_{H,t} = S_t^a C_t$. Combining the market-clearing in the informal and formal sectors and the international risk sharing, domestic consumption can expressed as:

$$C_t = \left(\frac{Y_{H,t}}{1-\alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} \left(C_t^* \Theta_t^{*\alpha} \right)^{\frac{a(1-\alpha)}{1-2\alpha}} \tag{28}$$

The terms of trade and the relative price of the informal good to the formal good can be expressed as:

$$\Theta_t = \left(\frac{Y_{I,t}}{\alpha C_t} \right)^{\frac{1}{\alpha-1}} \tag{29}$$

$$S_t = \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{\alpha}{(1-\alpha)(1-a)}} C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} \left(C_t^* \Theta_t^{*(-\alpha)} \right)^{\frac{1}{1-a}} \tag{30}$$

3.9 Marginal Costs and The Linkage Between Formal and Informal Sectors

The informal sector having access to public goods provided by the government through taxation of the formal sector makes the former dependent on the latter. The size of the public good is positively correlated with the size of the formal good sector for any given tax rate. By increasing informal sector productivity, public good effectively decreases the informal sector marginal cost. The formal firm's marginal cost chooses formal and

informal labor until their respective marginal cost are equalized. This implies a constant proportion of formal labor to informal labor given by:

$$N_t^F = \kappa^{\frac{1}{\psi}} N_t^I \tag{31}$$

which implies that formal production in terms of formal labor can be expressed as:

$$Y_{H,t} = \bar{v} A_t^F N_t^F G_t^{\chi} \tag{32}$$

where $\bar{v} = \left(1 + \kappa^{1+\frac{1}{\psi}}\right)$. The marginal cost in the formal sector can then be expressed as:

$$MC_t = \frac{W_t^F}{\bar{v} A_t^F G_t^{\chi} (1 - \tau_t)} \tag{33}$$

The above marginal cost is in terms of formal wage contracts, but it makes use of the optimal labor proportion so that the marginal cost in terms of the informal wage contract is identical. The informal firm's marginal cost can be written as:

$$MC_{I,t} = \frac{\Omega_t}{A_{I,t} \left(\delta G_t\right)^{\chi}} \tag{34}$$

Since prices are fully flexible in the informal sector, we have that $MC_{I,t} = 1$ and the output level is given by:

$$Y_{I,t} = A_{I,t} \left(\delta G_t\right)^{\chi} \alpha^{\frac{1}{1+\mu}} \tag{35}$$

G_t is a function of $Y_{H,t}$, so the informal sector output increases with the formal sector output. The presence of informality in the formal sector decreases the elasticity of the marginal cost with respect to output. An increase in demand (positive demand shock) causes the marginal cost to increase less as informality (in the formal sector) increases; this also means that inflation increases less. The tax levied by fiscal authorities increases formal sector marginal cost and the provision of public good decreases the marginal cost. The informal sector, on the other hand, benefits from a decrease in marginal cost without the tax burden. The current set-up of the model is such that the informal sector has no impact on the productivity of the public good. that can changed by making the public good less productive with an increase in the informal sector but not the formal sector, since that sector contributes to the financing of the public good. The public good is dependent on the size of the formal sector and the tax rate levied by the fiscal authorities. When $\chi = 0$, this equation boils down to the regular output level found in the New Keynesian literature; fully flexible price level of output is only a function of the productivity shocks.

3.10 The Rest of the World

The world economy is taken as given, and the home economy considered to be too small to have any impact on its dynamics. The rest of the world is populated by identical countries with two sectors of production, traded and non-traded (not necessarily informal). The traded sector, denoted by T, and the non-traded sector, denoted NT, are characterized by the following equations. The Marginal cost in the traded sector is:

$$MC_t^* = \frac{C_t^{*(\sigma+\psi)}}{A_t^*} \tag{36}$$

Firms in the traded sector that reset their prices choose the optimal price:

$$\Pi^*_{FF,t+k} = \left(\frac{\gamma}{\gamma - 1}\right) \frac{F^*_{1t}}{F^*_{2t}} \tag{37}$$

where:

$$F^*_{1t} = C^*_t MC^*_t + \theta\beta\Pi^{*(-1)}_{t+1}\Pi^{*(\gamma+1)}_{FF,t+1}F^*_{1,t+1}$$
$$F^*_{2t} = C^*_t + \theta\beta\Pi^{*(-1)}_{t+1}\Pi^{*\gamma}_{FF,t+1}F^*_{2,t+1}$$

The non-traded sector only produces a fraction of the traded sector output as follows:

$$Y^*_{NT,t} = \alpha C^*_t \tag{38}$$

The average marginal cost of a firm in the non-traded sector is given by:

$$MC^*_{NT,t} = \frac{Y^{*(\sigma+\psi)}_{NT,t}}{A^*_{NT}} \tag{39}$$

The respective inflation rates in the non-traded and traded sector are given by the expressions:

$$\Pi_{NT,t} = \frac{MC^*_{NT,t}}{MC^*_{NT,t-1}}$$
$$\Pi^*_t = \Pi^{\alpha}_{NT,t}\Pi^{*(1-\alpha)}_{FF,t}$$

where, for simplicity, I assume the non-traded sector to have fully flexible prices, just like the non-traded informal sector in the Home country. I take France as the rest of the world in this set-up, because the CFA-Franc used in these countries was originally pegged to the French Franc and now to the Euro (I also used calibration parameters of the Euro and the results did not change).

3.11 The Distortions

In this section I look at the different distortions in the economy.

Informality: The presence of informality distorts the labor market by reducing the amount of formal labor and allowing firms to replace formal labor with informal labor. This affects the marginal cost of production in the formal sector. Informality in goods decreases formal labor $(1 - \alpha)$ and labor informality increases formal labor $(v > 1)$.

Distortionary Taxation: The provision of the public through a unit sales tax increases the marginal cost of production, thus decreasing the level of output and level of employment in the formal sector, since only the formal sector is taxed.

Nominal Rigidity: The presence of nominal rigidity, namely price stickiness, induces dispersion in price levels and thus impacts real variables like output.

Monopolistic Distortion: Firms with monopolistic power produce less than the optimal level of output and create a lower level of employment.

3.12 Calibration

I pull the parameter values used to obtain the simulated results from several sources. The countries in the CFA zone are similar in terms of economic structure and characteristics. In that sense the parameters used to simulate the model can be extended to all the countries in the zone. I use the ratio of imports to GDP as a measure of openness and I obtain an estimate from the World Bank's World Development Indicator. For the size of the non-traded informal sector, I use Schneider (2005) as a reference and the average of the estimates reported for the CFA zone countries. Different estimates of the elasticity of output with respect to government spending are used to simulate sensitivity analysis. However, I use Yasin (2009) as a reference and set $\chi = 0.1$ for the remainder of the simulations. The remaining parameters are standard in the literature. Table 2 below summaries the parameters of the model and their sources.

4 Flexible Price Equilibrium and Optimal Fiscal Policy

Before I look at the conduct of monetary policy within the presence of nominal rigidity, I analyze the model under flexible prices. Under the assumption of flexible prices, the Ramsey optimal problem is static. I first consider the economy without informality and then I look at the implication of the presence of each type of informality separately.

4.1 Optimal Taxation and Informality

When there is no informality the optimal tax rate in the formal sector is well above zero at around 17.5%. This tax rate increases with χ, the elasticity of output with respect to the public good as reported in Table 5. The public good is financed by the tax proceed and enhances productivity, so as it becomes more productive, the authorities find it optimal to increase the tax rate since its positive effect is now higher. And the low volatility of the tax rate under flexible prices indicates that the government spreads the tax burden over time and does not change often.

To isolate the effect of each type of informality in order to better understand the channels through which they impact the economy, I introduce each type separately and explore how each type impacts the optimal tax rate in the formal sector. In particular I first introduce labor market informality into the formal sector ($v > 1$ or $x > 0$ and $\alpha = 0$). With informal labor, formal firms are able to decrease their production cost. As informal labor in the formal sector increases, the optimal tax rate increases (Table 6). As the marginal cost decreases, taxation becomes less and less distortionary and the Ramsey planner finds it optimal to increase the tax rate. An increase in labor informality in the formal sector has an impact on the optimal tax rate similar to that of an increase in the output elasticity of the public good.

By setting $v = 1$ or $x = 0$ and $\alpha > 0$, I shut off labor informality in the formal sector and introduce goods informality into the economy. The less productive non-traded sector that produces the informal good is not taxed, but the formal sector is. The optimal tax rate decreases with the expansion of the informal good sector (Table 7). The presence of the informal sector increases the marginal cost of the formal good production and makes it relatively more expensive than the informal good. The Ramsey planner then finds it optimal to decrease the optimal tax rate to decrease the marginal cost of producing the

formal good. This way, the formal good is not relatively more expensive compared to the informal good.

The two types of informality have opposite effect on the optimal tax rate in the formal sector. I introduce both into the model and I study the cumulative effect of both labor market informality and goods market informality on the optimal tax rate. In particular, I compare the economy with no informality ($\alpha = 0$ and $x = 0$) to the economy with both types of informality ($x = 0.7$ and $\alpha = 0.4$). Table 8 reports the simulated moments. The optimal tax rate is considerably lower in the presence of all types of informality, 0.72% compared to 17.5% when there is no informality. The impact of α, the size of the informal sector, on the optimal tax rate is much larger than the combined effects of χ and x, the output elasticity of government spending and the measure of labor informality. In particular as the size of the informal sector increases, the optimal Ramsey tax decreases (Figure1).

