## Is Exchange Rate Disconnected After All?\*

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#### Abstract

This paper assesses and compares the empirical relevance of macro-volatility shocks on one hand and shocks to international risk-sharing condition on the other in explaining exchange rate dynamics. We estimate a two-country New Keynesian model with recursive preferences and stochastic volatilities for the US and the Euro area, using third-order model approximation and data from 1987Q1 to 2008Q4. Inclusion of time-varying volatilities in monetary policy shocks can potentially account for the well-known forward premium or UIP puzzle, providing direct empirical support for the intuition/mechanism explored in earlier simulation-based papers: higher uncertainty in nominal conditions makes the home currency a good hedge, lowering its premium and at the same time, raises the nominal interest rate at home through higher money demands. But, our fullinformation Bayesian estimation shows that such volatility shocks offer little explanatory power for Dollar-Euro exchange rate dynamics. Instead, variance decompositions show that more than 70 percent of the fluctuations in the exchange rate are explained by a direct shock to the exchange rate, i.e. a shock to the international risk sharing condition. From this point of view, we find the exchange rate to be disconnected from other macroeconomic fundamentals, even if nonlinearities and stochastic volatilities are taken into account.

*JEL codes:* E52; F31; F41 *Keywords:* exchange rate; risk premium; international risk sharing; stochastic volatility; nonlinear estimation

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## 1 Introduction

The nominal exchange rate is an important driver of aggregate fluctuations as well as a key link between international goods and asset markets. However, endogenizing realistic exchange rate dynamics as observed in the data is a task that has alluded to international macroeconomists for decades. While various structural frameworks aim to understand how policies or the intrinsic shocks in one country spill over into other countries via the exchange rates, estimation efforts of such general equilibrium models typically find fluctuations in nominal exchange rates to be unrelated to macroeconomic forces.<sup>1</sup> Consequently, empirical evidence for the various transmission mechanisms of international policies and shocks through the exchange rate channel remains thin to non-existent, a pattern commonly referred to in the literature as the "exchange rate disconnect."

The exchange rate disconnect manifests itself into various empirical puzzles, each with its own vast literature exploring different reasons behind exchange rate fluctuations. This paper evaluates two recent alternative approaches by empirically estimating a full-fledged DSGE model that encompasses both sources of fluctuations: 1) direct shocks to exchange rate or international risking-sharing condition; and 2) macroe-conomic volatility shocks that induce time-varying risks in the exchange rates. We note that since the two approaches emphasize first-moment vs. 2nd-moment shocks, proper comparisons would thus require estimating the model up to a third order approximation, as well as evaluating them along the dimensions of both the means and the variances. In this paper, we look at how the two sources of shocks contribute to explaining the uncovered interest rate parity (UIP) puzzle and excess exchange rate volatility (relative to macro-fundamentals.)

To a first-order approximation (ignoring variance and covariance risk), the UIP as an no arbitrage condition implies that a country with high relative interest rates should expect to experience subsequent currency depreciation, ensuring zero expected excess returns from cross-border financial investments. As is well-known since Fama (1984), data consistently show significant and robust positive returns from "carry-trade" strategies that invest in the currency with higher interest rates, an empirical regularity known as the forward-premium puzzle or the UIP puzzle. There have been numerous attempts to solve the forward discount puzzle, though as pointed out in Itskhoki and Mukhin (2017), any proposed solutions must also account for the high volatilities present in the exchange rates, but absent in other macroeconomic variables.

This paper focuses on evaluating the following two mechanisms for explaining exchange rate dynamics: international risk sharing shocks, and time-varying risks in the macroeconomy. We first present a two-country New Open Economy Macro Economics (NOEM) model that encompass both channels, by adopting recursive preferences *a la* Epstein and Zin (1989) and stochastic volatilities, whose importance on ag-

<sup>&</sup>lt;sup>1</sup>See, for example, Lubik and Schorfheide (2006). Notable exception is Adolfson, Laseen, Linde, and Villani (2007) for the small open economy but they incorporate rather ad-hoc adjustment costs to capture risks in exchange rates.

gregate fluctuations have been emphasized in Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015).<sup>2</sup> While the vast majority of the literature using a DSGE setting relies on stimulation results, this paper estimates the model using pre-financial crisis US and the Euro area data, to evaluate which channel is the more important driver of the dollar-euro fluctuations.

The two mechanisms we emphasize capture arguments put forth in recent studies. For the first channel, Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017), for example, point out the importance of financial frictions in accounting for aggregate fluctuations in open economies. As frictions in financial transactions hinder international arbitrage through the exchange rates, they work as direct shocks to exchange rates themselves. We note that if such international financial friction shocks turn out to be the main driver behind exchange rate fluctuations in our estimation, one would conclude that exchange rates are indeed disconnected from other macroeconomic variables.

Alternatively, the empirical failure of the UIP may be the result of linear or firstorder approximation, as endogenous risk premium may arise from covariance between the stochastic discount factor and returns to international financial investments. Moving beyond a linearized framework, one can endogenously generate time-varying currency risks. For example, a structural or macroeconomic fundamental shock especially to volatilities—can simultaneously raises interest rates and appreciates the nominal exchange rates. If exchange rate fluctuations are mostly attributed to such endogenous risks, one would then infer that the exchange rate is not disconnected from macroeconomic fundamentals. Previous attempts to generate endogenous currency risk premiums through first-moment shocks have led to little success; this is why our paper considers shocks to the volatilities of macro variables.

From the literature that endogenizes exchange rate risks, the paper most closely related to ours is Benigno, Benigno, and Nistico (2011) (hereafter BBN). They examine the role of nominal and real stochastic volatilities in explaining exchange rate behavior by simulating a two-country NOEM model with recursive preferences. They find that a rise in the volatility of nominal shocks in the home country enhance the hedging properties of its currency relative to those of the foreign, thereby inducing endogenously a risk premium for foreign currency-holding. In addition, a rise in home nominal volatility tends to reduce domestic output and increase domestic producer inflation, while the domestic nominal interest rate declines proportionately more than the foreign one. Thus a negative correlation emerges between expected changes in nominal exchange rates and nominal interest rate differentials in response to volatility shocks, potentially over-turning the forward premium puzzle and accounting for the empirical regularities observed in exchange rate movements.

Our paper moves the evaluations of these mechanisms to an estimation framework and consider the fit to the data, instead of relying only on simulations with calibrated parameters. In fact, BBN already recognize this issue and argue that "the estima-

<sup>&</sup>lt;sup>2</sup>The representative models of NOEM can be found in Svensson and van Wijnbergen (1989), Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Clarida, Gali, and Gertler (2002), Benigno and Benigno (2003) and Devereux and Engel (2003).

tion of the model is really needed to evaluate its fit. To this purpose, an appropriate methodology should be elaborated to handle the features of our general second-order approximated solutions. We leave this research for future work."<sup>3</sup> We accomplish this task by first solving the two-country NOEM model using perturbation methods up to the third-order approximation so we can consider stochastic volatilities in the fundamental shocks more generally. We then conduct the full information maximum likelihood estimation in the general equilibrium setting. Note that to gauge the impact of stochastic volatilities, BBN employ the efficient method with second-order approximation proposed by Benigno, Benigno, and Nistico (2013), which can account for distinct and direct effects of volatility shocks, provided that shocks are conditionally linear. In contrast, by using the third order approximation, we can allow for thirdorder terms and hence take account of richer propagation mechanisms of structural shocks to exchange rate dynamics. To ensure stability in the model, we employ the pruning method developed by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018). We then estimate the model with a full information Bayesian approach. Because the model is non-linear, the standard Kalman filter is not applicable for evaluating the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).<sup>4</sup>

Our results are summarized as follows. Our estimated model can partly replicate some empirical regularities regarding volatility shocks as shown by BBN: (1) an increase in the volatility of the productivity shock induces an exchange rate depreciation; (2) an increase in the volatility of the monetary policy shock induces an exchange rate appreciation; and (3) an increase in the volatility of the monetary policy shock causes deviations from the UIP in the form of an increase in the excess return on the foreign currency. Despite the success in replicating these properties, the various volatility shocks that induce time-varying exchange rate risk premium, cannot account for the exchange rate volatility observed in the data. Our estimation results show that currency fluctuations are mostly explained by the direct shock to the exchange rate, i.e. shocks to the international risk-sharing condition, supporting the views offered by Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017). According to variance decompositions, the risk-sharing shock accounts for more than 70% of the variance of nominal exchange rate changes. We thus conclude that at least up to second-moment shocks, exchange rates appear to remain disconnected from macroeconomic fundamentals.

The remainder of this paper is organized as follows. Section 2 presents the model with recursive preferences and stochastic volatilities in open economies. Section 3 shows how we estimate the model in a nonlinear setting by a full-information Bayesian

<sup>&</sup>lt;sup>3</sup>In his comment to BBN, Uribe (2011) echoes the importance of a direct estimation of the model: "I would like to [suggest] an alternative identification approach. It consists of a direct estimation of a DSGE model. ... Admittedly, estimating DSGE models driven by time-varying volatility shocks is not a simple task."

<sup>&</sup>lt;sup>4</sup>Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

approach. Section 4 presents our main results and shows that fluctuations in exchange rates are mostly explained by the direct shock to exchange rates. Finally, Section 5 concludes.

## 2 Model

The model is basically the same as the one in BBN. The non-recursive preference *a la* Epstein and Zin (1989) is introduced together with stochastic volatilities in the otherwise standard NOEM model. There are three types of agents in each country: households, firms and the central bank.

