Currency Manipulation in a Model of Money, Banking, and Trade

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Can a large, emerging economy that runs current account surpluses resist real appreciation and improve its trade balance indefinitely by accumulating foreign currency reserves? I develop a two-country, monetary model in which heterogeneous lifecycle incomes give rise to private lending, while spatial separation, limited communication, and idiosyncratic liquidity risk create a role for banks and government issued country-specific fiat currencies. Nominal prices are fully flexible, the law of one price holds continuously for traded goods, and the equilibrium real exchange rate is the international relative price of non-traded goods. When governments are composite fiscal-monetary authorities, an emerging economy with a current account surplus can unilaterally target and sustain in a steady state equilibrium a real exchange rate depreciated relative to its equilibrium value. A targeting steady state exists both under capital controls (no loan trade) and free capital flows (international trade in loans). In this steady state, the targeting government accumulates reserves at a constant rate, and its non-tradable consumption rate endogenously declines so that sustained reserve accumulation is not inflationary. Only under capital controls does a relatively depreciated real exchange rate target improve a country's trade balance, providing a mercantilist rationale for the policy. Under free capital flows, however, a depreciated real exchange rate target can stabilize the real economy and nominal exchange rate.

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1. Introduction

The goal of this paper is to evaluate the case for persistent currency manipulation by China, by establishing theoretical conditions under which a large country can accomplish a sustained improvement in its trade balance through government purchases of foreign exchange in a monetary, general equilibrium model.

Accusations that China manipulates its currency have prevailed since the early 2000s. US scholars, businesses, and members of congress alike argue that China has deliberately suppressed a rise in the renminbi's value versus the US dollar by intervening in foreign exchange markets, accumulating dollar reserves, to increase exports and promote export-producing industries. The legal basis for objection to deliberate undervaluation of an otherwise strengthening currency equates it with an export subsidy, an effort to gain an unearned competitive advantage in trade. This violates the fairness principle that governs the world trading system. Proponents of this view, such as Bersten and Gagnon (2012), Gagnon (2012), and Porter (2017), assert that China manipulated the renminbi's value for mercantilist purposes consistently from at least 2003 until 2014 – under what was formally a flexible exchange rate regime from 2005 until 2014 – resulting in millions of US manufacturing sector job losses. Recently, claims of currency manipulation have proliferated, levied against and by not only other emerging economies, but also advanced countries with flexible exchange rates and relatively open capital accounts. The US secretary of the Treasury currently monitors more than 20 of the largest US trade partners for evidence of currency manipulation, biannually measuring their US bilateral trade balances, current account balances, and foreign currency purchases.

Yet classical international monetary theory implies that government accumulation of foreign exchange reserves need have no real effects whatsoever. Domestic currency creation must finance foreign currency purchases, and such purchases directly depreciate the nominal, not the real, exchange rate. They need have no significant, nor any lasting, effect for international relative prices of goods – effects needed to produce an improvement in trade competitiveness. With flexible prices, proportional nominal price adjustments arising from the monetary base expansion that funds the intervention eliminate any relative price consequences. Evidence – albeit controversial – suggests that a government's attempt to prevent this inflation by sterilizing the monetary expansion via open market bond sales results in a weak nominal, let alone real, exchange rate response to intervention. New Keynesian models with sticky nominal prices allow for highly correlated real and nominal exchange rate depreciations over short time periods, however, empirical evidence implies that price stickiness is insufficiently persistent to account for a decade-long effort by China to prevent its currency's appreciation. In this paper, I develop an alternative monetary model in which a government can attain a relatively depreciated real exchange rate and higher trade balance by purchasing foreign currency – and can do so for a sufficient duration to rationalize a decade of Chinese currency manipulation.

The two-country model I develop features overlapping generations of finitely lived private agents that value two types of imperfectly substitutable asset. National fiat monies, although return dominated, are valued for their liquidity. Illiquid consumption loans are valued for their interest earnings. That both "inside" and "outside" money circulate has several important implications. First, there is a realistic distinction in my model between the liquid asset (currency) accumulated as reserves by a government, and illiquid assets (loans) that are traded only privately within and/or across countries. Second, as long as there is some private sector international currency trade – however small – government reserve accumulation is not equivalent to a trade or current account balance policy under capital controls, as is true in non-monetary environments such as those studied by Jeanne (2013) and Choi and Taylor (2017). Third, both under capital controls and under free capital flows – with international borrowing and lending – a government can unilaterally establish and sustain a real exchange rate target depreciated relative to its steady state equilibrium value, by purchasing foreign currency at each date. Under free capital flows, the government's reserve accumulation, net private sector currency flows, and net private sector capital flows are each exactly determined; furthermore, there are substantive allocative consequences of the targeting policy as long as private agents also hold alternative currencies for liquidity in international trade.

A key result is that, under capital controls, a government's foreign reserve accumulation is associated with an endogenous rise in its country's trade balance supporting a mercantilist rational for reserve accumulation. The trade balance improvement is attributable to the equilibrium decline (rise) in the targeting country's (trade partner's) internal relative price of non-traded to traded goods. This reduces the tradable value of loans in the targeting country, increases the targeting country's real interest rate, and internal borrowing for tradable consumption contracts. The law of one price holds for the traded good, leaving no role for improvements in "competitiveness" arising from substitution effects of real depreciation; however, the negative "income" effect for lending of a lower relative price of non-traded goods induces a higher inter-temporal price of traded consumption in the targeting country. The converse effects afflict the targeting country's trade partner. Consequently the targeting country's tradable goods balance rises. Under free capital flows, internationally arbitraged real interest rates insulate borrowing for tradable consumption and trade balances from a targeting country's real exchange rate depreciation. However, targeting the real exchange rate can stabilize real activity relative to an equilibrium with no real exchange rate target or reserve accumulation, suggesting an alternative rationale for reserve accumulation by countries with open capital accounts.

I obtain these results in a pure exchange, spatial model of money and trade. Each country contains two symmetric locations inhabited by private agents, and a central location inhabited by a government. An infinite sequence of two-period lived overlapping generations inhabits each location, and each generation comprises a mass of lenders (workers) and borrowers (entrepreneurs). Lenders and borrowers can trade in one-period lived consumption loans. There are two types of good. One is freely tradable in frictionless markets characterized by perfect cross-location communication about the good, buyers, and sellers, and the law of one price. There is also a "non-traded" good, which agents can purchase only during "local trade" when there is no communication among agents across locations. Because of this, during local trade it is impossible to verify the value of private assets issued in other locations, and this gives rise to the private use of country-specific, government issued fiat currencies for liquidity. Specifically, workers are subject to idiosyncratic relocation shocks, which play the role of (currency-specific) liquidity shocks since relocated agents can use only currency in subsequent goods market exchange. I assume that deposit-taking banks arise to insure lenders against these shocks – banks that intermediate all savings, hold loans and currencies directly, and offer deposit returns contingent upon an agent's relocation status and ultimate destination. I focus on equilibria in which loans dominate currency in rate of return, so that banks hold currency solely to meet the liquidity needs of their depositors.

In each country, a composite fiscal/monetary policy authority sets a constant, exogenous growth rate for its currency that is outstanding in the hands of the public. The resulting seigniorage revenue supports an endogenously determined government consumption rate of non-traded goods and – under a unilateral real exchange rate targeting regime by either country – an endogenously determined real foreign reserve position. I consider two international capital account regimes. In the first, banks can trade internationally in currencies, but not in loans. I view this regime as one of bilateral capital controls. Contingent on parameter values, the model allows for international trade in currencies by banks to be very small, as we observe – for example – between China and the rest of the world. In the second regime, banks can trade internationally in loans and I view this regime as one of free capital flows. I explore the equilibrium consequences of a unilaterally established real exchange rate target under both regimes. If neither country establishes a real exchange rate target, the equilibrium bilateral real exchange rate equals the relative price of non-traded to traded goods across countries since the law-of-one-price holds for the traded good.

I assume that the domestic country is relatively poor and, under capital controls, runs current account surpluses. Several parameter restrictions assure these two features: The domestic country 1) has relatively low per capita endowments of traded and non-traded goods, and 2) has a relatively large portion of workers that are subject to liquidity shocks. The latter restriction implies that the domestic country is relatively cash-dependent and exhibits relatively little credit extension, with bank asset portfolios dominated by currency rather than loans. As a result, under capital controls, the domestic country's equilibrium interest rate is relatively high, domestic young borrowers therefore consume relatively few traded goods, and the country runs a trade surplus which funds positive net domestic bank purchases of foreign currency. Starting from an initial period, under capital controls and in the absence of a real exchange rate target, the world economy attains a unique steady state equilibrium with these properties at date 2. Under free capital flows in the absence of a real exchange rate target, given identical parameter values, real interest rates are arbitraged; the poor country's real interest rate falls and that of the rich country rises, relative to the equilibrium under capital controls, and this balances trade. From an initial period, the economy converges asymptotically to a unique steady state equilibrium. The unique transition path exhibits a monotonically depreciating domestic country real exchange rate.

Under the assumption that the foreign government is completely passive in response, the domestic country can use foreign reserve purchases to unilaterally establish, and sustain indefinitely in a unique steady state equilibrium, a constant real exchange rate target that is higher (more depreciated) than the steady state equilibrium real exchange rate. Attaining a targeting steady state is possible under either capital controls or free capital flows. The domestic government accomplishes this via an endogenously determined real foreign reserve purchase at every date, which is constant in the targeting steady state and associated with a constant growth rate of nominal reserve purchases equal to the foreign country's money growth rate. Under either capital market regime, the domestic government's non-tradable consumption declines, and foreign government consumption increases, relative to the no-targeting steady state. Thus, seigniorage revenue must be sufficiently high to guarantee non-negative domestic government consumption when the revenue must also finance reserve purchases. This implies that the real exchange rate target satisfy an upper bound, to limit the size of reserve purchases. Naturally, the higher is the domestic country's money growth rate, the looser is the upper bound constraint.

Under either capital account regime, the domestic government can attain a relatively depreciated real exchange rate target at any date when the economy has been previously in its unique, non-targeting steady state equilibrium. Thus, the establishment of a target is unanticipated from the perspective of private agents. Attainment of the unique targeting steady state is immediate under capital controls. Attainment of the targeting steady state occurs after just one period under free capital flows, so there are no transitional dynamics as there are in the absence of a target. Under capital controls, the steady state domestic country real interest rate is higher than that in the no-targeting steady state (because the domestic country tradable value of loans declines), and domestic country tradable borrowing and consumption fall so that its trade surplus rises. Under free capital flows, real interest rates are arbitraged and there is no impact of targeting for the trade balance. However, under free capital flows, a government can introduce a target at any finite date, and the same, unique, targeting steady state equilibrium is attainable after just one period. The target therefore completely stabilizes not only the real exchange rate, but the trade balance, financial balance, and entire real economy relative to the equilibrium transitional dynamics without a target.

The nominal exchange rate need not depreciate upon establishment of a depreciated real exchange rate target in the initial period of the target, nor will it depreciate at a faster rate in the unique steady state targeting equilibrium, relative to its non-targeting behavior. While domestic government accumulation of foreign exchange is a force for nominal depreciation of the domestic currency, a relatively depreciated real exchange rate target *reduces* the foreign non-traded value of domestic bank holdings of foreign currency and raises the domestic non-traded value of foreign bank holdings of domestic currency. Only if the portion of bank reserves held as local, rather than international, currency is relatively large does the domestic country's nominal exchange rate exhibit depreciation upon establishment of the target. In a targeting steady state equilibrium, under either capital account regime, the domestic country's nominal exchange rate depreciates at a unique, constant rate irrespective of whether there is a real exchange rate target. Namely, in *any* steady state equilibrium, the rate of the domestic currency's nominal depreciation must equal the difference between the domestic and foreign non-traded goods inflation rate, which equals the difference between the domestic and foreign constant money growth rates.

Similarly, persistent real depreciation accomplished via sustained reserve accumulation is not inflationary. By assumption, monetary policy is exogenous; each government sets a constant growth rate for the money supply outstanding in the private sector. For reserve accumulation to be consistent with the satisfaction of government budget constraints, therefore, requires endogenous adjustment in fiscal policy. Specifically, government consumption of non-traded goods in the targeting country declines to accommodate lower available seigniorage revenue, and the converse occurs in the foreign country. Mechanically, endogenous fiscal policy adjustments substitute for the excess money creation that would imply reserve accumulation generates inflation. Thus, real allocative consequences are associated with foreign exchange intervention even in this flexible price, monetary model. Intuitively, when the monetary and fiscal policy functions of government are relatively coordinated, as in China where the People's Bank of China is a department of the State Council, "sterilization" of the aggregate demand consequences of monetary base expansion can be accomplished by an increase in taxation or, as here, decline in government consumption. Thus, perfect coordination of monetary and fiscal policy, inflation targeting that disciplines money growth rates, and a prohibition on international borrowing and lending together can rationalize indefinite and successful currency manipulation by a relatively large poor country that runs trade surpluses.

This paper primarily contributes to two literatures. Several features of the model significantly generalize recent models of real exchange rate targeting, of which Choi and Taylor (2017), Jeanne (2013), and Korinek and Serven (2016) are notable contributions, with a key precursor being Calvo, Reinhart, and Vegh (1995). First, my model is explicitly monetary, including both liquid national currencies as well as interest bearing, illiquid loans. This illuminates the required coordination of monetary and fiscal policy to sustain a depreciated real exchange rate via reserve accumulation, and allows me to demonstrate that reserve accumulation need not be inflationary. The need for fiscal adjustment in my model reflects Eichengreen's (2007) observation that fiscal not monetary policy must accomplish systematic real exchange rate undervaluation. Second, I examine the general equilibrium (foreign country) consequences of a government's unilateral reserve accumulation. This extension from a small open economy environment suggests that the trade partner of a targeting country derives "seigniorage" benefits, which relaxes budgetary constraints on endogenously determined government consumption, and may rationalize a failure to retaliate. In addition, there are steady state welfare benefits for foreign workers purchasing local goods in domestic country markets, who enjoy a higher domestic country value of their currency holdings. Third, in my framework, public and private net foreign currency and net lending have explicit, distinct solutions under free capital flows. Relatedly, Ghironi (2006, 2008) explores the implications for a country's net foreign asset accumulation of Ricardian equivalence failure due to overlapping generations, although his motivation and model environments differ from mine. More generally, an overlapping generation environment typically has very different theoretical properties from that of the infinitely lived agent models of the extant literature. Nonetheless, the equilibria I study are unique, and money has no value if not for idiosyncratic liquidity shocks.

The paper also represents a contribution to the empirical and theoretical literature that explores mercantilist rationales for real exchange rate undervaluation. Dooley, Folkerts-Landau, and Garber (2004) make the mercantilist case for China and other emerging economies deliberately undervaluing their currencies, as do Bersten and Gagnon (2012) and Gagnon (2012), for example. In contrast, McKinnon (2006) argues that internal monetary stability motivated China's foreign exchange intervention in the 2000s rather than mercantilism (my choice of monetary policy specification makes it impossible to address this), while her external surpluses reflect a relatively high savings rate (an idea reflected in my assumption that the targeting country has a relatively high equilibrium real interest rate). Prasad and Wei (2005) also are skeptical of mercantilist motivation for China's reserve accumulation, attributing it instead to a surge in capital inflows until 2004 at least. Aizenmann and Lee (2007) present empirical evidence supporting a precautionary rather than mercantilist motivation for reserve accumulation in emerging markets, while Dominguez (2019) finds evidence to support systematic depreciation of nominal exchange rates via foreign exchange intervention, but no conclusive evidence of associated trade balance improvements. The paper also tangentially relates to a literature motivated by the Chinese experience that explores optimal monetary, reserve, and capital account policy in dynamic, optimizing environments. Bacchetta, Benhima, and Kalantzis (2013, 2014), Chang, Liu, and Spiegel (2015), and Liu and Spiegel (2015) represent some significant contributions. Here I conduct a purely positive analysis of currency manipulation, which includes evaluating the welfare consequences. However, the rich specification of monetary and fiscal policy, and of private banking behavior, in my model would lend itself easily to exploring a range of alternative monetary and financial policies and their relative welfare merits.

2. The Environment

I consider a two-country, two-good world economy. Time is discrete and indexed by *t*. An infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely lived government inhabit each country. I call the two countries the "domestic" and "foreign" country respectively, and use the superscript "*" to distinguish foreign from domestic country variables.

Within each country are two symmetric locations in which private agents reside, while the government inhabits a third central location. At every date *t=1,2,..*, a continuum of young agents with unit mass is assigned to each of the two symmetric locations. Of these young agents, a fraction ψ are *ex ante* identical workers. The remaining fraction, $1-\psi$, are identical entrepreneurs. In addition, at date *t=1*, a continuum of initial old agents with unit mass resides in each symmetric location. My assumptions guarantee that trade between locations 1 and 2 within a country preserves the symmetry of locations. In addition, I assume that location 1 (2) of the domestic country is always paired in trade with location 1 (2) of the foreign country. Internationally paired locations need not be at all symmetric, however.

There are two types of final, non-storable consumption good in the world economy; local goods and tradable goods. Workers produce and consume exclusively local goods. As I describe below, there is limited inter-location trade in local goods and, for expositional ease, I refer to them as "non-traded". Entrepreneurs produce and consume exclusively tradable goods, which are freely transportable across domestic and international locations. Both types of good are identical across locations.

There are also two types of asset. The government of each country issues a national fiat currency, and entrepreneurs and workers can issue and trade in one-period consumption loans. I assume that, while loans return-dominate currency, spatial separation of agents and limited communication among them within and across countries give rise to the need for currency for liquidity in inter-location exchange. For reasons that I describe below, all consumption loans are intermediated by deposit taking banks and banks hold all assets to back these deposits. I assume that any young worker can costlessly form a bank, and that free entry to banking and competition for depositors drives profits to zero.

2.1 Preferences, Endowments, and Technology

2.2.1 Preferences The domestic workers and entrepreneurs of generation *t* have lifetime expected utility functions, respectively,

$$u\left(c_{y,t}^{N}, c_{o,t+1}^{N}\right) = ln(c_{y,t}^{N}) + \beta E_t \ln\left(c_{o,t+1}^{N}\right), \tag{1a}$$

$$u\left(c_{y,t}^{T}, c_{o,t+1}^{T}\right) = ln(c_{y,t}^{T}) + \beta \ln(c_{o.t+1}^{T}). \tag{1b}$$

The superscript indicates the type of good consumed; "*N*" for the non-traded goods' consumption of workers, and "*T*" for the traded goods consumption of entrepreneurs. The first subscript indicates whether the agent's consumption is occurring during young age, "*y*", or old age, "o", and the second subscript denotes the date at which consumption occurs. In addition, the expectations operator appears in (1*a*) because workers are subject to idiosyncratic liquidity shocks prior to old age consumption, as I describe below in 2.3. In addition, initial old agents in the domestic country have the lifetime utility function,

$$u_0\left(c_{o,1}^{N}\right) = ln\left(c_{o,1}^{N}\right). \tag{2}$$

The preferences of foreign workers, entrepreneurs, and initial old agents are exactly analogous.

2.2.2 Endowments and Technology A young domestic (foreign) worker receives an endowment of labor when young, l (l^*), and supplies it in-elastically to the production of non-traded goods in his location. Each unit of labor produces $\frac{y}{l}$ $\left(\frac{y^*}{l^*}\right)$ units of output of the non-traded good. Workers have no other endowments of any other commodity at any date, and are retired when old. Consequently, total per worker output of non-traded goods in each location is simply y (y^*), and aggregate (per capita) non-traded output in each location of the domestic (foreign) country is ψy (ψy^*) $\forall t$.

Since workers value consumption of non-traded goods in both periods of their lifetime, but goods are not storable, they must save a portion of the non-traded output they produce when young in the form of some assets. The realization of idiosyncratic liquidity shocks at the end of date *t* determines whether currency or loan returns are valuable for a generation *t* worker in old age consumption, as I describe below.

Each young entrepreneur has an endowment of a technology for producing traded goods. Specifically, a young domestic (foreign) entrepreneur of generation *t* is endowed with a project at *t*, which generates q (q^*) units of traded final output at *t+1*. Entrepreneurs have no other endowments of commodities at any other date. Total per entrepreneur output of traded goods in each location is simply q (q^*), and aggregate (per capita) traded output in each location of the domestic (foreign) country is $(1-\psi)q$ $((1-\psi)q^*), \forall t$. As (1*b*) shows, a generation *t* entrepreneur values traded goods' consumption in both periods of life, and must therefore borrow at date *t* to accomplish young age consumption, repaying the debt using his project's output of traded goods at *t+1*. One period lived consumption loans from workers, which are intermediated through banks, are the vehicle for young entrepreneurs to borrow. I assume that the size of an entrepreneur's project, *q*, is large relative to the income of any individual worker, *y*, so that multiple workers fund each entrepreneur's loan.

The initial old generation in each location of the domestic (foreign) country has endowments comprising the initial outstanding per capita stock of fiat currency outstanding in the hands of the public of that country, $M_0(M_0^*)$ and claims to the entire per capita initial period output of traded goods. Since initial old agents care only about consumption of the non-traded good, initial young workers must accept the fiat currency of their country and/or initial tradable claims, in exchange for non-traded goods at date *1* in order that initial old agents consume.

2.3 Liquidity Shocks

Workers are subject to idiosyncratic uncertainty, which is resolved at the end of each period. With probability π $(\pi^*) > 0$, at the end of period *t*, a generation *t* domestic (foreign) worker is subject to relocation. Conditional on being subject to relocation, with probability $\varepsilon(\varepsilon^*) > 0$ a domestic (foreign) worker is relocated to the second domestic (foreign) location within his country, and with probability $(1-\varepsilon) > 0$ (or $(1-\varepsilon^*) > 0$) the domestic (foreign) worker is relocated internationally to the foreign (domestic) location paired in trade with his original location. With probability $(1-\pi) > 0$ $((1-\pi^*) > 0)$ a worker remains in his original location, and consumes the locally produced non-traded good in old age. The probabilities of stochastic relocation are constant over time, known by all agents, and *iid* across agents within a location; so there is no aggregate uncertainty. Further, net relocations within a country are zero so that within country locations retain symmetry, although locations paired in international trade need not be at all symmetric.

Relocated young workers must take with them some assets in order to purchase non-traded goods for old age consumption in their new location at $t+1$. I assume that currency is transportable between locations, but that privately issued loans held directly (with banks, checks written on bank deposits backed by loans) are not. In addition, by convention, a buyer must pay for purchases of non-traded goods in any location using the currency of the seller. Thus domestic young workers relocated domestically must carry with them domestic currency, and those relocated internationally must carry with them foreign currency. Analogous statements apply to foreign young workers.

The assumption that only currency is useful in inter-location exchange in spatial models – exchange between buyers and sellers originating in different locations – is well-established (Townsend (1987), Mitsui and Watanabe (1989), Champ, Smith, and Williamson (1992), Hornstein and Krusell (1993), Schreft and Smith (1997), and – in the open economy context – Betts and Smith (1997)). I motivate this role of currency by assuming that, during local goods market trade, young workers selling local goods cannot communicate with agents in remote locations. Consider a decentralized setting without banks. A young worker cannot verify the value of loan paper issued by entrepreneurs elsewhere during local trade, and such loan paper is therefore counterfeitable. Thus, if young workers lend to entrepreneurs only locally, a relocated old worker can use only the currency of his new location to purchase local goods from young workers. In the absence of banks, one can argue that local lending is the only private lending that occurs in equilibrium even if there are no capital controls limiting cross-location lending.[1] In an environment with banks, relocated old workers cannot write checks on their

[1] Workers will not lend directly to entrepreneurs in other locations because of counterfeit and default risk. When each worker's loan represents only a fraction of the total loan that an entrepreneur issues, it is generally impossible for a young worker to verify the value of an old worker's loan during local goods trade if there is inter-location lending and borrowing. Even if the borrower resides in the same country and location as the young worker attempting to verify the value of a loan, the latter must also contact other lenders to the project to corroborate the value of the claim. With inter-location lending,

deposits in remote banks to purchase local goods for the same reason that they cannot exchange locally issued loan paper; young workers selling goods cannot communicate with banks elsewhere, and hence only currency is useful in inter-location exchange. Young workers accept checks drawn on local *non-mover's* deposits in local banks, however, as they can verify the value of the bank's balance sheet. [2]

In either case, young workers accept only fiat currency from relocated old workers in exchange for non-traded goods, and currency therefore has liquidity advantages in inter-location exchange over privately issued assets. Relocation shocks play the role of liquidity preference shocks, such as those in Diamond and Dybvig (1983). Young workers subject to relocation want to liquidate holdings of any other assets they hold and use the proceeds to purchase the currency of the relevant country. Workers not relocated would prefer to hold only loans and to sell the return-dominated currencies they hold. The possibility of asset value losses makes it natural for banks to emerge to insure young workers against relocation risk by accepting their deposits, and offering state contingent deposit returns based on holdings of both types of currency, as well as privately issued loans.[3] Relocated workers withdraw their deposits in the form of the appropriate currency before moving at the end of period *t*, while non-

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other lenders to a project are located elsewhere with positive probability, and cannot be contacted. Even non-movers are then subject to the risk of rejection of their loan paper by young workers. Furthermore, there is no mechanism in a decentralized environment to insure non-movers against default by remote borrowers; the holder of loan paper issued by a local borrower, however, can seize project returns directly.

[2] When all bank lending and, hence, all borrowers are local – as is true under capital controls – this is straightforward. Is inter-location bank borrowing and lending possible? Obviously, this is irrelevant for the need for currency of relocated agents, as young workers cannot observe the balance sheet of a remote bank. However, young workers selling local goods to non-movers in exchange for a check written on a local bank deposit can observe the local bank's balance sheet, which backs the deposit. Nonetheless, they cannot contact remote borrowers to verify loan values. There are several possible resolutions to this verification problem. First, if – by contrast to an individual worker – a bank is sufficiently large relative to the size of a loan, loan diversification may eliminate the default risk confronted by individual lenders. Second, if banks can establish affiliates elsewhere with remote monitoring abilities, this may guarantee for young workers the value of foreign loans held by a local bank. Third, governments may act to guarantee lending by local banks, which they do not for individuals.

[3] There is at least one other natural institutional response to the possibility of private agents losing the value of assets due to unforeseen liquidity needs. Temporary local spot markets could open at the end of date *t*, allowing young workers with different relocation realizations to trade assets among themselves within a location.

movers can write checks against their deposits, which are backed solely by loans, during local goods market trade at $t+1$.

2.4 Nature and Timing of Trade

Each date contains two trading periods. At the beginning of t, local goods' market trade occurs autarkically within each location. Once local trade concludes, a "spatial trade" period occurs, with unrestricted inter-location exchange of traded goods and currencies and (under free capital flows) loans across locations.

During local trade, there is no movement of goods between locations and no communication among agents across locations. At the beginning of each date, young workers in any location consume a portion of the output of non-traded goods that they produce, and sell the remaining goods to old workers – at date 1, initial old agents – in return for some assets. Except in the initial period, some old workers have arrived from elsewhere, bringing with them only the currency of the seller to exchange for non-traded goods. Some old workers have not moved, and young workers can verify the value of local banks' balance sheets, accepting checks written against them in exchange for non-traded goods. Finally, each government offers newly printed units of its own currency in exchange for the non-traded good of each location within its country. Once local trade in non-traded goods within each location is complete, these markets close, and workers consume.

After workers' consumption is complete, old entrepreneurs' projects mature, producing traded goods. There is then a period of free, inter-location trade, domestically and internationally, in traded goods, currencies, and – if inter-location lending is permitted – consumption loans. During this spatial trade period, there is full communication among banks (on behalf of young workers), entrepreneurs, and governments within and across locations domestically, and internationally across locations paired in trade. Old entrepreneurs repay the bank loans they accepted at $t-1$, and banks receiving tradable loan returns offer new loans to young entrepreneurs. Under "capital controls", banks offer loans to local entrepreneurs only. Under "free capital flows", banks can offer loans to entrepreneurs in any location. Banks can reallocate their currency and loan portfolios in asset markets. Traded good and asset markets then close, and entrepreneurs consume.

At the end of each period t, following the conclusion of all trade and communication, young workers learn their relocation status. Relocated workers can contact their local banks at this time and make early withdrawals of domestic or foreign currency, depending on whether they confront domestic or international relocation. Workers not relocated write checks against their deposits to purchase local non-traded goods at the beginning of the following period. I depict the timing of trade in figure 1.

2.5 Banks

The risk of relocation implies that young workers want to save through intermediaries that take deposits, hold primary assets directly, and promise state contingent returns to depositors depending on their relocation status and ultimate destination. Under my assumptions, all savings are intermediated by such banks.

