

Altruism, Corruption, and Relative Performance Evaluation*

ALEXANDER HENKE†

Department of Economics

Grand Valley State University

Abstract

I construct a principal – agent – supervisor inspection model where the supervisor is altruistic towards the agent and therefore wishes to alter the signal in the agent’s favor. Even when side-contracting between the supervisor and the agent is infeasible, the threat of unilateral report manipulation from an altruistic supervisor leads to low-powered incentives and large transfers to the supervisor to induce truthful negative reports. When there are multiple agents whose production and related signals are entirely independent, the principal can use relative performance evaluation to mitigate the effect of the supervisor’s altruism, reducing wage compression and supervisor transfers. The power of relative performance evaluation diminishes when the supervisor is biased towards one agent and vanishes completely when the bias is completely in favor of one agent, as this eliminates the principal’s ability to balance the supervisor’s desires to see both agents do well. I show that a corrupt supervisor who bargains with all agents as a coalition presents an equivalent problem to a special, costly case of the problem with altruism, and I find that changes in a corrupt bargaining structure between agents and supervisor can alter incentives significantly.

Keywords: Relative performance evaluation, altruism, corruption, multiple agents

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† Email: henkeale@gvsu.edu

1. Introduction

When officials in charge of finding and reporting wrongdoing manipulate reports, it is natural to suspect that the official has obtained some illicit gain for his efforts in the form of a bribe. A citizen bribes an auditor to avoid punishment for tax evasion, or a factory owner bribes an inspector so that he may continue to pollute without paying fines. A student may offer higher teaching evaluations to an instructor who offers clear signals of high grades.

Some solutions to combatting corruption focus on inhibiting the technology of illicit side contracts. Corrupt relationships often rely on long-term reputation and observable side-transfers to properly enforce side-contracts. Rotating auditors, limiting long-term relationships, and limiting the visibility of potential side-transfers may deter collusive behavior without affecting the joint incentives of a corrupt coalition directly (Bardhan 1997).

Disrupting the technology of side-contracts only works, however, if side-contracting is the primary mechanism by which an official and an agent engage in cooperative behavior. In the setting of education, a pure quid pro quo arrangement between student and instructor would theoretically be a transfer of high grades for high student evaluations, named the leniency hypothesis in the education literature.¹ However, student evaluations tend to be anonymous, instructors are generally rotated away from particular students for practical scheduling reasons, and evaluations tend to occur before final grade submissions.

For these reasons, a direct pure quid pro quo agreement should suffer myriad credibility issues, where both the agent and supervisor have incentives to deviate without suffering reputational issues. In other settings, such as the management of

¹ See Gump (2007) and Spooren, Brockx, and Mortelmans (2013) for overviews of recent literature on the leniency hypothesis.

workers or pollution regulation, quid pro quo arrangements may be too easily detected and reported by third parties, effectively making them too costly. Instead, Tirole (1986) argues that more common forms of corruption involve non-pecuniary transfers, which include admiration and respect as they are easier to conceal. Still, Tirole focuses on reciprocal transfers, i.e. a maintenance of quid pro quo, when at times only one side can successfully enjoy such a non-pecuniary transfer.

When explicitly corrupt coalitions are easy to detect or thwart, a supervisor may still have the ability to unilaterally alter reports, if there is an internal incentive to do so. If a supervisor is altruistic towards an agent, he may take it upon himself to manipulate performance reports in the agent's favor without proper incentives. An instructor may increase a friendly student's grade to help that student achieve departmental obligations, or a manager may provide an excessively positive evaluation for a worker who appears unproductive.

Altruism is generally seen as a beneficial attribute as it relates to the provision of public goods², but it has clear drawbacks when the positive feelings are limited to a personal relationship between supervisor and agent. The principal clearly prefers a supervisor whose incentives are more closely aligned with its own. In effect, altruism represents a non-pecuniary conflict of interest between the supervisor's responsibilities and feelings towards those supervised.

One example of personal altruism representing such a potential conflict of interest is the family firm. Schulze, Lubatkin, and Dino (2003) examine family ownership of firms and find that familial altruism can create agency issues through excessive rewards and perverse incentives. While an ownership group that mutually cares about each other can have positive effects on production and stability, such a safe, stable relationship effectively reduces incentives and can potentially create entitlement in key workers (McConaughy et al. 1998; Sharma 2004).

² See Perry, Honddeghem, and Wise (2010) for a recent review of the literature on pro-social motivations for public service, and Dufwenberg and Kirchsteiger (2004) for a theoretical examination of reciprocal altruism.

The family literature highlights the reasons a principal may prefer to hire supervisors with such intrinsic motivations for reasons exogenous to this model. A manager or instructor's duties are not simply to verify whether a worker has exhibited sufficient effort; the supervisor may be intimately involved in the production process, and altruism may relieve strong issues of moral hazard³ that such a relationship would normally pose. This issue, while abstracted away in this paper, is a key reason to retain this type of supervisor as opposed to one that is indifferent to the agent's plight.

The principal has two tools to deter report manipulation when the supervisor is altruistic towards the agent. The first is direct compensation to the supervisor, similar to the principal's solution to a problem with corruption. The supervisor effectively becomes a bounty hunter a la Kofman and Lawarrée (1993), and is compensated when providing negative reports about the agent. Instead of deterring corruption, this compensates the supervisor's loss of utility for punishing the agent. An instructor's evaluations may be corrected for perceived course leniency. A manager may receive a bonus for keeping costs down instead of handing out bonuses to employees.

Another way to induce supervisor truth-telling is to create countervailing incentives. If the supervisor oversees multiple agents, as is common in an educational or managerial setting, the principal can use relative performance evaluation among agents even when production and monitoring between the agents is entirely independent. Employing grade distribution requirements, for instance, means that the instructor cannot reward all agents to the fullest extent. In particular, fabricating a successful report for one student reduces the utility of other students by lowering their grades directly, which in turn reduces the incentive for the supervisor to alter reports without having to resort to direct compensation.

³ See Kim, Lawarrée, and Shin (2004) for an examination of managerial moral hazard, Holmstrom (1982) for an examination of moral hazard with multiple agents, and Mookherjee and Png (1995) for an examination of moral hazard in inspection.

At its heart, altruism acts like a lesser form of corruption, which is more difficult to prevent. Both problems align the incentives of the supervisor with the agents as opposed to the principal; altruism does so imperfectly, but without the requirement of collaboration and credible side-contracting.

The paper proceeds as follows: Section 2 sets up the model, Section 3 goes over single-agent benchmarks, Section 4 describes the main results, Section 5 discusses extensions, and Section 6 concludes.

2. The setup

A principal hires two agents, A and B, to produce outputs x_A and x_B , which the agents privately choose to be 1 or 0. Output in this case could be the abatement of pollution by a factory, similar to Mookherjee and Png (1995), or scholarly engagement by a student in an educational setting, but the representative example will be production by a worker, supervised by a manager. An agent's objective function is $w_i^{0.5} - \psi x_i$, $i \in \{A, B\}$, where w is a wage paid to the agent by the principal, and ψ represents the agent's cost of output. The principal's value of x is large enough that the principal wishes to induce $x = 1$ whenever it is possible to do so. Output x_i is observed only by agent i , but the principal is the residual claimant of all x . Without supervision, the principal observes nothing and cannot effectively induce $x = 1$ for any agent regardless of wage scheme. Agents have limited liability such that $w \geq 0$.

The principal can hire a single supervisor to oversee the production of both agents. Without cost, the supervisor collects two signals, σ_A and σ_B , which provide information about the production of agents A and B respectively. The signal σ_i either displays the true level of production x_i for agent i with probability p , or no information (\emptyset) with probability $1-p$. Importantly, both the deterministic production of each agent and the signals of each agent collected by the supervisor are independent of each other. While the supervisor lacks costs, the principal can

pay the supervisor a transfer t_{σ_A, σ_B} , where σ_A and σ_B are the reported production levels of agents A and B respectively. The supervisor has limited liability such that $t \geq 0$.

While the supervisor is incorruptible, he is altruistic and can manipulate signal reports at no cost. His objective function is $t + \gamma(w_A^{0.5} + w_B^{0.5})$, where γ represents a fraction of the agents' utilities, $0 < \gamma < 1$. If the supervisor is made indifferent between reports, he will report the true signal⁴, but any difference in the agents' wages which is not otherwise compensated through supervisor transfers will induce the supervisor to produce a report most favorable to the agents.

While the signals of agents A and B are independent of each other, the principal may still base one agent's transfers on reports of another agent's production. Hence, for agent i , wages are denoted as $w_{i, \sigma_i, \sigma_j}$.

I define relative performance evaluation for agent i as any difference between $w_{i, \sigma_i, \sigma_j}$ and $w_{i, \sigma_i, \sigma'_j}$, $\sigma_j \neq \sigma'_j$, for some report σ_i of the production of agent i . To illustrate, suppose $\sigma_A = 1$. If the principal grants a constant reward to agent A $w_{A,1,1} = w_{A,1,\emptyset} = w_{A,1,0}$, then there is no relative performance evaluation. If $w_{A,1,\emptyset} > w_{A,1,1}$, for instance, there is relative performance evaluation, and the level of relative performance evaluation depends on the difference between the two transfers.

The order of the game is as follows:

1. The principal offers contracts to agents A and B consisting of wage schedules denoted by all combinations of $w_{i, \sigma_i, \sigma_j}$, along with transfers to the supervisor denoted by t_{σ_A, σ_B} , and the agents accept or reject the contract.
2. Production occurs; the agents choose $x_i = 0$ or $x_i = 1$, $i \in \{A, B\}$.
3. The supervisor collects signals σ_A and σ_B , and potentially alters reports of the signals.
4. The supervisor submits a potentially-altered report of σ_A and σ_B .

⁴ An equivalent assumption is that the cost of signal manipulation is a negligible amount ϵ .

5. The principal pays wages w and transfers t .

3. Benchmarks with one agent

3.1 Supervision with no manipulation

If the principal observes x directly with probability 1, the contract becomes the first best:

$$IR: w^{0.5} - \psi \geq 0$$

$$w = \psi^2$$

If the principal observes the imperfect signal directly, the principal minimizes expected wages subject to IC, IR, and LLC constraints:

$$\min_w pw_1 + (1-p)w_\emptyset$$

Subject to:

$$IR: pw_1^{0.5} + (1-p)w_\emptyset^{0.5} - \psi \geq 0$$

$$IC: pw_1^{0.5} + (1-p)w_\emptyset^{0.5} - \psi \geq pw_0^{0.5} + (1-p)w_\emptyset^{0.5}$$

$$LLC_i: w_i \geq 0, i \in \{0,1,\emptyset\}$$

Notice that IC simplifies to the following:

$$pw_1^{0.5} - \psi \geq pw_0^{0.5}$$

And that w_0 only enters negatively into IC, and positively into its own LLC.

The solution is as follows:

$$w_1 = \left(\frac{\psi}{p}\right)^2$$

$$w_0 = w_\emptyset = 0$$

Notice that the principal extracts all rent, but pays more than the first best due to risk compensation. In particular, the principal's wage bill in the first best is ψ^2 , whereas in the second best the expected wage bill is $\frac{\psi^2}{p}$.

3.2 External, altruistic supervisor with one agent

Clearly, if the principal does not offer transfers to the supervisor, an altruistic supervisor will simply pick a report that leads to the highest wage for the agent. In the single agent case, the principal's only option is to reward the supervisor for reports the supervisor does not prefer. These Supervisor Incentive Compatibility (SIC) constraints are much like Coalition Incentive Compatibility (CIC) constraints which would deter a corrupt auditor; the key difference is that the supervisor's incentives represents an imperfect combination of the utilities of the coalition, and hence incentives are more easily re-aligned. The SIC constraints can be described jointly as follows:

$$t_1 + \gamma w_1^{0.5} = t_\emptyset + \gamma w_\emptyset^{0.5} = t_0 + \gamma w_0^{0.5}$$

In other words, the supervisor must be made indifferent between reports once transfers are taken into consideration.

Note that, so long as *IC* and *SIC* hold, no report of 0 will occur on the equilibrium path. Therefore, any excess transfer t_0 the principal must pay the supervisor to ensure compliance bears no cost. Re-writing t_\emptyset as $t_\emptyset = t_1 + \gamma(w_1^{0.5} - w_\emptyset^{0.5})$, I set up the principal's new problem like so:

$$\min_{w,t} p(w_1 + t_1) + (1 - p) \left(w_\emptyset + t_1 + \gamma(w_1^{0.5} - w_\emptyset^{0.5}) \right)$$

Subject to:

$$IR: pw_1^{0.5} + (1 - p)w_\emptyset^{0.5} - \psi \geq 0$$

$$IC: pw_1^{0.5} + (1 - p)w_\emptyset^{0.5} - \psi \geq pw_\emptyset^{0.5} + (1 - p)w_0^{0.5}$$

$$LLC_{wi}: w_i \geq 0, i \in \{0,1,\emptyset\}$$

$$LLC_{ti}: t_i \geq 0, i \in \{0, 1, \emptyset\}$$

The derivation of the optimal contract is in Appendix A, and takes two forms depending on the size of γ :

Case 1: $\gamma < \frac{2\psi}{p}$

$$w_1 = \left(\frac{\psi}{p}\right)^2$$

$$w_\emptyset = \left(\frac{\gamma}{2}\right)^2$$

$$w_0 = t_1 = 0$$

$$t_\emptyset = \gamma \left(\frac{\psi}{p} - \frac{\gamma}{2}\right)$$

$$t_0 = \frac{\gamma\psi}{p}$$

Case 2: $\gamma \geq \frac{2\psi}{p}$

$$w_1 = w_\emptyset = \left(\frac{\psi}{p}\right)^2$$

$$w_0 = t_1 = t_\emptyset = 0$$

$$t_0 = \frac{\gamma\psi}{p}$$

In order to satisfy both the agent's and the supervisor's incentives, the principal must surrender rent in terms of w_\emptyset , i.e. costly wage compression, and t_\emptyset , i.e. costly supervisor incentives. Whether the principal splits rent between the supervisor and agent or surrenders all rent to the agent depends on the compensation required to make the supervisor indifferent, which depends on γ . In either case, the principal can use evidence that only occurs off the equilibrium path to incentivize the agent.

4. The two agent case

Recall that the signals collected by the supervisor on each agent are i.i.d., and the supervisor ultimately makes a joint report of both agents. For instance, a supervisor report of 1,0 would mean that the supervisor collected a signal of 1 for agent A, and 0 for agent B. Because both the signals and production are independent between agents, information about agent A does not provide any informative value regarding agent B, which leads to Lemma 1:

4.1 Irrelevance of relative performance in a two-tier hierarchy

Lemma 1:

When the principal is the supervisor, the two-agent case is a duplication of the single-agent case.

Proof: See Appendix A for the derivation of the optimal contract in the two-agent case. Notice that the optimal contract offers the same wage schedules as the single-agent case, and namely that the wages do not depend on the reported performance of the other agent.

In this context, relative performance evaluation does not serve any informative or risk-sharing purpose, as the signals between the two agents are independent. Rather, the purpose of relative performance evaluation in this context is to alleviate incentive issues with a third-party supervisor.

4.2 Solution to the main model with altruism and two agents

When there is an external supervisor with altruism, the existence of multiple agents provides an opportunity for the principal to reduce rent by linking transfers to all reports, including those of other agents. Recall that altruism takes the form γ , and wages for each agent take the form w_{i,σ_i,σ_j} , where i denotes agent A or B, σ_i denotes the agent's own report, and σ_j denotes the other agent's report. For instance, $w_{A,0,1}$ denotes the wage to agent A if the supervisor reports 0 for agent A and 1 for agent B. Transfers to the supervisor take the form of t_{σ_A,σ_B} .

