Optimal Tariff with Heterogeneous Firms, Variable Markups and Tariff-jumping FDI*

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Abstract

Variable markups and multinational production have gathered considerable attention in the trade literature, both because of their empirical prevalence and their welfare implication. In this paper, I study the optimal tariff in the presence of variable markups and foreign direct investment. I then identify conditions under which protectionist trade policy, by changing the distribution of markups, and by inducing tariff-jumping FDI, may affect welfare. Three policy implications stand out from the analysis. First, if the initial protection level is sufficiently high, an increase in home's tariff will increase the number of tariff-jumping foreign multinationals and decrease the number of foreign exporters, driving down the average markup in the home market, and creating a pro-competitive effect. Second, whether zero tariff is socially optimal depends on consumer's preference. Third, the promotion of FDI can reduce the non-cooperative tariff through a novel channel: reducing the misallocation in the economy.

Keywords: Optimal tariff, Firm heterogeneity, Misallocation, Variable markup, Foreign direct investment

JEL Codes: F12, F13, F23, F60, R13

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1 Introduction

What is the welfare implication of protectionist trade policy in an environment that features variable markups and foreign direct investment (FDI)? On the one hand, protectionism may hurt consumer welfare in the presence of variable markups if protection results in higher market concentration. This has been a concern since Adam Smith, and it has received increasing attention in recent years[1]. On the other hand, in a highly-integrated global market[2], foreign firms can avoid import tariffs by locating production within the destination market. Such "tariff-jumping" activities[3] can diminish the market power of domestic producers, thereby substantially mitigate welfare consequences of the original trade protection policy.

The goal of this paper is to study the optimal tariff in the context of monopolistic competition, heterogeneous firms, variable markups, and FDI. To this end, the paper introduces variable markups through quadratic quasi-linear preference, as in Melitz and Ottaviano (2008), into a two-country model with firm heterogeneity and FDI, as in Helpman et al. (2004). In the current framework, a firm needs to pay a fixed cost and draw its marginal production cost (which is inversely related to the firm's productivity) to enter the market. Post-entry, firms produce with different marginal cost levels. Exporters encounter two types of costs: iceberg-type trade cost and *ad valorem* tariff. Multinationals face an iceberg-type of *efficiency loss* as in Keller and Yeaple (2008). Firms formulate entry, export and FDI decisions based on expected profit. The difference in marginal cost preserves the sorting of firms[4]: the most productive firms access the foreign market through FDI, the less productive firms export and the least productive firms only serve their domestic market. An increase in foreign country's tariff affects the variable profit of home exporters and multinationals, making FDI a more profitable entry mode for the most productive exporters, inducing

[1]Outside of the academic literature, increasing market concentration has received significant attention, e.g., *A lapse in concentration* (The Economist, September 2016), CEA (2016). In the academic literature, see Asker et al. (2017), De Loecker and Eeckhout (2017) for recent evidence.

[2]Thanks to the growth of multinational firms. According to Antràs and Yeaple (2014), data from the U.S. Census Bureau indicates that roughly 90% of U.S. exports and imports flow through multinational firms, with close to one-half of U.S. imports transacted within the boundaries of multinational firms rather than across unaffiliated parties.

[3]With the improvement in micro-level data availability, tariff-jumping FDI has received increasing empirical support, see Blonigen (2002), Belderbos et al. (2004), Hijzen et al. (2008) and more recently, Pietrovito et al. (2013), Alfaro and Chen (2015, 2018).

[4]In Helpman et al. (2004), the sorting of firms is preserved by the combination of fixed cost and variable cost. Here, with bounded marginal utility, high-cost firms will not survive, even without such fixed costs. The difference in marginal cost is sufficient to generate the sorting. Adding fixed cost will substantially degrade the tractability of the model, without generating additional insight.

tariff-jumping FDI under the heterogeneous firm framework.

The analysis of the findings shows that the welfare implication of protectionist tariff crucially depends on the assumption of entry. If entry and exit are restricted[5], an increase in home import tariff makes it harder for the least productive foreign exporters to export. Those exporters will shut down their export department and only serve their domestic market. Meanwhile, an increase in home import tariff makes export a less desirable entry mode for the most productive foreign exporters. Those firms will switch to FDI simply because the variable profit of FDI is higher than that of export. In the current setup, if the level of protection is low, the reduction of foreign exporters will dominate the increase of foreign multinationals, resulting in a reduction of the total number of foreign firms in the home country. In equilibrium, the protectionist trade policy creates an easier environment for domestic firms to survive.

The optimal level of protection depends on the markup distribution in the economy. With a low protection level, an increase in home tariff reduces the total number of foreign firms, creating a less competitive environment. As a result, the markups of home's domestic producers, foreign exporters and FDI firms all go up. In addition, exporters can pass the tariff burden on to consumers[6]. The average markup in the economy is affected by the composition effect. As the protection level increases, the share of foreign exporters decreases, reducing the competition in the home market, and creating upward pressure on the average markup. At the same time, the share of foreign tariff-jumping multinational firms increases, increasing the competition in the home market, and creating downward pressure on the average markup. If the level of protection is high, the second effect can dominate the first, driving down the average markup in the economy. Protectionist trade policy can end up intensifying home market's competition.

The current framework yields important implication for the pro-competitive effect of trade. While recent studies on the welfare implication of trade liberalization[7] emphasize the importance of variable markup, the insight here is that we should not ignore the role of FDI. A decrease in home's import tariff makes it easier for the most productive foreign domestic firms to export, increasing the number of foreign

[5]Restricted entry may also provide an adequate description of a short-run equilibrium in which entry has not taken place yet and fixed costs are sunk, making exit never optimal. In this case, the economy is characterized by a fixed number and distribution of incumbents. These incumbents decide whether they should operate and produce-or shut down. If so, they can restart production without incurring the entry cost again.

[6] The degree of pass-through depends on firm's specific productivity. Pass-through rate is lower for more productive firms. This is inline with the empirical evidence from De Loecker et al. (2016).

[7]For example, Edmond et al. (2015) and Arkolakis et al. (2018).

exporters serving home market, and creating a downward pressure to the home average markup. At the same time, the reduction of tariff also makes it less desirable for the least productive foreign multinationals to do FDI, decreasing the number of foreign FDI firms, and generating upward pressure to the home average markup. If the initial protection level is sufficiently high, the decrease of multinational firms can dominate the increase of exporters, driving up the average markup in the home market, and generating a negative pro-competitive effect.

When entry and exit are unrestricted, and the tariff revenue is redistributed to consumers, the number of entrants and the number of firms in the economy are endogenously determined by the tariff level. With the free-entry condition, an increase in home import tariff makes the home country a more desirable place to do business, generating more domestic entry. Although the increase of home tariff still makes it harder for the least productive exporters to export, reduces the number of foreign exporters, and makes it easier for the most productive exporters to do FDI, and increases the number of foreign multinationals, the total impact on the number of firms in the home market is dominated by the domestic entry. Different from the restricted entry case, the protectionist trade policy here creates more entry, generates more competition in the home market, and makes it harder for local producers to survive.

With free-entry, trade policy implication depends on the efficiency of the economy. The market is not efficient due to several distortions: (1) inter-sectoral distortion: the markup-pricing in the differentiated-good sector distorts the allocation between the differentiated-good sector and homogeneous-good sector, implying an inefficiently small size of the monopolistically competitive sector; (2) intra-sectoral distortion: due to variable markups, if consumers have a strong preference toward the differentiated varieties, the market outcome can be inefficient in several dimensions compared to the socially optimal allocation: (i) weak selection in domestic, export cutoff, and over-selective in FDI cutoff, (ii) oversupplies high-cost varieties and undersupplies low-cost varieties, (iii) oversupplies the total number of varieties and features excessive entry

These market failures stem from several externalities: (i) on the supply side, both the markup-pricing and business-stealing effect tend to create too many varieties in the economy, (ii) on the demand side, the “love of variety” from the quadratic quasi-linear preference tends to create insufficient varieties in the economy, (iii) with variable markup, the market outcome oversupplies varieties produced by less productive firms, resulting in inefficiently large size for these firms. These externalities collectively result in the inefficiencies in the market outcome. In contrast, under CES preference, the market outcome produces the same allocation as the social planner.

Two general policy implications stand out from the analysis. First, free trade is not always socially optimal. Although a decrease in tariff can generate entry in the economy, and improve consumer welfare, it also takes away the profits of existing firms. If the relative demand for the differentiated varieties is sufficiently high, the negative effect on firms can outweigh the positive impact on consumers, thereby decreasing the social welfare. In this case, protectionist trade policy can be welfare-improving by deterring the excessive entry.

Second, the promotion of FDI can lower a country's non-cooperative tariff level when economy features misallocation. If the relative demand for the differentiated varieties is sufficiently high, the market economy oversupplies high-cost varieties. In this case, misallocation materializes as less productive firms are allocated with too many resources (labor). If the Pareto distribution parameter k is small (higher degree of firm heterogeneity), then there are relatively more productive firms in the economy. Since tariff-jumping FDI happens among the more productive firms along the marginal cost distribution, the tariff-jumping foreign firms now utilize more home labor. In this case, home labor is reallocated toward more productive firms, reducing the misallocation in the home economy. The reduction of misallocation has a more significant impact on the economy compared to the case when k is large (lower degree of firm heterogeneity). The Nash tariff under smaller k is lower than the Nash tariff under bigger k. This shows when the economy features a higher degree of firm heterogeneity (smaller k), hence a higher degree of misallocation, allowing firms to engage in FDI can lower the non-cooperative tariff level.

The rest of the paper proceeds as follows. Section 2 contrasts the current approach to the related literature. Section 3 describes the benchmark model and characterizes the equilibrium. Section 4 studies the equilibrium features of the model. Section 5 studies the composition effect under a tariff change, socially optimal tariff, and Nash tariff with and without FDI under symmetry. Section 6 further explores the role of variable markups in the current setup. Section 7 concludes.

2 Related Literature

The findings in this paper are related to, and have implications for, a large number of papers in the trade policy literature. Many authors have studied the trade policy implication with heterogeneous firms framework, for example: Demidova and Rodriguez-Clare (2009) use a Melitz-type model and a small country assumption to show the first-best outcome can be achieved through either a consumption subsidy, export tax, or an import tariff; Felbermayr et al. (2013) allow for Melitz-type large countries and characterize a link between the level of Nash import tariffs and parameters related to transportation costs and productivity dispersion; Bagwell and Lee (2015) study trade policy in Melitz and Ottaviano (2008) model and provides a rationale for the treatment of export subsidies within the World Trade Organization;Costinot et al. (2016) utilize a generalized Melitz model to characterize optimal unilateral tariffs both when tariffs are firm-specific and when they are industry-specific. They identify a central role for the terms-of-trade externality in their analysis of unilateral trade-policy intervention. Demidova (2017) studies the optimal tariff in the Melitz and Ottaviano (2008) environment without the outside good and finds protection is always desirable, and reductions in cost-shifting trade barriers are welfare-improving. A common feature of the aforementioned papers is their exclusive focus on domestic producers and exporters. A key message from the current analysis is that ignoring the multinational production may provide a misleading picture of the protectionist trade policy. The findings in this paper show that the promotion of FDI can effectively lower the non-cooperative tariff level.

A recent article by Cole and Davies (2011) is closely related to the current paper. The authors introduce ad valorem tariff and heterogeneous fixed costs into Helpman et al. (2004), and find equilibria in which both pure exporters and multinationals coexist, resolving a known puzzle[8] in the strategic tariff literature in the presence of multinationals. Heterogeneous fixed costs for exporters and multinationals is the key element to generate their result. In contrast, the coexistence of exporters and multinationals in the current framework comes from the different iceberg costs they are facing.

Despite the apparent similarity between the two frameworks, it should be clear that the two exercises are very different. First, Cole and Davies (2011) utilize quasi-linear CES preference, combining with monopolistic competition, yielding constant markups and complete pass-through in equilibrium. Despite its analytical tractability, the combination of CES and monopolistic competition has little merit, even as a first approximation, for welfare analysis. In contrast, the current framework utilizes quadratic quasi-linear preference to generate heterogeneous firms and incomplete pass-through for different firms, which is more suitable for pricing and welfare anal-

[8]In equilibrium, all foreign firms are either multinationals or exporters.

ysis. Second, Cole and Davies (2011) completely ignore the potential for tariffs to impact entry. According to Caliendo et al. (2017), the combination of ad valorem tariff and tariff rebate violates the macro assumption in ACR, the level of entry should not remain fixed in Cole and Davies (2011). In the current framework, the number of entrants is endogenously affected by tariff level, generating different welfare implication for protectionist trade policy in the short run and long run. Third, the presence of variable markup alters the free trade implication. Cole and Davies (2011) find socially optimal tariff is always to subsidize trade. This is because trade can foster competition and eliminate the least productive firms, increasing aggregate productivity. In the current framework, whether free trade is socially optimal depends on consumer's relative demand for the differentiated varieties. Subsidizing trade is desirable only when there is an insufficient entry in the economy.

The role of variable markup has received increasing attention in the international trade literature. For example, Arkolakis et al. (2018) show that the under a large class of demand function, the non-homothetic preference dampens the pro-competitive effect of trade liberalization (incurred by the change of iceberg-type trade cost) by increasing the degree of misallocation. Edmond et al. (2015) show that the size of the pro-competitive gain[9] can be quite large in the presence of significant misallocations and weak cross-country comparative advantage in individual sectors. Different from these two papers, the current framework shows that the pro-competitive effect of trade can be very different when FDI is incorporated.

Lastly, the welfare implication of FDI is an old topic in the field, see *e.g.,* Brecher and Alejandro (1977). Some recent papers have revisited the welfare impact of FDI, either analytically or quantitatively. Ramondo and Rodríguez-Clare (2013) show that when taking account of the multinational production, the gains from openness are around twice the gains calculated in trade-only models. Irarrazabal et al. (2013) extend Helpman et al. (2004) to allow intra-firm trade and structurally estimate their model using firm-level data from Norwegian manufacturing sector. Their counterfactual analysis indicates that impeding FDI has substantial effects on trade flows but not on welfare. Different from their exercises, this paper studies explicitly the welfare implication of FDI through the interaction with the tariff. The paper identifies a new source of welfare gain of FDI: through resource allocation by reducing the degree of misallocation in the economy.

[9]According to their setup, a pro-competitive gain is associated with a lower average markup.