On the asset side, banks behave competitively, viewing themselves as unable to influence the equilibrium returns to currency and loans. On the deposit side, they are Nash competitors, announcing schedules of state contingent returns as a function of relocation status and destination, taking the return schedules of other banks as given. With free entry into intermediation, competition for depositors implies that, in a Nash equilibrium, banks choose deposit returns to maximize the expected utility of a young worker, (1*a*), subject to balance sheet constraints. I focus on equilibria in which loans dominate the local currency in rate of return within each country. In these equilibria, banks hold domestic and foreign currency solely to meet the liquidity needs of domestically and internationally relocated workers.[4]

In the initial period, the savings that young workers deposit comprise the initial domestic (foreign) money stock, and claims to the entire output of traded goods of the location, which initial old agents have exchanged for non-traded goods. In all subsequent periods, young workers' savings comprise domestic currency exchanged by relocated old workers, and checks written on local bank deposits exchanged by non-movers. I denote by $d_t \equiv y - c_{y,t}^N$ the deposit of a domestic young worker, which is just his saving measured in non-traded goods. Domestic bank holdings of real per depositor (per worker) assets, measured in domestic non-traded goods must then satisfy the balance sheet constraint,

$$m_t^d + x_t m_t^f + \frac{(1-\psi)}{\psi}(l_{t+1}/p_t) \leq d_t, \forall t \geq 1. \tag{3}$$

In (3), $m_t^d \equiv \frac{M_t^d}{p_t^N}$ is domestic bank, per worker domestic currency holdings – M_t^d – measured in non-traded goods at *t*, where p_t^N is the price of a unit of a domestically produced non-traded good measured in domestic fiat currency. Domestic real balances held between *t* and *t+1* therefore have a

[4] Obviously, banks would never back the deposits of non-movers with the other country's currency, even if it were not return-dominated, because by convention only the local currency is acceptable in local trade.

non-traded return of $\left(\frac{p_t^N}{p_{t+1}^N}\right)$. Similarly, $x_t m_t^f \equiv \frac{M_t^f}{p_t^{N*}}$ is domestic bank, per worker foreign currency holdings – M_t^f – measured in domestic non-traded goods at *t*. Here, p_t^{*N} is the foreign currency price of a foreign-produced non-traded good, $x_t \equiv \left(e_t \frac{p_t^{N*}}{p_t^N}\right)$ is the relative price of a foreign non-traded good in terms of domestic non-traded goods, and e_t is the nominal exchange rate of the domestic country, measured in domestic currency units per foreign currency unit. This, $x_t = \left(e_t \frac{p_t^{N*}}{p_t^N}\right)$, is the real exchange rate of the domestic country. Since the traded good is identical across countries and there is free trade in these goods, the law of one price holds, $e_t = \frac{p_t^T}{p_t^{T*}}$, x_t is also just the international relative price of the internal relative price of non-traded in terms of traded goods,

$$x_t = \left(\frac{p_t^{N*}/p_t^{T*}}{p_t^N/p_t^T}\right) = \frac{p_t^*}{p_t}.$$

The real return foreign real balances held between *t* and *t+1* and measured in foreign non-traded goods at *t+1* per unit of foreign non-traded goods invested at *t* is just $\left(\frac{p_t^{*N}}{p_{t+1}^{*N}}\right)$. Hence $\left(\frac{p_t^{*N}}{p_{t+1}^{*N}}\right)\left(\frac{x_{t+1}}{x_t}\right) = \left(\frac{p_t^N}{p_{t+1}^N}\right)\left(\frac{e_{t+1}}{e_t}\right)$ is the return to foreign real balances measured in units of domestic non-traded goods at *t+1* per domestic non-traded good invested in foreign currency at *t*. In addition, l_{t+1} is the traded goods value of a domestic bank's date *t* per entrepreneur loans, and $\frac{(1-\psi)}{\psi} l_{t+1}$ is the per worker value of these tradable claims. Then $\frac{(1-\psi)}{\psi}(l_{t+1}/p_t)$ is the per worker value of a domestic banks' tradable claims measured in domestic non-traded goods, where $p_t \equiv \frac{p_t^N}{p_t^T}$ is the domestic relative price of a domestic non-traded good in terms of domestic traded goods, and p_t^T is the domestic currency price of a unit of domestically produced traded goods. If policy permits international bank lending, then the bank's total loan portfolio comprises both domestic (l_{t+1}^d) and foreign (l_{t+1}^f) loans,

$$l_{t+1} = l_{t+1}^d + l_{t+1}^f.$$

Each domestic loan of one traded good at *t* has a real gross return of R_{t+1}^T units of traded goods received at *t+1* per traded good loaned at *t*. Then, $R_{t+1}^N \equiv R_{t+1}^T \frac{p_t}{p_{t+1}}$ represents the gross return to a bank's one period domestic consumption loan measured in units of non-traded goods received at *t+1* per non-traded good invested at *t*. Each foreign loan of one traded good at *t* has a real gross return of R_{t+1}^{*T} units of traded goods received at *t+1* per traded good loaned at *t*. Then, $R_{t+1}^{*N} \equiv$

$R^{*T}_{t+1}\frac{p^*_t}{p^*_{t+1}}$ represents the gross return to this loan measured in units of foreign non-traded goods, and $R^{*N}_{t+1}\frac{x_{t+1}}{x_t} \equiv R^{*T}_{t+1}\frac{p^*_t}{p^*_{t+1}}\frac{x_{t+1}}{x_t}$ is the gross return to a foreign loan measured in domestic non-traded goods.

Domestic banks promise to pay domestically relocated, internationally relocated, and non-relocated young workers gross real returns on their deposits of $\rho_t^{\varepsilon\pi}, \rho_t^{(1-\varepsilon)\pi}$, and $\rho_t^{1-\pi}$ respectively. Since domestically relocated domestic young workers – of whom there are $\varepsilon\pi$ per depositer – require domestic currency in order to consume when old in their new location, gross payouts by domestic banks to these agents must satisfy

$$\rho_t^{\varepsilon\pi}\varepsilon\pi d_t \le m_t^d \frac{p_t^N}{p_{t+1}^N}. \tag{4}$$

Internationally relocated domestic young workers – of whom there are $(1-\varepsilon)\pi$ per depositer – require foreign currency in order to consume when old in their new location, so that gross payouts by domestic banks to these agents must satisfy

$$\rho_t^{(1-\varepsilon)\pi}(1-\varepsilon)\pi d_t \le x_t m_t^f \frac{p_t^N}{p_{t+1}^N}\frac{e_{t+1}}{e_t}. \tag{5}$$

Finally, banks back the deposits of non-movers solely by loans to entrepreneurs, under the assumption that the real return to loans dominates that of domestic currency measured in non-traded goods, $R_{t+1}^N > \frac{p_t^N}{p_{t+1}^N}$, where $R_{t+1}^N = R_{t+1}^T(p_t/p_{t+1})$. Hence, if there is no trade in loans,

$$\rho_t^{1-\pi}(1-\pi)d_t \le \left(\frac{1-\psi}{\psi}\right)\frac{l_{t+1}}{p_t}R_{t+1}^N, \tag{6}$$

and, if there is trade in loans,

$$\rho_t^{1-\pi}(1-\pi)d_t \le \left(\frac{1-\psi}{\psi}\right)\frac{l_{t+1}^d}{p_t}R_{t+1}^N + \left(\frac{1-\psi}{\psi}\right)\frac{l_{t+1}^f}{p_t}R_{t+1}^{*N}\frac{x_{t+1}}{x_t}. \tag{6'}$$

I define the domestic currency-deposit ratio of a domestic bank as $\gamma_t^{\varepsilon\pi} \equiv \frac{m_t^d}{d_t}$, the foreign currency-deposit ratio as $\gamma_t^{(1-\varepsilon)\pi} \equiv \frac{x_t m_t^f}{d_t}$, the domestic loan-deposit ratio as $\gamma_t^{d(1-\pi)} \equiv \frac{l_{t+1}^d(1-\psi)}{\psi d_t p_t}$, and the foreign loan deposit ratio as $1-\gamma_t^{\varepsilon\pi}-\gamma_t^{(1-\varepsilon)\pi} - \gamma_t^{d(1-\pi)} \equiv \frac{(l_{t+1}-l_{t+1}^d)(1-\psi)}{\psi d_t p_t}$. If loans are not traded, then $1-\gamma_t^{\varepsilon\pi}-\gamma_t^{(1-\varepsilon)\pi} = \gamma_t^{d(1-\pi)}$ and a domestic bank's budget constraints can be re-expressed as

$$\rho_t^{\varepsilon\pi} \le \frac{\gamma_t^{\varepsilon\pi}}{\varepsilon\pi}\frac{p_t^N}{p_{t+1}^N}, \tag{7a}$$

$$\rho_t^{(1-\varepsilon)\pi} \le \frac{\gamma_t^{(1-\varepsilon)\pi}}{(1-\varepsilon)\pi}\frac{p_t^N}{p_{t+1}^N}\frac{e_{t+1}}{e_t}, \tag{7b}$$

$$\rho_t^{1-\pi} \leq \frac{(1-\gamma_t^{\varepsilon\pi} - \gamma_t^{(1-\varepsilon)\pi})R_{t+1}^N}{1-\pi}. \tag{7c}$$

If there is inter-location lending, (*7c'*) replaces (*7c*),

$$\rho_t^{1-\pi} \leq \frac{\gamma_t^{d(1-\pi)}R_{t+1}^N + (1-\gamma_t^{\varepsilon\pi} - \gamma_t^{(1-\varepsilon)\pi} - \gamma_t^{d(1-\pi)})R_{t+1}^{*N}\left(\frac{x_{t+1}}{x_t}\right)}{1-\pi}. \tag{7c'}$$

Finally, the bank's holdings of both types of currency must be non-negative, so that $\gamma_t^{\varepsilon\pi}, \gamma_t^{(1-\varepsilon)\pi} \geq 0.$ Loan holdings are not constrained to be non-negative; banks can borrow from entrepreneurs in principle.

The decision problem for a domestic bank is

$$maximize_{\left(\rho_t^{\varepsilon\pi}, \rho_t^{(1-\varepsilon)\pi}, \rho_t^{1-\pi},, \gamma_t^{\varepsilon\pi}, \gamma_t^{(1-\varepsilon)\pi} \geq 0, \gamma_t^{(1-\pi)d}\right)} \Big\{ln(y-d_t)$$

$$+ \ \beta\left(\varepsilon\pi\, ln(d_t\rho_t^{\varepsilon\pi}) + (1-\varepsilon)\pi\, ln\left(d_t\rho_t^{(1-\varepsilon)\pi}\right) + (1-\pi)\, ln(d_t\rho_t^{1-\pi})\right)\Big\}$$

subject to (7a), (7b), (7c), and $\gamma_t^{\varepsilon\pi}, \gamma_t^{(1-\varepsilon)\pi} \geq 0$ *(if loans not traded),* *(P1)*

subject to (7a), (7b), (7c'), and $\gamma_t^{\varepsilon\pi}, \gamma_t^{(1-\varepsilon)\pi} \geq 0$ *(if loans are traded).* *(P1')*

Foreign banks face exactly analogous problems, (P1*) and (P1*'), which I omit here for the sake of brevity.

2.6 Individual Optimization

Given the existence of banks that solve problems (P1) and (P1'), a young worker of generation *t* need only decide an allocation of income between consumption at date *t* and bank deposits, taking as given the gross returns on deposits offered by banks. A domestic young worker solves the problem

$$max_{c_{y,t}^N \geq 0} \ln(c_{y,t}^N) + \beta\left(\varepsilon\pi\, ln\left((y-c_{y,t}^N)\rho_t^{\varepsilon\pi}\right) + (1-\varepsilon)\pi\, ln\left((y-c_{y,t}^N)\rho_t^{(1-\varepsilon)\pi}\right)\right.$$

$$\left. +(1-\pi)\ln\left((y-c_{y,t}^N)\rho_t^{1-\pi}\right)\right). \tag{P2}$$

Foreign workers solve an analogous problem, (P2*), omitted here for brevity. A domestic young entrepreneur of generation $t \geq 1$ solves the inter-temporal consumption/saving-borrowing problem,

$$max_{c_{y,t}^T, c_{o,t+1}^T \geq 0, l_{e,t+1} \leq 0} \ln(c_{y,t}^T) + \beta\ln(c_{o,t+1}^T),$$

$$\text{subject to } c_{y,t}^T + l_{e,t+1} \leq 0,$$

$$c_{o,t+1}^T \leq q + l_{e,t+1}R_{t+1}^T. \tag{P3}$$

Here, $l_{e,t+1}$ is an entrepreneur's net claims to traded goods at *t+1*. Obviously, $l_{e,t+1} < 0$ is required for a young entrepreneur to accomplish positive young age consumption. Since all worker savings are intermediated, young entrepreneurs borrow from banks. Foreign entrepreneurs confront an analogous problem, (P3*), omitted here. Finally, an initial old agent in the domestic country solves the problem

$$max_{c_{o,1}^N}\ ln\left(c_{o,1}^N\right),$$

$$\text{subject to } c_{o,1}^N \leq \frac{M_0}{p_1^N} + \frac{(1-\psi)q}{p_1}, \tag{P4}$$

and the initial old agent in the foreign country solves an analogous problem denoted (P4*).

2.7 Government Policy

I assume that a government comprises a composite fiscal and monetary authority; it consumes non-traded goods, may accumulate foreign exchange reserves, and prints national fiat currency.

I assume that the domestic (foreign) government carries into the initial period an endowment of a stock of the other country's currency, $F_0 > 0$ $(F_0^* > 0)$. At every date, the domestic (foreign) government accesses both domestic (foreign) locations to purchase non-traded goods during local trade. I denote the per capita amount purchased by the domestic (foreign) government by $g_t(g_t^*)$. In addition, the domestic (foreign) government may purchase foreign (domestic) country's currency, in the amount of F_t (F_t^*) per capita at $t \geq 1$. I abstract from taxes and government debt, so each government must generate enough seigniorage revenue from outside money creation to finance its consumption and any changes in its net reserve position. The domestic (foreign) government increases the quantity of money outstanding in the hands of the public, M_t (M_t^*), relative to the stock outstanding at *t-1*, $M_{t-1}(M_{t-1}^*)$, using new currency to purchase non-traded goods and finance any changes in its foreign exchange reserves. Measured in non-traded goods, the government budget constraints at any $t \geq 1$ are,

$$\frac{M_t - M_{t-1}}{p_t^N} = g_t + \frac{e_t}{p_t^N}(F_t - F_{t-1}) - \frac{1}{p_t^N}(F_t^* - F_{t-1}^*), \tag{8a}$$

$$\frac{M_t^* - M_{t-1}^*}{p_t^{*N}} = g_t^* + \frac{(F_t^* - F_{t-1}^*)}{e_t p_t^{*N}} - \frac{(F_t - F_{t-1})}{p_t^{*N}}. \tag{8b}$$

I consider the following choices of policies. Each government sets a constant growth rate of the stock of its money that is outstanding in the hands of the public. This stabilization is equivalent to an inflation-targeting regime in the sense that in any steady state equilibrium the inflation rate of nominal prices of both non-traded and traded goods equals the money growth rate. At date 1, the domestic and foreign governments each set a constant money growth rate for all time; hence,

$$\frac{M_t}{M_{t-1}} = \sigma > 1, t \geq 1,$$

$$\frac{M_t^*}{M_{t-1}^*} = \sigma^*, > 1\ t \geq 1.$$

In addition, I consider two alternative real exchange rate regimes. In the first, real and nominal exchange rates are entirely market determined and neither government manipulates the relative value of currencies by altering the endowed initial period net reserve position which. For the domestic (foreign) government, this initial net reserve position is $F_0^* - e_1 F_0 \left(\frac{F_0^*}{e_1} - F_0^*\right)$. Reserve adjustments are therefore zero at every date, $F_t = F_{t-1} = F_0 \forall t \geq 1,\quad F_t^* = F_{t-1}^* = F_0^* \; \forall t \geq 1,$ and all seigniorage revenue generated by money creation is devoted to government consumption. In the second regime, the domestic government unilaterally targets its bilateral real exchange rate, selecting a constant value, $x_t = \bar{x}, \forall\, t.$ The foreign government is entirely passive, in that it does not respond to this exchange rate targeting policy. Hence $, F_t^* = F_{t-1}^* = F_0^*, \forall t.$ By contrast, the domestic government manipulates its foreign currency holdings at every date to attain, and maintain, the real exchange rate target. Under both exchange rate regimes, each government's consumption of non-traded goods is endogenously determined, and under the second regime any changes in the domestic government's foreign reserve is endogenous to the target value.

Letting $m_t \equiv \frac{M_t}{p_t^N}$, and $m_t^* \equiv \frac{M_t^*}{p_t^{*N}}$, and using the fact that $F_t^* = F_{t-1}^* = F_0^*, \forall t,$ I can simplify (8*a*) and (8*b*) to

$$m_t \left(\frac{\sigma - 1}{\sigma}\right) = g_t + \frac{e_t}{p_t^N}(F_t - F_{t-1}), \tag{8a'}$$

$$m_t^* \left(\frac{\sigma^* - 1}{\sigma^*}\right) = g_t^* - \frac{(F_t - F_{t-1})}{p_t^{*N}}. \tag{8b'}$$

Finally, I consider two, bilateral capital account regimes. The first prohibits international bank lending. The second allows it.

3. Equilibrium

To make things concrete, I assume that while the domestic country is a sufficiently large economy that it can potentially influence world prices, it is relatively poor in comparison to the rest of the world, which is the foreign country. In particular, I assume that the per capita output of both traded and non-traded goods is lower in the domestic country. In addition, I assume that domestic workers are relatively highly dependent on liquid assets – cash – and hence domestic country credit extension is relatively low. This assumption implies that the domestic country runs a trade and current account surplus under capital controls. These assumptions reflect in the following restrictions on the relative sizes of parameters.

Assumption 1. $a)\; q^* > q;\;\; b)\;\; y^* > y;\;\; c)\; \pi > \pi^*.$

In addition, I define two critically low values of the domestic country's liquidity demand, and impose assumption 2, throughout much of the following analysis.

Definition 1. $a)\ \tilde{\pi} \equiv \frac{q^*(1-\pi^*\varepsilon^*)}{q^*(1-\pi^*\varepsilon^*)+q(1-\pi^*)(1-\varepsilon)}$;

$b)\ \hat{\pi} \equiv \frac{q^*\pi^*(1-\varepsilon^*)}{q^*\pi^*(1-\varepsilon^*)+q(1-\pi^*)(1-\varepsilon)}$.

Assumption 2. $a)\ \pi > \tilde{\pi}\ ;\ b)\ q^*(1-\varepsilon^*) > q(1-\varepsilon)$.

Notice that, as $\tilde{\pi} \geq \hat{\pi}$, assumption 2 implies that $\pi > \hat{\pi}$. It also implies a stronger restriction on the value of domestic liquidity than assumption 1 *c)*.

Definition 2. *An equilibrium is prices,* $\{p_t^N, p_t^{*N}, p_t^T, p_t^{*T}, e_t, p_t, p_t^*, x_t, R_{t+1}^T, R_{t+1}^{*T}, R_{t+1}^N, R_{t+1}^{*N}\}_{t=1}^{\infty}$, *, deposit returns,* $\left\{\rho_t^{\pi\varepsilon}, \rho_t^{*\pi\varepsilon}, \rho_t^{\pi(1-\varepsilon)}, \rho_t^{*\pi(1-\varepsilon)}, \rho_t^{(1-\pi)}, \rho_t^{*(1-\pi)}\right\}_{t=1}^{\infty}$, *an allocation for workers,* $\{c_{y,t}^N, c_{y,t}^{*N}, d_t, d_t^*, c_{o,t+1}^N, c_{o,t+1}^{*N},\}_{t=1}^{\infty}$, *for entrepreneurs,* $\{c_{y,t}^T, c_{y,t}^{*T}, c_{o,t+1}^T, c_{o,t+1}^{*T}, l_{e,t+1}, l_{e,t+1}^*\}_{t=1}^{\infty}$, *for initial old agents,* $\{c_{o,1}^N, c_{o,1}^{*N}\}$, *for banks,* $\left\{\gamma_t^{\pi\varepsilon}, \gamma_t^{*\pi\varepsilon}, \gamma_t^{\pi(1-\varepsilon)}, \gamma_t^{*\pi(1-\varepsilon)}, \gamma_t^{d(1-\pi)}, \gamma_t^{*d(1-\pi)}\right\}_{t=1}^{\infty}$, *and for governments,* $\{g_t, g_t^*, F_t, F_t^*\}_{t=1}^{\infty}$, *and policies,* $\{\sigma, \sigma^*, \bar{x}\}$, *such that:*

i) *Given prices, the deposit returns and allocation for banks solve (P1) and (P1*) if there is no loan trade and (P1') and (P1*') if there is trade in loans;*

ii) *Given prices and deposit returns, the allocation for workers solves (P2) and (P2*);*

iii) *Given prices and deposit returns, the allocation for entrepreneurs solves (P3) and (P3*);*

iv) *Given prices, the allocation for the initial old agents solves (P4) and (P4*);*

v) *Given prices and policies, the allocation for governments satisfies the budget constraints (8a') and (8b');*

vi) *Real interest rates satisfy return domination of money:* $R_{t+1}^N > \frac{p_t^N}{p_{t+1}^N}$; $R_{t+1}^{*N} > \frac{p_t^{*N}}{p_{t+1}^{*N}}\ \forall t \geq 1$;

vii) *Domestic and foreign currency markets, domestic and foreign loan markets, domestic and foreign non-traded goods markets, and the global market for traded goods must clear at every date,* $t \geq 1$.

3.1 Optimal allocations

The solution to banks' problem sets $\gamma_t^{\varepsilon\pi} = \varepsilon\pi, \gamma_t^{(1-\varepsilon)\pi} = (1-\varepsilon)\pi$, and $\left(1-\gamma_t^{\varepsilon\pi}-\gamma_t^{(1-\varepsilon)\pi}\right) = 1-\pi$. If there is international trade in loans, arbitrage equalizes loan returns across countries,

$$R_{t+1}^N = R_{t+1}^{*N}(x_{t+1}/x_t),$$

so that the composition of a bank's loan portfolio measured by $\gamma_t^{d(1-\pi)}$ and $(1-\gamma_t^{\varepsilon\pi}-\gamma_t^{(1-\varepsilon)\pi}-\gamma_t^{d(1-\pi)})$ is indeterminate. The state contingent gross deposit returns offered by domestic banks are $\rho_t^{\varepsilon\pi} = \frac{p_t^N}{p_{t+1}^N}$ to workers subject to domestic relocation, $\rho_t^{(1-\varepsilon)\pi} = \frac{p_t^N}{p_{t+1}^N}\frac{e_{t+1}}{e_t}$ for workers subject to international relocation, and $\rho_t^{1-\pi} = R_{t+1}^N$ for non-movers. The solutions for foreign banks are analogous.

The solution to the problem of a domestic worker, (P2), sets

$$c_{yt}^N = \frac{y}{1+\beta};\ d_t = \frac{y\beta}{1+\beta}.$$

Given the solutions to the bank's problem, the domestic non-traded goods value of a domestic generation *t* worker's old age consumption if domestically relocated is $c_{o,t+1}^{N\varepsilon\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{p_t^N}{p_{t+1}^N}\right)$, if internationally relocated is $c_{o,t+1}^{N(1-\varepsilon)\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{p_t^N}{p_{t+1}^N}\right)\left(\frac{e_{t+1}}{e_t}\right)$, and if a non-mover is $c_{o,t+1}^{N(1-\pi)} = \left(\frac{y\beta}{1+\beta}\right)R_{t+1}^N$. The solution to (P2*) for a foreign young worker is analogous. The optimal consumption and loan allocations to (P3) for a generation *t* domestic young entrepreneur are

$$c_{y,t}^T = \frac{q}{(1+\beta)R_{t+1}^T};\ l_{e,t+1} = \frac{-q}{(1+\beta)R_{t+1}^T};\ c_{o,t+1}^T = \frac{\beta q}{(1+\beta)}.$$

Analogous solutions obtain for foreign entrepreneurs solving (P3*). The solution to (P4) for an initial old agent in the domestic country simply sets $c_{o,1}^N = \frac{M_0}{p_1^N} + \frac{(1-\psi)q}{p_1}$, and an analogous solution obtains for foreign initial old agents.

3.2 Market clearing

3.2.1 Money markets In equilibrium, domestic and foreign bank per capita demand for domestic currency must equal the per capita supply of currency by the domestic government that is in the hands of the public. In per capita, domestic non-traded goods, $t \geq 1$

$$m_t = \frac{\varepsilon\pi\psi\beta y}{1+\beta} + \frac{(1-\varepsilon^*)\pi^*\psi\beta y^* x_t}{1+\beta}. \qquad (10a)$$

Similarly, domestic and foreign bank per capita demand for foreign currency must equal the per capita supply of currency by the foreign government. In per capita, foreign non-traded goods, $t \geq 1$

$$m_t^* = \frac{\varepsilon^*\pi^*\psi\beta y^*}{1+\beta} + \frac{(1-\varepsilon)\pi\psi\beta y/x_t}{1+\beta}. \qquad (10b)$$

3.2.2. Loan markets Loan markets must clear locally when capital controls are in place. Thus the per-entrepreneur supply of loans by banks within each country must equal the per entrepreneur demand for loans, measured in traded goods, in (each location of) that country. For, $t \geq 1$, $l_{e,t+1} + l_{t+1} = 0$ and $l_{e,t+1}^* + l_{t+1}^* = 0$. Given the optimal loan choices of young entrepreneurs and banks, the domestic and foreign loan market clearing conditions are, $\forall t \geq 1$,

$$\frac{(1-\pi)\psi\beta y p_t}{(1+\beta)} = \frac{(1-\psi)q}{(1+\beta)R_{t+1}^T}, \qquad (11a)$$

$$\frac{(1-\pi^*)\psi\beta y^* p_t^*}{(1+\beta)} = \frac{(1-\psi)q^*}{(1+\beta)R_{t+1}^{*T}}. \qquad (11b)$$

By contrast, under free capital flows, banks may lend to foreign entrepreneurs. Hence, there is a single loan market clearing condition,

$$\frac{(1-\pi)\psi\beta y p_t}{(1+\beta)} + \frac{(1-\pi^*)\psi\beta y^* p_t^*}{(1+\beta)} = \frac{(1-\psi)q}{(1+\beta)R_{t+1}^{T}} + \frac{(1-\psi)q^*}{(1+\beta)R_{t+1}^{*T}}. \qquad (11a')$$

In addition, the no-arbitrage condition for real returns offered to non-movers by banks, measured in non-traded goods holds, $R_{t+1}^N = R_{t+1}^{*N}(x_{t+1}/x_t)$, or, equivalently, real returns measured in traded goods are equal across countries,

$$R_{t+1}^T = R_{t+1}^N(p_{t+1}/p_t) = R_{t+1}^{*N}(x_{t+1}/x_t)(p_{t+1}/p_t) = R_{t+1}^{*N}(p_{t+1}^*/p_t^*) = R_{t+1}^{*T}. \qquad (11b')$$

Equations (11a') and (11b') replace (11a) and (11b) as equilibrium conditions in the economy with free capital flows. Notice that, combined, they imply

$$\frac{(1-\pi)\psi\beta y p_t}{(1+\beta)} + \frac{(1-\pi^*)\psi\beta y^* p_t^*}{(1+\beta)} = \frac{(1-\psi)(q+q^*)}{(1+\beta)R_{t+1}^{T}}. \qquad (11c)$$

3.2.3 Non-traded goods markets At date 1, the per capita supply of non-traded goods within each location must equal the per capita consumption of young workers and the government, plus the per capita consumption of initial old agents. The non-traded goods market clearing conditions in the domestic and foreign country respectively are therefore $\psi y = \frac{\psi y}{1+\beta} + \frac{M_0}{p_1^N} + \frac{(1-\psi)q}{p_1} + g_1$ and $\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{M_0^*}{p_1^{*N}} + \frac{(1-\psi)q^*}{p_1^*} + g_1^*$. Using the government budget constraints (8*a*') and (8*b*'), and substituting in date-1 real balances from money market clearing, I can rewrite the goods market clearing conditions as

$$\psi y = \frac{\psi y}{1+\beta} + \frac{\varepsilon\pi\psi\beta y}{1+\beta} + \frac{(1-\varepsilon^*)\pi^*\psi\beta y^* x_1}{1+\beta} + \frac{(1-\psi)q}{p_1} - \frac{e_1}{p_1^N}(F_1 - F_0), \qquad (12a)$$

$$\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{\varepsilon^*\pi^*\psi\beta y^*}{1+\beta} + \frac{(1-\varepsilon)\pi\psi\beta y/x_1}{1+\beta} + \frac{(1-\psi)q^*}{p_1^*} + \frac{1}{p_1^{*N}}(F_1 - F_0) \qquad (12b)$$

Equations (12*a*) and (12*b*) hold irrespective of the capital market regime.

