Is Exchange Rate Disconnected After All?*

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Abstract

This paper assesses and compares the empirical relevance of macro-volatility shocks on one hand and shocks to international risk-sharing condition on the other in explaining exchange rate dynamics. We estimate a two-country New Keynesian model with recursive preferences and stochastic volatilities for the US and the Euro area, using third-order model approximation and data from 1987Q1 to 2008Q4. Inclusion of time-varying volatilities in monetary policy shocks can potentially account for the well-known forward premium or UIP puzzle, providing direct empirical support for the intuition/mechanism explored in earlier simulation-based papers: higher uncertainty in nominal conditions makes the home currency a good hedge, lowering its premium and at the same time, raises the nominal interest rate at home through higher money demands. But, our full-information Bayesian estimation shows that such volatility shocks offer marginal explanatory power for Dollar-Euro exchange rate dynamics. Instead, variance decompositions show that more than half of the fluctuations in the exchange rate are explained by a direct shock to the exchange rate, i.e. a shock to the international risk sharing condition. From this point of view, we find the exchange rate to be mostly disconnected from other macroeconomic fundamentals, even if nonlinearities and stochastic volatilities are taken into account.

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1 Introduction

The nominal exchange rate is an important driver of aggregate fluctuations as well as a key link between international goods and asset markets. However, endogenizing realistic exchange rate dynamics as observed in the data is a task that has alluded to international macroeconomists for decades. While various structural frameworks aim to understand how policies or the intrinsic shocks in one country spill over into other countries via the exchange rates, estimation efforts of such general equilibrium models typically find fluctuations in nominal exchange rates to be unrelated to macroeconomic forces.[1] Consequently, empirical evidence for the various transmission mechanisms of international policies and shocks through the exchange rate channel remains thin to non-existent, a pattern commonly referred to in the literature as the "exchange rate disconnect."

The exchange rate disconnect manifests itself into various empirical puzzles, each with its own vast literature exploring different reasons behind exchange rate fluctuations. This paper evaluates two recent alternative approaches by empirically estimating a full-fledged DSGE model that encompasses both sources of fluctuations: 1) direct shocks to exchange rate or international risking-sharing condition; and 2) macroeconomic volatility shocks that induce time-varying risks in the exchange rates. We note that since the two approaches emphasize first-moment vs. 2nd-moment shocks, proper comparisons would thus require estimating the model up to a third order approximation, as well as evaluating them along the dimensions of both the means and the variances. In this paper, we look at how the two sources of shocks contribute to explaining the uncovered interest rate parity (UIP) puzzle and excess exchange rate volatility (relative to macro-fundamentals.)

To a first-order approximation (ignoring variance and covariance risk), the UIP as an no arbitrage condition implies that a country with high relative interest rates should expect to experience subsequent currency depreciation, ensuring zero expected excess returns from cross-border financial investments. As is well-known since Fama (1984), data consistently show significant and robust positive returns from "carry-trade" strategies that invest in the currency with higher interest rates, an empirical regularity known as the forward-premium puzzle or the UIP puzzle. There have been numerous attempts to solve the forward discount puzzle, though as pointed out in Itskhoki and Mukhin (2017), any proposed solutions must also account for the high volatilities present in the exchange rates, but absent in other macroeconomic variables.

This paper focuses on evaluating the following two mechanisms for explaining exchange rate dynamics: international risk sharing shocks, and time-varying risks in the macroeconomy. We first present a two-country New Open Economy Macro Economics (NOEM) model that encompass both channels, by adopting recursive preferences *a la* Epstein and Zin (1989) and stochastic volatilities, whose importance on aggregate fluctuations have been emphasized in Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015).[2] While the vast majority of the literature

[1]See, for example, Lubik and Schorfheide (2006). Notable exception is Adolfson, Laseen, Linde, and Villani (2007) for the small open economy but they incorporate rather ad-hoc adjustment costs to capture risks in exchange rates.

[2]The representative models of NOEM can be found in Svensson and van Wijnbergen (1989), Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Clarida, Gali, and Gertler (2002), Benigno and Benigno

using a DSGE setting relies on simulation results, this paper estimates the model using pre-financial crisis US and the Euro area data, to evaluate which channel is the more important driver of the dollar-euro fluctuations.

The two mechanisms we emphasize capture arguments put forth in recent studies. For the first channel, Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017), for example, point out the importance of financial frictions in accounting for aggregate fluctuations in open economies. As frictions in financial transactions hinder international arbitrage through the exchange rates, they work as direct shocks to exchange rates themselves. We note that if such international financial friction shocks turn out to be the main driver behind exchange rate fluctuations in our estimation, one would conclude that exchange rates are indeed disconnected from other macroeconomic variables.

Alternatively, the empirical failure of the UIP may be the result of linear or first-order approximation, as endogenous risk premium may arise from covariance between the stochastic discount factor and returns to international financial investments. Moving beyond a linearized framework, one can endogenously generate time-varying currency risks. For example, a structural or macroeconomic fundamental shock—especially to volatilities—can simultaneously raises interest rates and appreciates the nominal exchange rates. If exchange rate fluctuations are mostly attributed to such endogenous risks, one would then infer that the exchange rate is not disconnected from macroeconomic fundamentals. Previous attempts to generate endogenous currency risk premiums through first-moment shocks have led to little success; this is why our paper considers shocks to the volatilities of macro variables.

From the literature that endogenizes exchange rate risks, the paper most closely related to ours is Benigno, Benigno, and Nistico (2011) (hereafter BBN). They examine the role of nominal and real stochastic volatilities in explaining exchange rate behavior by simulating a two-country NOEM model with recursive preferences. They find that a rise in the volatility of nominal shocks in the home country enhance the hedging properties of its currency relative to those of the foreign, thereby inducing endogenously a risk premium for foreign currency-holding. In addition, a rise in home nominal volatility tends to reduce domestic output and increase domestic producer inflation, while the domestic nominal interest rate declines proportionately more than the foreign one. Thus a negative correlation emerges between expected changes in nominal exchange rates and nominal interest rate differentials in response to volatility shocks, potentially over-turning the forward premium puzzle and accounting for the empirical regularities observed in exchange rate movements.

Our paper moves the evaluations of these mechanisms to an estimation framework and consider the fit to the data, instead of relying only on simulations with calibrated parameters. In fact, BBN already recognize this issue and argue that "the estimation of the model is really needed to evaluate its fit. To this purpose, an appropriate methodology should be elaborated to handle the features of our general second-order approximated solutions. We leave this research for future work."[3] Moreover, BBN demonstrate that

(2003) and Devereux and Engel (2003).

[3] In his comment to BBN, Uribe (2011) echoes the importance of a direct estimation of the model: "I would like to [suggest] an alternative identification approach. It consists of a direct estimation of a DSGE model. ... Admittedly, estimating DSGE models driven by time-varying volatility shocks is not a simple

volatility shocks to monetary policy can replicate a negative correlation between the expected depreciation of the nominal exchange rate and nominal interest rate differentials as observed in the data when price stickiness is low and interest rate smoothing is high. Thus, it is of great importance to estimate related parameters as well as the size of shocks to evaluate the empirical relevance of volatility shocks to the exchange rate dynamics. We accomplish this task by first solving the two-country NOEM model using perturbation methods up to the third-order approximation so that we can consider stochastic volatilities in the fundamental shocks. Note that to gauge the impact of stochastic volatilities, BBN employ the efficient method with second-order approximation proposed by Benigno, Benigno, and Nistico (2013), which can account for distinct and direct effects of volatility shocks, provided that shocks are conditionally linear. In contrast, by using the third order approximation, we can allow for third-order terms and hence take account of richer propagation mechanisms of structural shocks to exchange rate dynamics. To ensure stability in the model, we employ the pruning method developed by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018). We then estimate the model with a full-information Bayesian approach. Because the model is non-linear, the standard Kalman filter is not applicable for evaluating the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).[4]

Our results are summarized as follows. Our estimated model can mostly replicate some empirical regularities regarding volatility shocks as shown by BBN: (1) an increase in the volatility of the productivity shock induces an exchange rate depreciation; (2) an increase in the volatility of the monetary policy shock induces an exchange rate appreciation; and (3) an increase in the volatility of the monetary policy shock causes deviations from the UIP in the form of an increase in the excess return on the foreign currency. Moreover, given our estimated parameters, several volatility shocks can generate a negative correlation between expected nominal exchange rate depreciation and nominal interest rate differentials as observed in the data. Despite the success in replicating these properties, the volatility shocks cannot be a major source of exchange rate fluctuations. Instead, currency fluctuations are mostly explained by the direct shock to the exchange rate, i.e. shocks to the international risk-sharing condition, supporting the views offered by Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017). According to variance decompositions, the risk-sharing shock accounts for more than half (57%) of the variance of nominal exchange rate changes. We thus conclude that at least up to second-moment shocks, exchange rates appear to remain in most part disconnected from macroeconomic fundamentals.

The remainder of this paper is organized as follows. Section 2 presents the model with recursive preferences and stochastic volatilities in open economies. Section 3 shows how we estimate the model in a nonlinear setting by a full-information Bayesian approach. Section 4 presents our main results and shows that fluctuations in exchange rates are mostly explained by the direct shock to exchange rates. Finally, Section 5 concludes.

task."

[4] Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

2 The Model

The model estimated in this paper is a two-country extension of the standard New Keynesian model but incorporates non-recursive preferences *a la* Epstein and Zin (1989) together with stochastic volatilities in various structural shocks. The world economy consists of the US (the domestic or home country) and the Euro area (the foreign country), which are assumed to be of the same size.[5] In each country, the representative household gains utility from aggregate consumption composed of home and foreign goods, and trades state contingent assets in both domestic and international asset markets. Monopolistically competitive firms produce differentiated goods, and are subject to Calvo (1983)-type staggered price-setting. Monetary authorities adjust the nominal interest rates in response to inflation and output growth. While we assume symmetric households preferences, the two regions differ in price-setting, monetary policy and fundamental shocks. The assumptions with regard to preferences, technology and complete financial markets give us a highly tractable framework for the open economy.

2.1 Household

A representative household in the domestic country maximizes the recursive utility:

$$V_t = \left[u\left(C_t, N_t\right)^{1-\sigma} + \beta \left(\mathbf{E}_t V_{t+1}^{1-\varepsilon} \right)^{\frac{1-\sigma}{1-\varepsilon}} \right]^{\frac{1}{1-\sigma}},$$

where σ measures the inverse of the intertemporal elasticity of substitution, and ε is the coefficient of relative risk aversion. N_t denotes labor supply. Aggregate consumption C_t is a composite of home- and foreign-produced goods, $C_{H,t}$ and $C_{F,t}$, given by

$$C_t := \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

with

$$C_{H,t} := \left[\int_0^1 C_{H,t}\left(j\right)^{1-\frac{1}{\mu}} \mathrm{d}j \right]^{\frac{\mu}{\mu-1}},$$
$$C_{F,t} := \left[\int_0^1 C_{F,t}\left(j^*\right)^{1-\frac{1}{\mu}} \mathrm{d}j^* \right]^{\frac{\mu}{\mu-1}},$$

where $C_{H,t}(j)$ and $C_{F,t}(j^*)$ are differentiated consumption goods produced by domestic and foreign firms, each of which are indexed by j and j^* respectively. The parameters α, η, and μ are the steady state share of the domestically produced goods consumption in the aggregate consumption, the elasticity of substitution between domestically produced and imported goods, the elasticity of substitution among differentiated products in each

[5] This assumption follows from Lubik and Schorfheide (2006). Indeed, the two regions are roughly the same size and have similar per capita income.

country. Following BBN, we specify the instantaneous utility as

$$u\left(C_t, N_t\right) := C_t^{\psi}\left(1-N_t\right)^{1-\psi}.$$

The household's utility maximization is subject to the budget constraint:

$$P_t C_t + B_t + \mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}] = R_{t-1}B_{t-1} + D_t + W_t N_t + T_t,$$

where P_t is the consumer price index, B_t is the holding of the domestic bond, $m_{t,t+1}$ is the real stochastic discount factor, $\pi_t := P_t/P_{t-1}$ is CPI inflation, D_t is the state-contingent payoff, R_t is the nominal interest rate, W_t is nominal wage, and T_t is the net transfer from firms and the government.

The optimality conditions for the home household lead to

$$C_{H,t} = (1-\alpha)\, p_{H,t}^{-\eta} C_t,$$

$$C_{F,t} = \alpha\left(p_{F,t}\right)^{-\eta} C_t,$$

$$C_t = \frac{\psi}{1-\psi}\left(1-N_t\right) w_t,$$

$$1 = \mathbf{E}_t m_{t,t+1}\frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta\left(\mathbf{E}_t V_{t+1}^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} V_{t+1}^{\sigma-\varepsilon}\frac{C_{t+1}^{\psi(1-\sigma)-1}\left(1-N_{t+1}\right)^{(1-\psi)(1-\sigma)}}{C_t^{\psi(1-\sigma)-1}\left(1-N_t\right)^{(1-\psi)(1-\sigma)}},$$

where $w_t := W_t/P_t$.

A representative household in the foreign country faces a symmetric utility maximization problem to the one in the home country.

2.2 Firms

In the home country, each firm, indexed by j, produces one kind of differentiated goods $Y_t(j)$ by choosing a cost-minimizing labor input $N_t(j)$, given the real wage w_t, subject to the production function:

$$Y_t(j) = A_{W_t} A_t N_t\left(j\right),$$

where A_t is a stationary and country-specific technology shock, and $A_{W,t}$ is a non-stationary worldwide technology component that grows at a constant rate γ, i.e.,

$$\frac{A_{W,t}}{A_{W,t-1}} = \gamma.$$

Firms set prices of their products on a staggered basis à la Calvo (1983). In each period, a fraction $1-\theta \in (0,1)$ of firms reoptimizes prices, while the remaining fraction θ indexes prices to a weighted average of the past inflation rate for the domestically produced goods $\pi_{H,t-1} := P_{H,t-1}/P_{H,t-2}$ and the steady-state inflation rate $\bar{\pi}$. Then, firms that reoptimize prices in the current period maximize their expected profit

Each firm sets its price in a monopolistically competitive market to maximize the present discounted value of their profits:

$$\mathbf{E}_t \sum_{n=0}^{\infty} \theta^n m_{t,t+n} \left[\frac{P_{H,t}(j)}{P_{t+n}} \prod_{i=1}^{n} \left(\bar{\pi}^{1-\iota} \pi_{H,t+i-1}^{\iota} \right) - \frac{w_{t+n}}{A_{W_{t+n}} A_{t+n}} \right] Y_{t+n}(j),$$

subject to the firm-level resource constraint

$$Y_t(j) = C_{H,t}(j) + G_{H,t}(j) + C_{H,t}^{*}(j),$$

and the downward sloping demand curves, which are obtained from the households' optimization problems,

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\mu} (C_{H,t} + G_t),$$

$$C_{H,t}^{*}(j) = \left[\frac{P_{H,t}^{*}(j)}{P_{H,t}^{*}} \right]^{-\mu} C_{H,t}^{*} = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\mu} C_{H,t}^{*},$$

where $\iota \in [0,1]$ denotes the weight of price indexation to past inflation relative to steady-state inflation, $C_{H,t}^{*}$ is export of the domestically produced goods, $P_{H,t}$ is the producer price index, $P_{H,t}^{*}$ is the export price of the domestically produced goods in the foreign currency, G_t ($G_{H,t}(j)$) is an external demand component other than consumption.[6] The last equality holds because we assume the law of one price:

$$P_{H,t}(j) = e_t P_{H,t}^{*}(j),$$

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency).