The SIC constraints can be written as follows:

$$\begin{aligned}
t_{0,0} + \gamma(w_{A,0,0}^{0.5} + w_{B,0,0}^{0.5}) &= t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5}) = t_{\emptyset,\emptyset} + \gamma(w_{A,\emptyset,\emptyset}^{0.5} + w_{B,\emptyset,\emptyset}^{0.5}) \\
&= t_{1,0} + \gamma(w_{A,1,0}^{0.5} + w_{B,0,1}^{0.5}) = t_{0,1} + \gamma(w_{A,0,1}^{0.5} + w_{B,1,0}^{0.5}) \\
&= t_{1,\emptyset} + \gamma(w_{A,1,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5}) = t_{\emptyset,1} + \gamma(w_{A,\emptyset,1}^{0.5} + w_{B,1,\emptyset}^{0.5}) \\
&= t_{\emptyset,0} + \gamma(w_{A,\emptyset,0}^{0.5} + w_{B,0,\emptyset}^{0.5}) = t_{0,\emptyset} + \gamma(w_{A,0,\emptyset}^{0.5} + w_{B,\emptyset,0}^{0.5})
\end{aligned}$$

With that, the principal solves the following minimization problem:

$$\begin{aligned}
\min_{w,t} p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\
+ p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} \\
+ \gamma(2w_{A,1,1}^{0.5} + 2w_{B,1,1}^{0.5} - w_{A,1,\emptyset}^{0.5} - w_{B,\emptyset,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{B,1,\emptyset}^{0.5})) \\
+ (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5}))
\end{aligned}$$

Subject to:

$$IR_i: p^2 w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \geq 0, i \in \{A, B\}$$

$$\begin{aligned}
IC_i: p^2 w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \\
\geq p(pw_{i,0,1}^{0.5} + (1-p)w_{i,0,\emptyset}^{0.5}) + (1-p)(pw_{i,\emptyset,1}^{0.5} + (1-p)w_{i,\emptyset,\emptyset}^{0.5}), i \in \{A, B\}
\end{aligned}$$

$$LLC_w: w \geq 0 \forall w$$

$$LLC_t: t \geq 0 \forall t$$

The derivation of the result is in Appendix B. For $\gamma < \frac{2\psi p}{p^2 - p + 2}$, the solution is characterized as follows:

$$t_{1,1} = w_{i,0,1} = w_{i,0,0} = w_{i,0,\emptyset} = 0$$

$$w_{i,\emptyset,1} = w_{i,\emptyset,\emptyset} = \left(\frac{\gamma}{2}\right)^2$$

$$w_{i,1,\emptyset} = \left(\frac{2\psi + \gamma}{2p}\right)^2$$

$$w_{i,1,1} = \left(\frac{2\psi p - (1-p)\gamma}{2p^2} \right)^2$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \gamma \left(\frac{2\psi p - (1-p)\gamma}{p^2} - \frac{2\psi + \gamma}{2p} - \frac{\gamma}{2} \right)$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{\gamma}{2p^2} (2\psi p - (p^2 - p + 2)\gamma)$$

$$t_{\emptyset,\emptyset} = \frac{\gamma}{p^2} (2\psi p - (1-p + p^2)\gamma)$$

In all cases, $i \in \{A, B\}$, as the results are symmetric for both agents. This leads to the following proposition:

4.3 Properties of the optimal contract

Proposition 1: Altruism leads to relative performance evaluation for positive reports

As γ increases, the difference between $w_{i,1,\emptyset}$ and $w_{i,1,1}$ increases.

The proof is simple and takes the following steps:

1. As $\gamma \rightarrow 0$, the contract resembles the two-agent contract where the principal is the supervisor. By Lemma 1, this contract lacks relative performance evaluation, which means $w_{i,1,1} = w_{i,1,\emptyset}$.
2. $w_{i,1,\emptyset}$ increases in γ , and $w_{i,1,1}$ decreases in γ .

Step 1:

Note that as $\gamma \rightarrow 0$, the solution resembles the case where the principal is the supervisor:

$$t_{i,1,1} = t_{i,1,\emptyset} \rightarrow \frac{\psi}{p}$$

$$t_{i,\emptyset,1} = t_{i,\emptyset,\emptyset} \rightarrow 0$$

Step 2:

The comparative statics of $w_{i,1,\emptyset}^{0.5}$ and $w_{i,1,1}^{0.5}$ with respect to γ are as follows:

$$\frac{\partial w_{i,1,\emptyset}^{0.5}}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[\frac{2\psi + \gamma}{2p} \right] = \frac{1}{2p} > 0$$

$$\frac{\partial w_{i,1,1}^{0.5}}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[\frac{2\psi p - (1-p)\gamma}{2p^2} \right] = -\frac{1-p}{2p^2} < 0$$

Notice that, as γ increases, $w_{i,1,\emptyset}$ increases, and $w_{i,1,1}$ decreases. This represents an increased use of relative performance evaluation, reserving the majority of the agent's reward for the case when the agent outperforms his peer. This reduces the temptation for the supervisor to report (1,1), i.e. that both agents performed well, which, in turn, allows the principal to more effectively punish agents for ambiguous reports.

Corollary to Proposition 1: The existence of multiple agents reduces wage compression

The wage for a signal of \emptyset is higher in the single agent case than in the two-agent case.

Compare the wages for the single agent case and the two-agent case respectively, for low γ :

$$w_{\emptyset}^{0.5} = \frac{0.5\gamma}{1-p}$$

$$w_{i,\emptyset,1}^{0.5} = w_{i,\emptyset,\emptyset}^{0.5} = \frac{\gamma}{2} < \frac{\gamma}{2(1-p)}$$

Relative performance evaluation decreases the need for wage compression to incentivize the supervisor.

Lemma 2: Altruism harms the principal

The principal's profits decrease in γ .

The proof is in Appendix B. The intuition is straightforward: The principal prefers a supervisor whose incentives are more easily aligned with its own. The more

altruism the supervisor holds towards the agent, the more the supervisor's incentives are aligned with the agent instead of the principal.

4.4 Discussion

A general property of the contract is $w_{i,1,\emptyset} > w_{i,1,1} > w_{i,\emptyset,1} = w_{i,\emptyset,\emptyset} > 0$. In essence, an agent obtains the highest reward when that agent's reported signal is superior to the other agent's reported signal, even though those signals are uncorrelated. The principal does this to reduce the supervisor's rent; the supervisor is more inclined to punish an agent by reporting \emptyset if there is a corresponding high reward for the other agent's signal of 1. The cost of enacting this policy is the excess wages required to compensate for an increased burden of risk on the agent. Specifically, varying one agent's wages based on another agent's signal increases the variance of that agent's wage distribution.

The effect of using differences in agent signals to improve supervisor incentives is also seen in the difference between $t_{i,1,\emptyset}$ and $t_{i,\emptyset,\emptyset}$. Because both agents are effectively punished when the supervisor reports (\emptyset, \emptyset) , the supervisor must be compensated heavily to report this truthfully. However, reporting $(\emptyset, 1)$ or $(1, \emptyset)$ does not require as much compensation, because $w_{1,\emptyset}$ is large.

5. Extensions

5.1 Corruption

I now examine the similarities and differences between a corrupt supervisor and an altruistic supervisor. The results are sensitive to how collusive side agreements work. If all agents and the supervisor must jointly cooperate to change a signal, the result is equivalent to a special case of the model with altruism. If the supervisor negotiates with agents separately, however, relative performance evaluation in the agents leads to perverse incentives for rent extraction, altering the optimal solution significantly.

Consider the same production and information structure as in the main model, with the following changes. The supervisor lacks any altruism, i.e. $\gamma = 0$. The supervisor is corruptible, in that the agents and the supervisor can engage in a collusive side-agreement to change the signal; so instead of falsifying the signal out of genuine altruism towards the agent, an indifferent supervisor receives side transfers in exchange for report falsification.

This examination relieves issues brought up by Laffont and Martimort (1997), who suggest that collusion between agents should alter incentives. Collusion between both agents and the supervisor somewhat resembles this case, and in fact aligns the incentives of the coalition against the principal.

5.1.1 Corruption with multi-lateral bargaining

Initially, I assume that the three non-principal players, i.e. both agents and the supervisor, engage in cooperative tri-lateral bargaining and side-agreements, with reports and side-transfers determined by standard Nash Bargaining in a multi-agent case. To alter the reports, cooperation is required by all agents, and hence the threat point is no report alteration. This assumption works well, for instance, if an agent who is harmed by a collusive side agreement between the other agent and the supervisor has a reasonable option to collect evidence and contest the supervisor's report.

To stop corruption in this case, the principal must eliminate gains from report alteration among the entire group. The Coalition Incentive Compatibility (CIC) constraints can be described as follows:

$$\begin{aligned}
t_{1,1} + w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} &= t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5} = t_{\emptyset,1} + w_{B,1,\emptyset}^{0.5} + w_{A,\emptyset,1}^{0.5} = t_{\emptyset,\emptyset} + w_{A,\emptyset,\emptyset}^{0.5} + w_{B,\emptyset,\emptyset}^{0.5} \\
&= t_{1,0} + w_{A,1,0}^{0.5} + w_{B,0,1}^{0.5} = t_{0,1} + w_{B,1,0}^{0.5} + w_{A,0,1}^{0.5} = t_{\emptyset,0} + w_{A,\emptyset,0}^{0.5} + w_{B,0,\emptyset}^{0.5} \\
&= t_{0,\emptyset} + w_{B,\emptyset,0}^{0.5} + w_{A,0,\emptyset}^{0.5} = t_{0,0} + w_{A,0,0}^{0.5} + w_{B,0,0}^{0.5}
\end{aligned}$$

Notice that this is exactly the same as the SIC constraints from the problem with altruism, for $\gamma = 1$, which leads to the following lemma:

Lemma 3: The multi-lateral corruption problem is a special case of altruism

The full proof is in Appendix C. The constraints which deter corruption are identical to the constraints which deter altruistic signal manipulation for $\gamma = 1$. This is because, after bargaining and side-transfers, the supervisor-agent-agent coalition effectively acts like a unilateral supervisor who takes each agent's utility fully into account.

The solution to the multi-lateral corruption problem is re-written here:

$$w_{i,\emptyset,1}^{0.5} = w_{i,\emptyset,\emptyset}^{0.5} = \frac{1}{2}$$

$$w_{i,1,\emptyset}^{0.5} = \frac{2\psi + 1}{2p} = \frac{\psi}{p} + \frac{1}{p}$$

$$w_{i,1,1}^{0.5} = \frac{2\psi p - (1 - p)}{2p^2} = \frac{\psi}{p} - \frac{1 - p}{2p^2}$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{1}{2p^2} (2\psi p - (p^2 - p + 2)) = \frac{\psi}{p} - \frac{p^2 - p + 2}{2p^2}$$

$$t_{\emptyset,\emptyset} = \frac{1}{p^2} (2\psi p - (1 - p + p^2)) = \frac{2\psi}{p} - \frac{1 - p + p^2}{p^2}$$

This result leads to the following proposition:

Proposition 2: Altruism is preferable to multi-lateral corruption for the principal

This follows from Lemma 2 and Lemma 3. The principal's costs increase in γ , the costs in the corruption contract are equivalent to an altruism problem where $\gamma = 1$, and $\gamma < 1$ in the problem with altruism. The incentives of an altruistic supervisor are only partly aligned with the agent, whereas a corrupt supervisor's incentives can be fully aligned with the agent via the bargaining process.

5.1.2 Bilateral bargaining

Suppose instead that the supervisor engages in two stages of bilateral Nash bargaining between agents over their individual reports. He first negotiates with agent A over his report, and then negotiates with agent B. In the first bargaining stage, the supervisor cannot commit to bargaining strategies in the second stage – he can only commit to that agent’s report falsification and side transfers. Hence, the threat point of the first stage is no falsification of the first report, along with the anticipated negotiated outcome between the supervisor and agent B.

Importantly, in a bilateral agreement between the supervisor and agent A, the supervisor can profit from a bribe and report alteration that is inefficient for the group as a whole. Conversely, there may be a set of reports that is efficient for the whole group but cannot be achieved through bilateral negotiations.

Take, for instance, the case where the “status quo” report is $(1, \emptyset)$. If the multi-lateral CIC constraints hold, then $t_{1,1} + w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} = t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5}$, and there is no joint incentive to change one signal. However, even if the multi-lateral CIC constraints hold, it is not necessarily the case that the bi-lateral coalition between the supervisor and agent B is deterred from bribery. In fact, if the multi-lateral CICs hold and $w_{A,1,1}^{0.5} < w_{A,1,\emptyset}^{0.5}$, which is true in the solution with multi-lateral Nash bargaining, it is guaranteed that the bi-lateral coalition has an incentive to manipulate the joint report to be $(1,1)$.

In essence, so long as there is relative performance evaluation, the supervisor is corrupt, and the supervisor can engage in bilateral agreements to alter a particular agent’s report, each agent suffers from a prisoner’s dilemma. Bribing the supervisor to manipulate the signal imposes a cost on the other agent, but is privately worthwhile. Ultimately these bribes act as an inefficient transfer from the agents to the supervisor. Therefore, there is no natural reason to lift the CIC constraints altogether, but because of the bilateral nature of corruption, the CICs change to bilateral CIC constraints:

$$CIC_A: t_{1,\sigma_B} + w_{A,1,\sigma_B}^{0.5} = t_{\emptyset,\sigma_B} + w_{A,\emptyset,\sigma_B}^{0.5} = t_{0,\sigma_B} + w_{A,0,\sigma_B}^{0.5}, \sigma_B \in \{0,1,\emptyset\}$$

$$CIC_B: t_{\sigma_A,1} + w_{B,1,\sigma_A}^{0.5} = t_{\sigma_A,\emptyset} + w_{B,\emptyset,\sigma_A}^{0.5} = t_{\sigma_A,0} + w_{B,0,\sigma_A}^{0.5}, \sigma_A \in \{0,1,\emptyset\}$$

These CIC constraints are more restrictive in certain ways and less restrictive in others. The principal's problem changes to the following:

$$\begin{aligned} \min_{w,t} p^2 (w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\ + p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}) + w_{A,1,1}^{0.5} \\ - w_{A,\emptyset,1}^{0.5}) \\ + (1-p)^2 (w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{1,1} + (w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5})) \end{aligned}$$

Subject to:

$$IR: p^2 w_{1,1}^{0.5} + p(1-p)(w_{1,\emptyset}^{0.5} + w_{\emptyset,1}^{0.5}) + (1-p)^2 w_{\emptyset,\emptyset}^{0.5} - \psi \geq 0$$

$$\begin{aligned} IC: p^2 w_{1,1}^{0.5} + p(1-p)(w_{1,\emptyset}^{0.5} + w_{\emptyset,1}^{0.5}) + (1-p)^2 w_{\emptyset,\emptyset}^{0.5} - \psi \\ \geq p(pw_{0,1}^{0.5} + (1-p)w_{\emptyset,\emptyset}^{0.5}) + (1-p)(pw_{\emptyset,1}^{0.5} + (1-p)w_{\emptyset,\emptyset}^{0.5}) \end{aligned}$$

$$LLC_w: w \geq 0 \forall w$$

$$LLC_t: t \geq 0 \forall t$$

$$CIC: w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{A,1,\emptyset}^{0.5} + w_{A,\emptyset,\emptyset}^{0.5} - w_{B,1,1}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5} + w_{B,1,\emptyset}^{0.5} = 0$$

The full derivation of the solution is in Appendix C. The solution is characterized as follows:

$$w_{i,1,1}^{0.5} = \frac{\psi}{p} - \left(\frac{1-p}{2p}\right)^2$$

$$w_{i,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{1-p}{4p}$$

$$w_{i,\emptyset,1}^{0.5} = \frac{1+p}{4p}$$

$$w_{i,\emptyset,\emptyset}^{0.5} = \frac{1}{4}$$

$$t_{1,1} = w_{i,0,\sigma_j} = 0$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{\psi}{p} - \frac{1-p+2p^2}{4p^2}$$

$$t_{\emptyset,\emptyset} = \frac{2\psi}{p} - \frac{1-2p+4p^2}{4p^2}$$

Comparing the bi-lateral solution to the multi-lateral solution, notice that while $w_{i,1,\emptyset} > w_{i,1,1}$, they are closer together under bilateral bargaining. Relative performance evaluation still has some value because of the link between agents provided by supervisor transfers, but the ability for the principal to use relative performance evaluation has diminished significantly. Because of this, the optimal contract differs significantly, and I arrive at the following proposition:

Proposition 3: Bi-lateral corruption is preferable to multi-lateral corruption

The proof is in Appendix C. Notice that relative performance evaluation decreases with bi-lateral corruption for two reasons: It is less effective at stopping corruption, but it is also less necessary, as the constraints on stopping corruption are less restrictive when the agents and supervisor cannot cooperate as effectively.

