Central Bank Digital Currency and Financial Frictions in Business Cycle and Monetary Policy Analysis*

Yizhan Huang

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Abstract

I explore how the introduction of a retail central bank digital currency (CBDC) affects the business cycle and welfare in the presence of financial frictions. In this paper, CBDC serves as an imperfect substitute for the private liquidity service, and it is the only household asset free from a default risk of the financial sector. Two CBDC regimes are discussed: intermediated CBDC and independent CBDC. Under the two regimes, I analyze second moments, business cycle propagation, and consumption-equivalent welfare with different CBDC monetary policy rules. The results show that the introduction of CBDC mitigates business cycle fluctuations and improves consumption-equivalence welfare across all CBDC economies compared to the non-CBDC baseline. The stabilization effects work differently among CBDC economies through a balance sheet channel and a substitution channel. The most significant improvement comes from the intermediated regime with a CBDC interest rule, in which mitigation effects from both channels are effective in response to macroeconomic shocks.

1. Introduction

Central banks often claim that, like traditional cash, central bank digital currency (CBDC) is a liability of the central bank, therefore it is immune to private credit and liquidity risk. At the

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same time, CBDC is technologically more advanced than traditional cash to support electronic transactions and grant central banks greater control over its security features [1]. Suppose these claims are true, it is reasonable to believe that such advantages of CBDC will incentivize households to demand CBDC for liquidity services. However, as households pivot to replace their private-issued assets with CBDC, would introducing such a safe asset for households bring more or less stability to the financial sector? Would it be beneficial to the entire economy? This paper aims to answer these questions through the lens of business cycle fluctuations and financial frictions.

Existing research on the business cycle implications of the introduction of CBDC is relatively thin. Among studies with a closed economy focus[2], Assenmacher et al. (2023) use a New Monetarist framework with sticky prices and financial frictions. They treat CBDC and bank deposits as perfect substitutes. In their model, the central bank can stabilize output and inflation by smoothing the spread between the interest rate on CBDC and bank deposits relative to the return on government bonds. Barrdear and Kumhof (2022) focus on a model with sticky prices and wages. CBDC and bank deposits are framed as imperfect substitutes. They find that CBDC improves business cycle stabilization by granting policymakers access to a second policy instrument. My study differs from these work by emphasizing CBDC as a safe liquidity service for households. With this emphasis, I uncover new mechanisms on how the introduction of CBDC affects the business cycle.

The model framework in this paper combines the classic money-in-the-utility function advanced by Sidrauski (1967) with the Gertler and Kiyotaki (2010) type of financial frictions. In the analysis, CBDC is an imperfect substitute for the private liquidity service (PvDC), and it is the only household asset that faces no default risk from financial intermediaries (banks). Within this framework, I analyze second moments, business cycle propagation, and consumption-equivalent welfare under two CBDC regimes - intermediated CBDC and independent CBDC, in comparison to a non-CBDC counterfactual.

The two CBDC regimes vary in how CBDC interacts with the financial sector. In the intermediated regime, CBDC is intermediated through banks, which can use CBDC as a funding source.

[1]We can see such claims in many research report or policy memos of central banks, such as Board of Governors and the Federal Reserve System (2022) and Reserve Bank of Australia and the Department of the Treasury (2024).

[2]There are studies on international business cycle that is less relevant to this work. For example, in an open economy setup, Ferrari Minesso et al. (2022) show that the presence of a CBDC amplifies the international spillovers of shocks and international linkages. Some other papers, such as Gross and Schiller (2021) and Piazzesi et al. (2022), have business cycle components, but the focus of their papers is more on monetary policy. I include them in the discussion of the monetary policy strand of CBDC literature.

This arrangement resembles a form of the central bank passing CBDC funding to private banks suggested by Brunnermeier and Niepelt (2019) and Niepelt (2022). To capture the safety aspect of CBDC, I assume that households can fully recover their CBDC holdings, whereas other household assets are divertible by banks in the event of default. In the independent regime, households hold CBDC directly with the central bank, so it does not enter banks' balance sheets.

I study the intermediated CBDC regime under an endogenous CBDC supply rule and a CBDC interest rule. Under the endogenous CBDC supply rule, banks directly influence CBDC demand and its return through their optimality conditions, and the central bank endogenously adjusts the money supply to meet money demand. When banks take CBDC demand and its return as given, I discuss the optimal monetary policy and employ a CBDC interest rule that aims to achieve the efficient allocation implied by a benevolent social planner. How the CBDC interest rule impacts the economy is discussed under both regimes.

Relative to the non-CBDC baseline, I find that business cycle volatility dampens across all CBDC economies. Crucially, I show that the stabilization works differently among CBDC economies through two channels: a balance sheet channel and a substitution channel. Under the intermediated CBDC regime with an endogenous CBDC supply rule, the stabilization effect arises from the balance sheet channel. Because CBDC is safe, the presence of CBDC makes it easier for households to recover funds in the event of bank default. As a result, households are more willing to lend to banks, which relaxes banks' borrowing constraints. The substitution channel under the endogenous CBDC supply rule works in the opposite direction. The optimality conditions of banks implies an excess return of PvDC over CBDC. When faced with shocks, the price of CBDC is amplified, leading to higher price and lower demand of money (defined as the sum of CBDC and PvDC) that increases business cycle volatility. By contrast, under the intermediated regime with an CBDC interest rate rule, the substitution channel complements the balance-sheet channel. The CBDC interest rate rule smooths the price and demand for CBDC. The price increase and demand drop of money are mitigated as households substitute between CBDC and PvDC. Under the independent CBDC regime, the balance-sheet channel is attenuated and the substitution channel becomes the primary source of stabilization.

The welfare implication of this paper is in line with the literature. Various studies, such as Bacchetta and Perazzi (2021), Barrdear and Kumhof (2022), Williamson (2022), Keister and Sanches

(2023), Paul et al. (2025), have shown that the introduction of CBDC can improve welfare[3]. I estimate the consumption-equivalence welfare across all CBDC economies compared to the non-CBDC baseline. In response to a combination of productivity and liquidity shocks, the best improvement is 1.18% from the intermediated CBDC regime with a CBDC interest rule.

There are other strands of literature relevant to this paper. On CBDC and financial intermediation, a core question is whether the introduction of CBDC disintermediates private banks by eroding deposit funding, thereby depressing real activity and amplifying financial instability. Existing studies, on the one hand, show that the introduction of a deposit-like CBDC tends to crowd out bank deposits under incomplete financial market (Whited et al. (2022), Keister and Sanches (2023), Kim and Kwon (2023), etc.). On the other hand, the extent of the disintermediation effect is likely policy-contingent[4] Moreover, even with the disintermediation effect, the benefits of CBDC, such as greater financial inclusion and welfare gains, can still make the adoption of CBDC desirable. (Brunnermeier and Niepelt (2019), Andolfatto (2021), Williamson (2022), etc.) In my analysis, disintermediation occurs under the independent CBDC regime because part of household liquidity services are in the form of CBDC that are not available for banks to use. However, the mitigation effect through the substitution channel between CBDC and PvDC dominates the disintermediation effect in stabilizing the economy when faced with macroeconomic shocks.

Another area of related work studies how the presence and design of CBDC shape monetary policy transmissions. For instances, Davoodalhosseini (2022) finds that, if CBDC and cash coexist, the central bank faces a tradeoff on either distorting the allocation relative to the first best under the cash-only scheme or having the agents incur the cost of carrying the CBDC under the CBDC-only scheme. Gross and Schiller (2021) show that monetary policy designs of CBDC are crucial to present destabilizing effects for the financial sector and CBDC monetary policy can provide additional tool to govern financial distress. George et al. (2022) demonstrate that counter-cyclical CBDC monetary policy improves welfare and economic stability. Piazzesi et al. (2022) show that different central bank operating procedures impact the costs of safety and liquidity for banks, leading to different

[3]These studies use different estimation methods, but the welfare gains are generally positive. For example, the welfare can improve 60 basis points of consumption in Bacchetta and Perazzi (2021), 0.34% of consumption in Davoodalhosseini (2022), 3% of GDP in Barrdear and Kumhof (2022).

[4]Several studies suggest that, if the central bank can pass CBDC funding to private banks, or properly set interest rule on CBDC, the disintermediation effect can be offset. See Brunnermeier and Niepelt (2019), Kim and Kwon (2023), Paul et al. (2025), etc.

pass-through to output and inflation. In this paper, I study the optimal monetary policy under the benevolent social planner. The optimal monetary policy implies all returns of household assets must be equal, which eliminates both monetary and financial frictions. I employ a CBDC interest rate rule to achieve the efficient allocation under both intermediated and independent CBDC regimes. The CBDC interest rate rule is crucial to obtain the mitigation effects from the substitution channel.

The rest of the paper is organized as follows. Section 2 describes CBDC models. Section 3 discusses optimal monetary policy. Section 4 investigates the business cycle impact of introducing CBDC with numerical exercises. Section 5 studies welfare implications. Section 6 concludes.

2. Model

This section describes three CBDC models. The benchmark model features an intermediated CBDC regime with an endogenous money supply rule. In this economy, the optimality conditions of banks directly influence CBDC demand and its return, while the central bank adjusts the CBDC supply endogenously to meet its demand. An alternative intermediated CBDC model considers the scenario in which banks take CBDC demand and its return as given. The last model discussed in this section is an independent CBDC model where CBDC does not enters banks' balance sheets.

2.1. Benchmark Model

The core framework is the DSGE model with financial frictions similar to Gertler and Kiyotaki (2010). The household setup resembles Piazzesi et al. (2022) with PvDC as an additional element. The bundle of money, denoted as M_t, follows a structure of constant elasticity of substitution (CES)[5]:

$$M_t = \left(\alpha \left(M_t^{CB} \right)^{\frac{\epsilon_m - 1}{\epsilon_m}} + (1 - \alpha) \left(M_t^{PV} \right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \right)^{\frac{\epsilon_m}{\epsilon_m - 1}}, \tag{1}$$

in which M_t^{CB} and M_t^{PV} denote CBDC and PvDC, respectively; α is the weight of CBDC in the money bundle; ϵ_m is the elasticity of substitution between CBDC and PvDC.

There exists an agency problem between households and banks. Banks can potentially divert funds received from households. If a bank chooses to divert these funds, depositors have the power

[5] Many existing CBDC papers, such as Bacchetta and Perazzi (2021), Li (2023), and Cirelli and Nyffenegger (2024), also apply this approach.

to force the bank into bankruptcy and recover a portion of their assets. That portion of assets include all their CBDC holding. As in Gertler and Kiyotaki (2010), price rigidity and labor market frictions are abstracted from the model.

Households

There is a representative household with a continuum of members of mass unity. The households consist of a fraction of ζ workers and $1-\zeta$ bankers. Each period, workers supply labor and return their wages to households. Each banker manages a bank. Bankers transfer all earnings back to households. Households provide funding to banks to obtain CBDC and PvDC for liquidity services and one-period private bonds for consumption-smoothing. CBDC, PvDC, and private bonds pay interests.

Let C_t be consumption, L_t be labor, and B_t be the amount of private bond holding during time t. Each period, the household chooses C_t, M_t^{CB}, M_t^{PV}, L_t, and B_t to maximize expected discounted utility:

$$\max_{\{C_t, M_t^{CB}, M_t^{PV}, L_t, B_t\}} \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \frac{1}{1-\frac{1}{\sigma}} \left[C_{t+\tau}^{1-\frac{1}{\eta}} + \omega \left(\frac{M_{t+\tau}}{P_{t+\tau}} \right)^{1-\frac{1}{\eta}} \right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi \frac{L_{t+\tau}^{1+\varphi}}{1+\varphi} \right\}, \tag{2}$$

subject to the budget constraint

$$P_t C_t + B_t + M_t^{CB} + M_t^{PV} \leq W_t L_t + (1 + i_{t-1}^B) B_{t-1} + \left(1 + i_{t-1}^{CB}\right) M_{t-1}^{CB} + \left(1 + i_{t-1}^{PV}\right) M_{t-1}^{PV} + D_t, \tag{3}$$

where P_t is the nominal price; W_t is the nominal wage; i_t^{CB}, i_t^{PV}, and i_t^B are nominal interest rates for CBDC, PvDC, and private bonds; D_t represents the net dividend transfer from the ownership of both financial and non-financial sectors. In the household preferences, β is the discount factor; η is the elasticity of money demand; ω is the weight on money in the utility; φ is the inverse Frisch elasticity of labor supply; ψ is the labor disutility scaling parameter; σ is the intertemporal elasticity of substitution between consumption-money bundles at different time periods.

Since price rigidity is absent from the model, I deflate all nominal variables by price level and adjust all nominal returns by inflation. The household first-order optimality conditions imply the opportunity cost for a bundle of CBDC and PvDC

$$q_t^M = \left[\alpha^{\varepsilon_m}(r_t^B - r_t^{CB})^{1-\varepsilon_m} + (1-\alpha)^{\varepsilon_m}(r_t^B - r_t^{PV})^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}}, \tag{4}$$

and the opportunity cost for a bundle of consumption and money

$$q_t^S = \left[1 + \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{5}$$

The opportunity cost of money impacts the money demands through the following three optimality conditions:

$$m_t = C_t \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{-\eta}, \tag{6}$$

$$m_t^{CB} = \alpha^{\varepsilon_m} \left(\frac{q_t^M}{r_t^B - r_t^{CB}}\right)^{\varepsilon_m} m_t, \tag{7}$$

and

$$m_t^{PV} = (1-\alpha)^{\varepsilon_m} \left(\frac{q_t^M}{r_t^B - r_t^{PV}}\right)^{\varepsilon_m} m_t. \tag{8}$$

According to the household optimality conditions, higher money-holding costs reduce the demand for liquidity services. These costs are determined by the interest rate differentials between private bonds and money. When the differentials widen, the opportunity cost of holding money rises, lowering households' money holdings.