Let $p_{H,t} = P_{H,t}/P_t$. Then, $\pi_{H,t} := P_{H,t}/P_{H,t-1}$ can be expressed as

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}}.$$

Moreover, with the auxiliary variables F_t and K_t, the optimal pricing decision can be written in the recursive form:

$$F_t = \frac{1}{2} p_{H,t} \left(C_{H,t} + G_t + C_{H,t}^{*} \right) + \theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota} \pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{1-\mu} F_{t+1},$$

$$K_t = \frac{1}{2} \frac{\mu}{\mu - 1} \frac{w_t}{A_t} \left(C_{H,t} + G_t + C_{H,t}^{*} \right) + \theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota} \pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{-\mu} K_{t+1}.$$

Under the present price-setting rule, the inflation rate for the domestically produced

[6] We assume that only domestically produced goods are used for external demand.

goods $\pi_{H,t}$ can be related to these auxiliary variables by

$$\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{1}{1-\mu}} F_t = K_t.$$

By aggregating the firm-level resource constraint, we have

$$Y_t = \Delta_t \left(C_{H,t} + G_t + C_{H,t}^*\right),$$

where $\Delta_t := \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\mu} dj$ represents price dispersion across firms. The price dispersion term evolves according to

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{\mu}{\mu-1}} + \theta\left(\frac{\pi_{H,t}}{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}\right)^{\mu}\Delta_{t-1}.$$

To specify measurement equations in the subsequent section, we define the output growth rate YGR_t:

$$YGR_t := \frac{Y_t}{Y_{t-1}}.$$

Foreign firms' profit maximization problems are symmetric to those presented above.

2.3 Monetary policy

The monetary authority in the home country adjusts the nominal interest Monetary policy in response to deviations of inflation and output growth from their steady state values.

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r)\left[\phi_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \log\left(\frac{Y_t}{\gamma Y_{t-1}}\right)\right] + \log(\varepsilon_{R,t}).$$

where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, and $\phi_\pi, \phi_y \geq 0$ are the degrees of monetary policy responses to inflation and output growth. $\varepsilon_{R,t}$ is an exogenous shock interpreted as an unsystematic component of monetary policy.

The monetary authority in the foreign country also controls the nominal interest rate following the same type of monetary policy rule.

2.4 Exchange rate and international linkage

Recall that the law of one price holds for prices of domestically produced goods:

$$P_{H,t} = e_t P_{H,t}^*,$$

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Note that * indicates variables in the foreign currency. We define the real exchange rate s_t as

$$s_t = \frac{e_t P_t^*}{P_t},$$

where P_t^* is the foreign aggregate price in the foreign currency. Let $p_{H,t} = P_{H,t}/P_t$ and $p_{H,t} = s_t p_{H,t}^*$. Then, we have

$$p_{H,t} = s_t p_{H,t}^*.$$

Similarly, we can obtain

$$p_{F,t} = s_t p_{F,t}^*.$$

From the definition of the real exchange rate, we have a expression for the nominal exchange rate depreciation d_t:

$$d_t := \frac{e_t}{e_{t-1}} = \frac{s_t \pi_t}{s_{t-1}\pi_t^*}.$$

Regarding the international asset market, the value of the asset in the foreign currency is given by

$$\mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}^* e_{t+1}]/e_t.$$

Thus, under the perfect risk sharing, the stochastic discount factor in the foreign currency $m_{t,t+1}^*$ must satisfy

$$\frac{m_{t,t+1}^*}{\pi_{t+1}^*} = \frac{m_{t,t+1}}{\pi_{t+1}} \frac{e_{t+1}}{e_t}.$$

Substituting the optimality conditions for the home and foreign households to this equation, we have

$$\begin{aligned} &\left(\frac{\left(V_{t+1}^*\right)^{1-\varepsilon} \mathbf{E}_t V_{t+1}^{1-\varepsilon}}{V_{t+1}^{1-\varepsilon} \mathbf{E}_t \left(V_{t+1}^*\right)^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} \left[\frac{C_{t+1}^{\psi} \left(1-N_{t+1}\right)^{(1-\psi)}}{\left(C_{t+1}^*\right)^{\psi} \left(1-N_{t+1}^*\right)^{(1-\psi)}}\right]^{1-\sigma} \frac{C_{t+1}^*}{C_{t+1}} s_{t+1} \\ = &\left[\frac{C_t^{\psi} \left(1-N_t\right)^{(1-\psi)}}{\left(C_t^*\right)^{\psi} \left(1-N_t^*\right)^{(1-\psi)}}\right]^{1-\sigma} \frac{C_t^*}{C_t} s_t. \end{aligned} \tag{1}$$

Let us denote

$$Q_t = \left[\frac{C_t^{\psi} \left(1-N_t\right)^{(1-\psi)}}{\left(C_t^*\right)^{\psi} \left(1-N_t^*\right)^{(1-\psi)}}\right]^{1-\sigma} \frac{C_t^*}{C_t} s_t. \tag{2}$$

Then, equation (1) can be written as

$$Q_{t+1} = Q_t \left(\frac{\left(V_{t+1}^*\right)^{1-\varepsilon} \mathbf{E}_t \left(V_{t+1}^{1-\varepsilon}\right)}{V_{t+1}^{1-\varepsilon} \mathbf{E}_t \left[\left(V_{t+1}^*\right)^{1-\varepsilon}\right]}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}}, \tag{3}$$

where we assume that $Q_0 = 1$, implying that the initial state-contingent wealth equalizes the marginal utilities across countries. If the preferences were non-recursive, i.e., $\sigma = \varepsilon$, then $Q_t = 1$ for all t, and hence the risk-sharing condition would be reduced to the one characterized by equation (2) with $Q_t = 1$. Thus, we regard equation (2) as the international risk-sharing condition and introduce a shock Ω_t to this condition as follows:

$$\Omega_t Q_t = \left[\frac{C_t^{\psi} (1 - N_t)^{(1-\psi)}}{(C_t^*)^{\psi} (1 - N_t^*)^{(1-\psi)}} \right]^{1-\sigma} \frac{C_t^*}{C_t} s_t, \tag{4}$$

Here, Ω_t works as the time-varying financial frictions considered in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017).

2.5 Exogenous shocks

The following variables are exogenous in the model: country-specific technology A_t, external demand g_t, monetary policy shock $\varepsilon_{R,t}$ in the home country, the corresponding foreign variables A_t^*, g_t^*, $\varepsilon_{R,t}^*$, and the risk-sharing shock Ω_t. The stochastic processes for these variables are given by

$$\begin{aligned}
\log(A_t) &= \rho_A \log(A_{t-1}) + \sigma_{A,t} u_{A,t}, \\
\log(g_t) &= (1 - \rho_g) \log \bar{g} + \rho_g \log(g_{t-1}) + \sigma_{g,t} u_{g,t}, \\
\log(\varepsilon_{R,t}) &= \sigma_{\varepsilon_R,t} u_{\varepsilon_R,t}, \\
\log(A_t^*) &= \rho_A^* \log\left(A_{t-1}^*\right) + \sigma_{A,t}^* u_{A,t}^*, \\
\log(g_t^*) &= \left(1 - \rho_g^*\right) \log \bar{g} + \rho_g^* \log\left(g_{t-1}^*\right) + \sigma_{g,t}^* u_{g,t}^*, \\
\log(\varepsilon_{R,t}^*) &= \sigma_{\varepsilon_R,t}^* u_{\varepsilon_R,t}^*, \\
\log(\Omega_t) &= \rho_\Omega \log(\Omega_{t-1}) + \sigma_{\Omega,t} u_{\Omega,t},
\end{aligned}$$

where $\rho_A, \rho_g, \rho_A^*, \rho_g^*, \rho_\Omega \in [0, 1)$ are the autoregressive parameters and $u_{A,t}, u_{g,t}, u_{\varepsilon_R,t}, u_{A,t}^*, u_{g,t}^*, u_{\varepsilon_R,t}^*, u_{\Omega,t} \sim$ i.i.d. $N(0, 1)$ are disturbances to the exogenous processes.

The stochastic processes for the volatilities of the shocks are given by

$$\begin{aligned}
\log(\sigma_{A,t}) &= (1 - \rho_{\sigma_A}) \log(\sigma_A) + \rho_{\sigma_A} \log(\sigma_{A,t-1}) + \tau_A z_{\sigma_A,t}, \\
\log(\sigma_{g,t}) &= \left(1 - \rho_{\sigma_g}\right) \log(\sigma_g) + \rho_{\sigma_g} \log(\sigma_{g,t-1}) + \tau_g z_{\sigma_g,t}, \\
\log(\sigma_{\varepsilon_R,t}) &= \left(1 - \rho_{\sigma_{\varepsilon_R}}\right) \log(\sigma_{\varepsilon_R}) + \rho_{\sigma_{\varepsilon_R}} \log(\sigma_{\varepsilon_R,t-1}) + \tau_{\varepsilon_R} z_{\sigma_{\varepsilon_R},t}, \\
\log\left(\sigma_{A,t}^*\right) &= \left(1 - \rho_{\sigma_A}^*\right) \log(\sigma_A^*) + \rho_{\sigma_A}^* \log\left(\sigma_{A,t-1}^*\right) + \tau_A^* z_{\tau_A,t}^*, \\
\log\left(\sigma_{g,t}^*\right) &= \left(1 - \rho_{\sigma_g}^*\right) \log\left(\sigma_g^*\right) + \rho_{\sigma_g}^* \log\left(\sigma_{g,t-1}^*\right) + \tau_g^* z_{\sigma_g,t}^*, \\
\log\left(\sigma_{\varepsilon_R,t}^*\right) &= \left(1 - \rho_{\sigma_{\varepsilon_R}}^*\right) \log\left(\sigma_{\varepsilon_R}^*\right) + \rho_{\sigma_{\varepsilon_R}}^* \log\left(\sigma_{\varepsilon_R,t-1}^*\right) + \tau_{\varepsilon_R}^* z_{\sigma_{\varepsilon_R},t}^*, \\
\log(\sigma_{\Omega,t}) &= (1 - \rho_{\sigma_\Omega}) \log(\sigma_\Omega) + \rho_{\sigma_\Omega} \log(\sigma_{\Omega,t-1}) + \tau_\Omega z_{\sigma_\Omega,t}.
\end{aligned}$$

where $\rho_{\sigma_A}, \rho_{\sigma_g}, \rho_{\sigma_{\varepsilon_R}}, \rho_{\sigma_A}^*, \rho_{\sigma_g}^*, \rho_{\sigma_{\varepsilon_R}}^*, \rho_{\sigma_\Omega} \in [0, 1)$ are the autoregressive parameters, $z_{\sigma_A}, z_{\sigma_g}, z_{\sigma_{\varepsilon_R}}, z_{\sigma_A}^*, z_{\sigma_g}^*, z_{\sigma_{\varepsilon_R}}^*, z_{\sigma_\Omega} \sim$ i.i.d. $N(0, 1)$ are the innovation to the stochastic volatilities, and $\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$ are their respective standard deviations.

2.6 Detrending

To make the model stationary and obtain the steady state, real variables in the home country are detrended by non-stationary worldwide technology component $A_{W,t}$ so that $v_t := V_t/A^{\psi}_{W,t}$, $y_t := Y_t/A_{W,t}$, $c_{H,t} := C_{H,t}/A_{W,t}$, $c_{F,t} := C_{F,t}/A_{W,t}$, $\tilde{w}_t := w_t/A_{W,t}$, and $g_t := G_t/A_{W,t}$. Foreign variables are also detrended in the same manner.

The Steady-state conditions in terms of detrended variables are presented in Appendix A, whereas the detrended system of equations are shown in Appendix B.

3 Solution and Estimation Methods

The model is solved using perturbation methods up to the third-order approximation in order to take account of the stochastic volatilities in the fundamental shocks. To ensure stability, we employ the pruning method developed by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018).

We estimate the model using a full-information Bayesian approach. Because the model is no longer linear, the standard Kalman filter is not applicable to evaluate the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).[7]

To approximate the posterior distribution, this paper exploits the generic Sequential Monte Carlo (SMC) algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015).[8] In the algorithm, a sequence of tempered posteriors are defined as

$$\varpi_n(\vartheta) = \frac{[p(X^T|\vartheta)]^{\tau_n} p(\vartheta)}{\int [p(X^T|\vartheta)]^{\tau_n} p(\vartheta) d\vartheta}, \quad n = 0, ..., N_\tau.$$

The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\chi$, where χ is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws and associated importance weights—which are called particles—from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; that is, at each stage, $\varpi_n(\vartheta)$ is represented by a swarm of particles $\{\vartheta_n^{(i)}, w_n^{(i)}\}_{i=1}^N$, where N denotes the number of particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\vartheta_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ through importance sampling.[9] Posterior inferences about parameters to be estimated are made based on the particles $\{\vartheta_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(X^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^N \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where $\tilde{w}_n^{(i)}$ is the incremental weight defined as $\tilde{w}_n^{(i)} = [p(X^T|\vartheta_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$. In the subse-

[7]Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

[8]Creal (2007) is the first that applied the SMC methods to the estimation of DSGE models.

[9]This process includes one step of a single-block RWMH algorithm.

quent empirical analysis, the SMC algorithm uses $N = 2,000$ particles and $N_\tau = 50$ stages. The parameter that controls the tempering schedule is set at $\chi = 2$ following Herbst and Schorfheide (2014, 2015).

Seven quarterly time series ranging from 1987Q1 to 2008Q4 are used for estimation: the per-capita real GDP growth rate ($100\Delta \log GDP_t, 100\Delta \log GDP_t^*$), the inflation rate of the GDP implicit price deflator ($100\Delta \log PGDP_t, 100\Delta \log PGDP_t^*$), and the three-month nominal interest rate(INT_t, INT_t^*), in the US and the Euro Area, and the nominal exchange rate depreciation of the US dollar against the Euro ($100\Delta \log EXR_t$). The construction of the data basically follows from Lubik and Schorfheide (2006): The US data are extracted from the FRED database maintained by the Federal Reserve Bank of St. Louis, whereas the Euro Area data and the exchange rate series are taken from the Area-Wide Model (AWM) database of the European Central Bank.[10] The observation equations that relate the data to model variables are given by

$$\begin{bmatrix} 100\Delta \log GDP_t \\ 100\Delta \log PGDP_t \\ INT_t \\ 100\Delta \log GDP_t^* \\ 100\Delta \log PGDP_t^* \\ INT_t^* \\ 100\Delta \log EXR_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ 0 \end{bmatrix} + \begin{bmatrix} 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100\hat{d}_t \end{bmatrix},$$

where $\bar{\gamma} = 100(\gamma - 1)$, $\bar{\pi} = 100(\pi - 1)$, $\bar{r} = 100(R - 1)$, and the hatted variables on the right hand side denote the log deviations from their steady-state values.

Before estimation, parameters regarding the share of foreign goods, the elasticity of substitution between home and foreign goods, the elasticity of substitution across the goods within each country, the share of external demand, the steady-state growth, inflation, and interest rates, and relative risk aversion are fixed at $\bar{\gamma} = 0.346$, $\bar{\pi} = 0.639$, $\bar{r} = 1.274$, $\bar{g}/\bar{y} = 0.18$, $\alpha = 0.13$, $\eta = 1.5$, $\mu = 6$, $\psi = 0.333$, $\epsilon = \epsilon^* = 5$, respectively, to avoid an identification issue. The values for $\bar{\gamma}$, $\bar{\pi}$, $\bar{r}$, and $\bar{g}/\bar{y}$ are set at the sample means of the corresponding data across the two countries so that the ergodic means of the model-implied observables tend to be close the sample means. The other parameter values are chosen based on the calibration in BBN. All the other parameters are estimated; their prior distributions are shown in Table 1. The priors are set according to those used in Smets and Wouters (2007) and the calibrated values in BBN. For the standard deviations of the stochastic volatilities ($\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$), the prior mean is set in line with the upper bound of the estimated standard deviation of the stochastic volatility regarding the technology shock reported in Fernández-Villaverde and Rubio-Ramírez (2007).