### 2.1 Household

A representative household in the domestic country maximizes welfare:

$$V_t = \left[ u \left( C_t, N_t \right)^{1-\sigma} + \beta \left( \mathbf{E}_t V_{t+1}^{1-\varepsilon} \right)^{\frac{1-\sigma}{1-\varepsilon}} \right]^{\frac{1}{1-\sigma}},$$

subject to the budget constraint:

$$P_t C_t + B_t + \mathbf{E}_t [\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}] = R_{t-1} B_{t-1} + D_t + W_t N_t + T_t,$$

and aggregators:

$$C_{t} := \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$
  

$$C_{H,t} := \left[ \int_{0}^{1} C_{H,t} (j)^{1-\frac{1}{\mu}} dj \right]^{\frac{\mu}{\mu-1}},$$
  

$$C_{F,t} := \left[ \int_{0}^{1} C_{F,t} (j^{*})^{1-\frac{1}{\mu}} dj^{*} \right]^{\frac{\mu}{\mu-1}},$$

where  $C_t$ ,  $N_t$ ,  $P_t$ ,  $B_t$ ,  $\pi_t$ ,  $D_t$ ,  $R_t$ ,  $W_t$ ,  $T_t$ ,  $C_{H,t}$ ,  $C_{F,t}$ , and,  $m_{t,t+1}$  denote the aggregate consumption, the labor supply, the consumer price index, the holdings of the domestic bond, CPI inflation rates, the dividend, nominal interest rates, the nominal wage, the sum of corporate profits and the lump-sum tax, the consumption of locally produced goods, the consumption of imported goods and the real stochastic discount factor, respectively. While  $\sigma$  measures the inverse of the intertemporal elasticity of substitution,  $\varepsilon$  is the coefficient of relative risk aversion. The parameters  $\alpha$ ,  $\eta$ , and  $\mu$  are the steady state share of the domestically produced goods consumption in the aggregate consumption, the elasticity of substitution between domestically produced and imported goods, the elasticity of substitution among differentiated products in each country. j and  $j^*$  are indices for domestic and foreign firms, respectively. Those with subscript \* are foreign variables.

The foreign representative household faces a similar welfare maximization problem.

#### 2.2 Firms

Firm *j* in the home country sets prices in a monopolistically competitive market to maximize the present discounted value of profits  $\Pi_t$ :

$$\mathbf{E}_{t}\sum_{n=0}^{\infty}\theta^{n}m_{t,t+n}\frac{\Pi_{t+n}\left(j\right)}{P_{t+n}},$$

where

$$n\Pi_{t+n}(j) = nP_{H,t}(j) C_{H,t}(j) + (1-n) e_t P_{H,t}^*(j) C_{H,t}^*(j) - W_t N_t(j),$$

subject to the production function:

$$Y_t(j) = A_{W_t} A_t N_t(j) \,,$$

the law of one price:

$$P_{H,t}\left(j\right) = e_t P_{H,t}^*\left(j\right),$$

the firm-level resource constraint:

$$nY_t(j) = n \left[ C_{H,t}(j) + G_{H,t}(j) \right] + (1-n) C^*_{H,t}(j),$$

the downward sloping demand curve which is obtained from households' problem:

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\mu} (C_{H,t} + G_t),$$
  
$$C_{H,t}^*(j) = \left[\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right]^{-\mu} C_{H,t}^* = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\mu} C_{H,t}^*,$$

and the indexation rule when price is not reoptimized:

$$P_{H,t+n}(j) = \tilde{P}_{H,t} \prod_{i=1}^{n} \bar{\pi}^{1-\iota} \pi_{H,t+i-1}^{\iota},$$

where  $e_t$ ,  $P_{H,t}$ ,  $P_{H,t}^*$ ,  $A_{W_t}$ ,  $A_t$ ,  $G_t$  ( $G_{H,t}$ ),  $\tilde{P}_{H,t}$ , and  $\bar{\pi}$ , denote nominal exchange rates, the producer price index and the export price of the domestically produced goods, the common technology, the country-specific technology, government expenditure, the reset price and the target level of CPI inflation rates, respectively. Inflation rates for

the domestically produced goods are given by

$$\pi_{H,t} := \frac{P_{H,t}}{P_{H,t-1}}.$$

Only domestically produced goods are used for domestic government expenditure. The parameter  $\theta$ , n and  $\iota$  denote the Calvo (1983) parameter for price stickiness, the domestic country size and the degree of the price indexation, respectively.

Foreign firms face a similar profit maximization problem.

#### 2.3 Preferences and aggregate conditions

We set the instantaneous utility as

$$u(C_t, N_t) := C_t^{\psi} (1 - N_t)^{1 - \psi}.$$

The world technological progress is assumed to be nonstationary:

$$\frac{A_{W,t}}{A_{W,t-1}} = \gamma_{t}$$

where  $\gamma$  denotes the global trend growth rate.

Monetary policy is determined by following a rule:<sup>5</sup>

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1 - \phi_r) \left[\phi_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \log\left(\frac{Y_t}{\gamma Y_{t-1}}\right)\right] + \log(\varepsilon_{R,t}).$$

Aggregating the firm-level resource constraint leads to

$$nY_t = \Delta_t \left[ n \left( C_{H,t} + G_t \right) + (1-n) C_{H,t}^* \right],$$

where the price dispersion  $\Delta_t$  is given by

$$\Delta_t := \int_0^1 \left[ \frac{P_{H,t}\left(j\right)}{P_{H,t}} \right]^{-\mu} dj.$$

Similar conditions are derived in the foreign country.

#### 2.4 International linkage and the exchange rate

The law of one price holds for aggregate prices:

$$P_{H,t} = e_t P_{H,t}^*,$$

<sup>&</sup>lt;sup>5</sup>Note that inflation rates here are given by the deviation from the trend inflation.

or

$$\frac{P_{H,t}}{P_t} = \frac{e_t P_t^*}{P_t} \frac{P_{H,t}^*}{P_t^*},$$

or

$$p_{H,t} = s_t p_{H,t}^*$$

where

$$s_t = \frac{e_t P_t^*}{P_t},$$
$$p_{H,t}^* := \frac{P_{H,t}^*}{P_t^*}.$$

Since the value of the asset in the foreign currency is given by

$$\mathbf{E}_t [\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}^* e_{t+1}]/e_t$$

Also, under the perfect risk sharing,

$$\frac{m_{t,t+1}^*}{\pi_{t+1}^*} = \frac{m_{t,t+1}}{\pi_{t+1}} \frac{e_{t+1}}{e_t},$$

and therefore

$$\begin{pmatrix} \left( \frac{\left( V_{t+1}^{*} \right)^{1-\varepsilon} \mathbf{E}_{t} V_{t+1}^{1-\varepsilon}}{V_{t+1}^{1-\varepsilon} \mathbf{E}_{t} \left( V_{t+1}^{*} \right)^{1-\varepsilon}} \right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \left[ \frac{C_{t+1}^{\psi} \left( 1-N_{t+1} \right)^{(1-\psi)}}{\left( C_{t+1}^{*} \right)^{\psi} \left( 1-N_{t+1}^{*} \right)^{(1-\psi)}} \right]^{1-\sigma} \frac{C_{t+1}^{*}}{C_{t+1}} s_{t+1} \\ = \left[ \frac{C_{t}^{\psi} \left( 1-N_{t} \right)^{(1-\psi)}}{\left( C_{t}^{*} \right)^{\psi} \left( 1-N_{t}^{*} \right)^{(1-\psi)}} \right]^{1-\sigma} \frac{C_{t}^{*}}{C_{t}} s_{t}.$$

Let us denote

$$Q_{t+1} = Q_t \left( \frac{\left( V_{t+1}^* \right)^{1-\varepsilon} \mathbf{E}_t \left( V_{t+1}^{1-\varepsilon} \right)}{V_{t+1}^{1-\varepsilon} \mathbf{E}_t \left[ \left( V_{t+1}^* \right)^{1-\varepsilon} \right]} \right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}},$$

~

with

$$Q_0 = 1.$$

Then the international risk sharing condition is given by

$$\Omega_t Q_t = \left[ \frac{C_t^{\psi} (1 - N_t)^{(1-\psi)}}{(C_t^*)^{\psi} (1 - N_t^*)^{(1-\psi)}} \right]^{1-\sigma} \frac{C_t^*}{C_t} s_t,$$

with the assumption of initial symmetry. Note that  $\Omega_t$  is a shock to the international risk sharing condition, which works as the time varying financial frictions considered in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017).

Nominal exchange rate depreciation is given by

$$d_t := \frac{e_t}{e_{t-1}} = \frac{s_t \pi_t}{s_{t-1} \pi_t^*}.$$

### 2.5 Detrended equilibrium conditions

To make the model stationary and obtain the steady state, domestic variables are detrended as  $v_t := V_t/A_{W,t}^{\psi}$ ,  $y_t := Y_t/A_{W,t}$ ,  $c_{H,t} := C_{H,t}/A_{W,t}$ ,  $c_{F,t} := C_{F,t}/A_{W,t}$ ,  $\tilde{w}_t := W_t/P_t/A_{W,t}$ ,  $p_{H,t} := P_{H,t}/P_t$ , and  $g_t := G_t/A_{W,t}$ . Real exchange rate is given by  $s_t := e_t P_t^*/P_t$ . Foreign variables are also detrended in a similar manner.

The system of equations consists of 33 equations as shown below. Steady-state conditions are presented in Appendix A.

#### 2.5.1 Domestic

$$\begin{split} c_t &:= \left[ (1-\alpha)^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \\ v_t^{1-\sigma} &= \left[ c_t^{\psi} \left( 1 - N_t \right)^{1-\psi} \right]^{1-\sigma} + \beta \gamma^{\psi} \left( \mathbf{E}_t \left[ v_{t+1}^{1-\varepsilon} \right] \right)^{\frac{1-\sigma}{1-\varepsilon}}, \\ \log \left( \frac{R_t}{R} \right) &= \phi_r \log \left( \frac{R_{t-1}}{R} \right) + (1-\phi_r) \left[ \phi_{\pi} \log \left( \frac{\pi_t}{\pi} \right) + \phi_y \log \left( \frac{y_t}{y_{t-1}} \right) \right] + \log(\varepsilon_{R,t}), \\ c_{H,t} &= (1-\alpha) p_{H,t}^{-\eta} c_t, \\ c_{F,t} &= \alpha \left( s_t p_{F,t}^* \right)^{-\eta} c_t, \\ c_t &= \frac{\psi}{1-\psi} \left( 1 - N_t \right) \tilde{w}_t, \\ 1 &= \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}}, \\ m_{t,t+1} &= \beta \left[ \mathbf{E}_t \left( v_{t+1} \right)^{1-\varepsilon} \right]^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \left( v_{t+1} \right)^{\sigma-\varepsilon} \gamma^{\psi(1-\sigma)-1} \frac{c_{t+1}^{\psi(1-\sigma)-1} \left( 1 - N_{t+1} \right)^{(1-\psi)(1-\sigma)}}{c_t^{\psi(1-\sigma)-1} \left( 1 - N_t \right)^{(1-\psi)(1-\sigma)}}, \\ \pi_{H,t} &= \frac{p_{H,t} \pi_t}{p_{H,t-1}}, \\ f_t &= p_{H,t} \left[ n \left( c_{H,t} + g_t \right) + (1-n) c_{H,t}^* \right] + \gamma \theta \mathbf{E}_t m_{t,t+1} \left( \frac{\pi^{1-\varepsilon} \pi_{H,t}^t}{\pi_{H,t+1}} \right)^{-\mu} f_{t+1}, \\ k_t &= \frac{\mu}{\mu-1} \frac{\tilde{w}_t}{A_t} \left[ n \left( c_{H,t} + g_t \right) + (1-n) c_{H,t}^* \right] + \gamma \theta \mathbf{E}_t m_{t,t+1} \left( \frac{\pi^{1-\varepsilon} \pi_{H,t}^t}{\pi_{H,t+1}} \right)^{-\mu} k_{t+1}, \\ y_t &= A_t N_t, \end{split}$$

$$\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{1}{1-\mu}}f_{t}=k_{t},$$
$$ny_{t}=\Delta_{t}\left[n\left(c_{H,t}+g_{t}\right)+(1-n)\,c_{H,t}^{*}\right],$$

and

$$\Delta_t = (1 - \theta) \left[ \frac{1 - \theta \left( \frac{\bar{\pi}^{1 - \iota} \pi_{H,t-1}^{\iota}}{\pi_{H,t}} \right)^{1 - \mu}}{1 - \theta} \right]^{\frac{\mu}{\mu - 1}} + \theta \left( \frac{\pi_{H,t}}{\bar{\pi}^{1 - \iota} \pi_{H,t-1}^{\iota}} \right)^{\mu} \Delta_{t-1}.$$