At all other dates, $t \geq 2$, the per capita supply of non-traded goods within each location must equal the per capita consumption of young workers and the government, plus the per capita consumption of old workers, some of which have been relocated from elsewhere bringing the entire outstanding per capita money supply of that location with them. Then, in the domestic and foreign country respectively, $\psi y = \frac{\psi y}{1+\beta} + m_{t-1}\left(\frac{p_{t-1}^N}{p_t^N}\right) + \frac{(1-\pi)\psi\beta\, y}{1+\beta}R_t^N + g_t$, and $\psi y^* = \frac{\psi y^*}{1+\beta} + m_{t-1}^*\left(\frac{p_{t-1}^{*N}}{p_t^{*N}}\right) +$

$\frac{\psi\beta\, y^*(1-\pi^*)}{1+\beta} R_t^{*N} + g_t^*$. Using the government budget constraints, and the fact that $m_{t-1}\left(\frac{p_{t-1}^N}{p_t^N}\right) = \frac{M_{t-1}}{p_{t-1}^N}\left(\frac{p_{t-1}^N}{p_t^N}\right) = \frac{M_t}{\sigma}\left(\frac{1}{p_t^N}\right) = \frac{m_t}{\sigma}\ \left[m_{t-1}^*\left(\frac{p_{t-1}^{*N}}{p_t^{*N}}\right) = \frac{m_t^*}{\sigma^*}\right]$,

$$\psi y = \frac{\psi y}{1+\beta} + \frac{\varepsilon\pi\psi\beta y}{1+\beta} + \frac{(1-\varepsilon^*)\pi^*\psi\beta y^* x_t}{1+\beta} + \frac{(1-\pi)\psi\beta y}{1+\beta} R_t^N - \frac{e_t}{p_t^N}(F_t - F_{t-1}), \qquad (13a)$$

$$\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{\varepsilon^*\pi^*\psi\beta y^*}{1+\beta} + \frac{(1-\varepsilon)\pi\psi\beta y}{(1+\beta)x_t} + \frac{(1-\pi^*)\psi\beta y^*}{1+\beta} R_t^{*N} + \frac{1}{p_t^{*N}}\ (F_t - F_{t-1}). \quad (13b)$$

Under free capital flows, because of arbitrage, (13*a*) and (13*b*) are

$$\psi y = \frac{\psi y}{1+\beta} + \frac{\varepsilon\pi\psi\beta y}{1+\beta} + \frac{(1-\varepsilon^*)\pi^*\psi\beta y^* x_t}{1+\beta} + \frac{(1-\pi)\psi\beta y}{1+\beta} R_t^N - \frac{e_t}{p_t^N}(F_t - F_{t-1}), \qquad (13a')$$

$$\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{\varepsilon^*\pi^*\psi\beta y^*}{1+\beta} + \frac{(1-\varepsilon)\pi\psi\beta y}{(1+\beta)x_t} + \frac{(1-\pi^*)\psi\beta y^*}{1+\beta} \frac{R_t^N x_{t-1}}{x_t} + \frac{(F_t - F_{t-1})}{p_t^{*N}}. \qquad (13b')$$

3.2.4 Traded goods market At date 1, the world traded goods market clearing condition requires that the supply of traded goods equals the demand for traded goods from young entrepreneurs in both the domestic and foreign country,

$$q + q^* = \frac{q}{(1+\beta)R_2^T} + \frac{q^*}{(1+\beta)R_2^{*T}}. \qquad (14)$$

At all other dates, the supply must equal demand from both young and old entrepreneurs in both countries,

$$q + q^* = \frac{q}{(1+\beta)R_{t+1}^T} + \frac{q^*}{(1+\beta)R_{t+1}^{*T}} + \frac{(q+q^*)\beta}{(1+\beta)}. \qquad (15)$$

Under free capital flows with a unified world loan market and interest rate, traded goods market clearing in the initial period must satisfy

$$q + q^* = \frac{q+q^*}{(1+\beta)R_2^T}, \qquad (14')$$

and traded goods market clearing at every date, $t > 1$, must satisfy

$$q + q^* = \frac{q+q^*}{(1+\beta)R_{t+1}^T} + \frac{(q+q^*)\beta}{(1+\beta)}. \qquad (15')$$

If there are capital controls, of the nine equilibrium conditions – seven market-clearing conditions and the two government budget constraints (8*a*') and (8*b*') – eight are independent. Under free capital flows, the nine equilibrium conditions constitute six market clearing conditions as there is a unified world loan market, the no-arbitrage condition, and the two government budget constraints. Which eight variables these eight independent equations determine under each capital market regime depends on the real exchange rate regime.

4. Capital Controls: No Real Exchange Rate Targeting

4.1 Steady State Equilibrium

I first investigate the existence and properties of steady state equilibria. In a steady state equilibrium, all of the conditions of definition 2 are satisfied, and all real endogenous variables are constant. The economy is stationary from date 2 onwards, and can attain a steady state equilibrium at this date.

For real money balances in each country to be constant over time, since the nominal money stock of each country grows at a constant policy determined rate, the non-traded goods price inflation rate of that country must grow at the same constant growth rate of the local money supply, $\frac{p_{t+1}^N}{p_t^N} = \sigma, \frac{p_{t+1}^{*N}}{p_t^{*N}} = \sigma^*$. Then, for the real exchange rate to be constant requires that the nominal exchange rate growth rate satisfy $\frac{e_{t+1}}{e_t} = \frac{x_{t+1}(p_{t+1}^N/p_{t+1}^{*N})}{x_t(p_t^N/p_t^{*N})} = \frac{(p_{t+1}^N/p_{t+1}^{*N})}{(p_t^N/p_t^{*N})} = \left(\frac{\sigma}{\sigma^*}\right)$. For a stationary relative price of non-traded goods in each country, traded good price levels must obviously grow at the same constant rates as non-traded good price levels; $\frac{p_{t+1}^T}{p_t^T} = \sigma, \frac{p_{t+1}^{*T}}{p_t^{*T}} = \sigma^*$.

Finally, constant real interest rates on tradable claims, $R_{t+1}^T = R^T$ and $R_{t+1}^{*T} = R^{*T}$, together with constant internal relative prices imply that real interest rates measured in non-traded goods equal real rates measured in traded goods within each country,

$$R_{t+1}^N = R_{t+1}^T \frac{p_t}{p_{t+1}} = R^T = R^N; \; R_{t+1}^{*N} = R_{t+1}^{*T} \frac{p_t^*}{p_{t+1}^*} = R^{*T} = R^{*N}.$$

Assuming that all real endogenous variables are constant, substituting the money market clearing conditions and government budget constraints into the non-traded goods market clearing conditions, combining these with the loan market clearing conditions, and using $p^* = xp$, yields two equations that jointly determine the steady state real exchange rate and domestic relative price of non-traded goods,

$$x = \frac{\psi\beta y(1-\varepsilon\pi) - (1-\psi)q/p}{\psi\beta y^*(1-\varepsilon^*)\pi^*}, \tag{16a}$$

$$x = \frac{\psi\beta y(1-\varepsilon)\pi + (1-\psi)q^*/p}{\psi\beta y^*(1-\varepsilon^*\pi^*)}. \tag{16b}$$

(16*a*) measures the relationship between the real exchange rate and relative price of non-traded goods in the domestic country. Given the available supply of domestic non-traded goods, an increase in the relative price of non-traded goods, *p*, by reducing the non-traded goods value of bank payouts to non-movers and hence their purchasing power requires an increase in the real exchange rate and hence the purchasing power over domestic non-traded goods of relocated foreign workers. (16*b*) measures the negative relationship between the real exchange rate and relative price of non-traded goods in the foreign country. Given the available supply of foreign non-traded goods, an increase in the domestic relative price of non-traded goods, p, increases the foreign relative price of non-traded goods $p^* = xp$. This reduces the non-traded goods value of bank payouts to foreign non-movers, and hence their purchasing power, which can be directly offset by a decrease in the real exchange rate. Such a decrease also fosters equilibrium by raising the purchasing power over foreign non-traded goods of relocated domestic workers. As $\frac{(1-\varepsilon)\pi}{(1-\varepsilon\pi^*)} < \frac{(1-\varepsilon\pi)}{(1-\varepsilon)\pi^*}$, there is a unique intersection of (16*a*) and (16*b*) at a strictly positive and finite value of the real exchange rate and of the relative price of domestic (and hence foreign) non-traded goods. Figure 2 illustrates this determination. Specifically,

$$0 < x \in \left(\frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*\pi^*)}, \frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right),$$

$$0 < p \in \left(\frac{(1-\psi)q}{(1-\varepsilon\pi)\psi\beta y}, \infty\right).$$

This implies there exists at most one steady state equilibrium, with existence conditional on return domination of money, which proposition 1 (below) addresses. In addition, there exist unique, strictly positive, and finite associated values of R^T and R^{*T} respectively that clear the loan markets (11*a*) and (11*b*) at the relative prices, $p, x,$ and $p^* = xp$ satisfying (16*a*) and (16*b*). Steady state solutions for all other endogenous real variables follow immediately. The steady state traded goods market clearing condition is not independent of the remaining conditions, and can be expressed as

$$R^T = \frac{q}{(q+q^*) - \frac{q^*}{R^{*T}}}. \tag{17}$$

Figure 3 depicts this relationship. Evidentally, $R^T \gtreqless 1$ iff $R^{*T} \lesseqgtr 1$.

Proposition 1. ***Steady state equilibrium under capital controls***

Let assumption 2 hold. Then there exists a unique steady state equilibrium with

$$R^T > 1, R^{*T} < 1, R^N > \frac{p_{t-1}^N}{p_t^N} = \frac{1}{\sigma}, \text{and } R^{*N} > \frac{p_{t-1}^{*N}}{p_t^{*N}} = \frac{1}{\sigma^*}$$

iff $$\sigma^* > \left(\left(\frac{q^*}{1-\pi^*}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}{q^*(1-\varepsilon\pi)+q(1-\varepsilon)\pi}\right)\right)^{-1} > 1.$$

Proof. The solutions for relative prices that satisfy all of the market clearing conditions are,

$$p = \left(\frac{(1-\psi)}{\psi\beta y}\right)\left(\frac{q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - (1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}\right),$$

$$x = \left(\frac{y}{y^*}\right)\left(\frac{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)}\right),$$

$$p^* = \left(\frac{(1-\psi)}{\psi\beta y^*}\right)\left(\frac{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}\right),$$

$$R^T = R^N = \left(\frac{q}{1-\pi}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi(1-\varepsilon)\pi^*(1-\varepsilon^*)}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$R^{*T} = R^{*N} = \left(\frac{q^*}{1-\pi^*}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi(1-\varepsilon)\pi^*(1-\varepsilon^*)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right).$$

The existence of a steady state equilibrium satisfying return domination of money in each country requires that $R^N > \frac{1}{\sigma}$ and $R^{*N} > \frac{1}{\sigma^*}$, in addition to satisfaction of the optimality, government budget constraint, and market clearing conditions of definition 2. As is apparent from an inspection of figure 2, since $\frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*} > \frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}$, the unique intersection of (16*a*) and (16*b*) always lies at strictly positive and finite values of *x* and *p*, which are necessary for the consumption and loan allocations resulting from the relative prices above, and satisfying all of the other conditions of equilibrium, to take admissible values. Hence, all that we need to show is the currency of each country is return-dominated by loans, so that banks hold currency solely to meet liquidity needs as definition 2 presumes, at the solution given by the intersection of (16*a*) and (16*b*). Manipulating the solution for $R^T = R^N$ and using definition 1, $R^T \gtreqless 1 \; iff \; \pi \gtreqless \hat{\pi}$. Hence, under assumption 2, $R^N = R^T > 1$, and so satisfies $R^T = R^N > \frac{1}{\sigma}$. However, under assumption 2, as manipulation of (17) verifies, $R^{*T} < 1$. Thus, currency is dominated in rate of return by loans in the foreign country at the solution to (16*a*) and (16*b*) iff $1 > R^{*N} = R^{*T} > \frac{1}{\sigma^*}$. The condition of proposition 1 follows immediately. ■

I record the complete set of steady state equilibrium private consumption and loan allocations, and steady state equilibrium bank and government allocations, in Appendix A. Manipulating the steady state solutions for relative prices yields the following proposition, the proof of which I omit for brevity.

Proposition 2. *Comparative statics*

a) An increase in the domestic country's relative supply of non-traded goods, y/y^*, *raises (depreciates) its real exchange rate, x.*

b) An increase in the domestic country's relative supply of traded goods, q/q^*, *reduces (appreciates) its real exchange rate, x.*

c) An increase in the domestic country's bank portfolio weight on liquid assets, π, *raises – depreciates – the domestic country's real exchange rate, x, iff the portion of the liquid portfolio weight assigned to domestic currency is sufficiently low and that of foreign currency sufficiently high: Specifically ,iff* $\varepsilon < \frac{q}{q+q^*}$.

d) An increase in the foreign country's bank portfolio weight on liquid assets, π^*, raises – depreciates – the domestic country's real exchange rate, x, iff the portion of the liquid portfolio weight assigned to foreign currency is sufficiently high and that of domestic currency sufficiently low: Specifically ,iff $\varepsilon^* > \frac{q^*}{q+q^*}$.

Proposition 2 *a)* and *b)* illustrate the classical nature of long-run real exchange rate determination of this economy. Parts *c)* and *d)* are intuitively clear, and show how banking and monetary factors – although not monetary policy – directly influence the steady state real exchange rate. The independence of the steady state real exchange rate from money growth rates contrasts sharply with the implications for real exchange rate determination of money growth rates in Betts and Smith (1997).

4.2 External Balance

At every date in the steady state equilibrium the relative size of real interest rates in the two countries determines which country runs a trade deficit and which a trade surplus in traded goods. The consequence of the domestic country exhibiting a relatively high steady state equilibrium real interest rate, under assumption 2, is that domestic young entrepreneurs borrow and consume relatively few traded goods, and the domestic country runs a trade surplus on these goods as a result. The steady state, per entrepreneur, external balance of the domestic country in traded goods is just $TB^T = q - c_y^T - c_o^T$. Substituting for young and old entrepreneurs' steady state equilibrium consumption, this balance is

$$TB^T = \left(\frac{1}{1+\beta}\right)\left(q - \frac{(1-\pi)\big(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\big)}{\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - (1-\varepsilon^*)\pi^*(1-\varepsilon)\pi\,\big)}\right). \tag{18}$$

The domestic country's steady state per worker financial balance is just the difference between foreign purchases of domestic currency and domestic bank purchases of foreign currency at every date, $FB = \frac{(1-\varepsilon^*)\pi^*\beta y^* x}{1+\beta} - \frac{(1-\varepsilon)\pi\beta y}{1+\beta}$. This difference represents inter-temporal trade between entrepreneurs and workers, intermediated by banks. Any trade surplus (deficit) funds (is funded by) a financial balance deficit (surplus), involving higher (lower) domestic purchases of foreign currency – a liability of the foreign government – than foreign purchases of domestic currency – a liability of the domestic government – measured in domestic non-traded goods. Redemption of each country's currency occurs internally, in that country's non-traded goods in the following period, thus, external balance is unaffected by redemptions and returns. Substituting the steady state equilibrium real exchange rate into the expression for the steady state financial balance of the domestic country yields

$$FB = \frac{\beta y}{1+\beta}\left((1-\varepsilon^*)\pi^*\left(\frac{q^*(1-\varepsilon\pi) + q(1-\varepsilon)\pi}{q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*))}\right) - (1-\varepsilon)\pi\right). \tag{19}$$

Proposition 3. ***Steady state trade balance under capital controls***

Let assumption 2 hold. Then $TB^T > 0$ *And* $FB < 0$.

Proof. From equation (18), $TB^T > 0$ iff $q > \frac{(1-\pi)\left(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\right)}{\left((1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - (1-\varepsilon^*)\pi^*(1-\varepsilon)\pi\right)}$. Using definition 1, this condition is equivalent to $\pi > \hat{\pi}$. Similarly, manipulating equation (19) gives $FB \gtreqless 0$ *iff* $\pi \lesseqgtr \hat{\pi}$. Under assumption 2, therefore, the domestic country's steady state financial balance is negative. ∎

It is straightforward to verify that the sum of the per capita values of the two balances measured in traded goods – the steady state equilibrium balance of payments of the domestic country – is zero as required for external equilibrium, $(1-\psi)TB^T + \psi FB \times p = 0$. In a steady state equilibrium, under assumption 2, the domestic (foreign) country permanently runs a trade surplus (deficit) on traded goods. This finances a domestic (foreign) country "financial balance" deficit (surplus). Specifically, the domestic country's trade surplus finances a higher domestic non-traded goods value of domestic purchases of foreign country currency than foreign country purchases of domestic currency.

4.3 The Initial Period and "Dynamic" Equilibrium

The economy cannot attain its steady state in the initial period because initial old agents exchange claims to the entire date 1 output of traded goods for non-traded goods with initial young workers. At every other date, old non-movers exchange claims to traded goods for non-traded goods by writing checks on bank loans to young entrepreneurs in the previous period. These claims reflect the optimal demand for consumption loans by young entrepreneurs in the previous period, which is only a fraction of the entire value of traded goods' output. However, all of the optimality, market clearing conditions, and government budget constraints are identical at every date from *t=2* onwards and are completely static; they are the steady state equilibrium conditions. The economy can thus attain the unique steady state equilibrium analyzed in sections 4.1 and 4.2 at date 2. Thus, there exists a "dynamic equilibrium" comprising the solutions to the initial period equilibrium conditions and an infinite sequence of steady state solutions from date 2 onwards, if the optimality and market clearing conditions, and the government budget constraints, are satisfied in period 1 at interest rates satisfying return domination of currency.

In Appendix B, I describe how the economy attains equilibrium solutions at date 1 in detail, and the determination of initial period external balance. Figure 4 depicts the determination of non-traded goods market clearing in the initial period, which is almost identical to that at every other date and in the steady state depicted in Figure 2, and Figure 5 shows the relation of initial period real interest rates. Here, I simply state the key results.

Proposition 4. ***Dynamic equilibrium under capital controls***

Let assumption 2 hold. Then there exists a unique "dynamic" equilibrium, with $R_2^T > \frac{1}{1+\beta}$, $R_2^{*T} < \frac{1}{1+\beta}$, $R_2^N > \frac{p_1^N}{p_2^N}$, $and\ R_2^N > \frac{p_1^N}{p_2^N}$; $\forall t > 1,\ R_{t+1}^T = R^T > 1, R_{t+1}^{*T} = R^{*T} < 1,\ \ R_{t+1}^N = R^N > \frac{1}{\sigma}$ *and* $R_{t+1}^{*N} = R^{*N} > \frac{1}{\sigma^*}$, *iff*

$$\sigma^* > \left(\left(\frac{q^*}{(1-\pi^*)}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}{q^*(1-\varepsilon\pi)+q(1-\varepsilon)\pi}\right)\right)^{-1}.$$

Proof. See Appendix B.

I record the full set of initial period consumption and asset allocations in Appendix B.

Proposition 5. ***Initial period trade balance under capital controls***

Let assumption 2 hold. Then $TB_1^T > 0$ *and* $FB_1 < 0$.

Proof. From equation (22), $TB_1^T > 0$ iff $q > \frac{(1-\pi)\left(q^*(1-\varepsilon^*)\pi^*+q(1-\varepsilon^*\pi^*)\right)}{\left((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi\ \right)}$. Using definition 1, this condition is equivalent to $\pi > \hat{\pi}$. Similarly, manipulating equation (19) gives $FB \gtreqless 0$ *iff* $\pi \lesseqgtr \hat{\pi}$. Under assumption 2, the domestic country's steady state financial balance is negative. ■

It is straightforward to verify that the sum of the per capita values of the two initial period balances measured in traded goods – the steady state equilibrium balance of payments of the domestic country – is zero, as required for external equilibrium, $(1-\psi)TB_1^T + \psi FB_1 \times p_1 = 0$.

5. Capital Controls: Real Exchange Rate Targeting

I assume that, at date $\hat{T}$, the domestic government unilaterally assumes a constant, bilateral real exchange rate target $\bar{x}_t, \bar{x}_t = \bar{x}, \forall t \geq \hat{T}$. Although the attainment and sustainability of targets that are more depreciated or "competitive" than the steady state equilibrium real exchange rate, $\bar{x} > x$, are of primary interest, wherever possible I derive results for all admissible target values. I assume that the economy has been in a steady state equilibrium with no real exchange rate target featuring the properties described in section 5.1, and that the policy takes effect unexpectedly from the perspective of private agents, including banks. However, from date $\hat{T}$ onwards, all agents have perfect foresight. I subscript variables determined in the last period of the steady state equilibrium by $\hat{T}-1$. In addition, to distinguish the values of endogenous variables under real exchange rate targeting from those in the absence of a real exchange rate target, I denote variable z by $\hat{z}$.

To foreshadow what follows, under this policy regime when $\bar{x} > x$ $(\bar{x} < x)$, in equilibrium the domestic country government purchases additional (sells) foreign currency at date $\hat{T}$, $\hat{F}_{\hat{T}} > F_0$ ($\hat{F}_{\hat{T}} < F_0$) to establish the target. When $\bar{x} > x$, the domestic government, by increasing its reserves by the same real value at every date, can maintain the real exchange rate target indefinitely in a steady state equilibrium. Furthermore, the economy can attain this steady state equilibrium at date $\hat{T}$. A one-time adjustment of price levels, relative prices, and allocations occurs at date $\widehat{T}$, in response to the policy shock, relative to the previous steady state, and thereafter no change in any real endogenous variable occurs. Furthermore, given the target value, $\bar{x} > x$, the steady state equilibrium is unique. By contrast, the domestic government cannot indefinitely *sell* reserves and maintain a constant real reserve adjustment value, as would be needed to sustain a more appreciated real exchange rate target, $\bar{x} < x$, in a steady state equilibrium. Consequently, although a government can potentially establish a relatively appreciated real exchange rate target at date $\widehat{T}$, such a policy is not part of any equilibrium. I now develop and formalize these results.

5.1 Initial Period of the Targeting Regime, $\widehat{\mathbf{T}}$

In the initial period of the targeting regime, the domestic (foreign) money market clearing condition, (10*a*) [(10*b*)], with money demand evaluated at $\bar{x}$, dictates the value of initial domestic (foreign) real balances consistent with the domestic government's real exchange rate target. For exogenously given money growth rates, this implies there exists a unique value of the date $\widehat{T}$ nominal price of domestic and foreign non-traded goods that is consistent with the real exchange rate target. Specifically, domestic and foreign money market clearing conditions imply that non-traded good price levels in period $\widehat{T}$ satisfy

$$\hat{p}^N_{\hat{T}} = \frac{\sigma M_{\hat{T}-1}(1+\beta)}{\psi\beta(\varepsilon\pi y+(1-\varepsilon^*)\pi^* y^* \bar{x})}, \tag{20a}$$

$$\hat{p}^{*N}_{\hat{T}} = \frac{\sigma^* M^*{}_{\hat{T}-1}(1+\beta)\bar{x}}{\psi\beta(\varepsilon^*\pi^* y^* \bar{x}+(1-\varepsilon)\pi y)}. \tag{20b}$$

For $\bar{x} > x$ ($\bar{x} < x$), the date $\widehat{T}$ domestic nominal price of non-traded goods is lower (higher) than it would have been in the non-targeting steady state equilibrium, accommodating the higher (lower) purchasing power of internationally relocated foreign workers holding domestic currency. The converse statements can be made of the foreign country nominal price of non-traded goods. The equilibrium value of the domestic country's initial nominal exchange rate is immediately determined for a given real exchange rate target, by $\hat{e}_{\hat{T}} = \frac{\bar{x}\hat{p}^N_{\hat{T}}}{\hat{p}^{*N}_{\hat{T}}}$,

$$\hat{e}_{\hat{T}} = \left(\frac{\sigma M_{\hat{T}-1}}{\sigma^* M^*_{\hat{T}-1}}\right)\left(\frac{\varepsilon^*\pi^* y^* \bar{x} + (1-\varepsilon)\pi y}{\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \bar{x}}\right). \tag{20c}$$

Proposition 6. $\frac{\partial \hat{e}_{\hat{T}}}{\partial \bar{x}} \gtreqless 0 \; iff \; \varepsilon \gtreqless 1-\varepsilon^*$.

Proof. This result follows from (20*c*), manipulation of which yields

$$\frac{\partial \hat{e}_{\hat{T}}}{\partial \bar{x}} = \left(\frac{\sigma M_{\hat{T}-1}}{\sigma^* M^*_{\hat{T}-1}}\right)\left(\frac{\pi y \pi^* y^* (\varepsilon+\varepsilon^*-1)}{(\varepsilon\pi y+(1-\varepsilon^*)\pi^* y^* \bar{x})^2}\right) \gtreqless 0 \; \textit{iff} \; \varepsilon \gtreqless 1-\varepsilon^*. \; \blacksquare$$

Proposition 6 implies that the domestic country's nominal exchange rate must depreciate (appreciate) at $\hat{T}$ with the establishment of a relatively depreciated (appreciated) real exchange rate target *if and only if* the fraction of domestic agents requiring domestic currency to consume is sufficiently high (low) relative to the fraction of foreign agents demanding domestic currency. The nominal price of domestic non-traded goods adjusts downwards (upwards) to accommodate a higher (lower) domestic real exchange rate, as we have seen, while the foreign country's price of non-traded goods rises (falls). Intuitively, only if the foreign private bank demand for domestic currency and domestic private bank demand for foreign currency is sufficiently "weak" ("strong") in the sense that $\varepsilon > 1 - \varepsilon^*$ ($\varepsilon < 1 - \varepsilon^*$) does the domestic country's currency depreciate (appreciate) in nominal terms to attain a depreciated (appreciated) real exchange rate target. For China, in light of the financial restrictions placed on domestic holdings of foreign currency and vice versa, it seems natural to assume that $\varepsilon > 1 - \varepsilon^*$ holds. Nonetheless, in general, while a positive nominal reserve adjustment needed to establish a more depreciated real exchange rate depreciates the domestic country's nominal exchange rate, the oft-assumed mechanism for currency manipulation, the equilibrium nominal exchange rate may appreciate.

As in any steady state equilibrium with a constant equilibrium real exchange rate, a constant real exchange rate target implies constant real balances in each country, as (10*a*) and (10*b*) show. After date $\hat{T}$, therefore, the domestic (foreign) nominal non-traded goods price rises at the rate of domestic (foreign) money growth, exactly the inflation rates that we observe in the economy with a market determined real exchange rate. Consequently, after $\hat{T}$, the nominal exchange rate must rise at the constant rate $\frac{\sigma}{\sigma^*}$ to maintain the real exchange rate target, exactly as the nominal exchange rate changes over time in the equilibria of the economy without a real exchange rate target. A single, date $\hat{T}$ adjustment in these nominal prices transitions the economy from the non-targeting steady state to a targeting regime with the same intertemporal nominal behavior.

For $\bar{x} > x$, in non-traded goods markets – given a fixed supply of goods – the higher purchasing power over domestic non-traded goods of relocated foreign workers holding domestic real balances at $\hat{T}$ must be offset by lower consumption of old domestic non-movers and/or lower domestic government consumption. The former would require a higher domestic country relative price of non-traded in terms of traded goods, to reduce the non-traded goods value of the tradable loan proceeds backing the checks of non-movers. This would imply a larger reduction in the date $\hat{T}$ domestic currency price of traded goods than of non-traded goods. However, the foreign relative price of non-traded goods would also then be higher, since $\hat{p}^*_{\hat{T}} = \bar{x}\hat{p}_{\hat{T}}$, reducing the foreign non-traded goods value of tradable loan proceeds backing the checks for foreign non-movers. This would aggravate the reduction in demand for foreign non-traded goods due to the lower purchasing power of domestic movers attributable to a relatively depreciated real exchange rate. On the other hand, since date $\hat{T}$ domestic real balances are higher than in the prior steady state, from the domestic government's budget constraint (8*a'*) government consumption declines *iff* the government uses some of the currency it prints to purchase additional foreign reserves. The domestic government's foreign currency purchase and associated decline in domestic government consumption serve to offset the increase in private demand for the domestic non-traded good. In addition, as seen in (8*b'*), by raising the foreign government's seigniorage revenue and hence foreign government consumption, the domestic government's reserve purchase offsets the decline in private demand for the foreign non-traded good. The converse statements apply if $\bar{x} < x$.

Using the fact that the foreign government maintains forever its period-0 reserve position, the date $\hat{T}$ government budget constraints are (8*a'*) and (8*b'*) evaluated at date $\hat{T}$. I denote the domestic government's nominal reserve adjustment at date $\hat{T}$ by $\left(\hat{F}_{\hat{T}} - F_0\right) \equiv \Delta\hat{F}_{\hat{T}}$. Define the domestic and foreign non-traded goods value of the domestic government's reserve adjustment by

$$\Delta\hat{f}_t^{dom} \equiv \left(\frac{\hat{e}_t}{\hat{p}_t^N}\right)\Delta\hat{F}_t; \qquad \Delta\hat{f}_t^{for} \equiv \left(\frac{1}{\hat{p}_t^{*N}}\right)\Delta\hat{F}_t = \frac{\Delta\hat{f}_t^{dom}}{\bar{x}}.$$

Using the steady state loan market clearing conditions which hold at $\hat{T}-1$, and $\hat{p}^*_{\hat{T}} = \bar{x}\hat{p}_{\hat{T}}$, the non-traded goods market clearing condition within each country can then be written as two equations in $\Delta\hat{f}_{\hat{T}}{}^{dom}$ and $\hat{p}_{\hat{T}}$,

$$\Delta\hat{f}_{\hat{T}}{}^{dom} = \frac{\psi\beta(y^*\bar{x}(1-\varepsilon^*)\pi^* - y(1-\epsilon\pi)) + \dfrac{(1-\psi)q}{\hat{p}_{\hat{T}}}}{1+\beta}, \tag{21a}$$

$$\Delta\hat{f}_{\hat{T}}{}^{dom} = \bar{x}\Delta\hat{f}_{\hat{T}}{}^{for} = \frac{\psi\beta(y^*\bar{x}(1-\varepsilon^*\pi^*) - y(1-\epsilon)\pi)) - \dfrac{(1-\psi)q^*}{\hat{p}_{\hat{T}}}}{1+\beta}. \tag{21b}$$

Given the values of $\hat{p}^N_{\hat{T}}$ and $\hat{e}_{\hat{T}}$ consistent with currency market clearing at $x = \bar{x}$, (21*a*) and (21*b*) jointly determine the real (and hence nominal) reserve adjustment, $\Delta\hat{f}_{\hat{T}}$, and relative price of non-traded goods, $\hat{p}_{\hat{T}}$, consistent with non-traded goods market clearing in the two countries. The foreign country's relative price of non-traded goods, $\hat{p}^*_{\hat{T}}$, as well as the nominal prices of traded goods in each country, follow immediately. Loan market clearing conditions (11*a*) and (11*b*) evaluated at $\hat{T}$ determine real tradable interest rates under the targeting regime. Date $\hat{T}$ consumption and loan allocations satisfying the market clearing conditions evaluated at $\bar{x}$ follow. In Appendix C, I document the properties of (21*a*) and (21*b*).