[10] For the nominal exchange rate series for the period prior to the introduction of the Euro in 1999, the USD-ECU (Euroepan Currency Unit) exchange rate is used.

Table 1: Prior distributions of parameters

Parameter	Distribution	Mean	S.D.
ε	Gamma	5.000	0.500
σ	Gamma	2.000	0.250
θ	Beta	0.667	0.100
ι	Beta	0.500	0.150
θ^*	Beta	0.667	0.100
ι^*	Beta	0.500	0.150
ϕ_r	Beta	0.750	0.100
ϕ_π	Gamma	1.500	0.200
ϕ_y	Gamma	0.125	0.050
ϕ_r^*	Beta	0.750	0.100
ϕ_π^*	Gamma	1.500	0.200
ϕ_y^*	Gamma	0.125	0.050
ρ_A	Beta	0.500	0.150
ρ_g	Beta	0.500	0.150
ρ_A^*	Beta	0.500	0.150
ρ_g^*	Beta	0.500	0.150
ρ_Ω	Beta	0.500	0.150
ρ_{σ_A}	Beta	0.500	0.150
ρ_{σ_g}	Beta	0.500	0.150
$\rho_{\sigma_{\epsilon_R}}$	Beta	0.500	0.150
$\rho_{\sigma_A}^*$	Beta	0.500	0.150
$\rho_{\sigma_g}^*$	Beta	0.500	0.150
$\rho_{\sigma_{\epsilon_R}}^*$	Beta	0.500	0.150
ρ_{σ_Ω}	Beta	0.500	0.150
$100\sigma_A$	Inverse Gamma	5.000	2.590
$100\sigma_g$	Inverse Gamma	5.000	2.590
$100\sigma_{\epsilon_R}$	Inverse Gamma	0.500	0.260
$100\sigma_A^*$	Inverse Gamma	5.000	2.590
$100\sigma_g^*$	Inverse Gamma	5.000	2.590
$100\sigma_{\epsilon_R}^*$	Inverse Gamma	0.500	0.260
$100\sigma_\Omega$	Inverse Gamma	5.000	2.590
τ_A	Inverse Gamma	1.000	0.517
τ_g	Inverse Gamma	1.000	0.517
τ_{ϵ_R}	Inverse Gamma	1.000	0.517
τ_A^*	Inverse Gamma	1.000	0.517
τ_g^*	Inverse Gamma	1.000	0.517
$\tau_{\epsilon_R}^*$	Inverse Gamma	1.000	0.517
τ_Ω	Inverse Gamma	1.000	0.517

4 Results

We first report the estimation result and then discuss how our model can account for aggregate fluctuations in open economies including exchange rate dynamics as observed in the data.

4.1 Parameter estimates

Table 2 and 3 reports the posterior estimates of parameters. For the purpose of comparison, a linearized version of the model and the models without the stochastic volatilities approximated up to the second and third order are also estimated. For each model, the posterior mean and 90 percent highest posterior density intervals for the estimated parameters are presented as well as the SMC-based approximation of log marginal data density $\log p(\mathcal{Y}^T)$.[11]

The marginal data densities $\log p(\mathcal{Y}^T)$ indicate that the higher-order approximation of the model does not contribute to improving the fit of the model to the data and the baseline model with stochastic volatilities deteriorates the fit even further. This is because the higher-order approximation with stochastic volatilities imposes such tighter cross-equation restrictions that do not necessarily improve the empirical performance of the model.

While the structural parameters are not much different across the four models, remarkable differences arise in the parameters related to shocks. First, as the degree of approximation is higher, the AR(1) coefficients for structural shocks tend to be smaller. In particular, the third-order approximation with stochastic volatilities results in substantially smaller estimates than the others, except for the coefficient on the technology shock in the Euro area ρ_A^*. Even in the baseline model (the third-order approximation with stochastic volatilities), however, the coefficient on the risk-sharing shock ρ_Ω is very large and close to unity. Second, the standard deviations of the structural shocks are not much different across the four models except for that of the risk-sharing shock, which is substantially lower in the baseline model.

Our model shares many similarities with the one in BBN and the same structural parameters appear in the two models. Table 4 compares our posterior mean estimates of parameters with the parameter values calibrated in BBN. While most of the parameters are very similar, the monetary policy responses to output growth are different across the two models. However, we have confirmed that the differences in these parameters do not lead to any qualitative differences in our main results presented below.

4.2 Impulse responses

This subsection demonstrates that our estimated model can mostly account for empirical regularities regarding volatility shocks as shown by BBN: (1) an increase in the volatility of the productivity shock induces an exchange rate depreciation; (2) an increase in the

[11] The risk aversion parameter ε in the recursive preferences does not appear in the linearized version of the model.

Table 2: Posterior distributions of parameters

	Linear		2nd order	
Parameter	Mean	90% interval	Mean	90% interval
ε	5.127	[4.308, 5.999]	5.007	[4.389, 5.705]
σ	2.184	[1.875, 2.501]	2.180	[1.962, 2.419]
θ	0.594	[0.495, 0.707]	0.710	[0.665, 0.761]
ι	0.193	[0.048, 0.313]	0.143	[0.048, 0.236]
θ^*	0.672	[0.603, 0.748]	0.633	[0.581, 0.680]
ι^*	0.119	[0.030, 0.199]	0.140	[0.047, 0.234]
ϕ_r	0.790	[0.754, 0.831]	0.817	[0.785, 0.850]
ϕ_π	1.946	[1.715, 2.190]	1.947	[1.703, 2.160]
ϕ_y	0.274	[0.164, 0.383]	0.207	[0.139, 0.275]
ϕ_r^*	0.768	[0.717, 0.815]	0.771	[0.732, 0.816]
ϕ_π^*	2.017	[1.812, 2.244]	2.113	[1.911, 2.307]
ϕ_y^*	0.249	[0.147, 0.347]	0.207	[0.130, 0.288]
ρ_A	0.667	[0.494, 0.813]	0.652	[0.560, 0.732]
ρ_g	0.943	[0.910, 0.977]	0.839	[0.786, 0.884]
ρ_A^*	0.618	[0.530, 0.722]	0.551	[0.453, 0.643]
ρ_g^*	0.954	[0.927, 0.979]	0.968	[0.947, 0.989]
ρ_Ω	0.997	[0.995, 0.999]	0.997	[0.996, 0.999]
$100\sigma_A$	2.138	[1.337, 2.969]	3.003	[2.126, 3.868]
$100\sigma_g$	8.339	[6.913, 9.566]	8.864	[7.495, 10.060]
$100\sigma_{\epsilon_R}$	0.159	[0.135, 0.185]	0.154	[0.133, 0.176]
$100\sigma_A^*$	2.980	[1.916, 4.115]	2.781	[2.055, 3.417]
$100\sigma_g^*$	7.781	[6.613, 8.969]	4.706	[4.108, 5.333]
$100\sigma_{\epsilon_R}^*$	0.160	[0.137, 0.185]	0.161	[0.140, 0.183]
$100\sigma_\Omega$	6.885	[6.059, 7.711]	8.591	[7.538, 9.648]
$\log p(\mathcal{Y}^T)$	-673.902		-683.774	

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathcal{Y}^T)$ represents the SMC-based approximation of log marginal data density.

Table 3: Posterior distributions of parameters (cont.)

Parameter	3rd order		3rd order with S.V.		No risk-sharing shock	
	Mean	90% interval	Mean	90% interval	Mean	90% interval
ε	4.388	[4.129, 4.625]	4.331	[3.993, 4.669]	4.139	[3.775, 4.439]
σ	2.615	[2.502, 2.774]	1.879	[1.682, 2.118]	2.427	[2.200, 2.628]
θ	0.708	[0.675, 0.742]	0.525	[0.473, 0.575]	0.521	[0.469, 0.565]
ι	0.140	[0.053, 0.256]	0.340	[0.212, 0.442]	0.587	[0.482, 0.673]
θ^*	0.495	[0.439, 0.539]	0.766	[0.713, 0.827]	0.840	[0.824, 0.858]
ι^*	0.330	[0.260, 0.416]	0.389	[0.248, 0.548]	0.616	[0.471, 0.792]
ϕ_r	0.749	[0.715, 0.793]	0.772	[0.703, 0.836]	0.685	[0.632, 0.725]
ϕ_π	2.208	[2.041, 2.360]	2.103	[1.893, 2.348]	1.803	[1.655, 1.946]
ϕ_y	0.123	[0.096, 0.152]	0.196	[0.164, 0.232]	0.103	[0.069, 0.139]
ϕ_r^*	0.745	[0.714, 0.772]	0.794	[0.733, 0.866]	0.699	[0.655, 0.739]
ϕ_π^*	1.428	[1.329, 1.489]	1.651	[1.462, 1.819]	1.380	[1.245, 1.499]
ϕ_y^*	0.085	[0.054, 0.116]	0.151	[0.099, 0.204]	0.089	[0.056, 0.122]
ρ_A	0.542	[0.456, 0.620]	0.481	[0.363, 0.590]	0.332	[0.126, 0.473]
ρ_g	0.983	[0.965, 1.000]	0.862	[0.757, 0.972]	0.553	[0.356, 0.701]
ρ_A^*	0.562	[0.486, 0.644]	0.822	[0.733, 0.928]	0.930	[0.903, 0.953]
ρ_g^*	0.947	[0.920, 0.988]	0.390	[0.245, 0.507]	0.581	[0.502, 0.649]
ρ_Ω	0.997	[0.995, 0.999]	0.955	[0.927, 0.990]	-	-
ρ_{σ_A}	-	-	0.683	[0.588, 0.780]	0.251	[0.090, 0.373]
ρ_{σ_g}	-	-	0.513	[0.373, 0.692]	0.386	[0.268, 0.512]
$\rho_{\sigma_{\epsilon_R}}$	-	-	0.739	[0.612, 0.882]	0.378	[0.304, 0.462]
$\rho_{\sigma_A}^*$	-	-	0.567	[0.454, 0.710]	0.105	[0.061, 0.146]
$\rho_{\sigma_g}^*$	-	-	0.337	[0.193, 0.461]	0.241	[0.156, 0.335]
$\rho_{\sigma_{\epsilon_R}}^*$	-	-	0.362	[0.189, 0.528]	0.356	[0.196, 0.501]
ρ_{σ_Ω}	-	-	0.389	[0.262, 0.498]	-	-
$100\sigma_A$	2.948	[2.218, 3.630]	2.048	[1.452, 2.528]	1.396	[1.014, 1.728]
$100\sigma_g$	8.108	[6.955, 9.136]	9.235	[8.100, 10.928]	4.616	[3.417, 5.520]
$100\sigma_{\epsilon_R}$	0.217	[0.172, 0.268]	0.144	[0.106, 0.186]	0.200	[0.143, 0.253]
$100\sigma_A^*$	1.749	[1.370, 2.117]	5.293	[4.034, 6.461]	11.140	[9.235, 13.468]
$100\sigma_g^*$	4.038	[3.405, 4.580]	7.734	[6.522, 8.799]	8.034	[6.393, 9.945]
$100\sigma_{\epsilon_R}^*$	0.285	[0.148, 0.430]	0.168	[0.107, 0.223]	0.179	[0.133, 0.227]
$100\sigma_\Omega$	6.589	[5.940, 7.360]	4.652	[3.833, 5.407]	-	[-, -]
τ_A	-	-	0.538	[0.408, 0.674]	1.087	[0.782, 1.427]
τ_g	-	-	0.862	[0.545, 1.115]	1.227	[0.851, 1.573]
τ_{ϵ_R}	-	-	1.339	[1.016, 1.686]	0.736	[0.570, 0.888]
τ_A^*	-	-	0.720	[0.582, 0.877]	0.987	[0.894, 1.121]
τ_g^*	-	-	1.162	[0.972, 1.338]	1.430	[1.142, 1.725]
$\tau_{\epsilon_R}^*$	-	-	1.287	[1.032, 1.553]	1.245	[0.930, 1.591]
τ_Ω	-	-	0.635	[0.486, 0.774]	-	-
$\log p(\mathcal{Y}^T)$	-775.060		-807.321		-919.449	

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathcal{Y}^T)$ represents the SMC-based approximation of log marginal data density.

Table 4: Comparison to BBN

Parameter	Our estimate	BBN's calibration
ε	4.33	5.00
σ	1.88	2.00
θ	0.53	0.66
θ^*	0.77	0.75
ϕ_r	0.77	0.76
ϕ_π	2.10	1.41
ϕ_y	0.20	0.66
ϕ_{r^*}	0.79	0.84
ϕ_{π^*}	1.65	1.37
ϕ_{y^*}	0.15	1.27

volatility of the monetary policy shock induces an exchange rate appreciation; and (3) an increase in the volatility of the monetary policy shock causes deviations from the UIP in the form of an increase in the excess return on the foreign currency.

The figures 1–2 show the impulse responses of the observed variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$) to the volatility shocks to home technology and home monetary policy, at the ergordic mean of state variables, given the posterior mean estimates of parameters in the baseline estimation. While the increase in volatility of home technology causes the appreciation of the exchange rate at the time of the shock, it causes its depreciation thereafter. In response to the volatility shock to home monetary policy, the exchange rate appreciates and the deviation from the UIP turns to be positive as demonstrated by BBN.

As shown in the next subsection, our estimation results indicates the importance of the risk-sharing shock in explaining the exchange rate fluctuations. The figure 3 depicts the impulse responses to the shock to the risk-sharing condition. This shock directly causes the depreciation of the dollar at the time of the shock, followed by the persistent negative deviation from the UIP in the form of a decrease in the excess return on the foreign currency. This shock also affects other macroeconomic variables to a substantial degree.