## 2.5.2 Foreign

$$\begin{split} c_t^* &:= \left[ (\alpha)^{\frac{1}{\eta}} \left( c_{H,t}^* \right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} \left( c_{F,t}^* \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \\ (v_t^*)^{1-\sigma} &= \left[ (c_t^*)^{\psi} \left( (1-N_t)^* \right)^{1-\psi} \right]^{1-\sigma} + \beta \gamma^{\psi} \left( \mathbf{E}_t \left[ \left( v_{t+1}^* \right)^{1-\varepsilon} \right] \right)^{\frac{1-\sigma}{1-\varepsilon}}, \\ \log \left( \frac{R_t^*}{R^*} \right) &= \phi_r^* \log \left( \frac{R_{t-1}}{R^*} \right) + (1-\phi_r^*) \left[ \phi_\pi^* \log \left( \frac{\pi_t^*}{\pi^*} \right) + \phi_y^* \log \left( \frac{y_t^*}{y_{t-1}^*} \right) \right] + \log(\varepsilon_{R,t}^*), \\ c_{H,t}^* &= \alpha \left( \frac{p_{H,t}}{S_t} \right)^{-\eta} c_t^*, \\ c_{H,t}^* &= \alpha \left( \frac{p_{H,t}}{1-\psi} \right)^{-\eta} c_t^*, \\ c_{F,t}^* &= (1-\alpha) \left( p_{F,t}^* \right)^{-\eta} c_t^*, \\ c_t^* &= \frac{\psi}{1-\psi} \left( 1-N_t^* \right) \tilde{w}_t^*, \\ 1 &= \mathbf{E}_t m_{t,t+1}^* \frac{R_t^*}{\pi_{t+1}^*}, \\ m_{t,t+1}^* &= \beta \left( \mathbf{E}_t \left( v_{t+1}^* \right)^{1-\varepsilon} \right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \left( v_{t+1}^* \right)^{\sigma-\varepsilon} \gamma^{\psi(1-\sigma)-1} \frac{\left( c_{t+1}^* \right)^{\psi(1-\sigma)-1} \left( 1-N_{t+1}^* \right)^{(1-\psi)(1-\sigma)}}{\left( c_t^* \right)^{\psi(1-\sigma)-1} \left( 1-N_t^* \right)^{(1-\psi)(1-\sigma)}}, \\ \pi_{F,t}^* &= \frac{p_{F,t}^* \pi_t^*}{p_{F,t-1}^*}, \\ f_t^* &= p_{F,t}^* \left[ nc_{F,t} + (1-n) \left( c_{F,t}^* + g_t^* \right) \right] + \gamma \theta^* \mathbf{E}_t m_{t,t+1}^* \left[ \frac{\left( \overline{\pi}^* \right)^{1-t} \left( \overline{\pi}_{F,t}^* \right)^t}{\pi_{F,t+1}^*} \right]^{-\mu} f_{t+1}^*, \\ k_t^* &= \frac{\mu}{\mu-1} \frac{w_t^*}{A_t^*} \left[ nc_{F,t} + (1-n) \left( c_{F,t}^* + g_t^* \right) \right] + \gamma \theta^* \mathbf{E}_t m_{t,t+1}^* \left[ \frac{\left( \overline{\pi}^* \right)^{1-t} \left( \overline{\pi}_{F,t}^* \right)^t}{\pi_{F,t+1}^*} \right]^{-\mu} k_{t+1}^*, \\ y_t^* &= A_t^* N_t^*, \end{split}$$

$$\begin{bmatrix} \frac{1 - \theta^* \left(\frac{(\bar{\pi}^*)^{1-\iota} \left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}^*}\right)^{1-\mu}}{1 - \theta^*} \end{bmatrix}^{\frac{1}{1-\mu}} f_t^* = k_t^*, \\ (1 - n) y_t^* = \Delta_t^* \left[nc_{F,t} + (1 - n) \left(c_{F,t}^* + g_t^*\right)\right],$$

and

$$\Delta_t^* = (1 - \theta^*) \left[ \frac{1 - \theta^* \left[ \frac{(\bar{\pi}^*)^{1-\iota} (\pi_{F,t-1}^*)^{\iota}}{\pi_{F,t}} \right]^{1-\mu}}{1 - \theta^*} \right]^{\frac{\mu}{\mu-1}} + \theta^* \left[ \frac{\pi_{F,t}^*}{(\bar{\pi}^*)^{1-\iota} (\pi_{F,t-1}^*)^{\iota}} \right]^{\mu} \Delta_{t-1}^*.$$

### 2.5.3 International

$$c_t^{\psi(1-\sigma)-1} \left(1 - N_t\right)^{(1-\psi)(1-\sigma)} s_t = \Omega_t Q_t \left(c_t^*\right)^{\psi(1-\sigma)-1} \left(1 - N_t^*\right)^{(1-\psi)(1-\sigma)},\tag{1}$$

$$Q_{t+1} = Q_t \left( \frac{\left( v_{t+1}^* \right)^{1-\varepsilon} \mathbf{E}_t \left( v_{t+1} \right)^{1-\varepsilon}}{\left( v_{t+1} \right)^{1-\varepsilon} \mathbf{E}_t \left( v_{t+1}^* \right)^{1-\varepsilon}} \right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}},$$
(2)

and

$$d_t = \frac{s_t \pi_t}{s_{t-1} \pi_t^*}.$$

### 2.5.4 Shocks

There are 14 structural shocks:

$$\log (A_t) = \rho_A \log (A_{t-1}) + \sigma_{A,t} u_{A,t},$$
  

$$\log (g_t) = (1 - \rho_g) \log \bar{g} + \rho_g \log (g_{t-1}) + \sigma_{g,t} u_{g,t},$$
  

$$\log (\varepsilon_{R,t}) = \sigma_{\varepsilon_R,t} u_{\varepsilon_R,t},$$
  

$$\log (A_t^*) = \rho_A^* \log (A_{t-1}^*) + \sigma_{A,t}^* u_{A,t}^*,$$
  

$$\log (g_t^*) = (1 - \rho_g^*) \log \bar{g} + \rho_g^* \log (g_{t-1}^*) + \sigma_{g,t}^* u_{g,t}^*,$$
  

$$\log (\varepsilon_{R,t}^*) = \sigma_{\varepsilon_R,t}^* u_{\varepsilon_R,t}^*,$$
  

$$\log (\Omega_t) = \rho_\Omega \log (\Omega_{t-1}) + \sigma_{\Omega,t} u_{\Omega,t},$$

and stochastic volatilities are given by

$$\begin{aligned} \sigma_{A,t} &= (1 - \rho_{\sigma_A}) \, \sigma_A + \rho_{\sigma_A} \sigma_{A,t-1} + \tau_A z_{\sigma_A,t}, \\ \sigma_{g,t} &= (1 - \rho_{\sigma_g}) \, \sigma_g + \rho_{\sigma_g} \sigma_{g,t-1} + \tau_g z_{\sigma_g,t}, \\ \sigma_{\varepsilon_R,t} &= (1 - \rho_{\sigma_{\varepsilon_R}}) \, \sigma_{\varepsilon_R} + \rho_{\sigma_{\varepsilon_R}} \sigma_{\varepsilon_R,t-1} + \tau_{\varepsilon_R} z_{\sigma_{\varepsilon_R},t}, \\ \sigma_{A,t}^* &= (1 - \rho_{\sigma_A}^*) \, \sigma_A^* + \rho_{\sigma_A}^* \sigma_{A,t-1}^* + \tau_A^* z_{\tau_A,t}^*, \\ \sigma_{g,t}^* &= (1 - \rho_{\sigma_g}^*) \, \sigma_g^* + \rho_{\sigma_g}^* \sigma_{g,t-1}^* + \tau_g^* z_{\sigma_g,t}^*, \\ \sigma_{\varepsilon_R,t}^* &= (1 - \rho_{\sigma_{\varepsilon_R}}^*) \, \sigma_{\varepsilon_R}^* + \rho_{\sigma_{\varepsilon_R}}^* \sigma_{\varepsilon_R,t-1}^* + \tau_{\varepsilon_R}^* z_{\sigma_{\varepsilon_R},t}^*, \\ \sigma_{\Omega,t}^* &= (1 - \rho_{\sigma_\Omega}) \, \sigma_\Omega + \rho_{\sigma_\Omega} \sigma_{\Omega,t-1} + \tau_\Omega z_{\sigma_\Omega,t}. \end{aligned}$$

### 3 Estimation

The model is solved using perturbation methods up to the third-order approximation in order to take account of the stochastic volatilities in the fundamental shocks. To ensure stability, we employ the pruning method developed by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018).