5.2 Favoritism

Suppose the supervisor's altruism towards agent A is γ_A , and the supervisor's altruism towards agent B is γ_B , $\gamma_A > \gamma_B \geq 0$.

I examine the case where the principal is aware of the nature of the favoritism and can take steps to correct for it. If the agents have observable factors that the supervisor historically prefers, for instance, the principal may be in a position to take steps to correct for the supervisor's biases. When they are pronounced enough, these biases can curtail the principal's ability to incentivize supervisor compliance.

The SIC changes to reflect the differences in γ :

$$\begin{aligned}
t_{0,0} + \gamma_A w_{A,0,0}^{0.5} + \gamma_B w_{B,0,0}^{0.5} &= t_{1,1} + \gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5} = t_{\emptyset,\emptyset} + \gamma_A w_{A,\emptyset,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,\emptyset}^{0.5} \\
&= t_{1,0} + \gamma_A w_{A,1,0}^{0.5} + \gamma_B w_{B,0,1}^{0.5} = t_{0,1} + \gamma_A w_{A,0,1}^{0.5} + \gamma_B w_{B,1,0}^{0.5} \\
&= t_{1,\emptyset} + \gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5} = t_{\emptyset,1} + \gamma_A w_{A,\emptyset,1}^{0.5} + \gamma_B w_{B,1,\emptyset}^{0.5} \\
&= t_{\emptyset,0} + \gamma_A w_{A,\emptyset,0}^{0.5} + \gamma_B w_{B,0,\emptyset}^{0.5} = t_{0,\emptyset} + \gamma_A w_{A,0,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,0}^{0.5}
\end{aligned}$$

Given this updated set of constraints, the principal solves the following problem:

$$\begin{aligned}
\min_{w,t} p^2 (w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\
+ p(1-p) (w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} + 2(\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) \\
- (\gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5}) - (\gamma_A w_{A,\emptyset,1}^{0.5} + \gamma_B w_{B,1,\emptyset}^{0.5})) \\
+ (1-p)^2 (w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{1,1} + (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) \\
- (\gamma_A w_{A,\emptyset,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,\emptyset}^{0.5}))
\end{aligned}$$

Subject to:

$$IR_i: p^2 w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \geq 0$$

$$\begin{aligned}
IC_i: p^2 w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \\
\geq p(pw_{i,0,1}^{0.5} + (1-p)w_{i,0,\emptyset}^{0.5}) + (1-p)(pw_{i,\emptyset,1}^{0.5} + (1-p)w_{i,\emptyset,\emptyset}^{0.5})
\end{aligned}$$

$$LLC_w: w \geq 0 \forall w$$

$$LLC_t: t \geq 0 \forall t$$

The full solution is derived in Appendix C, and is characterized as follows when the level of favoritism is relatively mild⁵:

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_A$$

$$w_{B,1,1}^{0.5} = \frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_B$$

⁵ This holds true when $\gamma_B > \frac{\gamma_A^2}{2\psi p - (1-p+p^2)\gamma_B}$.

$$w_{A,1,\emptyset}^{0.5} = \frac{2\psi + \gamma_A}{2p}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{2\psi + \gamma_B}{2p}$$

$$w_{A,\emptyset,1}^{0.5} = w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,1}^{0.5} = w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

$$w_{A,0,\sigma_B} = w_{B,0,\sigma_A} = 0 \quad \forall \sigma_A, \sigma_B$$

$$t_{1,1} = 0$$

$$t_{1,\emptyset} = \gamma_B \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_B \right) - \frac{\gamma_A^2}{2p^2}$$

$$t_{\emptyset,1} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_A \right) - \frac{\gamma_B^2}{2p^2}$$

$$t_{\emptyset,\emptyset} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_A \right) + \gamma_B \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_B \right)$$

Notice that this solution is identical to the original problem when $\gamma_A = \gamma_B = \gamma$. If there is a slight discernible level of favoritism, the principal will essentially treat each agent separately except for the supervisor's report. Agent A's wage schedule only depends on supervisor reports, costs, and altruism towards Agent A; the same logic applies to Agent B.

In cases of stronger favoritism, the principal begins linking one agent's transfers with both levels of altruism. The full range of solutions are in Appendix C, but this effect is clearest in the extreme case where $\gamma_A > 0$ and $\gamma_B = 0$, i.e. where the relative difference in altruism is strongest. The full solution for such a case is as follows:

$$w_{A,1,1}^{0.5} = w_{A,1,\emptyset}^{0.5} = w_{B,1,1}^{0.5} = w_{B,1,\emptyset}^{0.5} = \frac{\psi}{p}$$

$$w_{A,\emptyset,1}^{0.5} = w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,1}^{0.5} = w_{B,\emptyset,\emptyset}^{0.5} = 0$$

$$w_{A,0,\sigma_B} = w_{B,0,\sigma_A} = 0 \quad \forall \sigma_A, \sigma_B$$

$$t_{1,1} = t_{1,\emptyset} = t_{1,0} = 0$$

$$t_{\emptyset,1} = t_{\emptyset,\emptyset} = t_{\emptyset,0} = \gamma_A \left(\frac{\psi}{p} - \frac{\gamma_A}{2} \right)$$

This leads to the following proposition:

Proposition 4: Relative performance evaluation vanishes as favoritism increases

As $\gamma_B \rightarrow 0$, $(w_{A,1,\emptyset} - w_{A,1,1}) \rightarrow 0$ and $(w_{B,1,\emptyset} - w_{B,1,1}) \rightarrow 0$.

This is evident by the structure of the optimal contract.

One might expect the principal to abandon relative performance evaluation for Agent B as that case approaches the case of the principal-as-supervisor. As favoritism towards Agent A increases, however, it becomes increasingly difficult to use the reports from Agent B to incentivize the supervisor to report on the behavior of Agent A. In the extreme case, the principal abandons relative performance evaluation for both agents, instead opting to prevent signal manipulation via significant supervisor transfers and some rent to Agent A.

6. Conclusion

In many cases when report manipulation is a concern, the main problem to tackle is corruption. Countermeasures may attack the technological means of corruption through the prevention of long-term relationships between supervisor and agent, and by eliminating the means by which supervisor and agent can credibly provide each other benefits. When these interventions are not enough to deter report

manipulation, it may be that the supervisor acts, without any expectation of reciprocal reward from the agent.

In this case, the principal must take steps to combat signal manipulation with incentives for the supervisor. This can take the form of wage compression, relative performance evaluation, and supervisor compensation for reports that harm the agent.

Appendix A: Benchmark contracts

Note that all the benchmark contracts exist in Section 2.

A1: Single agent, principal as supervisor derivation

Here I derive the solution to the benchmark case where the principal is the supervisor and hires one agent.

First, note that *IC* and *LLC* jointly cover *IR*, so I will ignore *IR* when converting to the lagrangian.

$$\begin{aligned} L = & -pw_1 - (1-p)w_\emptyset \\ & +\phi_1(pw_1^{0.5} - \psi - pw_0^{0.5}) \\ & +\phi_2(w_1) \\ & +\phi_3(w_\emptyset) \\ & +\phi_4(w_0) \end{aligned}$$

Where ϕ_1 represents the shadow cost of IC and the other constraints represent limited liability on each agent transfer.

FOCs:

$$[w_1]: -p + \phi_1 0.5pw_1^{-0.5} + \phi_2 = 0$$

$$[w_\emptyset]: -(1-p) + \phi_3 = 0$$

$$[w_0]: -\phi_1 0.5pw_0^{-0.5} + \phi_4 = 0$$

If $\phi_1 = 0$, then $\phi_2 = p > 0$, and $w_1 = 0$. This violates *IC*. Hence, $\phi_1 > 0$ and $\phi_2 = 0$.

$$\phi_3 = 1 - p > 0$$

$$\phi_4 = \phi_1 0.5pw_0^{-0.5} > 0$$

$$w_0 = w_\emptyset = 0$$

By binding IC and $w_0 = 0$,

$$pw_1^{0.5} - \psi = 0$$

$$w_1^{0.5} = \frac{\psi}{p}$$

A2: External, altruistic supervisor derivation

This is the solution to the case where there is one agent and one altruistic supervisor.

Again note that IR is subsumed by IC and LLC. Additionally, I ignore certain LLC constraints and see that they hold in the optimal solution.

$$\begin{aligned} L = & -p(w_1 + t_1) - (1 - p)(w_\emptyset + t_1 + \gamma(w_1^{0.5} - w_\emptyset^{0.5})) \\ & + \phi_1(pw_1^{0.5} + (1 - p)w_\emptyset^{0.5} - \psi - pw_0^{0.5} - (1 - p)w_\emptyset^{0.5}) \\ & + \phi_2(w_\emptyset) \\ & + \phi_3(w_0) \\ & + \phi_4(t_1) \\ & + \phi_5(t_1 + \gamma(w_1^{0.5} - w_\emptyset^{0.5})) \end{aligned}$$

Where ϕ_1 is the shadow cost of IC, other constraints represent limited liability, and in particular ϕ_5 is the LLC of t_\emptyset , which is equal to $t_1 + \gamma(w_1^{0.5} - w_\emptyset^{0.5})$ by binding SIC constraints. Note that t_0 is not in the optimization problem because t_0 never occurs on the equilibrium path. Essentially, the principal can set $t_0 = \gamma w_1^{0.5}$, inducing the supervisor to report truthfully off the equilibrium path, without cost. This will apply to all future transfers with a truthful report of 0.

First order conditions:

$$[w_1]: -p - (1 - p)0.5\gamma w_1^{-0.5} + \phi_1 0.5p w_1^{-0.5} + \phi_5 0.5\gamma w_1^{-0.5} = 0$$

$$[w_\emptyset]: -(1 - p) + (1 - p)\gamma 0.5w_\emptyset^{-0.5} + \phi_2 - \phi_5 0.5\gamma w_\emptyset^{-0.5} = 0$$

$$[w_0]: -\phi_1 0.5p w_0^{-0.5} + \phi_3 = 0$$

$$[t_1]: -p - (1-p) + \phi_4 + \phi_5 = 0$$

Case 1, $\phi_5 = 0$

From $[w_1]$,

$$\phi_1 0.5 p w_1^{-0.5} = p + (1-p) 0.5 \gamma w_1^{-0.5} > 0.$$

From $[w_0]$,

$$\phi_3 = \phi_1 0.5 p w_0^{-0.5} > 0.$$

Because $\phi_3 > 0$ and represents LLC_{w_0} ,

$$w_0 = 0.$$

From $[t_1]$,

$$\phi_4 = 1 > 0.$$

Because $\phi_4 > 0$ and represents LLC_{t_1} ,

$$t_1 = 0.$$

From $[w_\emptyset]$,

$$0.5(1-p)\gamma w_\emptyset^{-0.5} = 1 - p - \phi_2.$$

Solving further,

$$w_\emptyset^{-0.5} = \frac{1 - p - \phi_2}{0.5(1-p)\gamma}$$

$$w_\emptyset^{0.5} = \frac{0.5(1-p)\gamma}{1 - p - \phi_2} \neq 0.$$

If $\phi_2 > 0$, then $w_\emptyset^{0.5} = 0$. This is impossible. Therefore, $\phi_2 = 0$. From this, we know

$$w_\emptyset^{0.5} = \frac{\gamma}{2}.$$

By binding IC and $w_0 = 0$,

$$pw_1^{0.5} - \psi = 0$$

$$w_1 = \left(\frac{\psi}{p}\right)^2$$

By binding $SIC_{\emptyset,1}$,

$$t_{\emptyset} = \gamma(w_1^{0.5} - w_{\emptyset}^{0.5}).$$

Plugging in values for w_1 and w_{\emptyset} ,

$$t_{\emptyset} = \gamma\left(\frac{\psi}{p} - \frac{\gamma}{2}\right).$$

Hence, $\phi_5 = 0$ if $\frac{\psi}{p} > \frac{\gamma}{2}$.

Finally, the value for t_0 can be determined by a binding $SIC_{0,1}$:

$$t_0 = \frac{\gamma\psi}{p}$$

Case 2: $\phi_5 > 0$

By binding $LLC_{t_{\emptyset}}$,

$$t_1 = \gamma(w_{\emptyset}^{0.5} - w_1^{0.5}).$$

By IC,

$$p(w_1^{0.5} - w_0^{0.5}) \geq \psi$$

If IC is slack, the principal can reduce w_1 until it is binding. Therefore, $\phi_1 > 0$.

From $[w_0]$,

$$\phi_3 = \phi_1 p > 0.$$

Because $\phi_3 > 0$ and represents LLC_{w_0} ,

$$w_0 = 0.$$

By $[w_1]$,

$$[\phi_1 - (1 - p - \phi_5)\gamma]w_1^{-0.5} = 2p$$

$$w_1^{0.5} = \frac{\phi_1 - (1 - p - \phi_5)\gamma}{2p}.$$

By $[w_\emptyset]$,

$$(1 - p - \phi_5)\gamma w_\emptyset^{-0.5} = 2(1 - p - \phi_2)$$

$$w_\emptyset^{0.5} = \frac{(1 - p - \phi_5)\gamma}{2(1 - p - \phi_2)}$$

Due to LLC_{w_\emptyset} , this means $\phi_5 \leq 1 - p$. Combined with $[t_1]$, this means $\phi_4 \geq p > 0$.