In the presence of informality, it is important to the Ramsey planner to a lower the tax rate in order to prevent the formal good from becoming relatively more expensive compared to the informal good. The policy implications suggest that fiscal authorities in countries with informal sectors should adopt lower tax rate so that goods from the (taxed) formal sector are relatively less expensive for consumers.

It is also worth noting that the extent to which the informal sector firms have access to the public good, as measured by the parameter δ, has no impact on the optimal tax rate. This suggests that the government should focus on the formal sector instead of spending resources to crack down on the informal sector.

4.1.1 Empirical Evidence

I empirically document the effect of the shadow economy on the tax rate and level in order to verify the steady results of the model. Figure 2 plots tax revenues (as a percentage of GDP) against the size of the informal sector; there is a negative relationship between these variables.

I collect data from the World Bank Development Indicators (WDI), the World Bank Government Indicators (WGI), and the IMF's World Economic Outlook. The variables used and their respective sources are listed in the appendix. I estimate the following two equations separately:

$$TRate_{i,t} = \alpha + \gamma IS_{i,t} + \psi X_{i,t} + \epsilon_{i,t} \tag{40}$$

where $TRate_t$ is the tax revenue collected as a percentage of total GDP. I use tax revenue as a percentage of GDP as a proxy for the tax rate levied on firms in the formal sector. IS_t is the size of the informal sector as estimated by Schneider et al. (2010). The panel data goes from 1999 to 2007 because estimates of the size of the informal sector is only available for those years. Due to missing data, the panel is unbalanced. Finally, X_t is a matrix of control variables known to have an impact on tax revenues.

Due to concerns over endogeneity, such as the tax rate inducing firms to relocate to the informal sector, I use general methods of moments (GMM) to estimate the model. This methodology also allows me to address other concerns such as measurement errors, especially about the size of the informal sector, and omitted variables. This identification method was also used by Elgin and Uras (2013).

Table 4 report the results of the estimations. As expected, the size of the informal sector negatively impacts impacts tax revenues. The estimated coefficient on the informal sector is significant.

4.2 Public Debt and Informality

So far the government budget is balanced, but one could change this specification and allow the government to borrow and study the impact of the informal sector on public debt. Each period, the authorities are faced with a random level of public spending that must be financed with tax revenues and debt. In addition they must also repay existing debt. Now the government's budget constraint becomes:

$$G_t + R_{t-1}D_t = \tau_t Y_{H,t} + D_{t+1} \tag{41}$$

Here the provision of the public good follows a random walk: $G_t = \rho_g G_{t-1} + \epsilon_t^g$. Assuming a constant long run constant tax rate, public debt rises with the size of the informal sector. Figure 4 plots long run public debt against the size of the informal sector for select number of countries.

4.2.1 Empirical Evidence

I empirically investigate the findings of the theoretical model. Figure 3 plots government debt and the size of the informal sector and the fitted line exhibit a positive slope. Using GMM I estimate the following equation:

$$Pdebt_t = \delta + \lambda IS_t + \beta X_t + \epsilon_t \tag{42}$$

where $Pdebt_t$ is the country's public debt as a percentage of the total GDP. IS_t is again the size of the informal and the panel data goes from 1999 to 2007 because estimates of the size of the informal sector is only available for those years. Z_t is a matrix of control variables known to have an impact on public debt.

Table 3 report the results of the estimations and the size of the informal sector positively impacts public debt.

4.3 Dynamic Adjustment Under Flexible Prices

I now analyze the response of some select macroeconomic variables to a productivity shock in the formal sector. Figure 5 plots the responses of the variables. In both cases, there is an increase in formal and informal outputs. The presence of informality has no impact on the dynamic adjustment of the formal output. The difference is observed in the adjustment of consumption and government spending. Consumption increases significantly in the presence of informality and government spending increases considerably in the absence of informality. The increase in consumption can be attributed to the increase in both the informal good and the formal good. The tax rate remains constant over the business cycle in both cases. The increase in the government spending is then justified by the increase in the formal good. The informal output responds to the formal sector productivity shock due to the public good provided through taxation of formal firms. This replicates the empirical documentation in Kanbur (2015), which shows that the informal sector has kept up with the expansion of the formal sector in emerging economies for the past three decades.

5 Sticky Price Equilibrium

This section introduces nominal rigidity in the form of price stickiness in the formal sector. I model price stickiness à la Calvo (1983). To close the model in this case, monetary and fiscal policy rules are needed. Here, I derive the optimal policies under commitment. Since a closed form solution is not attainable, I provide numerical simulations of the Ramsey allocations and optimal policies. The optimal monetary and fiscal policies derived in this section are used as benchmarks for comparing alternative monetary policy regimes, namely the optimal policy under discretion and an exchange rate peg policy. Nominal rigidity introduces forward-looking variables in the optimality conditions of the economy, so I first focus on the long run outcomes of the macro variables and then proceed to analyze their short run dynamic adjustments over the business cycle.

5.1 Optimal Fiscal and Monetary Policy in the Long Run

5.1.1 The Economy with No Informality

In the economy without informality, where $\alpha = \kappa = 0$, the Ramsey planner maximizes total welfare (lifetime utility) subject to market-clearing conditions and the FOCs of the households and firms:

$$\text{Max} \quad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^F\right)^{1+\psi}}{1+\psi} \right\}$$

s.t. households' and firms' optimality condition are satisfied and all markets clear.

The allocations under sticky prices are identical to the flexible price allocations and the optimal inflation is equal to zero when I assume that the government levies the optimal tax rate from the flexible price allocations and the monetary authority takes that as given and set monetary policy optimally (Ramsey commitment). The results are reported in Table 10. This is a standard result in the New Keynesian literature on monetary policy. When the tax is set at the flexible price optimal rate, the steady state is constrained efficient and the planner faces one distortion (nominal rigidity), so price stability is optimal and the flexible price allocation is replicated.

The long run outcomes become differ slightly when I assume that the Ramsey planner chooses the optimal tax rate and the optimal inflation rate simultaneously. The appendix contains the details of the derivation of the FOCs and the long run allocations are reported in Table 9. The long run optimal inflation is zero, which is consistent with most of the results in the New Keynesian literature on optimal monetary policy. One important observation is the sharp increase in the optimal tax rate with the introduction of nominal rigidity: the optimal tax rate under flexible prices stands at 17.52%, but in the presence of price stickiness it increases to 48%. The allocations of the remaining macroeconomic variables mirror quite well the allocations under flexible prices, but at the expense of higher taxation in the formal sector. Siu (2004) also report an increased optimal tax rate with increased price stickiness. In this situation, the steady state is neither efficient nor constrained efficient and the planner has only two instruments for more than two distortions. The authorities then set the tax rate higher than the optimal rate in the case of flexible prices to increase the size of the public good and increase output. This is necessary to replicate the flexible price allocations for the remaining macroeconomic variables.

5.1.2 The Economy with Informality

In the economy with informality, where α and x are both different than zero, the Ramsey planner solves the following problem:

$$\text{Max} \quad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^F\right)^{1+\psi}}{1+\psi} - \frac{\left(N_t^I\right)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t. the households' and firms' optimality conditions are satisfied and all markets clear.

The details of the derivations are contained in the appendix. As in the case with no informality, if the tax rate is set at the optimal rate under flexible prices and the Ramsey planner solves for the optimal monetary policy, the long run allocations mirror the flexible prices allocations. Table 12 reports the simulated means of some select variables.

Table 11 shows the long run mean of some variables when the Ramsey planner chooses the optimal tax rate and inflation simultaneously. The sticky price allocations are different than the flexible price allocation except for consumption. All the remaining variables are higher in the presence of sticky prices. The tax rate increases with the presence of nominal rigidity, from 0.72% to 2.8%. The increase in the tax rate erases the inefficiency caused by nominal rigidity and even increases output due the productive nature of the public good provided by the fiscal authority.

5.2 Optimal Fiscal and Monetary Policy in the Short Run

Table 14 reports the standard deviation of the different variables. The introduction of informality induces higher volatility in all of the variables except for the optimal tax rate in the formal sector. When there is no informality, volatility of both the formal good inflation and the tax rate are low at 0.03 and 0.05, respectively. This indicates that the policy authorities do not change these two policy instruments very often over the business cycle and tend to resort to tax changes in the formal sector slightly more often than inflation changes. In the presence of informality, the volatility of formal good inflation increases to 0.10 and that of the tax rate decreases to about 0.003. It is now costly to resort to fiscal policy changes when the economy has both informal labor and goods market, so the policy authority makes more usage of inflation changes.
The economies with and without informality respond differently to a productivity shock in the formal sector. Looking at the impulse responses under a technology shock in the formal sector, for the economy with no informality and that with informality, the tax rate in the formal sector remains unchanged when there is informality but drops then increases when there is informality. However, in both cases the optimal deviations from steady state replicate the deviations under flexible prices. Regardless of informality, the optimal policies call for deviations from price stability, which is in contrast with the results in the recent literature such as Galí and Monacelli (2005) and Benigno and Benigno (2003). Furthermore, even though the optimal policy calls for departure from price stability, the directions of the departure are opposite for Home formal inflation on impact. With no informality, the optimal response of formal inflation is to decrease then increase before returning to its steady state level. However, in the presence of informality, the opposite happens: formal goods inflation responds by increasing then decreasing (remaining negative) before returning to its steady state. Another key difference concerns government spending, which increases significantly less in the presence of informality. That is due to the low optimal tax rate levied by the authority in the presence of informality. CPI

inflation increases in both cases, but significantly more in the presence of informality due to the increases in the terms of trade, nominal exchange rate and informal price inflation. One can note that the magnitude of the responses tends to be larger in the presence of informality except for the responses of government spending and the tax rate. Informality induces higher macroeconomic volatility as reported in Table 14.