I assume that the utility function is non-separable in consumption and money ($\eta \neq \sigma$)[6]. Therefore, money has real effects through the optimality conditions of labor supply schedule

$$w_t = \psi L_t^\varphi C_t^{\frac{1}{\sigma}} (q_t^S)^{1-\frac{\eta}{\sigma}}, \tag{9}$$

and the Euler equation

$$\beta E_t \left[\left(\frac{q_{t+1}^S}{q_t^S}\right)^{\frac{\eta}{\sigma}-1} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\right] = \frac{1}{r_t^B}. \tag{10}$$

[6]See Piazzesi et al. (2022) for a discussion of separable and non-separable cases. We opt for non-separable setup because it is more empirically plausible.

Let $U_{C,t}$ denote the marginal utility of consumption. The stochastic discount factor is defined as

$$\Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}}. \tag{11}$$

Appendix A provides the detailed deviations of the household problem.

Banks

Consider a bank j with the following balance sheet:

$$N_{j,t} + B_{j,t} + M_{j,t}^{CB} + M_{j,t}^{PV} = Q_t^F S_{j,t}. \tag{12}$$

$N_{j,t}$ is the net worth of bank j. $S_{j,t}$ is the quantity of equity claims of bank j on non-financial firms, which equals the amount of capital bank j owns at the equilibrium. Q_t^F is the nominal price of capital. Bank j's net worth at time $t+1$ is the difference between the expected return of capital and the payments to its liabilities:

$$N_{j,t+1} = E_t(1 + i_{t+1}^K)Q_t^F S_{j,t} - (1 + i_{j,t}^B)B_{j,t} - (1 + i_{j,t}^{CB})M_{j,t}^{CB} - (1 + i_{j,t}^{PV})M_{j,t}^{PV}, \tag{13}$$

where $E_t(1+_{t+1}^K)$ is the expected gross nominal return on capital. Express the law of motion of bank j's net worth in real terms:

$$n_{j,t+1} = (E_t r_{t+1}^K - r_{j,t}^{CB})q_t^F S_{j,t} - (r_{j,t}^B - r_{j,t}^{CB})b_{j,t} - (r_{j,t}^{PV} - r_{j,t}^{CB})m_{j,t}^{PV} + r_{j,t}^{CB} n_{j,t}. \tag{14}$$

Following the literature (Gertler and Karadi (2011), Gertler and Kiyotaki (2010), etc.), to prevent bankers from accumulating enough wealth to overcome their financial constraints without the need to borrow from the household, a banker exits the financial sector and becomes a worker with i.i.d. probability $1 - \theta$ each period. The average duration of being a banker is thus $\frac{1}{1-\theta}$. The total number of banks is constant overtime as a fraction of $\zeta(1 - \theta)$ workers will randomly become bankers each period. Bank j's objective is to maximize the present value of its terminal real wealth

$$E_t \sum_{i=1}^{\infty} (1 - \theta)\theta^{i-1} \Lambda_{t,t+i} n_{j,t+i}. \tag{15}$$

Denote the value of bank j at period t as $V_{j,t}$. $V_{j,t}$ satisfies the Bellman equation

$$V_{j,t}\left(n_{j,t}\right) = E_t\Lambda_{t,t+1}\left\{(1-\theta)n_{j,t+1} + \theta \max_{m_{j,t}^{PV}}\left[\max_{S_{j,t},\, b_{j,t+1}} V_{j,t+1}(n_{j,t+1})\right]\right\} \tag{16}$$

The agency problem between households and banks discourages depositors from supplying funds to banks unless it is not optimal for the banks to divert the funds. To address this issue, the bank's optimization problem must include the following incentive constraint:

$$V_{j,t} \geq \xi_t(q_t^F S_{j,t} - \lambda^B b_{j,t} - \lambda^{PV} m_{j,t}^{PV} - m_{j,t}^{CB}). \tag{17}$$

This incentive constraint highlights two important aspects of banks. First, the value of banks at time t must exceed the amount of funds that can be diverted. As a result, banks have no incentive to misappropriate household assets. In addition, the abilities of banks to divert different types of funds vary. In the incentive constraint, λ^B and λ^{PV} ($\lambda^B, \lambda^{PV} \in (0,1)$) represent the fractions of private bonds and PvDC, respectively, that households can recover when banks run away. CBDC is entirely non-divertable. Following Sims and Wu (2021), I incorporate liquidity shock ξ_t in the incentive constraint, which is a key driver of business cycle fluctuations in this paper.

Bank j solves the Bellman equation subject to the law of motion of real net worth (eqn. 14) and the incentive constraint (eqn. 17). To solve the problem, I conjecture the value of bank is linear in net worth (e.g. $V_{j,t} = \nu_t n_{j,t}$)[7]. Banks are assumed to be identical as bank heterogeneity is not part of the discussion of this paper. The bank's first-order optimality conditions imply the premium returns over CBDC:

$$E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_{t+1}^K - r_t^{CB}\right) = \frac{\xi_t \mu_t}{1+\mu_t}, \tag{18}$$

$$E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_t^B - r_t^{CB}\right) = \frac{\xi_t \lambda^B \mu_t}{1+\mu_t}, \tag{19}$$

$$E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_t^{PV} - r_t^{CB}\right) = \frac{\xi_t \lambda^{PV} \mu_t}{1+\mu_t}, \tag{20}$$

[7] This method is straightforward in solving this type of bank problem. The solution yields clear excess returns on different assets. See Ozhan (2020), Sims and Wu (2021).

where the Lagrange multiplier

$$\mu_t = \frac{\xi_t(Q_t^f S_t - \lambda^B b_t - \lambda^{PV} m_t^{PV})}{E_t \Lambda_{t,t+1} \Omega_{t+1} r_t^{CB} n_t + \xi_t m_t^{CB}} - 1, \tag{21}$$

and the shadow marginal value of bank net worth

$$\Omega_{t+1} = 1 - \theta + \theta \nu_{t+1}. \tag{22}$$

The marginal net worth ν_t satisfies that

$$\nu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} r_{t+1}^{PV} (1 + \mu_t). \tag{23}$$

In the optimality conditions, the value of the Lagrange multiplier (eqn. 21) closely relates to the bank incentive constraint (eqn. 17). If the liquidity shock is positive or the price of capital increases, the incentive constraint gets tighten, rising the excess returns over CBDC through equation 18 - 20. On the contract, the increase in CBDC demand will relax the incentive constraint, lowering the value of the Lagrange multiplier and excess returns over CBDC. In the standard financial friction model, the increase in the excess return of capital plays a key role in amplifying the business cycle fluctuations[8]. The introduction of CBDC participates in this financial friction mechanism. Such effects is explained more with the numerical exercises in section 4.

The aggregate net worth for the banking sector is the sum of total net worth of existing and new entering bankers. Since the survival rate of bankers is θ, the total net worth of existing bankers is the survival rate times the net earnings of previous period's assets and liabilities:

$$n_t^E = \theta \left[z_t + (1-\delta) q_t^F \right] S_{t-1} - \theta (r_{t-1}^B b_{t-1} + r_{t-1}^{CB} m_{t-1}^{CB} + r_{t-1}^{PV} m_{t-1}^{PV}). \tag{24}$$

The net profit of equity claims $z_t + (1-\delta) q_t^F$ will be explained in detail under non-financial firms.

Upon entering the banking sector, it is assumed that new bankers receive a start-up funds from households, and the total amount of start-up funds equals to the fraction $\frac{\varpi}{1-\theta}$ of the final period

[8]For instance, in Gertler and Karadi (2011), a negative productivity shock worsens bank's balance sheet and increase the excess return of capital. As a result, the declines in real economic variables such as output, investment, and consumption are magnified.

assets of the exiting bankers. Therefore, the total net worth for new entering bankers is:

$$n_t^N = \left(\frac{\varpi}{1-\theta}\right)(1-\theta)\left[z_t + (1-\delta)q_t^F\right]S_{t-1}. \tag{25}$$

Adding the total net worth for existing and new entering bankers together gives the aggregate net worth for the entire banking sector:

$$n_t = (\varpi + \theta)\left[z_t + (1-\delta)q_t^F\right]S_{t-1} - \theta(r_{t-1}^B b_{t-1} + r_{t-1}^{CB} m_{t-1}^{CB} + r_{t-1}^{PV} m_{t-1}^{PV}). \tag{26}$$

Finally, financial market clearing condition implies

$$q_t^F S_t = n_t + + b_t + m_t^{CB} + m_t^{PV}. \tag{27}$$

A detailed solution to the banking sector is available in Appendix B.

Non-financial Firms

The non-financial sector consists two types of firms, good producers and capital producers. Both operate in competitive markets.

Good producers borrow loans from banks to fund new capital stock K_{t+1} that will be used from time t to t+1. After receiving revenues by selling goods and reselling used capitals, good producers pay wages to workers and return all profit to the banks through loan payments. Good producers aim to maximize expected lifetime discounted profit:

$$\max_{\{K_{t+1}, L_{t+1}\}} E_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}\{P_{t+i}Y_{t+i} + Q_{t+i}^F(1-\delta)K_{t+i} - W_{t+i}L_{t+i} - (1+i_{t+i}^K)Q_{t+i-1}^F S_{t+i-1}\} \tag{28}$$

subject to the production function:

$$Y_t = A_t K_t^{\gamma} L_t^{1-\gamma}, \tag{29}$$

and the non-arbitrage condition between the financial and non-financial sectors:

$$Q_t^F K_{t+1} = Q_t^F S_t, \tag{30}$$

where A_t represents the technology that follows an AR (1) process.

Solving good producers' maximization problem yields the expected return of capital

$$E_t \Lambda_{t,t+1} r_{t+1}^K = E_t \Lambda_{t,t+1} \frac{z_{t+1} + (1-\delta) q_{t+1}^F}{q_t^F}, \tag{31}$$

and the labor demand schedule

$$E_t \Lambda_{t,t+1} w_{t+1} = E_t \Lambda_{t,t+1} (1-\gamma) \frac{Y_{t+1}}{L_{t+1}}. \tag{32}$$

The real gross profit of capital

$$z_t = \frac{Y_t - w_t L_t}{K_t}. \tag{33}$$

Each period, capital producers maximize profit by selling new capital stock after purchasing existing capital and make investment I_t:

$$\max_{I_t} \quad E_t \sum_{i=1}^{\infty} \Lambda_{t,t+i} \{ Q_{t+i-1}^F K_{t+i} - Q_{t+i-1}^F (1-\delta) K_{t+i-1} - I_{t+i-1} \}, \tag{34}$$

subject to the law of motion of capital

$$K_{t+1} = (1-\delta) K_t + I_t - f\left(\frac{I_t}{I_{t-1}}\right) I_t \tag{35}$$

The capital adjustment cost $f\left(\frac{I_t}{I_{t-1}}\right)$ is a standard model twist to prevent the real capital price to be a unity constant. The capital producers' maximization problem implies that the real price of capital[9] satisfies

$$q_t^F \left[1 - f\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) \right] + E_t \Lambda_{t,t+1} q_{t+1}^F \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) = 1. \tag{36}$$

[9]The real price of capital is the Tobin's q that represents the relative price of installed capital in units of the consumption good. For example, how many units of the good the households are willing to give up at the margin to obtain one more unit of installed capital.

Equilibrium

Government spending and international trade are abstracted from the model for simplicity, therefore, the aggregate output is

$$Y_t = C_t + I_t. \tag{37}$$

The equilibrium of the economy is where all markets clear, and all agents are maximizing subject to their respective constraints, and the aggregate resource constraint is satisfied. In the benchmark model, the 24 equations 4 - 11, 18 - 23, 26, 27, 29 - 33, 35 - 37, together with the two exogenous processes of A_t and ξ_t, determine the 26 variables of the model.

2.2 Alternative CBDC Models

Alternative Interemdiated CBDC Model

In this model, CBDC demand and its return are exogenously given to banks' optimization problem. Banks' optimality conditions now imply the excess returns over PvDC instead of over CBDC:

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \left(r^K_{t+1} - r^{PV}_t \right) = \frac{\xi_t \mu_t}{1 + \mu_t}, \tag{38}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \left(r^B_t - r^{PV}_t \right) = \frac{\xi_t \lambda^B \mu_t}{1 + \mu_t}. \tag{39}$$

where the Lagrange multiplier

$$\mu_t = \frac{\xi_t (q^F_t S_t - \lambda^B b_t)}{E_t \Lambda_{t,t+1} \Omega_{t+1} [r^{PV}_t n_t + (r^{PV}_t - r^{CB}_t) m^{CB}_t] + \xi_t (\lambda^{PV} m^{PV}_t + m^{CB}_t)} - 1, \tag{40}$$

and the marginal value of banks

$$\nu_t = \frac{E_t \Lambda_{t,t+1} \Omega_{t+1} [r^{PV}_t n_t + (r^{PV}_t - r^{CB}_t) m^{CB}_t](1 + \mu_t) + \xi_t \mu_t (\lambda^{PV} m^{PV}_t + m^{CB}_t)}{n_t \xi_t}. \tag{41}$$

Compared to the benchmark model, one equation is dropped from the banking sector. A monetary policy rule, which is discussed in Section 3, is required to close the model.