3 The Model

This section introduces quadratic quasi-linear preference into Helpman et al. (2004) framework. There are two symmetric countries, home (H) and foreign (F). The markets are segmented, and international trade entails trade costs that take the form of transportation costs as well as ad valorem import tariffs. Tariff revenue is distributed equally across consumers in the tariff-imposing country. FDI incurs an iceberg-type of marginal cost (i.e., efficiency loss) in the spirit of Keller and Yeaple (2008). Different from Cole and Davies (2011), where firms' partition is induced by different fixed cost, the non-homothetic preference here induces different productivity cutoffs through different marginal cost.

3.1 Consumers

Consider the H economy with one unit of consumers, each supplies 1 unit of labor. Consumers in country H choose over q_0^H and q_i^H

$$U^H = q_0^H + \alpha \int_{i\in\Omega^H} q_i^H di - \frac{1}{2}\gamma \int_{i\in\Omega^H} \left(q_i^H\right)^2 di - \frac{1}{2}\eta \left(\int_{i\in\Omega^H} q_i^H di\right)^2$$

$$\text{subject to: } q_0^H + \int_{i\in\Omega^H} p_i^H q_i^H di \le I^H \equiv w^H + TR^H + \Pi^H$$

where α and η indicate the substitutability between the differentiated varieties and numéraire good, γ indicates the substitutability among the differentiated varieties. An increase in α and a decrease in η both shift out the demand for the differentiated varieties *relative* to the numéraire. Notice that different from Melitz and Ottaviano (2008), the tariff revenue and aggregate profit will enter into consumer's budget constraint through government transfer.

Assuming consumers have positive demands for the numéraire good $\left(q_0^H > 0\right)$, maximization of the above problem leads to the following inverse demand for each variety i:

$$p_i^H = \alpha - \gamma q_i^H - \eta Q^H \tag{1}$$

where $Q^H \equiv \int_{i\in\Omega^H} q_i^H d_i$ is the aggregate consumption of these varieties. Invert equation (1) to obtain the linear market demand for these varieties

$$
\begin{aligned}
q_i \equiv q_i^H &= \frac{\alpha}{\eta N^H + \gamma} - \frac{1}{\gamma} p_i^H + \frac{\eta N^H}{\eta N^H + \gamma} \frac{1}{\gamma} \bar{p}^H \\
&= \frac{1}{\gamma} \left(p_{\max}^H - p_i^H \right)
\end{aligned} \tag{2}
$$

where $p_{\max}^H = (\gamma \alpha + \eta N^H \bar{p}^H)/(\eta N^H + \gamma)$ represents the price at which demand for a variety is driven to 0, $\bar{p}^H \equiv (1/N^H) \int_{i \in \hat{\Omega}^H} p_i^H di$ is the average price of all consumed variety in country H, and $\hat{\Omega}^H$ is the consumed subset of Ω^H. Note that equation (1) also implies $p_{\max}^H \leq \alpha$. Different from CES preference, where the elasticity of demand is constant, the price elasticity of demand here is given by

$$
\varepsilon_i^H \equiv \left| \frac{\partial q_i^H}{\partial p_i^H} \times \frac{p_i^H}{q_i^H} \right| = \frac{1}{p_{\max}^H / p_i^H - 1} \tag{3}
$$

The lower the average price $\bar{p}^H$ or a larger number of competing varieties N^H induce a decrease in the price bound $p_{\max}^H$ and an increase in the price elasticity of demand ε_i^H at any given p_i^H. These all represent a "tougher" competitive environment, which can't be captured in an environment with constant elasticity of demand.

As in Melitz and Ottaviano (2008), welfare can be evaluated using the following indirect utility function:

$$
U^H = I^H + \frac{1}{2} \left(\frac{N^H}{\eta N^H + \gamma} \right) \left(\alpha - \bar{p}^H \right)^2 + \frac{1}{2} \frac{N^H}{\gamma} \sigma_{p^H}^2 \tag{4}
$$

where $\sigma_{p^H}^2 \equiv \left(1/N^H\right) \int_{i \in \hat{\Omega}^H} \left(p_i^H - \bar{p}^H \right)^2 di$ represents the variance of prices. To ensure positive demand levels for the numéraire, I assume that $I^H > \int_{i \in \hat{\Omega}^H} p_i^H q_i^H di = \bar{p}^H Q^H - N^H \sigma_{p^H}^2 / \gamma$. The welfare will be higher when the average price $\bar{p}^H$ is lower, the variance of prices $\sigma_{p^H}^2$ is higher and the number of variety N^H is larger.

3.2 Firms

Production in the economy only utilizes labor, which is supplied in an inelastic fashion in a competitive market. q_0^H is produced under a constant return to scale technology at unit cost. Thus the wage[10] in each country equals to one: $w^H = 1$. In the

[10] If I drop the numéraire good, wage will be endogenized and can be pinned down by trade balance condition.

differentiated-good sector, firms operate under monopolistic competition, and each firm produces a single variety. To enter the market, a firm needs to pay a fixed cost $f_E > 0$ and draws its marginal production cost c, which indicates the unit labor requirement. The cost is drawn from a Pareto distribution with cumulative distribution function $G(c) = (c/c_M)^k$, where $k \geq 1$ represents a shape parameter and $c_M > 0$ represents the upper bound of c. When $k = 1,$ the marginal cost distribution is uniform on $[0, c_M]$. As k increases, the relative number of low productivity firms increases, and the distribution is more concentrated at these lower productivity levels. I assume H and F share the same technology, hence the same upper bound c_M and the same f_E.

Depending on its productivity draw, a firm enters country H may exit, produce locally, export to country F or engage in the multinational activity. Following Melitz and Ottaviano (2008), I assume markets are segmented, and firms operate under monopolistic competition in each market. Therefore, a firm makes separate decisions about its prices at each market, taking the total number of varieties and the average price in a market as given.

3.2.1 Domestic Producer

A firm located in country H with cost level c selects its price in the domestic market, p_D^H , to maximize its domestic profit $\pi_D^H(c) = \left[p_D^H(c) - c\right] q_D^H(c).$ Together with equation (2), the optimal price, markup, quantity, hence profit are:

$$p_D^H(c) = \frac{1}{2}\left(c_D^H + c\right) \tag{5}$$

$$m_D^H(c) = \frac{1}{2c}\left(c_D^H + c\right) \tag{6}$$

$$q_D^H(c) = \frac{1}{2\gamma}\left(c_D^H - c\right) \tag{7}$$

$$\pi_D^H(c) = \frac{1}{4\gamma}\left(c_D^H - c\right)^2 \tag{8}$$

Let $c_D^H \equiv \sup\left\{c : \pi_D^H(c) > 0\right\}$ represent the cost of the firm who is indifferent about remaining in the market. This firm earns zero profit as its price is driven down to marginal cost, together with equation (2), $p_D^H\left(c_D^H\right) = c_D^H = p_{\max}^H$. Hence, a firm will only serve domestic market if $c \leq c_D^H$. As expected, lower cost firms set lower prices and earn higher profits. However, lower cost firms do not pass all of the cost differentials to the consumer, and they also charge higher markups (which is defined as $m(c) = p(c)/MC(c)$, decreasing in c).

3.2.2 Exporter

The exporter in country H will face an *ad valorem* import tariff imposed by country F, denoted as $t^F \geq 1$. On top of that[11], the exporter will also face a per-unit trade cost[12], denoted by τ^F. More specifically, the delivered cost of a unit cost c to country F is $\tau^F c$ where $\tau^F > 1$. A firm maximizes its profit $\pi_X^H(c) = \left[p_X^H(c)/t^F - \tau^F c\right] q_X^l(c)$ by choosing optimal price $p_X^H(c)$. Together with equation (2), the optimal price, markup, quantity, hence profit are:

$$p_X^H(c) = \frac{t^F \tau^F}{2} \left(c_X^H + c\right) \tag{9}$$

$$m_X^H(c) = \frac{t^F}{2c} \left(c_X^H + c\right) \tag{10}$$

$$q_X^H(c) = \frac{t^F \tau^F}{2\gamma} \left(c_X^H - c\right) \tag{11}$$

$$\pi_X^H(c) = \frac{t^F \left(\tau^F\right)^2}{4\gamma} \left(c_X^H - c\right)^2 \tag{12}$$

Let $c_X^H \equiv \sup\left\{c : \pi_X^H(c) > 0\right\}$ denotes the upper bound cost for exporters from H to F. Combine it with the definition of c_D^F (parallel to c_D^H), this cutoff then satisfies $c_X^H = c_D^F / t^F \tau^F$: tariffs and transportation cost make it harder for exporters to break even relative to the domestic market.

3.2.3 Multinational

To engage in the multinational activity, a firm located in country H with cost level c chooses its product price for consumers in country F, denoted as $p_{FDI}^H(c)$. Instead of serving foreign market through exports, it directly serves locally in country F, but doing so will incur a higher marginal cost[13], φ^F. Here, I **assume** $\varphi^F > \tau^F$ to ensure

[11] To ensure that when the net tariff is zero, there're still exporters in the economy, I need to introduce the iceberg-type of transportation cost.

[12] Following Melitz and Ottaviano (2008), I abstract from any fixed export cost, which would substantially reduce the tractability of the model without adding additional insights. With the bounded marginal utility, different marginal costs are enough to induce the sorting of firms.

[13] This feature is similar to Keller and Yeaple (2008), who shows that when technologies are complex, it is more difficult for US-owned foreign affiliates to substitute local production with imports from

there're still multinational firms in the economy even when the net tariff is zero. Multinational firm's profit function is as follow:

$$\pi_{FDI}^{H}(c) = \left[p_{FDI}^{H}(c) - \varphi^{F} c\right] q_{FDI}^{H}(c) \tag{13}$$

Together with equation (2), the optimal price, markup, quantity, hence profit are:

$$p_{FDI}^{H}(c) = \frac{1}{2}\left(c_D^F + \varphi^F c\right) \tag{14}$$

$$m_{FDI}^{H}(c) = \frac{1}{2\varphi^F c}\left(c_D^F + \varphi^F c\right) \tag{15}$$

$$q_{FDI}^{H}(c) = \frac{1}{2\gamma}\left(c_D^F - \varphi^F c\right) \tag{16}$$

$$\pi_{FDI}^{H}(c) = \frac{1}{4\gamma}\left(c_D^F - \varphi^F c\right)^2 \tag{17}$$

Let $c_{FDI}^{H} = \sup\left\{c : \pi_{FDI}^{H}(c) > \pi_X^H(c)\right\}$ denote the upper bound cost for multinational from H to F. Combine with the definition of c_D^F, this cutoff then satisfies $c_{FDI}^H = \xi^F c_D^F$, where $\xi^F \equiv (1-\sqrt{t^F})/(t^F\tau^F - \sqrt{t^F}\varphi^F)$ is derived by setting $\pi_{FDI}^H(c) = \pi_X^H(c)$.

Note, there are two possible cases in this solution, $c_{FDI}^H = (1 \pm \sqrt{t^F})/(t^F\tau^F \pm \sqrt{t^F}\varphi^F)c_D^F$, but only one of them is interesting and relevant here. According to the prediction in Helpman et al. (2004), for those firms that serve foreign markets, only the most productive ones engage in FDI[14]. In the current setup, this implies $c_{FDI}^H < c_X^H < c_D^H$. Compare the expression of c_X^H and c_{FDI}^H, both cases imply $\varphi^F > t^F\tau^F$, which incorporates the previous assumption that $\varphi^F > \tau^F$ since $t^F \geq 1$. However, for the case of $c_{FDI}^H = (1+\sqrt{t^F})/(t^F\tau^F + \sqrt{t^F}\varphi^F)c_D^F$, c_{FDI}^H will decrease in response to an increase in t^F, indicating the marginal multinationals will choose to become exporters when tariff increases. This is at odds with the empirical evidence[15] of tariff-jumping. Therefore, the other choice $c_{FDI}^H = (1-\sqrt{t^F})/(t^F\tau^F - \sqrt{t^F}\varphi^F)c_D^F$ makes more sense here since c_{FDI}^H will increase in response to an increase in t^F, in line with the empirical evidence of

multinational headquarter. φ^F can also stand for the information costs of working broad, transaction costs of dealing with FDI policy barriers, costs of maintaining the affiliate, servicing network costs, and other costs associated with technology costs in offshore production.

[14] This pattern also receives empirical support, see Doms and Jensen (1998) for the U.S. and Conyon et al. (2002) for the U.K, for more recent evidence, see Mataloni (2011).

[15] For example, Blonigen and Feenstra (1997), Barrell and Pain (1999) consistently find substantial tariff-jumping responses. Blonigen (2002) finds smaller average tariff-jumping responses and concludes that tariff-jumping is a realistic option for multinational firms from industrialized countries. Blonigen et al. (2004) find tariff-jumping in the form of new plants or plant expansion has significantly larger negative effects on U.S. domestic firms' profits.

productivity sorting and the tariff-jumping FDI. The following graph indicates the region that FDI will occur and the relation between t, φ, and τ for FDI to happen:

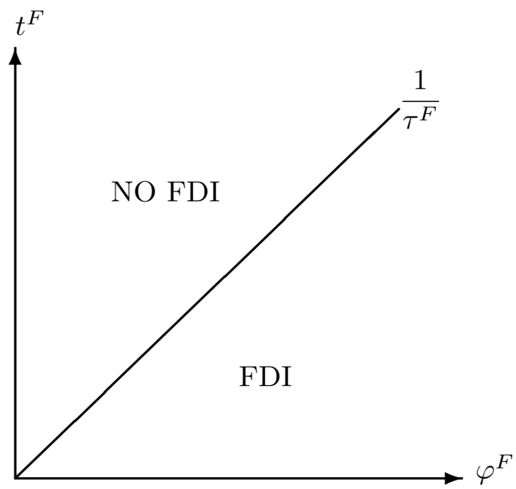

Figure 1: Minimum Tariff to Induce Tariff-jumping FDI

Discussion on firm's FDI motivation here is important. In Helpman et al. (2004), the sorting of firms is preserved under the assumption of $f_I > \tau^{\epsilon-1} f_X > f_D$. Export incurs a higher marginal cost (τ), but as long as the fixed cost of FDI, f_I, is sufficiently high, they can still guarantee the most productive firms find FDI more desirable than export. This is a classic proximity-concentration tradeoff in the spirit of Brainard (1997). The similar tradeoff is also present in Cole and Davies (2011), where the authors embed *ad valorem* and variable fixed cost into the Helpman et al. (2004) framework. They find as the tariff increases, the variable profit of exporter decreases while the differences in fixed cost remain the same. When the tariff level is sufficiently high, the gain from avoiding the tariff is higher than the fixed cost of becoming a multinational, and a firm prefers FDI over export as an entry mode. In the current setup, this is no longer the case. Comparing the profit function for exporter and multinational:

$$\pi_X^H(c) = \left[p_X^H(c)/t^F - \tau^F c\right] q_X^H(c) \tag{18}$$

$$\pi_{FDI}^H(c) = \left[p_{FDI}^H(c) - \varphi^F c\right] q_{FDI}^H(c) \tag{19}$$

As tariff increases, the revenue of exporter will drop, making export a less desirable mode of accessing foreign market. Eventually, FDI becomes a more desirable entry mode. Although the marginal cost of FDI is higher than export ($\varphi^F > \tau^F$), the operating profit of FDI exceeds the profit of export. The tradeoff between export and FDI is merely a comparison between the profits, no longer the conventional proximity-concentration tradeoff.