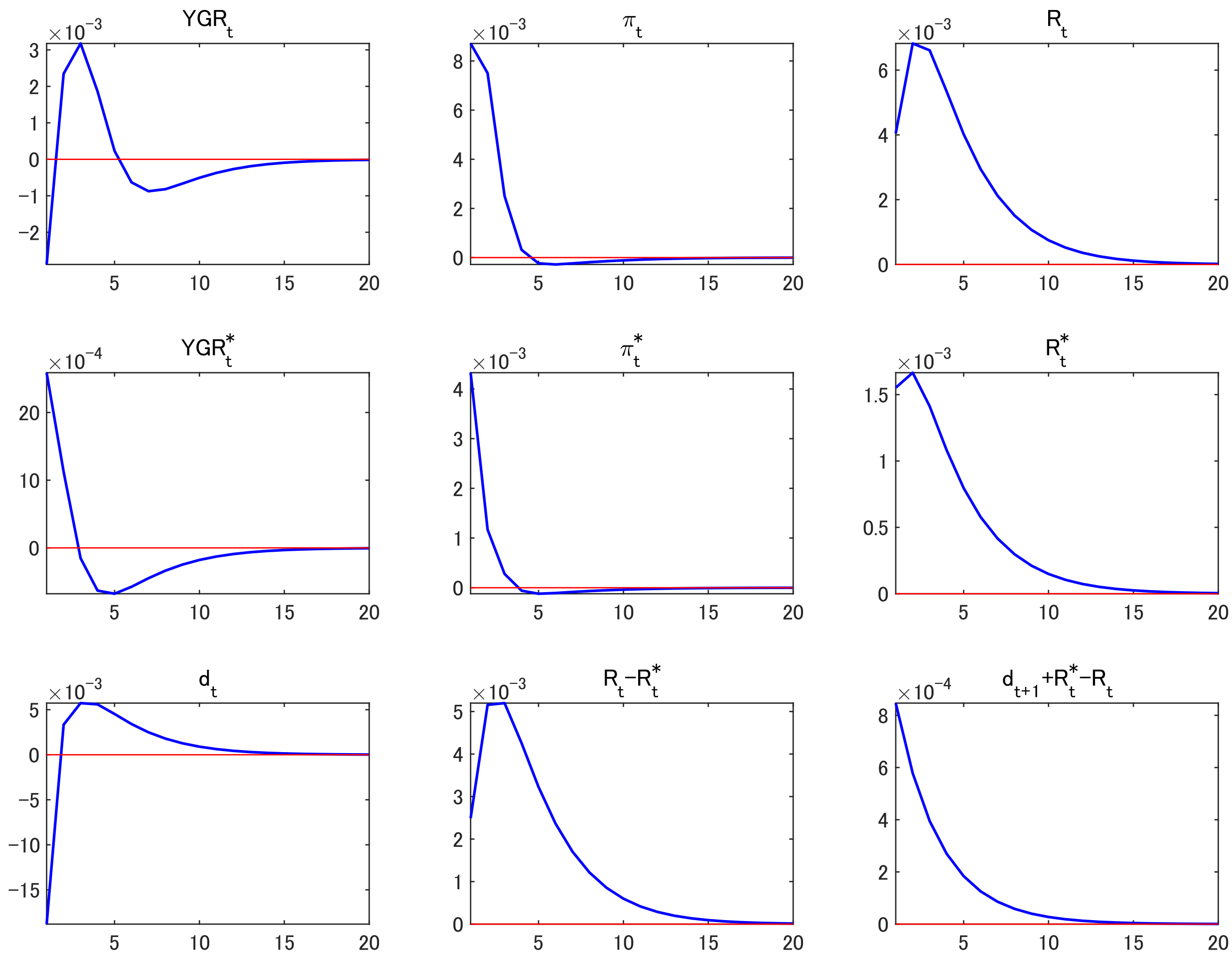
The impulse responses to the other shocks are presented in the Appendix.

4.3 Accounting for exchange rate dynamics

The log-linear approximation of the equilibrium conditions leads to the UIP condition incorporated with the risk-sharing shock Ω_t:

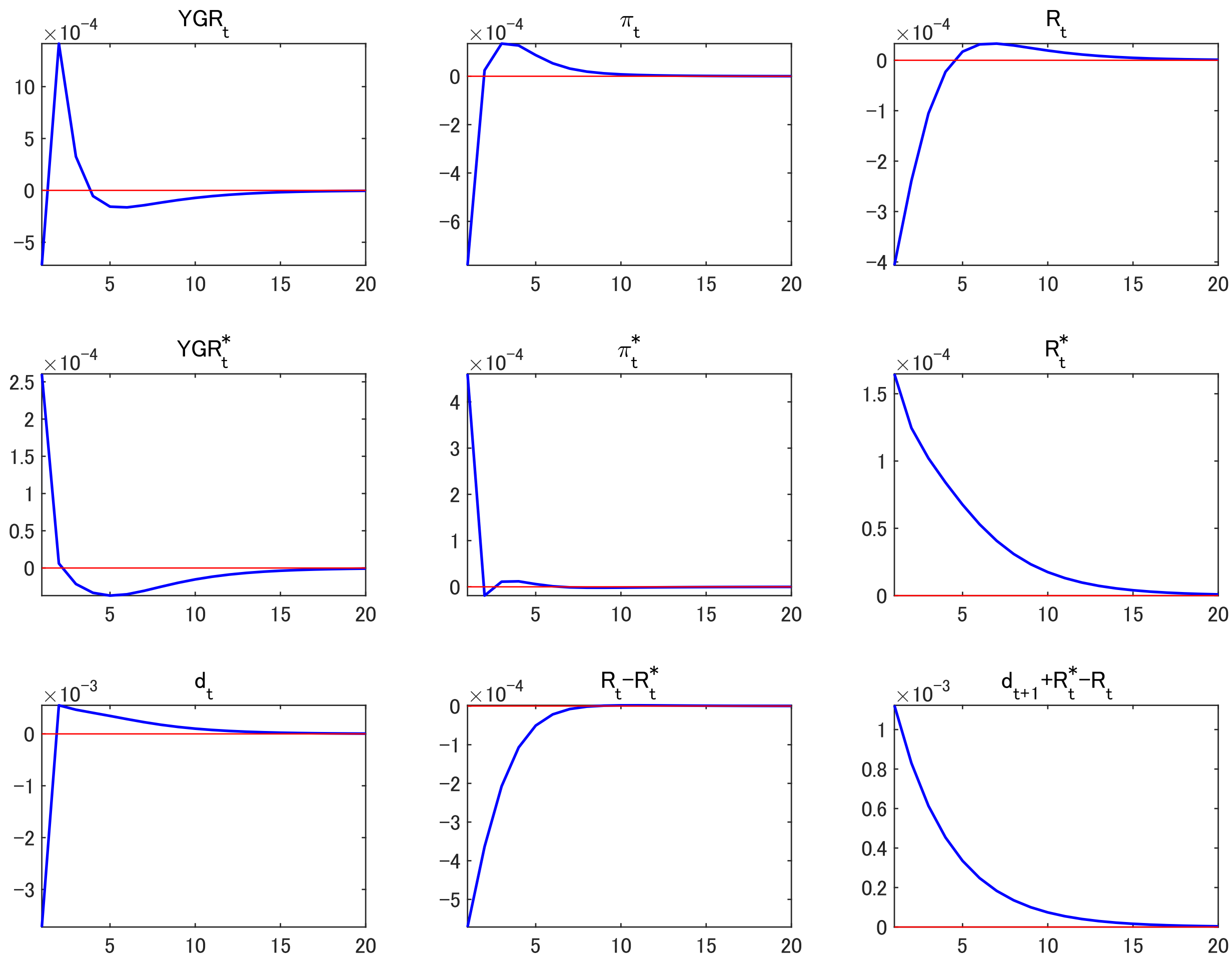
$$\hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} + \hat{\Omega}_t - \mathbf{E}_t \hat{\Omega}_{t+1}, \tag{5}$$

Figure 1: Responses to volatility shock to home technology



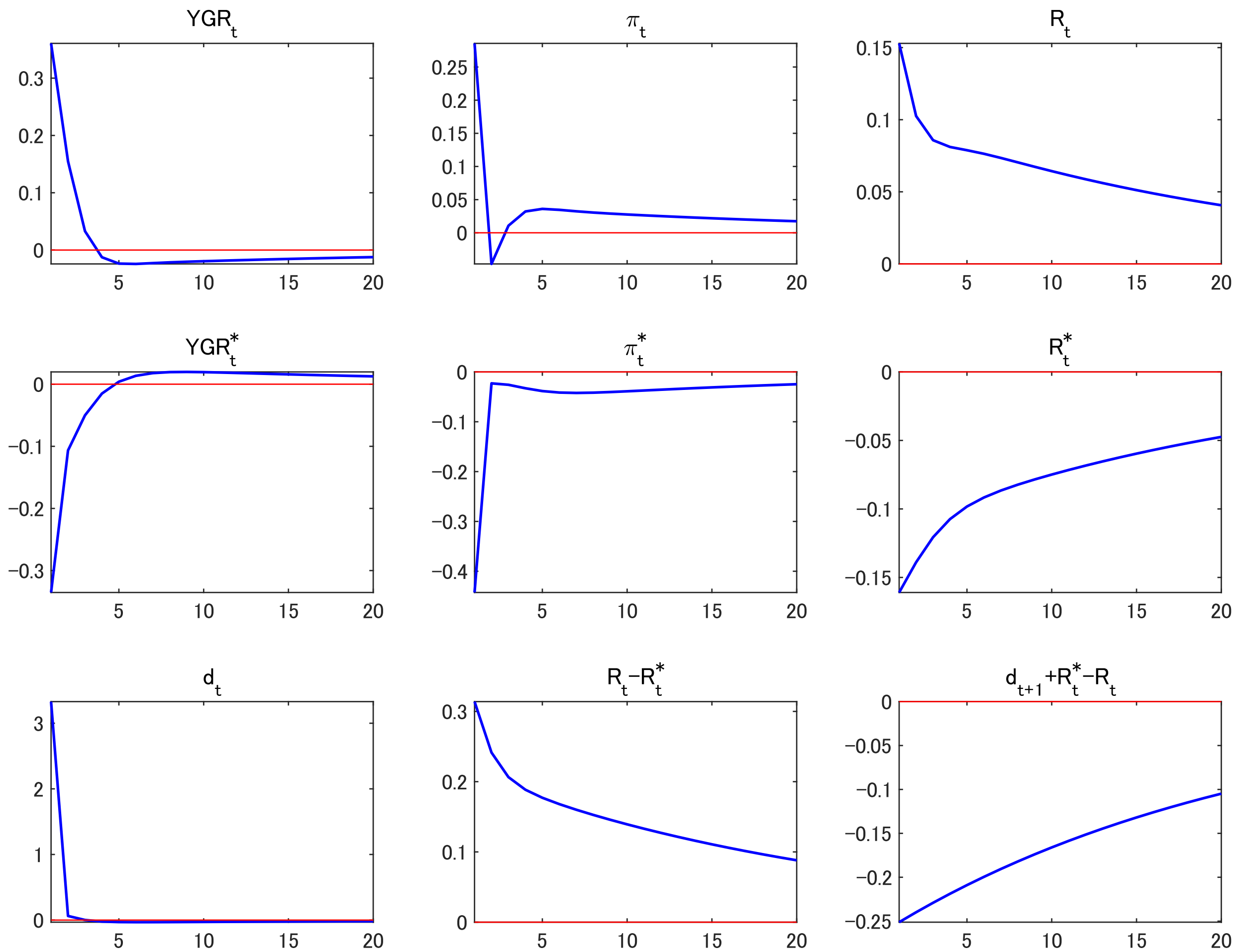
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 2: Responses to volatility shock to home monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 3: Responses to risk sharing shock



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.

where the circumflex denotes the log deviation from the steady state value. Then, the deviations from the UIP are captured by the risk-sharing shock. If data suggest sizable deviations from the UIP, the contribution of the risk-sharing shock to the exchange rate dynamics will be large. On the other hand, with the higher order approximation, the international risk-sharing condition is given by equations (4) with the time-varying Q_t which evolves according to (3). As a result, the deviation from the UIP can be partly captured by higher-order terms such as the endogenous risk premium and thus the contribution of the risk-sharing shock to exchange rate fluctuations is expected to decrease, compared with the linear case.

To examine this point, Table 5 shows the relative variances of the observables, i.e., the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation, excluding each shock,[12] given the posterior mean estimates of parameters in the estimation of a linearized version of the model, the models without the stochastic volatilities solved by the second- and third-order approximation, and the baseline model (with stochastic volatilities solved by the third-order approximation.)[13] Each number shows how much of fluctuations can be explained by excluding each structural shocks shown in the left column: u_A, u_g, u_{ϵ_R}, u_A^*, u_g^*, $u_{\epsilon_R}^*$, u_Ω, $z_{\sigma A}$, $z_{\sigma g}$, $z_{\sigma \epsilon_R}$, $z_{\sigma A}^*$, $z_{\sigma g}^*$, $z_{\sigma \epsilon_R}^*$, and $z_{\sigma\Omega}$ denotes the shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, and the volatility shocks to the respective shocks. Regarding the changes in nominal exchange rates d_t, the linear model excluding the international risk-sharing shock can explain only 14% of its volatility, implying that 86% of the exchange rate fluctuations are driven by the risk-sharing shock. While the second-order approximation does not contribute to the increase in the relative variance of the exchange rate, the third-order approximation without stochastic volatilities lead to the larger variance of 28% by taking account of non-linearities in the model. In the case with stochastic volatilities, the model can explain 43% of exchange rate fluctuations even if the shock to the risk-sharing condition is excluded. This result is consistent with BBN's argument; that is, time variation in uncertainty and risk can be an important source of exchange rate fluctuations. We, however, find that the risk-sharing shock is still a major source of the exchange rate dynamics, accounting for more than half (57%) of its fluctuations.

The finding above hints the importance of financial frictions as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017), who argue that frictions in financial transactions need to be incorporated to hinder the international arbitrage in account for exchange rate dynamics and aggregate fluctuations in open economies. The shock to the risk-sharing condition in the present model captures such frictions in a time-varying manner. To confirm the importance of the risk-sharing shock, we exclude this shock from the baseline model and estimate it. The last two columns of Table 3 show the estimation results without the risk-sharing shock. The price indexation parameters for both countries and several AR(1) coefficients become larger to compensate for missing persistency in the ob-

[12]We did not employ the standard variance decomposition because it can underestimate the contributions of each shock by ignoring cross-terms among shocks in nonlinear settings.

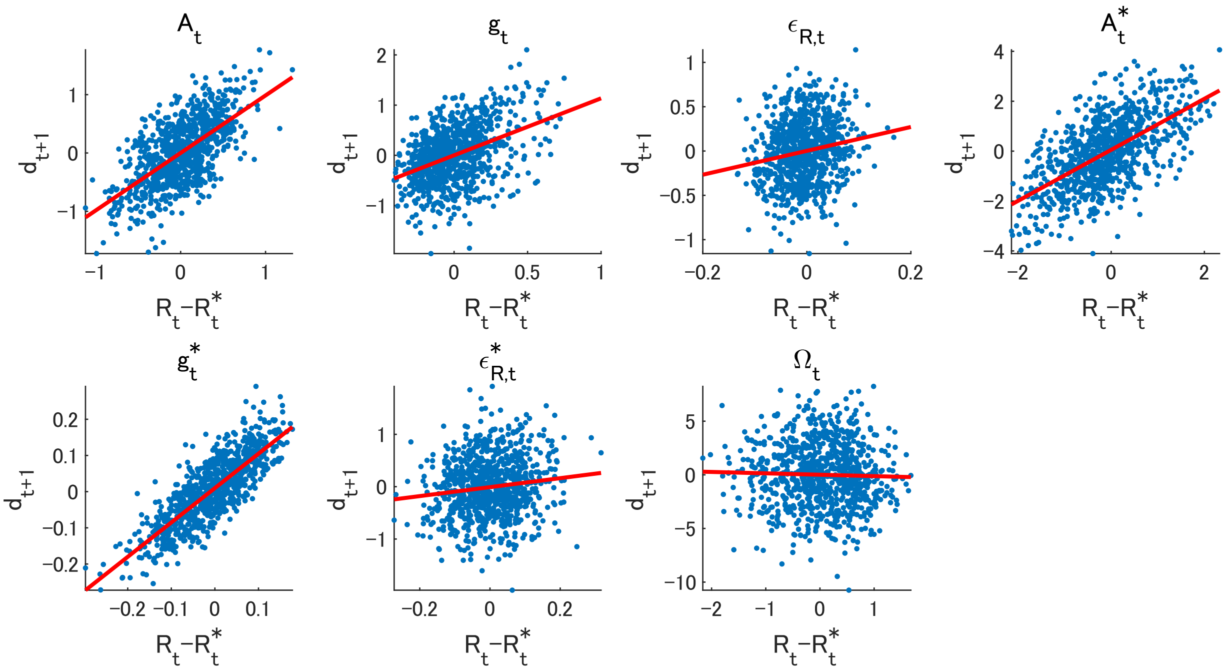
[13]To compute the variances, the model is simulated for 10,100 periods, and the first 100 observations are discarded.