Figure 6*a* depicts (21*a*) and (21*b*) for values of the target real exchange rate satisfying $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$ and $\bar{x} > x$. The domestic country's non-traded good market clearing condition is negatively sloped, and the foreign country's is positively sloped. The condition $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$ guarantees that (21*a*) asymptotes to a negative value of the initial period reserve adjustment as $\hat{p}_{\hat{T}} \uparrow \infty$, $\lim_{\hat{p}_{\hat{T}} \uparrow \infty} \Delta \hat{f}_{\hat{T}}^{dom} = \frac{\psi\beta}{1+\beta}((1-\varepsilon^*)\pi^* y^* \bar{x} - (1-\varepsilon\pi)y) < 0$, and that (24*b*) asymptotes to a strictly positive value of the reserve adjustment $-\lim_{\hat{p}_{\hat{T}} \uparrow \infty} \Delta \hat{f}_{\hat{T}}^{for} = \frac{\psi\beta}{1+\beta}((1-\varepsilon^*\pi^*)y^*\bar{x} - (1-\varepsilon)\pi y) > 0$. Thus, both loci cross the horizontal axis at a strictly positive, finite domestic relative price of non-traded goods. In addition, $\bar{x} > x$ guarantees that the domestic country's non-traded goods market-clearing locus cuts the horizontal axis at a higher value of $\hat{p}_{\hat{T}}$ than that of the foreign country. Figure 6*b* depicts (21*a*) and (21*b*) for the case of $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$, and $\bar{x} < x$, with the latter guaranteeing that the domestic goods market-clearing locus cuts the horizontal axis at a lower value of $\hat{p}_{\hat{T}}$ than the foreign country's locus. Figure 6*c* depicts (21*a*) and (21*b*) for the case of $\bar{x} = x$, in which case the loci intersect the horizontal axis at the same value of $\hat{p}_{\hat{T}}$. However, the restriction $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$ is not necessary for the existence of a unique intersection of (21*a*) and (21*b*) at a strictly positive relative price. For example, figure 6*d* depicts the configuration of (21*a*) and (21*b*) for $\bar{x} > \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*} > x$, and figure 6*e* depicts the configuration of (21*a*) and (21*b*) for $\bar{x} < \frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)} < x$, for $\hat{p}_{\hat{T}} \in (0,\infty)$. Proposition 7 follows.

Proposition 7. *Let* $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$. *Then there exists a unique solution to the date* $\hat{T}$ *non-traded goods market clearing conditions, satisfying* $\hat{p}_{\hat{T}} \in (0,\infty), \Delta\hat{f}_{\hat{T}} \in (-\infty,+\infty)$. *Specifically,*

a) *if* $\bar{x} > x$, *then* $0 < \hat{p}_{\hat{T}} \in \left(\frac{(1-\psi)q^*}{\psi\beta(y^*\bar{x}(1-\varepsilon^*\pi^*)-y(1-\epsilon)\pi)}, \frac{(1-\psi)q}{\psi\beta(y(1-\epsilon\pi)-y^*\bar{x}(1-\varepsilon^*)\pi^*)}\right)$, *and*
$$0 < \Delta\hat{f}_{\hat{T}} < \frac{\psi\beta(y^*\bar{x}(1-\varepsilon^*\pi^*) - y(1-\epsilon)\pi))}{1+\beta};$$

b) *if* $\bar{x} < x$, *then* $0 < \hat{p}_{\hat{T}} \in \left(\frac{(1-\psi)q}{\psi\beta(y(1-\epsilon\pi)-y^*\bar{x}(1-\varepsilon^*)\pi^*)}, \frac{(1-\psi)q^*}{\psi\beta(y^*\bar{x}(1-\varepsilon^*\pi^*)-y(1-\epsilon)\pi)}\right)$, *and* $0 > \Delta\hat{f}_{\hat{T}} > \frac{\psi\beta(y^*\bar{x}(1-\varepsilon^*)\pi^*-y(1-\epsilon\pi))}{1+\beta}$;

c) *if* $\bar{x} = x$, *then* $0 < \hat{p}_{\hat{T}} = \frac{(1-\psi)q}{\psi\beta(y(1-\epsilon\pi)-y^*\bar{x}(1-\varepsilon^*)\pi^*)} = \frac{(1-\psi)q^*}{\psi\beta(y^*\bar{x}(1-\varepsilon^*\pi^*)-y(1-\epsilon)\pi)}, 0 = \Delta\hat{f}_{\hat{T}}$.

Proof. See Appendix C.

The solution of (21*a)* and (21*b*) gives the date $\hat{T}$ value of the domestic government's real reserve adjustment. The latter is

$$\Delta\hat{f}_{\hat{T}} = (\bar{x} - x)\left(\frac{\beta}{1+\beta}\right)\left(\frac{y^*\big((1-\varepsilon^*\pi^*)q + (1-\varepsilon^*)\pi^* q^*\big)}{q+q^*}\right). \tag{22}$$

Equation (22) is an intuitively appealing representation of the date $\hat{T}$ reserve adjustment of the domestic government. First, (22) shows that $\Delta\hat{f}_{\hat{T}} \gtreqless 0$ *iff* $\bar{x} \gtreqless x$. If the domestic government wants to pursue a more "competitive" real exchange rate, it must purchase additional foreign currency in money markets at date $\hat{T}$. If the government wanted to pursue a stronger currency, in real terms, it must sell some of its initial foreign reserve, F_0, at date $\hat{T}$. Second, the absolute size of the reserve adjustment is increasing in the distance of the real exchange rate target from the non-targeting steady state real exchange rate. Third, higher foreign bank demand for domestic currency and foreign loans, given foreign bank demand for foreign currency, which is proportional to $(1-\varepsilon^*\pi^*)q + (1-\varepsilon^*)\pi^* q^*$, raises the required initial foreign reserve purchase by the domestic government when $\bar{x} > x$, and – by implying a lower steady state equilibrium domestic country real exchange rate – raises the required initial foreign reserve sale by the domestic government for any $\bar{x} < x$.

Then the date $\hat{T}$ relative price of non-traded to traded goods in each country that is the solution to (21*a*) and (21*b*) is

$$\hat{p}_{\hat{T}} = \frac{1}{\psi\beta}\left(\frac{(1-\psi)(q+q^*)}{y(1-\pi) + y^*\bar{x}(1-\pi^*)}\right) = \hat{p}^*_{\hat{T}}/\bar{x}, \tag{23}$$

Not surprisingly, these are increasing in the global supply of traded goods, and decreasing in global bank holdings of tradable loans which reflects the global demand for traded goods by young entrepreneurs. The implied nominal traded good prices that support the date $\hat{T}$ reserve adjustment are,

$$\hat{p}^T_{\hat{T}} = \left(\frac{M_{\hat{T}}(1+\beta)}{(1-\psi)(q+q^*)}\right)\left(\frac{y(1-\pi) + y^*\bar{x}(1-\pi^*)}{y\varepsilon\pi + y^*\bar{x}(1-\varepsilon^*)\pi^*}\right) = \frac{\hat{p}^{*T}_{\hat{T}}}{\hat{e}_{\hat{T}}}. \tag{24}$$

The nominal traded good price of a country is increasing in that country's money stock, decreasing in the global supply of traded goods, increasing in global bank holdings of loans and decreasing in the global bank demand for that country's currency. Finally, loan market clearing yields real interest rates on claims to tradable goods,

$$\hat{R}^T_{\hat{T}+1} = \frac{q\big(y(1-\pi) + y^*\bar{x}(1-\pi^*)\big)}{(q+q^*)(1-\pi)y}, \tag{25a}$$

$$\hat{R}^{*T}_{\hat{T}+1} = \frac{q^*\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}{(q+q^*)(1-\pi^*)y^*\bar{x}}, \tag{25b}$$

and the non-traded goods returns paid by banks to non-movers at $\hat{T}$ – which are subject to the shock of the real exchange rate target adoption – are

$$\hat{R}^N_{\hat{T}} \equiv \hat{R}^T_{\hat{T}}\frac{\hat{p}}{\hat{p}_{\hat{T}}} = \left(\frac{q}{1-\pi}\right)\left(\frac{y(1-\pi)+y^*\bar{x}(1-\pi^*)}{y(q+q^*)}\right), \tag{25c}$$

$$\hat{R}^{*N}_{\hat{T}} \equiv \hat{R}^{*T}_{\hat{T}}\frac{\hat{p}^*}{\hat{p}^*_{\hat{T}}} = \left(\frac{q^*}{1-\pi^*}\right)\left(\frac{y(1-\pi)+y^*\bar{x}(1-\pi^*)}{\bar{x}y^*(q+q^*)}\right). \tag{25d}$$

I record all date $\hat{T}$ consumption and asset allocations in Appendix D.

It is clear from the money market clearing conditions that $\left(\frac{\hat{p}^N_{\hat{T}-1}}{\hat{p}^N_{\hat{T}}}\right) \gtreqless 1/\sigma$ and $\left(\frac{\hat{p}^{*N}_{\hat{T}-1}}{\hat{p}^{*N}_{\hat{T}}}\right) \lesseqgtr \frac{1}{\sigma^*} iff\ \bar{x} \gtreqless x.$ There is a tradeoff between the external and internal real value of a country's currency. Domestic non-traded goods price inflation declines – and the internal value of its currency rises relative to the prior steady state equilibrium – and foreign non-traded goods price inflation rises when the domestic country establishes a more depreciated external value of its currency. There is, therefore, a date $\hat{T}$ welfare gain, for $\bar{x} > x,$ for domestic and foreign old workers who use domestic currency to consume domestic non-traded goods. There is a date $\hat{T}$ welfare loss for domestic and foreign old workers who require foreign currency to consume foreign non-traded goods. In addition, the date $\hat{T}$ real interest rate measured in non-traded goods paid by banks to non-movers rises (falls) in the domestic (foreign) country relative to its prior steady state value when $\bar{x} > x$ because the relative price of non-traded goods declines (rises) at date $\hat{T}$ relative to its steady state value at date $\hat{T}-1$. Thus, there is a consumption and welfare gain for domestic old non-movers and a loss for foreign old non-movers at period $\hat{T}$ relative to the prior steady state.

Given the solution for the period $\hat{T}$ relative price of non-traded goods and foreign reserve adjustment, period $\hat{T}$ government consumption of non-traded goods in each country is

$$\hat{g}_{\hat{T}} = (\bar{x}^{max}-\bar{x})\left(\frac{1}{\sigma}\right)\left[\frac{\psi\beta y^*\big(q^*(1-\varepsilon^*)\pi^*+q(1-\varepsilon^*\pi^*)+q(\sigma-1)(1-\pi^*)\big)}{(1+\beta)(q+q^*)}\right], \tag{26a}$$

$$\hat{g}^*_{\hat{T}} = \left(\bar{x}-\bar{x}^{min}\right)\left(\frac{1}{\sigma^*}\right)\left[\frac{\psi\beta y^*(q(\sigma^*-\varepsilon^*\pi^*)+q^*\pi^*(\sigma^*-\varepsilon^*))}{(1+\beta)(q+q^*)\bar{x}}\right], \tag{26b}$$

where

$$\bar{x}^{max} \equiv \left(\frac{y}{y^*}\right)\left(\frac{q^*(\sigma-\varepsilon\pi)+q\pi(\sigma-\varepsilon)}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)+(\sigma-1)q(1-\pi^*)}\right)$$

$$\bar{x}^{min} \equiv \left(\frac{y}{y^*}\right)\left(\frac{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)+q^*(\sigma^*-1)(1-\pi)}{q^*\pi^*(\sigma^*-\varepsilon^*)+q(\sigma^*-\varepsilon^*\pi^*)}\right).$$

Since government consumption must be non-negative, (26*a*) and (26*b*) imply that the value of the real exchange rate target must satisfy an upper and a lower bound ($\bar{x} \in [\bar{x}^{min}, \bar{x}^{max}]$), respectively, conditional on the domestic and foreign money growth rate. Note that $\bar{x}^{max}$ is strictly increasing in the domestic money growth rate, σ. The domestic government can establish a higher real exchange rate target the higher is its money growth rate, as higher seigniorage revenue relaxes the constraint on the value of domestic government consumption that reserve accumulation implies. Also note that $\bar{x}^{min}$ is strictly decreasing in the foreign money growth rate, σ^*. If the domestic government sought a more appreciated real exchange rate, it could establish a *lower* real exchange rate target the higher is the foreign country's money growth rate. This is because higher foreign seigniorage revenue relaxes the constraint on the value of foreign government consumption that domestic government foreign reserve sales implies. A more depreciated external value is possible the higher is the rate of internal depreciation dictated by domestic monetary policy, and a more appreciated external value is possible the higher is the rate of foreign currency internal depreciation dictated by foreign monetary policy.

Hence, admissible values of $\bar{x}$ should be bounded to guarantee non-negative government consumption. It turns out that, under some reasonable parameter restrictions, these bounds do not constrain the target value range more than the condition of proposition 7, namely that $\bar{x} \in \left(\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*}, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^* y^*}\right)$ – the range of real exchange rate values consistent with equilibrium in a no-targeting regime. Definition 3 and proposition 8 summarize these assumptions.

Definition 3. Let

a) $g(\pi^*) \equiv \frac{q(1-\pi^*)-q^*\pi^*(1-\varepsilon^*)}{q\pi^*(1-\varepsilon^*)+q(1-\pi^*)\varepsilon}$,

b) $h(\pi^*) \equiv \frac{q^*(1-\varepsilon^*\pi^*)}{q^*(1-\varepsilon^*\pi^*)+(q+q^*\pi^*)(1-\varepsilon)}$,

c) $\sigma^{min} \equiv \frac{(q^*+q)\pi^*(1-\varepsilon^*)}{(q^*+\pi q)\pi^*(1-\varepsilon^*)-q(1-\pi^*)(1-\varepsilon\pi)}$,

d) $\sigma^{*min} \equiv \frac{(q^*+q)\pi(1-\varepsilon)}{(q^*\pi^*+q)\pi(1-\varepsilon)-q^*(1-\pi)(1-\varepsilon^*\pi^*)}$.

Proposition 8. *Fiscal and monetary policy with a target under capital controls*

Let $\pi > max\left(g(\pi^*), h(\pi^*)\right)$. *Then* $\forall \bar{x} \in \left(\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*}, \frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)$, *a)* $\hat{g}_{\hat{T}} > 0$ *if* $\sigma \geq \sigma^{min}$, *b)* $\hat{g}^*_{\hat{T}} > 0$ *if* $\sigma^* \geq \sigma^{*min}$.

Proof. See Appendix D.

Proposition 8 establishes conditions under which sufficiently high money growth rates guarantee positive government consumption. For both countries, the guarantee of positive government consumption requires that π also be sufficiently high. Furthermore, the critical values of both domestic and foreign money growth are decreasing in π. Intuitively, higher domestic demand for domestic currency, and hence for domestic non-traded goods, relaxes the constraint on the domestic seigniorage revenue needed to support positive domestic government consumption when the real exchange rate target is higher than the equilibrium real exchange rate. It does so by raising the equilibrium value of domestic real balances or, equivalently, reducing the nominal price of domestic non-traded goods. When the target is lower than the equilibrium real exchange rate, foreign government seigniorage and consumption are constrained by domestic government reserve sales. Higher values of the domestic demand for foreign currency raise the equilibrium value of foreign real balances and relax this constraint. The panels of figure 7 illustrate the admissible combination of values of π and π^* such that the conditions of propositions 1 through 8 are all satisfied.

There are thus two crucial ingredients for the unilateral establishment of a real exchange rate target. First, there must be a sufficiently high money growth rate in one of the two countries to guarantee positive government consumption in that country, and the identity of the country depends on whether the target lies above or below the equilibrium real exchange rate. Second, the liquidity of the domestic country, π, must be sufficiently high relative to that of the foreign country.

5.2 Existence of steady state equilibrium

At every date $t > \hat{T}$, the equilibrium conditions take exactly the same form as they do in the initial period of the targeting regime, and – with the exception of government budget constraints, which include real reserve adjustments – are completely static. It is straightforward to verify that, if it exists, the only equilibrium for this economy comprises an infinite sequence of the stationary solutions for the real endogenous variables derived in Section 5.1 from date $\hat{T}$ onwards, together with constant growth rates of nominal prices. In other words, if it exists, the only equilibrium is a steady state equilibrium.

To see this, note that (21*a*) and (21*b*) are unchanged at every date and, as we have seen, yield a unique solution, given $\bar{x}$. Hence, $\hat{p}_{\hat{T}+i} = \hat{p}_{\hat{T}} = \hat{p}$ and $\Delta\hat{f}_{\hat{T}+i} = \Delta\hat{f}_{\hat{T}} = \Delta\hat{f}\ \forall i \geq 1$. Given $\hat{p}$ and $\hat{p}^* = \bar{x}\hat{p}$ loan market clearing yields constant solutions for real interest rates, $\hat{R}^T$, and $\hat{R}^{*T}$. Hence, real loan returns measured in non-traded goods, $\hat{R}^N$ and $\hat{R}^{*N}$, are also constant. As Section 5.1 establishes, a constant real exchange rate target implies that real balances are constant in each country, $\forall i \geq 1$, $\hat{m}_{\hat{T}+i} = \hat{m}_{\hat{T}} = \hat{m}, \hat{m}^*_{\hat{T}+i} = \hat{m}^*_{\hat{T}} = \hat{m}^*$. Since real balances and the real reserve adjustment are constant, the government budget constraints imply that government consumption is also constant, $\hat{g}_{\hat{T}+i} = \hat{g}_{\hat{T}} = \hat{g}$, and $\hat{g}^*_{\hat{T}+i} = \hat{g}^*_{\hat{T}} = \hat{g}^*, \forall i \geq 1$.

Since real balances are constant, the non-traded goods price of each country rises at the rate of that country's money growth rate, and since internal relative prices of non-traded goods are constant a country's traded good price also rises at that country's rate of money growth. The domestic country's nominal exchange rate rises at the ratio of domestic to foreign money growth to keep the real exchange rate constant. Then, all equilibrium private sector allocations, particularly those that depend on inflation rates, rates of nominal exchange rate growth, and real interest rates, are also constant.

Thus, the economy with a real exchange rate target can attain a steady state at $\hat{T}$, in which all real endogenous variables are constant at their date $\hat{T}$ values, and nominal variables change at constant rates. However, a steady state for this economy is possible only if the real exchange rate target is higher than the equilibrium real exchange rate. A target that is lower than the equilibrium real exchange rate results in a constant reserve loss valued in non-traded goods at every date, and – since non-traded goods prices rise at money growth rates that exceed one – this implies an increasing rate of *nominal* reserve loss. Increasing sales of nominal reserves culminating in the elimination of the stock at a finite date cannot be part of a stationary state comprising an infinite sequence of static conditions. In addition, existence of the steady state equilibrium, requires that money growth rates be sufficiently high to ensure that loans return-dominate money, and must feature non-negative government consumption.

Below I develop conditions for existence and uniqueness of a steady state equilibrium comprising an infinite sequence of the date $\hat{T}$ prices and allocations I have described $\forall\, \bar{x} \geq x$. Proposition 9 demonstrates that a more appreciated real exchange rate target than the equilibrium real exchange rate ($\bar{x} < x$) cannot be indefinitely sustained and hence cannot be part of any equilibrium.

Proposition 9. *Suppose that* $\bar{x} < x$. *Then no equilibrium exists.*

Proof. See Appendix E.

Identical to those at date $\hat{T}$, the non-traded goods market clearing conditions determine the domestic country's relative price of non-traded goods and its non-traded good value of the reserve adjustment. I denote these two variables by $\hat{p}$ and $\Delta\hat{f}$. The steady state non-traded goods market clearing conditions in the domestic and foreign country respectively are therefore identical to (21*a*) and (21*b*) given by $\hat{p}$ and $\Delta\hat{f}$. The unique solution of these two equations gives the steady state values of the domestic relative price of non-traded goods and the non-traded goods value of the domestic government's reserve adjustment, $\hat{p} = \hat{p}_{\hat{T}}$ and $\Delta\hat{f} = \Delta\hat{f}_{\hat{T}}$. I record steady state nominal and real price, private sector allocations satisfying the optimality and market clearing conditions of definition 2 in Appendix E.

The solutions for real returns to loans are

$$\hat{R}^T = \hat{R}^N = \frac{q\left(y(1-\pi)+y^*\bar{x}(1-\pi^*)\right)}{(1-\pi)y(q+q^*)}, \tag{27a}$$

$$\hat{R}^{*T} = \hat{R}^{*N} = \frac{q^*\left(y(1-\pi)+y^*\bar{x}(1-\pi^*)\right)}{(1-\pi^*)y^*\bar{x}(q+q^*)}. \tag{27b}$$

As is true at $\hat{T}$, the relative size of real returns across countries depends on the real exchange rate target, and the parameter restriction required for the steady state domestic interest rate to exceed, equal, or smaller than 1. Under assumption 2, $\frac{yq^*(1-\pi)}{y^*q(1-\pi^*)} < \frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}$. Then for any $\bar{x} \in \left(\frac{y(1-\varepsilon)\pi}{y^*(1-\varepsilon^*\pi^*)}, \frac{y(1-\varepsilon\pi)}{y^*(1-\varepsilon^*)\pi^*}\right)$, assumption 2 guarantees $\bar{x} > \frac{yq^*(1-\pi)}{y^*q(1-\pi^*)}$. Then, rearranging the expression for $\hat{R}^T$, clearly $\hat{R}^T > 1$ under assumption 2. Hence, $\hat{R}^{*T} < 1$.

The following proposition summarizes conditions for existence of a steady state equilibrium under a real exchange rate target. In this, I ignore the trivial case of $\bar{x} = x$.

Proposition 10. ***Steady state equilibrium with a target under capital controls***

Let i) $\pi \geq \max\{g(\pi^*), \tilde{\pi}\}$, *and ii)* $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$. *Then there exists a unique steady state equilibrium with positive government purchases,* $\hat{R}^T > 1, \hat{R}^{*T} < 1$, *and* $\hat{R}^N > \frac{p_{t-1}^N}{p_t^N} = \frac{1}{\sigma}$ *and* $\hat{R}^{*N} > \frac{p_{t-1}^{*N}}{p_t^{*N}} = \frac{1}{\sigma^*}$ *iff a)* $\sigma \geq \sigma^{min}$, *and b)* $\sigma^* > \left(\frac{q^*\left(y(1-\pi)+y^*\bar{x}(1-\pi^*)\right)}{(1-\pi^*)y^*\bar{x}(q+q^*)}\right)^{-1} = \hat{R}^{*N^{-1}} > 1$.

Proof. From an inspection of figure 6a, if the solution represented by the intersection of domestic and foreign non-traded goods market clearing condition for $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$ satisfies the remaining conditions for a steady state equilibrium, then the steady state equilibrium is unique. In addition, as is apparent from an inspection of figure 6*a*, the unique intersection of two non-traded goods market clearing conditions always lies at strictly positive and finite values of $\hat{p}$ and $\Delta\hat{f}$, while $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$ is also strictly positive and finite. Hence, the resulting private consumption and loan allocations satisfying all of the other conditions of equilibrium in definition 2 take admissible values. Government consumption must also satisfy non-negativity, however. Using the argument of Proposition 8, for $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$, $\bar{x} > x > \bar{x}^{min}$. Thus, from (26*b*), $\hat{g}^* > 0$. Proposition 8 also shows that if $\pi > g(\pi^*)$ and $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$, then $\hat{g} \geq 0$ iff $\sigma \geq \sigma^{min} > 1$. Part *a)* of the proposition is immediate. All that remains for existence of a steady state equilibrium is that the currency of each country is return-dominated by loans, so that banks hold currency solely to meet liquidity needs as definition 2 presumes, at the solution given by the intersection of steady-state version of (21*a*) and (21*b*). Return domination of money in each country requires that $\hat{R}^N > \frac{1}{\sigma}$ and $\hat{R}^{*N} > \frac{1}{\sigma^*}$. Since $\pi > \tilde{\pi}$, assumption 2 is satisfied. Then $\hat{R}^N = \hat{R}^T > 1$, and immediately satisfies $\hat{R}^N > \frac{1}{\sigma}$. However, under assumption 2, as through traded goods market clearing condition, $\hat{R}^{*N} = \hat{R}^{*T} < 1$. Thus, currency is dominated in rate of return by loans in the foreign country at the solution to steady-state version of (21*a*) and (21*b*) iff $1 > R^{*N} = R^{*T} > \frac{1}{\sigma^*}$. Part *b*) of the proposition follows. ■

5.3 External Balance

As at date $\hat{T}$, the relative size of the steady state real interest rate across the two countries determines which country runs a trade deficit and which a trade surplus in traded goods. Under the conditions of proposition 10, $\hat{R}^T > 1, \hat{R}^{*T} < 1$. In addition, (27*a*) and (27*b*) show that the higher the real exchange rate target, the higher the steady state domestic real interest rate and the lower is the steady state foreign real interest rate. The steady state trade balance is $\widehat{TB}^T > q - \hat{c}_y^T - \hat{c}_o^T$. Substituting for entrepreneurs' steady state equilibrium consumption (see Appendix E), this balance is just

$$\widehat{TB}^T = \left(\frac{1}{1+\beta}\right)\left(q - \frac{y(1-\pi)(q+q^*)}{\left(y(1-\pi) + y^*\bar{x}(1-\pi^*)\right)}\right). \tag{28}$$

Proposition 11. *Let assumption 2 hold and* $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^* y^*}\right)$. *Then* $\widehat{TB}^T > TB^T > 0$ *and* $\widehat{TB}^{*T} < 0$.

Proof. Under assumption 2, $\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*} > \frac{yq^*(1-\pi)}{y^*q(1-\pi^*)}$. Since $\bar{x} > x > \frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*}$, then $\bar{x} > \frac{yq^*(1-\pi)}{y^*q(1-\pi^*)}$ and $\widehat{TB}^T > 0$ follows. In a country with a real exchange rate target satisfying $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*}\right)$, a relatively high steady state real interest rate on consumption loans and relatively low bank credit results in relatively low borrower consumption, and a permanent trade surplus. In addition, the higher is the target value the larger is the trade surplus. In particular, the trade surplus is higher than its non-targeting steady state value for $\bar{x} > x$. Recall the equilibrium value of TB^T under no-targeting regime $TB^T = \left(\frac{1}{1+\beta}\right)\left(q - \frac{(1-\pi)\left(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\right)}{\left((1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - (1-\varepsilon^*)\pi^*(1-\varepsilon)\pi\right)}\right)$. Then $\widehat{TB}^T > TB^T$ iff $\frac{(1-\pi)\left(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\right)}{\left((1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - (1-\varepsilon^*)\pi^*(1-\varepsilon)\pi\right)} > \frac{y(1-\pi)(q+q^*)}{\left(y(1-\pi) + y^*\bar{x}(1-\pi^*)\right)}$, which after some manipulation, using the steady state solution for *x*, yields $\bar{x} > x$. The result follows. ■

When a government targets the real exchange rate by manipulating its foreign exchange reserve, a domestic-country trade surplus does not imply net private financial (currency) outflows. The domestic country's steady state per worker external financial balance is $\widehat{FB} = \frac{(1-\varepsilon^*)\pi^*\beta y^*\bar{x}}{1+\beta} - \frac{(1-\varepsilon)\pi\beta y}{1+\beta}$. Under real exchange rate targeting, the financial balance depends on the value of the target, and so is ambiguous. However, the domestic country's steady state per capita balance of payments must be permanently in surplus, since it equals the traded goods value of its reserve accumulation at each date, $\widehat{BOP} \equiv (1-\psi)\widehat{TB}^T + \psi\widehat{FB} \times \hat{p} = \Delta\hat{f}\hat{p} > 0.$ Balance-of-payment can be re-expressed as

$$\widehat{BOP} = (\bar{x} - x)\left(\frac{\psi\beta}{q+q^*}\right)\left(y^*\big((1-\varepsilon^*\pi^*)q + (1-\varepsilon^*)\pi^*q^*\big)\right)p.$$

Proposition 12 collects these results.

Proposition 12. ***Steady state external balance with a target under capital controls***
Let i) $\pi \geq \max\{g(\pi^*), \tilde{\pi}\}$, *and ii)* $\bar{x} \in \left(x, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^*y^*}\right)$ *Then, there exists a unique steady state equilibrium under a unilateral domestic country real exchange rate target, in which the domestic (foreign) country has a permanent trade surplus (deficit) exceeding that in the steady state equilibrium without targeting, a permanent balance of payments surplus (deficit), and the domestic country's financial balance satisfies*

$$\widehat{FB} \gtreqless 0 \; iff \; \bar{x} \gtreqless \frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*)\pi^*}.$$

Proof. See Appendix E.

6. Free Capital Flows

I now investigate how bank trade in loans – international capital flows – affects the properties of equilibria with and without real exchange rate targeting.

6.1 Market Determined Real Exchange Rate

6.1.1 Steady state equilibrium I first characterize and explore the conditions for existence of a steady state equilibrium for this economy, in which all of the real endogenous variables are constant. To distinguish variables from those under capital controls, I denote the value of variable x by $\check{x}$.

In order for real balances to be constant within each country, nominal non-traded price inflation rates, and the growth rate of the nominal exchange rate, exactly mimic their behavior in the steady state equilibria of the financially closed economy; $\frac{\check{p}_{t+1}^N}{\check{p}_t^N} = \sigma$, $\frac{\check{p}_{t+1}^{*N}}{\check{p}_t^{*N}} = \sigma^*$, and $\frac{\check{e}_{t+1}}{\check{e}_t} = \frac{\check{x}_{t+1}(\check{p}_{t+1}^N/\check{p}_{t+1}^{*N})}{\check{x}_t(\check{p}_t^N/\check{p}_t^{*N})} = \frac{(\check{p}_{t+1}^N/\check{p}_{t+1}^{*N})}{(\check{p}_t^N/\check{p}_t^{*N})} = \left(\frac{\sigma}{\sigma^*}\right), \forall t.$ For internal relative prices of non-traded goods to be constant, therefore, $\frac{\check{p}_{t+1}^T}{\check{p}_t^T} = \sigma$ and $\frac{\check{p}_{t+1}^{*T}}{\check{p}_t^{*T}} = \sigma^*, \forall t.$

Under free capital flows, equilibrium real interest rates measured in traded goods must be the same in the two countries. Then, in a steady state equilibrium, there must be a constant world real interest rate on tradable claims, $\check{R}_{t+1}^T = \check{R}^T = \check{R}^{*T}.$ Since the relative price of non-traded goods within each country is also constant, the constant steady state real interest rate measured in non-traded goods within each country equals the steady state world real interest rate measured in traded goods, $\breve{R}_{t+1}^{N} = \breve{R}^{N} = \breve{R}^{T}$; $\breve{R}_{t+1}^{*N} = \breve{R}^{*N} = \breve{R}^{*T} = \breve{R}^{T}$.