We estimate the model using a full-information Bayesian approach. Because the model is no longer linear, the standard Kalman filter is not applicable to evaluate the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).<sup>6</sup>

To approximate the posterior distribution, this paper exploits the generic Sequential Monte Carlo (SMC) algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015). In the algorithm, a sequence of tempered posteriors are defined as

$$\varpi_n(\vartheta) = \frac{[p(X^T|\vartheta)]^{\tau_n} p(\vartheta)}{\int [p(X^T|\vartheta)]^{\tau_n} p(\vartheta) d\vartheta}, \quad n = 0, ..., N_{\tau}.$$

The tempering schedule  $\{\tau_n\}_{n=0}^{N_{\tau}}$  is determined by  $\tau_n = (n/N_{\tau})^{\chi}$ , where  $\chi$  is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws and associated importance weights—which are called particles—from the sequence of posteriors  $\{\varpi_n\}_{n=1}^{N_{\tau}}$ ; that is, at each stage,  $\varpi_n(\vartheta)$  is represented by a swarm of particles  $\{\vartheta_n^i, w_n^i\}_{i=1}^N$ , where N denotes the number of particles. For  $n = 0, ..., N_{\tau}$ , the algorithm sequentially updates the swarm of particles  $\{\vartheta_n^i, w_n^i\}_{i=1}^N$  through importance sampling.<sup>7</sup> Posterior inferences about parameters to be estimated are made based on the particles  $\{\vartheta_{N_{\tau}}^i, w_{N_{\tau}}^i\}_{i=1}^N$  from the final importance sampling.

<sup>&</sup>lt;sup>6</sup>Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

<sup>&</sup>lt;sup>7</sup>This process includes one step of a single-block RWMH algorithm.

The SMC-based approximation of the marginal data density is given by

$$p(X^{T}) = \prod_{n=1}^{N_{\tau}} \left( \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{n}^{i} w_{n-1}^{i} \right),$$

where  $\tilde{w}_n^i$  is the incremental weight defined as  $\tilde{w}_n^i = [p(X^T | \vartheta_{n-1}^i)]^{\tau_n - \tau_{n-1}}$ . In the subsequent empirical analysis, the SMC algorithm uses N = 1,000 particles and  $N_{\tau} = 50$  stages. The parameter that controls the tempering schedule is set at  $\chi = 2$  following Herbst and Schorfheide (2014, 2015).

Seven quarterly time series ranging from 1987Q1 to 2008Q4 are used for estimation: the per-capita real GDP growth rate, the inflation rate of the GDP implicit price deflator, and the three-month nominal interest rate, in the US and the Euro Area, and the nominal exchange rate depreciation (USD to EUR). The construction of the series follows from Lubik and Schorfheide (2006).

Before estimation, parameters regarding the share of foreign goods, the elasticity of substitution between home and foreign goods, the elasticity of substitution across the goods within each country, the share of external demand, the steady-state growth, inflation, and interest rates, and relative risk aversion are fixed at  $\alpha = 0.13$ ,  $\eta = 1.5$ ,  $\mu = 6$ ,  $\psi = 0.333$ ,  $\bar{g}/\bar{y} = 0.18$ ,  $\bar{\gamma} = 0.346$ ,  $\bar{\pi} = 0.639$ ,  $\bar{r} = 1.274$ ,  $\epsilon = \epsilon^* = 5$ , respectively, to avoid an identification issue. All the other parameters are estimated; their prior distributions are shown in Table 1. The priors are set according to those used in Smets and Wouters (2007)and the calibrated values in BBN. For the standard deviations of the stochastic volatilities ( $\tau_x$ ,  $x \in \{A, g, \epsilon_R, A^*, g^*, \epsilon_{R^*}, \Omega\}$ ), the prior mean is set in line with the upper bound of the estimated standard deviation of the stochastic volatility regarding the technology shock reported in Fernández-Villaverde and Rubio-Ramírez (2007).

## 4 Results

We first report the estimation result and then discuss how our model can account for aggregate fluctuations in open economies including exchange rate dynamics as observed in the data.

#### 4.1 Parameter estimates

Table 2 and 3 reports the posterior estimates of parameters. For the purpose of comparison, a linearized version of the model and the models without the stochastic volatilities approximated up to the second and third order are also estimated. For each model, the posterior mean and 90 percent highest posterior density intervals for the estimated parameters are presented as well as the SMC-based approximation of log marginal data density  $\log p(\mathcal{Y}^T)$ .<sup>8</sup>

<sup>&</sup>lt;sup>8</sup>The risk aversion parameter  $\varepsilon$  in the recursive preferences does not appear in the linearized version of the model.

Parameter	Distribution	Mean	S.D.
ε	Gamma	5.000	0.500
$\sigma$	Gamma	2.000	0.250
$\theta$	Beta	0.667	0.050
ι	Beta	0.500	0.150
$\theta^*$	Beta	0.667	0.050
$\iota^*$	Beta	0.500	0.150
$\phi_r$	Beta	0.750	0.100
$\phi_{\pi}$	Gamma	2.000	0.100
	Gamma	0.125	0.050
$\phi_r^*$	Beta	0.750	0.100
$\phi_{\pi}^{*}$	Gamma	2.000	0.100
$egin{aligned} \phi_y \ \phi_r^* \ \phi_\pi^* \ \phi_y^* \end{aligned}$	Gamma	0.125	0.050
$\rho_A$	Beta	0.500	0.150
$\rho_g$	Beta	0.500	0.150
$ ho_A^*$	Beta	0.500	0.150
$ ho_g^*$	Beta	0.500	0.150
$\rho_{\Omega}$	Beta	0.500	0.150
$\rho_{\sigma_A}$	Beta	0.500	0.150
$\rho_{\sigma_g}$	Beta	0.500	0.150
	Beta	0.500	0.150
$\rho_{\sigma_A}^*$	Beta	0.500	0.150
$\rho_{\sigma_a}^*$	Beta	0.500	0.150
$ \begin{array}{l} \rho_{\sigma_{\epsilon_R}} \\ \rho^*_{\sigma_A} \\ \rho^*_{\sigma_g} \\ \rho^*_{\sigma_{\epsilon_R}} \end{array} $	Beta	0.500	0.150
$\rho_{\sigma_{\Omega}}$	Beta	0.500	0.150
$100\sigma_A$	Inverse Gamma	2.500	1.330
$100\sigma_q$	Inverse Gamma	2.500	1.330
$100\sigma_{\epsilon_R}$	Inverse Gamma	0.500	0.270
$100\sigma_{A}^{*}$	Inverse Gamma	2.500	1.330
$100\sigma_q^*$	Inverse Gamma	2.500	1.330
$100\sigma_{\epsilon_R}^{g}$	Inverse Gamma	0.500	0.270
$100\sigma_{\Omega}^{\epsilon_R}$	Inverse Gamma	2.500	1.330
$ au_A$	Inverse Gamma	1.250	0.640
$ au_g$	Inverse Gamma	1.250	0.640
$ au_{\epsilon_R}$	Inverse Gamma	1.250	0.640
$ au_A^{\epsilon_R}$	Inverse Gamma	1.250	0.640
$ au_g^{*}$	Inverse Gamma	1.250	0.640
$ au_{\epsilon_R}^{g}$	Inverse Gamma	1.250	0.640
$ au_R$ $ au_\Omega$	Inverse Gamma	1.250	0.640

Table 1: Prior distributions of parameters

		Linear		2	2nd order	
Parameter	Prior mean	Mean	90% interval	Mean	90% interval	
ε	5.000	-	-	3.806	[3.261, 4.300]	
$\sigma$	2.000	2.409	[2.076, 2.749]	1.915	[1.682, 2.133]	
heta	0.667	0.518	[0.437, 0.604]	0.736	[0.684, 0.779]	
ι	0.500	0.137	[0.032, 0.228]	0.251	[0.100, 0.405]	
$ heta^*$	0.667	0.591	[0.524, 0.672]	0.649	[0.580, 0.710]	
$\iota^*$	0.500	0.138	[0.028, 0.241]	0.262	[0.077, 0.446]	
$\phi_r$	0.750	0.782	[0.732, 0.826]	0.785	[0.746, 0.825]	
$\phi_{\pi}$	2.000	2.154	[1.966, 2.307]	1.878	[1.782, 1.961]	
$\phi_y$	0.125	0.215	[0.128, 0.310]	0.281	[0.198, 0.373]	
$\phi_r^*$	0.750	0.760	[0.710, 0.817]	0.747	[0.686, 0.807]	
$\phi^*_{\pi}$	2.000	2.052	[1.915, 2.206]	2.166	[2.065, 2.303]	
$\phi_y^*$	0.125	0.299	[0.187, 0.410]	0.141	[0.085, 0.194]	
$\rho_A^{j}$	0.500	0.722	[0.594, 0.861]	0.621	[0.520, 0.712]	
$ ho_g$	0.500	0.932	[0.895 <i>,</i> 0.968]	0.829	[0.770, 0.887]	
$\rho_A^*$	0.500	0.653	[0.565, 0.758]	0.498	[0.372, 0.623]	
$ ho_g^*$	0.500	0.945	[0.921, 0.969]	0.970	[0.948, 0.992]	
$\rho_{\Omega}$	0.500	0.997	[0.995 <i>,</i> 0.999]	0.993	[0.988, 0.996]	
$ ho_{\sigma_A}$	0.500	1.365	[0.927, 1.854]	4.133	[2.781, 5.349]	
$ ho_{\sigma_q}$	0.500	7.834	[6.800, 8.884]	9.638	[7.999, 11.137]	
$\rho_{\sigma_{\epsilon_R}}$	0.500	0.169	[0.144, 0.196]	0.150	[0.130, 0.172]	
$\rho^{*}_{\sigma_A}$	0.500	1.932	[1.239, 2.601]	3.481	[2.552, 4.440]	
$\rho^*_{\sigma_g}$	0.500	7.441	[6.530, 8.405]	4.624	[4.090, 5.098]	
$\rho^*_{\sigma_{\epsilon_R}}$	0.500	0.170	[0.143, 0.195]	0.186	[0.153, 0.222]	
$ ho_{\sigma_\Omega}$	0.500	7.184	[6.117, 8.240]	5.354	[4.916, 5.781]	
$\log p(\mathcal{Y}^T)$		-659.387		-746.225		

Table 2: Posterior distributions of parameters

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm. In the table,  $\log p(\mathcal{Y}^T)$  represents the SMC-based approximation of log marginal data density.