Table 5: Relative variances excluding each shock

		$\Delta \log Y_t$	$\log \pi_t$	$\log R_t$	$\Delta \log Y_t^*$	$\log \pi_t^*$	$\log R_t^*$	d_t
Linear								
w/o:	u_A	0.690	0.280	0.382	0.994	0.954	0.940	0.977
	u_g	0.423	0.962	0.793	1.000	0.992	0.997	0.970
	$u_{\epsilon R}$	0.959	0.920	0.984	1.000	0.996	1.000	0.983
	u_A^*	0.985	0.933	0.919	0.667	0.242	0.307	0.963
	u_g^*	0.999	0.995	0.997	0.456	0.966	0.840	0.968
	$u_{\epsilon R}^*$	1.000	0.996	0.999	0.955	0.952	0.985	0.989
	u_Ω	0.925	0.920	0.940	0.929	0.896	0.921	0.141
2nd order								
w/o:	u_A	0.837	0.331	0.377	0.986	0.952	0.913	0.979
	u_g	0.351	0.943	0.807	1.000	0.992	0.989	0.989
	$u_{\epsilon R}$	0.952	0.934	0.971	1.000	0.992	0.998	0.979
	u_A^*	0.979	0.936	0.951	0.590	0.262	0.270	0.965
	u_g^*	0.999	0.999	1.003	0.717	0.992	0.945	0.988
	$u_{\epsilon R}^*$	1.000	0.997	0.999	0.949	0.947	0.984	0.990
	u_Ω	0.886	0.815	0.880	0.730	0.903	0.923	0.105
3rd order								
w/o:	u_A	0.830	0.315	0.290	0.936	0.935	0.910	0.936
	u_g	0.342	0.946	0.868	0.975	0.985	0.967	0.898
	$u_{\epsilon R}$	0.931	0.925	0.959	0.999	0.995	0.998	0.970
	u_A^*	0.986	0.934	0.942	0.621	0.355	0.394	0.989
	u_g^*	0.999	0.998	1.000	0.765	0.976	0.885	0.991
	$u_{\epsilon R}^*$	0.999	0.988	0.997	0.821	0.790	0.964	0.932
	u_Ω	0.952	0.855	0.917	0.825	0.957	0.865	0.279
3rd order with SV								
w/o:	u_A	0.830	0.523	0.664	0.998	0.984	0.993	0.975
	u_g	0.295	0.932	0.722	1.000	0.994	0.994	0.956
	$u_{\epsilon R}$	0.907	0.673	0.956	1.000	0.996	1.000	0.915
	u_A^*	1.003	0.936	0.777	0.717	0.169	0.108	0.816
	u_g^*	0.999	0.998	0.998	0.338	0.992	0.988	0.997
	$u_{\epsilon R}^*$	1.000	0.985	0.992	0.940	0.927	0.975	0.926
	u_Ω	0.976	0.932	0.859	0.995	0.934	0.941	0.425
	$z_{\sigma A}$	0.905	0.734	0.815	0.998	0.991	0.997	0.984
	$z_{\sigma g}$	0.480	0.952	0.808	1.000	0.996	0.998	0.967
	$z_{\sigma \epsilon R}$	0.913	0.692	0.957	1.000	0.996	1.000	0.920
	$z_{\sigma A}^*$	0.996	0.963	0.871	0.823	0.457	0.427	0.877
	$z_{\sigma g}^*$	0.999	0.999	0.998	0.475	0.994	0.993	0.998
	$z_{\sigma \epsilon R}^*$	1.000	0.987	0.992	0.946	0.938	0.976	0.933
	$z_{\sigma \Omega}$	0.986	0.971	0.940	0.997	0.964	0.959	0.711

Notes: The table shows the variances of the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation excluding each shock, relative to those with all the shocks, given the posterior mean estimates of parameters.

Figure 4: UIP regressions based on simulated series driven by each level shock



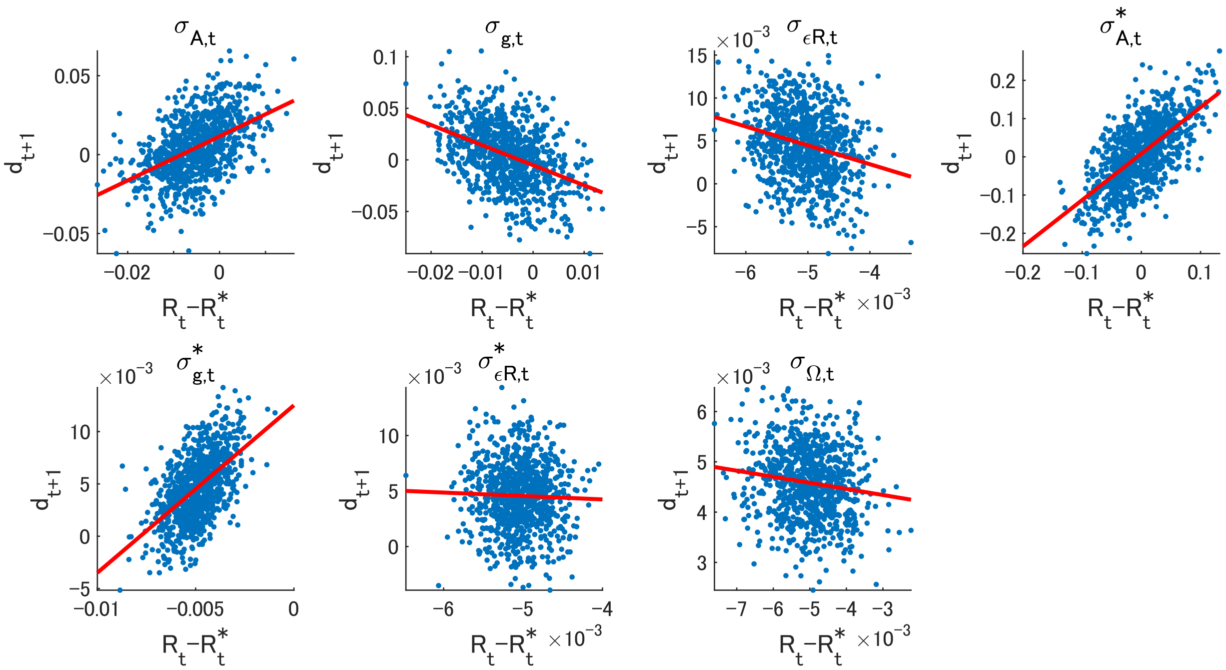
served variables which was captured by large AR(1) coefficient on the risk-sharing shock in the baseline specification. The log marginal data density $\log p(\mathcal{Y}^T)$ is substantially lower (-919.4) than that in the baseline estimation (-807.3), indicating far worse empirical performance. Thus, the risk-sharing shock plays an indispensable role in explaining the variability of observables in the US and the Euro area.

Why do the other shocks than the risk-sharing shock cannot be major sources of exchange rate fluctuations? To answer this question, we construct artificial time-series driven by each single shock and examine which shock can generate the negative correlation between the changes in the nominal exchange rate and nominal interest rate differentials—one of the empirical regularities in exchange rate dynamics, known as a negative slope in the UIP regression.

Figure 4 presents the results of the UIP regressions based on the simulated series of the exchange rate depreciation d_{t+1} and the nominal interest rate differentials $R_t - R_t^*$ driven by each level shock,[14] where A_t, g_t, $\varepsilon_{R,t}$, A_t^*, g_t^*, $\varepsilon_{R,t}^*$, and Ω_t denote the shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, respectively. The figure indicates that no other shocks except for the risk-sharing shock do not replicate any negative correlations between the expected changes in the nominal exchange rate and nominal interest rate differentials. This is indeed the mechanism stemming from the UIP: When nominal interest rates are high, exchange rates will depreciate

[14]Given the posterior mean estimates of parameters in the baseline estimation, the model is simulated for 1,100 periods, and the first 100 observations are discarded.

Figure 5: UIP regressions based on simulated series driven by each volatility shock



for no arbitrage.

The international risk sharing shocks can replicate a slightly negative correlation between them. In addition to this fact, we argue that the persistent dynamics in the risk-sharing shock increases its contribution to explaining the exchange rate fluctuations. As shown in Tables 2 and 3, the mean estimates of the AR(1) coefficient ρ_Ω are 0.99 with the first-, second-, and third-order approximation without stochastic volatilities and 0.96 even in the case of third-order approximation with stochastic volatilities. This is to account for the near random-walk process as observed in the data on exchange rates.

On the other hand, the volatility shocks may possibly replicate negative correlations between the expected changes in the nominal exchange rate and nominal interest rate differentials. As explored in BBN, more uncertainty in nominal shocks makes the home currency a good hedge and at the same time, leads to higher nominal interest rates, *i.e.* more demand for money, in the domestic country. Consequently, the carry trade may yield positive excess returns. The gains from the carry trade compensate for the risk of holding foreign currency to uncertainty in the conduct of monetary policy in the domestic country. BBN also discuss the interactions between monetary policy, price stickiness and stochastic volatilities on exchange rate dynamics. They find that the slope of the UIP regression can be negative when the parameter for price stickiness is small and that for interest rate smoothing is large.

Figure 5 depicts the results of the UIP regressions based on the simulated series of the exchange rate depreciation d_{t+1} and the nominal interest rate differentials $R_t - R_t^*$ driven by each volatility shock, where $\sigma_{A,t}$, $\sigma_{g,t}$, $\sigma_{\varepsilon R,t}$, $\sigma^*_{A,t}$, $\sigma^*_{g,t}$, $\sigma^*_{\varepsilon R,t}$, and $\sigma_{\Omega,t}$ are the volatility shocks to home technology, home external demand, home monetary policy, foreign technology, foreign external demand, foreign monetary policy, and the international risk-sharing condition, respectively. Given our estimated parameters, the volatility shocks to home external demand, home and foreign monetary policy, and risk-sharing condition can replicate slightly negative slopes of the UIP regressions. These volatility shocks partly contribute to explaining the exchange rate dynamics and diminish the role of the risk-sharing shock in accounting for the relative variance of the exchange rate shown in Table 5. According to the impulse responses presented above, however, the effect of the volatility shocks to the exchange rate is quite marginal in magnitude, compared with that of the level shock regarding the international risk-sharing condition. Therefore, the volatility shocks cannot be a major source of exchange rate fluctuations.

5 Conclusion

In this paper, we have estimated the two country New Keynesian model with the recursive preferences and stochastic volatilities using higher order approximation and the central difference Kalman filter. According to the estimation results, the shock to the international risk-sharing condition which represents the time-varying financial frictions that hinder the international arbitrage is a major driver in accounting for the exchange rate dynamics as well as aggregate fluctuations in open economies, whereas several volatility shocks partially contribute to explaining the exchange rate dynamics. Therefore, the exchange rate is in most part disconnected from macroeconomic fundamentals even if we allow for higher-order terms and volatility shocks.

Still several possibilities remain to reduce the importance of the shock to the risk-sharing shock, which is rather *ad-hocly* set in this paper. First, *news shocks* can be an important driver of the aggregate fluctuations as reported in Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012). The stochastic volatilities of news shocks may overturn our main result. Second, the exchange rate may be intrinsically indeterminate as advocated by Kareken and Wallace (1981). Taking account of *sunspot shocks* as in Lubik and Schorfheide (2004) and their volatility shocks must be an interesting challenge.

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A Steady state

To avoid nonstationarity, we need to assume

$$\bar{\pi} = \bar{\pi}^*$$

at the steady state.

We parameterize g/y instead of g. Thus, Then, g that appears in the subsequent steady-state conditions are given by

$$g = \frac{\frac{\psi(\mu-1)}{\mu-\psi}}{\left(\frac{\bar{g}}{\bar{y}}\right)^{-1} + \frac{\psi(\mu-1)}{\mu-\psi} - 1}.$$

A.1 Domestic

$$\pi = \bar{\pi},$$

$$p_H = 1,$$

$$\pi_H = \bar{\pi},$$

$$R = \frac{\bar{\pi}}{\beta\gamma^{\psi(1-\sigma)-1}},$$

$$m = \beta\gamma^{\psi(1-\sigma)-1},$$

$$\tilde{w} = \frac{\mu-1}{\mu},$$

$$c = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right),$$

$$N = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$y = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$c_H = \left(1-\alpha\right)c,$$

$$c_F = \alpha c,$$

$$v = \left\{\frac{\left[c^{\psi}\left(1-N\right)^{1-\psi}\right]^{1-\sigma}}{1-\beta\gamma^{\psi}}\right\}^{\frac{1}{1-\sigma}}$$

$$f = \frac{y}{2\left(1-\theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$k = \frac{y}{2\left(1-\theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$\Delta = 1,$$

$$YGR = \gamma.$$

A.2 Foreign

$$\pi^* = \bar{\pi}^*,$$

$$p_F^* = 1,$$

$$\pi_F^* = \bar{\pi}^*,$$

$$R^* = \frac{\bar{\pi}^*}{\beta\gamma^{\psi(1-\sigma)-1}},$$

$$m^* = \beta\gamma^{\psi(1-\sigma)-1},$$

$$\tilde{w}^* = \frac{\mu-1}{\mu},$$

$$c^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right),$$

$$N^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right) + g^*,$$

$$y^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right) + g^*,$$

$$c_H^* = \alpha c^*,$$

$$c_F^* = \left(1-\alpha\right)c^*,$$

$$v^* = \left\{ \frac{\left[c^{*\psi}\left(1-N^*\right)^{1-\psi}\right]^{1-\sigma}}{1-\beta\gamma^{\psi}} \right\}^{\frac{1}{1-\sigma}},$$

$$f^* = \frac{y^*}{2\left(1-\theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$k^* = \frac{y^*}{2\left(1-\theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$\Delta^* = 1,$$

$$YGR^* = \gamma.$$

A.3 International

$$s = 1,$$

$$Q = 1,$$

and

$$d = 1.$$

B Detrended system of equations

The detrended system of equations consists of 35 equations as shown below.