Therefore,

$$t_1 = 0,$$

Which brings us to the solution:

$$w_1 = w_\emptyset = \left(\frac{\psi}{p}\right)^2$$

$$t_1 = t_\emptyset = 0$$

A3: Proof of Lemma 1, derivation of the two-agent case with the principal as the supervisor

The two agents are labeled agents A and B. The wages are w_{i,σ_i,σ_j} , where i is agent type, σ_i is that agent's signal, and σ_j is the other agent's signal. The maximization problem is as follows:

$$\begin{aligned} \min_w \quad & p^2(w_{A,1,1} + w_{B,1,1}) + p(1 - p)(w_{A,1,\emptyset} + w_{B,\emptyset,1} + w_{A,\emptyset,1} + w_{B,1,\emptyset}) \\ & + (1 - p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset}) \quad s. t. \end{aligned}$$

$$IR_i: p^2 w_{i,1,1}^{0.5} + p(1 - p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1 - p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \geq 0$$

$$\begin{aligned} IC_i: \quad & p^2 w_{i,1,1}^{0.5} + p(1 - p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1 - p)^2 w_{i,\emptyset,\emptyset}^{0.5} - \psi \\ & \geq p(pw_{i,0,1}^{0.5} + (1 - p)w_{i,0,\emptyset}^{0.5}) + (1 - p)(pw_{i,\emptyset,1}^{0.5} + (1 - p)w_{i,\emptyset,\emptyset}^{0.5}) \end{aligned}$$

$$LLC_{i,\sigma_i,\sigma_j}: w_{i,\sigma_i,\sigma_j} \geq 0$$

Converting to a Lagrangian and ignoring constraints which will not be violated in the optimal solution:

$$\begin{aligned} L = & -p^2(w_{A,1,1} + w_{B,1,1}) - p(1-p)(w_{A,1,\emptyset} + w_{B,\emptyset,1} + w_{A,\emptyset,1} + w_{B,1,\emptyset}) \\ & - (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset}) \\ & + \phi_{1i} \left(p^2 w_{i,1,1}^{0.5} + p(1-p) w_{i,1,\emptyset}^{0.5} - \psi - p(p w_{i,0,1}^{0.5} + (1-p) w_{i,0,\emptyset}^{0.5}) \right) \\ & + \phi_{2i}(w_{i,0,1}) \\ & + \phi_{3i}(w_{i,0,\emptyset}) \\ & + \phi_{4i}(w_{i,\emptyset,1}) \\ & + \phi_{5i}(w_{i,\emptyset,\emptyset}) \end{aligned}$$

Where ϕ_{1i} represents both IC constraints for agents $i \in \{A, B\}$, and all other listed constraints represent limited liability in wages.

First order conditions:

$$[w_{i,1,1}]: -p^2 + \phi_{1i} 0.5 p^2 w_{i,1,1}^{-0.5} = 0$$

$$[w_{i,1,\emptyset}]: -p(1-p) + \phi_{1i} 0.5 p(1-p) w_{i,1,\emptyset}^{-0.5} = 0$$

$$[w_{i,0,1}]: -\phi_{1i} 0.5 p^2 w_{i,0,1}^{-0.5} + \phi_{2i} = 0$$

$$[w_{i,0,\emptyset}]: -\phi_{1i} 0.5 p(1-p) w_{i,0,\emptyset}^{-0.5} + \phi_{3i} = 0$$

$$[w_{i,\emptyset,1}]: -p(1-p) + \phi_{4i} = 0$$

$$[w_{i,\emptyset,\emptyset}]: -(1-p)^2 + \phi_{5i} = 0$$

Solving:

From $[w_{i,1,1}]$ and $[w_{i,1,\emptyset}]$,

$$w_{i,1,1}^{0.5} = w_{i,1,\emptyset}^{0.5} = \frac{\phi_{1i}}{2}.$$

From $[w_{i,0,1}]$,

$$\phi_{2i} = \phi_{1i} 0.5 p^2 w_{i,0,1}^{-0.5}.$$

From $[w_{i,0,\emptyset}]$,

$$\phi_{3i} = \phi_{1i} 0.5 p (1 - p) w_{i,0,\emptyset}^{-0.5}.$$

From $[w_{i,\emptyset,1}]$,

$$\phi_{4i} = p(1 - p) > 0.$$

Because $\phi_{4i} > 0$ and represents limited liability for $w_{i,\emptyset,1}$,

$$w_{i,\emptyset,1} = 0, i \in \{A, B\}.$$

From $[w_{i,\emptyset,\emptyset}]$,

$$\phi_{5i} = (1 - p)^2 > 0.$$

Because $\phi_{5i} > 0$ and represents limited liability for $w_{i,\emptyset,\emptyset}$,

$$w_{i,\emptyset,\emptyset} = 0, i \in \{A, B\}.$$

If $\phi_{1i} = 0$, then $w_{i,1,1} = w_{i,1,\emptyset} = 0$, which violates IC, because

$$-\psi < p(pw_{i,0,1}^{0.5} + (1 - p)w_{i,0,\emptyset}^{0.5}),$$

Noting that LLCs ensure the minimum of the right hand side is 0.

Therefore, $\phi_{1i} > 0$. This implies the following:

$$\phi_{2i} > 0$$

$$\phi_{3i} > 0$$

ϕ_{2i} and ϕ_{3i} represent limited liability for $w_{i,0,\emptyset}$ and $w_{i,0,1}$, so

$$w_{i,0,\emptyset} = w_{i,0,1} = 0.$$

By binding IC,

$$p^2 w_{i,1,1}^{0.5} + p(1-p)w_{i,1,\emptyset}^{0.5} - \psi = 0$$

Plugging in $w_{i,1,1}^{0.5} = w_{i,1,\emptyset}^{0.5} = \frac{\phi_{1i}}{2}$,

$$p \frac{\phi_{1i}}{2} = \psi$$

$$\frac{\phi_{1i}}{2} = \frac{\psi}{p}$$

$$w_{i,1,1}^{0.5} = w_{i,1,\emptyset}^{0.5} = \frac{\psi}{p}.$$

This is a replication of the single-agent case. The principal has no reason to link the transfers together except as a way to enforce proper reporting of the signal, which vanishes if the principal is the supervisor – or equivalently, if the supervisor isn't corruptible or altruistic.

Appendix B: Two agents, altruistic supervisor

B1: Derivation of the main model

Here I detail the derivation and solution to the main model with one altruistic supervisor and two agents, found in Section 4.

Converting the problem into a lagrangian:

$$\begin{aligned}
L = & -p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\
& - p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} \\
& + \gamma(2w_{A,1,1}^{0.5} + 2w_{B,1,1}^{0.5} - w_{A,1,\emptyset}^{0.5} - w_{B,\emptyset,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{B,1,\emptyset}^{0.5})) \\
& - (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5})) \\
& + \phi_{1A}(p^2w_{A,1,1}^{0.5} + p(1-p)(w_{A,1,\emptyset}^{0.5}) - \psi - p(pw_{A,0,1}^{0.5} + (1-p)w_{A,0,\emptyset}^{0.5})) \\
& + \phi_{1B}(p^2w_{B,1,1}^{0.5} + p(1-p)(w_{B,1,\emptyset}^{0.5}) - \psi - p(pw_{B,0,1}^{0.5} + (1-p)w_{B,0,\emptyset}^{0.5})) \\
& + \phi_{2A}(w_{A,0,1}) \\
& + \phi_{2B}(w_{B,0,1}) \\
& + \phi_{3A}(w_{A,0,\emptyset}) \\
& + \phi_{3B}(w_{B,0,\emptyset}) \\
& + \phi_4(t_{1,1}) \\
& + \phi_5(t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} - w_{A,1,\emptyset}^{0.5} - w_{B,\emptyset,1}^{0.5})) \\
& + \phi_6(t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} - w_{B,1,\emptyset}^{0.5} - w_{A,\emptyset,1}^{0.5})) \\
& + \phi_7(t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5}))
\end{aligned}$$

Where ϕ_{1A} and ϕ_{1B} are shadow costs for IC constraints of agents A and B respectively, ϕ_{2i} and ϕ_{3i} are shadow costs for limited liability constraints of the

listed wages, and ϕ_4 through ϕ_7 represent limited liability constraints on $t_{1,1}$, $t_{1,\emptyset}$, $t_{\emptyset,1}$, and $t_{\emptyset,\emptyset}$ respectively.

First Order Conditions:

$$[w_{i,1,1}]: -p^2 - p(1-p)\gamma w_{i,1,1}^{-0.5} - 0.5(1-p)^2\gamma w_{i,1,1}^{-0.5} + \phi_{1i}0.5p^2w_{i,1,1}^{-0.5} \\ + (\phi_5 + \phi_6 + \phi_7)0.5\gamma w_{i,1,1}^{-0.5} = 0$$

$$[w_{A,1,\emptyset}]: -p(1-p) + 0.5p(1-p)\gamma w_{A,1,\emptyset}^{-0.5} + \phi_{1A}0.5p(1-p)w_{A,1,\emptyset}^{-0.5} - \phi_50.5\gamma w_{A,1,\emptyset}^{-0.5} = 0$$

$$[w_{B,1,\emptyset}]: -p(1-p) + 0.5p(1-p)\gamma w_{B,1,\emptyset}^{-0.5} + \phi_{1B}0.5p(1-p)w_{B,1,\emptyset}^{-0.5} - \phi_60.5\gamma w_{B,1,\emptyset}^{-0.5} = 0$$

$$[w_{A,\emptyset,1}]: -p(1-p) + 0.5p(1-p)\gamma w_{A,\emptyset,1}^{-0.5} - \phi_6\gamma w_{A,\emptyset,1}^{-0.5} = 0$$

$$[w_{B,\emptyset,1}]: -p(1-p) + 0.5p(1-p)\gamma w_{B,\emptyset,1}^{-0.5} - \phi_5\gamma w_{B,\emptyset,1}^{-0.5} = 0$$

$$[w_{i,\emptyset,\emptyset}]: -(1-p)^2 + 0.5(1-p)^2\gamma w_{i,\emptyset,\emptyset}^{-0.5} - \phi_70.5\gamma w_{i,\emptyset,\emptyset}^{-0.5} = 0$$

$$[w_{i,0,1}]: -\phi_{1i}0.5p^2w_{i,0,1}^{-0.5} + \phi_{2i} = 0$$

$$[w_{i,0,\emptyset}]: -\phi_{1i}0.5p(1-p)w_{i,0,\emptyset}^{-0.5} + \phi_{3i} = 0$$

$$[t_{1,1}]: -1 + \phi_4 + (\phi_5 + \phi_6 + \phi_7)$$

Solving:

From $[w_{i,1,1}]$,

$$(4.1i): w_{i,1,1}^{-0.5} = \frac{2p^2}{\phi_{1i}p^2 + (\phi_5 + \phi_6 + \phi_7)\gamma - (1-p)(1+p)\gamma}$$

From $[w_{A,1,\emptyset}]$,

$$(4.2): w_{A,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1A}p(1-p) - \phi_5\gamma + p(1-p)\gamma}$$

From $[w_{B,1,\emptyset}]$,

$$(4.3): w_{B,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1B}p(1-p) - \phi_6\gamma + p(1-p)\gamma}$$

From $[w_{A,\emptyset,1}]$,

$$(4.4): w_{A,\emptyset,1}^{-0.5} = \frac{2p(1-p)}{p(1-p)\gamma - 2\phi_6\gamma}$$

From $[w_{B,\emptyset,1}]$,

$$(4.5): w_{B,\emptyset,1}^{-0.5} = \frac{2p(1-p)}{p(1-p)\gamma - 2\phi_5\gamma}$$

From $[w_{i,\emptyset,\emptyset}]$,

$$(4.6i): w_{i,\emptyset,\emptyset}^{-0.5} = \frac{2(1-p)^2}{(1-p)^2\gamma - 2\phi_5\gamma}$$

From $[w_{i,0,1}]$,

$$(4.7i): \phi_{1i}p^2w_{i,0,1}^{-0.5} = 2\phi_{2i}$$

From $[w_{i,0,\emptyset}]$,

$$(4.8i): \phi_{1i}p(1-p)w_{i,0,\emptyset}^{-0.5} = 2\phi_{3i}$$

From $[t_{1,1}]$,

$$(4.9): \phi_4 + (\phi_5 + \phi_6 + \phi_7) = 1$$

Case 1: $\phi_5 = \phi_6 = \phi_7 = 0$

Applying this case to equation (4.9),

$$(4.10) \phi_4 = 1$$

Because ϕ_4 represents limited liability on $t_{1,1}$,

$$(4.11) t_{1,1} = 0.$$

Applying this case and equation (4.10) to equation (4.1i):

$$(4.12i) w_{i,1,1}^{0.5} = \frac{\phi_{1i}}{2} - \frac{(1-p^2)\gamma}{2p^2}$$

Due to limited liability in $w_{i,1,1}$, the above implies that

$$\phi_{1i} > 0,$$

Which, when combined with equations (4.7i) and (4.8i), imply that

$$\phi_{2i} > 0$$

$$\phi_{3i} > 0.$$

Being limited liability constraints on $w_{i,0,1}$ and $w_{i,0,\emptyset}$ respectively, this generates the following equations:

$$(4.13i): w_{i,0,1} = 0$$

$$(4.14i): w_{i,0,\emptyset} = 0$$

Applying this case and equation (4.10) to equations (4.2) and (4.3):

$$(4.15i) w_{i,1,\emptyset}^{0.5} = \frac{\phi_{1i}}{2} + \frac{\gamma}{2}$$

Applying this case to equations (4.4), (4.5) and (4.6i):

$$(4.16i): w_{i,\emptyset,1}^{0.5} = \frac{\gamma}{2}$$

$$(4.17i): w_{i,\emptyset,\emptyset}^{0.5} = \frac{\gamma}{2}$$

Because we determined that IC_A and IC_B bind,

$$p^2 w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5}) = \psi.$$

Plugging in (4.12i) and (4.15i):

$$p^2 \left(\frac{\phi_{1i}}{2} - \frac{(1-p^2)\gamma}{2p^2} \right) + p(1-p) \left(\frac{\phi_{1i}}{2} + \frac{\gamma}{2} \right) = \psi$$

$$p\phi_{1i} - (1-p^2)\gamma + p(1-p)\gamma = 2\psi$$

$$(4.18i): \phi_{1i} = \frac{2\psi + (1-p)\gamma}{p}$$

Plugging (4.18i) back into (4.12i):

$$w_{i,1,1}^{0.5} = \frac{2\psi + (1-p)\gamma}{2p} - \frac{(1-p^2)\gamma}{2p^2}$$

$$(4.19i) \quad w_{i,1,1}^{0.5} = \frac{\psi}{p} - \frac{(1-p)\gamma}{2p^2}$$

Plugging (4.18i) back into (4.15i):

$$w_{i,1,\emptyset}^{0.5} = \frac{2\psi + (1-p)\gamma}{2p} + \frac{\gamma}{2}$$

$$(4.20i): w_{i,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{\gamma}{2p}$$

Plugging the optimal wages into binding SIC constraints:

$$t_{1,\emptyset} = t_{\emptyset,1} = \gamma \left(2 \frac{\psi}{p} - \frac{(1-p)\gamma}{p^2} - \frac{\psi}{p} - \frac{\gamma}{2p} - \frac{\gamma}{2} \right)$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \gamma \left(\frac{\psi}{p} - \frac{\gamma}{2p^2} (2(1-p) + p + p^2) \right)$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \gamma \left(\frac{\psi}{p} - \frac{\gamma}{2p^2} (p^2 - p + 2) \right)$$

$$(4.21): t_{1,\emptyset} = t_{\emptyset,1} = \frac{\gamma}{2p^2} (2\psi p - \gamma(p^2 - p + 2))$$

This case holds when $2\psi p > \gamma(p^2 - p + 2)$. To re-write,

$$\gamma < \frac{2\psi p}{p^2 - p + 2}$$

This is a parameter assumption in the model.