The underlying reason that FDI is a valid option for the firm is discussed in Mrázová and Neary (2018). They argue that "*...statements like "Only the more productive firms select into the higher fixed-cost activity" are often true, but always misleading: they are true given Super-modularity*[16]*, but otherwise may not hold. What matters for the direction of second-order selection effects* [17] *is not a trade-off between fixed and variable costs, but whether there is a complementarity between variable costs of production and of trade. Putting this differently, for FDI to be the preferred mode of market access, a firm must be able to afford the additional fixed costs of FDI* [18]*, but whether it can afford them or not depends on the cross-effect on profits of tariffs and production costs. When Super-modularity prevails, a more efficient firm has relatively higher operating profits in the FDI case, but when sub-modularity holds, the opposite may hold.* " The reason that the current setup can preserve the conventional sorting (i.e., second-order selection effect) is primarily due to the Super-modularity of profit function (as in Mrázová and Neary (2018), Section 6) since there exists complementarity between variable costs of production and of trade.

3.3 Free Entry Condition

Entry is unrestricted in both countries. Firms choose a production location before entry and paying the sunk entry cost. To restrict the analysis on the effects of trade costs differences, I assume that countries share the same technology[19] (i.e., the same entry cost f_E and the same cost distribution $G(c)$). Free entry of domestic firms

[16] For the definition of Super-modularity and later on, sub-modularity, please refer to Mrázová and Neary (2018).

[17] According to their description, this is referring to the choice of export vs. FDI.

[18] Their setup is a general preference, so they rely on fixed cost. In Appendix F, they mentioned Melitz and Ottaviano (2008) and pointed out the first-order selection effect (according to their description, this is referring to whether serve the foreign market or not) with quadratic quasi-linear preference needs the existence choke price.

[19] For implications of Ricardian comparative advantage, please refer to the Appendix in Melitz and Ottaviano (2008).

in country H implies zero expected profits in equilibrium, hence:

$$\int_0^{c_D^H} \pi_D^H(c)\, dG(c) + \int_{c_{FDI}^H}^{c_X^H} \pi_X^H(c)\, dG(c) + \int_0^{c_{FDI}^H} \pi_{FDI}^H(c)\, dG(c) = f_E \tag{20}$$

Given the Pareto assumption in both countries, the free entry condition for country H can be rewritten as:

$$\left(c_D^H\right)^{k+2} + \Phi_1^F \left(c_D^F\right)^{k+2} + \Phi_2^F \left(c_D^F\right)^{k+2} = \gamma\phi \tag{21}$$

where $\phi \equiv 2(k+1)(k+2)(c_M)^k f_E$ is a technology index that combines the effects of the better distribution of cost draws (lower c_M) and lower entry costs f_E. Moreover,

$$\begin{aligned}
\Phi_1^F \equiv \frac{(k+1)(k+2)\, t^F(\tau^F)^2}{2} & \Bigg\{ \left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)^2 \left(\xi^F\right)^k \\
& - \frac{2k}{k+1}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\frac{1}{t^F\tau^F}\right)\left(\xi^F\right)^{k+1}\right] + \frac{k}{k+2}\left[\left(\frac{1}{t^F\tau^F}\right)^{k+2} - \left(\xi^F\right)^{k+2}\right]\Bigg\} \\
\Phi_2^F \equiv \frac{(k+1)(k+2)\left(\xi^F\right)^k}{2} & \left[1 - \frac{2k\varphi^F\xi^F}{k+1} + \frac{k\left(\varphi^F\xi^F\right)^2}{k+2}\right]
\end{aligned}$$

are indices that combine the trade-off between tariff and higher marginal cost of FDI. The free entry condition is homogenous to degree $k+2$ regarding the cutoffs. This system (for H, F) can then be solved for the cutoffs in both countries:

$$c_D^H = \left[\gamma\phi \frac{1 - \left(\Phi_1^F + \Phi_2^F\right)}{1 - \left(\Phi_1^F + \Phi_2^F\right)\left(\Phi_1^H + \Phi_2^H\right)}\right]^{\frac{1}{k+2}} \tag{22}$$

Two observation stand out in comparison to Melitz and Ottaviano (2008): (1) This cutoff is lower[20] than the closed-economy cutoff[21], indicating the opening up of an economy via export and multinational activity will increase the aggregate productivity by forcing the least productive firms to exit. This result is similar to Melitz (2003) but works through product market competition, instead of factor market competition, as argued in Melitz and Ottaviano (2008). (2) This cutoff is even lower[22] than

[20]This is based on Lemma 1, see Appendix A.1.

[21]See equation (15) in Melitz and Ottaviano (2008), which is $c_D^H = (\gamma\phi)^{1/(k+2)}$. The difference is that I normalized the labor size to be one.

[22]See Appendix A.1 for the proof.

the open economy cutoff[23] generated in Melitz and Ottaviano (2008). The intuition is straightforward: the presence of FDI, here the most productive firms in the distribution, intensifies the competitive environment in the economy, forcing the least productive firms to exit and hence further increases aggregate productivity.

3.4 Prices, Product Variety, Number of Entrants and Welfare

To see more features in the current setup, I first compute $\bar{p}^H$. Notice, the cost of H's firms $c \in [0, c_D^H]$, the delivered cost of exporters $\tau^F c \in [0, c_D^H]$ and the cost of multinationals $\varphi^F c \in [0, c_D^H]$ all share identical distributions over the support given by $G^H(c) = (c/c_D^H)^k$. The price distribution of H's domestic firms, $p_D^H(c)$, and exporters producing in F, $p_X^F(c)$ and F's multinationals producing in H, $p_{FDI}^F(c)$, are therefore all identical. The average price in country H is thus given by:

$$\begin{aligned} \bar{p}^H &= \frac{1}{G\left(c_D^H\right)} \int_0^{c_D^H} p_D^H\left(c\right) dG\left(c\right) = \frac{1}{G\left(c_X^F\right)} \int_{c_{FDI}^F}^{c_x^F} p_X^F\left(c\right) dG\left(c\right) \\ &= \frac{1}{G\left(c_{FDI}^F\right)} \int_0^{c_{FDI}^F} p_{FDI}^F\left(c\right) dG\left(c\right) = \frac{2k+1}{2k+2} c_D^H \end{aligned} \tag{23}$$

Combining this with the definition of $p_{\max}^H$ and $p_{\max}^F$, the number of firms selling in country H is:

$$N^H = \frac{2\gamma\left(\alpha - c_D^H\right)(k+1)}{\eta c_D^H} \tag{24}$$

From this expression, it must be the case that $\alpha > c_D^H$ so that the number of firms selling in country H is positive in equilibrium. The number of product variety in country H is composed of domestic producers, exporters, and multinationals from country F. Given a positive mass of entrants N_E in both countries, there are $G(c_D^H)N_E^H$ domestic producers, $[G(c_X^F) - G(c_{FDI}^F)]N_E^F$ exporters, and $G(c_{FDI}^F)N_E^F$ multinationals selling in H satisfying:

$$G\left(c_D^H\right) N_E^H + \left[G\left(c_X^F\right) - G\left(c_{FDI}^F\right)\right] N_E^F + G\left(c_{FDI}^F\right) N_E^F = N^H \tag{25}$$

[23]See equation (23) in Melitz and Ottaviano (2008), which is $c_D^H = \left[\gamma\phi\left(1-\rho^F\right) / \left(1-\rho^H\rho^F\right)\right]^{1/(k+2)}$ when labor is normalized to one. The open economy cutoff is slightly different due to the ad valorem tariff, see Appendix A.1 for details.

Solving this system (for H and F) will give us the number of entrants in country H:

$$N_E^H = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1-\delta^H\delta^F\right)}\left[\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} - \delta^H \frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}}\right] \tag{26}$$

where $\delta^l = (t^l\tau^l)^{-k}$, for $l \in \{H, F\}$Notice, to ensure positive mass of entry in the equilibrium, it is straightforward to show that $\alpha > c_D^l$ for $l = H, F$. This implication is crucial in understanding the equilibrium feature of the model. I will come back to this point in Section 3. Following Melitz and Ottaviano (2008), combine (4), (23), (24) and the definition of $\sigma^2_{p^H}$, it is straightforward to show the consumer welfare in H equals to:

$$U^H = I^H + \underbrace{\frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right)}_{\equiv CS^H} \tag{27}$$

Once again, welfare changes monotonically with the domestic cutoff, which captures the effect of an increase in product variety and a decrease in the average price. Also notice, consumer surplus in country H is given by the second term in equation (27).

3.5 Tariff Revenue and National Welfare

This part will be particularly important when analyzing socially optimal tariff and Nash tariff. Note that tariff revenue is also a component of consumer income I^H through the redistribution from the government. I define the pre-tax value of country H's import as:

$$\begin{aligned} IM^H &= N_E^F \int_{c_{FDI}^F}^{c_X^F} \frac{p_X^F(c)}{t^F} q_X^F(c)\, dG(c) \\ &= N_E^F \frac{t^H\left(\tau^H\right)^2\left(c_D^H\right)^{k+2}}{4\gamma(k+2)\left(c_M\right)^k}\left[2\left(\frac{1}{t^H\tau^H}\right)^{k+2} - \frac{k+2}{\left(t^H\tau^H\right)^2}\left(\xi^H\right)^k + k\left(\xi^H\right)^{k+2}\right] \end{aligned} \tag{28}$$

Therefore, the total important tariff revenue of country H is defined as

$$\begin{aligned} TR^H &\equiv \left(t^H - 1\right) \times IM^H \\ &= N_E^F \frac{t^H - 1}{t^H}\frac{\left(c_D^H\right)^{k+2}}{4\gamma(k+2)\left(c_M\right)^k}\left[2\left(\frac{1}{t^H\tau^H}\right)^k - (k+2)\left(\xi^H\right)^k + k\left(\xi^H\right)^{k+2}\left(t^H\tau^H\right)^2\right] \end{aligned} \tag{29}$$

From the trade-policy perspective, the government will consider the following consumer welfare function as its criterion:

$$U_n^H = \underbrace{w^H + (t^H - 1) \times IM^H + \Pi^H}_{\equiv I^H} + \underbrace{\frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right)}_{\equiv CS^H} \tag{30}$$

Therefore tariff affects consumer welfare from two channels: (1) consumer surplus, which is directly affected by the change in c_D^H in response to tariff; (2) tariff revenue, which is affected by both the tariff level (t^H) and the tariff base (IM^H). Due to free-entry condition, aggregate profit Π^H is driven to zero in equilibrium. Notice due to the presence of numéraire good, $w^H = 1$. Consumers will not take t^H into consideration when maximizing their utility. However, the government does choose the optimal tariff level to achieve highest national welfare objective function.

With the model above, I will now discuss the equilibrium features of this economy, contrast it with an economy features heterogeneous firms, FDI but constant markups, as in Cole and Davies (2011). Then I will discuss the welfare implication for trade policy in the current setup.

4 Equilibrium Conditions

The presence of FDI tends to intensify the degree of competition in the economy.

Lemma 1. *The presence of FDI makes the economy more competitive, the domestic cutoff is lower compared to the case when there is no FDI:*

$$c_{D1}^H = \left[\gamma\phi\frac{1-\left(\Phi_1^F + \Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}} < c_{D2}^H = \left[\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right]^{\frac{1}{k+2}}$$

Proof. See Appendix A.1 □

In the Appendix A.1, I showed $\Phi_1^l + \Phi_2^l > \psi^l$ for $l \in \{H, F\}$. The sum of Φ can be viewed as a measure of "openness". The presence of FDI makes the country more "open" compared to the case when FDI is not an option. In Melitz and Ottaviano (2008), ψ^l [24]measures the "freeness" of trade. The presence of FDI intensifies the competition, making it harder to survive. The marginally surviving firm needs to be

[24]More precisely, the freeness of trade is measured by τ^{-k} in Melitz and Ottaviano. Here, due to the presence of tariff, this term is augmented to incorporate tariff, $\tau^{-k}t^{-(k+1)}$.

more productive. Openness (either through export or FDI) increases competition[25] in the domestic product market, shifting up residual demand price elasticities for all firms at any given demand level. This forces the least productive firms to exit. This effect is very similar to an increase in market size in the closed economy: the increased competition induces a downward shift in the distribution of markups across firms. Although only relatively more productive firms survive (with higher markups than the less productive firms who exit), the average markup is reduced.

4.1 Restricted Entry

Now I study the impact of tariff when entry is restricted. When entry and exit are allowed; incumbents firms decide whether to produce or shut-down. Home country is characterized by a fixed number of incumbents $\bar{N_I^H}$ with cost distribution $\bar{G^H}$ on $[0, \bar{c}_M]$. I continue to assume that the productivity $1/c$ is distributed with Pareto shape k, implying $\bar{G}^H(c) = (c/\bar{c}_M)^k$. A Home firm produces if it can earn non-negative profits from sales from either its domestic market, export market or FDI market. This leads to the cutoff conditions for sales:$c_D^H = \sup\{c : \pi_D^H(c) \geq 0$ and $c \leq \bar{c}_M\}$, $c_X^H = \sup\{c : \pi_X^H(c) \geq 0$ and $c \leq \bar{c}_M\}$, and $c_{FDI}^H = \sup\{c : \pi_{FDI}^H(c) \geq \pi_X^H(c)$ and $c \leq \bar{c}_M\}$. As long as the cutoffs satisfies the above conditions, the following threshold price conditions must be true:

$$N^H = \frac{2\left(k+1\right)\gamma}{\eta}\frac{\alpha - c_D^H}{c_D^H}$$
$$N^F = \frac{2\left(k+1\right)\gamma}{\eta}\frac{\alpha - t^F\tau^F c_X^H}{t^F\tau^F c_X^H}$$

where N^H, N^F represent the endogenous number of sellers in country H, F in the short run. Notice that the different cutoffs satisfy the same condition as in the long run.