From the traded goods market clearing conditions (14') and (15'), the initial period world real interest rate is $\frac{1}{1+\beta}$, while at every other date, in any equilibrium, it is constant and equal to one. Hence, as was true under capital controls, the economy cannot attain its steady state at date 1. As we would expect, the steady state equilibrium real interest rate lies between the equilibrium interest rates under capital controls.

Given a steady state real interest rate of one, either of the steady state non-traded goods market clearing conditions, (13a') and (13b') evaluated at constant endogenous variables ($\frac{\psi y\beta}{1+\beta} = \frac{\varepsilon\pi\psi y\beta}{1+\beta} + \frac{(1-\varepsilon^{*})\pi^{*}\psi\beta y^{*}\breve{x}}{1+\beta} + \frac{\psi\beta\, y(1-\pi)}{1+\beta}\breve{R}^{T}$ in the domestic country and $\frac{\psi y^{*}\beta}{1+\beta} = \frac{\psi\varepsilon^{*}\pi^{*}y^{*}\beta}{1+\beta} + \frac{\psi(1-\varepsilon)\pi\beta y/\breve{x}}{1+\beta} + \frac{\psi\beta\, y^{*}(1-\pi^{*})}{1+\beta}\breve{R}^{T}$ in the foreign country) yields the steady state real exchange rate, and the second is redundant. The domestic country's steady state relative price of non-traded goods, *p*, follows from the global loan market clearing condition, (11'), evaluated at $\breve{R}^{T} = 1$. Steady state real balances follow from money market clearing conditions, (16*a*) and (16*b*), and government consumption from the budget constraints (17*a*) and (17*b*).

Proposition 13. ***Steady state equilibrium under free capital flows***

There exists a unique steady state equilibrium with $\breve{R}^{N} = \breve{R}^{*N} > \max\left(\frac{1}{\sigma}, \frac{1}{\sigma^{*}}\right)$.

Proof. It is evident from (15) that there is a unique world steady state real interest rate consistent with traded goods market clearing, which is equal to one and, since relative prices are constant in a steady state, the real interest rate measured in non-traded goods also take the unique value of one. Then there is a unique solution for $\breve{x}$ satisfying (13*a'*) or (13*b'*) evaluated at constant endogenenous variables, and hence a unique solution for $\breve{p}$ satisfying (11') evaluated at $\breve{R}^{T} = 1$. If these solutions for $\breve{R}^{T} = \breve{R}^{*T} = \breve{R}^{N} = \breve{R}^{*N}$, $\breve{x}$, and $\breve{p}$ satisfy all of the other conditions of a steady state equilibrium, then the steady state equilibrium is unique. The solutions for relative prices are

$$\breve{x} = \left(\frac{y}{y^{*}}\right)\left(\frac{\pi(1-\varepsilon)}{\pi^{*}(1-\varepsilon^{*})}\right),$$

$$\breve{p} = \left(\frac{(1-\psi)(q+q^{*})}{\psi\beta y}\right)\left(\frac{\pi^{*}(1-\varepsilon^{*})}{(1-\pi)\pi^{*}(1-\varepsilon^{*}) + (1-\pi^{*})\pi(1-\varepsilon)}\right),$$

$$\breve{p}^{*} = \left(\frac{(1-\psi)(q+q^{*})}{\psi\beta y^{*}}\right)\left(\frac{\pi(1-\varepsilon)}{(1-\pi)\pi^{*}(1-\varepsilon^{*}) + (1-\pi^{*})\pi(1-\varepsilon)}\right).$$

These solutions are strictly positive and finite, implying strictly positive, finite values for real balances, government consumption, and all private consumption, loan, and bank allocations satisfying the optimality, market clearing conditions, and government budget constraints of definition 2. Then all that is required for existence of a steady state equilibrium is that real interest rates satisfy return-domination of money, $\breve{R}^N > \frac{\breve{p}_{t-1}^N}{\breve{p}_t^N}$ and $\breve{R}^{*N} = \breve{R}^N > \frac{\breve{p}_{t-1}^N}{\breve{p}_t^N}$. Since $\breve{R}^T = \breve{R}^N = \breve{R}^{*N} = 1$, and steady state inflation rates are $\frac{\breve{p}_t^N}{\breve{p}_{t-1}^N} = \sigma > 1, \frac{\breve{p}_t^{*N}}{\breve{p}_{t-1}^{*N}} = \sigma^* > 1$, return domination is always satisfied, since $\breve{R}^N = \breve{R}^{*N} = 1 > \max\left(\frac{1}{\sigma}, \frac{1}{\sigma^*}\right)$. ■

I document the full set of steady state private sector and government allocations under free capital flows in Appendix F. Manipulating the solution for the steady state real exchange rate yields the following proposition, which I state without proof.

Proposition 14. ***Comparative statics***

a) An increase in the domestic country's relative supply of non-traded goods, y/y^**, raises –depreciates – its real exchange rate,* $\breve{x}$*.*

b) An increase in the domestic country's bank portfolio weight on liquid assets, π*, raises – depreciates – its real exchange rate,* $\breve{x}$*.*

c) An increase in the foreign country's bank portfolio weight on liquid assets, π^**, reduces – appreciates – the domestic country's real exchange rate,* $\breve{x}$*.*

Note that the steady state equilibrium real exchange rate under free capital flows is independent of relative national supplies of traded goods, by contrast to that under capital controls. Now, the inter-temporal prices of traded and non-traded goods are arbitraged, in addition to the intra-temporal price of traded goods, and this completely insulates the relative price of non-traded goods across countries from the traded goods market. In addition, there is no longer any ambiguity in the effect of higher liquidity demand for the real exchange rate; it no longer requires sufficiently strong "own" relative to "foreign" demand for the currency to depreciate in value with higher domestic liquidity demand and appreciate with higher foreign liquidity demand.

The steady state, per entrepreneur, external balance of the domestic country in traded goods is $\widetilde{TB}^T = q - \breve{c}_y^T - \breve{c}_o^T$. Since the real interest rate is one, $\breve{c}_y^T = \frac{q}{1+\beta}$, and the trade balance is

$$\widetilde{TB}^T = q - \left(\frac{q}{1+\beta}\right) - \left(\frac{q\beta}{1+\beta}\right) = 0.$$

In a steady state equilibrium, trade is permanently balanced. The domestic country's steady state per worker financial balance measured in domestic non-traded goods, is

$$\widetilde{FB} = \frac{\pi^*(1-\varepsilon^*)\beta y^* \breve{x}}{1+\beta} - \frac{\pi(1-\varepsilon)\beta y}{1+\beta}.$$

Under free capital flows, this balance includes zero change in net foreign bank lending to the domestic country at each date, at a gross real interest rate of one, $\left[\left(\frac{(1-\pi^*)\psi\beta y^* \breve{x}\breve{p}}{(1+\beta)} - \frac{(1-\psi)q^*}{(1+\beta)\breve{R}^T}\right) - \left(\frac{(1-\pi^*)\psi\beta y^* \breve{x}\breve{p}}{(1+\beta)} - \frac{(1-\psi)q^*}{(1+\beta)\breve{R}^T}\right) = 0\right]$, and the current account which equals the sum of the trade balance and net interest received on foreign loans is obviously zero. Substituting the steady state equilibrium real exchange rate and interest rate into the expression for the financial balance yields

$$\widetilde{FB} = \frac{\beta y}{1+\beta}\left(\pi^*(1-\varepsilon^*)\left(\frac{\pi(1-\varepsilon)}{\pi^*(1-\varepsilon^*)}\right) - \pi(1-\varepsilon)\right) = 0.$$

The solutions for the steady state equilibrium real exchange rate and real interest rate imply that, with free capital flows, net inter-location trade in currencies and loans is zero, and there is balanced trade for traded goods.

Proposition 15. *Let assumption 2 hold. Then the steady state equilibrium real exchange rate under free capital flows is higher (more depreciated) than that under capital controls.*

Proof. A comparison of the expressions for the steady state equilibrium real exchange rate under capital controls, $\breve{x} = \frac{y}{y^*}\left(\frac{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right)$, and that under free capital flows, $\breve{x} = \left(\frac{y}{y^*}\right)\left(\frac{\pi(1-\varepsilon)}{\pi^*(1-\varepsilon^*)}\right)$ implies that the latter is higher than the former if $\left(\frac{\pi(1-\varepsilon)}{\pi^*(1-\varepsilon^*)}\right) > \left(\frac{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right)$. Imposing assumption 2 proves the proposition. ■

Since the steady state domestic country real interest rate is equal to one and hence, under assumption 2, lower than in the steady state equilibrium with capital controls, the demand for domestic non-traded goods of non-movers who write checks backed by loans is also lower. A relatively depreciated real exchange rate increases the purchasing power of foreign consumers over domestic non-traded goods, offsetting the decline in domestic demand from non-movers. Further, a comparison of the steady state solutions for internal relative prices shows that, under assumption 2, a relatively depreciated real exchange rate under free capital flows reflects in a lower domestic relative price of non-traded goods, and higher foreign relative price of non-traded goods, relative to those under capital controls. At these relative prices, the steady state private consumption and loan allocations of the economy are identical to those of a non-monetary, autarkic economy, owing to completely balanced trade, in goods, currencies, and changes in net lending.

The welfare of young workers, of workers subject to relocation earning rates of return to currencies, and of old borrowers are each identical under free capital flows to that under capital controls. However, under assumption 2, a lower real interest rate under free capital flows implies that the welfare of domestic (foreign) non-movers is lower (higher), and that of domestic (foreign) young borrowers is higher (lower). In addition, under assumption 2, steady state domestic (foreign) real balances, seigniorage, and hence government consumption are higher (lower) under free capital flows relative to those under capital controls, due to the a relatively depreciated domestic country real exchange rate, and the concomitant increase (decrease) in purchasing power for foreign (domestic) agents over domestic (foreign) non-traded goods. There are both internal and international steady state distributional consequences of allowing international capital flows when there is no real exchange rate target – domestic borrowers and foreign lenders gain, and domestic lenders and foreign borrowers lose.

6.1.2 The initial period and dynamic equilibrium The economy can never attain its steady state in the initial period because the world real interest rate must accommodate the absence of old entrepreneurs in the global market for traded goods, as is true under capital controls. At date 1, using (14), $\check{R}_2^T = \check{R}_2^{*T} = \frac{1}{1+\beta}$. The initial period non-traded goods market clearing conditions (12*a'*) and (12*b'*), setting $\check{F}_1 = F_0$, are identical to those under capital controls, and we know that they yield the following, unique solutions for $\check{p}_1, \check{x}_1$, and $\check{p}_1^* = \check{p}_1\check{x}_1$,

$$\check{x}_1 = \left(\frac{y}{y^*}\right)\left(\frac{q^*(1-\varepsilon\pi)+q(1-\varepsilon)\pi}{q^*(1-\varepsilon^*)\pi^*+q(1-\varepsilon^*\pi^*)}\right), \tag{29a}$$

$$\check{p}_1 = \left(\frac{(1-\psi)(1+\beta)}{\psi\beta y}\right)\left(\frac{q^*(1-\varepsilon^*)\pi^*+q(1-\varepsilon^*\pi^*)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}\right), \tag{29b}$$

$$\check{p}_1^* = \left(\frac{(1-\psi)(1+\beta)}{\psi\beta y^*}\right)\left(\frac{q^*(1-\varepsilon\pi)+q(1-\varepsilon)\pi}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-(1-\varepsilon^*)\pi^*(1-\varepsilon)\pi}\right). \tag{29c}$$

Since the initial period real exchange rate is identical to that under capital controls, the initial period nominal prices of non-traded goods that clear money markets, given exogenous initial money stocks and money growth rates, are also unchanged. Then, since the initial period relative price of non-traded goods in each country and nominal non-traded good prices are unchanged relative to those under capital controls, the initial period nominal traded good prices is also unchanged relative to the economy with capital controls, as is the initial period nominal exchange rate. Thus, the initial period solutions under capital controls in proposition 4 all hold under free capital flows, except for initial real tradable interest rates, which are arbitraged under free capital flows. The arbitraging of real interest rates is irrelevant for the initial period equilibrium real exchange rate, relative prices and allocations, because the demand for non-traded goods arises from the initial young and the initial old, who have no interest income. As is true of the steady state equilibrium interest rate under free capital flows, the initial world real interest rate in traded goods lies between the equilibrium initial period real interest rates of the two countries under capital controls. Under assumption 2, the domestic country realizes a relatively high initial period real interest rate under capital controls. Thus, under free capital flows, the initial period real interest rate in the domestic country is lower, and that in the foreign country is higher, than in the financially closed economy.

The initial period world real interest rate balances trade. Each country's output of the traded good is exactly equal to the value of that country's consumption of the traded good. Young entrepreneurs are the only agents whose initial period allocation depends on the status of international loan trade. Under assumption 2, domestic young entrepreneurs face a lower real interest rate than they do in the financially closed economy, and consume more than they do under capital controls, while foreign young entrepreneurs face a higher real interest rate and consume less. These changes in consumption produce initial period balanced trade. The per-entrepreneur, external balance of the domestic country in traded goods is $\widetilde{TB}_1^T = q - \check{c}_{y,1}^T$. Since the real interest rate equals $\frac{1}{1+\beta}$, $\check{c}_y^T = q$, and the trade balance is $\widetilde{TB}_1^T = q - q = 0$.

The domestic country's initial period per worker financial balance measured in non-traded goods, including the establishment of initial period net foreign bank lending, is

$$\widetilde{FB}_1 = \frac{\pi^*(1-\varepsilon^*)\beta y^* \check{x}_1}{1+\beta} - \frac{\pi(1-\varepsilon)\beta y}{1+\beta} + \frac{q(1-\psi)}{\check{p}_1 \psi(1+\beta)\check{R}_2^T} - \frac{(1-\pi)\beta y}{(1+\beta)}. \tag{30}$$

Substituting the initial period solutions for the real exchange rate, real interest rate, and relative price of non-traded goods into this expression yields $\widetilde{FB}_1 = 0$. The solution for the initial real exchange rate implies that – under assumption 2 – net inter-location trade in currencies has a negative balance, $\left(\frac{\beta}{1+\beta}\right)$ $(\pi^*(1-\varepsilon^*)y^*\check{x}_1 - \pi(1-\varepsilon)y) < 0$. Imports of foreign currency exceed exports of domestic currency. This net purchase of foreign currency (accumulation of foreign assets) is funded by a surplus in net foreign borrowing via one period consumption loans, measured by $\left(\frac{1}{1+\beta}\right)\left(\frac{(1-\psi)(1+\beta)q}{\check{p}_1} - (1-\pi)\psi\beta y\right) > 0$.

In order that the initial period solutions to the equilibrium conditions be part of a dynamic equilibrium, of course, requires that money be return-dominated by loans between periods 1 and 2, as I discuss below. At every other date, things are rather different compared to the financially closed economy. Since there is a single, global loan market, it is no longer the case that one can substitute country-specific loan market clearing conditions (11*a*) and (11*b*) into the domestic and foreign non-traded goods market clearing conditions (13*a*) and (13*b*) to eliminate $\check{R}_t^N$ and yield two equations in $\check{x}_t$ and $\check{p}_t$. Instead, (13*a'*) and (13*b'*) must be solved at every date.

The real interest rates measured in non-traded goods that appear in the non-traded goods market clearing conditions at each date $t \geq 2$ are $\check{R}_t^N = \check{R}_t^T \frac{\check{p}_{t-1}}{\check{p}_t}$ and $\check{R}_t^{*N} = \check{R}_t^N \frac{\check{x}_{t-1}}{\check{x}_t} = \check{R}_t^T \frac{\check{p}_{t-1}}{\check{p}_t} \frac{\check{x}_{t-1}}{\check{x}_t}$. Here $\check{R}_t^T$, $\check{p}_{t-1}$, and $\check{x}_{t-1}$ are pre-determined, so (13*a'*) and (13*b'*), determine the date *t* real exchange rate and date *t* domestic country relative price of non-traded goods (and hence $\check{p}_t^* = \check{x}_t \check{p}_t$), and therefore $\check{R}_t^N$ and $\check{R}_t^{*N}$. The world real interest rate on traded goods between *t* and *t+1*, $\check{R}_{t+1}^T$, is determined from traded goods market clearing, as always, and the loan market clearing condition is redundant. Given the real exchange rate, $\check{x}_t$, money market clearing yields nominal non-traded goods price levels and, hence, the nominal exchange rate that is consistent with these prices and the real exchange rate $\check{e}_t = \check{x}_t \check{p}_t^N / \check{p}_t^{*N}$. The relative price of non-traded goods in each country then yields the nominal traded goods price level in that country, $\check{p}_t^T = \check{p}_t \check{p}_t^N, \check{p}_t^{*T} = \check{p}_t^* \check{p}_t^{*N}$. Date *t* allocations follow.

There is no immediate attainment of a steady state in this environment, due to the appearance of the dynamic variables $\frac{\check{p}_{t-1}}{\check{p}_t}$ and $\frac{\check{x}_{t-1}}{\check{x}_t}$ in (13*a'*) and (13*b'*). It is clear that the real interest rates, $\check{R}_2^N$ and $\check{R}_2^{*N} = \check{R}_2^N \left(\frac{\check{x}_1}{\check{x}_2}\right)$ cannot be equal at date 2 – and cannot be equal to the world real interest rate – as they are in a steady state equilibrium, because the initial real exchange rate is not equal to the steady state equilibrium real exchange rate. Since this is the case, the real exchange rate that solves (13*a'*) and (13*b'*) at date 2 is also not the steady state equilibrium real exchange rate. I now explore the implied dynamic equilibria.

Can this economy converge to its steady state equilibrium, asymptotically? Are there dynamic equilibria? First, it is evident from the traded goods' market equilibrium condition that the equilibrium world real interest rate is constant and equal to one at every date. Then the non-traded goods market clearing conditions at every date, $t \geq 2$, can be expressed as

$$\psi y = \frac{\psi y}{1+\beta} + \frac{\psi\beta\, \varepsilon\pi y}{1+\beta} + \frac{\psi\beta\,(1-\varepsilon^*)\pi^* y^* \check{x}_t}{1+\beta} + \frac{\psi\beta\, y(1-\pi)}{1+\beta}\frac{\check{p}_{t-1}}{\check{p}_t} \tag{31a}$$

$$\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{\psi\beta\, \varepsilon^*\pi^* y^*}{1+\beta} + \frac{\psi\beta\,(1-\varepsilon)\pi y}{(1+\beta)\check{x}_t} + \frac{\psi\beta\, y^*(1-\pi^*)}{1+\beta}\frac{\check{p}_{t-1}}{\check{p}_t}\frac{\check{x}_{t-1}}{\check{x}_t}. \tag{31b}$$

From (31*a*) we obtain the following expression for the rate of change of the domestic non-traded goods price,

$$\frac{\check{p}_t}{\check{p}_{t-1}} = \frac{(1-\pi)y}{(1-\varepsilon\pi)y-(1-\varepsilon^*)\pi^* y^* \check{x}_t}, \forall t \geq 2. \tag{32}$$

Notice that (32) is always less than (greater than) one for $\check{x}_t < (>) \check{x}$. In particular, for $\check{x}_t < \check{x}$, the domestic relative price declines over time, although the gross growth rate of the domestic relative price of non-traded goods rises over time converging to one as $\check{x}_t$ approaches its steady state value. Substituting (32) into (31*b*) yields the following law of motion for the real exchange rate,

$$\check{x}_{t+1} = \left(\frac{y}{y^*}\right)\left(\frac{y\pi(1-\varepsilon)(1-\pi) + y^*(1-\varepsilon\pi)(1-\pi^*)\check{x}_t}{y(1-\varepsilon^*\pi^*)(1-\pi) + y^*\pi^*(1-\varepsilon^*)(1-\pi^*)\check{x}_t}\right), t \geq 1. \quad (33)$$

I depict this law of motion, configured under assumption 2, in figure 8. Evidently, the unique steady state equilibrium is asymptotically stable. I now state this formally.

Proposition 16. ***Asymptotic stability of steady state equilibrium under capital controls***
The law of motion for $\check{x}_t$ is monotone increasing, and crosses the 45-degree line from above.

Proof. See Appendix G.

Since there exists a unique initial period solution for the real exchange rate, the asymptotic stability of the steady state equilibrium implies that there exists a unique perfect foresight equilibrium path of the real exchange rate. Furthermore, under assumption 2, the steady state equilibrium real exchange rate is higher than the initial period equilibrium real exchange rate. Then under assumption 2, the equilibrium trajectory exhibits a permanently depreciating real exchange rate. A country with relatively high use of liquid assets and low credit extension, that is open to international capital flows, will experience a monotonically depreciating real exchange rate in the absence "shocks" that shift the law of motion. Conversely, a country with relatively low liquidity and high credit extension would exhibit a permanently appreciating real exchange rate.

Once the date $t \geq 2$ real exchange rate is determined by this law of motion, date t real balances follow from money market clearing, (32) yields the domestic country's relative price of non-traded goods (and $\check{p}_t^* = \check{x}_t \check{p}_t$), and government consumption is determined by the government budget constraints. The dynamic behavior of the foreign country relative price of non-traded goods is, from (31*b*),

$$\frac{\check{p}_t^*}{\check{p}_{t-1}^*} = \frac{\check{p}_t}{\check{p}_{t-1}} \frac{\check{x}_t}{\check{x}_{t-1}} = \frac{y^*(1-\pi^*)}{y^*(1-\varepsilon^*\pi^*) - y\pi(1-\varepsilon)/\check{x}_t}. \quad (34)$$

If $\check{x}_t$ is rising over time towards its steady state value, as it is under assumption 2, $\frac{\check{p}_t^*}{\check{p}_{t-1}^*} > 1$, although the (net) rate of increase in the foreign relative price falls over time, converging to zero as the real exchange rate approaches its steady state value.

We can now determine real interest rates paid to non-movers in dynamic equilibrium. Since $\check{R}_t^N = \check{R}_t^T \frac{\check{p}_{t-1}}{\check{p}_t} = \frac{\check{p}_{t-1}}{\check{p}_t}$ and $\check{R}_t^{*N} = \check{R}_t^{*T} \frac{\check{p}_{t-1}^*}{\check{p}_t^*} = \frac{\check{p}_{t-1}^*}{\check{p}_t^*}$ then, under assumption 2, the domestic real interest rate in non-traded goods rises over time towards its steady state value of one, since $\frac{\check{p}_{t-1}}{\check{p}_t} > 1$, while the foreign real non-traded return falls over time towards its steady state value. For existence of dynamic equilibrium, it must be the case that at every date on the trajectory these interest rates satisfy $\check{R}_t^N = \frac{\check{p}_{t-1}}{\check{p}_t} > \frac{\check{p}_{t-1}^N}{\check{p}_t^N}$, and $\check{R}_t^{*N} = \frac{\check{p}_{t-1}^*}{\check{p}_t^*} > \frac{\check{p}_{t-1}^{*N}}{\check{p}_t^N}$. Then existence of dynamic equilibrium requires that $\frac{\check{p}_t^T}{\check{p}_{t-1}^T} > 1$ and $\frac{\check{p}_t^{*T}}{\check{p}_{t-1}^{*T}} > 1, \forall t \geq 2.$ From the domestic money market clearing condition,

$$\frac{\check{p}_t^N}{\check{p}_{t-1}^N} = \sigma\left(\frac{\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_{t-1}}{\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_t}\right), t \geq 2. \tag{35a}$$

If, as is true under assumption 2, $\check{x}_t > \check{x}_{t-1} \forall t \geq 2,$ then the domestic country's non-traded goods inflation rate is *lower* than the rate of domestic nominal money growth at all dates, however, it rises over time in converging to the nominal money growth rate as the rate of increase of the real exchange rate declines. For $\frac{\check{p}_t^N}{\check{p}_{t-1}^N} > 1$ at every date, σ must be sufficiently high. Similarly, in the foreign country,

$$\frac{\check{p}_t^{*N}}{\check{p}_{t-1}^{*N}} = \sigma^*\left(\frac{\varepsilon^*\pi^* y^* + (1-\varepsilon)\pi y/\check{x}_{t-1}}{\varepsilon^*\pi^* y^* + (1-\varepsilon)\pi y/\check{x}_t}\right), t \geq 2. \tag{35b}$$

Under assumption 2, $\check{x}_t > \check{x}_{t-1}, \forall t \geq 2,$ and the foreign country non-traded goods price level rises at a rate *greater* than its nominal money growth rate. Then $\frac{\check{p}_t^{*N}}{\check{p}_{t-1}^{*N}} > 1, \forall t \geq 2.$ I impute the inflation rates of traded goods prices from the inflation rate of the nominal non-traded goods price and the rate of change of the relative price of non-traded goods for each country. Specifically,

$$\frac{\check{p}_t^T}{\check{p}_{t-1}^T} = \left(\frac{\check{p}_t^N}{\check{p}_{t-1}^N} / \frac{\check{p}_t}{\check{p}_{t-1}}\right) = \sigma\left(\frac{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_{t-1})\big((1-\varepsilon\pi)y - (1-\varepsilon^*)\pi^* y^* \check{x}_t\big)}{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_t)(1-\pi)y}\right), \tag{35c}$$

$$\frac{p_t^{*T}}{p_{t-1}^{*T}} = \left(\frac{p_t^{*N}}{p_{t-1}^{*N}} / \frac{p_t^*}{p_{t-1}^*}\right) = \sigma^*\left(\frac{(\varepsilon^*\pi^* y^* \check{x}_{t-1} + (1-\varepsilon)\pi y)\big((1-\varepsilon\pi)y - (1-\varepsilon^*)\pi^* y^* \check{x}_t\big)}{(1-\pi)y(\varepsilon^*\pi^* y^* \check{x}_t + (1-\varepsilon)\pi y)}\right). \tag{35d}$$

Under assumption 2, domestic country traded goods prices increase more quickly than domestic country non-traded goods prices, because the domestic country relative price of non-traded goods falls over time with a depreciating real exchange rate, from (32). However, the domestic money growth rate must be sufficiently high to ensure that money is return dominated. This is because the domestic money growth rate must be sufficiently high to ensure that $\frac{\check{p}_t^N}{\check{p}_{t-1}^N} \geq 1.$ Under assumption 2, foreign traded good prices increase less quickly than foreign country non-traded good prices, because the relative price of foreign non-traded goods rises over time. In this case, we know that $\frac{\check{p}_t^{*N}}{\check{p}_{t-1}^{*N}} > 1.$ Then, $\frac{\check{p}_t^{*T}}{\check{p}_{t-1}^{*T}} > 1, \forall t \geq 2,$ provided that the foreign money growth rate is sufficiently high.

Using the law of motion for the real exchange rate and the fact that $\frac{\check{x}_t}{\check{x}_{t-1}} = \frac{\check{e}_t}{\check{e}_{t-1}}\left(\frac{\check{p}_t^{*N}}{\check{p}_{t-1}^{*N}} / \frac{\check{p}_t^{N}}{\check{p}_{t-1}^{N}}\right)$, the rate of nominal exchange rate depreciation of the domestic country is

$$\frac{\check{e}_t}{\check{e}_{t-1}} = \frac{\sigma}{\sigma^*}\frac{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_{t-1})(\varepsilon^*\pi^* y^* \check{x}_t + (1-\varepsilon)\pi y)}{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \check{x}_t)(\varepsilon^*\pi^* y^* \check{x}_{t-1} + (1-\varepsilon)\pi y)}. \qquad (35e)$$

From (35*e*), if $\varepsilon + \varepsilon^* > 1$, $\frac{\check{e}_t}{\check{e}_{t-1}} \gtrless \frac{\sigma}{\sigma^*}$ *iff* $\frac{\check{x}_t}{\check{x}_{t-1}} \gtrless 1$. The converse is true if $\varepsilon + \varepsilon^* < 1$. Under assumption 2, the initial real exchange rate lies below the steady state real exchange rate, and the real exchange rate increases over time, $\frac{\check{x}_t}{\check{x}_{t-1}} > 1\ \forall t$. Then if $\varepsilon + \varepsilon^* > 1$, the nominal exchange rate of the domestic country depreciates at a faster rate than the relative nominal money growth rate of the domestic country – faster than its steady state growth rate. If $\varepsilon + \varepsilon^* < 1$ it depreciates more slowly. The intuition for this result is that if the portion of domestic liquidity demand that is demand for domestic currency exceeds the portion of foreign liquidity demand that is demand for domestic currency, the external value of the currency depreciates more quickly and vice versa. In the former (latter) case, the rate of nominal depreciation declines (increases) monotonically towards the relative money growth rate of the domestic country as the economy approaches the steady state.

6.2 Real Exchange Rate Targeting

I now consider conditions under which it is feasible for the domestic government to establish and maintain indefinitely a real exchange rate target $\bar{x}$ starting at some date $\hat{T}$. I assume that the economy has been in a steady state equilibrium until period $\hat{T}$. Establishment of the target at $\hat{T}$ surprises private agents, however, agents have perfect foresight at every other date. Below, I show that there exists a unique steady state equilibrium for $\bar{x} > \check{x}$, attainable from period $\hat{T} + 1$ onwards. Furthermore, the steady state equilibrium is the only equilibrium that the economy can attain following the successful establishment of a real exchange rate target at $\hat{T}$. Thus a real exchange rate target under free capital flows is not associated with the type of equilibrium dynamics that I described in section 6.1.2. In fact, if establishment of the target occurs at date 1, or any finite date following date 1, this eliminates all of the dynamic equilibria converging to a steady state in the absence of a target that I analyzed in 6.1.2.