Continuing to plug wages into binding SIC constraints:

$$t_{\emptyset,\emptyset} = 2\gamma \left(\frac{\psi}{p} - \frac{(1-p)\gamma}{2p^2} - \gamma \right)$$

$$t_{\emptyset,\emptyset} = \frac{\gamma}{p^2} (2\psi p - (1-p)\gamma - 2p^2\gamma)$$

$$(4.22): t_{\emptyset,\emptyset} = \frac{\gamma}{p^2} (2\psi p - \gamma(2p^2 - p + 1))$$

This case holds when

$$\gamma < \frac{2\psi p}{2p^2 - p + 1}.$$

This is less restrictive than the previous condition on γ .

B2: Proof of Lemma 2

The principal's expected wages and transfers paid are as follows:

$$\begin{aligned} E(w + t) &= p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\ &\quad + p(1-p)(w_{A,1,\emptyset} + w_{B,\emptyset,1} + t_{1,\emptyset} + w_{A,\emptyset,1} + w_{B,1,\emptyset} + t_{\emptyset,1}) \\ &\quad + (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{\emptyset,\emptyset}) \\ E(w + t) &= p^2 \left(2 \left(\frac{\psi}{p} - \frac{(1-p)\gamma}{2p^2} \right)^2 \right) \\ &\quad + p(1-p) \left(2 \left(\frac{\psi}{p} + \frac{\gamma}{2p} \right)^2 + \frac{\gamma^2}{2} + \frac{\gamma}{2p^2} (2\psi p - \gamma(p^2 - p + 2)) \right) \\ &\quad + (1-p)^2 \left(\frac{\gamma^2}{2} + \frac{\gamma}{p^2} (2\psi p - \gamma(2p^2 - p + 1)) \right) \end{aligned}$$

Taking the comparative statics with respect to γ :

$$\begin{aligned} \frac{\partial E(w + t)}{\partial \gamma} &= -2(1-p) \left(\frac{\psi}{p} - \frac{(1-p)\gamma}{2p^2} \right) + 2(1-p) \left(\frac{\psi}{p} + \frac{\gamma}{2p} \right) + p(1-p)\gamma + (1-p)\psi \\ &\quad - \frac{1-p}{p} (p^2 - p + 2)\gamma + (1-p)^2\gamma + \frac{(1-p)^2}{p} \psi - \left(\frac{1-p}{p} \right)^2 (2p^2 - p + 1)\gamma \end{aligned}$$

$$\begin{aligned} \frac{\partial E(w+t)}{\partial \gamma} = (1-p) & \left[\frac{(1-p)\gamma}{p^2} - \frac{2\psi}{p} + \frac{2\psi}{p} + \frac{\gamma}{2p} + p\gamma + \psi - \frac{p^2-p+2}{p}\gamma + (1-p)\gamma \right. \\ & \left. + \frac{1-p}{p}\psi - \frac{1-p}{p^2}(2p^2-p+1)\gamma \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial E(w+t)}{\partial \gamma} = \frac{1-p}{p^2} & [(1-p)\gamma + \gamma p + p^3\gamma + p^2\psi - p(p^2-p+2)\gamma + (1-p)p^2\gamma \\ & + (1-p)p\psi - (1-p)(2p^2-p+1)\gamma] \end{aligned}$$

$$\frac{\partial E(w+t)}{\partial \gamma} = \frac{1-p}{p^2} [-\gamma p^2(1-p) + \psi p]$$

$$\frac{\partial E(w+t)}{\partial \gamma} = (1-p) \left[\frac{\psi}{p} - \gamma(1-p) \right] > 0$$

Therefore, expected transfers increase in γ .

Appendix C: Extensions

C1: Proof of Lemma 2

Re-writing the principal's problem with an altruistic supervisor and two agents:

$$\begin{aligned} \min_{w,t} p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) + p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + t_{1,\emptyset} + t_{\emptyset,1}) \\ + (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{\emptyset,\emptyset}) \end{aligned}$$

Subject to:

$$IR_i: p^2w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2w_{i,\emptyset,\emptyset}^{0.5} - \psi \geq 0, i \in \{A, B\}$$

$$\begin{aligned} IC_i: p^2w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2w_{i,\emptyset,\emptyset}^{0.5} - \psi \\ \geq p(pw_{i,0,1}^{0.5} + (1-p)w_{i,0,\emptyset}^{0.5}) + (1-p)(pw_{i,\emptyset,1}^{0.5} + (1-p)w_{i,\emptyset,\emptyset}^{0.5}), i \in \{A, B\} \end{aligned}$$

$$LLC_w: w \geq 0 \forall w$$

$$LLC_t: t \geq 0 \forall t$$

$$\begin{aligned} SIC: t_{0,0} + \gamma(w_{A,0,0}^{0.5} + w_{B,0,0}^{0.5}) = t_{1,1} + \gamma(w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5}) = t_{\emptyset,\emptyset} + \gamma(w_{A,\emptyset,\emptyset}^{0.5} + w_{B,\emptyset,\emptyset}^{0.5}) \\ = t_{1,0} + \gamma(w_{A,1,0}^{0.5} + w_{B,0,1}^{0.5}) = t_{0,1} + \gamma(w_{A,0,1}^{0.5} + w_{B,1,0}^{0.5}) \\ = t_{1,\emptyset} + \gamma(w_{A,1,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5}) = t_{\emptyset,1} + \gamma(w_{A,\emptyset,1}^{0.5} + w_{B,1,\emptyset}^{0.5}) \\ = t_{\emptyset,0} + \gamma(w_{A,\emptyset,0}^{0.5} + w_{B,0,\emptyset}^{0.5}) = t_{0,\emptyset} + \gamma(w_{A,0,\emptyset}^{0.5} + w_{B,\emptyset,0}^{0.5}) \end{aligned}$$

Setting $\gamma = 1$, the principal's problem is as follows:

$$\begin{aligned} \min_{w,t} p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) + p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + t_{1,\emptyset} + t_{\emptyset,1}) \\ + (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{\emptyset,\emptyset}) \end{aligned}$$

Subject to:

$$IR_i: p^2w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2w_{i,\emptyset,\emptyset}^{0.5} - \psi \geq 0, i \in \{A, B\}$$

$$\begin{aligned} IC_i: p^2w_{i,1,1}^{0.5} + p(1-p)(w_{i,1,\emptyset}^{0.5} + w_{i,\emptyset,1}^{0.5}) + (1-p)^2w_{i,\emptyset,\emptyset}^{0.5} - \psi \\ \geq p(pw_{i,0,1}^{0.5} + (1-p)w_{i,0,\emptyset}^{0.5}) + (1-p)(pw_{i,\emptyset,1}^{0.5} + (1-p)w_{i,\emptyset,\emptyset}^{0.5}), i \in \{A, B\} \end{aligned}$$

$$LLC_w: w \geq 0 \forall w$$

$$LLC_t: t \geq 0 \forall t$$

$$\begin{aligned} SIC: t_{1,1} + w_{A,1,1}^{0.5} + w_{B,1,1}^{0.5} &= t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} + w_{B,\emptyset,1}^{0.5} = t_{\emptyset,1} + w_{B,1,\emptyset}^{0.5} + w_{A,\emptyset,1}^{0.5} \\ &= t_{\emptyset,\emptyset} + w_{A,\emptyset,\emptyset}^{0.5} + w_{B,\emptyset,\emptyset}^{0.5} = t_{1,0} + w_{A,1,0}^{0.5} + w_{B,0,1}^{0.5} = t_{0,1} + w_{B,1,0}^{0.5} + w_{A,0,1}^{0.5} \\ &= t_{\emptyset,0} + w_{A,\emptyset,0}^{0.5} + w_{B,0,\emptyset}^{0.5} = t_{0,\emptyset} + w_{B,\emptyset,0}^{0.5} + w_{A,0,\emptyset}^{0.5} = t_{0,0} + w_{A,0,0}^{0.5} + w_{B,0,0}^{0.5} \end{aligned}$$

This is identical to the problem with a corrupt supervisor and multi-lateral bargaining.

C2: Bilateral CIC constraints

Writing out the explicit bilateral CIC constraints from Section 5.1.2:

$$CIC_{A1}: t_{1,1} + w_{A,1,1}^{0.5} = t_{\emptyset,1} + w_{A,\emptyset,1}^{0.5} = t_{0,1} + w_{A,0,1}^{0.5}$$

$$CIC_{A\emptyset}: t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} = t_{\emptyset,\emptyset} + w_{A,\emptyset,\emptyset}^{0.5} = t_{\emptyset,0} + w_{A,0,0}^{0.5}$$

$$CIC_{A0}: t_{1,0} + w_{A,1,0}^{0.5} = t_{\emptyset,0} + w_{A,\emptyset,0}^{0.5} = t_{0,0} + w_{A,0,0}^{0.5}$$

$$CIC_{B1}: t_{1,1} + w_{B,1,1}^{0.5} = t_{1,\emptyset} + w_{B,\emptyset,1}^{0.5} = t_{1,0} + w_{B,0,1}^{0.5}$$

$$CIC_{B\emptyset}: t_{\emptyset,1} + w_{B,1,\emptyset}^{0.5} = t_{\emptyset,\emptyset} + w_{B,\emptyset,\emptyset}^{0.5} = t_{\emptyset,0} + w_{B,0,0}^{0.5}$$

$$CIC_{B0}: t_{0,1} + w_{B,1,0}^{0.5} = t_{0,\emptyset} + w_{B,\emptyset,0}^{0.5} = t_{0,0} + w_{B,0,0}^{0.5}$$

Solving, focusing on on-equilibrium-path reports:

$$t_{1,1} + w_{A,1,1}^{0.5} = t_{\emptyset,1} + w_{A,\emptyset,1}^{0.5}$$

$$t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} = t_{\emptyset,\emptyset} + w_{A,\emptyset,\emptyset}^{0.5}$$

$$t_{1,1} + w_{B,1,1}^{0.5} = t_{1,\emptyset} + w_{B,\emptyset,1}^{0.5}$$

$$t_{\emptyset,1} + w_{B,1,\emptyset}^{0.5} = t_{\emptyset,\emptyset} + w_{B,\emptyset,\emptyset}^{0.5}$$

$$t_{\emptyset,1} = t_{1,1} + w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}$$

$$t_{\emptyset,\emptyset} = t_{1,\emptyset} + w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}$$

$$t_{1,\emptyset} = t_{1,1} + w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}$$

$$t_{\emptyset,\emptyset} = t_{1,1} + (w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5})$$

$$t_{\emptyset,1} = t_{\emptyset,\emptyset} + w_{B,\emptyset,\emptyset}^{0.5} - w_{B,1,\emptyset}^{0.5} = t_{1,1} + w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}$$

$$t_{1,1} + (w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}) + (w_{B,\emptyset,\emptyset}^{0.5} - w_{B,1,\emptyset}^{0.5}) = t_{1,1} + w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}$$

$$w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{A,1,\emptyset}^{0.5} + w_{A,\emptyset,\emptyset}^{0.5} - [w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5} - w_{B,1,\emptyset}^{0.5} + w_{B,\emptyset,\emptyset}^{0.5}] = 0$$

C3: Solution to the corruption problem with bilateral negotiation

This is the derivation of the solution to the problem detailed in Section 5.1.2, first converted to a lagrangian:

$$\begin{aligned} L = & -p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\ & - p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}) \\ & + (w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5})) \\ & - (1-p)^2(w_{A,\emptyset,\emptyset} \\ & + [w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{A,1,\emptyset}^{0.5} + w_{A,\emptyset,\emptyset}^{0.5} - w_{B,1,1}^{0.5} + w_{B,\emptyset,1}^{0.5} + w_{B,1,\emptyset}^{0.5}]^2 + t_{1,1} \\ & + (w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5})) \\ & + \phi_{1A}(p^2 w_{A,1,1}^{0.5} + p(1-p)w_{A,1,\emptyset}^{0.5} - \psi - p(pw_{A,0,1}^{0.5} + (1-p)w_{A,0,\emptyset}^{0.5})) \\ & + \phi_{1B}(p^2 w_{B,1,1}^{0.5} + p(1-p)w_{B,1,\emptyset}^{0.5} - \psi - p(pw_{B,0,1}^{0.5} + (1-p)w_{B,0,\emptyset}^{0.5})) \\ & + \phi_{2A}w_{A,0,1} \\ & + \phi_{2B}w_{B,0,1} \\ & + \phi_{3A}w_{A,0,\emptyset} \end{aligned}$$

$$\begin{aligned}
& +\phi_{3B}w_{B,0,\emptyset} \\
& +\phi_4t_{1,1} \\
& +\phi_5(t_{1,1} + w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}) \\
& +\phi_6(t_{1,1} + w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}) \\
& +\phi_7(t_{1,1} + (w_{A,1,\emptyset}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}))
\end{aligned}$$

First order conditions:

Writing $[w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5} - w_{A,1,\emptyset}^{0.5} + w_{A,\emptyset,\emptyset}^{0.5} - w_{B,1,1}^{0.5} + w_{B,\emptyset,1}^{0.5} + w_{B,1,\emptyset}^{0.5}]$ as K ,

$$[w_{A,1,1}]: -p^2 - 0.5p(1-p)w_{A,1,1}^{-0.5} + \phi_{1A}0.5p^2w_{A,1,1}^{-0.5} + \phi_50.5w_{A,1,1}^{-0.5} - (1-p)^2Kw_{A,1,1}^{-0.5} = 0$$

$$\begin{aligned}
[w_{B,1,1}]: & -p^2 - 0.5p(1-p)w_{B,1,1}^{-0.5} - 0.5(1-p)^2w_{B,1,1}^{-0.5} + \phi_{1B}0.5p^2w_{B,1,1}^{-0.5} + \phi_60.5w_{B,1,1}^{-0.5} \\
& + (1-p)^2Kw_{B,1,1}^{-0.5} = 0
\end{aligned}$$

$$\begin{aligned}
[w_{A,1,\emptyset}]: & -p(1-p) - 0.5(1-p)^2w_{A,1,\emptyset}^{-0.5} + \phi_{1A}0.5p(1-p)w_{A,1,\emptyset}^{-0.5} + \phi_70.5w_{A,1,\emptyset}^{-0.5} \\
& + (1-p)^2Kw_{A,1,\emptyset}^{-0.5} = 0
\end{aligned}$$

$$[w_{B,1,\emptyset}]: -p(1-p) + \phi_{1B}0.5p(1-p)w_{B,1,\emptyset}^{-0.5} - (1-p)^2Kw_{B,1,\emptyset}^{-0.5} = 0$$

$$[w_{A,\emptyset,1}]: -p(1-p) + 0.5p(1-p)w_{A,\emptyset,1}^{-0.5} - \phi_50.5w_{A,\emptyset,1}^{-0.5} + (1-p)^2Kw_{A,\emptyset,1}^{-0.5} = 0$$

$$\begin{aligned}
[w_{B,\emptyset,1}]: & -p(1-p) + 0.5p(1-p)w_{B,\emptyset,1}^{-0.5} + 0.5(1-p)^2w_{B,\emptyset,1}^{-0.5} - \phi_60.5w_{B,\emptyset,1}^{-0.5} - \phi_70.5w_{B,\emptyset,1}^{-0.5} \\
& - (1-p)^2Kw_{B,\emptyset,1}^{-0.5} = 0
\end{aligned}$$

$$[w_{A,\emptyset,\emptyset}]: -(1-p)^2 + 0.5(1-p)^2w_{A,\emptyset,\emptyset}^{-0.5} - \phi_70.5w_{A,\emptyset,\emptyset}^{-0.5} - (1-p)^2Kw_{A,\emptyset,\emptyset}^{-0.5} = 0$$

$$[w_{i,0,1}]: -\phi_{1i}0.5p^2w_{i,0,1}^{-0.5} + \phi_{2i} = 0$$

$$[w_{i,0,\emptyset}]: -\phi_{1i}0.5p(1-p)w_{i,0,\emptyset}^{-0.5} + \phi_{3i} = 0$$

$$[t_{1,1}]: -1 + \phi_4 + (\phi_5 + \phi_6 + \phi_7) = 0$$

Solving:

Case 1: $\phi_5 = \phi_6 = \phi_7 = 0$

$$\phi_4 = 1; t_{1,1} = 0$$

$$w_{A,1,1}^{-0.5} = \frac{2p^2}{\phi_{1A}p^2 - 2K(1-p)^2 - p(1-p)}$$

$$\phi_{1A} > 0$$

$$w_{B,1,1}^{-0.5} = \frac{2p^2}{\phi_{1B}p^2 + 2K(1-p)^2 - (1-p)}$$

$$w_{A,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1A}p(1-p) + 2K(1-p)^2 - (1-p)^2}$$

$$w_{B,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1B}p(1-p) - 2K(1-p)^2}$$

$$\phi_{1B} > 0$$

$$w_{A,\emptyset,1}^{-0.5} = \frac{2p(1-p)}{p(1-p) + 2K(1-p)^2}$$

$$w_{B,\emptyset,1}^{-0.5} = \frac{2p(1-p)}{(1-p) - 2K(1-p)^2}$$

$$w_{A,\emptyset,\emptyset}^{-0.5} = \frac{2(1-p)^2}{(1-p)^2 - 2K(1-p)^2}$$

$$\phi_{1i} 0.5 p^2 w_{i,0,1}^{-0.5} = \phi_{2i} > 0$$

$$w_{i,0,1} = 0$$

$$\phi_{1i} 0.5 p(1-p) w_{i,0,\emptyset}^{-0.5} = \phi_{3i} > 0$$

$$w_{i,0,\emptyset} = 0$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p}{2p^2} (p + 2K(1-p))$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p}{2p^2}(1-2K(1-p))$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p}{2p}(1-2K)$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p}{p}K$$

By binding IC_A

$$p^2 w_{A,1,1}^{0.5} + p(1-p)w_{A,1,\emptyset}^{0.5} = \psi$$

$$\phi_{1A}p - (1-p)(p + 2K(1-p)) - (1-p)^2(1-2K) = 2\psi$$

$$\phi_{1A}p - (1-p) = 2\psi$$

$$\phi_{1A} = \frac{2\psi + (1-p)}{p}$$

By binding IC_B

$$p^2 w_{B,1,1}^{0.5} + p(1-p)w_{B,1,\emptyset}^{0.5} = \psi$$

$$\phi_{1B}p - (1-p)(1-2K(1-p)) - (1-p)^2 2K = 2\psi$$

$$\phi_{1B} = \frac{2\psi + (1-p)}{p} = \phi_{1A}$$

$$w_{A,1,1}^{0.5} = \frac{2\psi + (1-p)}{2p} - \frac{1-p}{2p^2}(p + 2K(1-p))$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} + \frac{(1-p)p}{2p^2} - \frac{(1-p)p}{2p^2} - \frac{K(1-p)^2}{p^2}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} - K\left(\frac{1-p}{p}\right)^2$$

$$w_{B,1,1}^{0.5} = \frac{2\psi + (1-p)}{2p} - \frac{1-p}{2p^2}(1-2K(1-p))$$

$$w_{B,1,1}^{0.5} = \frac{\psi}{p} + \frac{p(1-p)}{2p^2} - \frac{1-p}{2p^2} + K \left(\frac{1-p}{p} \right)^2$$

$$w_{B,1,1}^{0.5} = \frac{\psi}{p} - \left(\frac{1-2K}{2} \right) \left(\frac{1-p}{p} \right)^2$$

$$w_{A,1,\emptyset}^{0.5} = \frac{2\psi + (1-p)}{2p} - \frac{1-p}{2p} (1-2K)$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{1-p}{2p} - \frac{1-p}{2p} + K \frac{1-p}{p}$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} + K \frac{1-p}{p}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{2\psi + (1-p)}{2p} - K \frac{1-p}{p}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{1-p}{p} \left(\frac{1}{2} - K \right)$$

$$w_{A,\emptyset,1}^{0.5} = \frac{1}{2} + K \frac{1-p}{p}$$

$$w_{B,\emptyset,1}^{0.5} = \frac{1}{2p} - K \frac{1-p}{p}$$

$$w_{A,\emptyset,\emptyset}^{0.5} = \frac{1}{2} - K$$

$$K = (w_{A,1,1}^{0.5} - w_{B,1,1}^{0.5}) + (w_{B,\emptyset,1}^{0.5} - w_{A,\emptyset,1}^{0.5}) + (w_{B,1,\emptyset}^{0.5} - w_{A,1,\emptyset}^{0.5}) + w_{A,\emptyset,\emptyset}^{0.5}$$

$$\begin{aligned} K &= \left(\frac{\psi}{p} - K \left(\frac{1-p}{p} \right)^2 - \frac{\psi}{p} + \left(\frac{1-2K}{2} \right) \left(\frac{1-p}{p} \right)^2 \right) + \left(\frac{1}{2p} - K \frac{1-p}{p} - \frac{1}{2} - K \frac{1-p}{p} \right) \\ &\quad + \left(\frac{\psi}{p} + K \frac{1-p}{p} - \frac{\psi}{p} - \frac{1-p}{p} \left(\frac{1}{2} - K \right) \right) + \frac{1}{2} - K \end{aligned}$$

$$K = \left(\frac{1-4K}{2}\right)\left(\frac{1-p}{p}\right)^2 + \left(\frac{1-4K}{2}\right)\left(\frac{1-p}{p}\right) - \left(\frac{1-4K}{2}\right)\left(\frac{1-p}{p}\right) + \frac{1}{2} - K$$

$$\left(\frac{1-4K}{2}\right)\left(\frac{p^2 - (1-p)^2}{p^2}\right) = 0$$

Assuming $p \neq \frac{1}{2}$, the only solution is $K = \frac{1}{4}$.

$$w_{i,1,1}^{0.5} = \frac{\psi}{p} - \left(\frac{1-p}{2p}\right)^2$$

$$w_{i,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{1-p}{4p}$$

$$w_{i,\emptyset,1}^{0.5} = \frac{1+p}{4p}$$

$$w_{i,\emptyset,\emptyset}^{0.5} = \frac{1}{4}$$

$$t_{1,1} = w_{i,0,\sigma_j} = 0$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{\psi}{p} - \left(\frac{1-p}{2p}\right)^2 - \frac{1+p}{4p} = \frac{\psi}{p} - \frac{1-p+2p^2}{4p^2}$$

$$t_{\emptyset,\emptyset} = \left(\frac{\psi}{p} + \frac{1-p}{4p} - \frac{1}{4}\right) + \frac{\psi}{p} - \left(\frac{1-p}{2p}\right)^2 - \frac{1+p}{4p}$$

$$t_{\emptyset,\emptyset} = \frac{2\psi}{p} - \frac{1-2p+4p^2}{4p^2}$$

C4: Proof of Proposition 3

The proof takes the following steps:

- i. I simplify and compare the costs in the bilateral solution with the multilateral solution.
- ii. The difference in bilateral and multilateral costs increases in p .
- iii. The difference in bilateral and multilateral costs is 0 when $p = 1$.

Step i:

Bi-lateral solution:

$$w_{i,1,1}^{0.5} = \frac{\psi}{p} - \left(\frac{1-p}{2p}\right)^2$$

$$w_{i,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{1-p}{4p}$$

$$w_{i,\emptyset,1}^{0.5} = \frac{1+p}{4p}$$

$$w_{i,\emptyset,\emptyset}^{0.5} = \frac{1}{4}$$

$$t_{1,1} = w_{i,0,\sigma_j} = 0$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{\psi}{p} - \frac{1-p+2p^2}{4p^2}$$

$$t_{\emptyset,\emptyset} = \frac{2\psi}{p} - \frac{1-2p+4p^2}{4p^2}$$

Multi-lateral solution:

$$w_{i,\emptyset,1}^{0.5} = w_{i,\emptyset,\emptyset}^{0.5} = \frac{1}{2}$$

$$w_{i,1,\emptyset}^{0.5} = \frac{2\psi+1}{2p} = \frac{\psi}{p} + \frac{1}{p}$$

$$w_{i,1,1}^{0.5} = \frac{2\psi p - (1-p)}{2p^2} = \frac{\psi}{p} - \frac{1-p}{2p^2}$$

$$t_{1,\emptyset} = t_{\emptyset,1} = \frac{1}{2p^2} (2\psi p - (p^2 - p + 2)) = \frac{\psi}{p} - \frac{p^2 - p + 2}{2p^2}$$

$$t_{\emptyset,\emptyset} = \frac{1}{p^2} (2\psi p - (1 - p + p^2)) = \frac{2\psi}{p} - \frac{1 - p + p^2}{p^2}$$

Bi-lateral cost:

$$\begin{aligned}
& p^2 \left(2 \left(\frac{\psi}{p} - \left(\frac{1-p}{2p} \right)^2 \right)^2 \right) + 2p(1-p) \left(\left(\frac{\psi}{p} + \frac{1-p}{4p} \right)^2 + \left(\frac{1+p}{4p} \right)^2 + \frac{\psi}{p} - \frac{1-p+2p^2}{4p^2} \right) \\
& + (1-p)^2 \left(\frac{2}{16} + \frac{2\psi}{p} - \frac{1-2p+4p^2}{4p^2} \right) \\
& \frac{4p^3 - 16p^2\psi - 5p^2 + 16p\psi^2 + 16p\psi + 2p - 1}{8p^2} \\
& \frac{2\psi}{p} (\psi + 1 - p) + \frac{4p^3 - 5p^2 + 2p - 1}{8p^2} = E(w + t|BL)
\end{aligned}$$

Multi-lateral profit:

$$\begin{aligned}
& p^2 \left(2 \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \right)^2 \right) + 2p(1-p) \left(\left(\frac{\psi}{p} + \frac{1}{p} \right)^2 + \frac{1}{4} + \frac{\psi}{p} - \frac{p^2 - p + 2}{2p^2} \right) \\
& \frac{p^4 - 4p^3\psi - 3p^3 + 3p^2 + 4p\psi^2 + 4p\psi - 2p + 1}{2p^2} \\
& \frac{2\psi}{p} (\psi + 1 - p^2) + \frac{p^4 - 3p^3 + 3p^2 - 2p + 1}{2p^2} = E(w + t|ML)
\end{aligned}$$

Step ii:

$$E(w + t|BL) - E(w + t|ML) = \Delta E(w + t) = \frac{-4p^4 + 16p^3 - 17p^2 + 10p - 5}{8p^2} - 2\psi(1-p)$$

$$\begin{aligned}
\frac{\partial}{\partial p} (\Delta E(w + t)) &= \frac{\partial}{\partial p} \left(\frac{-4p^4 + 16p^3 - 17p^2 + 10p - 5}{8p^2} - 2\psi(1-p) \right) \\
&= \frac{\partial}{\partial p} \left(-\frac{p^2}{2} + 2p - \frac{17}{8} + \frac{5}{4p} - \frac{5}{8p^2} - 2\psi(1-p) \right) \\
&= -p + 2 - \frac{5}{4p^2} + \frac{5}{4p^3} + 2\psi \\
&= \frac{5(1-p) - 4p^2 + 2p^3}{4p^3} + 2\psi > 0
\end{aligned}$$

Step iii:

If $p = 1$,

$$\Delta E(w + t) = \frac{-4 + 16 - 17 + 10 - 5}{8} - 0 = 0$$

Therefore, at any $p < 1$, $E(w + t|BL) < E(w + t|ML)$.

C5: Solution to the problem with favoritism

This is the derivation of the solution to the problem with altruism and favoritism detailed in Section 5.2. I will ignore unlisted constraints and see that they hold in the optimal solution.

$$\begin{aligned}
L = & -p^2(w_{A,1,1} + w_{B,1,1} + t_{1,1}) \\
& - p(1-p)(w_{A,1,\emptyset} + w_{B,1,\emptyset} + w_{A,\emptyset,1} + w_{B,\emptyset,1} + 2t_{1,1} + 2(\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5})) \\
& - (\gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,1,\emptyset}^{0.5}) - (\gamma_A w_{A,\emptyset,1}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5}) \\
& - (1-p)^2(w_{A,\emptyset,\emptyset} + w_{B,\emptyset,\emptyset} + t_{1,1} + (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5})) \\
& - (\gamma_A w_{A,\emptyset,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,\emptyset}^{0.5}) \\
& + \phi_{1A}(p^2 w_{A,1,1}^{0.5} + p(1-p)(w_{A,1,\emptyset}^{0.5}) - \psi - p(p w_{A,0,1}^{0.5} + (1-p)w_{A,0,\emptyset}^{0.5})) \\
& + \phi_{1B}(p^2 w_{B,1,1}^{0.5} + p(1-p)(w_{B,1,\emptyset}^{0.5}) - \psi - p(p w_{B,0,1}^{0.5} + (1-p)w_{B,0,\emptyset}^{0.5})) \\
& + \phi_{2A}(w_{A,0,1}) \\
& + \phi_{2B}(w_{B,0,1}) \\
& + \phi_{3A}(w_{A,0,\emptyset}) \\
& + \phi_{3B}(w_{B,0,\emptyset}) \\
& + \phi_4(t_{1,1}) \\
& + \phi_5(t_{1,1} + (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) - (\gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,1,\emptyset}^{0.5})) \\
& + \phi_6(w_{B,\emptyset,1})
\end{aligned}$$