Independent CBDC Model

Under the independent CBDC regime, no CBDC element enters the bank balance sheets, therefore the optimality conditions for banks become:

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \left(r^K_{t+1} - r^{PV}_t \right) = \frac{\xi_t \mu_t}{1+\mu_t}, \tag{42}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \left(r^B_t - r^{PV}_t \right) = \frac{\xi_t \lambda^B \mu_t}{1+\mu_t}, \tag{43}$$

$$\mu_t = \frac{\xi_t (q^f_t S_t - \lambda^B b_t)}{E_t \Lambda_{t,t+1} \Omega_{t+1} r^{PV}_t n_t + \xi_t \lambda^{PV} m^{PV}_t} - 1, \tag{44}$$

$$\nu_t = \frac{E_t \Lambda_{t,t+1} \Omega_{t+1} r^{PV}_t n_t (1+\mu_t) + \xi_t \mu_t \lambda^{PV} m^{PV}_t}{n_t \xi_t}. \tag{45}$$

Like in the alternative intermediated CBDC model, a monetary policy rule is required to close the model.

3. Monetary Policy

This section explores optimal monetary policy rules that the central bank can implement to intervene the economy. The efficient allocation under the social planner is presented first.

3.1. Efficient Allocation

The efficient allocation is achieved under a hypothetical benevolent social planner seeking to maximize the utility of the representative household. The social planner's problem is:

$$\max_{\{C_t, m_t, L_t, K_{t+1}, I_t\}} \quad E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ \frac{1}{1-\frac{1}{\sigma}} \left[C_{t+\tau}^{1-\frac{1}{\eta}} + \omega (m_{t+r})^{1-\frac{1}{\eta}} \right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi \, \frac{L_{t+\tau}^{1+\varphi}}{1+\varphi} \right\} \tag{46}$$

subject to the aggregate resource constraint

$$C_t + I_t = A_t K_t^\gamma L_t^{1-\gamma}, \tag{47}$$

and law of motion of capital

$$K_{t+1} = (1-\delta)K_t + I_t - f\left(\frac{I_t}{I_{t-1}}\right) I_t. \tag{48}$$

The first-order optimality conditions of the social planners imply:

$$\frac{U_{m_t}}{U_{C_t}} = 0, \tag{49}$$

$$-\frac{U_{L_t}}{U_{C_t}} = (1-\gamma)A_t K_t^{\gamma} L_t^{-\gamma}, \tag{50}$$

$$q_t^F = E_t \Lambda_{t,t+1}(\gamma A_{t+1} K_{t+1}^{\gamma-1} L_{t+1}^{1-\gamma} + (1-\sigma) q_{t+1}^F), \tag{51}$$

where q_t is the Tobin's q that follows the same form of equation 36. See Appendix C for a detailed solution of the social planner's problem.

The social planner's problem implies that the marginal utility of holding money is zero. For the central bank that operates in a market economy, to achieve the efficient allocation, the optimal monetary policy must ensure that the optimality conditions of the social planner and all agents in the economy are simultaneously satisfied. From households' optimality conditions (See Appendix A for more details):

$$\frac{U_{m_t^{CB}}}{U_{C_t}} = \frac{r_t^B - r_t^{CB}}{r_t^B}, \tag{52}$$

and

$$\frac{U_{m_t^{PV}}}{U_{C_t}} = \frac{r_t^B - r_t^{PV}}{r_t^B}. \tag{53}$$

Equations 49, 52, and 53 imply the returns of household assets must be equal:

$$r_t^{CB} = r_t^B = r_t^{PV}, \tag{54}$$

or in nominal terms:

$$i_t^{CB} = i_t^B = i_t^{PV}. \tag{55}$$

Equation 55 is a modified version of the Friedman Rule. The intuition for this implication is the same as in the standard model: the opportunity costs of holding money must be equal to the social planner's cost of producing money, which is essentially zero, to bring about the efficiency of holding money. It is also worth noting that, when the returns of household assets are equal, the Lagrange multiplier in the banks' optimality conditions must also be zero ($\mu_t = 0$). This implies that the expected return on capital equals to the returns of household assets:

$$i_t^{CB} = i_t^{B} = i_t^{PV} = E_t i_{t+1}^{K}. \tag{56}$$

For the last equality, since the returns of household assets are the same, the recovery ability of each household asset must be the same when they are held in banks. This is only true when none of the asset is divertable by banks so that the agency problem between households and banks does not exist ($\mu_t = 0$). Therefore, both monetary and financial frictions are eliminated under the efficient allocation.

Unlike in the standard model, the long run inflation rate is not necessary negative under the optimal monetary policy rule. From the Euler equation (equation 10), when the long-term nominal return of bonds $i^B = \frac{1}{\beta} - 1$, the long-term inflation rate is stabilized at zero.

3.2. Optimal Monetary Policy Rule

Equation 54 ensures that the economy reaches the efficient allocation implied by a benevolent social planner, however, simply setting all returns of household assets equal leads to the indeterminacy of the system[10]. To address this issue, I characterize an optimal CBDC interest rule that leads to a unique equilibrium of the model under the efficient allocation.

Suppose that the central bank's budget constraint is

$$M_t^{CB} = M_{t-1}^{CB} + \Delta_t. \tag{57}$$

When $\Delta_t > 0$, the central bank is expanding CBDC supply in period t. When $\Delta_t < 0$, the central bank is contracting CBDC supply. Let g_t^{CB} be the growth rate of CBDC supply, the central bank's

[10]This implication is the same as in the standard money in the utility model. One common solution for this problem is to characterize a feedback monetary policy rule that leads to unique equilibrium of the model. See Chapter 2, Galí (2008).

budget constraint can be written as

$$M_t^{CB} = (1 + g_t^{CB})M_{t-1}^{CB}. \tag{58}$$

Combine with equations 6 and 7 from households' optimality conditions:

$$1 + g_t^{CB} = (1 + \pi_t)\frac{C_t}{C_{t-1}}\left(\frac{q_t^{CB}}{q_{t-1}^{CB}}\right)^{\epsilon_m}\left(\frac{p_t^M}{p_{t-1}^M}\right)^{-\eta}, \tag{59}$$

where

$$q_t^{CB} = \left[\alpha^{\varepsilon_m} + (1-\alpha)^{\varepsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}} \tag{60}$$

is the opportunity cost of money measured by unit of CBDC,

$$p_t^M = \frac{q_t^M}{r_t^B} \tag{61}$$

is the real price of money, and π_t is the inflation rate. At the steady state, $g = \pi$.

Together with the households' optimality conditions, C_t, m_t, and L_t can be expressed as functions of g_t^{CB}:

$$C_t = C(g_t^{CB}) = C_{t-1}\left(\frac{1+g_t^{CB}}{1+\pi_t}\right)\left(\frac{q_t^{CB}}{q_{t-1}^{CB}}\right)^{-\epsilon_m}\left(\frac{p_t^M}{p_{t-1}^M}\right)^{\eta}, \tag{62}$$

$$m_t = m(g_t^{CB}) = \omega^\eta C_{t-1}\left(\frac{1+g_t^{CB}}{1+\pi_t}\right)\left(\frac{q_t^{CB}}{q_{t-1}^{CB}}\right)^{-\epsilon_m}(p_{t-1}^M)^{-\eta}, \tag{63}$$

and

$$L_t = L(g_t^{CB}) = \left[\frac{w_t}{\psi}(q_t^S)^{\frac{\eta}{\sigma}-1}C(g_t^{CB})^{-\frac{1}{\sigma}}\right]^{\frac{1}{\varphi}}. \tag{64}$$

The optimal monetary policy problem of the central bank is choosing the growth rate of CBDC supply g_t^{CB} to maximize the representative household's expected lifetime discounted utility:

$$\max_{g_t^{CB}} \quad E_t\sum_{\tau=0}^{\infty}\beta^\tau U(C_{t+\tau}, m_{t+\tau}, L_{t+\tau}). \tag{65}$$

Taking the derivative with respect to g_t^{CB}, the optimality condition of the central bank's monetary policy problem implies

$$U_{C_t}C_{g_t^{CB}} + U_{m_t}m_{g_t^{CB}} + U_{L_t}L_{C_t}C_{g_t^{CB}} = 0. \tag{66}$$

A monetary policy rule that guarantees the nominal opportunity cost of money $Q_t^M = 0$ at the equilibrium is sufficient to satisfy the optimality conditions of the central bank and the social planner[11]. Consider the following CBDC interest rate rule:

$$i_t^{CB} = i_t^B - \{\phi_\pi \left[(1+\pi_t)q_{t-1}^M\right]^{\phi_s} - \phi_i(i_t^B - i_t^{PV})^{\phi_s}\}^{\frac{1}{\phi_s}}, \tag{67}$$

under which the cost of CBDC responds positively to the price of money and the cost of PvDC. Let $\phi_\pi = \frac{\phi_q^{1-\epsilon_m}}{\alpha^{\epsilon_m}}$, $\phi_i = \left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}$, and $\phi_s = 1 - \epsilon_m$. The CBDC interest rule implies the difference equation:

$$E_t Q_{t+1}^M = \phi_q Q_t^M. \tag{68}$$

For any $\phi_q > 1$, $Q_t^M = 0$ is the only stationary solution to equation 68[12]. Therefore, the optimal CBDC interest rule (equation 67) can imply the optimality conditions of the central bank (66) and the social planner (equation 49)[13].

Since g_t^{CB} is not solved directly, the budget constraint of the central bank (equation 58) cannot be used to determine the equilibrium of the model[14]. Therefore, we need another equation to close the model. Because interest on CBDC is the only payment that the central bank needs to make, the budget constraint of the central bank can be written as

$$M_t^{CB} = (1 + i_{t-1}^{CB})M_{t-1}^{CB} + e_t^{CB}, \tag{69}$$

where e_t^{CB} is the CBDC supply shock that follows a zero-mean white-noise process. Taking the

[11] $Q_t^M = q_t^M E_t(1+\pi_{t+1}) = \left[\alpha^{\varepsilon_m}(i_t^B - i_t^{CB})^{1-\varepsilon_m} + (1-\alpha)^{\varepsilon_m}(i_t^B - i_t^{PV})^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}}$.

[12] In numerical exercises, I set $Q_t^M \approx log(1+Q_t^M)$ for a small value of Q_t^M.

[13] Note that other optimality conditions of the social planner, equations 50 and 51, are consistent with the market optimality conditions. Thus, the central bank policy rule does not need to address these two conditions.

[14] Solving g_t^{CB} results in a highly non-linear function of g_t^{CB} that cannot be easily interpreted. For better understanding of the monetary policy rule, I characterize the CBDC interest rate rule instead of solving g_t^{CB} directly.

expectation of the CBDC supply gives the expected CBDC growth path:

$$E_t M_{t+1}^{CB} = (1 + i_t^{CB}) M_t^{CB}, \tag{70}$$

which will be used to determine the equilibrium of the model[15].

4. Numerical Exercises

In this section, I begin with the assessment of how well the model fits into data before CBDC is introduced by comparing the second moments of a non-CBDC regime to the data. The model of the non-CBDC regime simply subtracts the CBDC element from the benchmark model (see Appendix A). The business cycle drivers are a combination of productivity and liquidity shocks. When comparing the CBDC to non-CBDC regimes, I also analyze the impulse response functions to a liquidity shock and a technology shock. The numerical experiments are meant only to be suggestive.

4.1. Calibration

Table 1 summarizes the parameter values, descriptions, and sources. Most of the parameter values are conventional and shared across models. Some parameter values, which I focus on explaining in this section, are relatively less standard and some of them vary across models. For convenience, I use $\widetilde{m_t^{CB}}$ and $\widetilde{i_t^{CB}}$ to represent the endogenous CBDC supply rule and the CBDC interest rate rule, respectively.

Similar to Sims and Wu (2021), I target a steady-state excess return of capital over the PvDC, $r^K - r^{PV}$, of 75 basis points and that of the excess return of private bonds over the PvDC, $r^B - r^{PV}$, of 25 basis points (both are on quarterly basis). This results in the fraction of recoverable PvDC is $\lambda^{PV} \approx 0.07$, and the fraction of recoverable private bonds is $\lambda^B \approx 0.33$ under the non-CBDC and the independent CBDC regimes when CBDC has no direct impact on banks' optimality conditions. To maintain tractability, I assume the relative recovery ability of PvDC to bonds, $\frac{\lambda^{PV}}{\lambda^B}$, remains the same across models. This helps to keep the values of λ^B and λ^{PV} in a reasonable range. When

[15] Following the model section, the CBDC monetary policy rule and the expected CBDC growth are required to close the alternative intermediated and independent CBDC models, where CBDC and its return are exogenous to the banks.