I have demonstrated that a more appreciated real exchange rate than the steady state real exchange rate is not sustainable indefinitely, in a steady state equilibrium, under capital controls. The same intuition and almost identical mechanics generate the same result under free capital flows, so I ignore the case of $\bar{x} \leq \check{x}$ in what follows for the sake of brevity. To distinguish the values of variables under real exchange rate targeting from those without a real exchange rate target, I denote the value of variable z under the targeting regime by $\hat{\check{z}}$.

6.2.1 The initial period of the targeting regime First note from the money market clearing conditions that, with a constant real exchange rate target, as is true under capital controls the real balances of both currencies are constant at every date, $t \geq \hat{T}$. Consequently, the inflation rate of non-traded goods in a country for $t \geq \hat{T}+1$ is constant and equal to that country's money growth rate. At date $\hat{T}$, for $\bar{x} > \check{x}$, the non-traded goods value of domestic (foreign) real balances held by banks rises (falls) so the $\hat{T}$ domestic (foreign) country price level of non-traded goods must fall (rise) relative to the value it would have taken in the preceding steady state equilibrium. Namely, as in establishing a real exchange rate target under capital controls, the date $\hat{T}$ domestic (foreign) non-traded goods inflation rate is lower (higher) than σ (σ^*). For $t \geq \hat{T}+1$, however, both real balances and inflation rates of non-traded good price levels are constant.

In addition, as at all other dates under free capital flows, traded goods market clearing is independent of the real exchange rate at date $t \geq \hat{T}$, and hence of the real exchange rate regime. Together with the no arbitrage condition, traded goods market clearing implies that the period $\hat{T}+1$ world real interest rate equals its steady state value and period $\hat{T}$ value, $\hat{\check{R}}^T_{\hat{T}+1} = \hat{\check{R}}^{*T}_{\hat{T}+1} = 1, \hat{\check{R}}^T_{\hat{T}} = \hat{\check{R}}^{*T}_{\hat{T}} = \check{R}^T = 1.$ As in the absence of a target, the world real interest rate is constant at this value for all periods thereafter, $\hat{\check{R}}^T_{\hat{T}+t} = 1, \forall t \geq 0.$

The world loan market clearing condition, (11*c*), using $\hat{\check{p}}^*_{\hat{T}} = \bar{x}\hat{\check{p}}_{\hat{T}}$ is

$$\frac{(1-\pi)\psi\beta y\hat{\check{p}}_{\hat{T}}}{(1+\beta)} + \frac{(1-\pi^*)\psi\beta y^*\bar{x}\hat{\check{p}}_{\hat{T}}}{(1+\beta)} = \frac{(1-\psi)(q+q^*)}{(1+\beta)\hat{\check{R}}^T_{\hat{T}+1}}. \qquad (11c')$$

Equation (11*c*') exhibits both a constant real tradable interest rate equal to one, and hence equal to its previous period's steady state value, and a constant real exchange rate that is higher than its equilibrium value in the preceding steady state. Thus, a constant, unique value of the domestic relative price of non-traded goods solves (11*c*') at every date in a targeting regime $t \geq \hat{T}$. Furthermore, it cannot equal its preceding steady state value since $\bar{x} > \check{x}$. Specifically, it must be *lower* than its preceding steady state value at $\hat{T}-1$ to clear the loan market. Solving (11*c*') yields

$$\hat{\check{p}}_{\hat{T}} = \frac{(1-\psi)(q+q^*)}{\psi\beta}\left(\frac{1}{(1-\pi)y+(1-\pi^*)y^*\bar{x}}\right). \qquad (36a)$$

Then, $\hat{\hat{p}}^*_{\hat{T}} = \bar{x}\hat{\hat{p}}_{\hat{T}}$ must be higher than its preceding steady state equilibrium value for (11*c'*) to hold. In addition, the date $\hat{T}$ real interest rate measured in *non-traded* goods received by old non-movers from banks within each country also does not equal its steady state value of one, but is given by

$$\hat{\hat{R}}^N_{\hat{T}} = \hat{\hat{R}}^T_{\hat{T}} \frac{\hat{\hat{p}}_{\hat{T}-1}}{\hat{\hat{p}}_{\hat{T}}} = \frac{\hat{\hat{p}}_{\hat{T}-1}}{\hat{\hat{p}}_{\hat{T}}} = \left(\frac{y(1-\pi)\pi^*(1-\varepsilon^*) + y^*\bar{x}(1-\pi^*)\pi^*(1-\varepsilon^*)}{y(1-\pi)\pi^*(1-\varepsilon^*) + y(1-\pi^*)\pi(1-\varepsilon)}\right) > 1, \qquad (36b)$$

$$\hat{\hat{R}}^{*N}_{\hat{T}} = \hat{\hat{R}}^{*T}_{\hat{T}} \frac{\hat{\hat{p}}^*_{\hat{T}-1}}{\hat{\hat{p}}^*_{\hat{T}}} = \frac{\check{x}\hat{\hat{p}}_{\hat{T}-1}}{\bar{x}\hat{\hat{p}}_{\hat{T}}} = \left(\frac{y(1-\pi)\pi(1-\varepsilon) + y^*\bar{x}(1-\pi^*)\pi(1-\varepsilon)}{\bar{x}y^*(1-\pi)\pi^*(1-\varepsilon^*) + \bar{x}y^*(1-\pi^*)\pi(1-\varepsilon)}\right) < 1. \qquad (36c)$$

At every date after $\hat{T}$, however, since the relative price of non-traded goods that clears the loan market is constant, and given by its date $\hat{T}$ value (36*a*), non-traded returns in each country are constant and equal to one.

Substituting (36b) into the date $\hat{T}$ domestic non-traded goods market clearing condition, or (36c) into the date $\hat{T}$ foreign non-traded goods market clearing condition, yields the unique value of the reserve adjustment at date $\hat{T}$ that clears both non-traded goods markets at these interest rates,

$$\psi y = \frac{\psi y}{1+\beta} + \frac{\varepsilon\pi\psi\beta y}{1+\beta} + \frac{(1-\varepsilon^*)\pi^*\psi\beta y^*\bar{x}}{1+\beta} + \frac{(1-\pi)\psi\beta y}{1+\beta} R^N_{\hat{T}} - \Delta\hat{\hat{f}}_{\hat{T}},$$

$$\psi y^* = \frac{\psi y^*}{1+\beta} + \frac{\varepsilon^*\pi^*\psi\beta y^*}{1+\beta} + \frac{(1-\varepsilon)\pi\psi\beta y}{(1+\beta)\bar{x}} + \frac{(1-\pi^*)\psi\beta y^*}{1+\beta} R^{*N}_{\hat{T}} + \Delta\hat{\hat{f}}_{\hat{T}}/\bar{x}.$$

This solution is

$$\Delta\hat{\hat{f}}_{\hat{T}} = \left(\bar{x} - \hat{\hat{x}}\right)\left(\frac{\psi\beta}{1+\beta}\right)\pi^*(1-\varepsilon^*)y^*\left(1 + \frac{(1-\pi)(1-\pi^*)}{(1-\pi)\pi^*(1-\varepsilon^*)+(1-\pi^*)\pi(1-\varepsilon)}\right) > 0. \qquad (36d)$$

Substituting (36d) into the date $\widehat{T}$ government budget constraints, and real balances evaluated at the real exchange rate target, yields government consumption

$$\hat{\hat{g}}_{\hat{T}} = (\bar{x}^{max} - \bar{x})\left(\frac{1}{\sigma}\right)\left(\frac{\psi\beta}{1+\beta}\right)\pi^*(1-\varepsilon^*)y^*(1+\Omega), \qquad (36e)$$

$$\hat{\hat{g}}^*_{\hat{T}} = \left(\bar{x} - \bar{x}^{min}\right)\left(\frac{1}{\sigma^*}\right)\left(\frac{\psi\beta}{1+\beta}\right)\frac{\pi^*(1-\varepsilon^*)y^*}{\bar{x}}\left(\left(\frac{\sigma^*-\varepsilon^*}{1-\varepsilon^*}\right)\Omega + \sigma^*(1-\pi)(1-\pi^*)\right), \qquad (36f)$$

where $\Omega \equiv (1-\pi)\pi^*(1-\varepsilon^*) + (1-\pi^*)\pi(1-\varepsilon) \in (0,1)$, and

$$\bar{x}^{max} = \check{x}\left(\frac{\left(\frac{\sigma-\varepsilon}{1-\varepsilon}\right)\Omega + \left(\frac{\sigma}{1-\varepsilon}\right)(1-\pi)(1-\pi^*)}{\Omega + \sigma(1-\pi)(1-\pi^*)}\right) > \check{x},$$

$$\bar{x}^{min} = \check{x}\left(\frac{\Omega + \sigma^*(1-\pi)(1-\pi^*)}{\left(\frac{\sigma^*-\varepsilon^*}{1-\varepsilon^*}\right)\Omega + \sigma^*(1-\pi)(1-\pi^*)}\right) < \check{x}.$$

It is straightforward to verify that, as usual, the upper bound for the target value that guarantees non-negative domestic government consumption when $\bar{x} > \check{x}$, $\bar{x}^{max}$, is increasing in the domestic money growth rate, σ. Similarly, the lower bound for the real exchange rate target that guarantees non-negative foreign government consumption when $\bar{x} < \check{x}$, $\bar{x}^{min}$, is decreasing in the foreign money growth rate, σ^*.

The domestic government can establish a real exchange rate target relatively depreciated compared to the steady state equilibrium real exchange rate under free capital flows provided its money growth rate is sufficiently high to guarantee that its own consumption is non-negative. For the target and the date $\widehat{T}$ solutions to be consistent with equilibrium requires, of course, that money be return-dominated by loans in both countries at $\widehat{T}$. I return to this issue below. I record the private sector date $\widehat{T}$ consumption and asset solutions in Appendix I.

6.2.2 Existence of steady state equilibrium with $\bar{x} > \check{x}$ As I noted above, since the real exchange rate is irrelevant for traded goods market clearing, the unique world real interest rate that clears this market when there are no arbitrage opportunities equals one at every date (except date 1) in *any* equilibrium, including a steady state equilibrium (if it exists). Then, in a steady state equilibrium, given the target $\bar{x}$, the world loan market clearing condition (11*c*) yields a unique, constant solution for the relative price of non-traded goods in each country,

$$\hat{\hat{p}} = \frac{1}{\psi\beta}\left(\frac{(1-\psi)(q+q^*)}{y(1-\pi)+y^*\bar{x}(1-\pi^*)}\right) = \frac{\hat{\hat{p}}^*}{\bar{x}}. \qquad (37a)$$

Note that these stationary solutions are identical to those under capital controls with a real exchange rate target. They are also the solutions for relative prices in the economy with free capital flows and a real exchange rate target at every date from $\hat{T}$ onwards. Since $\hat{\hat{R}}^{*T} = \hat{\hat{R}}^{T} = 1$, and using the fact that relative prices are constant in a steady state, then

$$\hat{\hat{R}}^N_{t+1} = \hat{\hat{R}}^T \frac{\hat{\hat{p}}_{t-1}}{\hat{\hat{p}}_t} = \hat{\hat{R}}^T = \hat{\hat{R}}^N = 1, \qquad (37b)$$

$$\hat{\hat{R}}^{*N}_{t+1} = \hat{\hat{R}}^{*T} \frac{\hat{\hat{p}}_{t-1}\hat{\hat{x}}_{t-1}}{\hat{\hat{p}}_t\hat{\hat{x}}_t} = \hat{\hat{R}}^{*T} = \hat{\hat{R}}^{*N} = 1. \qquad (37c)$$

Substituting these solutions into either of the non-traded goods market clearing conditions, and combining these conditions with the money market clearing conditions and government budget constraints, yields a unique solution for the domestic government's steady state reserve adjustment $\Delta\hat{\hat{f}}$. The common value that solves either non-traded goods market clearing condition is

$$\Delta\hat{\hat{f}} = (\bar{x} - \check{x})\left(\frac{\psi\beta}{1+\beta}\right)(\pi^*(1-\varepsilon^*)y^*) < \Delta\hat{\hat{f}}_{\hat{T}}. \tag{37d}$$

This is smaller than $\Delta\hat{\hat{f}}_{\hat{T}}$, because the steady state domestic (foreign) non-tradable loan return is lower (higher) than in the initial period of the targeting regime, yielding lower (higher) purchasing power over non-traded goods for old workers writing checks than at $\hat{T}$. Then higher domestic and lower foreign government consumption is required to clear local markets in the steady state than at $\hat{T}$, a and a smaller domestic reserve adjustment results. The steady state solutions for real balances are identical to those that obtain at every date with a constant real exchange rate, while the constant value of government consumption in each country derives from the government budget constraints and I discuss these below.

These steady state solutions satisfy the optimality and market clearing conditions, and the government budget constraints of definition 2. In addition, all of the endogenous variables take admissible values at every date in this steady state. The solutions (37*a*) through (37*d*) and the assumption that $\bar{x} > \check{x}$, guarantee admissible values for all of the endogenous variables except government consumption. In particular, the government's real and nominal reserve adjustment is always positive, and hence sustainable. In addition, money is return dominated in a steady state equilibrium because real loan interest returns measured in non-traded goods equal one in each country at all dates. I record the private sector steady state equilibrium consumption and asset allocations in Appendix H. The steady state government consumption that satisfies the government budget constraint in each country, and which must be non-negative in equilibrium, is

$$\hat{\hat{g}} = \left(\bar{\bar{x}}^{max} - \bar{x}\right)\left(\frac{1}{\sigma}\right)\left(\frac{\psi\beta}{1+\beta}\right)\pi^*(1-\varepsilon^*)y^*, \tag{37e}$$

$$\hat{\hat{g}}^* = \left(\bar{x} - \bar{\bar{x}}^{min}\right)\left(\frac{1}{\sigma^*}\right)\left(\frac{\psi\beta}{1+\beta}\right)\frac{\pi^*(\sigma^*-\varepsilon^*)y^*}{\bar{x}}, \tag{37f}$$

where

$$\bar{\bar{x}}^{max} = \frac{y}{y^*}\frac{\pi(\sigma-\varepsilon)}{\pi^*(1-\varepsilon^*)} > \check{x},$$

$$\bar{\bar{x}}^{min} = \frac{y}{y^*}\frac{\pi(1-\varepsilon)}{\pi^*(\sigma^*-\varepsilon^*)} < \check{x}.$$

For government consumption to be non-negative, (37*e*) and (37*f*) imply that the target value must satisfy $\bar{x} \in [\bar{\bar{x}}^{min}, \bar{\bar{x}}^{max}]$. Again, the upper bound $\bar{\bar{x}}^{max}$ is increasing in the domestic money growth rate while $\bar{\bar{x}}^{min}$ is decreasing in the foreign money growth rate. It is clear that, since $\bar{\bar{x}}^{min} < \check{x}$, the lower bound is irrelevant for all targets that yield sustainable (positive) reserve adjustments $(\bar{x} >$

$\breve{x}$); steady state foreign government consumption increases by the foreign non-traded goods value of the positive domestic government reserve adjustment. A higher domestic money growth rate therefore simply increases the range of target values, above the steady state equilibrium real exchange rate, that are consistent with a targeting steady state equilibrium with non-negative domestic government consumption. Proposition 17 establishes conditions under which the upper bound of sustainable real exchange rate targets under free capital flows is not more constrained than that under capital controls.

Proposition 17. ***Fiscal and monetary policy with a target under free capital flows***

Let $\sigma \geq \frac{1}{\pi}$. *Then* $\forall \bar{x} \in \left(\breve{x}, \frac{y}{y^*}\frac{1-\varepsilon\pi}{\pi^*(1-\varepsilon^*)}\right), \hat{\bar{g}} > 0, \hat{\bar{g}}^* > 0$.

Proof. Manipulating the expression for $\bar{\bar{x}}^{max}$ and comparing it to $\frac{y}{y^*}\frac{1-\varepsilon\pi}{\pi^*(1-\varepsilon^*)}$, it is evident that if $\sigma \geq \frac{1}{\pi}$, then $\bar{\bar{x}}^{max} \geq \frac{y}{y^*}\frac{1-\varepsilon\pi}{\pi^*(1-\varepsilon^*)}$. Then $\forall \bar{x} \in \left(\breve{x}, \frac{y}{y^*}\frac{1-\varepsilon\pi}{\pi^*(1-\varepsilon^*)}\right)$, $\bar{\bar{x}}^{max} > \bar{x}$, and from (37*a*) $\hat{\bar{g}} > 0$. Since $\bar{x} > \breve{x} > \bar{\bar{x}}^{min}$, then from (37*b*) $\hat{\bar{g}}^* > 0$. ■

Proposition 18 summarizes the foregoing, and I state it without proof.

Proposition 18. ***Steady state equilibrium with a target under free capital flows***

Let $\bar{x} \in \left(\breve{x}, \frac{y}{y^*}\frac{1-\varepsilon\pi}{\pi^*(1-\varepsilon^*)}\right)$. *Then there exists a unique, steady state equilibrium with* $\hat{\bar{g}} > 0, \hat{\bar{g}}^* > 0$, $1 = \hat{\bar{R}}^N > \frac{\hat{\bar{p}}_{t-1}^N}{\hat{\bar{p}}_t^N} = \frac{1}{\sigma}$ *and* $1 = \hat{\bar{R}}^{*N} > \frac{\hat{\bar{p}}_{t-1}^{*N}}{\hat{\bar{p}}_t^{*N}} = \frac{1}{\sigma^*}$ *iff* $\sigma \geq \frac{1}{\pi}$.

As in the steady state without a real exchange rate target, since the real interest rate is equal to one, $\hat{\bar{c}}_y^T = \frac{q}{1+\beta}$, and this balances trade,

$$\widehat{\overline{TB}}^T = q - \left(\frac{q}{1+\beta}\right) - \left(\frac{q\beta}{1+\beta}\right) = 0.$$

The domestic country's steady state per worker financial balance measured in domestic non-traded goods, is

$$\widehat{\overline{FB}} = \frac{\pi^*(1-\varepsilon^*)\beta y^*\bar{x}}{1+\beta} - \frac{\pi(1-\varepsilon)\beta y}{1+\beta}.$$

At a gross real interest rate of one there is no change in net lending between any two periods in the steady state, $\left[\left(\frac{(1-\pi^*)\psi\beta y^*\bar{x}\hat{\bar{p}}}{(1+\beta)} - \frac{(1-\psi)q^*}{(1+\beta)\hat{\bar{R}}^T}\right) - \left(\frac{(1-\pi^*)\psi\beta y^*\bar{x}\hat{\bar{p}}}{(1+\beta)} - \frac{(1-\psi)q^*}{(1+\beta)\hat{\bar{R}}^T}\right) = 0\right]$ and the current account balance is zero. As $\bar{x} > \breve{x}$, $\widehat{\overline{FB}} = \frac{\beta y^*}{1+\beta}(\bar{x} - \breve{x})\pi^*(1-\varepsilon^*) > 0$. The domestic government's net accumulation of foreign currency exactly offsets this positive balance. In short, the per capita, non-traded goods value of the steady state balance of payments is just $\widehat{\overline{BOP}} = \Delta\hat{\bar{f}} = \psi\widehat{\overline{FB}}$.

6.2.3 Dynamic equilibrium and stabilization of real activity At every date $t \geq \hat{T} + 1$, the equilibrium conditions and their unique solutions are identical to those in the steady state equilibrium.

Therefore if the economy satisfies the conditions of Proposition 18, it attains the unique steady state equilibrium at $\hat{T}+1$. Then there exists a dynamic equilibrium comprising the date $\widehat{T}$ solutions recorded in 6.2.1, followed by an infinite sequence of the steady state equilibrium solutions recorded in 6.2.2, if money is return dominated at every relevant date. Since real interest rates measured in non-traded goods equal one in the steady state, it is immediate that return domination of money is satisfied for interest rates $\hat{\hat{R}}_t^N$ and $\hat{\hat{R}}_t^{*N}$ from date $t = \hat{T}+1$ onwards, and banks in period $\hat{T}$ hold money only to meet liquidity needs. We also know that any steady state equilibrium satisfies $\hat{\hat{R}}_{\hat{T}}^N > 1 > \frac{1}{\sigma}$ and $\frac{1}{\sigma^*} < \hat{\hat{R}}_{\hat{T}}^{*N} < 1$, by definition of an equilibrium. Since the institution of the target is a "surprise", banks in the prior steady state equilibrium behave as though return domination is satisfied between $\hat{T}-1$ and $\hat{T}$, and hold money solely to satisfy liquidity needs of movers. Ex post, of course, real interest rates deviate from their steady state values, as do inflation rates, but this has no implications for bank portfolio choices.

As long as the real exchange rate target is constant, there are no equilibrium dynamics from $\hat{T}+1$ onwards. This is also true if the government establishes a target, $\bar{x} > \check{x}$, at any date during the unique dynamic equilibrium path of real depreciation observed without a target, or at date 1, when it is fully anticipated by private agents. In either case, there is one period of adjustment, when internal real loan returns measured in non-traded goods, and internal relative prices of non-traded goods, adjust to accommodate the higher real exchange rate than that period's equilibrium real exchange rate. Thereafter, the real exchange rate, the internal relative price(s) of non-traded goods that clear the global loan market, and - hence – non-tradable loan returns are all constant at their steady state values. Consequently, establishing a real exchange rate target at any finite date under free capital flows completely stabilizes real activity relative to the dynamic equilibrium that otherwise obtains. In addition, since in the targeting regime the inflation rate of a country's non-traded good price equals its constant money growth rate from $\hat{T}+1$ onwards, the rate of nominal exchange rate depreciation is just its steady state equilibrium value, $\frac{\hat{\hat{e}}_{\hat{T}+t}}{\hat{\hat{e}}_{\hat{T}+t-1}} = \frac{\bar{x}}{\bar{x}}\frac{\hat{\hat{p}}_{\hat{T}+t}^N/\hat{\hat{p}}_{\hat{T}+t-1}^N}{\hat{\hat{p}}_{\hat{T}+t}^{*N}/\hat{\hat{p}}_{\hat{T}+t-1}^{*N}} = \frac{\sigma}{\sigma^*}$, $t \geq 1$. In the dynamic equilibrium that we would otherwise observe in finite time, the rate of nominal depreciation may be higher than this. Specifically, from (35*e*), if $\varepsilon + \varepsilon^* > 1$, $\frac{\check{e}_{t+1}}{\check{e}_t} > \frac{\sigma}{\sigma^*}$. If private agents trade little in currencies, then for the domestic country, establishing a real exchange rate target under free capital flows may also stabilize the rate of nominal exchange rate depreciation.

7. Conclusion

I develop a two-country, monetary, dynamic general equilibrium model with flexible prices in which the composite fiscal-monetary authority of a country can unilaterally establish and sustain indefinitely a relatively depreciated real exchange rate target through foreign reserve accumulation. If government consumption endogenously adjusts to sterilize the consequences of the intervention for private consumption of non-traded goods, the policy is not inflationary. Nonetheless, money growth rates in both countries must be "sufficiently high" to support the equilibria I analyze, guaranteeing that loans dominate money in rate of return (so that banks hold currency solely for its liquidity as I assume), and that domestic government consumption is non-negative. Under capital controls, the real exchange rate targeting regime improves the targeting country's trade balance, supporting a mercantilist rationale for the policy. Under free capital flows, it has no impact for the trade balance but – if introduced at date 1, for example – stabilizes real activity and, potentially, the nominal exchange rate, relative to their equilibrium behavior at finite time horizons in the absence of a target.

The model in which I obtain these results is, obviously, highly stylized, and many of its assumptions strong. A natural extension would separate the budget constraints of the fiscal and monetary policy authority, with government consumption funded by taxes and bond sales, and the central bank's seigniorage revenue funding reserve accumulation and government bond purchases. An evaluation of the effects of traditional sterilization of reserve accumulation, via central bank sales of bonds to private banks, would be possible in this environment. A second extension would seek to rationalize the absence of an international response to the targeting regime, by analyzing the implications of retaliation through competitive devaluation, or the imposition of tariffs. A third modification would relax the assumption that there is no aggregate uncertainty in liquidity demand can be relaxed, by allowing randomness in π and π^*. Alternatively, the assumption of no aggregate uncertainty in alternative currency demands can be relaxed, by allowing randomness in ε and ε^*. These extensions would permit evaluation of whether the management of reserves to maintain a target is feasible when there is risk of an aggregate liquidity or currency crisis, and an analysis of whether reserve management and capital controls can stem financial crises, in an environment where agents hold country-specific currencies solely for their liquidity. Finally, introducing capital formation and endogenous growth would enable an analysis of how real exchange rate targeting, currency manipulation, and capital controls affect a country's long-run growth prospects. Beyond the scope of the current paper, I leave these extensions to future research.

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Appendix

A. ***Steady state equilibrium with capital controls and no targeting***

The steady state consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, and real balances and government consumption in each country are;

$$c_y^N = \frac{y}{(1+\beta)}, \qquad c_y^{*N} = \frac{y^*}{(1+\beta)},$$

$$c_o^{N,\varepsilon\pi} = \frac{\beta y}{1+\beta}\left(\frac{1}{\sigma}\right), \qquad c_o^{*N,\varepsilon^*\pi^*} = \frac{\beta y^*}{1+\beta}\left(\frac{1}{\sigma^*}\right),$$

$$c_o^{N,(1-\varepsilon)\pi} = \frac{\beta y}{1+\beta}\left(\frac{1}{\sigma}\right), \qquad c_o^{*N,(1-\varepsilon^*)\pi^*} = \frac{\beta y^*}{1+\beta}\left(\frac{1}{\sigma^*}\right),$$

$$c_o^{N,1-\pi} = \left(\frac{\beta y}{1+\beta}\right)\left(\frac{q}{1-\pi}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$c_o^{*N,1-\pi^*} = \left(\frac{\beta y^*}{1+\beta}\right)\left(\frac{q^*}{1-\pi^*}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right),$$

$$c_y^T = \frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}{(1+\beta)\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)},$$

$$c_y^{*T} = \frac{(1-\pi^*)\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)}{(1+\beta)\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)},$$

$$l_e = -\frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}{(1+\beta)\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)},$$

$$l_e^* = -\frac{(1-\pi^*)\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)}{(1+\beta)\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)},$$

$$c_o^T = \frac{\beta q}{(1+\beta)}, \qquad c_o^{T*} = \frac{\beta q^*}{(1+\beta)},$$

$$m^d = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), m^f = \left(\frac{(1-\varepsilon)\pi\beta y}{1+\beta}\right), m^{*f} = \left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right), m^{*d} = \left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right),$$

$$m = \left(\frac{\psi\beta y}{1+\beta}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*)+q\pi((1-\pi^*)\varepsilon+\pi^*(1-\varepsilon^*))}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$m^* = \left(\frac{\psi\beta y^*}{1+\beta}\right)\left(\frac{q^*\pi^*\big((1-\pi)\varepsilon^*+\pi(1-\varepsilon)\big)+q\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right),$$

$$g = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\psi\beta y}{1+\beta}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*)+q\pi((1-\pi^*)\varepsilon+\pi^*(1-\varepsilon^*))}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$g^* = \left(\frac{\sigma^*-1}{\sigma^*}\right)\left(\frac{\psi\beta y^*}{1+\beta}\right)\left(\frac{q^*\pi^*\big((1-\pi)\varepsilon^*+\pi(1-\varepsilon)\big)+q\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right).$$

B. ***The initial period and dynamic equilibrium under capital controls and no targeting***

Equations (12*a*) and (12*b*) show the initial period non-traded goods market clearing conditions. With no real exchange rate targeting, reserve movements are zero, and these conditions can be re-expressed as the following two equations in the initial real exchange rate and initial domestic relative price of non-traded goods,

$$x_1 = \frac{(1-\varepsilon\pi)\psi\beta y - (1-\psi)q(1+\beta)/p_1}{(1-\varepsilon^*)\pi^*\psi\beta y^*}, \qquad (A.1)$$

$$x_1 = \frac{(1-\varepsilon)\pi\psi\beta y + (1-\psi)q^*(1+\beta)/p_1}{(1-\varepsilon^*\pi^*)\psi\beta y^*}. \qquad (A.2)$$

Figure 4 depicts the two loci implied by (A.1) and (A.2), which are very similar to those characterizing the steady state equilibrium in (16*a*) and (16*b*). The loci have a unique intersection at strictly positive and finite values of x_1 and p_1, which implies there exists at most one initial period solution satisfying market clearing and optimality for these two variables. Specifically,

$$0 < x_1 \in \left(\frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*\pi^*)}, \frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right),$$

$$0 < p_1 \in \left(\left(\frac{1+\beta}{\beta}\right)\frac{(1-\psi)q}{\psi(1-\varepsilon\pi)y}, \infty\right).$$

The only difference relative to the joint determination of these variables at every other date, and in the steady state equilibrium, is the appearance of $1+\beta$ in the numerator of the value of p_1 at which (20*a*) intersects the horizontal axis. The lowest possible solution for p_1 which satisfies non-negativity of the real exchange rate and domestic non-traded goods market clearing is *higher* at date 1 than at any other date. This is because the quantity of traded good claims held by initial old agents is higher than at any other date. The real exchange rate and hence demand for non-traded goods deriving from the real money balances held by initial old agents that are consistent with this higher value must be lower at any given relative price of non-traded goods – (20*a*) is lower than (16*a*).

Given the solution to (20*a*) and (20*b*), $p_1^* = p_1 x_1$ follows, (11*a*) and (11*b*) yield the initial loan market clearing real interest rates that are consistent with the initial relative price of non-traded goods of each country, and all other solutions for initial period endogenous variables follow immediately. The initial period traded goods market clearing condition is not independent of the remaining initial period equilibrium conditions, and is satisfied at the real interest rates that clear loan markets. Rearranging the condition yields the following relationship between domestic and foreign country real interest rates in the initial period,

$$R_2^T = \frac{q}{(q+q^*)(1+\beta) - \frac{q^*}{R_2^{*T}}}. \qquad (A.3)$$

Figure 5 depicts this relationship. It is evident from (21) than $R_2^T \gtrless \frac{1}{1+\beta}\ iff\ R_2^{*T} \lessgtr \frac{1}{1+\beta}$.