There are $\bar{N}_I^H G\left(c_D^H\right)$ producers in H who sell in their domestic market, $\bar{N}_I^F \left[G\left(c_X^F\right)\right.$ $\left.-G\left(c_{FDI}^F\right)\right]$ exporters from H to F and $\bar{N}_I^F G\left(c_{FDI}^F\right)$ FDI firms in F. These numbers must add up to the total number of producers in country H. Similar equation also holds for country F:

$$N^H = \bar{N}_I^H G\left(c_D^H\right) + \bar{N}_I^F\left[G\left(c_X^F\right) - G\left(c_{FDI}^F\right)\right] + \bar{N}_I^F G\left(c_{FDI}^F\right)$$
$$N^F = \bar{N}_I^F G\left(c_D^F\right) + \bar{N}_I^H\left[G\left(c_X^H\right) - G\left(c_{FDI}^H\right)\right] + \bar{N}_I^H G\left(c_{FDI}^H\right)$$

[25]Compared to the case when export is the only option to access foreign market.

Combining this with the threshold price conditions yield expressions for the cost cutoffs in both countries:

$$\frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^H}{\bar{c}_M^k} + \left[\left(\frac{1}{t^H\tau^H}\right)^k - \left(\xi^H\right)^k\right]\bar{N}_I^F + \left(\xi^H\right)^k\bar{N}_I^F\right\}$$
$$\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} = \frac{\eta}{2\left(k+1\right)\gamma}\left\{\frac{\bar{N}_I^F}{\bar{c}_M^k} + \left[\left(\frac{1}{t^F\tau^F}\right)^k - \left(\xi^F\right)^k\right]\bar{N}_I^H + \left(\xi^F\right)^k\bar{N}_I^H\right\}$$

This condition clearly highlights the protection role played by import tariff in the short run. An increase in H's tariff will make it harder for the foreign exporters to access home market, so the number of exporters from F to H will decrease. At the same time, an increase in H's tariff will induce tariff-jumping FDI among the foreign exporters, so the number of foreign firms that access home market through FDI will increase. In the current setup, the decrease of exporters surpasses the increase of FDI firms, so the right-hand side of the first equation is decreasing in t^H, indicating an increase in H's domestic cost cutoff. Therefore, an increase in H's tariff reduces the total number of foreign firms (exporters and FDI firms) accessing home market, making it easier for home producers to survive. This effect, however, will be offset when entry is unrestricted.

Notice, according to equation (6), (10) and (15), markups respond to tariff in various ways. For an increase in t^H, tariff affects *home domestic producer's* markup $m_D^H(c)$ through the equilibrium effect on c_D^H. Protection makes it easier for home producers and results in a higher c_D^H, meaning a higher markup for all the domestic sellers. When entry is unrestricted, this effect will be reversed. For *foreign exporters*, their markup $m_X^F(c)$ is affected by tariff both directly and indirectly. An increase in t^H directly increases m_X^F, meaning foreign exporters will pass the tariff burden to the consumers by increasing price. It indirectly affects m_X^F through the equilibrium effect on c_D^H. With restricted entry, these two effects are in the same direction. With unrestricted entry, these two effects are in the opposite direction and the general impact on m_X^F is ambiguous. For *foreign FDI firms*, tariff affect $m_{FDI}^F(c)$ through the equilibrium effect on c_D^H. Protection results in a less competitive home environment, this benefits the foreign FDI firms and allow them to charge higher markups. Just as the domestic producers, the effect of protection will be the opposite with unrestricted entry.

4.2 Unrestricted Entry

With unrestricted entry, firms can freely enter and exit the market. Since the ad valorem tariff revenue are rebated to the consumers, the number of entrants in the economy are endogenously affected by the level of tariff[26]. All the equilibrium features are analyzed and contrasted with the environment in Cole and Davies (2011). A change in tariff has quite a different impact on productivity cutoffs, and this is mainly due to its impact on the competitive environment in the economy.

Lemma 2. *An increase in country H's import tariff results in a decrease in the cutoff cost level in country $H's$ domestic market and an increase in the cutoff cost level in country F's domestic market:*

$$\frac{\partial c_D^H}{\partial t^H} < 0 < \frac{\partial c_D^F}{\partial t^H}$$

Proof. See Appendix A.2 □

This result is different from Cole and Davies (2011). They find an increase in the import tariff in country H will increase the protection level in country F, shield country H's firm from competition, making domestic surviving firms less productive, i.e., $\partial c_D^H/\partial t^H > 0$ (equation (11) in Cole and Davies (2011)). Here, the story is different. Although an increase in the import tariff increases the protection level in country H, it also fosters a more extensive entry from domestic firms over time. Lemma 5 from below further demonstrates this point. With free-entry, the larger entry will generate a higher competition in the domestic market, driving out the least productive firms and making the marginally surviving firms more productive.

Lemma 3. *An increase in country H's import tariff results in an increase in the export cutoff cost level in country H and a decrease in the export cutoff cost level in country F:*

$$\frac{\partial c_X^H}{\partial t^H} > 0 > \frac{\partial c_X^F}{\partial t^H}$$

Proof. See Appendix A.3 □

[26] Balistreri et al. (2011) first found entry is no longer necessarily fixed when either (i) ad valorem revenue tariffs are imposed rather than iceberg transport costs, or (ii) there are multiple sectors. In the current setup, both the ad valorem tariff and the quadratic quasi-linear preference are contributing to the endogenous level of entrants

This result is the same as in Cole and Davies (2011), the increase in import tariff in country H will make the least productive exporters in country F quit exporting, only serve its domestic market. This is because the increase in tariff reduces exporter's revenue (hence profit), making it less desirable for the least productive exporters to serve H's market. With their exit, the marginal surviving exporter is more productive, hence a lower c_X^F.

Lemma 4. *An increase in country H's import tariff results in an increase in the FDI cutoff cost level in country F:*

$$\frac{\partial c_{FDI}^F}{\partial t^H} > 0$$

Proof. See Appendix A.4 □

The result is similar to the findings in Cole and Davies (2011), the most productive exporters from F, when facing an increase in import tariff in H, will find it less desirable to access H's market through export, hence choose FDI as an entry mode. This is mainly because the profit of FDI outweighs the profit of export when t^H increases. Hence the marginally surviving multinationals from country F become less productive (previously they were exporters), hence a higher c_{FDI}^F.

To sum up and contrast with Cole and Davies (2011), I construct the following graph: when t^H increases, in Cole and Davies, through their equation (11)-(13):

Country F

$c_{FDI}^F \Rightarrow \quad \Leftarrow c_X^F \quad c_D^F \longrightarrow$

Country H

$c_{FDI}^H \quad c_X^H \quad c_D^H \Rightarrow \longrightarrow$

but in the current setup with variable markups:

Country F

$c_{FDI}^F \Rightarrow \quad \Leftarrow c_X^F \quad c_D^F \longrightarrow$

Country H

$c_{FDI}^H \quad c_X^H \quad \Leftarrow c_D^H \longrightarrow$

Figure 2 : Comparison with Cole and Davies (2011)

In Cole and Davies (2011), an increase in t^H leads the least productive foreign exporters to exit the domestic market (c_X^F decreases) and the most productive foreign exporters to become multinationals (c_{FDI}^F increases). This change makes the composition of domestic foreign firms (F's exporters and multinationals) more productive. Due to the protection, the domestic market is shielded from foreign competition. Hence domestic firms are easier to survive (c_D^H increases).

In the current setup, an increase in t^H will similarly lead the least productive foreign exporters to exit the domestic market (c_X^F decreases) and the most productive foreign exporters to become multinationals (c_{FDI}^F increases), also making the composition of domestic foreign firms more productive. But home protection will attract more firms to enter (N_E^H increases), making home country's environment more competitive, so that the domestic firms need to be more productive to survive (c_D^H decreases).

In both cases, we have tariff-jumping FDI. In Cole and Davies (2011), tariff-jumping intensifies the competitive environment in the domestic market of H, but this effect is dominated by the protection effect raised through tariff. So the outcome is an environment easier to survive. In the current setup, the tariff-jumping FDI intensifies the competitive environment in the domestic market. The excess entry generated by protection also makes the domestic environment more competitive. These two effects together result in a tougher environment in the home market, making it harder to survive. Based on this result, the following must be correct:

Corollary 1. *Under the assumption that $\varphi^H > \tau^H$[27], an increase in $H's$ import tariff results in a tougher competitive environment in the domestic market over time, this effect will be exacerbated by the presence of FDI:*

$$\left| \frac{\frac{\partial c_D^H}{\partial t^H}\Big|_{without\ FDI}}{\frac{\partial c_D^H}{\partial t^H}\Big|_{with\ FDI}} \right| < 1$$

Lemma 5. *An increase in H´s import tariff results in an increase in the number of entrants in H and a decrease in the number of entrants in F. Over time, this contributes to an increase in the number of varieties in H and a decrease in the number of varieties*

[27] The domestic cutoff without FDI but with ad valorem tariff is $c_D^H = \left[\gamma\phi\left(1-\rho^F\right)/\left(1-\rho^H\rho^F\right)\right]^{1/(k+2)}$, where $\rho^H = (\tau^H)^{-k}(t^H)^{-(k+1)}$. The domestic cutoff with FDI is defined in equation (22).

in F:

$$\frac{\partial N^H}{\partial t^H} > 0 > \frac{\partial N^F}{\partial t^H}$$
$$\frac{\partial N_E^H}{\partial t^H} > 0 > \frac{\partial N_E^F}{\partial t^H}$$

Proof. See Appendix A.5 □

This result is the same as in Melitz and Ottaviano (2008). If one investigates the impact of a tariff change in the short-run[28], following Melitz and Ottaviano, one can show:

$$\frac{\partial c_D^H}{\partial t^H}|_{\text{short-run}} > 0$$

This means when entry is restricted, the protection of H country will shield the home firms from foreign competition, making domestic market easier to survive. Therefore the cutoff level is higher, similar to the finding in Cole and Davies (2011). However, combined with Lemma 2, this effect is offset by entry. This classic "delocation" result has been studied extensively in previous work(see, for example, Venables (1985), Helpman and Krugman (1989), Baldwin et al. (2003)) and here is also confirmed in the heterogeneous firm framework with FDI.

5 Optimal Trade Policy

This section studies the optimal trade policy in the current setup. I first investigate the composition effect of a tariff change and identify the conditions under which protectionist trade policy can increase welfare. I then investigate if free trade is socially optimal and find the outcome crucially depends on the competitive environment in the economy. Then I move on to study the non-cooperative tariff policy when the demand for differentiated varieties is sufficiently high. At last, I explore whether the presence of FDI will affect a country's Nash tariff choice.

[28] According to Melitz and Ottaviano (2008), section 3.7, no entry and exit is possible in the short-run. Therefore each country is characterized by a fixed number of incumbents $\bar{N_D}^l$ for $l = H, F$.

5.1 Average Markup

Proposition 1. *If the level of protection is high, the increase of tariff-jumping foreign multinational firms, which creates downward pressure on average markup, can dominate the decrease of foreign exporter firms, which creates upward pressure on average markup. The average markup in the economy decreases as protection level increases. Therefore, protectionist trade policy can end up intensifying home market's competition.*

Proof. See Appendix A.6. □

According to (Edmond et al., 2015), looking only at the markups of domestic producers may be misleading. Since it could be the case that a reduction in trade barriers leads to lower domestic markups (as Home producers lose market share), combined with higher markups on imported goods (as Foreign producers gain market share), the overall markup dispersion increases and misallocation are worse. In this case, the pro-competitive gains from trade would be negative.

In the current setup, a unilateral fall in iceberg trade cost (τ) does not affect the average markup[29] due to the assumption of Pareto cost distribution. But the *ad valorem* tariff does have the ability to affect the average markup. The average markup in country H under the current setup is as follow:

$$\bar{m}^H = \frac{1}{N_D^H + N_X^F + N_{FDI}^F}\left[N_D^H \int_0^{c_D^H} m_D^H(c)\,\frac{dG(c)}{G(c_D^H)} + N_X^F \int_{c_{FDI}^F}^{c_X^F} m_X^F(c)\,\frac{dG(c)}{G(c_X^F)} + N_{FDI}^F \int_0^{c_{FDI}^F} m_{FDI}^F(c)\,\frac{dG(c)}{G(c_{FDI}^F)}\right] \tag{31}$$

After imposing symmetry, the average markup can be rewritten as follow (for detailed

[29]See Melitz and Ottaviano (2008) Section 3.2.

derivation, see Appendix A.6)

$$\bar{m} = \underbrace{\frac{1}{1+(t\tau)^{-k}} \times \frac{2k-1}{2k-2}}_{\text{weighted expected markup in domestic}} + \underbrace{\underbrace{\frac{(t\tau)^{-k}-\xi^k}{1+(t\tau)^{-k}}}_{\text{share of foreign exporters}} \times t \underbrace{\left\{\frac{1}{2}\left[1-(t\tau\xi)^k\right] + \frac{k}{2k-2}\left[1-(t\tau\xi)^{k-1}\right]\right\}}_{\text{expected markup of foreign exporter}}}_{\text{weighted expected markup of foreign exporters}}$$

$$+ \underbrace{\underbrace{\frac{\xi^k}{1+(t\tau)^{-k}}}_{\text{share of foreign FDI}} \times \underbrace{\left(\frac{k}{2k-2}\frac{1}{\varphi\xi} + \frac{1}{2}\right)}_{\text{expected markup of foreign FDI}}}_{\text{weighted expected markup of foreign FDI}}$$

For illustrative purpose, I again focused on the symmetric case. This is similar to the bilateral liberalization studied in Section 4.1[30] in Melitz and Ottaviano (2008). As t increases, the weighted expected markup from domestic firms (the first term) is increasing, this is due to the fact that protection reduces the competition level and makes the domestic environment less competitive, so expected markup will increase due to the reduced competition. The weighted expected markup from foreign exporters (the second term) will decrease as t increases. This is due to two channels, the decreasing of share of foreign exporters (extensive margin, based on Lemma 3 and Lemma 4) and the expected markup[31]. The weighted expected markup of foreign FDI (the third term) will increase as t increases. This also comes from two channels, the increasing share of foreign FDI (extensive margin) and the increasing expected markup (intensive margin).

The first and third term will dominate the second term at the beginning, but as t continually increases, the second term will dominate the other two terms, dragging down the average markup. For illustration purpose, I choose the same parameter as in Section 5.4, with $k = 2$, the average markup term without FDI (m0, plotted in the green line) and the average markup with FDI (m1, plotted in the blue line) are plotted as follow:

[30] Note, different from the unrestricted entry result, bilateral reduction in tariff deliver the same results as in the restricted entry case, i.e., liberalization increases competition and decreases the domestic cutoff level, making it harder for a firm to survive.