Table 1: Parameterization

Parameter	Value	Description	Source
Households			
β	0.995	Discount factor	Standard
η	0.22	Elasticity of money demand	Piazzesi et al. (2022)
ω	0.14	Weight of money in the utility	Piazzesi et al. (2022)
φ	0.1	Inverse Frisch elasticity of labor supply	Gertler and Kiyotaki (2010)
ψ	3.409	Labor disutility scaling parameter	Gertler and Karadi (2011)
σ	1	Intertemporal elasticity of substitution	Standard
CBDC related			
ϵ_m	6	Elasticity of substitution between CBDC and PvDC	Mid-range in literature
α	0.5	Weight of CBDC in the utility	No CBDC bias
Non-financial firms			
γ	0.33	Effective capital share	Standard
δ	0.025	Depreciation rate	Standard
ϕ_K	2	Capital adjustment cost coefficient	Standard
ϕ_a	0.95	AR productivity	Standard
Banks			
θ	0.95	Bank survival probability	Sims and Wu (2019)
λ^B	0.33	Fraction of recoverable bonds (see table notes)	Spread targets
	0.39	λ^B (intermediated CBDC, $\widetilde{m_t^{CB}}$)	Fixed $\frac{\lambda^{PV}}{\lambda^B}$
	0.50	λ^B (intermediated CBDC, $\widetilde{i_t^{CB}}$)	long-term i^{CB} =0
λ^{PV}	0.07	Fraction of recoverable PvDC (see table notes)	Spread targets
	0.08	λ^{PV} (intermediated CBDC with $\widetilde{m_t^{CB}}$)	Fixed $\frac{\lambda^{PV}}{\lambda^B}$
	0.10	λ^{PV} (intermediated CBDC with $\widetilde{i_t^{CB}}$)	long-term i^{CB} =0
ϖ	0.045%	Fraction of bank start-up transfer	Small effect of banker entry-exit
ϕ_ξ	0.95	AR liquidity	Standard

Notes: The baseline values for the fraction of recoverable bonds and PvDC apply to the non-CBDC and independent CBDC model where CBDC does not appear on banks' balance sheets.

a monetary policy rule is endogenous, the steady-state real CBDC return is determined by the bank's optimality conditions; when a monetary policy rule is exogenous, the steady-state CBDC rate is set to 0 to match zero steady-state inflation rate. With the spread targets and steady-state returns, I obtain $\lambda^B \approx 0.39$ under intermediated CBDC with $\widetilde{m_t^{CB}}$ and $\lambda^B \approx 0.50$ with $\widetilde{i_t^{CB}}$. The corresponding fractions of recoverable PvDC are $\lambda^{PV} \approx 0.08$ with $\widetilde{m_t^{CB}}$ and $\lambda^{PV} \approx 0.10$ with $\widetilde{i_t^{CB}}$. I choose 0.95 for θ, the survival probability for banks, and 0.045% for ϖ, a small proportional transfers to new entering banks. They are in line with the literature.

For the CBDC-related parameters, α is assumed to be 0.5, which indicates no CBDC bias when households choose liquidity services. The elasticity of substitution between CBDC and PvDC ϵ_m is set at 6. This value is around the median in CBDC literature[16]. A robustness check of ϵ_m is available in Appendix D[17].

[16]See Cirelli and Nyffenegger (2024).

[17]The values of α and ϵ_m must satisfy the arbitrage condition between CBDC and PvDC implied by equations 7 and 8. Assigning values for α and ϵ_m is confined by the steady-state returns of household assets.

4.2. Second Moments

Table 2 shows how the the model fit into the data before CBDC is introduced. Data moments are calculated based on U.S. data from 1983Q1 to 2007Q3. Data volatilities for output growth ($\widehat{Y_t}$), consumption growth ($\widehat{C_t}$), investment growth ($\widehat{I_t}$), and labor hour growth ($\widehat{L_t}$), are unconditional standard deviations of their quarterly growth data expressed at annualized percentage rate. Cyclicalities are correlations with output growth. For moments in the models, the business cycle drivers are a combination of productivity and liquidity shocks that follow AR(1) processes. The standard deviations of the productivity and liquidity shocks are 0.3% and 2.6% receptively. I choose these numbers to loosely match the moments from the selected dataset. Due to the high non-linearity of the models, I obtain the implied moments by simulating the models for one hundred thousand periods. As shown in Table 2, the volatilities and cyclicalities of the key macroeconomic variables from the non-CBDC model are reasonably in line with the data.

Table 2: Volatility and Cyclicality: Data v.s. Non-CBDC Regime

		$\widehat{Y_t}$	$\widehat{C_t}$	$\widehat{I_t}$	$\widehat{L_t}$
Volatility	Data	2.27	1.62	8.66	2.57
	No CBDC	2.40	1.60	8.42	2.05
Cyclicality	Data	1	0.51	0.83	0.61
	No CBDC	1	0.58	0.86	0.76

Notes: For output, investment, consumption, I compute real values by dividing the GDP deflator. Labor represents the hours worked for all workers in the nonfarm business sector. The volatility of data for $\widehat{Y_t}$, $\widehat{C_t}$, $\widehat{I_t}$, $\widehat{L_t}$ corresponds to their growth rates measured at annualized percentage rate from 1983 Q1 and ending in 2007 Q3. Data for Output, investment (gross domestic investment plus durable good consumption), consumption (personal consumption expenditure less durables), and GDP deflator is from the U.S. Bureau of Economic Analysis. Data for labor hours is from the Bureau of Labor Statistics. Values for volatility are expressed in percentage term.

Table 3 compares volatilities under the non-CBDC and all the CBDC regimes. It shows that all CBDC regimes reduce volatility relative to the non-CBDC baseline. The independent CBDC regime and the intermediated CBDC regime with an endogenous policy rule deliver similar, moderate improvements: output, consumption, and labor volatilities each fall by less than 4%, and investment volatility declines by less than 11%. In comparison, intermediated CBDC regimes with the CBDC interest rate rule achieve substantially larger reductions. Output volatility falls by around 12%, consumption by around 9%, investment by around 32%, and labor by over 22%.

Table 3. Volatility: Non-CBDC v.s. CBDC Regimes

		$\widehat{Y_t}$	$\widehat{C_t}$	$\widehat{I_t}$	$\widehat{L_t}$
Volatility	No CBDC	2.40	1.60	8.42	2.05
	Intermediated CBDC ($\widetilde{m_t^{CB}}$)	2.34	1.54	7.51	2.00
	Intermediated CBDC ($\widetilde{i_t^{CB}}$)	2.11	1.44	5.78	1.60
	Independent CBDC ($\widetilde{i_t^{CB}}$)	2.36	1.56	7.60	1.99

Notes: $\widetilde{m_t^{CB}}$ denotes the regime under the endogenous CBDC supply rule. $\widetilde{i_t^{CB}}$ denotes the regime under the CBDC interest rate rule. All values represent standard deviations expressed in percentage term.

4.3. Impulse Response Functions

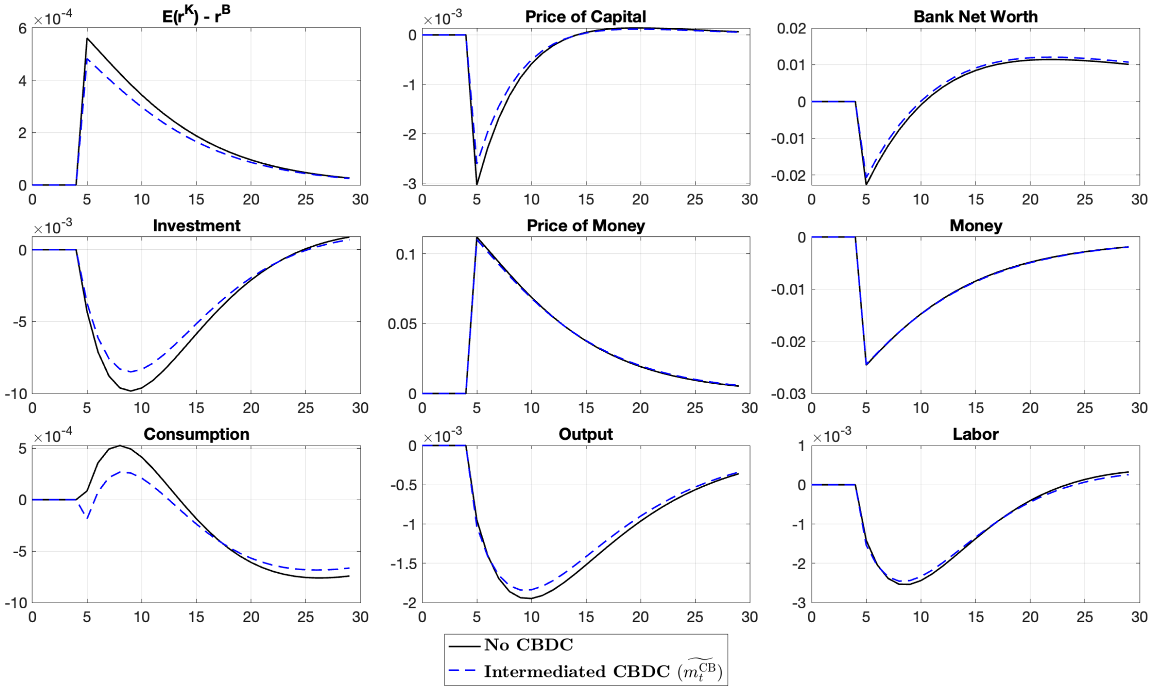
The second moments from the models show that the introduction of CBDC dampens business cycle volatilities in comparison to the non-CBDC baseline. In this subsection, I analyze the impulse response functions and explain the mechanism behind the mitigation effects. The analysis focuses on two exogenous shocks: a liquidity shock and a technology shock. The size of liquidity shock is a 1% standard deviation and the persistence is 0.95. Such a positive shock makes it harder for households to recover their funds in the event of bank failure and, other things equal, reduces their willingness to lend to banks. The size of technology shock is -1%, also with persistence 0.95. This negative shock lowers total factor productivity and thus reduces the efficiency of both capital and labor in production. In each of the figures below, the shock kicks in period 5. The vertical axis represents the percentage change from the steady state. The price of money at period t is defined as $\frac{q_t^M}{r_t^B}$, where money is the sum of m_t^{CB} and m_t^{PV}. Appendix E complements this analysis by providing impulse responses for a -1% liquidity shock and a 1% technology shock.

Across these experiments, I find that the introduction of CBDC affects business cycle fluctuations through two main channels: a balance sheet channel and a substitution channel. The strength and implications of these channels differ across the CBDC regimes.

Intermediated CBDC ($\widetilde{m_t^{CB}}$)

Figure 1 compares the responses of the intermediated CBDC regime with an endogenous money supply rule and the non-CBDC regime under 1% liquidity shock. The solid black line represents the non-CBDC regime and the blue dashed line represents the intermediated CBDC regime ($\widetilde{m_t^{CB}}$). The liquidity shock makes households less willing to lend to banks, tightening banks' borrowing constraint. As in the standard model, this tighter constraint triggers the sale of capital, leading to a drop in the price of capital. The fall in capital prices induces a decline in investment and output, while the expected excess return on capital rises. The higher cost of capital further depresses investment and output. At the same time, the liquidity shock increases the prices of both PvDC and CBDC, causing the demands for both liquidity services to fall.

Figure 1: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under 1% Liquidity Shock



Comparing the intermediated CBDC regime ($\widetilde{m_t^{CB}}$) to the non-CBDC regime, the introduction of CBDC relaxes banks' borrowing constraint through a balance sheet channel: because CBDC is non-divertible, its presence on banks' balance sheets makes it easier for households to recover funds in the event of bank default. As a result, households are more willing to lend to banks when CBDC is held by banks. This mechanism mitigates the declines in output and investment. On the other hand, the banks' optimality conditions also imply an excess return of PvDC over CBDC. The increase in the price of money and the decline in money demand are larger in the regime with CBDC than in the regime with no CBDC. To see this, express the price of money under CBDC regimes as

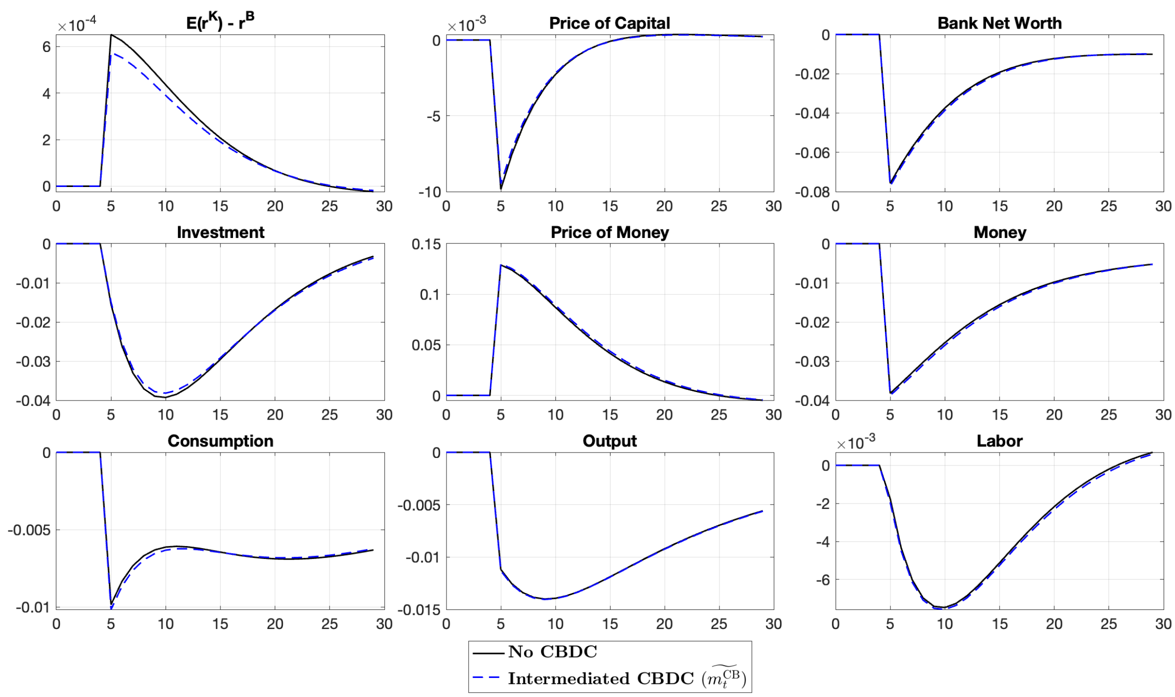
$$\frac{q_t^M}{r_t^B} = \left[\alpha^{\varepsilon_m}\left(\frac{r_t^B - r_t^{CB}}{r_t^B}\right)^{1-\varepsilon_m} + (1-\alpha)^{\varepsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B}\right)^{1-\varepsilon_m}\right]^{\frac{1}{1-\varepsilon_m}}, \tag{71}$$

where $\frac{r_t^B - r_t^{CB}}{r_t^B}$ and $\frac{r_t^B - r_t^{PV}}{r_t^B}$ are price of CBDC and price of PvDC, respectively. While in the non-CBDC regime, the price of money is simply $\frac{r_t^B - r_t^{PV}}{r_t^B}$. Given the price of PvDC increases in the same degree, a larger increase in the price of CBDC than the price of PvDC under the CBDC regimes makes the overall price of money more elevated.