B.1 Domestic

$$c_t := \left[(1-\alpha)^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

$$v_t^{1-\sigma} = \left[c_t^{\psi} (1-N_t)^{1-\psi} \right]^{1-\sigma} + \beta \gamma^{\psi} \left(\mathbf{E}_t \left[v_{t+1}^{1-\varepsilon} \right] \right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r)\left[\phi_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \log\left(\frac{y_t}{y_{t-1}}\right) \right] + \log(\varepsilon_{R,t}),$$

$$c_{H,t} = (1-\alpha)\, p_{H,t}^{-\eta} c_t,$$

$$c_{F,t} = \alpha \left(s_t p_{F,t}^* \right)^{-\eta} c_t,$$

$$c_t = \frac{\psi}{1-\psi} (1-N_t)\, \tilde{w}_t,$$

$$1 = \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta \left[\mathbf{E}_t (v_{t+1})^{1-\varepsilon} \right]^{\frac{\varepsilon-\sigma}{1-\varepsilon}} (v_{t+1})^{\sigma-\varepsilon} \gamma^{\psi(1-\sigma)-1} \frac{c_{t+1}^{\psi(1-\sigma)-1} (1-N_{t+1})^{(1-\psi)(1-\sigma)}}{c_t^{\psi(1-\sigma)-1} (1-N_t)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}},$$

$$f_t = \frac{1}{2} p_{H,t} \left(c_{H,t} + g_t + c_{H,t}^* \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{1-\mu} f_{t+1},$$

$$k_t = \frac{1}{2} \frac{\mu}{\mu-1} \frac{\tilde{w}_t}{A_t} \left(c_{H,t} + g_t + c_{H,t}^* \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{-\mu} k_{t+1},$$

$$y_t = A_t N_t,$$

$$\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{1}{1-\mu}} f_t = k_t,$$

$$y_t = \Delta_t \left(c_{H,t} + g_t + c_{H,t}^* \right),$$

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{\mu}{\mu-1}} + \theta\left(\frac{\pi_{H,t}}{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}\right)^{\mu}\Delta_{t-1},$$

$$YGR_t := \gamma\frac{y_t}{y_{t-1}}.$$

B.2 Foreign

$$c_t^* := \left[(\alpha)^{\frac{1}{\eta}}\left(c_{H,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}}\left(c_{F,t}^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$\left(v_t^*\right)^{1-\sigma} = \left[\left(c_t^*\right)^{\psi}\left((1-N_t)^*\right)^{1-\psi}\right]^{1-\sigma} + \beta\gamma^{\psi}\left(\mathbf{E}_t\left[\left(v_{t+1}^*\right)^{1-\varepsilon}\right]\right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t^*}{R^*}\right) = \phi_r^*\log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\phi_r^*)\left[\phi_\pi^*\log\left(\frac{\pi_t^*}{\bar{\pi}^*}\right) + \phi_y^*\log\left(\frac{y_t^*}{y_{t-1}^*}\right)\right] + \log(\varepsilon_{R,t}^*),$$

$$c_{H,t}^* = \alpha\left(\frac{p_{H,t}}{s_t}\right)^{-\eta}c_t^*,$$

$$c_{F,t}^* = (1-\alpha)\left(p_{F,t}^*\right)^{-\eta}c_t^*,$$

$$c_t^* = \frac{\psi}{1-\psi}\left(1-N_t^*\right)\tilde{w}_t^*,$$

$$1 = \mathbf{E}_t m_{t,t+1}^*\frac{R_t^*}{\pi_{t+1}^*},$$

$$m_{t,t+1}^* = \beta\left(\mathbf{E}_t\left(v_{t+1}^*\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\left(v_{t+1}^*\right)^{\sigma-\varepsilon}\gamma^{\psi(1-\sigma)-1}\frac{\left(c_{t+1}^*\right)^{\psi(1-\sigma)-1}\left(1-N_{t+1}^*\right)^{(1-\psi)(1-\sigma)}}{\left(c_t^*\right)^{\psi(1-\sigma)-1}\left(1-N_t^*\right)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{F,t}^* = \frac{p_{F,t}^*\pi_t^*}{p_{F,t-1}^*},$$

$$f_t^* = \frac{1}{2}p_{F,t}^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{1-\mu}f_{t+1}^*,$$

$$k_t^* = \frac{1}{2}\frac{\mu}{\mu-1}\frac{\tilde{w}_t^*}{A_t^*}\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{-\mu}k_{t+1}^*,$$

$$y_t^* = A_t^*N_t^*,$$

$$\left[\frac{1-\theta^*\left(\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}^*}\right)^{1-\mu}}{1-\theta^*}\right]^{\frac{1}{1-\mu}}f_t^* = k_t^*,$$

$$y_t^* = \Delta_t^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right),$$

$$\Delta_t^* = (1-\theta^*)\left[\frac{1-\theta^*\left[\frac{(\bar{\pi}^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}}\right]^{1-\mu}}{1-\theta^*}\right]^{\frac{\mu}{\mu-1}} + \theta^*\left[\frac{\pi_{F,t}^*}{(\bar{\pi}^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}\right]^{\mu}\Delta_{t-1}^*,$$

$$YGR_t^* := \gamma\frac{y_t^*}{y_{t-1}^*}.$$

B.3 International

$$c_t^{\psi(1-\sigma)-1}\left(1-N_t\right)^{(1-\psi)(1-\sigma)} s_t = \Omega_t Q_t \left(c_t^*\right)^{\psi(1-\sigma)-1}\left(1-N_t^*\right)^{(1-\psi)(1-\sigma)},$$

$$Q_{t+1} = Q_t\left(\frac{\left(v_{t+1}^*\right)^{1-\varepsilon}\mathbf{E}_t\left(v_{t+1}\right)^{1-\varepsilon}}{\left(v_{t+1}\right)^{1-\varepsilon}\mathbf{E}_t\left(v_{t+1}^*\right)^{1-\varepsilon}}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}},$$

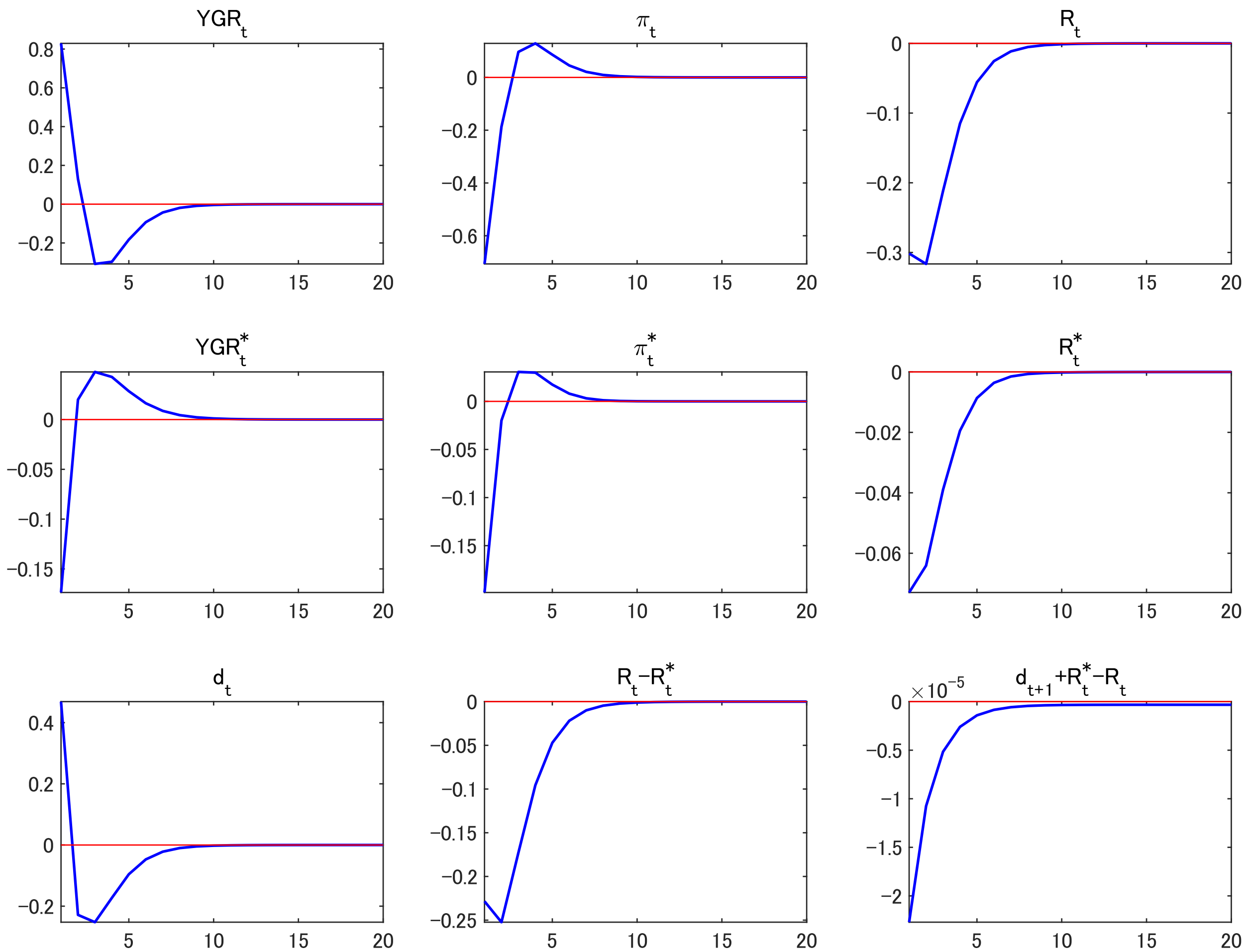
and

$$d_t = \frac{s_t\pi_t}{s_{t-1}\pi_t^*}.$$

C Impulse responses to the other shocks

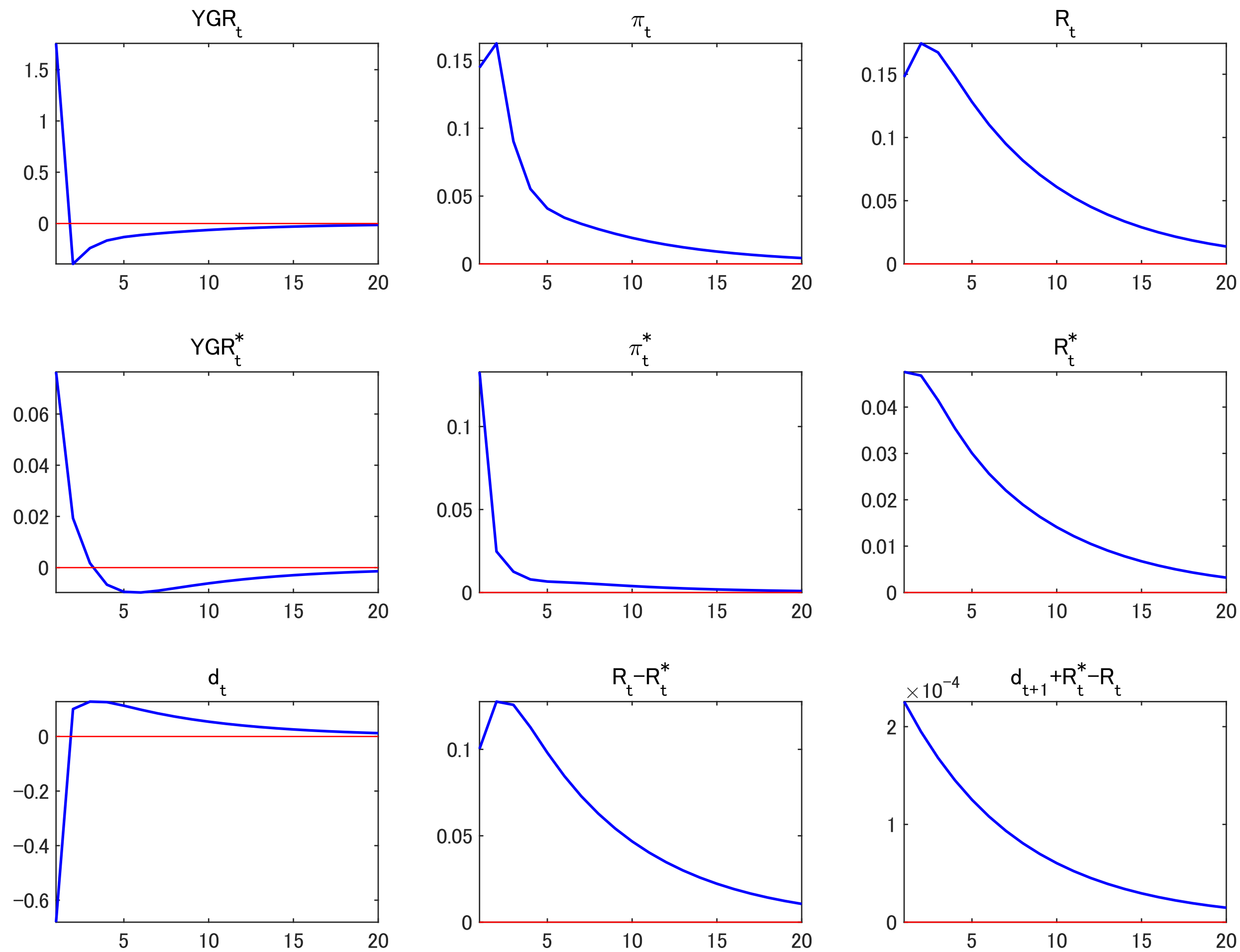
In what follows, the figures 6–16 show the impulse responses of the observed variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$) to the other shocks that are not reported in Section 4.2, given the posterior mean estimates of parameters in the baseline estimation.

Figure 6: Responses to home technology shock



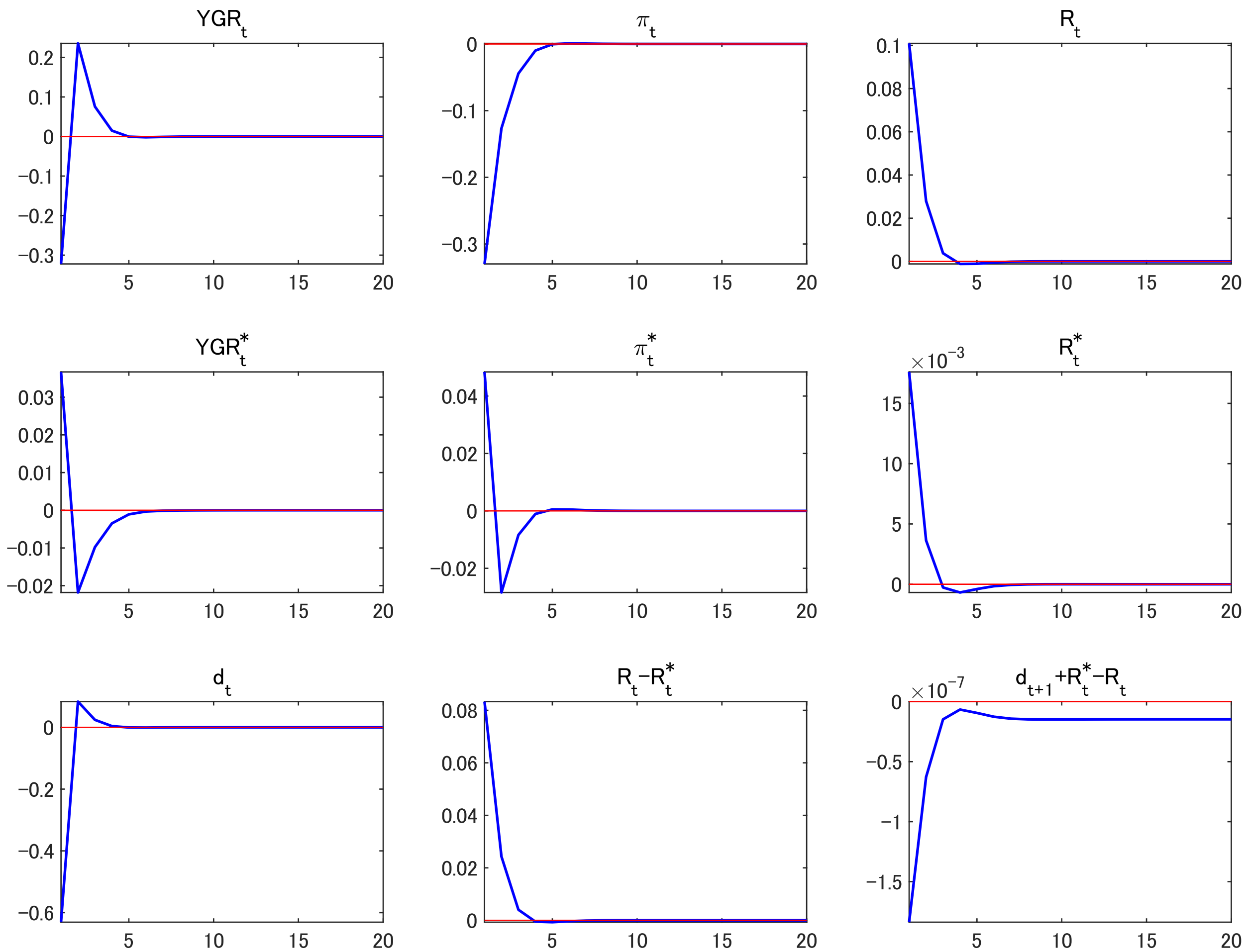
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 7: Responses to home external demand shock