		3	Brd order	3rd order with S.V.		
Parameter	Prior mean	Mean 90% interval		Mean		
ε	5.000	4.344	[4.043, 4.558]	4.085	[3.621, 4.574]	
$\sigma$	2.000	1.770	[1.640, 1.967]	2.377	[2.211, 2.591]	
$\theta$	0.667	0.466	[0.433, 0.499]	0.593	[0.569, 0.620]	
ι	0.500	0.058	[0.015, 0.097]	0.294	[0.169, 0.450]	
$ heta^*$	0.667	0.662	[0.637, 0.690]	0.585	[0.551, 0.624]	
$\iota^*$	0.500	0.143	[0.050, 0.216]	0.230	[0.151, 0.363]	
$\phi_r$	0.750	0.663	[0.602, 0.720]	0.808	[0.768, 0.845]	
$\phi_{\pi}$	2.000	2.294	[2.245, 2.346]	1.967	[1.815, 2.071]	
$\phi_y$	0.125	0.027	[0.014, 0.040]	0.115	[0.069, 0.148]	
$\phi_r^*$	0.750	0.702	[0.653, 0.743]	0.790	[0.749, 0.829]	
$\phi^*_{\pi}$	2.000	2.082	[2.017, 2.171]	2.075	[2.008, 2.124]	
$\phi_y^*$	0.125	0.371	[0.342, 0.406]	0.017	[0.005, 0.029]	
$\rho_A$	0.500	0.846	[0.791, 0.927]	0.358	[0.251, 0.448]	
$ ho_g$	0.500	0.824	[0.753, 0.898]	0.738	[0.620, 0.839]	
$ ho_A^*$	0.500	0.126	[0.044, 0.193]	0.540	[0.459, 0.626]	
$ ho_g^*$	0.500	0.899	[0.855, 0.931]	0.544	[0.456, 0.639]	
$\rho_{\Omega}$	0.500	0.989	[0.984, 0.993]	0.964	[0.940, 0.987]	
$ ho_{\sigma_A}$	0.500	-	-	0.402	[0.247, 0.528]	
$ ho_{\sigma_g}$	0.500	-	-	0.498	[0.414, 0.565]	
$\rho_{\sigma_{\epsilon_R}}$	0.500	-	-	0.558	[0.501, 0.626]	
$\rho^*_{\sigma_A}$	0.500	-	-	0.385	[0.243, 0.507]	
$\rho^*_{\sigma_g}$	0.500	-	-	0.441	[0.368, 0.505]	
$\rho^*_{\sigma_{\epsilon_R}}$	0.500	-	-	0.527	[0.423, 0.652]	
$ ho_{\sigma_\Omega}$	0.500	-	-	0.531	[0.452, 0.618]	
$100\sigma_A$	2.500	1.421	[1.076, 1.658]	2.427	[1.715, 2.920]	
$100\sigma_g$	2.500	10.633	[9.675, 11.653]	5.398	[4.811, 6.059]	
$100\sigma_{\epsilon_R}$	0.500	0.207	[0.171, 0.237]	0.143	[0.109, 0.178]	
$100\sigma_A^*$	2.500	4.385	[3.911, 4.967]	2.357	[2.049, 2.704]	
$100\sigma_g^*$	2.500	6.001	[4.672, 7.125]	4.447	[3.837, 5.443]	
$100\sigma_{\epsilon_R}^{s}$	0.500	0.189	[0.162, 0.218]	0.220	[0.168, 0.271]	
$100\sigma_{\Omega}$	2.500	5.258	[4.809, 5.745]	4.688	[4.047, 5.079]	
$ au_A$	1.250	-	-	0.557	[0.419, 0.737]	
$ au_{g}$	1.250	-	-	0.681	[0.491, 0.827]	
$ au_{\epsilon_R}$	1.250	-	-	1.623	[1.212, 2.007]	
$ au_A^*$	1.250	-	-	0.818	[0.665, 0.978]	
$ au_g^*$	1.250	-	-	1.754	[1.406, 2.123]	
$ au_g^* \  au_{\epsilon_R}^*$	1.250	-	-	0.467	[0.335, 0.600]	
$ au_{\Omega}^{arepsilon_{R}}$	1.250	-	-	0.595	[0.360, 0.786]	
$\log p(\mathcal{Y}^T)$			-766.862	-	-852.741	

Table 3: Posterior distributions of parameters (cont.)

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 1,000 particles from the final importance sampling in the SMC algorithm. In the table,  $\log p(\mathcal{Y}^T)$  represents the SMC-based approximation of log marginal data density.

Parameter	Our estimate	BBN's calibration
ε	4.09	5.00
$\sigma$	2.38	2.00
heta	0.59	0.66
$ heta^*$	0.59	0.75
$\phi_r$	0.81	0.76
$\phi_{\pi}$	1.97	1.41
$\phi_{oldsymbol{y}}$	0.12	0.66
$\phi_{r^*}$	0.79	0.84
$\phi_{\pi^*}$	2.08	1.37
$\phi_{y^*}$	0.02	1.27

Table 4: Comparison to BBN

The marginal data densities indicate that the higher-order approximation of the model does not contribute to improving the fit of the model to the data and the baseline model with stochastic volatilities deteriorates the fit even further. This is because the higher-order approximation with stochastic volatilities imposes such tighter crossequation restrictions that are at odds with the data.

While the structural parameters are not much different across the four models, remarkable differences arise in the parameters related to shocks. First, as the degree of approximation is higher, the AR(1) coefficients for structural shocks tend to be smaller. In particular, the third-order approximation with stochastic volatilities results in substantially smaller estimates than the others, except for the coefficient on the technology shock in the Euro area  $\rho_A^*$ . Even in the baseline model, however, the coefficient on the risk-sharing shock  $\rho_{\Omega}$  is very close to unity, indicating the almost unit-root process. Second, the standard deviations of the structural shocks become substantially lower in the baseline model because the stochastic volatilities are incorporated into the model.

Our model shares many similarities with the one in BBN and the same structural parameters appear in the two models. Table 4 compares our posterior mean estimates of parameters with the parameter values calibrated in BBN. While the parameters on households' preferences and firms' price settings are not much different, the monetary policy responses to inflation and output growth are different across the two models. However, we have confirmed that the differences in these parameters do not lead to any qualitative differences in our main results presented below.

#### 4.2 Impulse responses

This subsection demonstrates that our estimated model can partly account for empirical regularities regarding volatility shocks as shown by BBN: (1) an increase in the volatility of the productivity shock induces an exchange rate depreciation; (2) an increase in the volatility of the monetary policy shock induces an exchange rate appreciation; and (3) an increase in the volatility of the monetary policy shock causes deviations from the UIP in the form of an increase in the excess return on the foreign currency.

The figures 1–2 show the generalized impulse responses of the observed variables  $(YGR_t, \pi_t, R_t, YGR_t^*, \pi_t^*, R_t^*, d_t)$ , nominal interest rate differential  $(R_t - R_t^*)$ , and the excess return on the foreign currency  $(E_t d_{t+1} + R_t^* - R_t)$  to the volatility shocks to home technology and home monetary policy, given the posterior mean estimates of parameters in the baseline estimation. While the increase in volatility of home technology causes the appreciation of the exchange rate at the time of the shock, it causes its depreciation. From the second period after the volatility shock to home monetary policy, the exchange rate appreciates and the deviation from the UIP turns to be positive, although these directions are opposite in the first period when the shock occurs.

As shown in the next subsection, our estimation results indicates the importance of the risk-sharing shock in explaining the exchange rate fluctuations. The figure 3 depicts the impulse responses to the shock to the risk-sharing condition. This shock directly causes the deviation from the UIP and the depreciation of the dollar at the time of the shock followed by its persistent appreciation. This shock also affects other macroeconomic variables to a substantial degree.

The impulse responses to the other shocks are presented in the Appendix.

#### 4.3 Accounting for exchange rate dynamics

The log-linear approximation of the equilibrium conditions leads to the UIP condition incorporated with the risk-sharing shock  $\Omega_t$ :

$$\hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} + \hat{\Omega}_t - \mathbf{E}_t \hat{\Omega}_{t+1}, \tag{3}$$

where the circumflex denotes the log deviation from the steady state value. Then, the deviations from the UIP are captured by the risk-sharing shock. If data suggest sizable deviations from the UIP, the contribution of the risk-sharing shock to the exchange rate dynamics will be large. On the other hand, with the higher order approximation, the international risk-sharing condition are given by equations (1) and (2). As a result, the deviation from the UIP can be partly captured by higher-order terms such as the endogenous risk premium and thus the contribution of the risk-sharing shock is expected to decrease, compared with the linear case.

To examine this point, Table 5 shows the relative variances excluding each shock,<sup>9</sup> given the posterior mean estimates of parameters in the estimation of a linearized version of the model, the models without the stochastic volatilities solved by the secondand third-order approximation, and the baseline model (with stochastic volatilities solved by the third-order approximation.) Each number shows how much of fluc-

<sup>&</sup>lt;sup>9</sup>We did not employ the standard variance decomposition because it can underestimate the contributions of each shock by ignoring cross-terms among shocks in nonlinear settings.

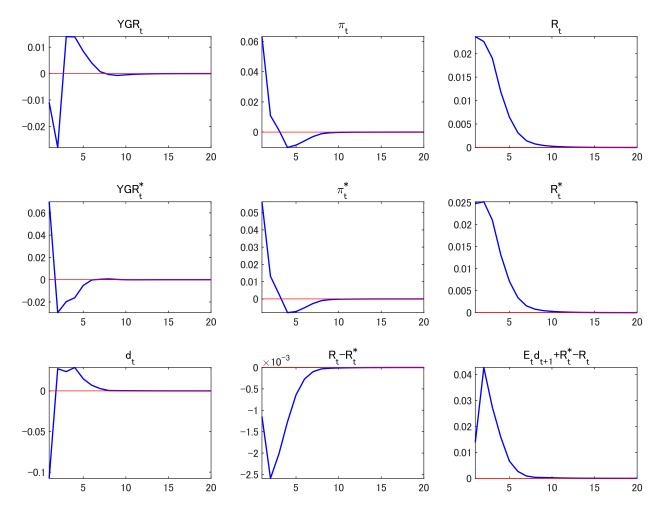


Figure 1: Responses to volatility shock to home technology

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home technology, given the posterior mean estimates of parameters in the baseline model.