Due to the excess return of PvDC over CBDC under the interemdeiated CBDC ($\widetilde{m_t^{CB}}$), the price of CBDC increases more than the price of PvDC[18]. Therefore, the substitution effect between PvDC and CBDC leads households to rebalance their money holdings toward more PvDC. Since CBDC is easier for households to recover, reducing demand for CBDC tightens banks' borrowing constraints. In this case, the substitution channel amplifies, rather than mitigates, the decline in output and investment. In figure 1, this amplification effect is translated to nearly indifferent responses of price of money and money demand under both regimes even if the responses of other variables under intermediated CBDC are mitigated.

Overall, under the 1% liquidity shock, in the intermediated CBDC regime ($\widetilde{m_t^{CB}}$), the mitigating effect through the balance sheet channel dominates the amplifying effect through the substitution channel: the drops in output and investment are reduced, and the volatility of other variables such as consumption, labor, and bank net worth is dampened.

Figure 2: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under -1% Technology Shock



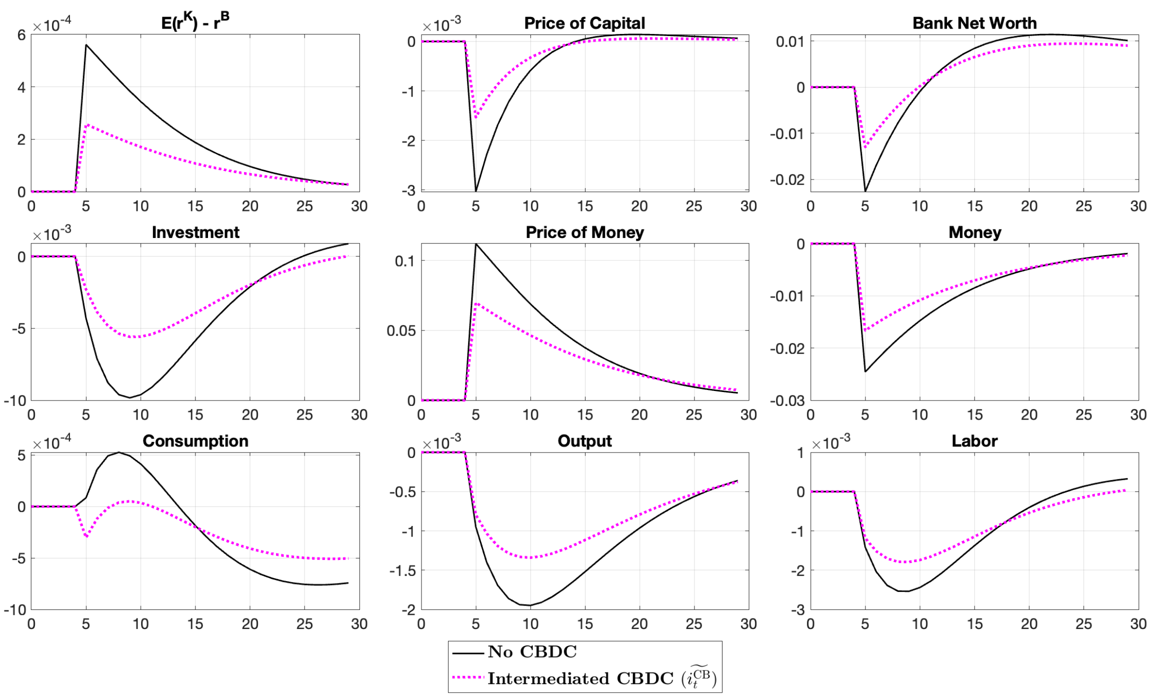
[18]Given a small value of λ^{PV} (0.08), this effect is relatively weak.

Figure 2 illustrates the responses to a -1% technology shock. The technology shock directly dampens output and investment, and the borrowing constraint amplifies this effect by raising the cost of capital and reducing the price of capital, which further depresses investment and output. As a result, the declines in investment and output are more significant than under the liquidity shock. However, unlike the liquidity shock, the technology shock does not directly affect how easily households can recover funds, so the presence of CBDC only provides a modest mitigation effect through the balance sheet channel. At the same time, the tightening effect arising from lower demand for CBDC dominates this limited balance-sheet mitigation, causing the initial drops in consumption, output, and labor to be slightly larger under the intermediated CBDC regime ($\widetilde{m_t^{CB}}$).

Intermediated CBDC ($\widetilde{i_t^{CB}}$)

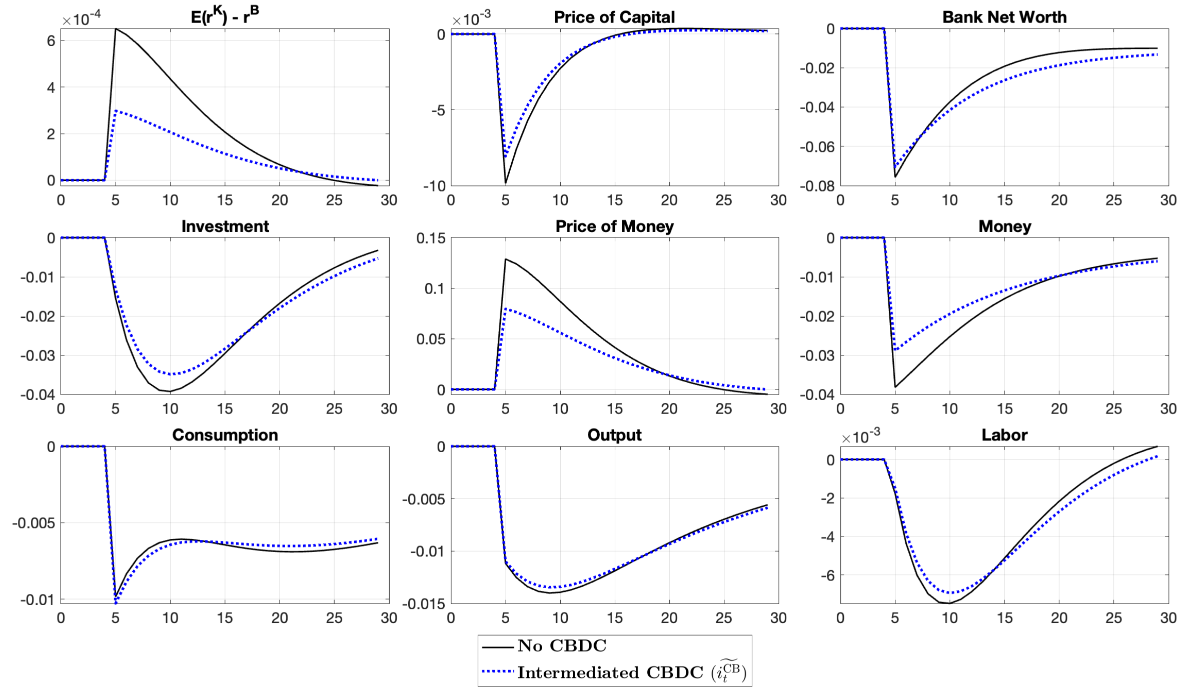
Figures 3 and 4 compare the responses of the intermediated CBDC regime ($\widetilde{i_t^{CB}}$) and the non-CBDC regime to a 1% liquidity shock and a -1% technology shock, respectively. In both figures, the solid black line represents the non-CBDC regime, and the magenta dotted line represents the intermediated CBDC regime ($\widetilde{i_t^{CB}}$).

Figure 3: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under 1% Liquidity Shock



Similar to the intermediated CBDC regime ($\widetilde{m_t^{CB}}$), the introduction of CBDC relaxes banks'

Figure 4: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under -1% Technology Shock



borrowing constraints through the balance sheet channel. Additionally, in the intermediated CBDC regime ($\widetilde{i_t^{CB}}$), the banks' optimality conditions no longer imply an excess return of PvDC over CBDC. With the CBDC interest rate rule, the price and demand for CBDC are stabilized, which in turn smooths the price and demand of PvDC through the substitution channel. As a result of this substitution effect, the overall increase in the price of money and the drop in money demand are mitigated, which further alleviates the declines in output and investment and dampens the volatilities of other variables such as consumption, labor, and bank net worth. Compared to the responses under the intermediated CBDC regime ($\widetilde{m_t^{CB}}$), the mitigation effects due to the introduction of CBDC under the intermediated CBDC regime ($\widetilde{i_t^{CB}}$) are more salient as both balance sheet and substitution channels are effective.

Independent CBDC Regime ($\widetilde{i_t^{CB}}$)

Under the independent regime, since CBDC does not appear on banks' balance sheets, the balance sheet channel is muted, and the mitigation arises entirely through the substitution channel. In figure 5 and 6, the solid black line corresponds to the non-CBDC regime, and the red dot–dash line depicts the independent CBDC regime ($\widetilde{i_t^{CB}}$). Overall, the magnitude of mitigation is similar

Figure 5: non-CBDC v.s. Independent CBDC ($\widetilde{i_t^{CB}}$) under 1% Liquidity Shock

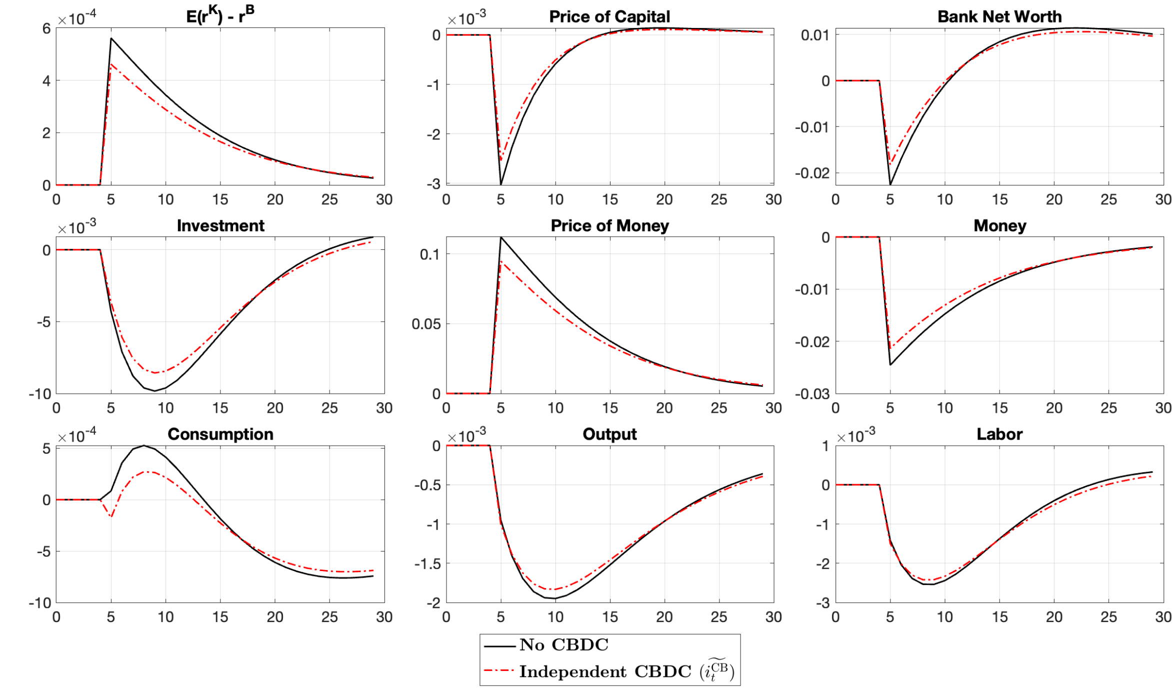
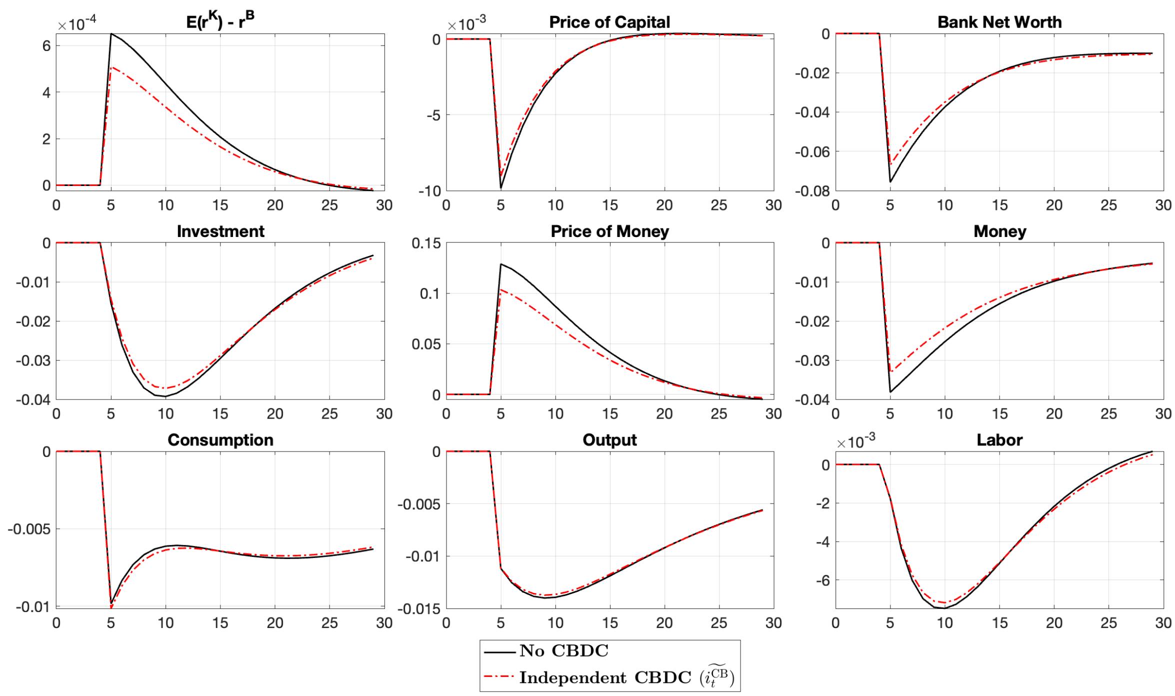


Figure 6: non-CBDC v.s. Independent CBDC ($\widetilde{i_t^{CB}}$) under -1% Technology Shock



to that observed under the intermediated CBDC regime ($\widetilde{m_t^{CB}}$), however, the increase in price of money and the decline in money demand are mitigated under the independent regime.