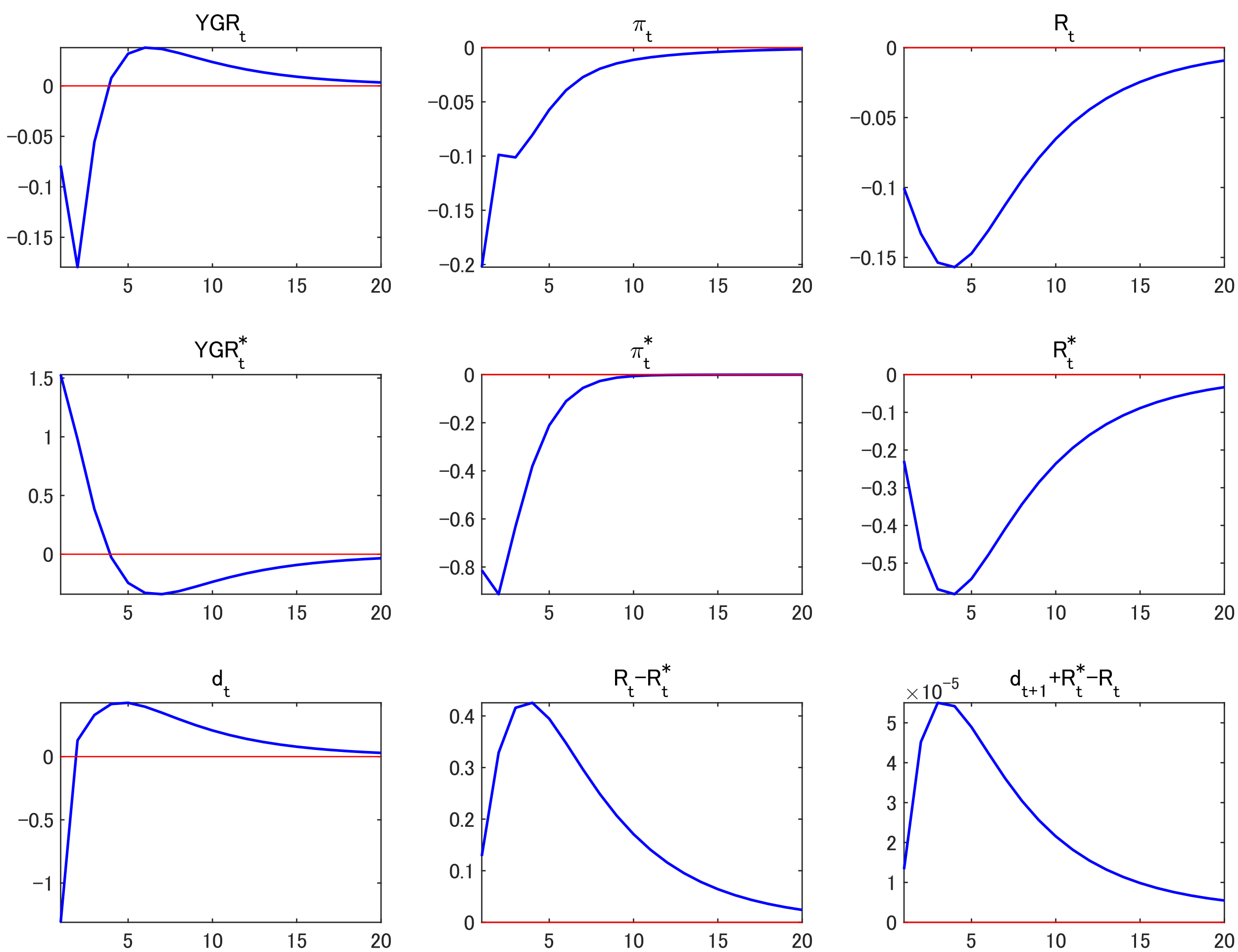
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 8: Responses to home monetary policy shock



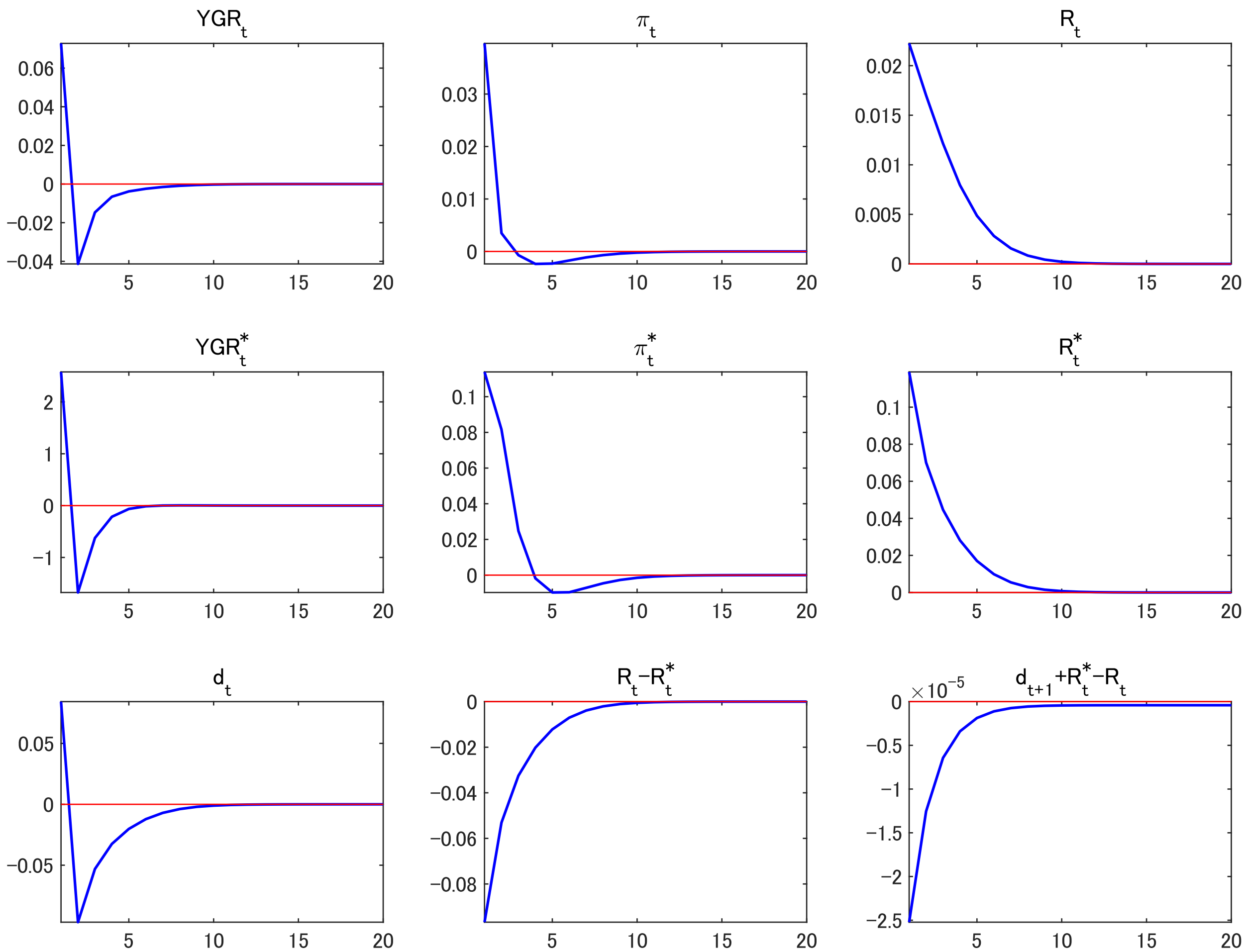
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 9: Responses to foreign technology shock



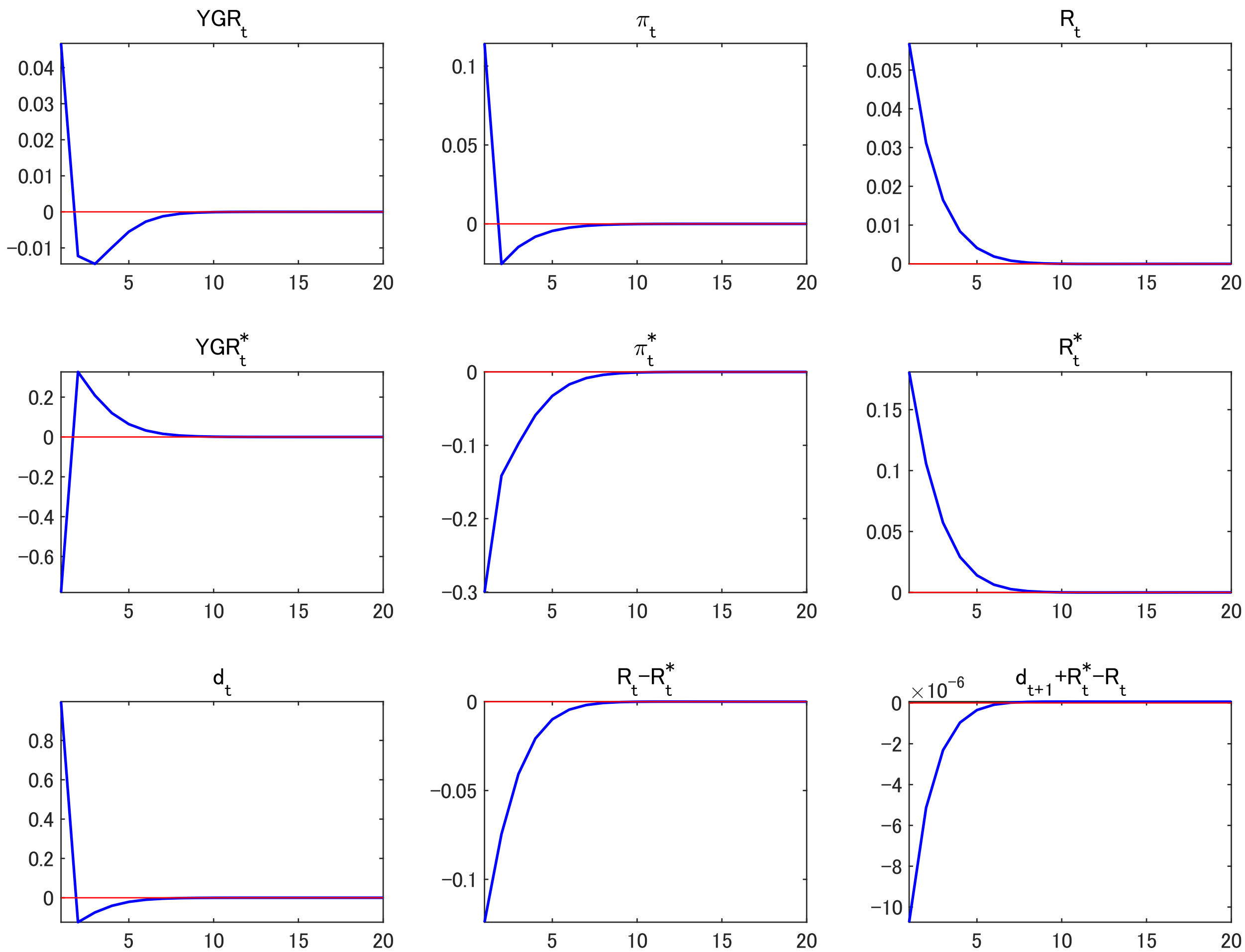
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 10: Responses to foreign external demand shock



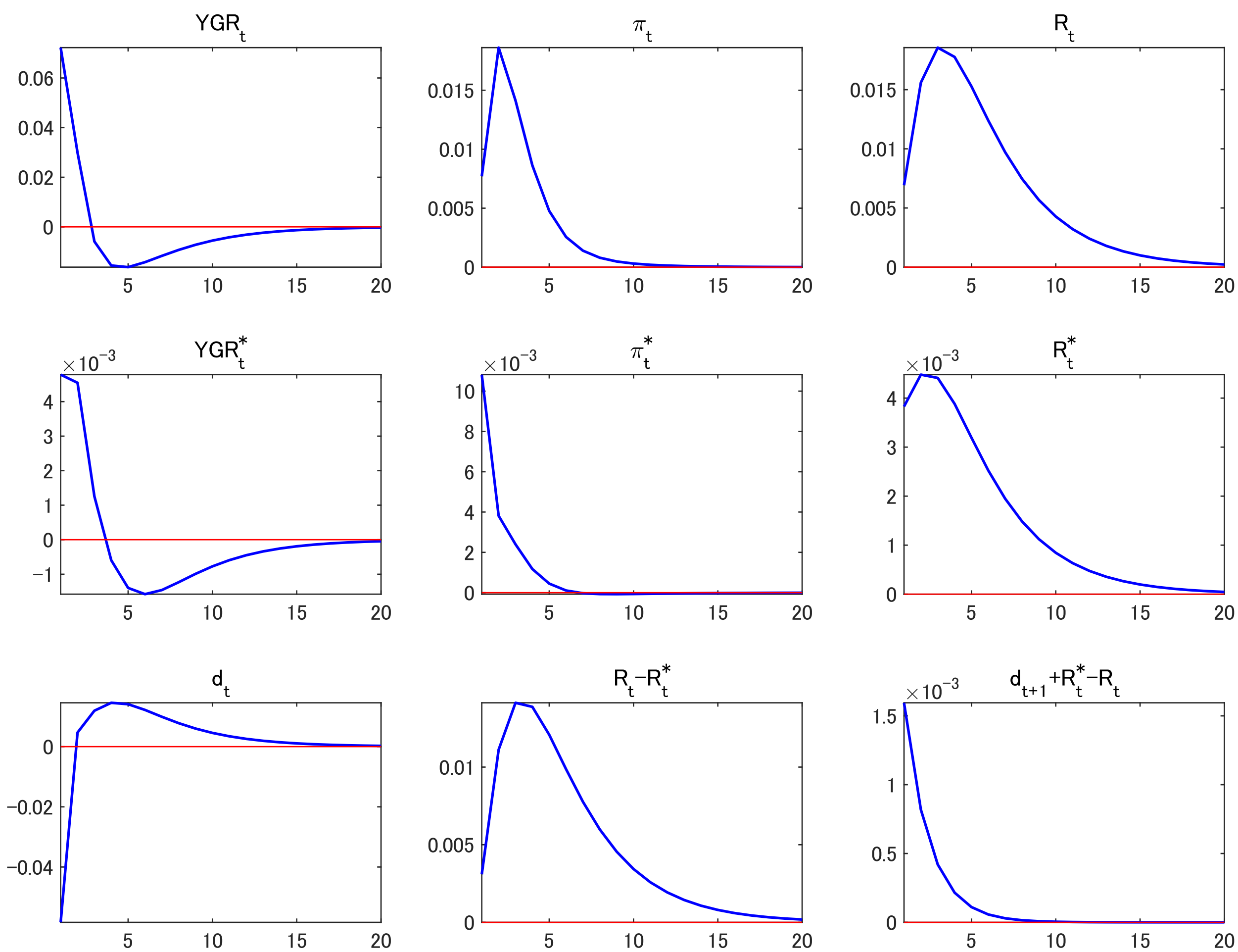
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 11: Responses to foreign monetary policy shock