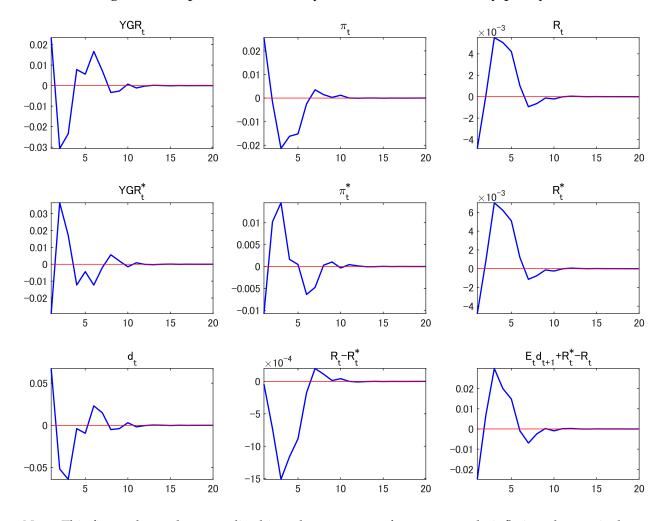


Figure 2: Responses to volatility shock to home monetary policy

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

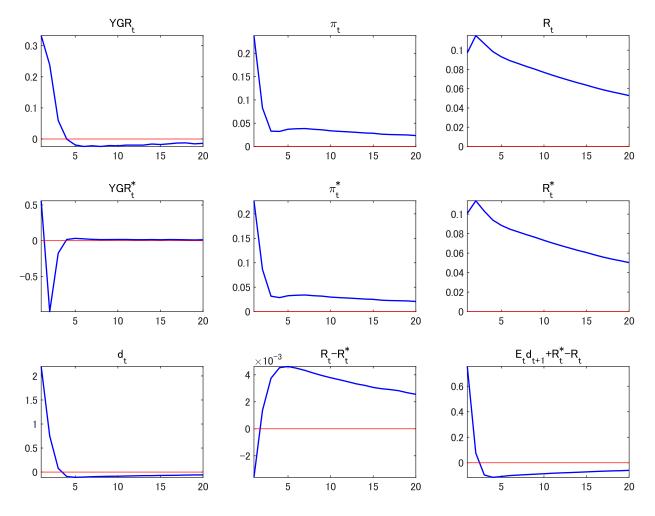


Figure 3: Responses to risk sharing shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.

		$\Delta \log Y_t$	$\log \pi_t$	$\log R_t$	$\Delta \log Y_t^*$	$\log \pi_t^*$	$\log R_t^*$	$d_t$
Linear								
w/o:	$u_A$	0.714	0.380	0.437	0.996	0.962	0.946	0.979
	$u_g$	0.418	0.950	0.758	1.000	0.990	0.994	0.972
	$u_{\epsilon_R}$	0.969	0.858	0.984	1.000	0.996	0.999	0.986
	$u_A^*$	0.990	0.922	0.914	0.672	0.269	0.378	0.975
	$u_g^*$	0.998	0.992	0.993	0.466	0.954	0.772	0.967
	$u_{\epsilon_R}^s$	1.000	0.995	0.999	0.963	0.923	0.987	0.990
	$u_{\Omega}$	0.895	0.911	0.933	0.904	0.900	0.911	0.123
2nd or	der							
w/o:	$u_A$	0.794	0.252	0.350	0.991	0.954	0.919	0.955
	$u_g$	0.263	0.946	0.813	1.000	0.993	0.993	0.977
	$u_{\epsilon_R}$	0.961	0.962	0.978	1.000	0.995	0.999	0.975
	$u_A^*$	0.988	0.933	0.948	0.379	0.185	0.192	0.919
	$u_g^*$	0.997	1.000	1.001	0.743	0.995	0.974	0.985
	$u_{\epsilon_R}^*$	1.000	0.997	0.999	0.934	0.949	0.981	0.981
	$u_{\Omega}$	0.984	0.901	0.921	0.935	0.948	0.952	0.202
3rd ord	ler							
w/o:	$u_A$	0.671	0.583	0.434	0.995	0.967	0.924	0.904
	$u_g$	0.394	0.798	0.715	1.003	0.983	0.973	0.962
	$u_{\epsilon_R}$	0.990	0.844	0.987	1.000	0.998	1.000	0.986
	$u_A^*$	0.992	0.865	0.937	0.688	0.187	0.388	0.976
	$u_g^*$	1.000	0.994	0.997	0.448	0.978	0.860	0.975
	$u_{\epsilon_R}^*$	1.000	0.995	0.998	0.937	0.960	0.982	0.987
	$u_{\Omega}$	0.975	0.929	0.946	0.911	0.941	0.877	0.202
3rd ord	ler wit	h SV						
w/o:	$u_A$	0.874	0.569	0.597	0.996	0.976	0.976	0.986
	$u_g$	0.501	0.968	0.882	1.000	0.997	0.996	0.994
	$u_{\epsilon_R}$	0.698	0.608	0.872	1.001	0.986	0.993	0.825
	$u_A^*$	0.979	0.936	0.914	0.891	0.299	0.310	0.941
	$u_g^*$	0.988	0.986	0.975	0.132	0.886	0.869	0.984
	$u_{\epsilon_R}^*$	1.000	0.998	0.999	0.989	0.927	0.988	0.979
	$u_{\Omega}$	0.960	0.918	0.753	0.991	0.929	0.859	0.287
	$z_{\sigma A}$	0.941	0.800	0.819	0.998	0.988	0.990	0.993
	$z_{\sigma g}$	0.699	0.980	0.935	1.000	0.999	0.999	0.996
	$z_{\sigma\epsilon_R}$	0.715	0.630	0.877	1.001	0.987	0.993	0.836
	$z_{\sigma A}^*$	0.986	0.959	0.949	0.932	0.534	0.549	0.961
	$z_{\sigma g}^*$	0.990	0.988	0.977	0.185	0.893	0.880	0.986
	$z^*_{\sigma\epsilon_R}$	1.000	1.000	1.000	0.995	0.971	0.994	0.991
	$z_{\sigma\Omega}$	0.979	0.964	0.878	0.994	0.963	0.922	0.640

Table 5: Relative variances excluding each shock

Notes: The table shows the variances of the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation excluding each shock, relative to those with all the shocks, given the posterior mean estimates of parameters.

tuations can be explained by excluding each structural shocks shown in the left column:  $u_A$ ,  $u_g$ ,  $u_{\epsilon_R}$ ,  $u_A^*$ ,  $u_g^*$ ,  $u_{\epsilon_R}^*$ ,  $u_\Omega$ ,  $z_{\sigma A}$ ,  $z_{\sigma g}$ ,  $z_{\sigma \epsilon_R}$ ,  $z_{\sigma g}^*$ ,  $z_{\sigma e_R}^*$ , and  $z_{\sigma \Omega}$  denotes the shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, and the volatility shocks to the respective shocks. Regarding the changes in nominal exchange rates  $d_t$ , the linear model excluding the international risk-sharing shock can explain only 12% of its volatility, implying that 88% of the nominal exchange rate fluctuations are driven by the risk-sharing shock. The second- and third-order approximation without stochastic volatilities lead to the increases in the relative variances by taking account of nonlinearities in the model, but the increases are limited to 20 %. Even in the case with stochastic volatilities, the model can explain only 29% of exchange rate fluctuations if the shock to the risk-sharing condition is excluded. Therefore, even the stochastic volatility, whose importance on aggregate fluctuations have been emphasized in the form of uncertainty shocks by Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015), cannot explain the exchange rate dynamics as observed in the data.

These results altogether imply that nominal exchange rates are disconnected from the macroeconomic fundamentals. The variance decomposition from our estimation hints the importance of financial frictions as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017). Frictions in financial transactions need to be incorporated to hinder the international arbitrage in account for exchange rate dynamics and aggregate fluctuations in open economies.

Why do the other shocks than the risk-sharing shock cannot be major sources of exchange rate fluctuations? To answer this question, we construct artificial time-series driven by each single shock and examine which shock can generate the negative correlation between the expected changes in the nominal exchange rate and nominal interest rate differentials—one of the empirical regularities in exchange rate dynamics, known as a negative slope in the UIP regression.

Figure 4 presents the scatter plots and the UIP regression simulated with each level shock, where  $A_t$ ,  $g_t$ ,  $\varepsilon_{R,t}$ ,  $A_t^*$ ,  $g_t^*$ ,  $\varepsilon_{R,t}^*$ , and  $\Omega_t$  denote the shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, respectively. The figure indicates that no other shocks except for the risk-sharing shock do not replicate any negative correlations between the expected changes in the nominal exchange rate and nominal interest rate differentials. This is indeed the mechanism stemming from the UIP: When nominal interest rates are high, exchange rates will depreciate for no arbitrage.

The international risk sharing shocks can replicate a slightly negative correlation between them. In addition to this fact, we argue that the persistent dynamics in the risk-sharing shock increases its contribution to explaining the exchange rate fluctuations. As shown in Tables 2 and 3, the mean estimates of the AR(1) coefficient  $\rho_{\Omega}$  are 0.99 with the first-, second-, and third-order approximation without stochastic volatilities and 0.96 even in the case of third-order approximation with stochastic volatilities. This is to account for the near random-walk process found in real exchange rates in

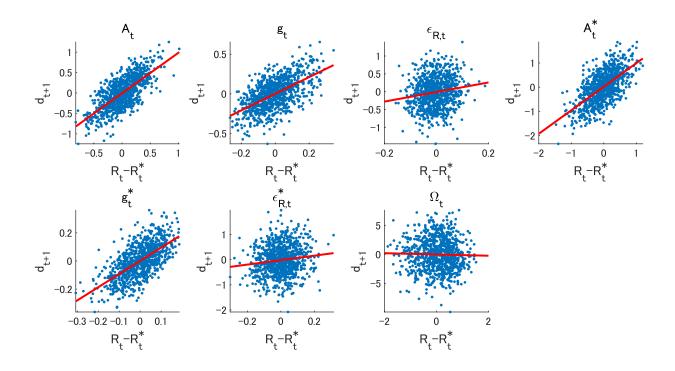


Figure 4: UIP regressions based on simulated series driven by each level shock

the sample period, examined in this paper. As a result, the current shock and its expectation cancel out each other at the UIP condition augmented with the international risk sharing condition as in equation (3).

On the other hand, the volatility shocks may possibly replicate negative correlations between the expected changes in the nominal exchange rate and nominal interest rate differentials. As explored in BBN, more uncertainty in nominal shocks makes the home currency a good hedge and at the same time, leads to higher nominal interest rates, *i.e.* more demand for money, in the domestic country. Consequently, the carry trade may yield positive excess returns. The gains from the carry trade compensate for the risk of holding foreign currency to uncertainty in the conduct of monetary policy in the domestic country. BBN also discuss the interactions between monetary policy, price stickiness and stochastic volatilities on exchange rate dynamics. They find that interest rate smoothing,  $\phi_r$ , and the price stickiness,  $\theta$ , are key parameters to determine the size of the deviation from the UIP: The more (less) interest rate smoothing and stickier (more flexible) the price becomes, the more negative (positive) the UIP coefficient becomes. Sizable negative coefficient in the Fama regression emerges with the estimated values for the interest rate smoothing and the price stickiness.