6 Alternative Policies/Incentive to Join a Currency Union

I explore the implications of alternative policies to the optimal Ramsey policy under commitment. Namely I examine the historical policy that has been adopted by the countries of the CFA zone and also the Ramsey problem under discretion. Soffritti and Zanetti (2008) perform a similar exercise and conclude that in the case in which the country cannot credibly commit to a policy, pegging its exchange rate is better than adopting a discretionary policy. What are the implications of the presence of informality in the choice of an exchange rate regime?

6.1 Exchange Rate Peg and Ramsey Optimal Fiscal Policy

The first alternative policy I consider is a policy of exchange rate peg coupled with a optimal Ramsey taxation in the formal sector.

$$\text{Max} \qquad E_0 \left\{ \sum_{t=0}^{\infty} \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^F\right)^{1+\psi}}{1+\psi} - \frac{\left(N_t^I\right)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t. the households' and firms' optimality conditions are satisfied, all markets clear and $\Delta E_t = 1$.

6.2 Historical Policies: Exchange Rate Peg and Flat Tax Rate

The historical policies followed by these countries have been a flat tax rate and a fixed exchange rate. The common currency is currently pegged to the Euro; however it was pegged to the French Franc at the time of its creation. The parity was changed only once (January 1994) since its inception. The historical policy can be summarized by:

$$\tau_t = 0.18$$
$$\Delta E_t = 1$$

This flat tax is very close to the optimal tax rate under flexible price and no informality, which is 17.5%. This is just a mere coincidence and a different calibration would yield different values for the optimal tax rate under flexible prices, but would not change the reported results. The simulated moments of the variables under this historical regime, with and without informality, are reported in Table 15. One thing is very clear: in the long run, the allocations under the historical policy are almost identical to the allocations under the optimal Ramsey policy when there is no informality. Under the optimal Ramsey policy with commitment, the tax rate and formal inflation rate have very low volatility. Under the historical policy, the tax rate is constant so it mimics the optimal fiscal policy

under commitment. However, the volatility of the formal good inflation is higher in the presence of informality than the volatility under the optimal commitment policy. Without the changes in the nominal exchange rate, the planner resorts to more changes in the formal inflation rate.

6.3 Discretionary Policy: Fiscal and Monetary

Under discretion, the monetary authorities choose the optimal policies in every period. I establish the optimal discretionary policy and calculate the welfare level under different degrees of informality. With $\alpha = \kappa = 0$, there is no informality in the economy and the planner solves the following problem:

$$\text{Max} \quad \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^F\right)^{1+\psi}}{1+\psi} \right\}$$

s.t the HH and firms' optimality conditions are satisfied and all markets clear.

If instead the economy is now characterized by both labor and good informality, the planner solves the following problem:

$$\text{Max} \quad \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^F\right)^{1+\psi}}{1+\psi} - \frac{\left(N_t^I\right)^{1+\psi}}{1+\psi} - \frac{L_t^{1+\mu}}{1+\mu} \right\}$$

s.t the HH and firms' optimality conditions are satisfied and all markets clear.

The simulated moments of the variables under this discretionary regime for both economies with and without informality are reported in Table 16. The long run optimal inflation, as expected, is positive. Under discretionary policy the monetary authorities use inflation as a tool to increase output. With no informality, the tax rate and formal inflation have zero standard deviation. In the presence of informality, the volatility of the formal inflation is 0.04 and that of the tax rate is 0.3. Under discretion, the policy authority resorts to more changes in tax policy than inflation policy, which is contrary to the policy prescription under the optimal commitment policy. Recall that under the optimal commitment policy the standard deviation of the formal inflation is 0.1 while that of the optimal tax rate is 0.003.

6.4 Welfare Ranking

I calculate the long run welfare level under different policy regimes: the optimal Ramsey policy under commitment, the optimal Ramsey policy under discretion and an exchange rate peg. I use the long run level of welfare under the optimal Ramsey commitment policy and then calculate the increase in consumption needed under the alternative regimes in order to close the gap in terms of welfare.

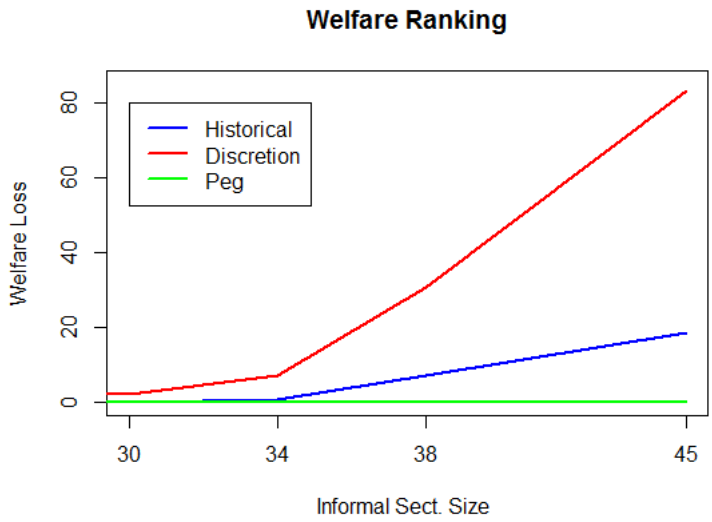
$$W^c = \frac{U\left[\left(1+\frac{\Delta^p}{100}\right)C^p, N_F^p, N_I^d, L^p\right]}{1-\beta} \tag{43}$$

where W^c is the long run welfare under the Ramsey commitment policy and p refers to the alternative policy being evaluated (i.e exchange rate peg). Figure **??** below plots the

Δ^p for different sizes of the informal sector. The welfare loss increases with the size of the informal sector when the country pegs its currency.

The optimal Ramsey fiscal and monetary policy under commitment yield the highest welfare. This is a standard result in the literature on optimal monetary policy suggesting that policy authorities should follow a credible commitment policy over time. The policy of an exchange rate peg coupled with the optimal Ramsey fiscal policy performs as well as the policy of Ramsey fiscal and monetary policy. The welfare loss is greatest under the optimal Ramsey optimal fiscal and monetary policy under discretionary regardless of the presence of informality. The historic regime of an exchange rate peg and fixed tax rate, in turn outperforms the discretionary regime, and the gap is amplified in the presence of informality. As the size of the informal sector increases, the welfare loss under the historical policy and the discretionary policy rises. Hence the informal sector decreases a country's incentive to peg its currency and should make use of rule based fiscal and monetary policy.

However, If I assume that these countries' central banks do not have the indepedence and credibility for commitment and that the choice is then between the historical policy of pegging their exchange rate plus a constant tax rate and conducting discretionary policy every period, then I can conclude that the presence of informality *increases* the incentive of these countries to join a currency union. The ability to credibily and effectively of commit to an optimal fiscal and monetary policy hinges on the level of independence of the central bank and its strength. Dincer and Barry Eichengreen (2014) reports regional central transparency and independence and the African as a whole had a score of 0.34 (out of 100) in 2010. Giavazzi and Pagano (1988) and Soffritti and Zanetti (2008) also arrive at a similar conclusions. These previous results are strengthened in the presence of informality.



Different calibrations of the model yield different allocations and welfare losses, but do not alter the welfare rankings. The welfare loss is sensitive to the elasticity of government spending; higher elasticity of public goods causes larger welfare loss under all sub-optimal policy regimes, particularly under the discretionary policy regime.

In conclusion, fiscal and monetary authorities should never adopt a discretionary policy regime, particularly if the economy is characterized by informality as the level

of welfare worsens. Batini et al. (2011) report a similar result: the time inconsistency problem, observed under the discretionary regime, worsens with the introduction of informality. Under the discretionary regime, both the formal inflation and tax rate have higher volatility compared to the historical regime and the optimal commitment regime. Furthermore, the volatility of the tax rate is higher than that of the formal inflation, which is the opposite of the the prescription under the optimal regime. This can help explain why the discretionary regime performs worse than both the optimal commitment regime and the historical regime. The welfare cost of the historical regime comes from the too much volatility in the formal sector inflation while the welfare cost of the discretionary regime results from high volatility of the formal tax rate and low volatility of the formal sector inflation. Figures 7 and 8 plot the impulse responses of the three different regimes under a productivity shock in the formal sector for the economy with no informality and the economy with informality, respectively. One can see that while the tax rate remains unchanged under the historical policy and optimal commitment policy, it increases considerably under the discretionary regime before returning to its steady state rate. Consumption increases in all the policy regimes, though it does so more under the optimal regime and less under discretion. This is one of the main sources of low level of welfare under the discretionary regime.

7 Conclusion

This paper developed a small open DSGE model characterized by informality in the goods market and labor market. The paper shows that informality significantly decreases the optimal tax rate levied on the formal sector. Policy makers in countries with significant informality should aim to keep taxes low and steady in the formal sector in order to avoid formal goods becoming relatively more expensive than the informal good. Even though it does not affect the long run optimal inflation, informality affects the long run levels of other macro variables and increases the volatility of all variables except the tax rate. If countries have neither the technology nor the credibility to commit to the optimal policy, it is desirable to peg the nominal exchange and adopt a flat tax rate.