5. Welfare Analysis

In this section, I estimate the consumption-equivalence welfare (CE welfare) of all CBDC regimes in comparison to the non-CBDC baseline, following the standard CE welfare calculation method. I calculate a λ^W such that the household utilities under both scenarios are equal: $V_t^{CBDC} = V_t^{NoCBDC}(\lambda^W)$, where

$$V_t^{CBDC} = \frac{1}{1-\frac{1}{\sigma}}\left[(C_t)^{1-\frac{1}{\eta}} + \omega\left(\frac{M_t}{P_t}\right)^{1-\frac{1}{\eta}}\right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi\frac{L_t^{1+\varphi}}{1+\varphi} + \beta V_{t+1}^{CBDC}, \tag{72}$$

$$V_t^{NoCBDC}(\lambda^W) = \frac{1}{1-\frac{1}{\sigma}}\left[((1+\lambda^W)C_t)^{1-\frac{1}{\eta}} + \omega\left(\frac{M_t}{P_t}\right)^{1-\frac{1}{\eta}}\right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi\frac{L_t^{1+\varphi}}{1+\varphi} + \beta V_{t+1}^{NoCBDC}(\lambda^W). \tag{73}$$

If λ^W is greater than zero, it indicates that the CBDC regime improves CE welfare compared to the non-CBDC regime. Welfare is computed using a second-order approximation over 100,000 periods. Because CBDC models assume a CES structure between PvDC and CBDC, the steady-state real balance entering the utility function is lower than that of the non-CBDC baseline, which also implies a lower steady-state level of consumption. To isolate the welfare effects arising from policy and not from this modeling artifact, I scale the steady-state values of real balances and consumption in the CBDC regimes so that they match those in the non-CBDC model during the simulation.

Table 4. CE Welfare: Non-CBDC v.s. CBDC Regimes

CE Welfare	No CBDC	Baseline
	Intermediated CBDC ($\widetilde{m_t^{CB}}$)	0.82%
	Intermediated CBDC ($\widetilde{i_t^{CB}}$)	1.18%
	Independent CBDC ($\widetilde{i_t^{CB}}$)	0.52%

Notes: The welfare results are estimated under second order approximation over one hundred thousand periods. The steady state real balances and consumptions in the CBDC regimes are adjuested to match those in the non-CBDC baseline.

As shown in Table 3, all CBDC regimes raise consumption-equivalence welfare, with the gains generally larger under the intermediated CBDC regimes. The intermediated CBDC regimes with the CBDC interest rate rule delivers the best outcome, with CE welfare rising by 1.18%.

6. Conclusion

This paper has shown that introducing a CBDC into the economy can help mitigate the effects of financial frictions, stabilize business cycle fluctuations, and improve welfare. These benefits rely crucially on the safety advantage that central banks often claim for CBDC, but this advantage must be combined with suitable CBDC design features. When central banks intermediate CBDC through banks and allow banks to use it as a funding source, the safety of CBDC works through a balance sheet channel that promotes stabilization: because CBDC is safe, its presence on banks' balance sheets makes it easier for households to recover their funds in the event of a bank default, thereby households are more willing to lend to banks. This balance sheet channel is not effective if central banks do not intermediate CBDC through financial intermediaries. In that case, a CBDC monetary policy rule becomes essential, with which the stabilization operating through the substitution channel between CBDC and PvDC that smooths the price and demand for money. In my analysis, a CBDC interest rate rule that aims to achieve the efficient allocation under the benevolent social planner is sufficient to generate this substitution effect. The quantitative results in this paper are robust. Yet, given the limited empirical evidence on CBDC at the time of writing this paper, several aspects of the long-term properties of a CBDC economy remain to be explored. For example, how would the introduction of CBDC affect long-term asset returns? What would be the degree of substitutability between CBDC and other liquidity services? Would households exhibit a systematic preference for CBDC when choosing among liquidity services? Only once CBDC becomes widely adopted in everyday life would these questions be fully answered. Nonetheless, despite these limitations, this paper contributes to our understanding of how the introduction of CBDC can affect the economy both qualitatively and quantitatively.

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Appendix A. Households

For convenience, I directly solve the household maximization problem under the real terms. The solving process with nominal terms is similar. The household maximization problem is:

$$\max_{\{C_t, m_t^{CB}, m_t^{PV}, b_t, L_t\}} \quad E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \frac{1}{1-\frac{1}{\sigma}} \left[C_{t+\tau}^{1-\frac{1}{\eta}} + \omega \left(m_{t+\tau}\right)^{1-\frac{1}{\eta}} \right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi \frac{L_{t+\tau}^{1+\varphi}}{1+\varphi} \right\}$$

$$s.t. \quad C_t + m_t^{CB} + m_t^{PV} + b_t \leq w_t L_t + r_{t-1}^{CB} m_{t-1}^{CB} + r_{t-1}^{PV} m_{t-1}^{PV} + r_{t-1}^{B} b_{t-1} + d_t,$$

where $m_t = \left[\alpha \left(m_t^{CB}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} + (1-\alpha) \left(m_t^{PV}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \right]^{\frac{\epsilon_m}{\epsilon_m - 1}}$ is the real balance composite.

Denote the bundle of consumption and real balance as:

$$X_t = \left[C_t^{1-\frac{1}{\eta}} + \omega \left(m_t\right)^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}}.$$

The first-order optimality conditions imply:

$$\begin{aligned}
[C_t] \quad & X_t^{\frac{1}{\eta}-\frac{1}{\sigma}} C_t^{-\frac{1}{\eta}} = \lambda_t, \\
[L_t] \quad & \psi L_t^{\varphi} = \lambda_t w_t, \\
[m_t^{CB}] \quad & \alpha \omega X_t^{\frac{1}{\eta}-\frac{1}{\sigma}} \left(m_t\right)^{\frac{1}{\epsilon_m}-\frac{1}{\eta}} \left(m_t^{CB}\right)^{-\frac{1}{\epsilon_m}} = \lambda_t - \beta r_t^{CB} E_t \lambda_{t+1}, \\
[m_t^{PV}] \quad & (1-\alpha) \omega X_t^{\frac{1}{\eta}-\frac{1}{\sigma}} \left(m_t\right)^{\frac{1}{\epsilon_m}-\frac{1}{\eta}} \left(m_t^{PV}\right)^{-\frac{1}{\epsilon_m}} = \lambda_t - \beta r_t^{PV} E_t \lambda_{t+1}, \\
[b_{t+1}] \quad & \beta r_{t+1}^{B} E_t \lambda_{t+1} = \lambda_t.
\end{aligned}$$

where λ_t is the Lagrange multiplier. Arrange the above equations:

$$\begin{aligned}
\alpha \omega \left(m_t\right)^{\frac{1}{\epsilon_m}-\frac{1}{\eta}} \left(m_t^{CB}\right)^{-\frac{1}{\epsilon_m}} &= C_t^{-\frac{1}{\eta}} \left(\frac{r_t^{B} - r_t^{CB}}{r_t^{B}} \right), \\
and \quad (1-\alpha) \omega \left(m_t\right)^{\frac{1}{\epsilon_m}-\frac{1}{\eta}} \left(m_t^{PV}\right)^{-\frac{1}{\epsilon_m}} &= C_t^{-\frac{1}{\eta}} \left(\frac{r_t^{B} - r_t^{PV}}{r_t^{B}} \right).
\end{aligned}$$

The above two equations give the arbitrage condition between CBDC and PvDC:

$$\frac{\alpha}{1-\alpha}\left(\frac{m_t^{PV}}{m_t^{CB}}\right)^{\frac{1}{\varepsilon_m}} = \frac{r_t^B - r_t^{CB}}{r_t^B - r_t^{PV}}.$$

Plug the arbitrage condition into the real balance composite equation, we get:

$$\begin{aligned}
m_t &= \left[\alpha\left(m_t^{CB}\right)^{\frac{\epsilon_m-1}{\epsilon_m}} + (1-\alpha)\left(m_t^{PV}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\right]^{\frac{\epsilon_m}{\epsilon_m-1}} \\
&= \left\{\alpha\left(m_t^{CB}\right)^{\frac{\epsilon_m-1}{\epsilon_m}} + (1-\alpha)\left[m_t^{CB}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{-\epsilon_m}\left(\frac{\alpha}{1-\alpha}\right)^{-\epsilon_m}\right]^{\frac{\epsilon_m-1}{\epsilon_m}}\right\}^{\frac{\epsilon_m}{\epsilon_m-1}} \\
&= \left\{\alpha\left(m_t^{CB}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]\right\}^{\frac{\epsilon_m}{\epsilon_m-1}}.
\end{aligned}$$

Define the cost for holding one unit of m_t^{CB} as $r_t^B - r_t^{CB}$ and the cost for holding one unit of m_t^{PV} as $r_t^B - r_t^{PV}$. The total cost of real balance holding is:

$$\begin{aligned}
O_t &= \left(r_t^B - r_t^{CB}\right)m_t^{CB} + \left(r_t^B - r_t^{PV}\right)m_t^{PV} \\
\Longrightarrow \frac{O_t}{r_t^B - r_t^{CB}} &= m_t^{CB} + \left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)m_t^{PV} \\
\Longrightarrow \frac{O_t}{r_t^B - r_t^{CB}} &= m_t^{CB} + m_t^{CB}\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m} \\
&= m_t^{CB}\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right] \\
\Longrightarrow m_t^{CB} &= \left(\frac{O_t}{r_t^B - r_t^{CB}}\right)\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]^{-1}.
\end{aligned}$$

Combine it with the real balance composite equation:

$$\begin{aligned}
m_t &= \left\{\alpha\left(m_t^{CB}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]\right\}^{\frac{\epsilon_m}{\epsilon_m-1}} \\
&= \left\{\alpha\left(\frac{O_t}{r_t^B - r_t^{CB}}\right)^{\frac{\epsilon_m-1}{\epsilon_m}}\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]^{\frac{1}{\epsilon_m}}\right\}^{\frac{\epsilon_m}{\epsilon_m-1}} \\
&= \alpha^{\frac{\epsilon_m}{\epsilon_m-1}}\left(\frac{O_t}{r_t^B - r_t^{CB}}\right)\left[1+\left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m}\left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]^{\frac{1}{\epsilon_m-1}}
\end{aligned}$$

$$= O_t \left[\alpha^{\epsilon_m}(r_t^B - r_t^{CB})^{1-\epsilon_m} + (1-\alpha)^{\epsilon_m}(r_t^B - r_t^{PV})^{1-\epsilon_m}\right]^{\frac{1}{\epsilon_m - 1}}.$$

Denote q_t^M as the optimal cost of purchasing one unit of m_t:

$$q_t^M = O_t|_{m_t=1} = \left[\alpha^{\epsilon_m}(r_t^B - r_t^{CB})^{1-\epsilon_m} + (1-\alpha)^{\epsilon_m}(r_t^B - r_t^{PV})^{1-\epsilon_m}\right]^{\frac{1}{1-\epsilon_m}}.$$

Also,

$$\begin{aligned} m_t &= \left\{\alpha \left(m_t^{CB}\right)^{\frac{\epsilon_m - 1}{\epsilon_m}} \left[1 + \left(\frac{1-\alpha}{\alpha}\right)^{\epsilon_m} \left(\frac{r_t^B - r_t^{PV}}{r_t^B - r_t^{CB}}\right)^{1-\epsilon_m}\right]\right\}^{\frac{\epsilon_m}{\epsilon_m - 1}} \\ &= \alpha^{-\epsilon_m}(r_t^B - r_t^{CB})^{\epsilon_m}(q_t^M)^{-\epsilon_m} m_t^{CB}. \end{aligned}$$

Combined with the first-order conditions, the optimal demand for real balance composite is given by

$$m_t = C_t \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{-\eta}.$$

Define the cost of making one unit of consumption C_t as r_t^B. The total cost of consumption and real balance bundle is:

$$F_t = r_t^B C_t + q_t^M m_t^B.$$

For convenience, measure the cost of bundle in terms of unit of consumption:

$$\begin{aligned} F_t' &= \frac{F_t}{r_t^B} = C_t + \frac{q_t^M}{r_t^B} m_t^B \\ \implies X_t &= \left[C_t^{1-\frac{1}{\eta}} + \omega \left(C_t \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{-\eta}\right)^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\frac{1}{\eta}}} \\ &= C_t \left[1 + \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{1-\eta}\right]^{\frac{1}{1-\frac{1}{\eta}}} \\ &= \frac{F_t'}{\left[1 + \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{1-\eta}\right]} \left[1 + \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{1-\eta}\right]^{\frac{1}{1-\frac{1}{\eta}}}. \end{aligned}$$