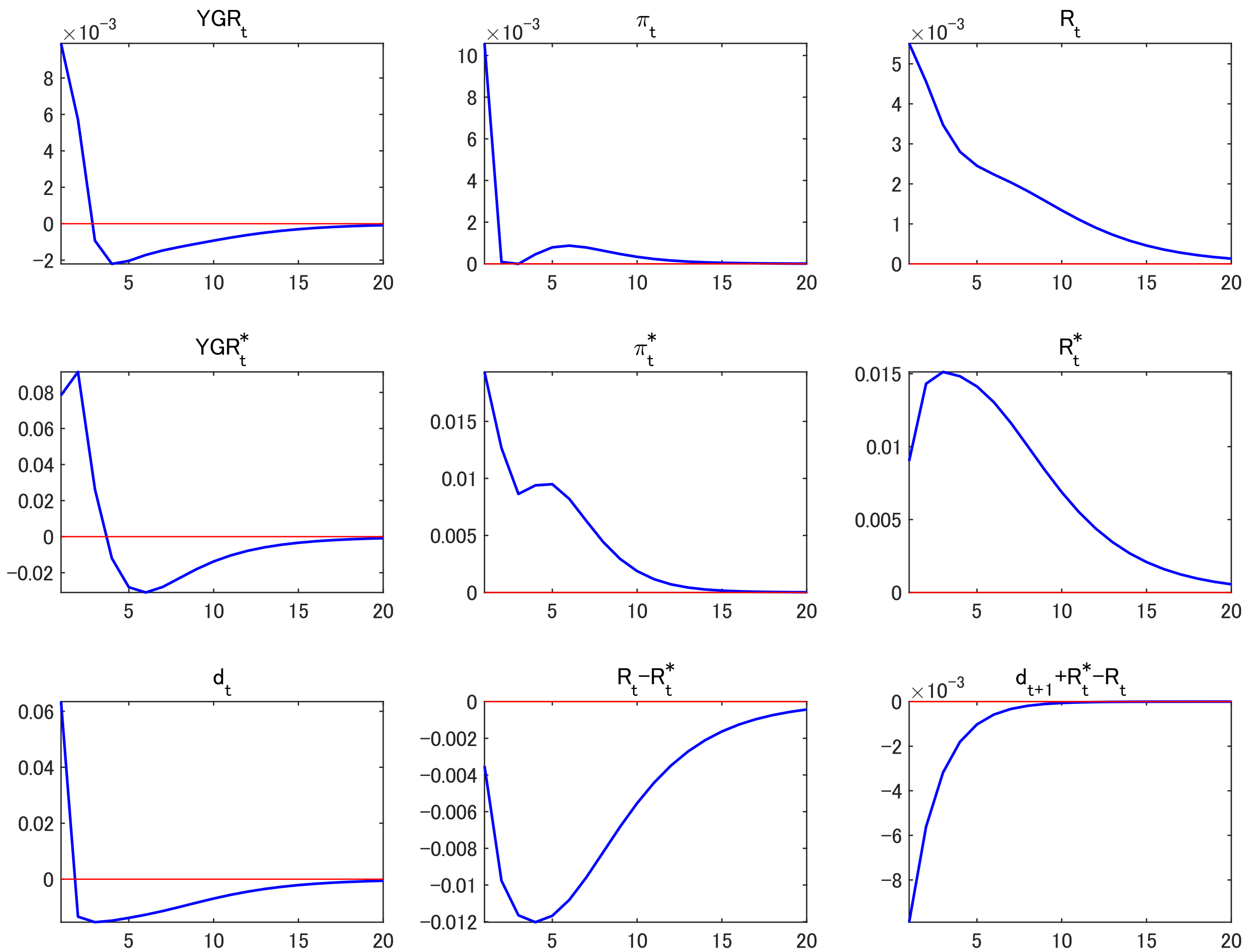
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 12: Responses to volatility shock to home external demand



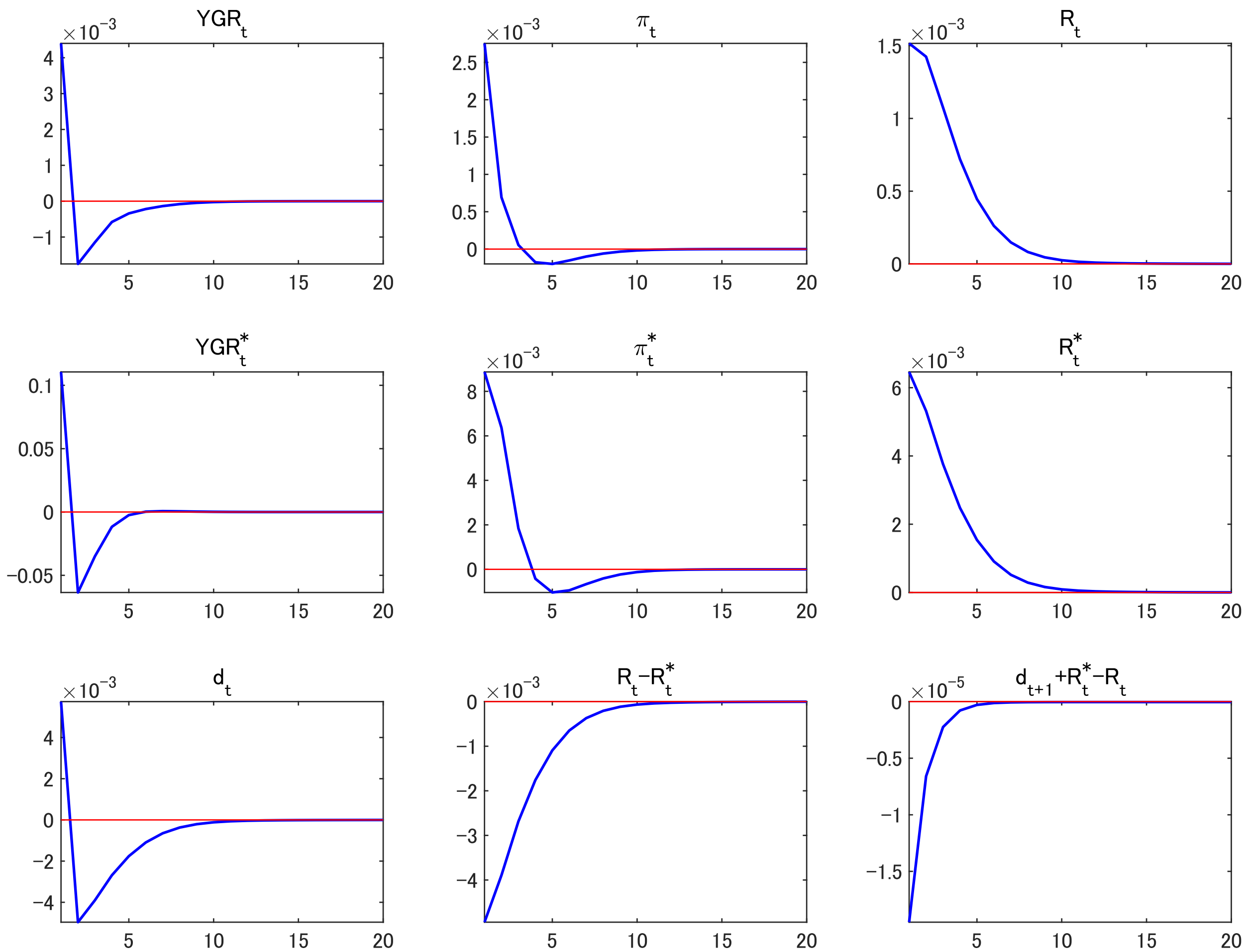
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home external demnad, given the posterior mean estimates of parameters in the baseline model.

Figure 13: Responses to volatility shock to foreign technology



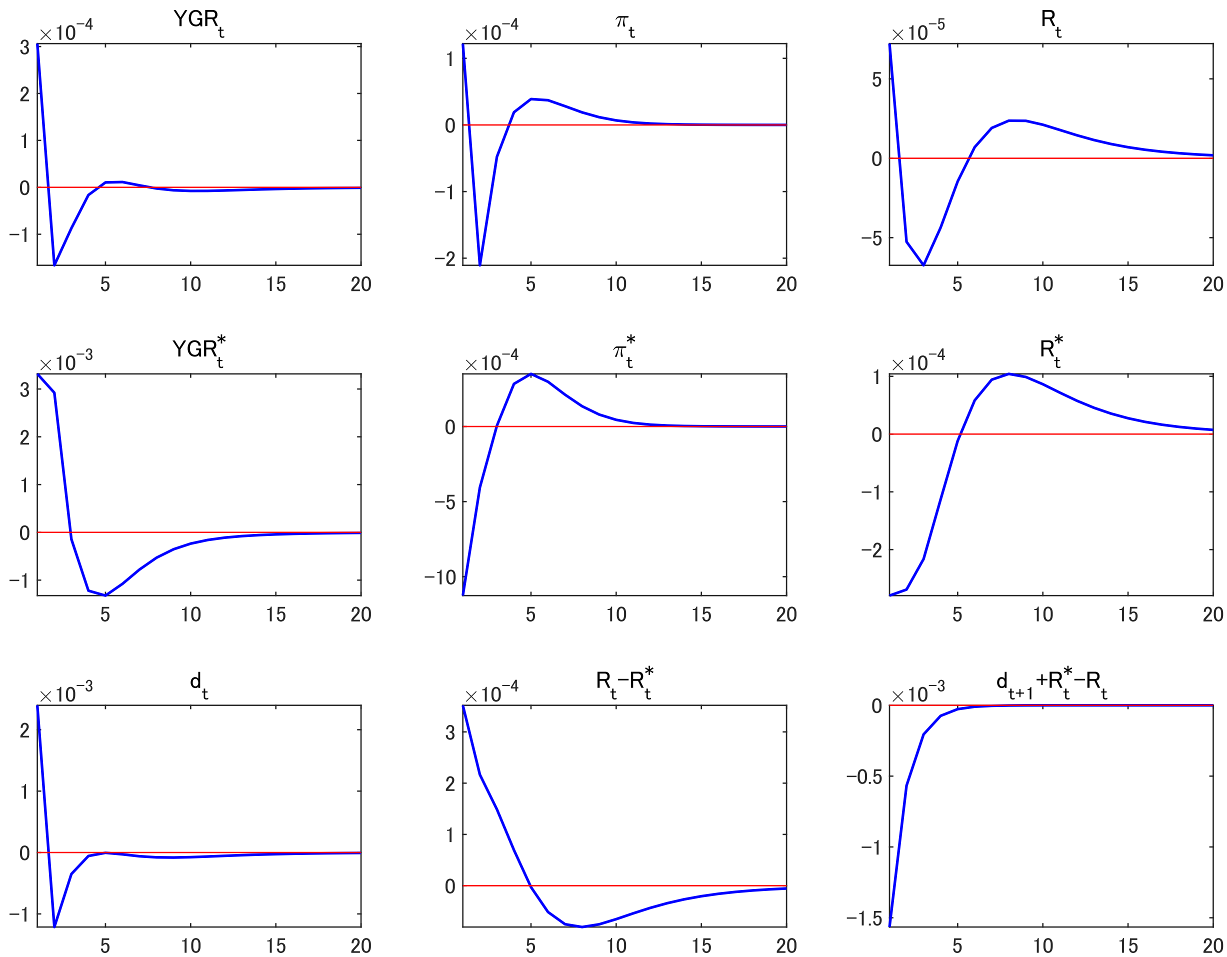
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 14: Responses to volatility shock to foreign external demand



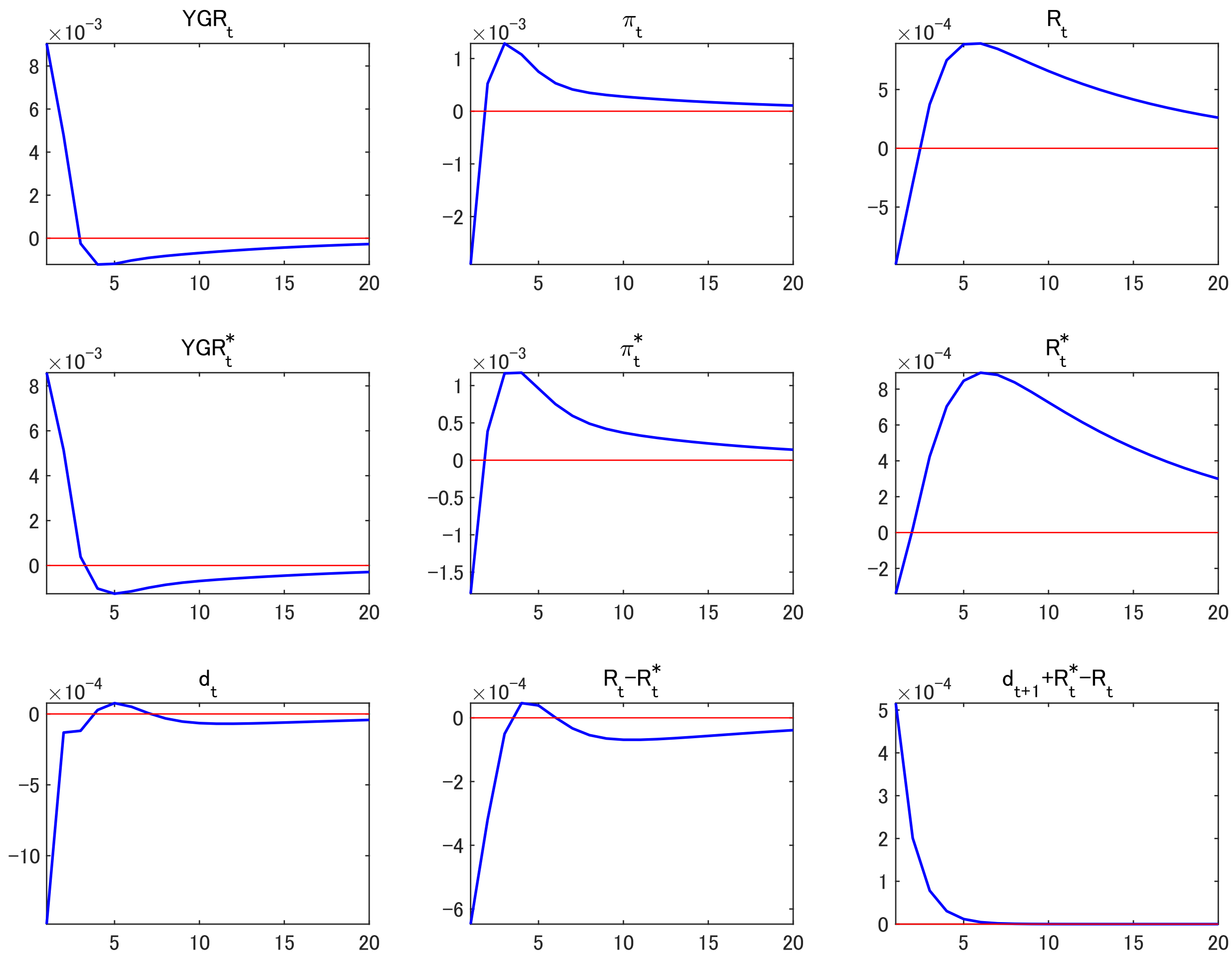
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 15: Responses to volatility shock to foreign monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 16: Responses to volatility shock to risk sharing



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.