The paper contribution to research on the implications of informality for policy making is two-fold. First, I shed light on the impact of informality on the optimal fiscal and monetary policy. Every economy has some degree of informality, but it is more pervasive in developing countries, and understanding its impact on policy making is of first order importance. Second, I re-evaluate policy prescription on optimal exchange rate volatility for small open economies and show that an exchange peg coupled with a flat tax rate outperforms a discretionary regime.

The current setup of the model can be extended in several directions. An especially important one will be to relax the assumption of financial autarky, explore the consequences of capital flows, and allow the government to run budget deficits. This would give us insights into the dynamic of public debt under a currency peg with informality-constrained fiscal revenues. Another interesting extension would be to allow the productivity of the public good to decrease with the size of the informal sector. Existence of the informal sector would then decrease the positive effect of tax proceeds, which would have implications for optimal taxation. I intend to explore these extensions in future work.

References

Phillipe Aghion, Phillipe Bachetta, Romaine Rancière, and Kenneth Rogoff. Exchange rate volatility and economic growth: The role of financial development. *Journal of Monetary Economics*, 56:494–513, 2009.

ANSD. Enquête nationale sur le secteur informel au senegal, censis 2011. *Agence Nationale de la Statistique et de la Démographie*, November 2013.

Grossman Herschel Azuma, Yoshiaki. A theory of the informal sector. *Economics and Politics*, 20(1):62–79, 2008.

Nicoletta Batini, Paul Levine, Emanuela Lotti, and Bae-Kim Young. Informal labour and credit markets; a survey. *IMF Working Paper*, February 2010.

Nicoletta Batini, Paul Levine, Emanuela Lotti, and Bo Yang. Monetary and fiscal policy in the presence of informal labour markets. *Working Papers*, 2011.

Gianluca Benigno and Pierpaolo Benigno. Price stability in open economies. *Review of Economic Studies*, 70(4):743–764, 2003.

Paul Castillo and Carlos Montoro. Inflation dynamics in the presence of informal labour markets. *BIS Working Paper*, February 2012.

Martha Alter Chen. Rethinking the informal economy: Linkages with the formal economy and the formal regularoty environment. *DESA Working Paper No. 46*, July 2007.

Nergiz Dincer and Barry Barry Eichengreen. Central bank transparency and independence: Updates and new measures. *International Journal of International Banking*, 2014.

Ceyhun Elgin and Burak R. Uras. Public debt, sovereign default risk and shadow economy. *Journal of Financial Stability*, 9(4), 2013.

Norbert Fiess, Marco Fugazza, and William Maloney. The informal sector, nominal rigidities and real exchange rates. *Working Paper*, July 2001.

Jordi Galí and Tommaso Monacelli. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, (72):707–734, 2005.

Francesco Giavazzi and Marco Pagano. The advantage of tying one's hands: Ems discipline and central bank credibility. *Europena Economic Review*, 32:1055–1075, 1988.

Ravi Kanbur. Informality: Causes, consequences and policy responses. *CEPR Discussion Paper No. DP10509*, March 2015.

Eduardo Levy-Yeyati and Federico Sturzenegger. To float or to fix: Evidence on the impact of exchange rate regimes on growth. *American Economic Review*, 93(4):1173–1193, 2003.

V Norman Loayza. The economics of the informal sector: a simple model and some empirical evidence from latin america. *Carnegie-Rochester Conference Series on Public Policy*, 45:129–162, December 1996.

Sugata Marjit and Saibal Kar. A contemporary perspective on the informal labour market: Theory, policy and the indian experience. *Economic and Political Weekly*, 44(11): 60–71, April 2009.

Margaret McMillan and Dani Rodrik. Globalization, structural change and productivity growth. *IFPRI Discussion Paper 01160*, February 2012.

James E. Rauch. Modelling the informal sector formally. *Journal of Development Economics*, 35:33–47, 1991.

Stephanie Schmitt-Gohé and Martin Uríbe. Optimal fiscal policy and monetary policy under sticky price. *Journal of Economic Theory*, 2004.

Friederich Schneider, Andreas Buehn, and Claudio E. Montenegro. Shadow economies all over the world. *Policy Research Working Paper*, 2010.

Friedrich Schneider. Shadow economies around the world: what do we really know? *European Journal of Political Economy*, 21(3):598–642, September 2005.

Alan Finkelstein Shapiro. Institutions, informal labour markets and business cycle volatility. *Economia*, 2015.

E. Henry Siu. Optimal fiscal policy with sticky prices. *Journal of Monetary Economics*, 2004.

Mirco Soffritti and Francesco Zanetti. The advantage of tying one's hands: revisited. *International Journal of Finance and Economics*, 13(2):135–149, 2008.

Áureo de Paula and José A. Scheinkman. The informal sector. (13486), October 2007.

Colin C. Williams. *Confronting The Shadow Economy*. Edward Elgar, 2014.

Mesghena Yasin. Public spending and economic growth: Empirical investigation of subsaharan africa. *Southwestern Economic Review*, 2009.

Tables

Calibration

Table 2: Calibration

	Parameter	Value
Openness	a	0.3
Informality	α	0.4
Output elasticity of Gov. Sp.	χ	0.01
Formal Labor elasticity	ψ	3
Informal Labor elasticity	μ	3
Formal Sector Informalization	κ	0.7
Informal Sector Access to Public Good	δ	0.6
Price stickiness	θ	2/3
Coef. of autocor. Formal Output	ρ_F	0.9
Coef. of autocor. Informal Output	ρ_I	0.7

Table 3: GMM Regression with all countries

	Dependent variable: Government Debt
Size of Informal sector	**6.237*****
	(0.823)
GDP per capita	**0.001**
	(0.002)
GDP per capita growth	**0.168**
	(0.388)
Inflation	**0.203*****
	(0.059)
Current accounts	**0.036**
	(0.203)
Openness (trade)	**−0.419*****
	(0.086)
Corruption Control	-11.600^{*}
	(6.361)
Voice & Accountability	-13.168^{**}
	(5.279)
Observations	366
R^2	0.372
Adjusted R^2	0.303
F Statistic	19.612^{***} (df = 9; 298)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 4:

	Dependent variable: Tax Revenue (% of GDP))
Size of informal Sector	$\mathbf{-0.157}^{*}$
	(0.085)
GDP per Capita	$\mathbf{0.002}^{***}$
	(0.0004)
Foreign aid	$\mathbf{-0.052}^{**}$
	(0.026)
GDP per capita growth	**0.026**
	(0.037)
Inflation	**0.016**
	(0.027)
Current accounts	**0.007**
	(0.023)
Openness (Trade)	$\mathbf{0.047}^{***}$
	(0.012)
Rule of Law	**0.564**
	(0.860)
Corruption Control	**−0.555**
	(0.632)
Governement Effectiveness	$\mathbf{-2.170}^{**}$
	(0.833)
Voice & accountability	**−0.355**
	(0.580)
Observations	222
R^2	0.459
Adjusted R^2	0.348
F Statistic	12.973^{***} (df = 11; 168)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 5: Optimal Tax: Varying Elasticity χ

Elasticity χ	Optimal Tax Rate
0.1	17.5%
0.2	32.8%
0.3	45.2%
0.4	54.7%
0.5	62%

The optimal tax rate under flexible price, at varying degrees of output elasticity of government spending.

Table 6: Optimal Tax: Varying Degree of Labor Informality κ

Labor Informality κ	Optimal Tax Rate
0.1	17.8%
0.2	18.3%
0.3	18.8%
0.4	19.3%
0.5	19.8%

The optimal tax rate under flexible price, with varying degrees of labor market informality.

Table 7: Optimal Tax: Varying Size of the Informal Good α

Goods Informality α	Optimal Tax Rate
0.1	21%
0.2	13.5%
0.3	6.7%
0.4	0.7%

The optimal tax rate under flexible price, with varying degrees of goods informality.

Table 8: Flexible Price (Ramsey) Allocations

Variable	No Informality	Informality ($\alpha = 0.4$ and $\kappa = 0.7$)
Consumption	0.80	1.1
Tax	17.5%	0.72%
Gov. Spending	0.13	0.5%
Formal output	0.74	0.71
Formal Labor	0.91	0.74
Informal Output	-	0.44

Flexible price allocation with informality and without informality.

Sticky Prices

Table 9: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price
CPI Inflation	–	1.0002
Formal Inflation	–	1.0002
Consumption	0.80	0.79
Tax	17.5%	48%
Gov. Spending	0.13	0.35
Formal output	0.74	0.73

Ramsey allocation (no informality) when choosing both monetary and fiscal policies simultaneously.

Table 10: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price ($\tau = 17.5\%$)
CPI Inflation	–	1.00
Formal Inflation	–	1.00
Consumption	0.80	0.80
Tax	17.5%	17.5%
Gov. Spending	0.13	0.13
Formal output	0.74	0.74

Ramsey allocations (no informality) when holding the optimal tax rate at the rate under flexible price.

Table 11: Long Run (Ramsey) Allocations With Sticky Prices

Variable	Flexible Price	Sticky Price
CPI Inflation	–	1.0003
Formal Inflation	–	1.0003
Informal Inflation	–	1.0003
Consumption	1.1	1.1
Tax	0.72%	2.8%
Gov. Spending	0.0052	0.023
Formal output	0.71	0.82
Informal Output	0.44	0.52

Ramsey allocation (with informality) when choosing both monetary and fiscal policies simultaneously.