Denote q_t as the optimal cost of purchasing one unit of X_t:

$$q_t^S = F_t'|_{X_t=1} = \left[1 + \omega^\eta \left(\frac{q_t^M}{r_t^B}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Then, $X_t = (q_t^S)^{-\eta} C_t$, which solves the real CBDC and PvDC demands, labor supply schedule, and Euler equation:

$$\begin{aligned}
m_t^{CB} &= \alpha^{\varepsilon_m} \left(\frac{q_t^M}{r_t^B - r_t^{CB}}\right)^{\varepsilon_m} m_t^B, \\
m_t^{PV} &= (1-\alpha)^{\varepsilon_m} \left(\frac{q_t^M}{r_t^B - r_t^{PV}}\right)^{\varepsilon_m} m_t^B, \\
w_t &= \psi L_t^\varphi C_t^{\frac{1}{\sigma}} (q_t^S)^{1-\frac{\eta}{\sigma}}, \\
\beta E_t &\left[\left(\frac{q_{t+1}^S}{q_t^S}\right)^{\frac{\eta}{\sigma}-1} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\right] r_{t+1}^B = 1.
\end{aligned}$$

When there is no CBDC, m_t is simply m_t^{PV}, the opportunity cost of money is $r_t^B - r_t^{PV}$. Following similar solving process, the first-order optimality conditions imply: the opportunity cost for a bundle of consumption and money

$$q_t^S = \left[1 + \omega^\eta \left(\frac{r_t^B - r_t^{PV}}{r_t^B}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

and real money demand

$$m_t = C_t \omega^\eta \left(\frac{r_t^B - r_t^{PV}}{r_t^B}\right)^{-\eta}.$$

Conditions for labor supply schedule and Euler equation remain the same as those with CBDC.

Appendix B. Banks

For interemediated CBDC regime with endogenous CBDC supply rule, the dynamic program of banker j is given by

$$V_{j,t}\left(n_{j,t}\right) = E_t\Lambda_{t,t+1}\left\{(1-\theta)n_{j,t+1} + \theta \max_{m_{j,t}^{PV}}\left[\max_{S_{j,t},b_{j,t}} V_{j,t+1}(n_{j,t+1})\right]\right\},$$

subject to

$$\begin{aligned}
q_t^F S_{j,t} =& n_{j,t} + m_{j,t}^{PV} + m_{j,t}^{CB} + b_{j,t},\\
n_{j,t+1} =& (E_t r_{t+1}^K - r_{j,t}^{CB})q_t^F S_{j,t} - (r_{j,t}^B - r_t^{CB})b_{j,t} - (r_{j,t}^{PV} - r_{j,t}^{CB})m_{j,t}^{PV} + r_{j,t}^{CB} n_{j,t},\\
V_{j,t} \geq& \xi_t(q_t^F S_{j,t} - \lambda^B b_{j,t} - \lambda^{PV} m_{j,t}^{PV} - m_{j,t}^{CB}).
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
\mathcal{L} =& (1+\mu_t)E_t\Lambda_{t,t+1}\Big\{(1-\theta)\Big[(r_{t+1}^K - r_{j,t}^{CB})q_t^F S_{j,t} - (r_{j,t}^{PV} - r_{j,t}^{CB})m_{j,t}^{PV} - (r_{j,t}^B - r_{j,t}^{CB})b_{j,t}\\
&+ r_{j,t}^{CB} n_{j,t}\Big] + \theta V_{j,t+1}\Big\} - \mu_t(q_t^F S_{j,t} - \lambda^B b_{j,t} - \lambda^{PV} m_{j,t}^{PV} - m_{j,t}^{CB}).
\end{aligned}$$

With identical banks, the first-order optimality conditions imply:

$$\begin{aligned}
[S_t] \quad & E_t\Lambda_{t,t+1}\left[(1-\theta)\left(r_{t+1}^K - r_t^{CB}\right) + \theta\frac{\partial V_{t+1}}{\partial S_t}\right] = \frac{\xi_t\mu_t}{1+\mu_t},\\
[b_t] \quad & E_t\Lambda_{t,t+1}\left[(1-\theta)\left(r_{t+1}^B - r_t^{CB}\right) + \theta\frac{\partial V_{t+1}}{\partial b_t}\right] = \frac{\lambda^B\xi_t\mu_t}{1+\mu_t},\\
[m_t^{PV}] \quad & E_t\Lambda_{t,t+1}\left[(1-\theta)\left(r_t^{PV} - r_t^{CB}\right) + \theta\frac{\partial V_{t+1}}{\partial m_t^{PV}}\right] = \frac{\lambda^{PV}\xi_t\mu_t}{1+\mu_t}.
\end{aligned}$$

Guess the value function is linear in bank net worth, $V_{t+1} = \xi_t\nu_t n_t$, the first-order conditions can be written as:

$$\begin{aligned}
[S_t] \quad & E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_{t+1}^K - r_t^{CB}\right) = \frac{\xi_t\mu_t}{1+\mu_t},\\
[b_t] \quad & E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_t^B - r_t^{CB}\right) = \frac{\lambda^B\xi_t\mu_t}{1+\mu_t},
\end{aligned}$$

$$[m_t^{PV}] \quad E_t\Lambda_{t,t+1}\Omega_{t+1}\left(r_t^{PV} - r_t^{CB}\right) = \frac{\lambda^{PV}\xi_t\mu_t}{1+\mu_t}.$$

where $\Omega_{t+1} = 1 - \theta + \theta\nu_{t+1}$. Substitute the first-order conditions into the value function:

$$V_t = \frac{\xi_t\mu_t}{1+\mu_t}q_t^F S_t - \frac{\lambda^B\xi_t\mu_t}{1+\mu_t}b_t - \frac{\lambda^{PV}\xi_t\mu_t}{1+\mu_t}m_t^{PV} + E_t\Lambda_{t,t+1}\Omega_{t+1}r_t^{CB}n_t,$$

which implies:

$$\xi_t\nu_t n_t = \frac{\mu_t}{1+\mu_t}(\xi_t\nu_t n_t + \xi_t m_t^{CB}) + E_t\Lambda_{t,t+1}\Omega_{t+1}r_t^{CB}n_t.$$

Solve for ν_t and μ_t:

$$\nu_t = E_t\Lambda_{t,t+1}\Omega_{t+1}r_{t+1}^{PV}(1+\mu_t),$$

$$\mu_t = \frac{\xi_t(Q_t^f S_t - \lambda^B b_t - \lambda^{PV}m_t^{PV})}{E_t\Lambda_{t,t+1}\Omega_{t+1}r_t^{CB}n_t + \xi_t m_t^{CB}} - 1.$$

Under the intermediated CBDC regime that CBDC elements are given, the key difference is the law of motion for bank net worth. Since CBDC demand and return are exogenous to the banks, bank j's net worth evolves as:

$$n_{j,t+1} = (E_t r_{t+1}^K - r_{j,t}^{CB})q_t^F S_{j,t} - (r_{j,t}^B - r_t^{PV})b_{j,t} + (r_{j,t}^{PV} - r_{j,t}^{CB})m_{j,t}^{CB} + r_{j,t}^{PV}n_{j,t}.$$

Following similar setup and take first-order conditions only with respect to S_t and b_t will yield the banks' optimality condition under the regime.

For independent CBDC regime, simply subtract m_t^{CB} from bank balance sheet and follow similar steps. Under both interdemiated CBDC regime with the CBDC interest rate rule and independent CBDC regime, conditions for m_t^{PV} will be endogenously determined without banks taking first-order conditions with respect to it.

Appendix C. Social Planner

The social planner's maximization problem is

$$\max_{\{C_t, m_t, L_t, K_{t+1}, I_t\}} \quad U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left\{ \frac{1}{1-\frac{1}{\sigma}} \left[C_{t+\tau}^{1-\frac{1}{\eta}} + \omega (m_{t+\tau})^{1-\frac{1}{\eta}} \right]^{\frac{1-\frac{1}{\sigma}}{1-\frac{1}{\eta}}} - \psi \frac{L_{t+\tau}^{1+\varphi}}{1+\varphi} \right\}$$

subject to the aggregate resource constraint and law of motion of capital:

$$C_t + I_t = A_t K_t^{\gamma} L_t^{1-\gamma},$$

$$K_{t+1} = (1-\delta) K_t + I_t - f\left(\frac{I_t}{I_{t-1}}\right) I_t.$$

The first-oder optimality conditions imply:

$$
\begin{aligned}
[C_t] \quad & U_{C_t} = \lambda_{1,t}, \\
[m_t] \quad & U_{m_t} = 0, \\
[L_t] \quad & -U_{L_t} = \lambda_{1,t} (1-\gamma) A_t K_t^{\gamma} L_t^{-\gamma}, \\
[K_{t+1}] \quad & \beta \lambda_{1,t+1} \gamma A_{t+1} K_{t+1}^{\gamma-1} L_{t+1}^{1-\gamma} - \lambda_{2,t} + \beta \lambda_{2,t+1} (1-\sigma) = 0, \\
[I_t] \quad & -\lambda_{1,t} + \lambda_{2,t} - \lambda_{2,t} \left[\frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) + f\left(\frac{I_t}{I_{t-1}}\right) \right] - \beta \lambda_{2,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) = 0,
\end{aligned}
$$

where $\lambda_{1,t}$ is the Lagrange multiplier on the aggregate resource constraint and $\lambda_{2,t}$ is the Lagrange multiplier on the law of motion of capital. Arrange the above conditions, we have:

$$\frac{U_{m_t}}{U_{C_t}} = 0,$$

$$-\frac{U_{L_t}}{U_{C_t}} = (1-\gamma) A_t K_t^{\gamma} L_t^{-\gamma},$$

$$q_t^F = E_t \Lambda_{t,t+1} (\gamma A_{t+1} K_{t+1}^{\gamma-1} L_{t+1}^{1-\gamma} + (1-\sigma) q_{t+1}^F),$$

$$q_t^F \left[1 - f\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) \right] + E_t \Lambda_{t,t+1} q_{t+1}^F \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) = 1,$$

where the stochastic discount factor is defined as

$$\Lambda_{t,t+1} = \beta \frac{U_{C_{t+1}}}{U_{C_t}},$$

and q_t^F is Tobin's q defined as $\frac{\lambda_{2,t}}{\lambda_{1,t}}$.

Appendix D. Robustness of ϵ_m

The choice of ϵ_m must satisfy the arbitrage condition implied by households' optimality conditions (equations 7 and 8):

$$\frac{r_t^B - r_t^{CB}}{r_t^B - r_t^{PV}} = \frac{1-\alpha}{\alpha} \left(\frac{m_t^{PV}}{m_t^{CB}} \right)^{\frac{1}{\epsilon}}.$$

Thus, the choice of ϵ_m is confined by the steady-state returns on household assets and the value of alpha. Because empirical evidence on how the introduction of CBDC will affect these parameters in the long term is currently blank, the robustness check is conducted under the assumption that these values remain unchanged. The goal is to understand whether a reasonable change of ϵ_m will change the main conclusions of this paper or not. The table below shows the changes of ϵ_m only produce small deviations from the quantitative outcomes in the benchmark estimation and it does not alter the conclusions of this paper.