As in the steady state equilibrium, the consequence of a relatively high real interest rate is that domestic young entrepreneurs borrow and consume relatively few traded goods, and the domestic country runs a trade surplus on these goods as a result. Recall that only young entrepreneurs consume traded goods in the initial period. The initial period per entrepreneur, external balance of the domestic country in traded goods is, therefore, $TB_1^T = q - c_{y,1}^T$. Substituting for young entrepreneurs' initial period equilibrium consumption, $c_{y,1}^T = \frac{q}{(1+\beta)R_2^T}$, this balance is just

$$TB_1^T = q - \frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)\big)}{\big((1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)}. \qquad (A.4)$$

The domestic country's initial period financial balance measured in non-traded goods is equal to its steady state value, since the initial period real exchange rate equals its steady state value,

$$FB_1 = FB = \frac{\beta y}{1+\beta}\left((1-\varepsilon^*)\pi^*\left(\frac{q^*(1-\varepsilon\pi)+q(1-\varepsilon)\pi}{q^*(1-\varepsilon^*)\pi^*+q(1-\varepsilon^*\pi^*))}\right) - (1-\varepsilon)\pi\right).$$

However, its traded good value is larger than its steady state value in absolute terms, since the relative price of non-traded goods is higher in the initial period than in the steady state. See section 4.3 for proposition and proof of dynamic equilibrium, and proposition concerning initial period external balances.

Proof of Proposition 4. *Dynamic equilibrium under capital controls*

The solutions for relative prices that satisfy all of the market clearing conditions at *t=1* are,

$$p_1 = \left(\frac{(1-\psi)(1+\beta)}{\psi\beta y}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)}{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right)$$

$$x_1 = \left(\frac{y}{y^*}\right)\left(\frac{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)}\right),$$

$$p_1^* = \left(\frac{(1-\psi)(1+\beta)}{\psi\beta y^*}\right)\left(\frac{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right),$$

$$R_2^T = \left(\frac{q}{(1+\beta)(1-\pi)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)}\right),$$

$$R_2^{*T} = \left(\frac{q^*}{(1+\beta)(1-\pi^*)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}\right),$$

In addition, initial period nominal prices are

$$p_1^N = \left(\frac{\sigma M_0(1+\beta)}{\psi\beta y}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*) + q(1-\varepsilon^*\pi^*)}{q^*\pi^*(1-\varepsilon^*) + q\pi\big(\varepsilon(1-\pi^*) + \pi^*(1-\varepsilon^*)\big)}\right),$$

$$p_1^{*N} = \left(\frac{\sigma^* M_0^*(1+\beta)}{\psi\beta y^*}\right)\left(\frac{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}{q^*\pi^*\big(\varepsilon^*(1-\pi) + \pi(1-\varepsilon)\big) + q\pi(1-\varepsilon)}\right).$$

$$p_1^T = \left(\frac{\sigma M_0}{(1-\psi)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*) + q\pi\big(\varepsilon(1-\pi^*) + \pi^*(1-\varepsilon^*)\big)}\right),$$

$$p_1^{*T} = \left(\frac{\sigma^* M_0^*}{(1-\psi)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi) - \pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*\pi^*\big(\varepsilon^*(1-\pi) + \pi(1-\varepsilon)\big) + q\pi(1-\varepsilon)}\right),$$

$$e_1 = \frac{p_1^T}{p_1^{*T}} = \left(\frac{\sigma M_0}{\sigma^* M_0^*}\right)\left(\frac{q^*\pi^*\big(\varepsilon^*(1-\pi) + \pi(1-\varepsilon)\big) + q\pi(1-\varepsilon)}{q^*\pi^*(1-\varepsilon^*) + q\pi\big(\varepsilon(1-\pi^*) + \pi^*(1-\varepsilon^*)\big)}\right).$$

At every $t \geq 2$, the equilibrium conditions yield exactly the steady state solutions for relative prices; $p_t = p$, $x_t = x$, $p_t^* = p^*$, $R_{t+1}^T = R^T = R^N$, $R_{t+1}^{*T} = R^{*T} = R^{*N} = \left(\frac{q^*}{1-\pi^*}\right)\left(\frac{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - \pi(1-\varepsilon)\pi^*(1-\varepsilon^*)}{q^*(1-\varepsilon\pi) + q\pi(1-\varepsilon)}\right)$. Initial period real interest rates measured in terms of non-traded goods are thus equal to their steady state equilibrium values, $R_2^N = R_2^T \frac{p_1}{p_2} = R_2^T \frac{p_1}{p} = R_2^T(1+\beta) = R^N$, $R_2^{*N} = R_2^{*T}\frac{p_1^*}{p_2^*} = R_2^{*T}\frac{p_1^*}{p^*} = R_2^{*T}(1+\beta) = R^{*N}$. Since the real exchange rate and real balances are constant in both countries from date 1 onwards, nominal non-traded prices must rise at money growth rates, $p_{t+1}^N = \sigma p_t^N$; $p_{t+1}^{*N} = \sigma^* p_t^{*N}, \forall t \geq 1$. Since the real exchange rate is constant from *t=1* onwards, the nominal exchange rate between any two periods must depreciate at a rate equal to the relative domestic money growth rate, $\frac{e_{t+1}}{e_t} = \frac{\sigma}{\sigma^*}$. However, from $p_2 = \frac{1}{1+\beta} p_1$, the rate of growth of nominal traded goods prices between dates 1 and 2 must satisfy $p_2^T = \sigma(1+\beta)p_1^T$, $p_2^{*T} = \sigma^*(1+\beta)p_1^{*T}$, while since p and p^* are constant from date 2 onwards, $p_{t+1}^T = \sigma p_t^T$; $p_{t+1}^{*T} = \sigma^* p_t^{*T}, \forall t > 1$. From period *t=2* onwards, real and nominal variables take on their steady state values.

From figure 4, there exists at most one initial period real exchange rate and initial period domestic relative price of non-traded goods (and hence all other initial period endogenous variables) satisfying the optimality conditions of definition 2 and initial period government budget constraint and market clearing conditions. This solution must also satisfy return domination of currency between dates 1 and 2 to be part of an equilibrium. Manipulating the solution for R_2^T and using definition 1, it is evident that $R_2^T \gtrless \frac{1}{1+\beta}$ iff $\pi \gtrless \hat{\pi}$. Hence, under assumption 2, $R_2^T > \frac{1}{1+\beta}$. Then $R_2^N = R_2^T\left(\frac{p_1}{p}\right) = R_2^T(1+\beta) > 1$. It is immediate that $R_2^N > \frac{p_1^N}{p_2^N} = \frac{1}{\sigma}$. Assumption 2 implies that $R_2^{*T} < \frac{1}{1+\beta}$. Hence, $R_2^{*N} = R_2^{*T}\left(\frac{p_1^*}{p_2^*}\right) = R_2^{*T}(1+\beta) < 1$. Then, the foreign real interest rate satisfies return domination of currency between dates 1 and 2 $iff\ R_2^{*N} = R_2^{*T}(1+\beta) > \frac{p_1^{*N}}{p_2^{*N}} = \frac{1}{\sigma^*}$. Using the solution for R_2^{*T} above this condition is equivalent to

$$\sigma^* > \left(\left(\frac{q^*}{(1-\pi^*)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right)\right)^{-1}.$$

The solutions for all endogenous variables satisfying all of the equilibrium conditions for the economy from date 2 onwards are identical to those in a steady state equilibrium, the existence of which proposition 1 establishes. As illustrated by (16*a*), (16*b*) and figure 2, these solutions are unique. Then all that is required for the initial period solutions, and an infinite sequence of steady state solutions from date 2 onwards to constitute a dynamic equilibrium is that currency be return dominated by loans within each country from date 2 onwards. Proposition 1 and its proof establish that this condition is satisfied, under assumption 2, *iff*

$$\sigma^* > \left(\left(\frac{q^*}{(1-\pi^*)}\right)\left(\frac{(1-\varepsilon^*\pi^*)(1-\varepsilon\pi)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right)\right)^{-1}. \blacksquare$$

The initial period consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, and real balances and government consumption in each country are;

$$c_{y,1}^N = \frac{y}{(1+\beta)}, \qquad c_{y,1}^{*N} = \frac{y^*}{(1+\beta)}.$$

$$c_{o,1}^N = \frac{M_0}{\psi p_1^N} + \frac{(1-\psi)q}{\psi p_1} = \frac{M_0}{\psi p_1^N} + \frac{q\beta y\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)}{(1+\beta)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)},$$

$$c_{o,1}^N = \frac{\varepsilon\pi y\beta+(1-\varepsilon^*)\pi^*\beta y^* x_1}{(1+\beta)\sigma} + \frac{\beta y\big(q(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-q\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)\big)}{(1+\beta)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)},$$

$$c_{o,1}^N = \left(\frac{\beta y}{1+\beta}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*)+q\big((\sigma(1-\varepsilon^*\pi^*)-\pi(\sigma-1)(\pi^*(1-\varepsilon^*)(1-\varepsilon))\big)}{\sigma\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}\right),$$

$$c_{o,1}^{*N} = \left(\frac{\beta y^*}{1+\beta}\right)\frac{q\pi(1-\varepsilon)+q^*\left(\sigma^*(1-\varepsilon\pi)-\pi^*(\sigma^*-1)\big((1-\pi)\varepsilon^*+\pi(1-\varepsilon)\big)\right)}{\sigma^*\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)},$$

$$c_{y,1}^T = \frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)},$$

$$c_{y,1}^{*T} = \frac{(1-\pi^*)\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)},$$

$$l_{e,2} = -\left(\frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right),$$

$$l_{e,2}^{*} = -\left(\frac{(1-\pi^*)\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right),$$

$$l_2 = \left(\frac{(1-\pi)\big(q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right),$$

$$l_2^{*} = \left(\frac{(1-\pi^*)\big(q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)\big)}{(1-\varepsilon\pi)(1-\varepsilon^*\pi^*)-\pi^*(1-\varepsilon^*)\pi(1-\varepsilon)}\right),$$

$$m_1^d = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), m_1^f = \left(\frac{(1-\varepsilon)\pi\beta y}{1+\beta}\right), m_1^{*f} = \left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right), m_1^{*d} = \left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right),$$

$$m_1 = \left(\frac{\psi\beta y}{1+\beta}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*)+q\pi((1-\pi^*)\varepsilon+\pi^*(1-\varepsilon^*))}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$m_1^{*} = \left(\frac{\psi\beta y^*}{1+\beta}\right)\left(\frac{q^*\pi^*\big((1-\pi)\varepsilon^*+\pi(1-\varepsilon)\big)+q\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right),$$

$$g_1 = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\psi\beta y}{1+\beta}\right)\left(\frac{q^*\pi^*(1-\varepsilon^*)+q\pi((1-\pi^*)\varepsilon+\pi^*(1-\varepsilon^*))}{q^*\pi^*(1-\varepsilon^*)+q(1-\varepsilon^*\pi^*)}\right),$$

$$g_1^{*} = \left(\frac{\sigma^*-1}{\sigma^*}\right)\left(\frac{\psi\beta y^*}{1+\beta}\right)\left(\frac{q^*\pi^*\big((1-\pi)\varepsilon^*+\pi(1-\varepsilon)\big)+q\pi(1-\varepsilon)}{q^*(1-\varepsilon\pi)+q\pi(1-\varepsilon)}\right).$$

C. *Initial period of a targeting regime with capital controls*

Properties of (21a) and (21b) in initial period of a targeting regime with capital controls

$$a)\ \frac{\partial\Delta f_{\hat{T}}^{dom}}{\partial p_{\hat{T}}} < 0, \frac{\partial^2\Delta f_{\hat{T}}^{dom}}{\partial p_{\hat{T}}^2} > 0,\ \forall p_{\hat{T}} \in (0,\infty);\ \lim_{p_{\hat{T}}\downarrow 0}\frac{\partial\Delta f_{\hat{T}}^{dom}}{\partial p_{\hat{T}}} = -\infty, \lim_{p_{\hat{T}}\uparrow\infty}\frac{\partial\Delta f_{\hat{T}}^{dom}}{\partial p_{\hat{T}}} = 0;$$

$$b)\ \frac{\partial\Delta f_{\hat{T}}^{for}}{\partial p_{\hat{T}}} > 0, \frac{\partial^2\Delta f_{\hat{T}}^{for}}{\partial p_{\hat{T}}^2} < 0,\ \forall p_{\hat{T}} \in (0,\infty);\ \lim_{p_{\hat{T}}\downarrow 0}\frac{\partial\Delta f_{\hat{T}}^{for}}{\partial p_{\hat{T}}} = +\infty, \lim_{p_{\hat{T}}\uparrow\infty}\frac{\partial\Delta f_{\hat{T}}^{for}}{\partial p_{\hat{T}}} = 0;$$

$$c) \lim_{p_{\hat{T}}\downarrow 0}\Delta f_{\hat{T}}^{dom} = \infty, \lim_{p_{\hat{T}}\uparrow\infty}\Delta f_{\hat{T}}^{dom}\frac{\psi\beta}{1+\beta}((1-\varepsilon^*)\pi^*y^*\bar{x}-(1-\varepsilon\pi)y);$$

$$d) \lim_{p_{\hat{T}}\downarrow 0}\Delta f_{\hat{T}}^{for} = -\infty, \lim_{p_{\hat{T}}\uparrow\infty}\Delta f_{\hat{T}}^{for} = \tfrac{\psi\beta}{1+\beta}((1-\varepsilon^*\pi^*)y^*\bar{x}-(1-\varepsilon)\pi y) > \lim_{p_{\hat{T}}\uparrow\infty}\Delta f_{\hat{T}}^{dom};$$

$$e)\ p_{\hat{T}}|_{\Delta f_{\hat{T}}^{dom}=0} = \left(\frac{(1-\psi)q}{\psi\beta}\right)\left(\frac{1}{(1-\varepsilon\pi)y-(1-\varepsilon^*)\pi^*y^*\bar{x}}\right);$$

$$f)\ p_{\hat{T}}|_{\Delta f_{\hat{T}}^{for}=0} = \left(\frac{(1-\psi)q^*}{\psi\beta}\right)\left(\frac{1}{(1-\varepsilon^*\pi^*)y^*\bar{x}-(1-\varepsilon)\pi y}\right).$$

Proof of Proposition 7. Equations (21*a*) and (21*b*) show, and as I have documented above, that as $\hat{p}_{\hat{T}} \downarrow 0$ from above, the domestic country's locus is strictly higher than the foreign country's locus, while as $\hat{p}_{\hat{T}} \uparrow \infty$, the foreign country's locus lies strictly above that of the domestic country. In addition, the functions (21*a*) and (21*b*) of $\hat{p}_{\hat{T}}$ are continuous, and continuously differentiable, on $\hat{p}_{\hat{T}} \in$

$(0, \infty)$. These four facts, together with the curvature of (21*a*) and (21*b*) imply that there is a single intersection of the two loci on $\hat{p}_{\hat{T}} \in (0, \infty)$. Whether $\Delta\hat{f}_{\hat{T}}$ is positive, negative, or zero, and the value of $\hat{p}_{\hat{T}}$, depend on the locations of (21*a*) and (21*b*) in ($\Delta\hat{f}_{\hat{T}}, \hat{p}_{\hat{T}}$) space, which depend (in part) on the value of the real exchange rate target. It is obvious from an inspection of (21*a*) and (21*b*) that, *ceteris paribus*, both loci shift up with higher values of the target real exchange rate, and down for lower values. The loci intersect on the horizontal axis at $\Delta\hat{f}_{\hat{T}}^{dom} = \Delta\hat{f}_{\hat{T}}^{for} = 0$ when $\bar{x} = x$. We know that $x \in \left(\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*}, \frac{(1-\varepsilon\pi)y}{(1-\varepsilon^*)\pi^* y^*}\right)$. Then, when $\bar{x} = x$, $\lim_{\hat{p}_{\hat{T}}\uparrow\infty} \Delta\hat{f}_{\hat{T}}^{dom} = \frac{\psi\beta}{1+\beta}((1-\varepsilon^*)\pi^* y^*\bar{x} - (1-\varepsilon\pi)y) < 0$, $\lim_{\hat{p}_{\hat{T}}\uparrow\infty} \Delta\hat{f}_{\hat{T}}^{for} = \frac{\psi\beta}{1+\beta}((1-\varepsilon^*\pi^*)y^*\bar{x} - (1-\varepsilon)\pi y) > 0$. Since both loci shift up with higher values of the target, they must intersect *above* the horizontal axis when $\bar{x} > x$, where $\Delta\hat{f}_{\hat{T}}^{dom} = \Delta\hat{f}_{\hat{T}}^{for} > 0$, and intersect below it when $\bar{x} < x$, where $\Delta\hat{f}_{\hat{T}}^{dom} = \Delta\hat{f}_{\hat{T}}^{for} < 0$. ■

D. *Initial period of a targeting regime with capital controls*

The initial period consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, and real balances under a targeting regime with capital controls are;

$$\hat{c}^N_{y,\hat{T}} = \frac{y}{(1+\beta)}, \qquad \hat{c}^{*N}_{y,\hat{T}} = \frac{y^*}{(1+\beta)},$$

$$\hat{c}^{N,\varepsilon\pi}_{o,\hat{T}} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{\hat{p}^N_{\hat{T}-1}}{\hat{p}^N_{\hat{T}}}\right) = \left(\frac{\beta}{1+\beta}\right)\left(\frac{1}{(1+\beta)\sigma}\right)\left(\frac{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^*\bar{x})\big(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\big)}{\big(q^*(1-\varepsilon^*)\pi^* + q\big(\varepsilon\pi(1-\pi^*) + \pi^*\pi(1-\varepsilon^*)\big)\big)}\right),$$

$$\hat{c}^{*N,\varepsilon^*\pi^*}_{o,\hat{T}} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{\hat{p}^{*N}_{\hat{T}-1}}{\hat{p}^{*N}_{\hat{T}}}\right) = \left(\frac{\beta}{1+\beta}\right)\left(\frac{1}{(1+\beta)\sigma^*}\right)\left(\frac{(\varepsilon^*\pi^* y^*\bar{x} + (1-\varepsilon)\pi y)\big(q(1-\varepsilon)\pi + q^*(1-\varepsilon\pi)\big)}{\bar{x}\Big(q(1-\varepsilon)\pi + q^*\big(\varepsilon^*\pi^*(1-\pi) + \pi\pi^*(1-\varepsilon)\big)\Big)}\right),$$

$$\hat{c}^{N,(1-\varepsilon)\pi}_{o,\hat{T}} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{1}{(1+\beta)\sigma}\right)\left(\frac{(\varepsilon^*\pi^* y^*\bar{x} + (1-\varepsilon)\pi y)\big(q^*(1-\varepsilon^*)\pi^* + q(1-\varepsilon^*\pi^*)\big)}{\Big(q(1-\varepsilon)\pi + q^*\big(\varepsilon^*\pi^*(1-\pi) + \pi\pi^*(1-\varepsilon)\big)\Big)}\right),$$

$$\hat{c}^{*N,(1-\varepsilon^*)\pi^*}_{o,\hat{T}} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{1}{(1+\beta)\sigma^*}\right)\left(\frac{(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^*\bar{x})\big(q(1-\varepsilon)\pi + q^*(1-\varepsilon\pi)\big)}{\bar{x}\Big(q(1-\varepsilon)\pi + q^*\big(\varepsilon^*\pi^*(1-\pi) + \pi\pi^*(1-\varepsilon)\big)\Big)}\right),$$

$$\hat{c}^{N,1-\pi}_{o,\hat{T}} = \left(\frac{y\beta}{1+\beta}\right)\hat{R}^N_{\hat{T}} = \left(\frac{y\beta}{1+\beta}\right)\hat{R}^T_{\hat{T}}\frac{\hat{p}_{\hat{T}-1}}{\hat{p}_{\hat{T}}} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{q}{1-\pi}\right)\left(\frac{y(1-\pi) + y^*\bar{x}(1-\pi^*)}{y^*(q+q^*)}\right)$$

$$\hat{c}^{*N,1-\pi^*}_{o} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{q^*}{1-\pi^*}\right)\left(\frac{y(1-\pi) + y^*\bar{x}(1-\pi^*)}{\bar{x}\, y^*(q+q^*)}\right),$$

$$\hat{c}^T_{y,\hat{T}} = \frac{y(1-\pi)(q+q^*)}{(1+\beta)\big(y(1-\pi) + y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{c}^{*T}_{y,\hat{T}} = \frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi) + y^*\bar{x}(1-\pi^*)\big)}$$

$$\hat{l}_{e,\hat{T}+1}=-\frac{y(1-\pi)(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{l}^*_{e,\hat{T}+1}=-\frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{l}_{\hat{T}+1}=\frac{y(1-\pi)(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{l}^*_{\hat{T}+1}=\frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\widehat{m}^d_{\hat{T}}=\left(\frac{\varepsilon\pi\beta y}{1+\beta}\right),\widehat{m}^f_{\hat{T}}=\left(\frac{(1-\varepsilon)\pi\beta y}{\bar{x}(1+\beta)}\right),\widehat{m}^{*d}_{\hat{T}}=\left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right),\widehat{m}^{*f}_{\hat{T}}=\left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right)\bar{x},$$

$$\widehat{m}_{\hat{T}}=\left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon\pi y+(1-\varepsilon^*)\pi^*y^*\bar{x}),$$

$$\widehat{m}^*_{\hat{T}}=\left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon^*\pi^*y^*+(1-\varepsilon)\pi y/\bar{x}).$$

Proof of Proposition 8. ***Fiscal and monetary policy with a target under capital controls***

a) It is straightforward to verify, using definition 3 *a)* and definition 3 *c)*, that i) if $\pi>g(\pi^*),\sigma^{min}>1$, ii) if $\pi<g(\pi^*),\sigma^{min}<0$, iii) $\lim_{\pi\to g(\pi^*)}\sigma^{min}=\infty$. It is also the case that, from the definition of $\bar{x}^{max}$, $iv)$ $if\ \pi\geq g(\pi^*)$, $\bar{x}^{max}\geq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)iff\ \sigma\geq\sigma^{min}$, $v)$ $if\ \pi<g(\pi^*),\bar{x}^{max}\geq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)iff\ \sigma\leq\sigma^{min}$. Since $\sigma\in(1,\infty)$ is the range of admissible money growth rates, then *i)* through *v)* imply that the restriction $\bar{x}^{max}\geq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)$ can be satisfied at an admissible money growth rate only if $\pi>g(\pi^*)$ holds. Specifically, if $\pi>g(\pi^*)$, $\bar{x}^{max}\geq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)iff\ \sigma\geq\sigma^{min}>1$. Then for any $\bar{x}\in\left(\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)y^*},\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)$, if $\pi>g(\pi^*)$ and $\sigma\geq\sigma^{min}$, $then\ \bar{x}^{max}>\bar{x}$. From (26*a*), $\bar{x}^{max}>\bar{x}$ iff $\hat{g}_{\hat{T}}>0$.

b) Similarly, it is straightforward to verify, using the definition of σ^{*min}, that $i)\ if\ \pi>h(\pi^*),\sigma^{*min}>1$, $ii)\ if\ \pi<h(\pi^*),\sigma^{*min}<0$, and $iii)$ $\lim_{\pi^*\to h(\pi)}\sigma^{*min}=\infty$. It is also the case that, using the definition of $\bar{x}^{min}$, $iv)\ if\ \pi\geq h(\pi^*),\bar{x}^{min}\leq\left(\frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*\pi^*)}\right)iff\ \sigma^*\geq\sigma^{*min}$, and $v)\ if\ \pi^*<h(\pi),\bar{x}^{min}\leq\left(\frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*\pi^*)}\right)iff\ \sigma^*\leq\sigma^{*min}$. Since $\sigma^*\in(1,\infty)$ is the range of admissible money growth rates, then *i)* through *v)* imply that the restriction $\bar{x}^{min}\leq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)$ can be satisfied at an admissible money growth rate only if $\pi>h(\pi^*)$ holds. Specifically, if $\pi>h(\pi^*)$, $\bar{x}^{min}\leq\left(\frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)iff\ \sigma^*\geq$

$\sigma^{*min} > 1.$ Then for any $\bar{x} \in \left(\frac{(1-\varepsilon)\pi y}{(1-\varepsilon^*\pi^*)}, \frac{y}{y^*}\frac{(1-\varepsilon\pi)}{(1-\varepsilon^*)\pi^*}\right)$, if $\pi > h(\pi^*)$ and $\sigma^* \geq \sigma^{*min}, then\ \bar{x}^{min} < \bar{x}$. From (26*b*), $\bar{x}^{min} < \bar{x}\ iff\ \hat{g}^*_{\hat{T}} > 0.$ ■

E*. Steady state equilibrium in a targeting regime under capital controls*

Proof of Proposition 9.

As the real value of this adjustment measured in (domestic) non-traded goods must be constant in equilibrium, then we must have $\frac{\Delta\hat{F}_{\hat{T}+i}\hat{e}_{\hat{T}+i}\hat{p}^N_{\hat{T}+i-1}}{\Delta\hat{F}_{\hat{T}+i-1}\hat{e}_{\hat{T}+i-1}\hat{p}^N_{\hat{T}+i}} = 1, \forall i \geq 1$. Then the domestic government's nominal foreign reserve adjustment must satisfy $\Delta\hat{F}_{\hat{T}+i} = \sigma^*\Delta\hat{F}_{\hat{T}+i-1}, \forall i \geq 1$. The date $\hat{T}$ reserve movement consistent with attainment of a target more appreciated than the equilibrium real exchange rate is $\Delta\hat{F}_{\hat{T}} \equiv \hat{F}_{\hat{T}} - F_0 = (\bar{x} - x)\left(\frac{M^*_{\hat{T}}}{q+q^*}\right)\left(\frac{y^*((1-\varepsilon^*\pi^*)q+(1-\varepsilon^*)\pi^*q^*)}{\varepsilon^*\pi^*y^*\bar{x}+(1-\varepsilon)\pi y}\right) < 0$, so $\hat{F}_{\hat{T}} < F_0$. Since $\frac{\Delta\hat{F}_{\hat{T}+i}}{\Delta\hat{F}_{\hat{T}+i-1}} = \sigma^* \forall i \geq 1$, the domestic government's nominal foreign reserve level at $\hat{T}+1$ satisfies $\hat{F}_{\hat{T}+1} = (1+\sigma^*)\hat{F}_{\hat{T}} - \sigma^*F_0$, and, iterating, this implies $\hat{F}_{\hat{T}+i} = \hat{F}_{\hat{T}+i-1} + \sigma^{*i}(\hat{F}_{\hat{T}} - F_0), \forall i \geq 1$. The reserve change is increasingly negative relative to the initial period of establishment of the target. Consider the conditions under which the domestic government's reserve approaches zero at some date $\hat{T}+\hat{\imath}$, $\hat{F}_{\hat{T}+\hat{\imath}} = \hat{F}_{\hat{T}+\hat{\imath}-1} + \sigma^{*\hat{\imath}}(\hat{F}_{\hat{T}} - F_0) \rightarrow 0$. Then it must be that $\hat{F}_{\hat{T}+\hat{\imath}-1} \rightarrow \sigma^{*\hat{\imath}}(F_0 - \hat{F}_{\hat{T}}) > 0$. Then, $\hat{F}_{\hat{T}+\hat{\imath}-1} = \hat{F}_{\hat{T}+\hat{\imath}-2} + \sigma^{*\hat{\imath}-1}(\hat{F}_{\hat{T}} - F_0) \rightarrow \sigma^{*\hat{\imath}}(F_0 - \hat{F}_{\hat{T}})$, or $\hat{F}_{\hat{T}+\hat{\imath}-2} \rightarrow (\sigma^{*\hat{\imath}} + \sigma^{*\hat{\imath}-1})(F_0 - \hat{F}_{\hat{T}})$. Iterating, $\hat{F}_{\hat{T}+1} = \hat{F}_{\hat{T}} + \sigma^*(\hat{F}_{\hat{T}} - F_0) \rightarrow \sum_{j=2}^{\hat{\imath}}\sigma^{*j}(F_0 - \hat{F}_{\hat{T}})$. This implies that the current reserve level approaches zero at $\hat{T}+\hat{\imath}$, $\hat{F}_{\hat{T}+\hat{\imath}} \rightarrow 0$, iff $\hat{F}_{\hat{T}} \rightarrow F_0\left(\frac{\sum_{j=1}^{\hat{\imath}}\sigma^{*j}}{\sum_{j=0}^{\hat{\imath}}\sigma^{*j}}\right) = F_0\left(\frac{\sigma^*-\sigma^{*\hat{\imath}+1}}{1-\sigma^{*\hat{\imath}+1}}\right)$. Then the initial reserve movement satisfies

$$(\hat{F}_{\hat{T}} - F_0) \rightarrow F_0\left(\frac{\sum_{j=1}^{\hat{\imath}}\sigma^{*j}}{\sum_{j=0}^{\hat{\imath}}\sigma^{*j}} - 1\right) = F_0\left(\frac{\sigma^*-1}{1-\sigma^{*\hat{\imath}+1}}\right) \leq 0.$$