[31] The expected markup of foreign exporter (the intensive margin) increases first and then decreases, but the increase is dominated by the extensive margin (the share of foreign exporters), so the overall weighted expected markup of foreign exporters is decreasing.

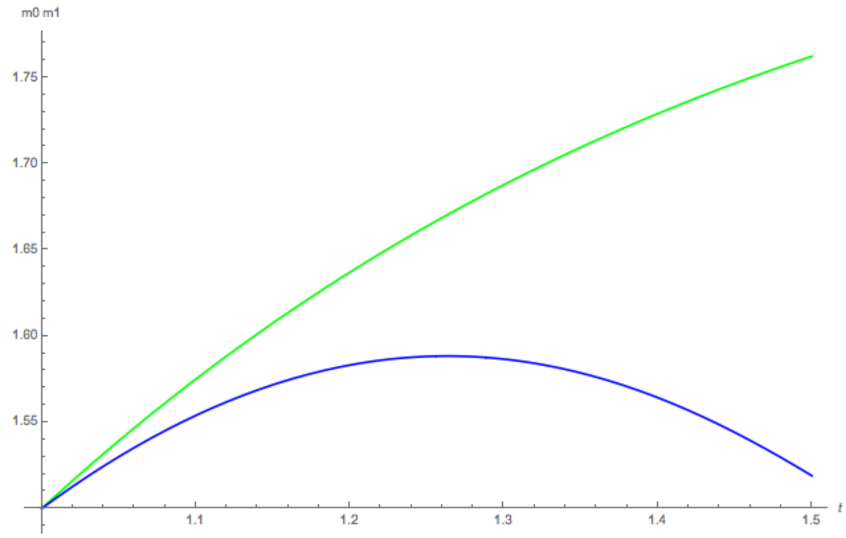

Figure 3: Average markup without FDI (m0, green line) and with FDI (m1, blue line)

When FDI is not an option, the average markup increases as the tariff protection level increases, this confirms the result in Section 4.1 of Melitz and Ottaviano (2008). When FDI is an option, as argued earlier, the markup will increase first and then decrease. According to Edmond et al. (2015), a reduction in trade barrier can potentially create negative pro-competitive effect if average markup goes up. This is also true in the current setup due to the presence of FDI. Although the number of imported varieties is increasing (hence giving a downward pressure to average markup) due to tariff liberalization (for example when t reduces from 1.4 to 1.3), the exiting of multinationals (which reduces the competition in domestic market and gives an upward pressure to average markup) is also contributing to the increase in average markup.

5.2 Is Free Trade Socially Optimal?

To answer this question, I set $t^H = t^F = 1$ (free trade) and study the joint welfare in H and F:

$$\mathbb{W} \equiv \mathbb{U}_n^H|_{t^H=1} + \mathbb{U}_n^F|_{t^F=1}$$

And I find the following proposition:

Proposition 2. *Free Trade is in general not socially optimal. If H and F start with free trade ($t^H = t^F = 1$), then a small symmetric decrease in import tariff increases* *social welfare if and only if $\tilde{c}_D > \alpha/2$, decreases social welfare if and only if $\tilde{c}_D < \alpha/2$ and has no effect on social welfare if and only if $\tilde{c}_D = \alpha/2$, where $\tilde{c}_D$ is the domestic cutoff under symmetry when net tariff is zero.*

Proof. See Appendix A.7. □

Interestingly, free trade is not always socially optimal here. If α, which measures relative demand between the differentiated varieties and numéraire good, is sufficiently high, then the strong demand for differentiated variety will drive large entry into the market over time. Under this condition, this large entry of firms will create negative externality to the economy, making free trade a less desirable choice for the social planner. The optimal choice in the presence of large entry is to tax (i.e., impose import tariff) the firms so that entry can be reduced to the socially optimal level. If the relative demand for differentiated varieties, is not that strong, then there will be less entry into the market. Additional entry, in this case, will create positive externality to the economy. In this situation, positive import tariff will decrease the social welfare. The social planner should subsidize trade. If the relative demand for differentiated is exactly equal to the threshold value, then free trade is socially optimal.

To gain more intuition of the threshold , the social planner problem can be written as the following:

$$\mathbb{W} = \max_{\{N_E^H, N_E^F\}} CS^H + CS^F + \Pi^H + \Pi^F$$

Following Mankiw and Whinston (1986), here I consider a second-best problem faced by a social planner who cannot affect the market outcome for any given number of firms. This is particularly relevant under the current heterogeneous firm framework since we cannot reach the first-best outcome for two reasons: (i) According to Dhingra and Morrow (2012), the market allocation is efficient under the combination of CES preference and monopolistic competition. In the current framework, the market allocation will not be efficient due to the quadratic quasi-linear preference. (ii) The presence of numéraire good adds an extra distortion to the model. There is no markup in the numéraire-good sector, but in the differentiated-good sector, producers charge prices above their marginal costs due to their monopoly power. As pointed out by Bhagwati (1969), the presence of distortions can result in the breakdown of Pareto-optimality of laissez-faire.

The planner chooses the optimal level of entry to maximize welfare, which is composed of consumer surplus (defined in equation (27)) and aggregate profit. Under free entry condition, the marginal entrant does not consider the possible externality it generates toward the consumer, so the market entry level might not be socially desirable. After imposing *symmetry* and $t = 1$, the above objective function can be rewritten as:

$$\max_{\{N_E\}} CS + \Pi$$

Consumers take the number of entrants as given and maximize their choice concerning differentiated varieties defined in equation (2). The consumer surplus in (27) can be rewritten in terms of the optimized choice of variety i as follow:

$$CS = \frac{1}{2}\gamma \int_{i\in\Omega^H} (\hat{q}_i)^2 \, di + \frac{1}{2}\eta \left(\int_{i\in\Omega^H} \hat{q}_i di \right)^2$$

According to Ottaviano et al. (2002), the first term corresponds to the sum of consumer surplus at each variety i, and the second term reflects the variety effect that brings to consumer surplus. To highlight the role of the marginal entrant and its impact on the welfare, I rewrite the above equation in the following way:

$$CS = \underbrace{N_E \times \frac{\gamma}{2} \int_0^{\tilde{c}_D} (q_D(c))^2 \, dG(c)}_{N_E \times \text{Average CS for a variety}} + \underbrace{\frac{(\alpha - \tilde{c}_D)}{2\eta} \left[\alpha - \frac{(k+1)\left(1+\tau^{-k}\right)+1}{(k+2)\left(1+\tau^{-k}\right)} \tilde{c}_D \right]}_{\text{Variety Effect (VE)}}$$

where I utilized the solution of CS in terms of c_D defined in equation (27) to back out the exact expression of variety effect. Therefore the social planner's problem can be further rewritten as

$$\max_{\{N_E\}} N_E \times \textbf{Avg.CS} + \textbf{VE} + \Pi$$

The first order condition related to entry will give us:

$$\underbrace{\textbf{Avg. CS} + N_E \frac{\partial \textbf{Avg. CS}}{\partial N_E} + \frac{\partial \textbf{VE}}{\partial N_E} + N_E \frac{\partial \bar{\pi}}{\partial N_E}}_{\neq 0 \text{ due to variable markup}} + \underbrace{\bar{\pi} - f_E}_{\text{Free entry}} = 0 \tag{32}$$

The free entry will only take care of the last item, and that is why it is not guaranteed to deliver the socially desirable level of entry. According to the seminal work[32]

[32] For recent related discussions under heterogeneous firms framework, see Dhingra and Morrow (2012), Weinberger (2015), Bagwell and Lee (2015) and Behrens et al. (2018)

by Mankiw and Whinston (1986), the first term (> 0) represents the average consumer surplus gain from a new variety. The second term (< 0) represents the average consumer surplus loss for existing varieties when a new variety becomes available (substitution effect). The third item (> 0) represents the variety effect/benefit from a new variety, and the fourth item (< 0) represents the "business-stealing" effect since it measures how the new entrant affects the average profit of existing firms. These four items add up together gives the externality of firms' entry. In the Appendix A.8, I showed that this externality effect is positive when $\alpha < 2\tilde{c}_D$, negative when $\alpha > 2\tilde{c}_D$ and exactly equal to zero when $\alpha = 2\tilde{c}_D$.

The result here is different from Cole and Davies (2011), where the authors find the socially optimal tariff in their setting is always a subsidy. This is because opening up to trade will expose domestic firms to foreign competition, driving out the least productive firms and reallocating resources to the more productive firms. When trade barrier is a choice variable, the social planner will have an additional incentive to promote trade since trade-liberalization can boost aggregate productivity. In the current setup, their conclusion only holds when $\alpha < 2\tilde{c}_D$, this is because when the demand for differentiated varieties is not high enough, the social planner has an incentive to promote trade because encouraging entry creates a positive externality. For the rest of the analysis, I will focus on the case that there exists excessive entry, i.e. $\alpha > 2\tilde{c}_D$ so that the optimal tariff level will be above one.

5.3 Socially Optimal Tariff vs. Nash Tariff

The socially optimal tariff (chosen by a social planner) maximizes the sum of the two countries' consumer utility:

$$\max_{\{t^H,t^F\}} \mathbb{W} = \max_{\{t^H,t^F\}} \mathbb{U}^H + \mathbb{U}^F$$

Looking at the tariff level for H specifically, the optimal level of t^H should satisfy

$$\frac{\partial \mathbb{W}}{\partial t^H} = \underbrace{IM^H + (t^H - 1) \times \frac{\partial IM^H}{\partial t^H}}_{\text{Effect on } H\text{'s tariff revenue}} + \underbrace{\left(t^F - 1\right) \times \frac{\partial IM^F}{\partial t^H}}_{\text{Effect on } F\text{'s tariff revenue}} + \underbrace{\frac{\partial CS^H}{\partial t^H} + \frac{\partial CS^F}{\partial t^H}}_{\text{Effect on } CS^H \text{ and } CS^F} = 0 \qquad (33)$$

It can is straightforward to show the socially optimal tariff level of country H is given by

$$t_S^H - 1 = -\frac{\frac{\partial CS^H}{\partial t^H} + IM^H + \frac{\partial CS^F}{\partial t^H} + \left(t^F - 1\right) \times \frac{\partial IM^F}{\partial t^H}}{\frac{\partial IM^H}{\partial t^H}} \tag{34}$$

In contrast, the Nash tariff level for H is defined as follow:

$$\max_{\{t^H\}} \mathbb{U}^H$$

The optimal non-cooperative policy should satisfy:

$$\frac{\partial \mathbb{U}^H}{\partial t^H} = \underbrace{IM^H + (t^H - 1) \times \frac{\partial IM^H}{\partial t^H}}_{\text{Effect on } H\text{'s tariff revenue}} + \underbrace{\frac{\partial CS^H}{\partial c_D^H} \frac{\partial c_D^H}{\partial t^H}}_{\text{Effect on } CS^H} = 0 \tag{35}$$

Obviously, Nash tariff ignores the impact[33] of its tariff on the other country. Assuming $\alpha > 2\tilde{c}_D$, which will implies the socially optimal tariff is greater than one. This condition is to restrict the range of investigation, for the case $\alpha < 2\tilde{c}_D$, all the conclusions here shall hold in a symmetric way. It can be shown that the Nash tariff level for country H is as follow:

$$t_N^H - 1 = -\frac{\frac{\partial CS^H}{\partial t^H} + IM^H}{\frac{\partial IM^H}{\partial t^H}} \tag{36}$$

Comparing equation (34) and (36), I obtained the following proposition:

Proposition 3. *When the demand for differentiated varieties is sufficiently high ($\alpha > 2\tilde{c}_D$), the Nash tariff (t_N)*[34] *is higher than the socially optimal tariff (t_S).*

Proof. See Appendix A.9 □

This finding confirms Proposition 2 in Cole and Davies (2011), but it is established in the Melitz and Ottaviano (2008) framework with FDI. Under the current setting, there is no terms of trade effect, since I am assuming symmetry between H and F. Also, the quasi-linear utility pushes the income changes onto the numéraire good. If I drop the numéraire good, then wage in both countries will be endogenously determined, then there will be terms of trade effect.

[33]Both on F's tariff revenue and on F's consumer surplus.

[34]At this stage, the existence and uniqueness of Nash tariff can only be supported by computation.

This result confirms the finding in Cole and Davies (2011). Utilizing the F's free-entry condition as in equation (20), we can get:

$$\underbrace{\left(c_D^F\right)^{k+2}}_{\uparrow \text{in } t^H} + \underbrace{\Phi_1^H}_{\downarrow \text{in } t^H} \underbrace{\left(c_D^H\right)^{k+2}}_{\downarrow \text{in } t^H} + \underbrace{\Phi_2^H}_{\uparrow \text{in } t^H} \left(c_D^H\right)^{k+2} = \gamma\phi$$

where the first term on the left is the expected profit of being a domestic producer in F, the second term is the expected profit of being an exporter in F and the third term is the expected profit of being multinational firm in F. When H country set its tariff, according to Lemma 2-4, F's exporter cutoff level ($c_X^F = c_D^H / t^H \tau^H$) is decreasing in t^H. According to the definition of Φ, it is straightforward to show Φ_1^H is decreasing in t^H, Φ_2^H is increasing in t^H. So if t^H increases, the expected profit of an exporter in F goes down, the cutoff also goes down. In fact, the sum of the expected profit of exporter and multinational in F goes down. When H set its Nash tariff, it ignores the impact of its tariff on F's exporter and multinational, purely focusing on the impact on H's welfare, thereby setting a tariff level higher than what the social planner would choose.