FOCs:

$$[w_{A,1,1}]: -p^2 - p(1-p)\gamma_A w_{A,1,1}^{-0.5} - 0.5(1-p)^2 \gamma_A w_{A,1,1}^{-0.5} + \phi_{1A} 0.5p^2 w_{A,1,1}^{-0.5} + \phi_5 0.5\gamma_A w_{A,1,1}^{-0.5} = 0$$

$$[w_{B,1,1}]: -p^2 - p(1-p)\gamma_B w_{B,1,1}^{-0.5} - 0.5(1-p)^2 \gamma_B w_{B,1,1}^{-0.5} + \phi_{1B} 0.5p^2 w_{B,1,1}^{-0.5} + \phi_5 0.5\gamma_B w_{B,1,1}^{-0.5} = 0$$

$$[w_{A,1,\emptyset}]: -p(1-p) + 0.5p(1-p)\gamma_A w_{A,1,\emptyset}^{-0.5} + \phi_{1A} 0.5p(1-p)w_{A,1,\emptyset}^{-0.5} - \phi_5 0.5\gamma_A w_{A,1,\emptyset}^{-0.5} = 0$$

$$[w_{B,1,\emptyset}]: -p(1-p) + 0.5p(1-p)\gamma_B w_{B,1,\emptyset}^{-0.5} + \phi_{1B} 0.5p(1-p)w_{B,1,\emptyset}^{-0.5} = 0$$

$$[w_{A,\emptyset,1}]: -p(1-p) + 0.5p(1-p)\gamma_A w_{A,\emptyset,1}^{-0.5} = 0$$

$$[w_{B,\emptyset,1}]: -p(1-p) + 0.5p(1-p)\gamma_B w_{B,\emptyset,1}^{-0.5} - \phi_5 0.5\gamma w_{B,\emptyset,1}^{-0.5} + \phi_6 = 0$$

$$[w_{A,\emptyset,\emptyset}]: -(1-p)^2 + 0.5(1-p)^2 \gamma_A w_{A,\emptyset,\emptyset}^{-0.5} = 0$$

$$[w_{B,\emptyset,\emptyset}]: -(1-p)^2 + 0.5(1-p)^2 \gamma_A w_{B,\emptyset,\emptyset}^{-0.5} = 0$$

$$[w_{A,0,1}]: -\phi_{1A} 0.5p^2 w_{A,0,1}^{-0.5} + \phi_{2A} = 0$$

$$[w_{B,0,1}]: -\phi_{1B} 0.5p^2 w_{B,0,1}^{-0.5} + \phi_{2B} = 0$$

$$[w_{A,0,\emptyset}]: -\phi_{1A} 0.5p(1-p)w_{A,0,\emptyset}^{-0.5} + \phi_{3A} = 0$$

$$[w_{B,0,\emptyset}]: -\phi_{1B} 0.5p(1-p)w_{B,0,\emptyset}^{-0.5} + \phi_{3B} = 0$$

$$[t_{1,1}]: -1 + \phi_4 + \phi_5 = 0$$

Solving:

$$w_{A,1,1}^{-0.5} = \frac{2p^2}{\phi_{1A}p^2 - (1+p)(1-p)\gamma_A + \phi_5\gamma_A}$$

$$w_{B,1,1}^{-0.5} = \frac{2p^2}{\phi_{1B}p^2 - (1+p)(1-p)\gamma_B + \phi_5\gamma_B}$$

$$w_{A,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1A}p(1-p) + p(1-p)\gamma_A - \phi_5\gamma_A}$$

$$w_{B,1,\emptyset}^{-0.5} = \frac{2p(1-p)}{\phi_{1B}p(1-p) + p(1-p)\gamma_B}$$

$$w_{A,\emptyset,1}^{-0.5} = \frac{2p(1-p)}{p(1-p)\gamma_A}$$

$$w_{B,\emptyset,1}^{-0.5} = \frac{2p(1-p) - 2\phi_6}{p(1-p)\gamma_B - \phi_5\gamma_B}$$

$$w_{A,\emptyset,\emptyset}^{-0.5} = \frac{2(1-p)^2}{(1-p)^2\gamma_A}$$

$$w_{B,\emptyset,\emptyset}^{-0.5} = \frac{2(1-p)^2}{(1-p)^2\gamma_B}$$

$$\phi_{2A} = \phi_{1A}p^2w_{A,\emptyset,1}^{-0.5} > 0$$

$$\phi_{2B} = \phi_{1B}p^2w_{B,\emptyset,1}^{-0.5} > 0$$

$$\phi_{3A} = \phi_{1A}0.5p(1-p)w_{A,\emptyset,\emptyset}^{-0.5} > 0$$

$$\phi_{3B} = \phi_{1B}0.5p(1-p)w_{B,\emptyset,\emptyset}^{-0.5} > 0$$

$$\phi_4 + \phi_5 = 1$$

Case 1: $\phi_5 = 0$

$$\phi_4 = 1; t_{1,1} = 0$$

$$\phi_{1A} > 0; \phi_{1B} > 0$$

$$w_{A,0,1} = w_{B,0,1} = w_{A,0,\emptyset} = w_{B,0,\emptyset} = 0$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2}\gamma_A$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2} \gamma_B$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \frac{1}{2} \gamma_A$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\phi_{1B}}{2} + \frac{1}{2} \gamma_B$$

By binding IC_A ,

$$p^2 w_{A,1,1}^{0.5} + p(1-p)(w_{A,1,\emptyset}^{0.5}) = \psi$$

$$\phi_{1A} p - ((1-p^2) - p(1-p)) \gamma_A = 2\psi$$

$$\phi_{1A} = \frac{2\psi + (1-p)\gamma_A}{p}$$

By binding IC_B ,

$$\phi_{1B} = \frac{2\psi + (1-p)\gamma_B}{p}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} + \left(\frac{p(1-p) - (1-p^2)}{2p^2} \right) \gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_A$$

$$w_{B,1,1}^{0.5} = \frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_B$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} + \left(\frac{1}{2} + \frac{1-p}{2p} \right) \gamma_A$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} + \frac{\gamma_A}{2p} = \frac{2\psi + \gamma_A}{2p}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{2\psi + \gamma_B}{2p}$$

$$w_{A,\emptyset,1}^{0.5} = w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,1}^{0.5} = \frac{p(1-p)\gamma_B}{2p(1-p) - 2\phi_6} \neq 0$$

$$\phi_6 = 0$$

$$w_{B,\emptyset,1}^{0.5} = w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

$$\begin{aligned} t_{1,\emptyset} &= (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) - (\gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5}) \\ &= \gamma_A (w_{A,1,1}^{0.5} - w_{A,1,\emptyset}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5}) \end{aligned}$$

$$t_{1,\emptyset} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_A - \frac{\psi}{p} - \frac{\gamma_A}{2p} \right) + \gamma_B \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_B - \frac{\gamma_B}{2} \right)$$

$$t_{1,\emptyset} = \gamma_B \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_B \right) - \frac{\gamma_A^2}{2p^2}$$

$$t_{\emptyset,1} = (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) - (\gamma_A w_{A,\emptyset,1}^{0.5} + \gamma_B w_{B,1,\emptyset}^{0.5})$$

$$t_{\emptyset,1} = \gamma_A (w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,1,\emptyset}^{0.5})$$

$$t_{\emptyset,1} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_A - \frac{\gamma_A}{2} \right) + \gamma_B \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_B - \frac{\psi}{p} - \frac{\gamma_B}{2p} \right)$$

$$t_{\emptyset,1} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_A \right) - \frac{\gamma_B^2}{2p^2}$$

$$t_{\emptyset,\emptyset} = (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) - (\gamma_A w_{A,\emptyset,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,\emptyset}^{0.5})$$

$$t_{\emptyset,\emptyset} = \gamma_A (w_{A,1,1}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5})$$

$$t_{\emptyset,\emptyset} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_A - \frac{\gamma_A}{2} \right) + \gamma_B \left(\frac{\psi}{p} - \frac{1-p}{2p^2} \gamma_B - \frac{\gamma_B}{2} \right)$$

$$t_{\emptyset,\emptyset} = \gamma_A \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_A \right) + \gamma_B \left(\frac{\psi}{p} - \frac{1-p+p^2}{2p^2} \gamma_B \right)$$

By parameter assumptions, this is positive.

Case 2: $\phi_5 > 0, \phi_4 > 0$ (Note: This occurs when there is a significant difference between γ_A and γ_B , especially when γ_B approaches 0 and γ_A remains positive)

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1 - p^2 - \phi_5}{2p^2} \gamma_A$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1 - p^2 - \phi_5}{2p^2} \gamma_B$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\phi_{1B} + \gamma_B}{2}$$

$$w_{A,\emptyset,1}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,1}^{0.5} = \frac{p(1-p)\gamma_B - \phi_5\gamma_B}{2p(1-p) - 2\phi_6}$$

$$w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

If $\phi_{1A} = 0$, then

$$w_{A,1,1}^{0.5} = \frac{\phi_5 - 1 + p^2}{2p^2} \gamma_A \leq \frac{p(1-p) - 1 + p^2}{2p^2} \gamma_A = -\frac{1-p}{2p^2} \gamma_A < 0$$

Therefore,

$$\phi_{1A} > 0$$

If $\phi_{1B} = 0$, then

$$w_{B,1,1}^{0.5} = \frac{\phi_5 - 1 + p^2}{2p^2} \gamma_B \leq \frac{p(1-p) - 1 + p^2}{2p^2} \gamma_B < 0$$

Therefore,

$$\phi_{1B} > 0$$

Meaning

$$w_{A,0,\sigma_B} = w_{B,0,\sigma_A} = 0 \quad \forall \sigma_A, \sigma_B$$

$$t_{1,\emptyset} = t_{1,1} + (\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5}) - (\gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5}) = 0$$

$$\gamma_A w_{A,1,1}^{0.5} + \gamma_B w_{B,1,1}^{0.5} = \gamma_A w_{A,1,\emptyset}^{0.5} + \gamma_B w_{B,\emptyset,1}^{0.5}$$

If $\phi_6 = 0$,

$$\begin{aligned} & \gamma_A \left(\frac{\phi_{1A}}{2} - \frac{1-p^2-\phi_5}{2p^2} \gamma_A \right) + \gamma_B \left(\frac{\phi_{1B}}{2} - \frac{1-p^2-\phi_5}{2p^2} \gamma_B \right) \\ &= \gamma_A \left(\frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A \right) + \gamma_B \left(\frac{\phi_{1B}}{2} - \frac{\gamma_B \phi_5}{2p(1-p)} \right) \end{aligned}$$

$$\begin{aligned} & \phi_5 [\gamma_A^2(1-p) + \gamma_B^2(1-p) + \gamma_A^2 p + \gamma_B^2 p] \\ &= \gamma_A^2(1-p^2)(1-p) + \gamma_B^2(1-p^2)(1-p) + \gamma_A^2 p^2(1-p) + \gamma_B^2 p^2(1-p) \\ & \quad - \gamma_B p^2(1-p) \phi_{1B} \end{aligned}$$

$$\phi_5 [\gamma_A^2 + \gamma_B^2] = [\gamma_A^2 + \gamma_B^2](1-p) - \gamma_B p^2(1-p) \phi_{1B}$$

$$\phi_5 = 1-p - \frac{\gamma_B}{\gamma_A^2 + \gamma_B^2} p^2(1-p) \phi_{1B} = (1-p) \left(1 - \frac{\gamma_B}{\gamma_A^2 + \gamma_B^2} p^2 \phi_{1B} \right)$$

From binding IC_A and IC_B :

$$p^2 w_{A,1,1}^{0.5} + p(1-p) w_{A,1,\emptyset}^{0.5} = \psi$$

$$p^2 w_{B,1,1}^{0.5} + p(1-p) w_{B,1,\emptyset}^{0.5} = \psi$$

$$\phi_{1A} p - (1-p^2-\phi_5-p(1-p)+\phi_5) \gamma_A = 2\psi$$

$$\phi_{1A} = \frac{2\psi + (1-p)\gamma_A}{p}$$

$$\phi_{1B} = \frac{2\psi + (1-p)\gamma_B}{p}$$

Solving for ϕ_5 :

$$\phi_5 = 1-p - \frac{\gamma_B}{\gamma_A^2 + \gamma_B^2} p^2(1-p) \frac{2\psi + (1-p)\gamma_B}{p} = 1-p - \frac{\gamma_B p(1-p)}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B)$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2-\phi_5}{2p^2}\gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2}\gamma_A + \frac{\phi_5}{2p^2}\gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2-1+p}{2p^2}\gamma_A + \frac{\gamma_A\gamma_B}{2(\gamma_A^2+\gamma_B^2)}(1-p)\phi_{1B}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} + \frac{(1-p)p\gamma_A}{2p^2} - \frac{p(1-p)}{2p^2}\gamma_A + \frac{\gamma_A\gamma_B}{2(\gamma_A^2+\gamma_B^2)}(1-p)\frac{2\psi+(1-p)\gamma_B}{p}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} \left(1 + \frac{\gamma_A\gamma_B(1-p)}{\gamma_A^2+\gamma_B^2} \right) + \frac{p(1-p)\gamma_A}{2p^2} \left(\frac{\gamma_B^2(1-p)}{\gamma_A^2+\gamma_B^2} - 1 \right)$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} \left(1 + \frac{\gamma_A\gamma_B(1-p)}{\gamma_A^2+\gamma_B^2} \right) - \frac{1-p}{2p}\gamma_A \left(\frac{\gamma_A^2-p\gamma_B^2}{\gamma_A^2+\gamma_B^2} \right)$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2}\gamma_B + \frac{\phi_5}{2p^2}\gamma_B$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2}\gamma_B + \frac{\gamma_B}{2p^2} \left(1-p - \frac{\gamma_B}{\gamma_A^2+\gamma_B^2}p^2(1-p)\phi_{1B} \right)$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} \left(1 - \frac{\gamma_B^2(1-p)}{\gamma_A^2+\gamma_B^2} \right) - \frac{\gamma_B}{2p^2}(1-p^2-1+p)$$

$$w_{B,1,1}^{0.5} = \frac{2\psi+(1-p)\gamma_B}{p} \left(1 - \frac{\gamma_B^2(1-p)}{\gamma_A^2+\gamma_B^2} \right) - \frac{\gamma_B}{2p^2}p(1-p)$$

$$w_{B,1,1}^{0.5} = \frac{2\psi}{p} \left(1 - \frac{\gamma_B^2(1-p)}{\gamma_A^2+\gamma_B^2} \right) + \frac{(1-p)\gamma_B}{2p} \left(1 - 2\frac{\gamma_B^2(1-p)}{\gamma_A^2+\gamma_B^2} \right)$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{1 - \frac{\gamma_B}{\gamma_A^2+\gamma_B^2}p^2\phi_{1B}}{2p} \right) \gamma_A$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p}{2p}\gamma_A + \frac{\gamma_A\gamma_B}{\gamma_A^2+\gamma_B^2}p\phi_{1B}$$

$$w_{A,1,\emptyset}^{0.5} = \frac{2\psi + (1-p)\gamma_A}{2p} - \frac{1-p}{2p}\gamma_A + \frac{\gamma_A\gamma_B}{\gamma_A^2 + \gamma_B^2}p \frac{2\psi + (1-p)\gamma_B}{p}$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} \left(1 + 2p \frac{\gamma_A\gamma_B}{\gamma_A^2 + \gamma_B^2} \right) + \frac{\gamma_A\gamma_B}{\gamma_A^2 + \gamma_B^2} (1-p)\gamma_B$$

$$w_{B,1,\emptyset}^{0.5} = \frac{2\psi + (1-p)\gamma_B}{2p} + \frac{\gamma_B}{2}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{2\psi + \gamma_B}{2p}$$

$$w_{A,\emptyset,1}^{0.5} = w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

$$w_{B,\emptyset,1}^{0.5} = \frac{p(1-p)\gamma_B - \phi_5\gamma_B}{2p(1-p)} = \gamma_B \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right)$$

$$w_{B,\emptyset,1}^{0.5} = \gamma_B \left(\frac{1}{2} - \frac{\left(1 - p - \frac{\gamma_B p(1-p)}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B) \right)}{2p(1-p)} \right)$$

$$w_{B,\emptyset,1}^{0.5} = \frac{\gamma_B}{2p} (p - 1 + \gamma_B)$$

$$w_{B,\emptyset,1}^{0.5} = \frac{\gamma_B}{2} - \frac{\gamma_B \left(1 - \frac{\gamma_B p}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B) \right)}{2p}$$

$$w_{B,\emptyset,1}^{0.5} = \frac{\gamma_B}{2p} \left(p - 1 + \frac{\gamma_B p}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B) \right)$$

$$w_{B,\emptyset,1}^{0.5} = \frac{\gamma_B}{2p} \left(\frac{\gamma_B}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B) - (1-p) \right)$$