Figure 5 shows the results based on the volatility shocks, where  $\sigma_{A,t}$ ,  $\sigma_{g,t}$ ,  $\sigma_{\varepsilon R,t}$ ,  $\sigma_{A,t}^*$ ,  $\sigma_{g,t}^*$ ,  $\sigma_{\varepsilon R,t}^*$ , and  $\sigma_{\Omega,t}$  are the volatility shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, respectively. While

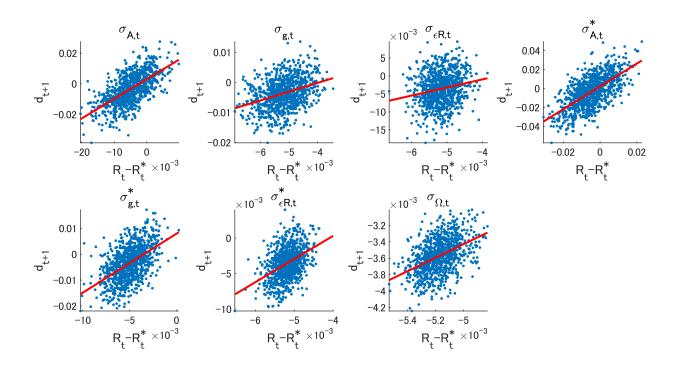


Figure 5: UIP regressions based on simulated series driven by each volatility shock

BBN demonstrate that the negative correlations can emerge with a volatility shock to monetary policy under some parameter settings, we find that our estimated model cannot produce the same result.

## 5 Conclusion

In this paper, we have estimated the two country New Keynesian model with the recursive preferences and stochastic volatilities using higher order approximation and the central difference Kalman filter. According to the estimation results, the shock to the international risk-sharing condition which represents the time-varying financial frictions that hinder the international arbitrage is a major driver in accounting for the realistic exchange rate dynamics as well as aggregate fluctuations in open economies. The exchange rate is disconnected from macroeconomic fundamentals even if we allow for higher-order terms and volatility shocks.

Still several possibilities remain to reduce the importance of the shock to the international risk sharing condition, which is rather *ad-hocly* set in this paper. First, the *news shock* is an important driver of the aggregate fluctuations as reported in Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012). The stochastic volatilities of news shocks may overturn our main result. Second, exchange rate may be intrinsically indeterminate as advocated by Kareken and Wallace (1981). Interaction between the structural shocks and the *sunspot shocks* as in Lubik and Schorfheide (2004) or volatility shocks to sunspot shocks must be an interesting challenge.

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## A Steady state

Symmetric steady states between two countries with  $n = n^* = .5$  are assumed. To avoid nonstationarity, we need to assume

$$\bar{\pi} = \bar{\pi}^*.$$

We parameterize  $\frac{\bar{g}}{\bar{y}}$  instead of g. Then,  $y = g/(\frac{\bar{g}}{\bar{y}})$ . Substitute this expression into the steady-state equation for y:

$$y = \frac{\psi(\mu - 1)}{\mu - \psi} (1 - g) + g,$$

leads to

$$\begin{split} g/(\frac{\bar{g}}{\bar{y}}) &= \frac{\psi(\mu-1)}{\mu-\psi} \left(1-g\right) + g \\ \Leftrightarrow g &= \frac{\bar{g}}{\bar{y}} \frac{\psi(\mu-1)}{\mu-\psi} \left(1-g\right) + \frac{\bar{g}}{\bar{y}}g \\ \Leftrightarrow \left[1 + \frac{\bar{g}}{\bar{y}} \frac{\psi(\mu-1)}{\mu-\psi} - \frac{\bar{g}}{\bar{y}}\right] g &= \frac{\bar{g}}{\bar{y}} \frac{\psi(\mu-1)}{\mu-\psi} \\ \Leftrightarrow g &= \frac{\frac{\bar{g}}{\bar{y}} \frac{\psi(\mu-1)}{\mu-\psi}}{1 + \frac{\bar{g}}{\bar{y}} \frac{\psi(\mu-1)}{\mu-\psi} - \frac{\bar{g}}{\bar{y}}} \\ \Leftrightarrow g &= \frac{\frac{\psi(\mu-1)}{\mu-\psi}}{\left(\frac{\bar{g}}{\bar{y}}\right)^{-1} + \frac{\psi(\mu-1)}{\mu-\psi} - 1}. \end{split}$$

### A.1 Domestic

$$\begin{split} \pi &= \bar{\pi}, \\ p_H &= 1, \\ \pi_H &= \bar{\pi}, \\ R &= \frac{\bar{\pi}}{\beta \gamma^{\psi(1-\sigma)-1}}, \\ m &= \beta \gamma^{\psi(1-\sigma)-1}, \\ \tilde{w} &= \frac{\mu-1}{\mu}, \\ c &= \frac{\psi(\mu-1)}{\mu-\psi} \left(1-g\right), \\ &= \frac{\psi(\mu-1)}{\mu-\psi} \left(1-g\right) + g, \end{split}$$

N

$$y = \frac{\psi(\mu - 1)}{\mu - \psi} (1 - g) + g,$$
$$c_H = (1 - \alpha) c,$$
$$c_F = \alpha c,$$
$$v = \left\{ \frac{\left[ c^{\psi} (1 - N)^{1 - \psi} \right]^{1 - \sigma}}{1 - \beta \gamma^{\psi}} \right\}^{\frac{1}{1 - \sigma}}$$
$$f = \frac{y}{2 (1 - \theta \beta \gamma^{\psi(1 - \sigma)})},$$
$$k = \frac{y}{2 (1 - \theta \beta \gamma^{\psi(1 - \sigma)})},$$

and

 $\Delta = 1.$ 

# A.2 Foreign

$$\begin{split} \pi^* &= \bar{\pi}^*, \\ p_F^* &= 1, \\ \pi_F^* &= \bar{\pi}^*, \\ R^* &= \frac{\bar{\pi}^*}{\beta \gamma^{\psi(1-\sigma)-1}}, \\ m^* &= \beta \gamma^{\psi(1-\sigma)-1}, \\ \tilde{w} &= \frac{\mu - 1}{\mu}, \\ c^* &= \frac{\psi(\mu - 1)}{\mu - \psi} \left(1 - g^*\right), \\ N^* &= \frac{\psi(\mu - 1)}{\mu - \psi} \left(1 - g^*\right) + g^*, \\ y^* &= \frac{\psi(\mu - 1)}{\mu - \psi} \left(1 - g^*\right) + g^*, \\ c_H^* &= \alpha c^*, \\ c_F^* &= (1 - \alpha) c^*, \\ v^* &= \left\{ \frac{\left[c^{*\psi} \left(1 - N^*\right)^{1-\psi}\right]^{1-\sigma}}{1 - \beta \gamma^{\psi}} \right\}^{\frac{1}{1-\sigma}}, \end{split}$$

$$f^* = \frac{y^*}{2(1 - \theta^* \beta \gamma^{\psi(1-\sigma)})},$$
$$k^* = \frac{y^*}{2(1 - \theta^* \beta \gamma^{\psi(1-\sigma)})},$$
$$\Delta^* = 1.$$

and

## A.3 International

and

d = 1.

s = 1,

Q = 1,

## **B** Impulse responses to the other shocks

In what follows, the figures 6–16 show the generalized impulse responses of the observed variables ( $YGR_t$ ,  $\pi_t$ ,  $R_t$ ,  $YGR_t^*$ ,  $\pi_t^*$ ,  $R_t^*$ ,  $d_t$ ), nominal interest rate differential ( $R_t - R_t^*$ ), and the excess return on the foreign currency ( $E_t d_{t+1} + R_t^* - R_t$ ) to the other shocks that are not reported in Section 4.2, given the posterior mean estimates of parameters in the baseline estimation.

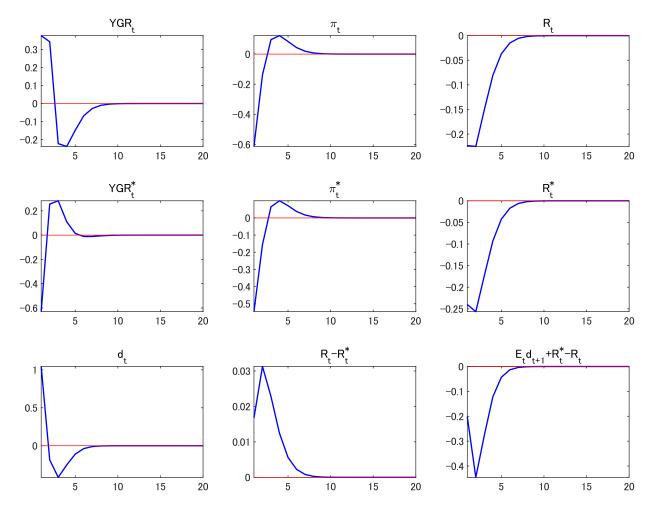


Figure 6: Responses to home technology shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home technology, given the posterior mean estimates of parameters in the baseline model.

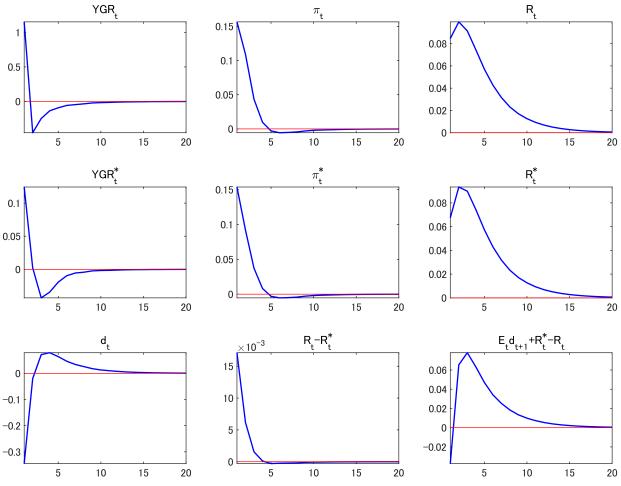


Figure 7: Responses to home external demand shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home external demand, given the posterior mean estimates of parameters in the baseline model.