5.4 Interaction of Variable Markup and FDI

However, after combining with (22), (27) and (28), neither the optimal social tariff nor the Nash tariff has a closed-form analytical solution due to the non-linear feature of the equations. So I computed the above equation under reasonable parameter restrictions as in Behrens et al. (2011) in Mathematica. The parameterization is as follow:

Parameter Values

α	12	Intensity of preferences for the differentiated product
η	0.1	Substitutability among the varieties
k	To be varied	Pareto shape parameter
c_M	5	Upper bound of c in Pareto distribution
γ	0.6	Degree of love for variety
φ	1.9	Iceberg-type cost of FDI
τ	1.1	Iceberg-type transportation cost
f_E	0.1	Fixed cost of entry

Table 1: Parameter values based on Behrens et al. (2011)

For illustration purpose, I plotted the optimal non-cooperative tariff level computed from equation (35) as a function of φ and α[35] . For $k = 1$, the Nash tariff without FDI (indicated in green color) and the Nash tariff with FDI (indicated in blue color) are plotted as follow :

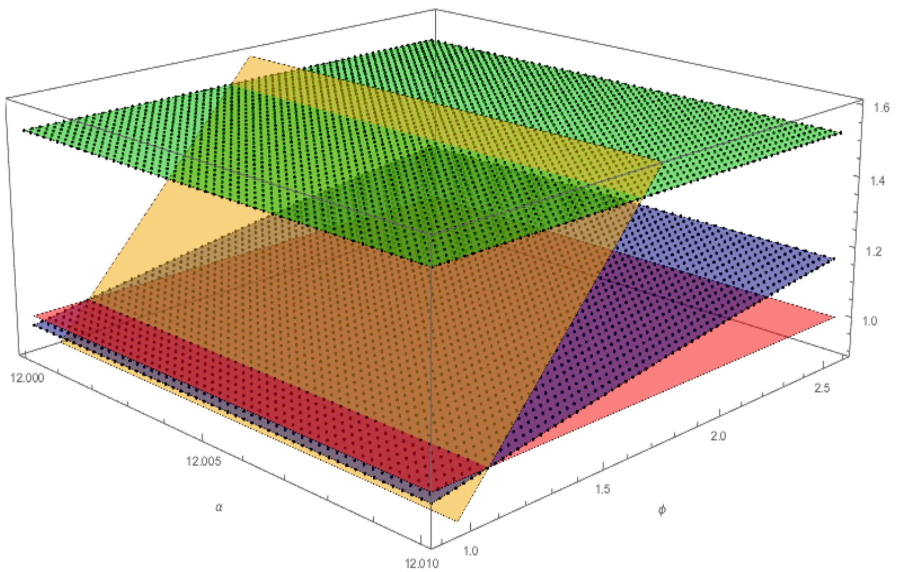

Figure 4: Three-dimensional Nash Tariff with $k = 1$

The yellow plane separates the space. The area above indicates no FDI activity since $\varphi < t\tau$ and the area below indicates FDI occurs since $\varphi > t\tau$. The red plane indicates zero net tariff, i.e. $t = 1$ since α is chosen such that the optimal tariff when FDI occurs is greater than 1. Then it makes sense that the blue plane lies in between yellow plane and the red plane. Several observations stand out from the graph:

- As stated in Proposition 3, Nash tariff without FDI is always higher than the Nash tariff with FDI. This confirms the finding in Cole and Davies (2011). The gain from implementing tariff is smaller due to the tariff-jumping multinationals who lower the tariff base. This can be seen from Figure 2, where the mass of foreign exporters is reduced due to the decrease of exporter cutoff and the increase of multinational cutoff.

- φ does not affect the Nash tariff level without FDI since when FDI is not an option, φ does not affect any exporter behavior. However, according to the Figure

[35]Again, here α is chosen to be larger than $2\tilde{c}_D$ so that the optimal tariff level will be greater than 1.

4 in Cole and Davies (2011), the fixed cost parameter (λ) affects the Nash tariff level without FDI, as well as with FDI. This is because, in their setup, both the fixed costs of export and FDI are affected by λ. Hence the change of λ will have a direct impact on the incentive to implement tariff. In the current setup, the change in φ will only affect the tariff level when FDI occurs.

- As φ increases, the Nash tariff with FDI is increasing, and it is getting closer to the Nash Tariff without FDI. One one hand, if φ approaches infinity, then the FDI cutoff will be zero, indicating foreign firms only excess home country via exports. Hence the Nash tariff level returns to the Nash tariff without FDI case. On the other hand, when FDI is an option, Nash tariff level is increasing in φ. This is similar to the results in Cole and Davies (2011) regarding λ, the higher φ reduces the cutoff of multinational (c_{FDI}), making the least productive multinationals to become exporters, increasing the tariff base, hence increasing the incentive of imposing a higher tariff.

- For high φ, the Nash tariff level (indicated by blue plane) lies below the yellow plane, indicating FDI will occur in equilibrium, confirming the finding in Cole and Davies (2011). As can be seen from the two-dimensional graph below, the bold line indicates the country's choice of Nash tariff. Same with Cole and Davies, the corner solution exists:

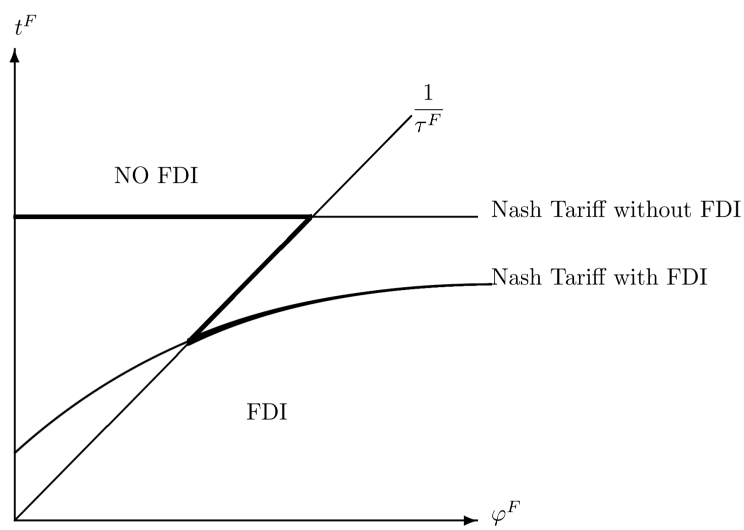

Figure 5: Two-dimensional Nash Tariff

- Different from Cole and Davies (2011), in the current setup, we can see how the competitive environment affects the Nash tariff choice of the policymaker. For illustration purpose, I contrast the following two cases: $k = 1$ and $k = 1.05$.

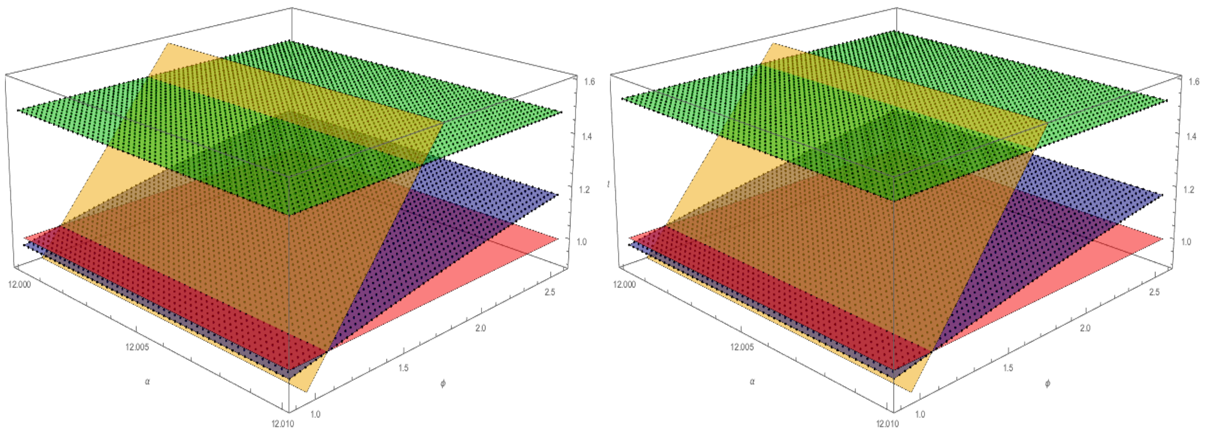

Figure 6: Three-dimensional Nash Tariff for $k = 1.05$ (left) and $k = 1$ (right)

According to discussion in Section 6.2, with the current quadratic quasi-linear preference, the market equilibrium tends to undersupplies varieties from firms with higher productivity/lower marginal cost, and oversupplies varieties from firms with lower productivity/higher marginal cost. The degree of firm heterogeneity affects the choice of noncooperative tariff level. Here, I will briefly highlight the role of firm heterogeneity and its interplay with FDI under variable markup. More intuition will be discussed in Section 6.2.

- Nash tariff without FDI ($k = 1$) > Nash tariff without FDI ($k = 1.05$). When k goes from 1 to 1.05, this tends to foster the under-provision of variety in the market equilibrium. One can show that the number of varieties in the market outcome is decreasing in k[36]. Together with the fact that demand for differentiated varieties is sufficiently high, this explains why Nash tariff ($k = 1$) > Nash tariff ($k = 1.05$). When $k = 1$, there is relatively more varieties in the economy, the economic environment is "too competitive". This gives the policymaker an incentive to increase tariff to deter entry, hence improving welfare. When $k = 1.05$, the market provides relatively fewer varieties in the economy, the policy maker has smaller incentive to deter entry. Hence the tariff level is lower. In contrast, Cole and Davies (2011) are not able to discuss the role of firm heterogeneity and its impact on Nash tariff. This is due to their specific variety/productivity assumption[37] and the fact that under CES and monopolistic competition, firm heterogeneity does not create any externality in the economy. While this makes

[36] See Appendix
[37] See their footnote 11.

their presentation simple and neat, it also removes the possibility to study the impact of competitive environment (through the interaction of quadratic quasi-linear preference and productivity distribution) on Nash tariff.

- Nash tariff with FDI ($k = 1$, green) < Nash tariff with FDI ($k = 1.05$, blue). According to the discussion in Section 5.1 and 5.3, the presence of FDI can potentially reduce misallocation. In the following graph, I use $k = 1$ (the green plane) to represent an economy with larger misallocation and $k = 1.05$ (blue plane) to represent an economy with smaller misallocation. When the misallocation is large ($k = 1$), the Nash tariff with FDI is lower compared to the case when misallocation is small ($k = 1.05$). This is due to the reduction of misallocation triggered by tariff change in the presence of tariff-jumping FDI.

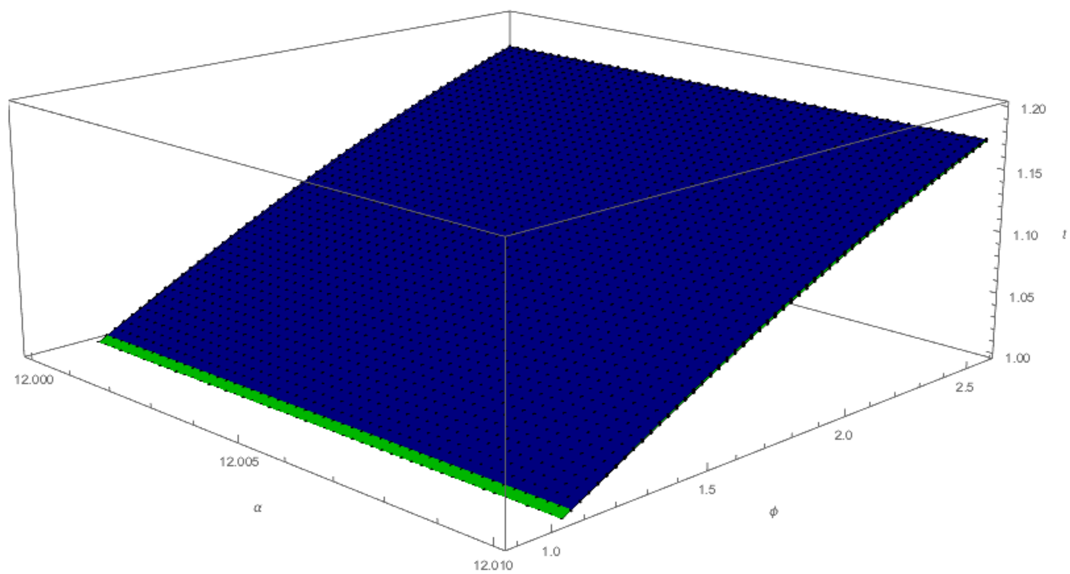

Figure 7: Nash Tariff with FDI when $k = 1.05$ (blue) and $k = 1$ (green)

6 Role of Variable Markup

This section further investigates the role of variable markup in the current setup. I first study how would a change in ad valorem tariff affect the covariance between firm-level markup and change in firm-level employment share, which is crucial in studying the pro-competitive effect of trade in Arkolakis et al. (2018). I also investigate the impact of FDI on this covariance. Then I study how would a change in ad valorem tariff affect the average markup distribution.

6.1 Misallocation

According to Arkolakis et al. (2018), variable markups can create a new source of gain or loss from trade liberalization, depending on whether low-cost firms, which charge high markups and under-supply their varieties, end up growing in size. According to their Appendix A.4, the effect of trade liberalization on the welfare of country j depends on the sign of the covariance of the markup, charged by a firm in country j that produces variety for market i, and a change in its labor share that is needed to produce this variety for this market:

$$\text{cov}\left(m^{i}\left(\omega\right), \frac{dl^{i}\left(\omega\right)}{L^{j}}\right) \tag{37}$$

where $l^i(\omega)$ is the total employment associated with a production of variety ω in country j for sales in country i. In other words, if this covariance is positive, then trade liberalization has an additional positive effect on welfare in country j through a reduction in misallocation. In their setup, without considering the choice of FDI, equation (37) becomes:

$$\begin{aligned}\text{cov}\left(m^{i}\left(\omega\right), \frac{dl^{i}\left(\omega\right)}{L^{j}}\right) &= N_D^H \int_0^{c_D^H} \frac{p_D^H\left(c\right)}{c} \frac{d\left[c q_D^H\left(c\right)\right]}{L^H} \frac{dG\left(c\right)}{G\left(c_D^H\right)} \\ &+ N_X^H \int_0^{c_X^H} \frac{p_X^H\left(c\right)}{\tau^F c} \frac{d\left[c \tau^F q_X^H\left(c\right)\right]}{L^H} \frac{dG\left(c\right)}{G\left(c_X^H\right)}\end{aligned}$$

It is important to notice that this covariance is at the firm-level. Therefore it not only includes firm's domestic production decision but also relates to its export decision. In their setting, this covariance is negative, so the presence of variable markups reduces the welfare gain from trade. This is because a decrease in trade costs makes exporting firms relatively more productive, which leads to changes in markups. When demand is log-concave, as assumed in Krugman (1979), higher markups implies incomplete pass-through of changes in marginal costs to prices, which tends to lower the welfare gains from trade.