Then, if the $\hat{T}+\hat{\imath}$ reserve approaches zero, the initial period reserve movement $(\hat{F}_{\hat{T}} - F_0) = (\bar{x} - x)\left(\frac{\sigma^{*\hat{T}}M^*_0}{q+q^*}\right)\left(\frac{y^*((1-\varepsilon^*\pi^*)q+(1-\varepsilon^*)\pi^*q^*)}{\varepsilon^*\pi^*y^*\bar{x}+(1-\varepsilon)\pi y}\right)$ satisfies

$$(\bar{x} - x)\left(\frac{\sigma^{*\hat{T}}M^*_0}{q+q^*}\right)\left(\frac{y^*((1-\varepsilon^*\pi^*)q+(1-\varepsilon^*)\pi^*q^*)}{\varepsilon^*\pi^*y^*\bar{x}+(1-\varepsilon)\pi y}\right) \rightarrow F_0\left(\frac{\sigma^*-1}{1-\sigma^{*\hat{\imath}+1}}\right).$$

This shows that if $\hat{\imath} = \infty$, then $F_0\left(\frac{\sigma^*-1}{1-\sigma^{*\hat{\imath}+1}}\right)$, and hence the initial reserve movement, must equal zero, but the latter is possible iff $(\bar{x} - x) = 0$. Thus, for $(\bar{x} - x) < 0$, the nominal reserve level must approach zero in finite time, and the economy cannot sustain indefinitely a constant real value of the reserve. ■

The steady state consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, real balances, and government consumption under a targeting regime with capital controls are;

$$\hat{c}_y^N = \frac{y}{(1+\beta)}, \qquad \hat{c}_y^{*N} = \frac{y^*}{(1+\beta)},$$

$$\hat{c}_o^{N,\varepsilon\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{1}{\sigma}\right), \qquad \hat{c}_o^{*N,\varepsilon^*\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{1}{\sigma^*}\right),$$

$$\hat{c}_o^{N,(1-\varepsilon)\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{1}{\sigma^*}\right), \qquad \hat{c}_o^{*N,(1-\varepsilon^*)\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{1}{\sigma}\right),$$

$$\hat{c}_o^{N,1-\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}{y(q+q^*)}\right),$$

$$\hat{c}_o^{*N,1-\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}{\bar{x}\,y^*(q+q^*)}\right),$$

$$\hat{c}_y^T = \frac{(1-\pi)y(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}, \qquad \hat{c}_y^{*T} = \frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{l}_e = -\frac{(1-\pi)y(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}, \qquad \hat{l}_e^* = -\frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)},$$

$$\hat{l} = \frac{(1-\pi)y(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}, \qquad \widehat{l^*} = \frac{(1-\pi^*)y^*\bar{x}(q+q^*)}{(1+\beta)\big(y(1-\pi)+y^*\bar{x}(1-\pi^*)\big)}$$

$$\widehat{m}^d = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), \widehat{m}^f = \left(\frac{(1-\varepsilon)\pi\beta y}{\bar{x}(1+\beta)}\right), \widehat{m}^{*d} = \left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right), \widehat{m}^{*f} = \left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right)\bar{x},$$

$$\widehat{m} = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^*\bar{x}), \qquad \widehat{m}^* = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon^*\pi^* y^* + (1-\varepsilon)\pi y/\bar{x}),$$

$$\hat{g} = (\bar{x}^{max}-\bar{x})\left(\frac{1}{\sigma}\right)\left(\frac{\psi\beta}{(1+\beta)(q+q^*)}\right)y^*\big((q+\pi^*q^*)\sigma - \pi^*(\sigma-1)(q+q^*) - \varepsilon^*\pi^*(q+q^*)\big),$$

$$\hat{g}^* = \big(\bar{x}-x^{min}\big)\left(\frac{1}{\sigma^*}\right)\left(\frac{\psi\beta}{(1+\beta)(q+q^*)}\right)\frac{y^*}{\bar{x}}\big((q+\pi^*q^*)\sigma^* - \varepsilon^*\pi^*(q+q^*)\big).$$

Proof of Proposition 12. ***Steady state external balance with a target under capital controls***

Proposition 10 establishes that there exists a unique steady state equilibrium under conditions *i)* and *ii)*. Proposition 11 establishes the value of the trade balance when assumption 2 holds, which is equivalent to $\pi \geq \max\{g(\pi^*), \tilde{\pi}\}$, and it demonstrates that it is larger – more positive – than in the non-targeting steady state. In addition, for $\pi > \tilde{\pi}, \frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*)\pi^*} > x$. Then the domestic country's financial balance is ambiguous, and satisfies $\widehat{FB} \gtreqless 0$ iff $\bar{x} \gtreqless \frac{y}{y^*}\frac{(1-\varepsilon)\pi}{(1-\varepsilon^*)\pi^*}$. ∎

F. *Steady state equilibrium under free capital flows and no targeting*

The steady state consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, real balances, and government consumption under free capital flows are;

$$\check{c}_y^N = \frac{y}{(1+\beta)}; \qquad \check{c}_y^{*N} = \frac{y^*}{(1+\beta)},$$

$$\check{c}_o^{N,\varepsilon\pi} = \frac{y\beta}{1+\beta}\left(\frac{1}{\sigma}\right), \quad \check{c}_o^{*N,\varepsilon^*\pi^*} = \frac{y^*\beta}{1+\beta}\left(\frac{1}{\sigma^*}\right),$$

$$\check{c}_o^{N,(1-\varepsilon)\pi} = \frac{y\beta}{1+\beta}\left(\frac{1}{\sigma}\right), \quad \check{c}_o^{*N,(1-\varepsilon^*)\pi^*} = \frac{y^*\beta}{1+\beta}\left(\frac{1}{\sigma^*}\right),$$

$$\check{c}_o^{N,1-\pi} = \left(\frac{y\beta}{1+\beta}\right), \quad \check{c}_o^{*N,1-\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right),$$

$$\check{c}_y^T = \frac{q}{(1+\beta)}, \qquad \check{l}_e = -\check{l} = -\frac{q}{(1+\beta)}, \qquad \check{c}_o^T = \frac{\beta q}{(1+\beta)},$$

$$\check{c}_y^{*T} = \frac{q^*}{(1+\beta)}, \qquad \check{l}_e^* = -\check{l}^* = -\frac{q^*}{(1+\beta)}, \qquad \check{c}_o^{T*} = \frac{\beta q^*}{(1+\beta)},$$

$$\check{m}^d = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), \check{m}^f = \left(\frac{(1-\varepsilon)\pi\beta y}{1+\beta}\right), \check{m}^{*f} = \left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right), \check{m}^{*d} = \left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right),$$

$$\check{m} = \left(\frac{\pi\psi\beta y}{1+\beta}\right), \quad \check{m}^* = \left(\frac{\pi^*\psi\beta y^*}{1+\beta}\right);$$

$$\check{g} = \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\pi\psi\beta y}{1+\beta}\right), \qquad \check{g}^* = \left(\frac{\sigma^*-1}{\sigma^*}\right)\left(\frac{\pi^*\psi\beta y^*}{1+\beta}\right).$$

G. *Dynamic equilibrium under fee capital flows without a target*

Proof of Proposition 16. *Asymptotic stability of steady state equilibrium under capital controls*

First, note that (33) has an intercept at $\check{x}_{t+1} = \left(\frac{y}{y^*}\right)\left(\frac{\pi(1-\varepsilon)}{(1-\varepsilon^*\pi^*)}\right) > 0$, and this value lies below the steady state equilibrium value of $\check{x}$. Second, differentiation of (33) yields

$$\frac{d\check{x}_{t+1}}{d\check{x}_t} = \left(\frac{y}{y^*}\right)\frac{y(1-\pi)y^*(1-\pi^*)\big((1-\varepsilon\pi)(1-\varepsilon^*\pi^*) - \pi(1-\varepsilon)\pi^*(1-\varepsilon^*)\big)}{(y(1-\varepsilon^*\pi^*)(1-\pi) + y^*\pi^*(1-\varepsilon^*)(1-\pi^*)\check{x}_t)^2},$$

which is strictly positive for all finite values of $\check{x}_t$. Moreover, from (33), $\lim_{\check{x}_t\to\infty} \frac{d\check{x}_{t+1}}{d\check{x}_t} = 0$ and $\frac{d^2\check{x}_{t+1}}{d\check{x}_t{}^2} < 0$. Thus, (33) must intersect the 45-degree line only once, and it must clearly cross that line from above. The steady state equilibrium is, therefore, asymptotically stable. ■

H. *Steady state equilibrium under a targeting regime with free capital flows*

The steady state consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, and real balances under a targeting regime with free capital flows are;

$$\hat{\check{c}}_y^N = \frac{y}{(1+\beta)}, \qquad \hat{\check{c}}_y^{*N} = \frac{y^*}{(1+\beta)},$$

$$\hat{\hat{c}}_o^{N,\varepsilon\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{1}{\sigma}\right), \qquad \hat{\hat{c}}_o^{*N,\varepsilon^*\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{1}{\sigma^*}\right),$$

$$\hat{\hat{c}}_o^{N,(1-\varepsilon)\pi} = \left(\frac{y\beta}{1+\beta}\right)\left(\frac{1}{\sigma^*}\right), \qquad \hat{\hat{c}}_o^{*N,(1-\varepsilon^*)\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right)\left(\frac{1}{\sigma}\right),$$

$$\hat{\hat{c}}_o^{N,1-\pi^*} = \left(\frac{y\beta}{1+\beta}\right), \qquad \hat{\hat{c}}_o^{*N,1-\pi^*} = \left(\frac{y^*\beta}{1+\beta}\right),$$

$$\hat{\hat{c}}_y^{T} = \frac{q}{1+\beta}, \qquad \hat{\hat{c}}_y^{*T} = \frac{q^*}{1+\beta},$$

$$\hat{\hat{l}}_e = -\hat{\hat{l}} - \frac{q}{1+\beta}, \qquad \hat{\hat{l}}_e^* = -\hat{\hat{l}}^* = -\frac{q^*}{1+\beta}.$$

$$\hat{\hat{c}}_o^{T} = \frac{q\beta}{1+\beta}, \qquad \hat{\hat{c}}_y^{*T} = \frac{q^*\beta}{1+\beta}.$$

$$\hat{\hat{m}}^d = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), \hat{\hat{m}}^f = \left(\frac{(1-\varepsilon)\pi\beta y}{1+\beta}\right), \hat{\hat{m}}^{*f} = \left(\frac{\varepsilon^*\pi^*\beta y^*}{1+\beta}\right), \hat{\hat{m}}^{*d} = \left(\frac{(1-\varepsilon^*)\pi^*\beta y^*}{1+\beta}\right),$$

$$\hat{\hat{m}} = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \bar{x}), \qquad \hat{\hat{m}}^* = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon^*\pi^* y^* + (1-\varepsilon)\pi y/\bar{x}).$$

I. *Initial period allocations under a targeting regime with free capital flows*

The initial period consumption and loan allocations for workers and entrepreneurs, asset allocations for banks, and real balances under a targeting regime with free capital flows are;

$$\hat{\hat{c}}_{y,\hat{T}}^{N} = \frac{y}{(1+\beta)}, \qquad \hat{\hat{c}}_{y,\hat{T}}^{*N} = \frac{y^*}{(1+\beta)},$$

$$\hat{\hat{c}}_{o,\hat{T}}^{N,\varepsilon\pi} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{1}{\sigma}\right)\left(\frac{\varepsilon\pi y + (1-\varepsilon^*)\pi^* y^* \bar{x}}{\pi}\right),$$

$$\hat{\hat{c}}_{o,\hat{T}}^{N,(1-\varepsilon)\pi} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{y^*(1-\varepsilon^*)}{\sigma^*}\right)\left(\frac{\varepsilon^*\pi^* y^* \bar{x} + (1-\varepsilon)\pi y}{\pi(1-\varepsilon)}\right),$$

$$\hat{\hat{c}}_{o,\hat{T}}^{*N,(1-\varepsilon^*)\pi^*} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{y(1-\varepsilon)}{\sigma}\right)\left(\frac{\varepsilon\pi y/\bar{x} + (1-\varepsilon^*)\pi^* y^*}{\pi^*(1-\varepsilon^*)}\right),$$

$$\hat{\hat{c}}_{o,\hat{T}}^{N,1-\pi} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{(1-\varepsilon^*)\pi^*\big(y(1-\pi) + y^*\bar{x}(1-\pi^*)\big)}{(1-\pi)\pi^*(1-\varepsilon^*) + (1-\pi^*)\pi(1-\varepsilon)}\right),$$

$$\hat{\hat{c}}_{o}^{*N,1-\pi^*} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{(1-\varepsilon)\pi\big(y(1-\pi) + y^*\bar{x}(1-\pi^*)\big)}{\bar{x}\big((1-\pi)\pi^*(1-\varepsilon^*) + (1-\pi^*)\pi(1-\varepsilon)\big)}\right),$$

$$\hat{\hat{c}}_{y,\hat{T}}^{T} = \frac{q}{1+\beta}, \qquad \hat{\hat{c}}_{y,\hat{T}}^{*T} = \frac{q^*}{1+\beta},$$

$$\hat{\hat{l}}_{e,\hat{T}+1} = -\hat{\hat{l}}_{\hat{T}+1} = -\frac{q}{1+\beta}, \qquad \hat{\hat{l}}_{e,\hat{T}+1}^* = -\hat{\hat{l}}_{\hat{T}+1}^* = -\frac{q^*}{1+\beta},$$

$$\hat{\hat{c}}_{o,\hat{T}}^{T} = \frac{q\beta}{1+\beta}, \qquad \hat{\hat{c}}_{y,\hat{T}}^{*T} = \frac{q^*\beta}{1+\beta},$$

$$\hat{\hat{m}}_{\hat{T}}^{d} = \left(\frac{\varepsilon\pi\beta y}{1+\beta}\right), \hat{\hat{m}}_{\hat{T}}^{f} = \left(\frac{(1-\varepsilon)\pi\beta y}{1+\beta}\right), \hat{\hat{m}}_{\hat{T}}^{*f} = \left(\frac{\varepsilon^{*}\pi^{*}\beta y^{*}}{1+\beta}\right), \hat{\hat{m}}_{\hat{T}}^{*d} = \left(\frac{(1-\varepsilon^{*})\pi^{*}\beta y^{*}}{1+\beta}\right),$$

$$\hat{\hat{m}}_{\hat{T}} = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon\pi y + (1-\varepsilon^{*})\pi^{*}y^{*}\bar{x}), \qquad \hat{\hat{m}}_{\hat{T}}^{*} = \left(\frac{\psi\beta}{1+\beta}\right)(\varepsilon^{*}\pi^{*}y^{*} + (1-\varepsilon)\pi y/\bar{x}).$$

Figure 1: Timing of Transactions

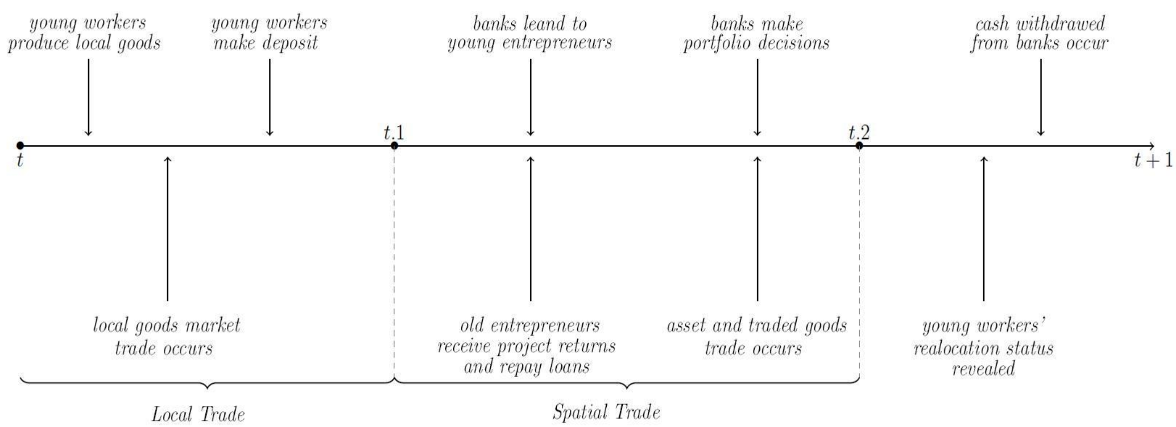

Figure 2: Steady State Equilibrium

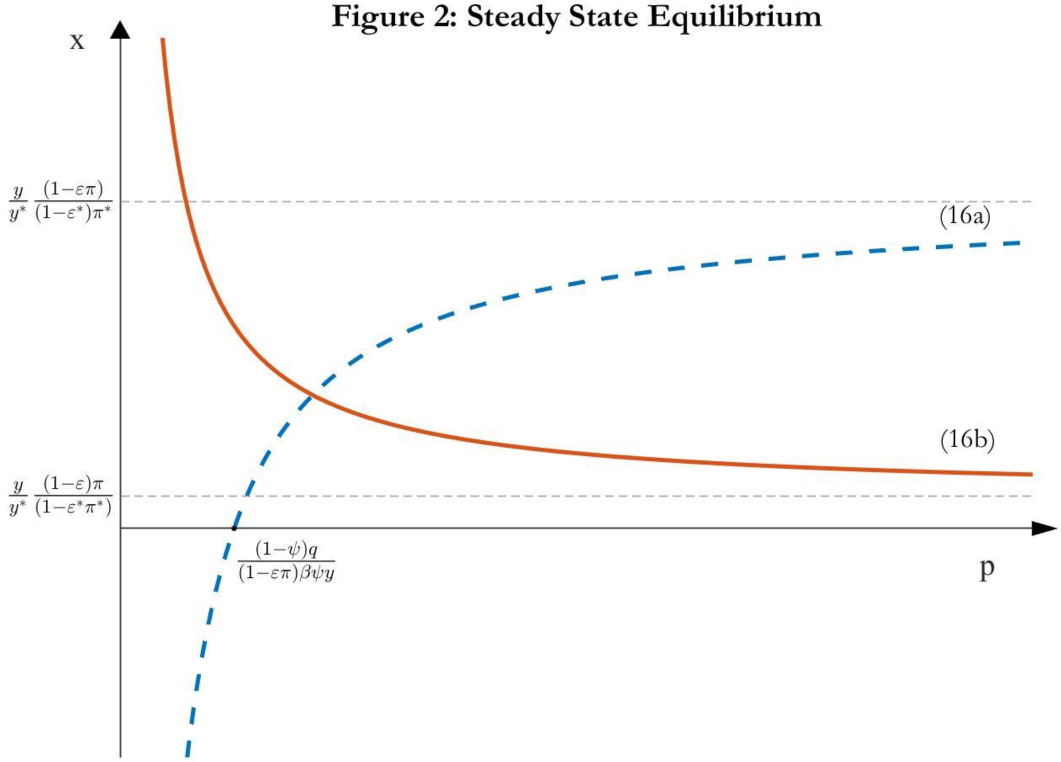

Figure 3: Steady State Real Interest Rates

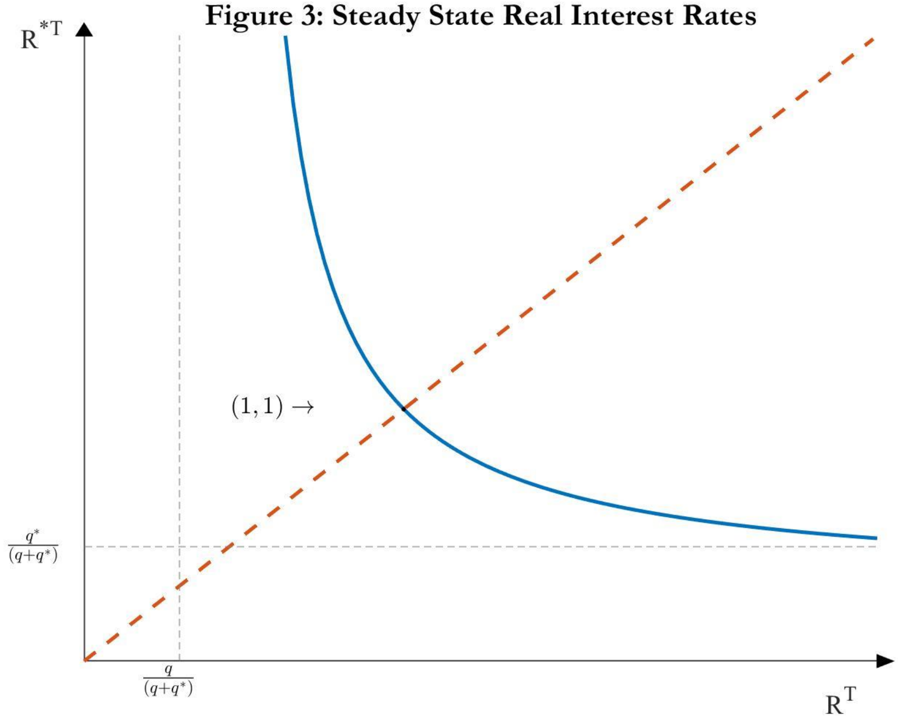

Figure 4: Initial Period Non-Traded Goods Markets

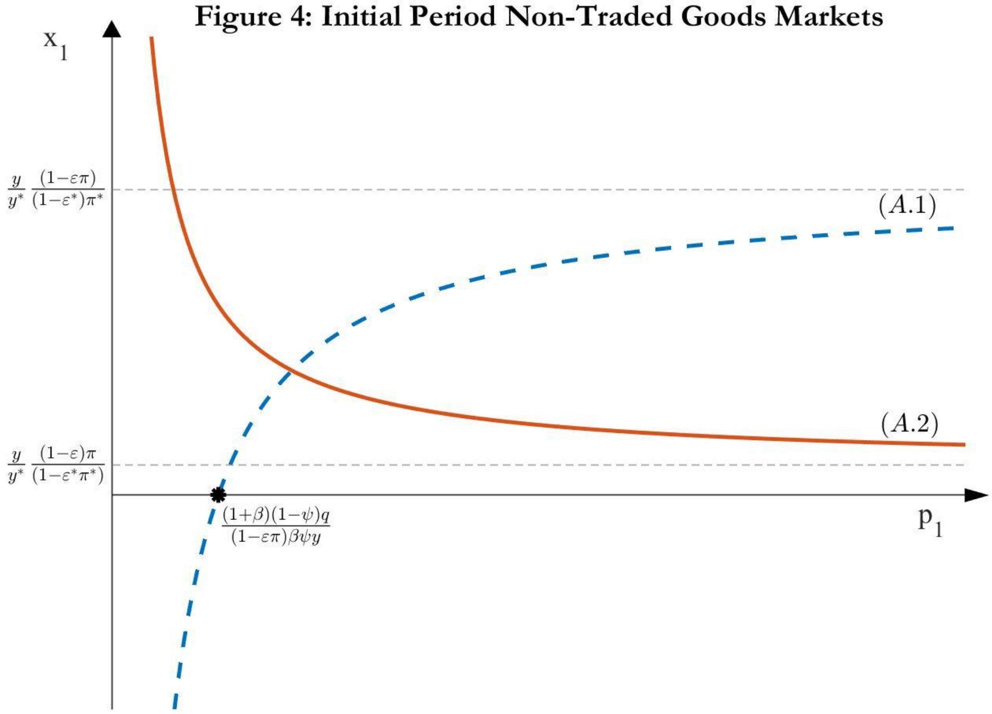

Figure 5: Initial Period Real Interest Rates

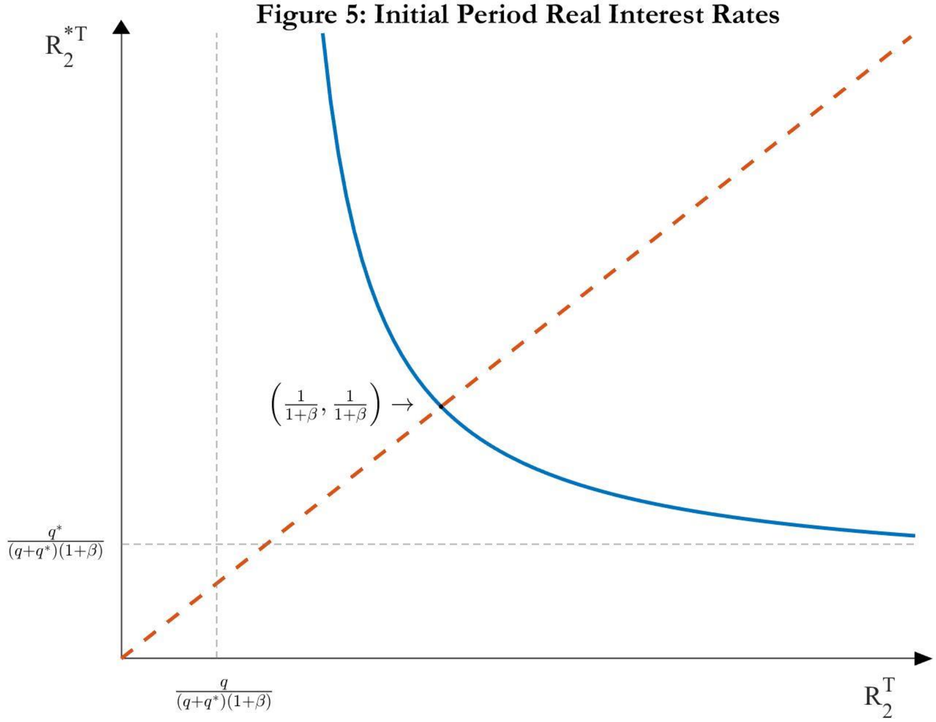

Figure 6: Establishing a Real Exchange Rate Target

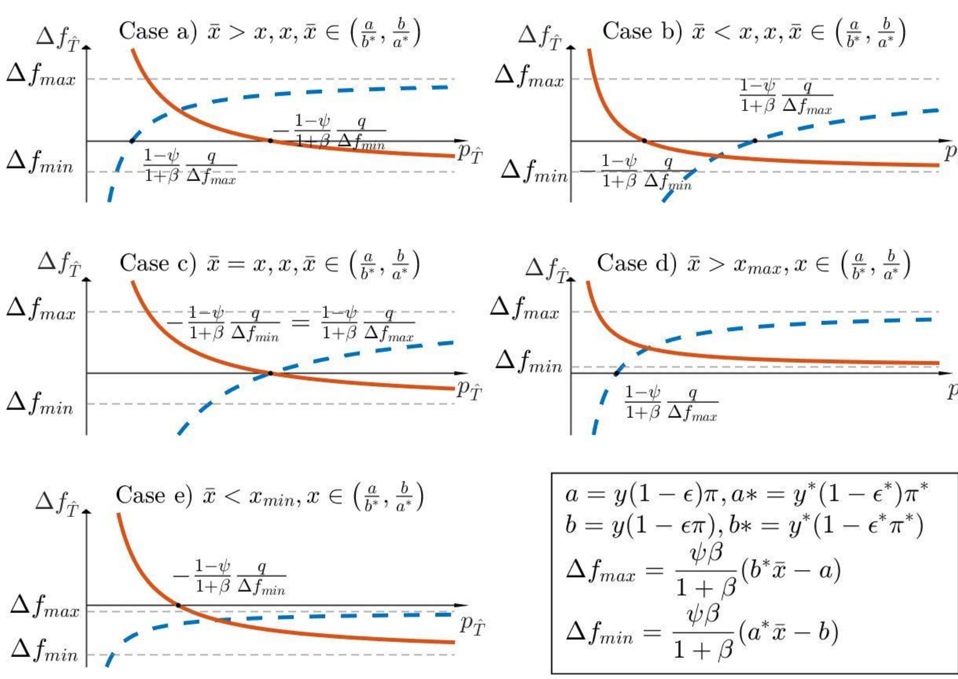

Figure 7: Admissible Region of π and π^*

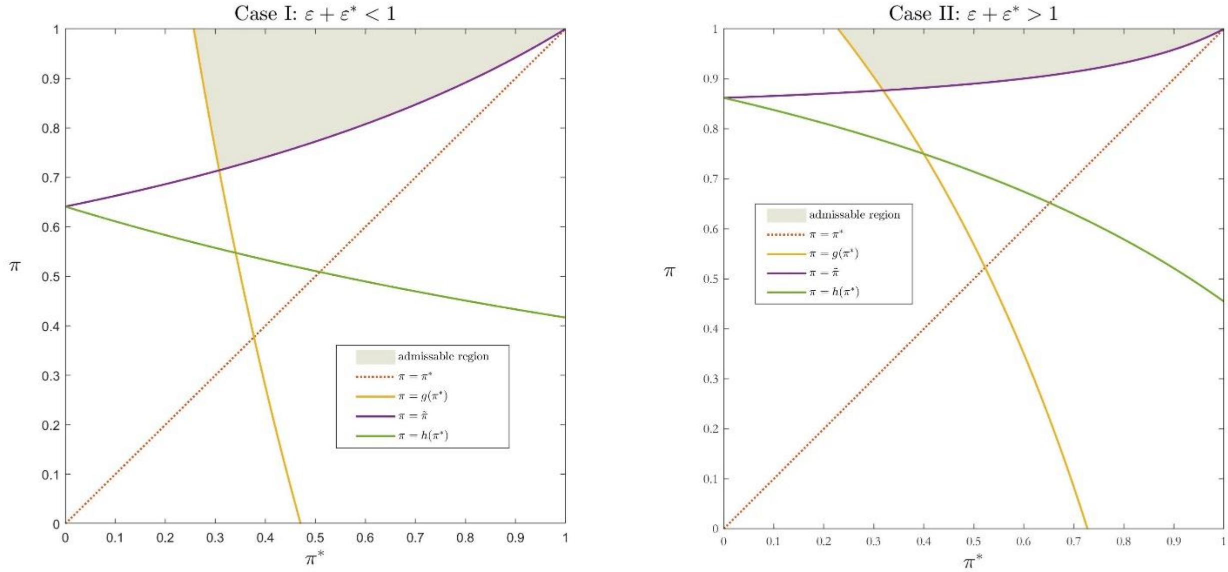

Figure 8: Transition Path for the Real Exchange Rate