Table 12: Long Run (Ramsey) Allocations

Variable	Flexible Price	Sticky Price($\tau = 0.72\%$)
CPI Inflation	–	1.0003
Formal Inflation	–	1.0003
Informal Inflation	–	1.0003
Consumption	1.1	1.1
Tax	0.72%	0.72%
Gov. Spending	0.0052	0.0051
Formal output	0.71	0.71
Informal Ouput	0.44	0.44

Ramsey allocation (with informality) when holding tax rate fixed.

Table 13: Long Run (Ramsey) Allocations

Variable (Mean)	No Informality	Informality ($\alpha = 0.4$ and $x = 0.7$)
CPI Inflation	1.0002	1.0003
Formal Inflation	1.0002	1.0003
Tax	48%	2.80%
Gov. Spending	0.35	0.023
Formal output	0.73	0.82
Consumption	0.80	1.10
Informal Ouput	-	0.52

Ramsey allocation (mean): Informality vs No Informality

Table 14: Standard Deviation of Some Select Variables

Variable (Sdt Deviation)	No Informality	Informality ($\alpha = 0.4$ and $x = 0.7$)
CPI Inflation	0.3	2.30
Formal Inflation	0.03	0.10
Tax	0.05	0.0028
Gov. Spending	0.47	0.03
Formal output	1.03	1.25
Consumption	0.78	3.21
Informal Ouput	-	0.07

Ramsey allocation (std deviation): Informality vs No Informality

Table 15: Historic Policy

Variable	No Informality	Informality
CPI Inflation	1.00	1.00
Formal Inflation	1.00	1.00
Informal Inflation	–	1.00
Formal output	0.74	0.96
Informal output	–	0.63
Conumption	0.80	1.03
Gov. spending	0.13	0.17
Tax	0.18	0.18

Long run allocation under the historical policy: No Informality Vs Informality

Table 16: Discretionary Policy

Variable	No Informality	Informality
Π	1.07	1.07
Π_{HF}	1.07	1.07
Π_I	—	1.07
Y_H	0.28	0.26
Y_I	–	0.53
ΔE	1.07	1.07
C	0.41	0.10
G	0.18	0.03
τ	0.63	0.12

Ramsey long run allocations under discretionary policy: Non Informality Vs Informality