In the current setup, the covariance term is:

$$\textbf{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = N_D^H \int_0^{c_D^H} \frac{p_D^H(c)}{c} \frac{d\left[c q_D^H(c)\right]}{L^H} \frac{dG(c)}{G(c_D^H)}$$
$$+ N_X^H \int_{c_{FDI}^H}^{c_X^H} \frac{p_X^H(c)}{\tau^F c} \frac{d\left[c\tau^F q_X^H(c)\right]}{L^H} \frac{dG(c)}{G(c_X^H)}$$
$$+ N_{FDI}^H \int_0^{c_{FDI}^H} \frac{p_{FDI}^H(c)}{\varphi^F c} \frac{d\left[c\varphi^F q_{FDI}^H(c)\right]}{L^H} \frac{dG(c)}{G(c_{FDI}^H)}$$

Several differences compared to Arkolakis et al. (2018): (1) trade liberalization can also take the form of tariff reduction, generating pro-competitive effect; (2) the covariance has an additional item due to the choice of FDI, which means now the welfare implication of a change in tariff also depends on firm's FDI activity. In Appendix A.11, I analytically derived the covariance term under symmetry:

$$\textbf{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) = \frac{(\alpha - c_D)\, dc_D}{2\eta\left(1 + t^{-k}\tau^{-k}\right)} \left\{2k + 1 + \left[(t\tau)^{-k} - \xi^k\right]\right.$$
$$\left. \times \left[2k + 1 - k(1 - t\tau\xi)(t\tau\xi)^k - (t\tau\xi)^k\right] + \xi^k(k + k\varphi + 1)\right\}$$

Since $dc_D/dt > 0$[38], that means for $dt > 0$, the covariance term is *positive*, indicating a reduction in misallocation through protection. As discussed in Section 4, when $\alpha > 2\tilde{c}_D$, demand for the differentiated varieties is sufficiently high, the optimal tariff under this case is greater than one, which means an increase in tariff raises the welfare. The presence of variable markup and FDI results in a positive covariance term between firm-level markup and change in firm-level employment share, which means the gain from protection, in this case, is even larger due to the reduction in misallocation. Since the current quadratic quasi-linear preference satisfies the basic assumptions[39] in the general demand structure in Arkolakis et al. (2018), that means an increase in tariff will decrease the relative demand for high-cost varieties. This, in turn, triggers a reallocation of labor away from these goods. Intuitively, the change in tariff induces tariff-jumping in the economy, triggering labor reallocation toward the multinationals, whose goods are originally under-supplied and who charge higher markups. Misallocation is reduced since the market becomes more concentrated after the change in tariff. Therefore, generating a positive correlation between markups and the labor share, hence increase the gains from the change in tariff.

[38]See Appendix A.11

[39]See their 2012 working paper, Section 5.

6.2 Social Optimum vs. Market Outcome

To further understand the role of variable markup and its impact on tariff choice, I follow Nocco et al. (2014) to study the social optimum in the current economy. Since quadratic quasi-linear utility implies transferable utility, social welfare may be expressed as the sum of all consumers' utilities. The planner chooses the number of entrants (N_E^H, N_E^F) and output level $(q_0^H, q_0^F, q_i^H, q_i^F)$ to achieve the first-best outcome[40]:

$$\max_{\{N_E, q_0, q_i\}} \mathbb{W}$$

$$\text{s.t. } q_0^H + q_0^F + f\left(N_E^H + N_E^F\right) + N_E^H \int_0^{c_M} \left[c q_D^H(c) + \tau^F c q_X^H(c) + \varphi^F c q_{FDI}^H(c)\right] dG(c)$$
$$+ N_E^F \int_0^{c_M} \left[c q_D^F(c) + \tau^H c q_X^F(c) + \varphi^H c q_{FDI}^F(c)\right] dG(c) = 2 + \bar{q}_0^H + \bar{q}_0^F$$

Compared to the social optimum, the market outcome produces multiple failures: (1) the home domestic market selection is weaker than the socially optimum selection $\left(c_D^{HM} > c_D^{HO}\right)$, (2) the home exporter market selection is weaker than the socially optimum selection $\left(c_X^{HM} > c_X^{HO}\right)$, (3) the home FDI market selection is stronger than the socially optimum selection $\left(c_{FDI}^{HM} < c_{FDI}^{HO}\right)$, (4) market outcome undersupplies varieties with low marginal production cost[41], $q_D^{HM} < q_D^{HO}$ when $c < \left[2 - (2/\triangle)^{1/(k+2)}\right] c_D^{HO}$, (5) depending on the relative demand for the differentiated varieties, the market outcome does not always yield the socially optimal level of the total number of varieties (N^H, N^F) and the number of entrants (N_E^H, N_E^F).

The inefficiencies in the market outcome originate from multiple externalities in the economy. The monopoly power in the differentiated-good sector allow a firm to price over its marginal cost, under the free-entry condition, this externality tends to create too many varieties. The new entrant will also cut the profits of existing firms, and this business-stealing effect also tends to create too many varieties. With quadratic quasi-linear preference, the consumers display "love of variety", yet firms do not consider this fact when making entry decision. This tends to create not enough varieties in the economy. With variable markup, firm heterogeneity also creates an externality. Since markup is decreasing in marginal cost, this results in the low marginal cost firms (more productive) being inefficiently small and high marginal

[40] For detailed solution of first-best outcome, please refer to the Appendix.
[41] Please refer to the Appendix for a detailed expression of $\triangle$.

cost firms (less productive) being inefficiently large in the market outcome. All these externalities work together to generate market failures in the current economy.

According to Dhingra and Morrow (2012), when monopolistic competition is combined with CES preference, the market outcome coincides with the first-best outcome. The externalities mentioned above exactly cancel with each other[42]. The current economy deviates from this benchmark due to the quadratic quasi-linear preference. The forces that generate externality do not cancel with each other. The market outcome does not coincide with the first-best outcome.

More importantly, entry in the current economy is not socially optimal as in Melitz (2003); Melitz and Ottaviano (2008). According to Arkolakis et al. (2012), entry level is not affected by the change in transportation cost due to one of their "macro" assumptions-aggregate profits in any country is a constant share of aggregate revenue, which is satisfied in the special case of a Pareto distribution on productivity draws. Balistreri et al. (2011) were the first to notice that entry[43] is no longer fixed if *ad valorem* tariff is introduced. In the current framework, based on equation (26), entry is affected by the tariff level. An increase in home tariff can bring the market outcome closer to the unconstrained optimum:

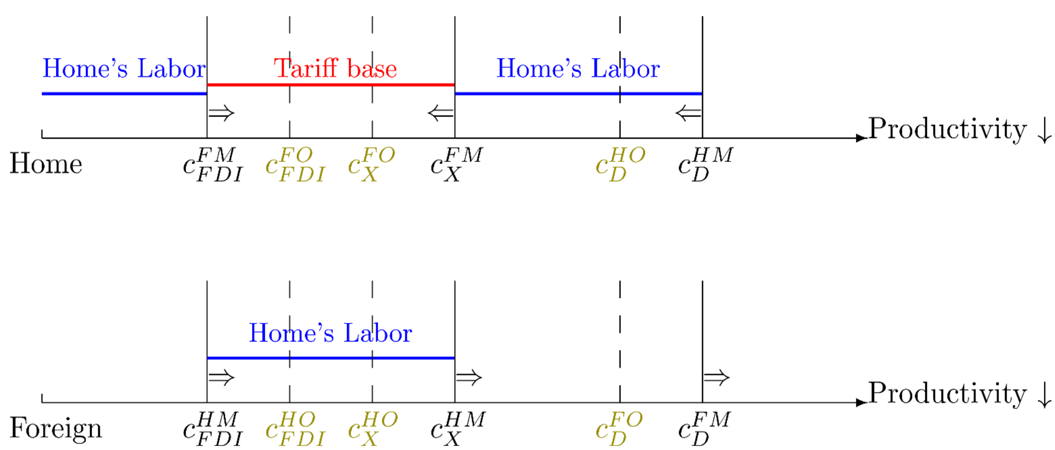

Figure 8: Cutoff responses to an increase in t^H

When the relative demand for the differentiated varieties is sufficiently high, the social planner has an incentive to encourage protection since it can bring all the

[42]Note, firm heterogeneity does not create externality since all firms charge identical markup

[43]For a detailed discussion of entry in heterogeneous firm framework, see Caliendo et al. (2017). However, in Cole and Davies (2011), although *ad valorem* tariff is introduced, the entry margin is entirely ignored. This is likely due to the fact that they do not obtain a closed-form solution of the model.

cutoffs closer to their first-best level, reducing the misallocation in the economy. This is because a higher tariff will reduce the markup of all the firms, but disproportionally those firms charge higher markups, causing these firms to pass their cost advantage to prices.

Turning back to the results in Section 5.4, where the uncooperative tariff level depends on the shape of Pareto distribution k, the degree of firm-heterogeneity decreases as k increases. If $k \to \infty$, the marginal cost distribution becomes degenerate at its lower bound c_M, firm heterogeneity reaches zero, and all the firms have identical marginal cost draw. As k decreases, the degree of firm heterogeneity increases. If $k \to 0$, the marginal cost distribution becomes uniform distribution over $[0, c_M]$, firm heterogeneity reaches maximum since firms have equal chance to draw any marginal cost level over the support. If k is small, which means there are relatively more productive firms in the economy, since tariff-jumping FDI happens among the more productive firms (exporters and FDI firms) along the marginal cost distribution, the reduction of misallocation has a more substantial impact on the economy. If k is large, which means there are relatively less productive firms in the economy, the reduction of misallocation through tariff-jumping has a smaller impact on the economy. In Section 5.4, I find Nash tariff without FDI (small k) > Nash tariff without FDI (big k) and Nash tariff with FDI (small k) < Nash tariff with FDI (big k), this shows under the variable markup, the degree of firm heterogeneity affects the choice noncooperative level. When the economy features a higher degree of firm heterogeneity (small k), allowing firms to engage in FDI can lower the tariff competition by reducing misallocation.

7 Conclusion

In this paper, I study the trade policy implication of revenue tariffs with rebates to consumers in the presence of variable markups and firm's endogenous choice to engage in multinational production. I introduced *ad valorem* tariff and horizontal FDI into the Melitz and Ottaviano (2008) framework. My conclusions can be broadly summarized as follows. First, I find protectionist trade policy can create pro-competitive effect if the increase of tariff-jumping multinationals dominates the reduction of foreign exporters. Second, I find that in contrast to Cole and Davies (2011), the welfare implication of protectionist tariffs crucially depends on consumer's preference. If the demand for the differentiated varieties is sufficiently high, this will drive excess firm entry. The business-stealing effect and substitution effect will dominate the positive impact on consumers generated through the entry, creating negative externality in the economy. In this case, the social planner can improve the welfare by properly deterring entry through tariff. Third, complementing the results in Arkolakis et al. (2018), I find a change in *ad valorem* tariff can affect the degree of misallocation in the economy by reallocating labor toward the the more productive firms, hence affecting the choice of Nash tariff choice. Lastly, the presence of FDI affects a country's non-cooperative tariff choice. In particular, it can reduce the non-cooperative tariff through a novel channel: reducing the degree of misallocation in the economy.

Given these results, there are several potential avenues for future research. For example, deviating from Pareto distribution. As discussed by Feenstra (2018), if the support of cost distribution become bounded, other channels that will affect the pro-competitive effect of trade will begin to work, delivering different welfare implication of trade. On the other hand, the combination of Melitz and Pareto implies that trade cost will only affect export through extensive margin, but this is at odds with the empirical fact that most of the adjustments happen along the intensive margin. This can be reconciled by introducing log-normal distribution[44].

Another interesting path would be to investigate if the current trade policy results still hold under alternative demand structures that generate variable markups. Several approaches are being considered: (1) Deviating from CES and quadratic quasi-linear preference. For instance, QMOR (quadratic mean of order r) preference as in Feenstra (2018). This preference nested CES and the Melitz and Ottaviano (2008) preference. Incorporating ad valorem tariff and FDI into this framework will provide a direct comparison of the Nash tariff level under these two special cases. Other preferences include IA (indirectly additive) preference as in Bertoletti et al. (2018), AQS (additively quasi-separable) preference as in Behrens and Murata (2007), and translog preference as in Rodriguez-Lopez (2011). (2) Dropping the *monopolistic competition* in Cole and Davies (2011), this will generate variable markups under the CES preference and can also allow comparison the Nash tariff level (at least numerically). For example, by assuming Bertrand or Cournot competition, as in Bernard et al. (2003), Atkeson and Burstein (2008), and Edmond et al. (2015).

Lastly, the trade policy implication here is primarily focusing on the monopolistically competitive sector. It would be interesting and relevant to see how would a multi-sector framework affects the policy results. For example, Spearot (2016) ex-

[44]See Fernandes et al. (2017) for a detailed discussion

tended Melitz and Ottaviano (2008) by incorporating multiple countries and multiple industries with heterogeneity in the country-by-industry shape parameters of the Pareto cost distributions and provided quantitative implication for unilateral tariff liberalization. The excess entry in the current framework might have different implication here since the mass of entrants in a particular sector now depends on the relative expenses on goods produced there. I leave these questions for future work.