Meaning $\phi_6 = 0$ if

$$\frac{\gamma_B}{\gamma_A^2 + \gamma_B^2} (2\psi + (1-p)\gamma_B) > 1-p$$

If $\phi_6 > 0$,

$$w_{B,\emptyset,1} = 0$$

$$\gamma_A \left(\frac{\phi_{1A}}{2} - \frac{1-p^2 - \phi_5}{2p^2} \gamma_A \right) + \gamma_B \left(\frac{\phi_{1B}}{2} - \frac{1-p^2 - \phi_5}{2p^2} \gamma_B \right) = \gamma_A \left(\frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A \right)$$

$$\frac{\phi_5}{2p^2(1-p)} ((\gamma_A^2 + \gamma_B^2)(1-p) + p\gamma_A^2) = (\gamma_A^2 + \gamma_B^2) \frac{1-p^2}{2p^2} - \gamma_B \frac{\phi_{1B}}{2} + \frac{\gamma_A^2}{2}$$

$$\frac{\phi_5}{2p^2(1-p)} (\gamma_A^2 + \gamma_B^2(1-p)) = \frac{\gamma_A^2 + \gamma_B^2(1-p^2)}{2p^2} - \gamma_B \frac{\phi_{1B}}{2}$$

$$\phi_5 = \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B})}{\gamma_A^2 + \gamma_B^2(1-p)}$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2} \gamma_A + \frac{\phi_5 \gamma_A}{2p^2}$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2} \gamma_A + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B})}{2p^2(\gamma_A^2 + \gamma_B^2(1-p))} \gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B}) - (1-p^2)(\gamma_A^2 + \gamma_B^2(1-p))}{2p^2(\gamma_A^2 + \gamma_B^2(1-p))} \gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{\gamma_B p^2(1-p)\phi_{1B} + \gamma_A^2 p(1-p)}{2p^2(\gamma_A^2 + \gamma_B^2(1-p))} \gamma_A$$

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{\gamma_A^3(1-p)}{2p(\gamma_A^2 + \gamma_B^2(1-p))} - \frac{\gamma_A \gamma_B(1-p)}{2(\gamma_A^2 + \gamma_B^2(1-p))} \phi_{1B}$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2} \gamma_B + \frac{\phi_5}{2p^2} \gamma_B$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2} \gamma_B + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B})}{2p^2(\gamma_A^2 + \gamma_B^2(1-p))} \gamma_B$$

$$w_{B,1,1}^{0.5} = \frac{\phi_{1B}}{2} - \frac{\gamma_B p^2 (1-p) \phi_{1B} + \gamma_A^2 p (1-p)}{2p^2 (\gamma_A^2 + \gamma_B^2 (1-p))} \gamma_B$$

$$w_{B,1,1}^{0.5} = \frac{\gamma_A^2 \phi_{1B}}{2(\gamma_A^2 + \gamma_B^2 (1-p))} - \frac{\gamma_A^2 \gamma_B (1-p)}{2p(\gamma_A^2 + \gamma_B^2 (1-p))}$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A$$

$$= \frac{\phi_{1A}}{2} + \frac{\gamma_A}{2} - \frac{(1-p)(\gamma_A^2 + \gamma_B^2 (1-p^2)) - \gamma_B p^2 \phi_{1B}}{\gamma_A^2 + \gamma_B^2 (1-p)} \frac{\gamma_A}{2p(1-p)}$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \frac{p(\gamma_A^2 + \gamma_B^2 (1-p)) - \gamma_A^2 - \gamma_B^2 (1-p^2) + \gamma_B p^2 \phi_{1B}}{2p(\gamma_A^2 + \gamma_B^2 (1-p))} \gamma_A$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\phi_{1A}}{2} + \frac{p\gamma_A \gamma_B \phi_{1B}}{2(\gamma_A^2 + \gamma_B^2 (1-p))} - \frac{(\gamma_A^2 + \gamma_B^2)(1-p)\gamma_A}{2p(\gamma_A^2 + \gamma_B^2 (1-p))}$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\phi_{1B} + \gamma_B}{2}$$

$$w_{A,\emptyset,1}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

By binding IC_A :

$$p^2 w_{A,1,1}^{0.5} + p(1-p) w_{A,1,\emptyset}^{0.5} = \psi$$

$$p^2 \left(\frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2} \gamma_A + \frac{\phi_5 \gamma_A}{2p^2} \right) + p(1-p) \left(\frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A \right) = \psi$$

$$p\phi_{1A} - (1-p^2 - p(1-p))\gamma_A + (0)\phi_5 \gamma_A = 2\psi$$

$$p\phi_{1A} - (1-p)\gamma_A = 2\psi$$

$$\phi_{1A} = \frac{2\psi + (1-p)\gamma_A}{p}$$

By binding IC_B :

$$p^2 \left(\frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2} \gamma_B + \frac{\phi_5}{2p^2} \gamma_B \right) + p(1-p) \left(\frac{\phi_{1B} + \gamma_B}{2} \right) = \psi$$

$$p\phi_{1B} - (1-p^2 - p(1-p))\gamma_B + \phi_5\gamma_B = 2\psi$$

$$p\phi_{1B} - (1-p)\gamma_B + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B})}{\gamma_A^2 + \gamma_B^2(1-p)} \gamma_B = 2\psi$$

$$\left(\frac{p(\gamma_A^2 + \gamma_B^2(1-p)) - \gamma_B^2(1-p)p^2}{\gamma_A^2 + \gamma_B^2(1-p)} \right) \phi_{1B} + \frac{(1-p)\gamma_B}{\gamma_A^2 + \gamma_B^2(1-p)} (\gamma_A^2 + \gamma_B(1-p^2) - \gamma_A^2 - \gamma_B(1-p)) = 2\psi$$

$$\phi_{1B} = \frac{2\psi(\gamma_A^2 + \gamma_B^2(1-p)) - \gamma_B^2 p(1-p)^2}{p(\gamma_A^2 + \gamma_B^2(1-p)) - \gamma_B^2(1-p)p^2}$$

$$\phi_{1B} = \frac{2\psi(\gamma_A^2 + \gamma_B^2(1-p)) - \gamma_B^2 p(1-p)^2}{p\gamma_A^2 + \gamma_B^2 p(1-p)^2}$$

When $\gamma_B = 0$, $\phi_{1B} = \frac{2\psi}{p}$

Solving for wages:

$$w_{A,1,1}^{0.5} = \frac{\phi_{1A}}{2} - \frac{1-p^2}{2p^2} \gamma_A + \frac{\phi_5 \gamma_A}{2p^2}$$

$$w_{A,1,1}^{0.5} = \frac{2\psi + (1-p)\gamma_A}{2p} - \frac{1-p^2}{2p^2} \gamma_A + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_B p^2 \phi_{1B})}{\gamma_A^2 + \gamma_B^2(1-p)} \frac{\gamma_A}{2p^2}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} + \frac{(1-p)p - 1 + p^2}{2p^2} \gamma_A + \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2))}{(\gamma_A^2 + \gamma_B^2(1-p))2p^2} \gamma_A - \frac{\gamma_A \gamma_B (1-p) \phi_{1B}}{2(\gamma_A^2 + \gamma_B^2(1-p))}$$

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} - \frac{(1-p)(\gamma_A^2 + \gamma_B^2(1-p^2) - \gamma_A^2 - \gamma_B^2(1-p))}{(\gamma_A^2 + \gamma_B^2(1-p))2p^2} \gamma_A - \frac{\gamma_A \gamma_B (1-p)}{2(\gamma_A^2 + \gamma_B^2(1-p))} - \frac{2\psi(\gamma_A^2 + \gamma_B^2(1-p)) - \gamma_B^2 p(1-p)^2}{p\gamma_A^2 + \gamma_B^2 p(1-p)^2}$$

$$\begin{aligned}
w_{A,1,1}^{0.5} &= \frac{\psi}{p} \left(1 - \frac{\gamma_A \gamma_B (1-p)}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \right) - \frac{\gamma_A \gamma_B^2 (1-p)^2}{(\gamma_A^2 + \gamma_B^2 (1-p)) 2p} \\
&\quad + \frac{\gamma_A \gamma_B (1-p)}{2p(\gamma_A^2 + \gamma_B^2 (1-p))} \frac{\gamma_B^2 p (1-p)^2}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \\
w_{A,1,1}^{0.5} &= \frac{\psi}{p} \left(1 - \frac{\gamma_A \gamma_B (1-p)}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \right) + \frac{\gamma_A \gamma_B^2 (1-p)^2}{(\gamma_A^2 + \gamma_B^2 (1-p)) 2p} \left(\frac{\gamma_B p (1-p) - \gamma_A^2 - \gamma_B^2 (1-p)^2}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \right) \\
w_{A,1,\emptyset}^{0.5} &= \frac{\phi_{1A}}{2} + \left(\frac{1}{2} - \frac{\phi_5}{2p(1-p)} \right) \gamma_A = \frac{\phi_{1A}}{2} + \frac{\gamma_A}{2} - \frac{(\gamma_A^2 + \gamma_B^2 (1-p^2) - \gamma_B p^2 \phi_{1B}) \gamma_A}{\gamma_A^2 + \gamma_B^2 (1-p)} \frac{\gamma_A}{2p} \\
w_{A,1,\emptyset}^{0.5} &= \frac{\psi}{p} + \frac{\gamma_A \gamma_B}{(\gamma_A^2 + \gamma_B^2 (1-p)) 2p} \phi_{1B} \\
w_{A,1,\emptyset}^{0.5} &= \frac{\psi}{p} + \frac{\gamma_A \gamma_B}{(\gamma_A^2 + \gamma_B^2 (1-p)) 2p} \frac{2\psi(\gamma_A^2 + \gamma_B^2 (1-p)) - \gamma_B^2 p (1-p)^2}{p\gamma_A^2 + \gamma_B^2 p (1-p)^2} \\
w_{A,1,\emptyset}^{0.5} &= \frac{\psi}{p} \left(1 + \frac{\gamma_A \gamma_B}{p\gamma_A^2 + \gamma_B^2 p (1-p)^2} \right) - \frac{\gamma_A \gamma_B^3}{2p(\gamma_A^2 + \gamma_B^2 (1-p))(p\gamma_A^2 + \gamma_B^2 p (1-p)^2)} \\
w_{B,1,1}^{0.5} &= \frac{\phi_{1B}}{2} - \frac{1-p^2}{2p^2} \gamma_B + \frac{(1-p)(\gamma_A^2 + \gamma_B^2 (1-p^2) - \gamma_B p^2 \phi_{1B}) \gamma_B}{\gamma_A^2 + \gamma_B^2 (1-p)} \frac{\gamma_B}{2p^2} \\
w_{B,1,1}^{0.5} &= \frac{\gamma_A^2 - p\gamma_B^2}{2(\gamma_A^2 + \gamma_B^2 (1-p))} \phi_{1B} - \frac{1-p}{2p} \gamma_B \\
w_{B,1,1}^{0.5} &= \frac{\gamma_A^2 - p\gamma_B^2}{2(\gamma_A^2 + \gamma_B^2 (1-p))} \frac{2\psi(\gamma_A^2 + \gamma_B^2 (1-p)) - \gamma_B^2 p (1-p)^2}{p\gamma_A^2 + \gamma_B^2 p (1-p)^2} - \frac{1-p}{2p} \gamma_B \\
w_{B,1,1}^{0.5} &= \frac{\psi}{p} \frac{\gamma_A^2 - p\gamma_B^2}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \\
&\quad - \frac{(\gamma_A^2 - p\gamma_B^2) \gamma_B (1-p)^2 + (1-p)(\gamma_A^2 + \gamma_B^2 (1-p))(\gamma_A^2 + \gamma_B^2 (1-p)^2)}{2p(\gamma_A^2 + \gamma_B^2 (1-p))(\gamma_A^2 + \gamma_B^2 (1-p)^2)} \gamma_B \\
w_{B,1,\emptyset}^{0.5} &= \frac{\phi_{1B} + \gamma_B}{2} = \frac{2\psi(\gamma_A^2 + \gamma_B^2 (1-p)) - \gamma_B^2 p (1-p)^2}{2(p\gamma_A^2 + \gamma_B^2 p (1-p)^2)} + \frac{\gamma_B}{2}
\end{aligned}$$

Full solution:

$$w_{A,1,1}^{0.5} = \frac{\psi}{p} \left(1 - \frac{\gamma_A \gamma_B (1-p)}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \right) + \frac{\gamma_A \gamma_B^2 (1-p)^2}{(\gamma_A^2 + \gamma_B^2 (1-p)) 2p} \left(\frac{\gamma_B p (1-p) - \gamma_A^2 - \gamma_B^2 (1-p)^2}{\gamma_A^2 + \gamma_B^2 (1-p)^2} \right)$$

$$w_{A,1,\emptyset}^{0.5} = \frac{\psi}{p} \left(1 + \frac{\gamma_A \gamma_B}{p \gamma_A^2 + \gamma_B^2 p (1-p)^2} \right) - \frac{\gamma_A \gamma_B^3}{2p(\gamma_A^2 + \gamma_B^2 (1-p))(p \gamma_A^2 + \gamma_B^2 p (1-p)^2)}$$

$$w_{B,1,1}^{0.5} = \frac{\psi}{p} \frac{\gamma_A^2 - p \gamma_B^2}{\gamma_A^2 + \gamma_B^2 (1-p)^2} - \frac{(\gamma_A^2 - p \gamma_B^2) \gamma_B (1-p)^2 + (1-p)(\gamma_A^2 + \gamma_B^2 (1-p))(\gamma_A^2 + \gamma_B^2 (1-p)^2)}{2p(\gamma_A^2 + \gamma_B^2 (1-p))(\gamma_A^2 + \gamma_B^2 (1-p)^2)} \gamma_B$$

$$w_{B,1,\emptyset}^{0.5} = \frac{\phi_{1B} + \gamma_B}{2} = \frac{2\psi(\gamma_A^2 + \gamma_B^2 (1-p)) - \gamma_B^2 p (1-p)^2}{2(p \gamma_A^2 + \gamma_B^2 p (1-p)^2)} + \frac{\gamma_B}{2}$$

$$w_{A,\emptyset,1}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,\emptyset}^{0.5} = \frac{\gamma_B}{2}$$

$$w_{B,\emptyset,1} = 0$$

$$t_{1,1} = 0$$

$$t_{1,\emptyset} = \gamma_A (w_{A,1,1}^{0.5} - w_{A,1,\emptyset}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,\emptyset,1}^{0.5})$$

$$t_{\emptyset,1} = \gamma_A (w_{A,1,1}^{0.5} - w_{A,\emptyset,1}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,1,\emptyset}^{0.5})$$

$$t_{\emptyset,\emptyset} = \gamma_A (w_{A,1,1}^{0.5} - w_{A,\emptyset,\emptyset}^{0.5}) + \gamma_B (w_{B,1,1}^{0.5} - w_{B,\emptyset,\emptyset}^{0.5})$$

C6: Proof of Proposition 4

When $\gamma_B = 0$, the solution is as follows:

$$w_{A,1,1}^{0.5} = w_{B,1,1}^{0.5} = w_{A,1,\emptyset}^{0.5} = w_{B,1,\emptyset}^{0.5} = \frac{\psi}{p}$$

$$w_{A,\emptyset,1}^{0.5} = w_{A,\emptyset,\emptyset}^{0.5} = \frac{\gamma_A}{2}$$

$$w_{B,\emptyset,\emptyset} = w_{B,\emptyset,1} = t_{1,1} = t_{1,\emptyset} = 0$$

$$t_{\emptyset,1} = t_{\emptyset,\emptyset} = \gamma_A \left(\frac{\psi}{p} - \frac{\gamma_A}{2} \right)$$

Define relative performance evaluation as the difference between $w_{i,1,1}$ and $w_{i,1,\emptyset}$ for $i \in \{A, B\}$. As $\gamma_B \rightarrow 0$, i.e. as the relative level of favoritism increases, the level of relative performance evaluation used goes to zero, even though the principal is still altruistic towards agent A.

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