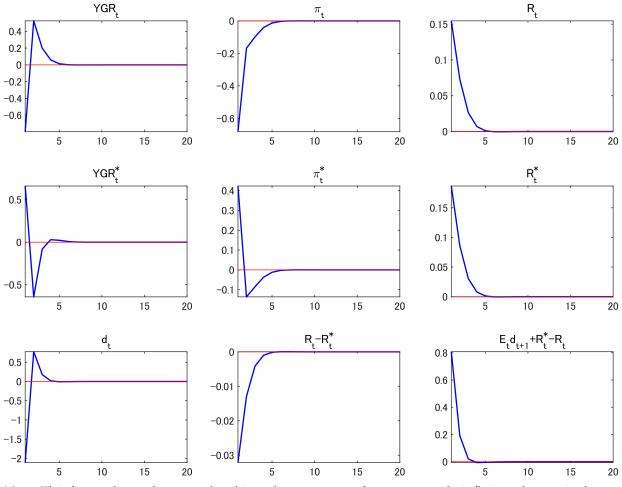


Figure 8: Responses to home monetary policy shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

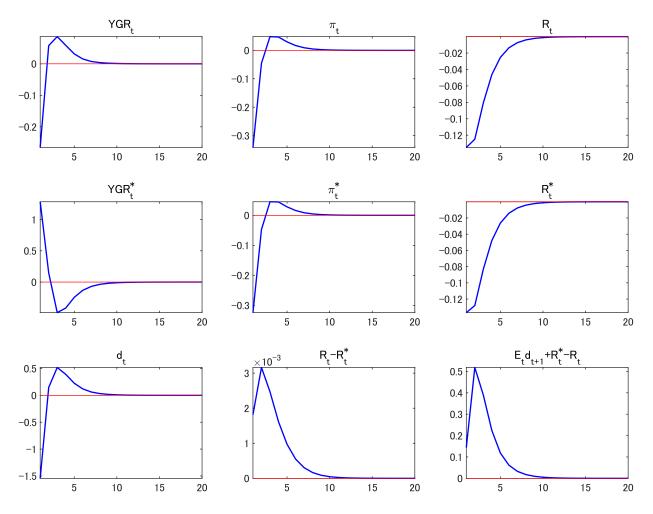


Figure 9: Responses to foreign technology shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

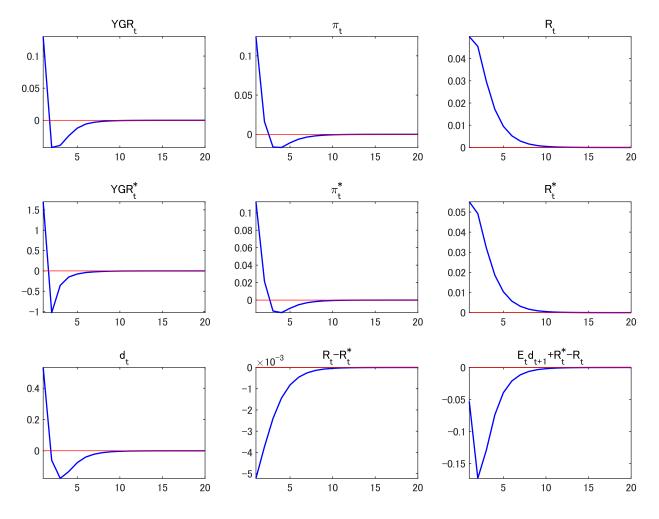


Figure 10: Responses to foreign external demand shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

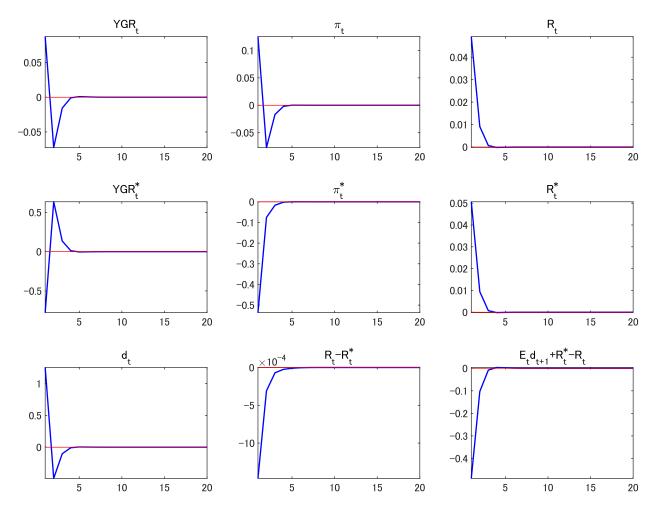


Figure 11: Responses to foreign monetary policy shock

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

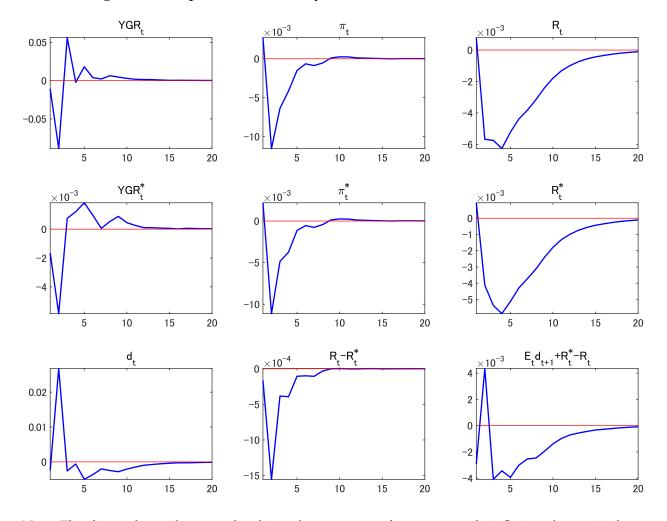


Figure 12: Responses to volatility shock to home external demand

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home external demnad, given the posterior mean estimates of parameters in the baseline model.

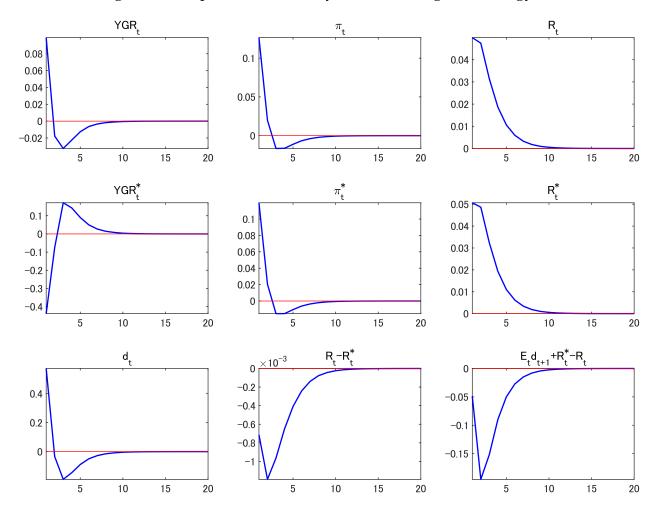


Figure 13: Responses to volatility shock to foreign technology

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

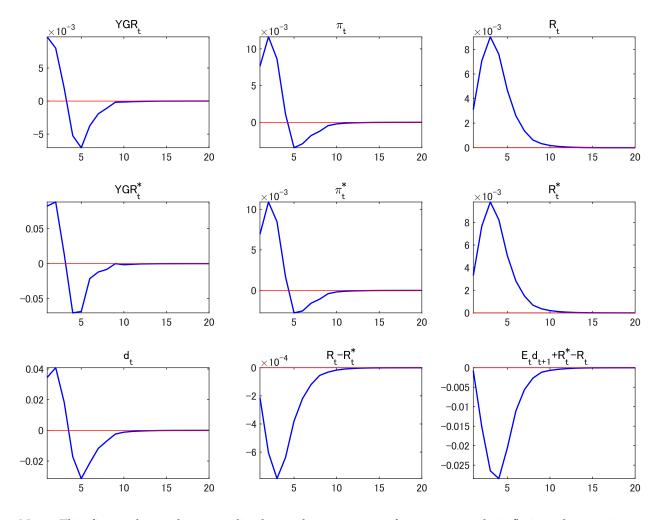


Figure 14: Responses to volatility shock to foreign external demand

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a onestandard-deviation volatility shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

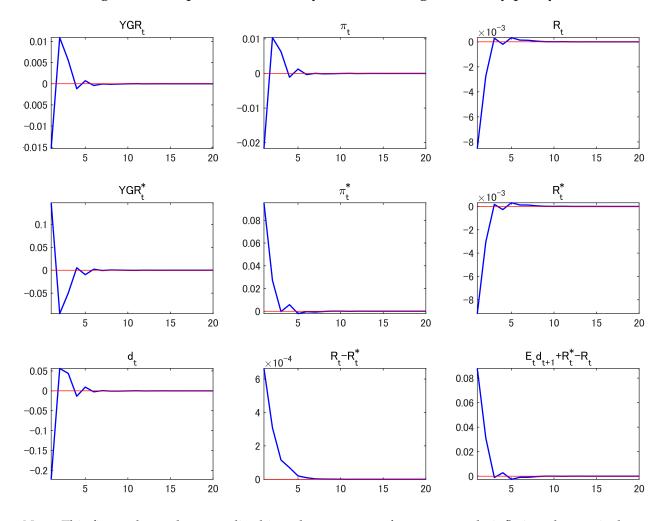


Figure 15: Responses to volatility shock to foreign monetary policy

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

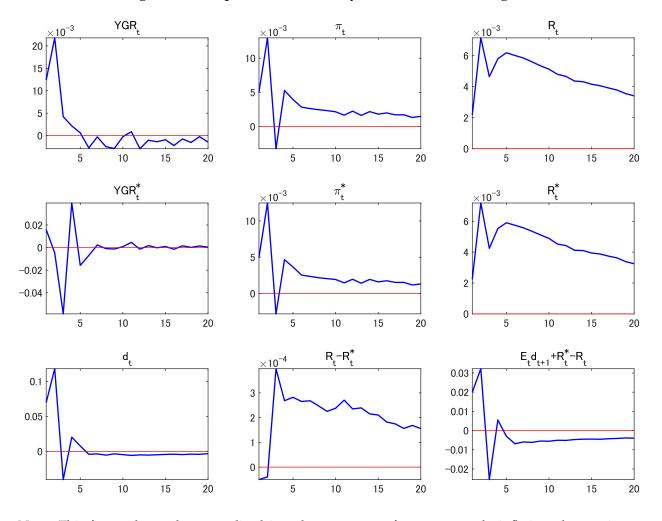


Figure 16: Responses to volatility shock to risk sharing

Note: This figure shows the generalized impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a onestandard-deviation volatility shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.