Figures

Figure 1: Optimal Ramsey Tax Rate and Informal Sector

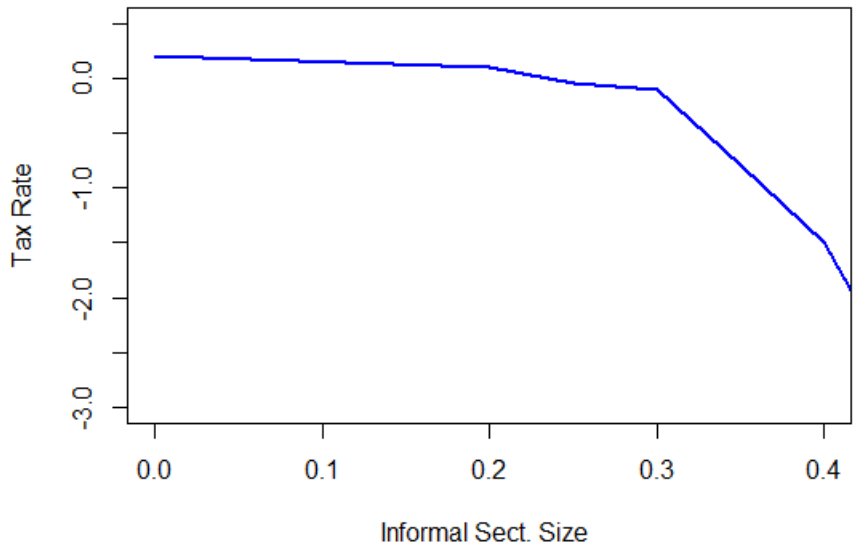


Figure 2: Tax Revenues and the Size of the Informal Sector

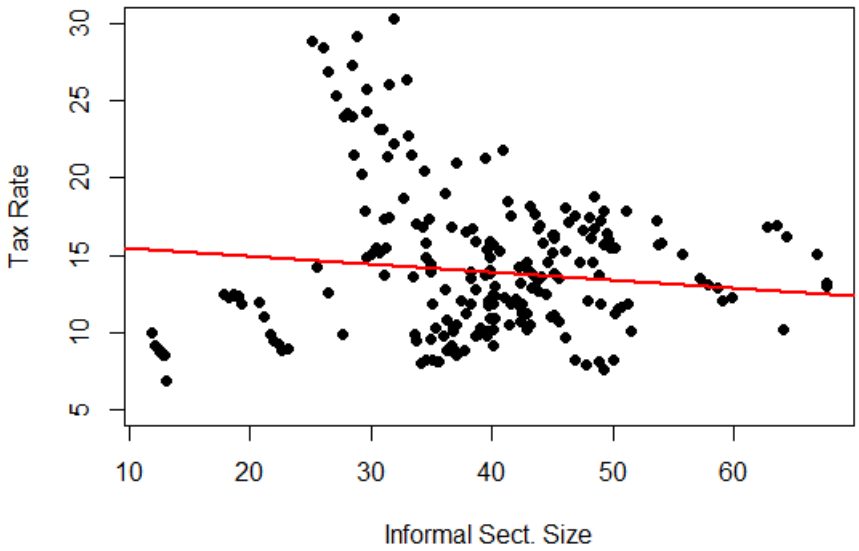


Figure 3: Government Debt and the Size of the Informal Sector

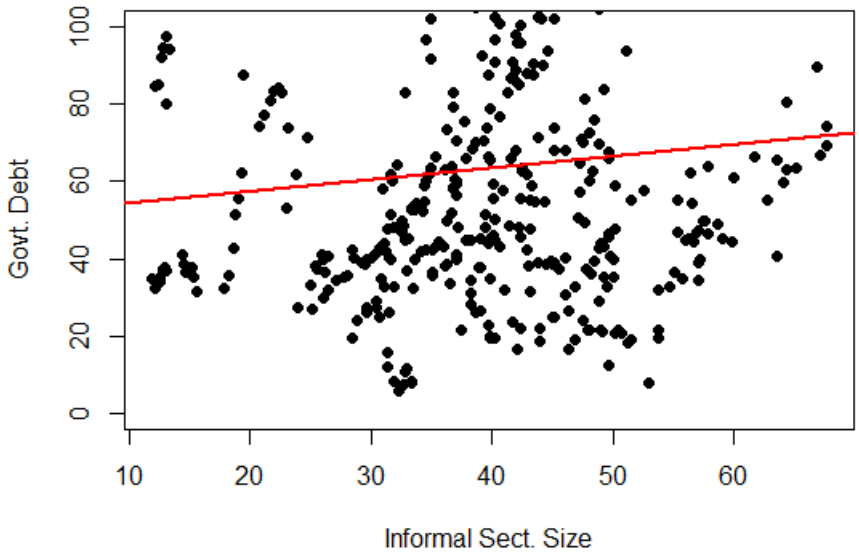


Figure 4: Optimal Ramsey Public Debt and Informal Sector

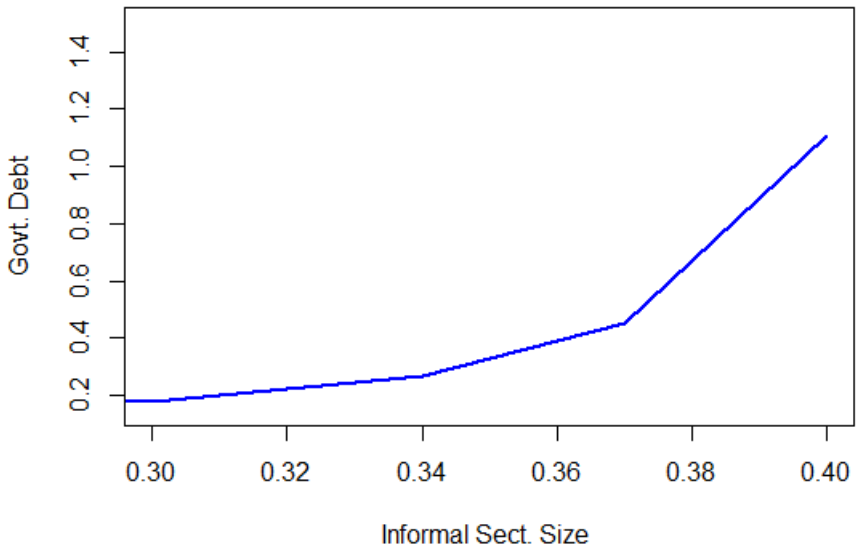
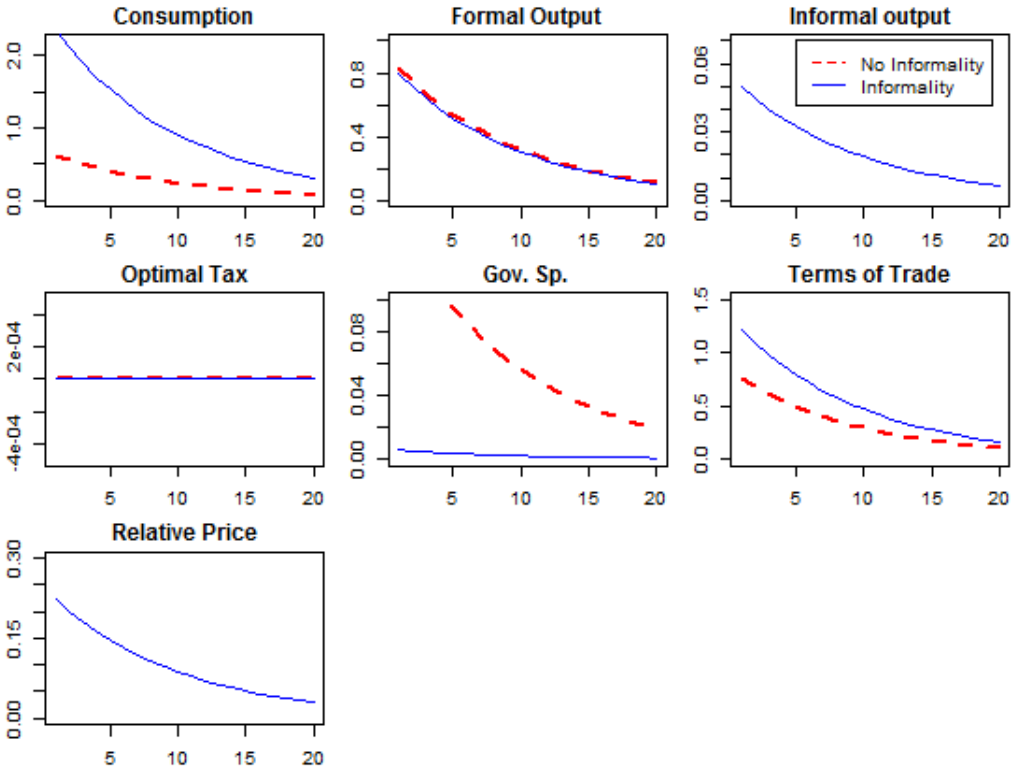


Figure 5: Impulse responses to a productivity shock in the formal sector under flexible prices



Appendix

A- Flexible Price Equilibrium without Informality

Iin this section, I present the equilibrium conditions of the economy with no informality $\alpha = x = 0$ and $v = 1$) under flexible prices.

Ramsey Problem with Lump Sum Tax:

When the public good is financed through lump sum taxation, the equilibrium conditions in the economic are:

$$
\begin{aligned}
Y_{H,t} &= m^{\frac{1}{\psi+1}} A_t G_t^{\chi} \\
N_t^F &= m^{\frac{1}{\psi+1}} \\
C_t &= Y_{H,t}^{1-a} \left(C_t^*\right)^a \\
G_t &= T_t
\end{aligned}
$$

Figure 6: Impulse responses to a productivity shock in the formal sector under the optimal ramsey commitment

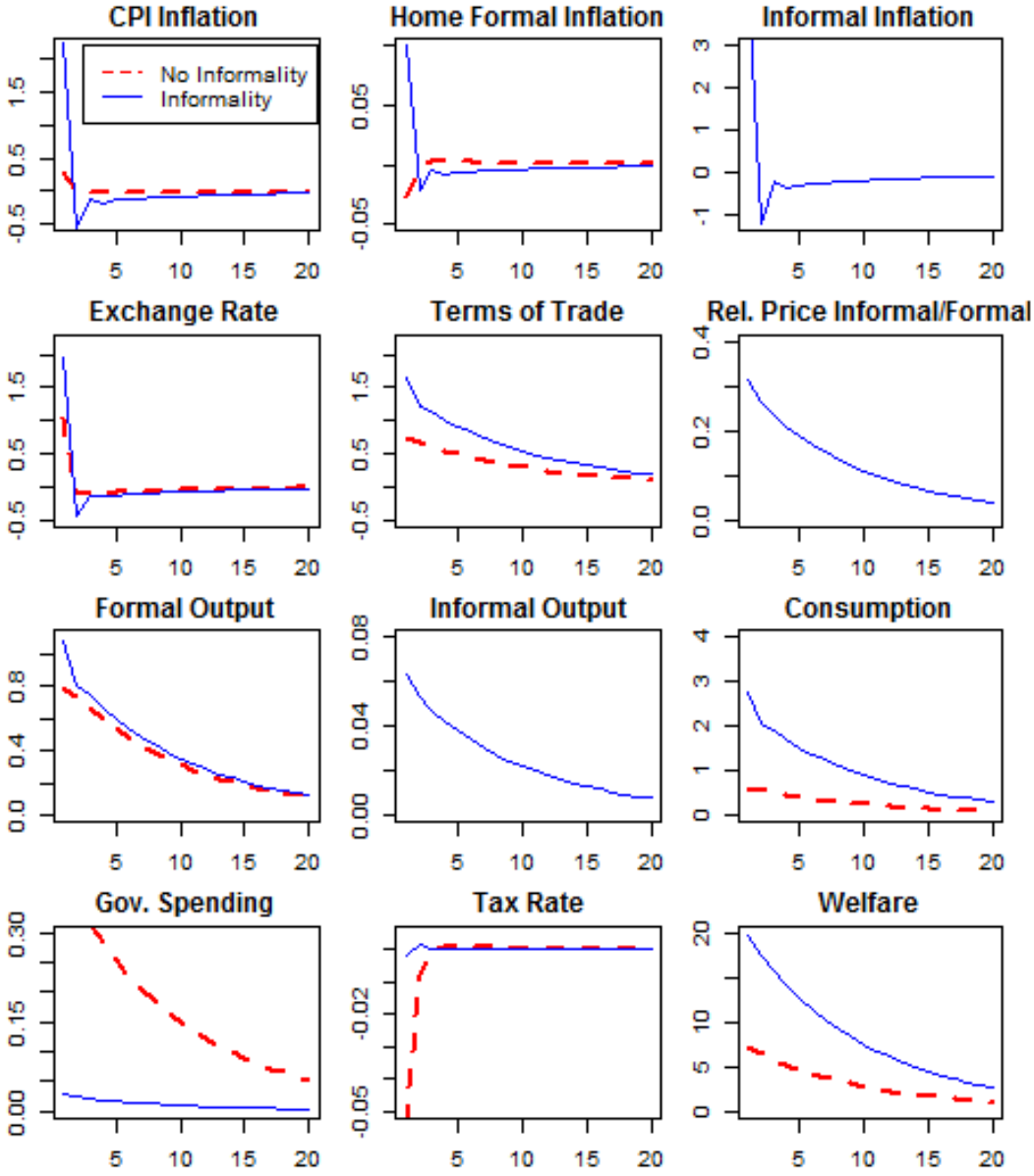


Figure 7: Impulse responses to a productivity under three different policy regimes with no informality