Table 5: Robustness of ϵ_m

Benchmark:	$\boldsymbol{\epsilon_m = 6}$	$\widehat{Y_t}$	$\widehat{C_t}$	$\widehat{I_t}$	$\widehat{L_t}$	Welfare
Volatility	No CBDC	2.40	1.60	8.42	2.05	Baseline
	Intermediated CBDC ($\widetilde{m_t^{CB}}$)	2.34	1.54	7.51	2.00	0.82%
	Intermediated CBDC ($\widetilde{i_t^{CB}}$)	2.11	1.44	5.78	1.60	1.18%
	Independent CBDC ($\widetilde{i_t^{CB}}$)	2.36	1.56	7.60	1.99	0.52%
	$\boldsymbol{\epsilon_m = 3}$	$\widehat{Y_t}$	$\widehat{C_t}$	$\widehat{I_t}$	$\widehat{L_t}$	Welfare
Volatility	Intermediated CBDC ($\widetilde{m_t^{CB}}$)	2.38	1.54	7.66	2.11	0.99%
	Intermediated CBDC ($\widetilde{i_t^{CB}}$)	2.05	1.42	5.48	1.50	1.41%
	Independent CBDC ($\widetilde{i_t^{CB}}$)	2.35	1.56	7.53	1.98	0.57%
	$\boldsymbol{\epsilon_m = 10}$	$\widehat{Y_t}$	$\widehat{C_t}$	$\widehat{I_t}$	$\widehat{L_t}$	Welfare
Volatility	Intermediated CBDC ($\widetilde{m_t^{CB}}$)	2.32	1.54	7.46	1.94	0.67%
	Intermediated CBDC ($\widetilde{i_t^{CB}}$)	2.16	1.46	6.03	1.68	1.05%
	Independent CBDC ($\widetilde{i_t^{CB}}$)	2.36	1.56	7.65	2.00	0.47%

Notes: $\widetilde{m_t^{CB}}$ *denotes the regime under the endogenous CBDC supply rule.* $\widetilde{i_t^{CB}}$ *denotes the regime under the CBDC interest rate rule. All values represent standard deviations expressed in percentage term.*

Appendix E. Additional Impulse Response Functions

Figure 7: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under -1% Liquidity Shock

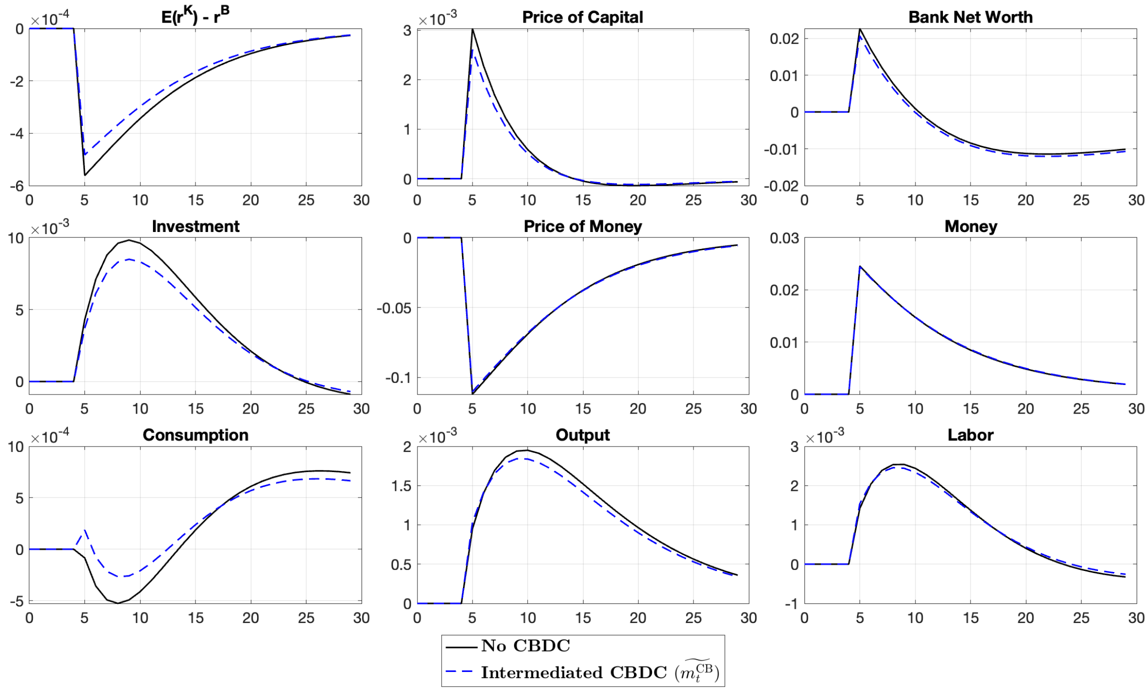

Figure 8: non-CBDC v.s. Intermediated CBDC ($\widetilde{i_t^{CB}}$) under -1% Liquidity Shock

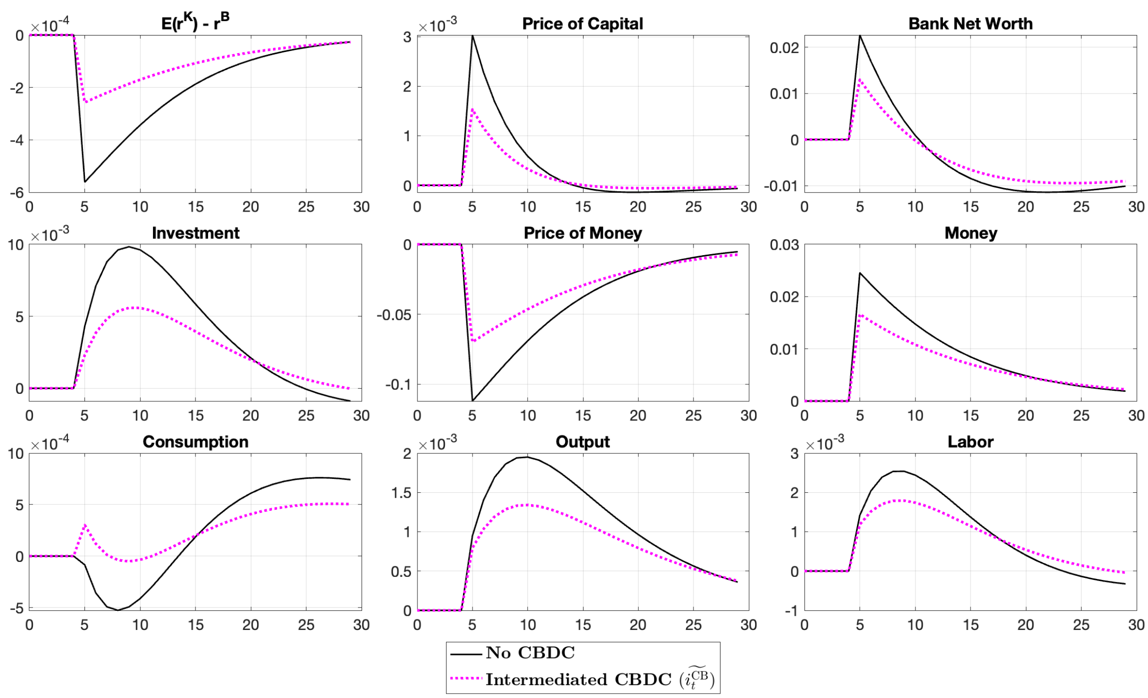

Figure 9: non-CBDC v.s. Independent CBDC ($\widetilde{i_t^{CB}}$) under -1% Liquidity Shock

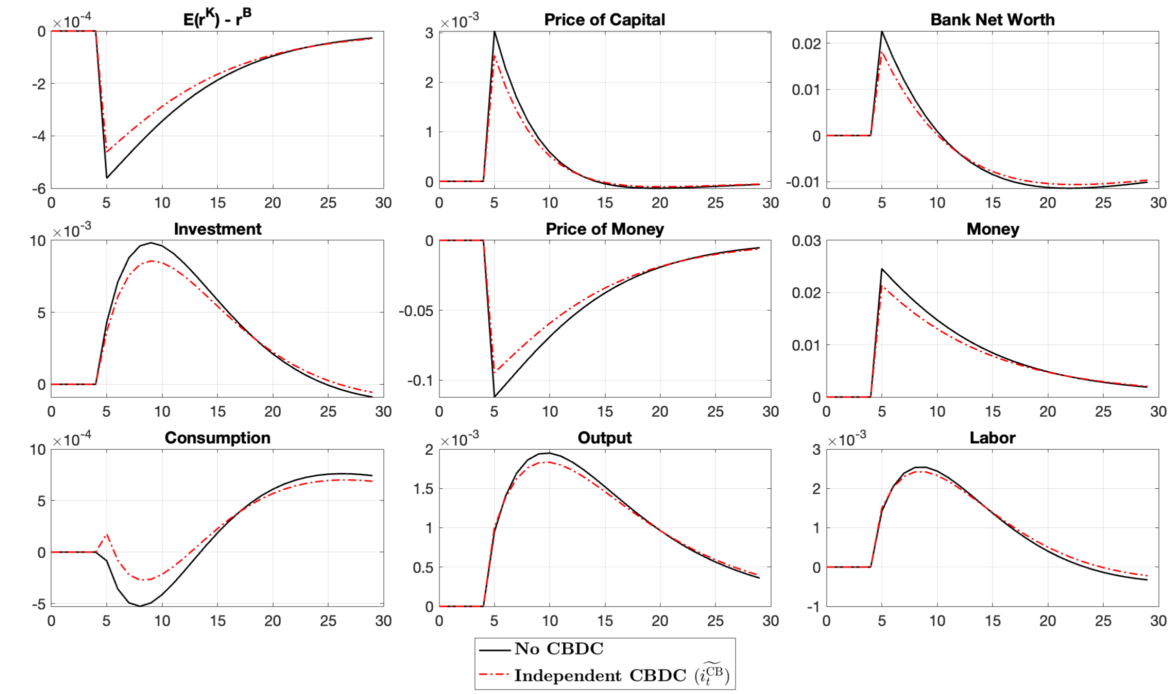

Figure 10: non-CBDC v.s. Intermediated CBDC ($\widetilde{m_t^{CB}}$) under 1% Technology Shock

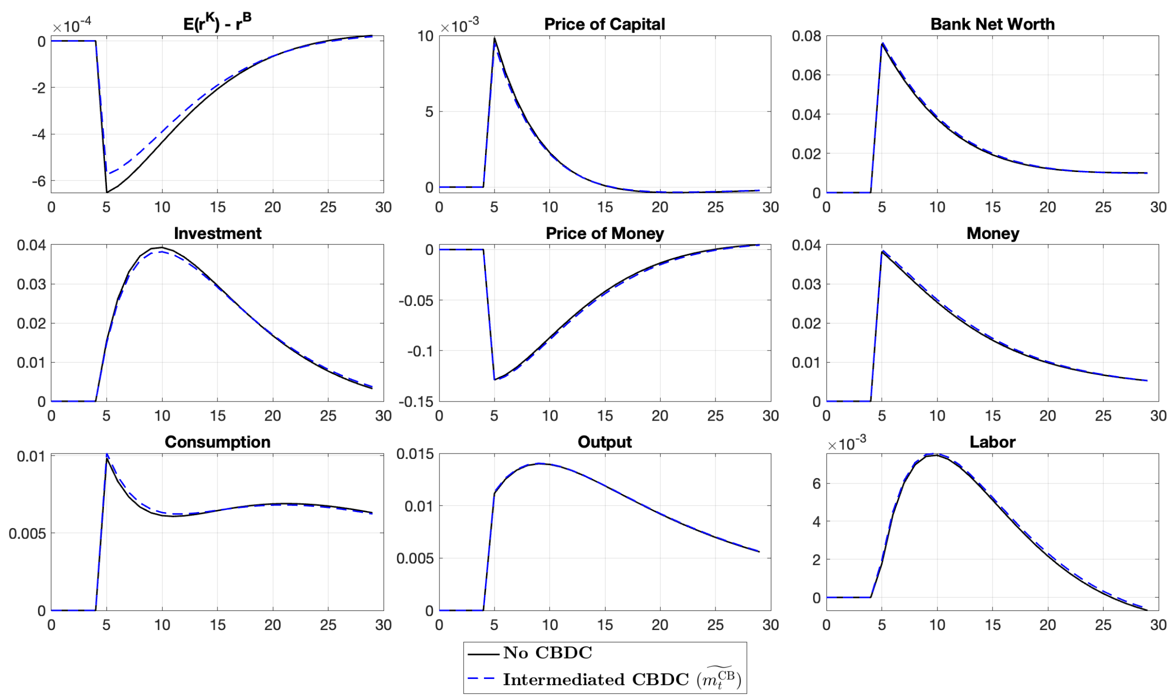

Figure 11: non-CBDC v.s. Intermediated CBDC ($\widetilde{i_t^{CB}}$) 1% Technology Shock

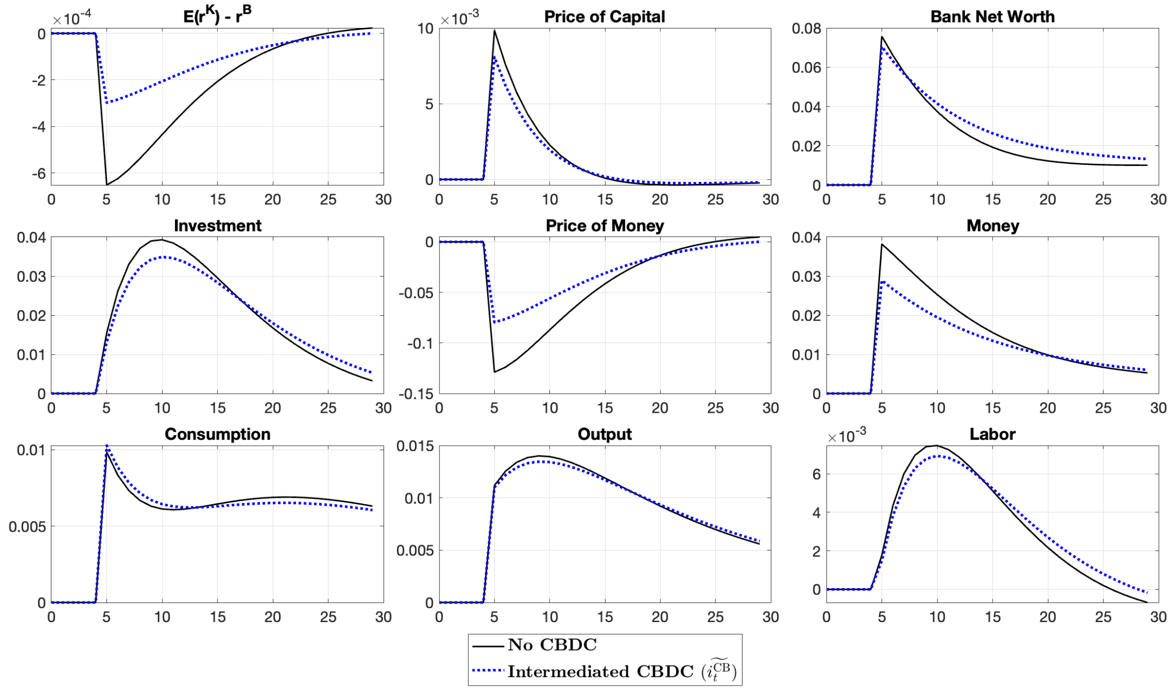

Figure 12: non-CBDC v.s. Independent CBDC ($\widetilde{i_t^{CB}}$) under 1% Technology Shock