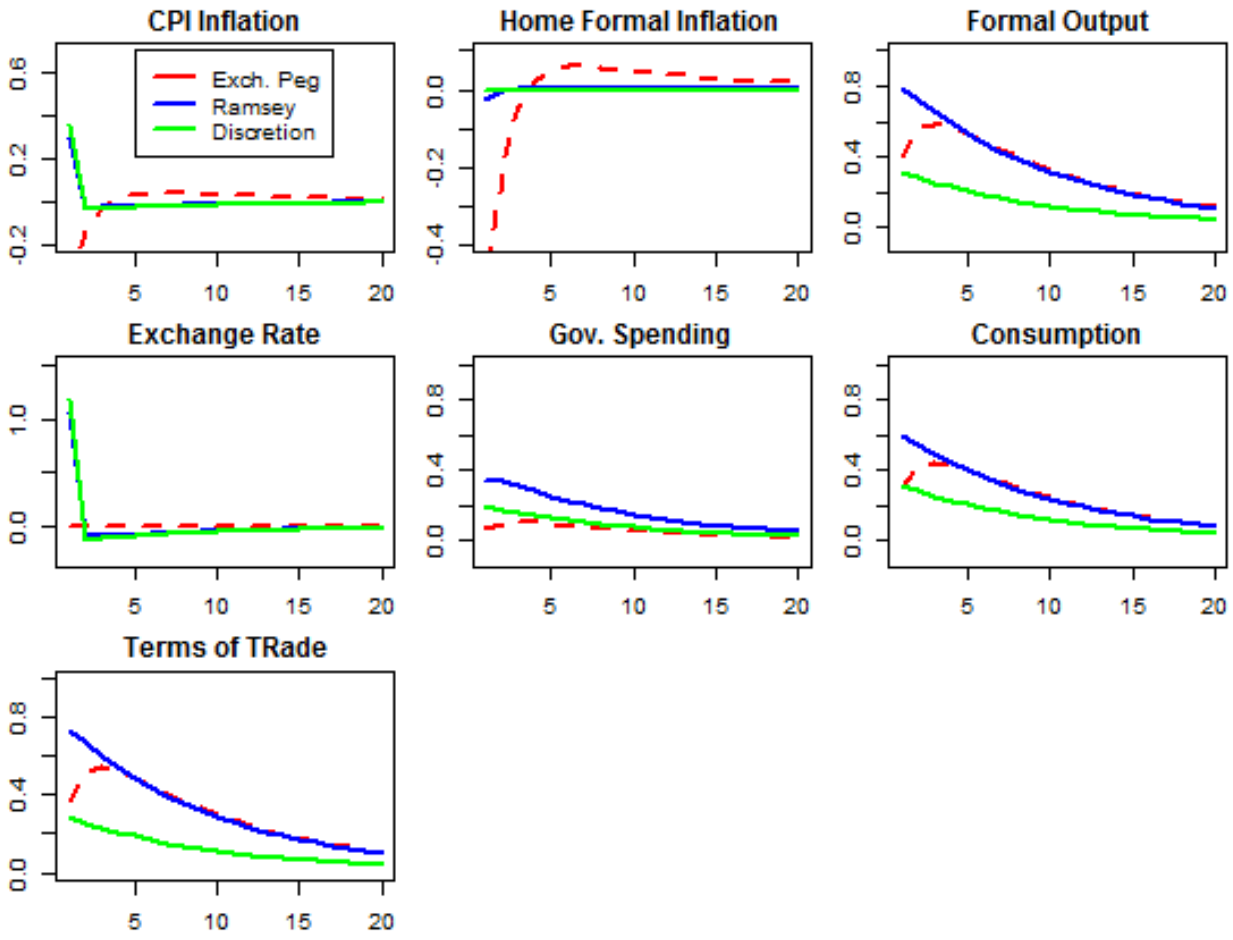
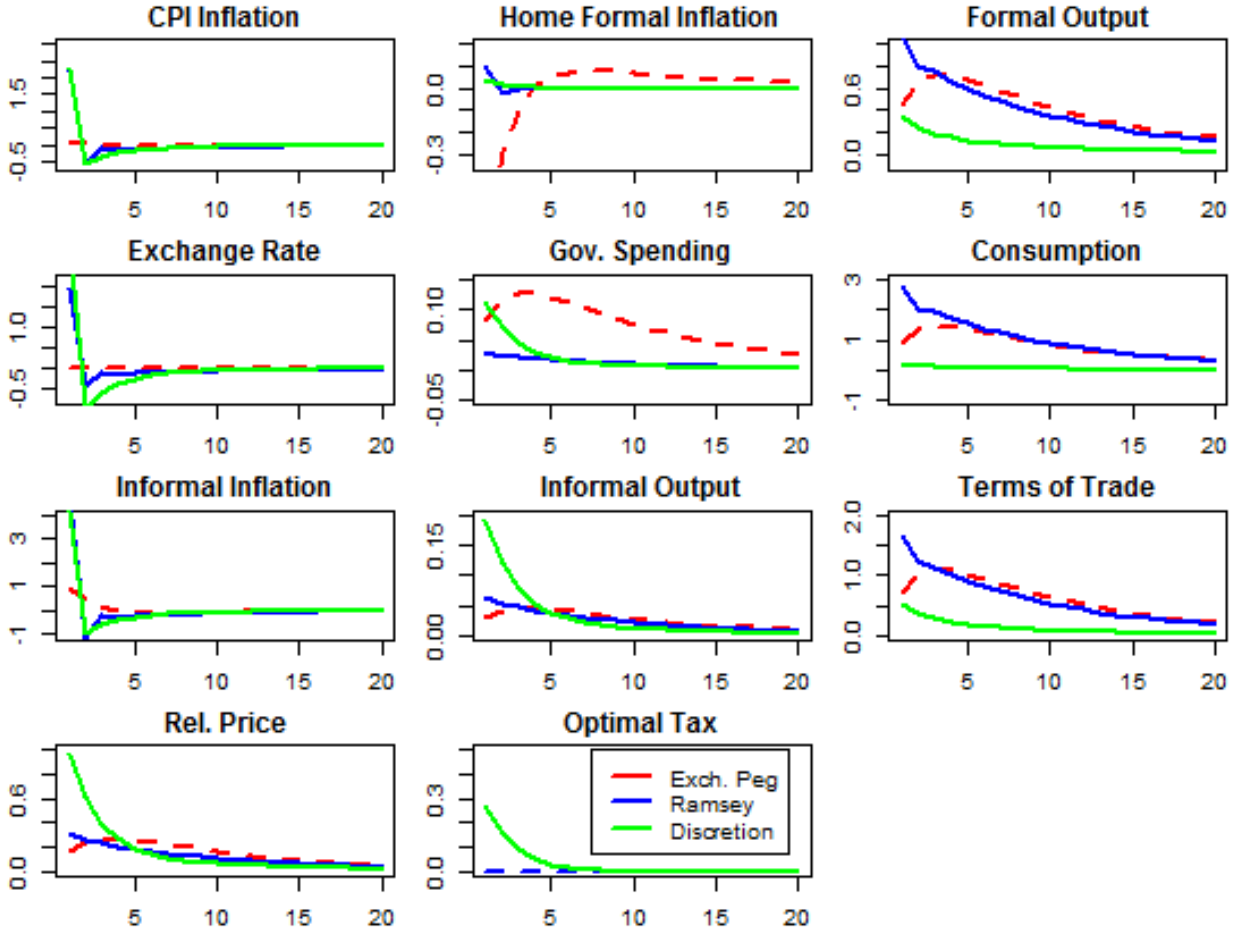


Figure 8: Impulse responses to a productivity shock in the formal sector with informality



Ramsey Problem with Distortionary Tax:

The equilibrium conditions when the public good is financed by a unit tax on formal output are as follow:

$$\begin{aligned} Y_{H,t} &= (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} A_t G_t^{\chi} \\ N_t^F &= m^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} \\ C_t &= Y_{H,t}^{1-a} (C_t^*)^a \\ G_t &= \tau_t Y_{H,t} \end{aligned}$$

The Ramsey planner chooses the unit tax rate by solving the following problem:

$$\begin{aligned} \text{Max} \quad & \sum_{t=0}^{\infty} \left\{ log(C_t) - \frac{N_t^{F(1+\psi)}}{1+\psi} \right\} \\ s.t \quad & Y_{H,t} = (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} A_t G_t^{\chi} \\ & N_t^F = m^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} \\ & C_t = Y_{H,t}^{1-a} (C_t^*)^a \\ & G_t = \tau_t Y_{H,t} \end{aligned}$$

B- Flexible Price Equilibrium with Informality

In this situation, prices are flexible in both the formal sector and the informal sector, so no nominal rigidity and no tax distortions. The existing distortions are: informality in the goods market with $(0 < \alpha < 1)$, informality in the labor market where $(0 < x < 1)$ and monopoly power with $(0 < m < 1)$.

Ramsey Problem with Lump Sum Tax:

If the Ramsey planner finances the public good through lump sum taxation, then the equilibrium conditions are:

$$\begin{aligned} Y_{H,t} &= (1-\alpha)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^{\chi} \\ Y_{I,t} &= \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^{\chi} \\ N_t^F &= \left(\frac{m(1-\alpha)}{v} \right)^{\frac{1}{\psi+1}} \\ N_t^I &= x^{\frac{1}{\psi}} \left(\frac{m(1-\alpha)}{v} \right)^{\frac{1}{\psi+1}} \\ L_t &= \alpha^{\frac{1}{1+\mu}} \\ C_t &= \left(\frac{Y_{H,t}}{1-\alpha} \right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha} \right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Theta_t^{*\alpha})^{\frac{a(1-\alpha)}{1-2\alpha}} \\ G_t &= T_t \end{aligned}$$

Ramsey Problem with Distortionary Tax:

If the public good is financed through a unit tax on formal output then the equilibrium conditions in the economy are:

$$
\begin{aligned}
Y_{H,t} &= (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^{\chi} \\
Y_{I,t} &= \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^{\chi} \\
N_t^F &= (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
N_t^I &= x^{\frac{1}{\psi}} (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
L_t &= \alpha^{\frac{1}{1+\mu}} \\
C_t &= \left(\frac{Y_{H,t}}{1-\alpha}\right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha}\right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Gamma_t^{*\alpha})^{\frac{a}{1-\alpha}} \\
G_t &= \tau_t Y_{H,t}
\end{aligned}
$$

The Ramsey planner chooses the optimal unit tax by solving the following problem:

$$
\begin{aligned}
\text{Max} \quad & \sum_{t=0}^{\infty} \left\{ log\,(C_t) - \left(\frac{1+x^{\frac{1}{\psi}}}{1+\psi}\right) \left(N_t^F\right)^{1+\psi} - \frac{\alpha}{1+\mu} \right\} \\
s.t \quad & Y_{H,t} = (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{\psi}{\psi+1}} A_t G_t^{\chi} \\
& Y_{I,t} = \alpha^{\frac{1}{1+\mu}} A_{I,t} (\delta G_t)^{\chi} \\
& N_t^F = (1-\alpha)^{\frac{1}{\psi+1}} (1-\tau_t)^{\frac{1}{\psi+1}} m^{\frac{1}{\psi+1}} v^{\frac{-1}{\psi+1}} \\
& C_t = \left(\frac{Y_{H,t}}{1-\alpha}\right)^{\frac{(1-\alpha)(1-a)}{1-2\alpha}} \left(\frac{Y_{I,t}}{\alpha}\right)^{\frac{-\alpha}{1-2\alpha}} (C_t^* \Gamma_t^{*\alpha})^{\frac{a(1-\alpha)}{1-2\alpha}} \\
& G_t = \tau_t Y_{H,t}
\end{aligned}
$$

C- Sticky Price Equilibrium without Informality

With Lump Sum Tax:

When the public good is financed through a lump sum tax, the equilibrium conditions are:

$$
\begin{aligned}
C_t &= Y_{H,t}^{1-a}\left(C_t^*\right)^a \\
\Delta_{p_h,t} &= \theta\Pi_{HF,t}^{\gamma}\Delta_{p_h,t-1} + (1-\theta)\,\Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi}Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi}A_t^{F(1+\psi)}G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma}F_{1,t+1} \\
F_{2t} &= Y_{H,t} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma-1}F_{2,t+1} \\
1 &= (1-\theta)\,\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^*\right)^a\left(\Delta E_t\right)^a \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*}\right)\left(\frac{C_{t-1}}{C_t}\right)^{\frac{1}{a}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}
\end{aligned}
$$

With Distortionary Tax:

When the public good is financed through a unit tax on formal output, the equilibrium conditions are:

$$
\begin{aligned}
C_t &= Y_{H,t}^{1-a}\left(C_t^*\right)^a \\
\Delta_{p_h,t} &= \theta\Pi_{HF,t}^{\gamma}\Delta_{p_h,t-1} + (1-\theta)\,\Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^* &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi}Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi}A_t^{F(1+\psi)}G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma}F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma-1}F_{2,t+1} \\
1 &= (1-\theta)\,\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^*\right)^a\left(\Delta E_t\right)^a \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^*}\right)\left(\frac{C_{t-1}}{C_t}\right)^{\frac{1}{a}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}
\end{aligned}
$$

The Ramsey Planner chooses the optimal tax and inflation by solving the following

problem:

$$\text{Max} \quad \sum_{t=0}^{\infty} \left\{ log\left(C_t\right) - \left(\frac{1}{1+\psi}\right)\left(\frac{\Delta_{p_h,t} Y_{H,t}^{F}}{A_t^{F} G_t^{\chi}}\right)^{1+\psi} \right\}$$

$$\begin{aligned}
s.t \quad C_t^{\frac{1}{(1-a)}} &= Y_{H,t}\left(C_t^{*}\right)^{\frac{a}{1-a}} \\
\Delta_{p_h,t} &= \theta \Pi_{HF,t}^{\gamma} \Delta_{p_h,t-1} + (1-\theta)\,\Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^{*} &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi} Y_{H,t}^{\psi+2}}{(1-\alpha) v^{\psi} A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma} F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t}^{\gamma-1} F_{2,t+1} \\
1 &= (1-\theta)\,\Pi_{HF,t+1}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^{*}\right)^{a}\left(\Delta E_t\right)^{a} \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^{*}}\right)\left(\frac{C_{t-1}}{C_t}\right)^{\frac{1}{a}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}
\end{aligned}$$

D- Sticky Price Equilibrium with Informality

With Lump Sum Tax:

When the public good is financed through a lump sum tax on the formal sector, the equilibrium of the economy are as follow:

$$
\begin{aligned}
C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} &= \left(\frac{Y_{H,t}}{1-\alpha}\right)\left(\frac{Y_{I,t}}{\alpha}\right)^{\frac{-\alpha}{(1-\alpha)(1-a)}}\left(C_t^{*}\Theta_t^{*\alpha}\right)^{\frac{a}{1-a}} \\
Y_{I,t} &= A_{I,t}\left(\delta G_t\right)^{\chi}\alpha^{\frac{1}{1+\mu}} \\
\Delta_{p_h,t} &= \theta\Pi_{HF,t}^{\gamma}\Delta_{p_h,t-1} + (1-\theta)\,\Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^{*} &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi}Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi}A_t^{F(1+\psi)}G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma}F_{1,t+1} \\
F_{2t} &= Y_{H,t} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t}^{\gamma-1}F_{2,t+1} \\
1 &= (1-\theta)\,\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \Pi_{I,t}^{\alpha}\Pi_{F,t}^{1-\alpha} \\
\Pi_{I,t}^{1-\alpha} &= \left(\frac{G_{t-1}}{G_t}\right)^{\chi}\left(\frac{C_t}{C_{t-1}}\right)\left(\frac{A_{i,t-1}}{A_{i,t}}\right)\Pi_{F,t}^{1-\alpha} \\
\Pi_{F,t} &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^{*}\right)^{a}\left(\Delta E_t\right)^{a} \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^{*}}\right)\left(\frac{C_t}{C_{t-1}}\right)^{\frac{2\alpha-1}{a(1-\alpha)}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}\left(\frac{Y_{I,t}}{Y_{I,t-1}}\right)^{\frac{\alpha}{a(1-\alpha)}}
\end{aligned}
$$

With Distortionary Tax:

The equilibrium of the economy when the public good is financed through a unit tax on formal output:

$$
\begin{aligned}
C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} &= \left(\frac{Y_{H,t}}{1-\alpha}\right)\left(\frac{Y_{I,t}}{\alpha}\right)^{\frac{-\alpha}{(1-\alpha)(1-a)}}\left(C_t^*\Theta_t^{*\alpha}\right)^{\frac{a}{1-a}} \\
Y_{I,t} &= A_{I,t}\left(\delta G_t\right)^{\chi}\alpha^{\frac{1}{1+\mu}} \\
\Delta_{p_h,t} &= \theta\Pi_{HF,t}^{\gamma}\Delta_{p_h,t-1} + (1-\theta)\,\Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^{*} &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi}Y_{H,t}^{\psi+2}}{(1-\alpha)v^{\psi}A_t^{F(1+\psi)}G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma}F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t}^{\gamma-1}F_{2,t+1} \\
1 &= (1-\theta)\,\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \Pi_{I,t}^{\alpha}\Pi_{F,t}^{1-\alpha} \\
\Pi_{I,t}^{1-\alpha} &= \left(\frac{G_{t-1}}{G_t}\right)^{\chi}\left(\frac{C_t}{C_{t-1}}\right)\left(\frac{A_{i,t-1}}{A_{i,t}}\right)\Pi_{F,t}^{1-\alpha} \\
\Pi_{F,t} &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^{*}\right)^{a}\left(\Delta E_t\right)^{a} \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^{*}}\right)\left(\frac{C_t}{C_{t-1}}\right)^{\frac{2\alpha-1}{a(1-\alpha)}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}\left(\frac{Y_{I,t}}{Y_{I,t-1}}\right)^{\frac{\alpha}{a(1-\alpha)}}
\end{aligned}
$$

The Ramsey Planner chooses the optimal tax and inflation by solving the following

problem:

$$\text{Max} \quad \sum_{t=0}^{\infty} \left\{ log\left(C_t\right) - \left(\frac{1+x^{\frac{1}{\psi}}}{1+\psi}\right)\left(\frac{\Delta_{p_h,t} Y_{H,t}^F}{v A_t^F G_t^{\chi}}\right)^{1+\psi} - \frac{\alpha}{1+\mu} \right\}$$

$$\begin{aligned}
s.t \quad C_t^{\frac{1-2\alpha}{(1-\alpha)(1-a)}} &= \left(\frac{Y_{H,t}}{1-\alpha}\right)\left(\frac{Y_{I,t}}{\alpha}\right)^{\frac{-\alpha}{(1-\alpha)(1-a)}} \left(C_t^* \Theta_t^{*\alpha}\right)^{\frac{a}{1-a}} \\
Y_{I,t} &= A_{I,t}\left(\delta G_t\right)^{\chi} \alpha^{\frac{1}{1+\mu}} \\
\Delta_{p_h,t} &= \theta \Pi_{HF,t}^{\gamma} \Delta_{p_h,t-1} + \left(1-\theta\right) \Pi_{HF,t}^{*(-\gamma)}; \\
\Pi_{HF,t}^{*} &= \left(\frac{\gamma}{\gamma-1}\right)\frac{F_{1t}}{F_{2t}} \\
F_{1t} &= \frac{\Delta_{p_h,t}^{\psi} Y_{H,t}^{\psi+2}}{(1-\alpha) v^{\psi} A_t^{F(1+\psi)} G_t^{\chi(1+\psi)}} + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma} F_{1,t+1} \\
F_{2t} &= Y_{H,t} - G_t + \theta\beta\left(\frac{C_t}{C_{t+1}}\right)\Pi_{t+1}^{-1}\Pi_{HF,t+1}^{\gamma-1} F_{2,t+1} \\
1 &= \left(1-\theta\right)\Pi_{HF,t}^{*(1-\gamma)} + \theta\Pi_{HF,t}^{\gamma-1} \\
\Pi_t &= \Pi_{I,t}^{\alpha}\Pi_{F,t}^{1-\alpha} \\
\Pi_{I,t}^{1-\alpha} &= \left(\frac{G_{t-1}}{G_t}\right)^{\chi}\left(\frac{C_t}{C_{t-1}}\right)\left(\frac{A_{i,t-1}}{A_{i,t}}\right)\Pi_{F,t}^{1-\alpha} \\
\Pi_{F,t} &= \left(\Pi_{HF,t}\right)^{(1-a)}\left(\Pi_{FF,t}^{*}\right)^{a}\left(\Delta E_t\right)^{a} \\
\Delta E_t &= \left(\frac{\Pi_{HF,t}}{\Pi_{FF,t}^{*}}\right)\left(\frac{C_t}{C_{t-1}}\right)^{\frac{2\alpha-1}{a(1-\alpha)}}\left(\frac{Y_{H,t}}{Y_{H,t-1}}\right)^{\frac{1}{a}}\left(\frac{Y_{I,t}}{Y_{I,t-1}}\right)^{\frac{\alpha}{a(1-\alpha)}}
\end{aligned}$$