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A Proofs

A.1 Proof of Lemma 1

To prove this Lemma, I first prove the following condition:

$$\Phi_1^l + \Phi_2^l > \psi^l \in (0,1) \text{ for } l \in \{H, F\}$$

Given $\psi^l \equiv \left(\tau^l\right)^{-k} \left(t^l\right)^{-(k+1)}$ and

$$\Phi_1^l \equiv \frac{(k+1)(k+2)\, t^l (\tau^l)^2}{2} \left\{ \left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\frac{1}{t^l \tau^l}\right)^2 \left(\xi^l\right)^k \right.$$
$$\left. - \frac{2k}{k+1} \left[\left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\frac{1}{t^l \tau^l}\right) \left(\xi^l\right)^{k+1} \right] + \frac{k}{k+2} \left[\left(\frac{1}{t^l \tau^l}\right)^{k+2} - \left(\xi^l\right)^{k+2} \right] \right\}$$
$$\Phi_2^l \equiv \frac{(k+1)(k+2)\left(\xi^l\right)^k}{2} \left[1 - \frac{2k\varphi^l \xi^l}{k+1} + \frac{k\left(\varphi^l \xi^l\right)^2}{k+2} \right]$$

It is then straightforward to show

$$\Phi_1^l + \Phi_2^l = \psi^l + \frac{(k+1)(k+2)\left(\xi^l\right)^k}{2} \left\{ \left(1 - \frac{1}{t^l}\right) \right.$$
$$\left. - \frac{2k}{k+1} \xi^l \left(\varphi^l - \tau^l\right) + \frac{k}{k+2} \left(\xi^l\right)^2 \left(\left(\varphi^l\right)^2 - t^l \left(\tau^l\right)^2 \right) \right\}$$

To show that $\Phi_1^l + \Phi_2^l > \psi^l$, it is equivalent to show that

$$\left(1 - \frac{1}{t^l}\right) - \frac{2k}{k+1} \xi^l \left(\varphi^l - \tau^l\right) + \frac{k}{k+2} \left(\xi^l\right)^2 \left(\left(\varphi^l\right)^2 - t^l \left(\tau^l\right)^2 \right) > 0$$

Based on the definition of $\xi^l \equiv \left(\sqrt{t^l} - 1\right) / \left(\sqrt{t^l}\varphi^l - t^l \tau^l\right)$, the above equation becomes:

$$\left(1-\frac{1}{t^l}\right)-\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\left(\varphi^l-\tau^l\right)+\frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)>0$$

$$\Leftrightarrow\left(1-\frac{1}{t^l}\right)+\frac{k}{k+2}\left(\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\right)^2\left(\left(\varphi^l\right)^2-t^l\left(\tau^l\right)^2\right)>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}\varphi^l-t^l\tau^l}\left(\varphi^l-\tau^l\right)$$

$$\Leftrightarrow\frac{t^l-1}{t^l}+\frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l+\sqrt{t^l}\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l-\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}$$

$$\Leftrightarrow\frac{t^l-1}{t^l}+\frac{k}{k+2}\frac{\left(\sqrt{t^l}-1\right)^2}{t^l}\frac{\varphi^l+\sqrt{t^l}\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}>\frac{2k}{k+1}\frac{\sqrt{t^l}-1}{\sqrt{t^l}}\frac{\varphi^l-\tau^l}{\varphi^l-\sqrt{t^l}\tau^l}$$

Multiply both sides by $(k+1)(k+2)\,t^l\left(\varphi^l-\sqrt{t^l}\tau^l\right)$, I have

$$\begin{aligned}&\left(k^2+3k+2\right)\left(\sqrt{t^l}+1\right)\left(\varphi^l-\sqrt{t^l}\tau^l\right)+\\&\qquad\left(k^2+k\right)\left(\sqrt{t^l}-1\right)\left(\varphi^l+\sqrt{t^l}\tau^l\right)>\left(2k^2+4k\right)\sqrt{t^l}\left(\varphi^l-\tau^l\right)\\&\qquad\qquad\Leftrightarrow 2\sqrt{t^l}\varphi^l-2\sqrt{t^l}\tau^l+2\left(k+1\right)\left(\varphi^l-t^l\tau^l\right)>0\\&\qquad\qquad\Leftrightarrow 2\sqrt{t^l}\left(\varphi^l-\tau^l\right)+2\left(k+1\right)\left(\varphi^l-t^l\tau^l\right)>0\end{aligned}$$

This is obviously true when $\varphi^l > t^l\tau^l$ (note $t^l > 1$), which is the assumption we made to guarantee the existence of tariff-jumping FDI.

Compare the cutoff expressions, for $l \in \{H, F\}$

Open economy, with tariff, export and FDI: $c_{D1}^H=\left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$

Open economy, with tariff and export: $c_{D2}^H=\left(\gamma\phi\frac{1-\psi^F}{1-\psi^F\psi^H}\right)^{\frac{1}{k+2}}$, $\psi^l=\left(\tau^l\right)^{-k}\left(t^l\right)^{-(k+1)}$

Closed economy: $c_{D3}^H=\left(\gamma\phi\right)^{\frac{1}{k+2}}$, as in MO(2008) Section 2

With the proved condition, it is straightforward to show that

$$c_{D3}^H > c_{D2}^H > c_{D1}^H$$

□

A.2 Proof of Lemma 2

Based on the solution of $c_D^H = \left[\gamma\phi \frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{\frac{1}{k+2}}$, I have

$$\begin{aligned}
\frac{\partial c_D^H}{\partial t^H} &= \frac{\gamma\phi}{k+2}\left[\gamma\phi\frac{1-\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}} \frac{\left(\Phi_1^F+\Phi_2^F\right)\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H} \\
&= \underbrace{\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}
\end{aligned}$$

So the sign crucially depends on $\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}$. It is straightforward to show that

$$\begin{aligned}
\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H} &= -\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}\Bigg\{\frac{2}{\left(t^H\tau^H\right)^k}\left[t^H\left(\tau^H\right)^2-2\varphi^H\sqrt{t^H}\tau^H+\left(\varphi^H\right)^2\right] \\
&+\left(\xi^H\right)^k\left[-2t^H\left(\tau^H\right)^2-2k\sqrt{t^H}t^H\left(\tau^H\right)^2+k\left(t^H\tau^H\right)^2\right. \\
&\left.+2\left(k+2\right)\sqrt{t^H}\tau^H\varphi^H-\left(k+2\right)\left(\varphi^H\right)^2\right]\Bigg\} \\
&= \underbrace{-\frac{k+1}{2t^H\left(\sqrt{t^H}\varphi^H-t^H\tau^H\right)^2}}_{<0}\Bigg\{2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right] \\
&-\left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\Bigg\}
\end{aligned}$$

The expression within the big bracket is greater than zero for all $k\in[1,+\infty)$ when $\varphi^H > t^H\tau^H$, to see this, it is equivalent to show

$$2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\left(t^H\tau^H\right)^{-k}-\left(\xi^H\right)^k\right] > \left(\xi^H\right)^k k\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)$$

For $k=1$, the expression become

$$
\begin{aligned}
&2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left[\frac{1}{(t^H\tau^H)}-\xi^H\right]>\xi^H\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow&2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\left(\frac{1}{\xi^H t^H\tau^H}-1\right)>\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow&2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2\frac{\varphi^H-t^H\tau^H}{\sqrt{t^H}\tau^H\left(\sqrt{t^H}-1\right)}>\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow&2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2>\sqrt{t^H}\tau^H\left(\sqrt{t^H}-1\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)\\
\Leftrightarrow&2\left(\varphi^H\right)^2-3\sqrt{t^H}\tau^H\varphi^H+3\sqrt{t^H}t^H\left(\tau^H\right)^2-t^H\tau^H\varphi^H-\left(t^H\tau^H\right)^2>0\\
\Leftrightarrow&\left(\varphi^H-t^H\tau^H\right)\left(2\varphi^H+t^H\tau^H-3\sqrt{t^H}\tau^H\right)>0
\end{aligned}
$$

which is obviously true.

For k approach infinity, to prove the equation, it is equivalent to show

$$
\frac{2\left(\varphi^H-\sqrt{t^H}\tau^H\right)^2}{\left(\varphi^H-t^H\tau^H\right)\left(\varphi^H+t^H\tau^H-2\sqrt{t^H}\tau^H\right)}>\frac{k}{\left(\xi^H t^H\tau^H\right)^{-k}-1}
$$

As $k\to\infty$, the limit of right-hand side is 0. It means as long as the left-hand side is positive, the equation is true for $k\to\infty$. The left hand side is obviously positive given $\varphi^H>t^H\tau^H$. Therefore,

$$
\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}<0\Rightarrow\frac{\partial c_D^H}{\partial t^H}<0
$$

To show $\dfrac{\partial c_D^F}{\partial t^H}$ is easier, note that

$$
\begin{aligned}
\frac{\partial c_D^F}{\partial t^H}&=\left[\gamma\phi\frac{1-\left(\Phi_1^H+\Phi_2^H\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\right]^{-\frac{k+1}{k+2}}\frac{\gamma\phi\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]/(k+2)}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]^2}\frac{-\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}\\
&=\underbrace{\frac{c_D^F}{k+2}\frac{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\right]}{\left[1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)\right]\left[1-\left(\Phi_1^H+\Phi_2^H\right)\right]}}_{>0}\underbrace{\frac{-\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}_{>0}>0
\end{aligned}
$$

□

A.3 Proof of Lemma 3

Based on the cutoff relations, it is straightforward to show

$$\begin{aligned}
\frac{\partial c_X^H}{\partial t^H} &= \frac{\partial\left(c_D^F/t^F\tau^F\right)}{\partial t^H} = \frac{1}{t^F\tau^F}\frac{\partial c_D^F}{\partial t^H} > 0 \\
\frac{\partial c_X^F}{\partial t^H} &= \frac{\partial\left(c_D^H/t^H\tau^H\right)}{\partial t^H} = \frac{1}{\left(t^H\right)^2\tau^H}\left(\frac{\partial c_D^H}{\partial t^H}t^H - c_D^H\right)
\end{aligned}$$

To show $\frac{\partial c_D^H}{\partial t^H}t^H - c_D^H < 0$ is equivalent to show

$$\begin{aligned}
&\frac{1}{k+2}c_D^H\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}t^H < c_D^H \\
\Leftrightarrow\ &\underbrace{\frac{1}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}t^H}_{<0} < 1
\end{aligned}$$

Which is obviously true. Therefore, Lemma 3 is proved. □

A.4 Proof of Lemma 4

This part is not easy to prove, notice

$$\begin{aligned}
\frac{\partial c_{FDI}^F}{\partial t^H} &= \frac{\partial\left(c_D^H\xi^H\right)}{\partial t^H} = \frac{\partial c_D^H}{\partial t^H}\xi^H + \frac{\partial\xi^H}{\partial t^H}c_D^H \\
&= \underbrace{\frac{c_D^H\xi^H}{k+2}\frac{\left(\Phi_1^F+\Phi_2^F\right)}{1-\left(\Phi_1^F+\Phi_2^F\right)\left(\Phi_1^H+\Phi_2^H\right)}}_{>0}\underbrace{\frac{\partial\left(\Phi_1^H+\Phi_2^H\right)}{\partial t^H}}_{<0} + \underbrace{\frac{\partial\xi^H}{\partial t^H}c_D^H}_{>0}
\end{aligned}$$

It is straightforward to verify that under the parameter choice in Section 4.3, the second term dominates the first term, so foreign country's FDI cutoff level (c_{FDI}^F)is strictly increasing as Home country's tariff (t^H) increases. Do notice, the other choice of ξ, which is $\xi = \left(\sqrt{t}+1\right)/\left(\sqrt{t}\varphi + t\tau\right)$ will make the second item negative, thereby making c_{FDI}^F decreasing in response to t^H's increase, hence no tariff jumping. □

A.5 Proof of Lemma 5

Based on equation (24), it's straightforward to show

$$\frac{\partial N^H}{\partial t^H} = \frac{2\gamma(k+1)}{\eta} \frac{-\frac{\partial c_D^H}{\partial t^H} c_D^H - \frac{\partial c_D^H}{\partial t^H}\left(\alpha - c_D^H\right)}{\left(c_D^H\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c_D^H\right)^2} \frac{\partial c_D^H}{\partial t^H} > 0$$

$$\frac{\partial N^F}{\partial t^H} = \frac{2\gamma(k+1)}{\eta} \frac{-\frac{\partial c_D^F}{\partial t^H} c_D^F - \frac{\partial c_D^F}{\partial t^H}\left(\alpha - c_D^F\right)}{\left(c_D^F\right)^2} = -\frac{2\gamma\alpha(k+1)}{\eta\left(c_D^F\right)^2} \frac{\partial c_D^F}{\partial t^H} < 0$$

Now based on equation (26), as t^H increases

$$N_E^F = \frac{2\left(c_M\right)^k (k+1)\gamma}{\eta\left(1 - \delta^H \delta^F\right)} \left[\frac{\alpha - c_D^F}{\left(c_D^F\right)^{k+1}} - \delta^F \frac{\alpha - c_D^H}{\left(c_D^H\right)^{k+1}}\right]$$

δ^H decreases (hence the coefficient in front of the bracket decreases), c_D^F increases (the first item in the bracket decreases), c_D^H decreases (the second item in the bracket increases). Hence the whole expression on the right decreases, therefore $\partial N_E^F / \partial t^H < 0$. Now utilizing the free-entry condition, which is equation (25)

$$G\left(c_D^H\right) N_E^H + G\left(c_X^F\right) N_E^F = N^H$$

As t^H increases, N^H increases, it means the left-side also needs to increase. Notice N_E^F decreases, c_X^F decreases, c_D^H decreases, it then must be true that N_E^H increases. Hence, $\partial N_E^H / \partial t^H > 0$. □

A.6 Proof of Proposition 1

Once again, to simplify the proof, I assume symmetry, following Appendix A.11, $c_D = c_D^H = c_D^F$ and

$$N_E = \frac{2\gamma c_M^k (k+1)\left(\alpha - c_D\right)}{\eta\left(c_D\right)^{k+1}\left(1 + t^{-k}\tau^{-k}\right)}$$

$$N_D = \frac{2\gamma (k+1)\left(\alpha - c_D\right)}{\eta\left(1 + t^{-k}\tau^{-k}\right) c_D}$$

$$N_X = \frac{2\gamma (k+1)\left(\alpha - c_D\right)}{\eta\left(1 + t^{-k}\tau^{-k}\right) c_D} \left[(t\tau)^{-k} - \xi^k\right]$$

$$N_{FDI} = \frac{2\gamma (k+1)\left(\alpha - c_D\right)}{\eta\left(1 + t^{-k}\tau^{-k}\right) c_D} \xi^k$$

Together with equation (6), (10) and (15), the average markup in (31) can be written as follow:

$$
\begin{aligned}
\bar{m} &= \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \int_0^{c_D} m_D(c) \frac{dG(c)}{G(c_D)} \right. \\
&+ \left. N_X \int_{c_{FDI}}^{c_X} m_X(c) \frac{dG(c)}{G(c_X)} + N_{FDI} \int_0^{c_{FDI}} m_{FDI}(c) \frac{dG(c)}{G(c_{FDI})} \right] \\
&= \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \int_0^{c_D} \frac{c_D + c}{2c} \frac{kc^{k-1}}{c_D^k} dc \right. \\
&+ \left. N_X \int_{c_{FDI}}^{c_X} \frac{t(c_X + c)}{2c} \frac{kc^{k-1}}{c_X^k} dc + N_{FDI} \int_0^{c_{FDI}} \frac{c_D + \varphi^H c}{2\varphi^H c} \frac{kc^{k-1}}{c_{FDI}^k} dc \right] \\
&= \frac{1}{N_D + N_X + N_{FDI}} \left[N_D \times \frac{2k-1}{2k-2} + N_X \times t \right. \\
&\times \left. \left(\frac{2k-1}{2k-2} - \frac{k}{2k-2} (t\tau\xi)^{k-1} - \frac{1}{2} (t\tau\xi)^k \right) + N_{FDI} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right) \right] \\
&= \frac{1}{1 + (t\tau)^{-k}} \times \frac{2k-1}{2k-2} + \frac{t\left[(t\tau)^{-k} - \xi^k \right]}{1 + (t\tau)^{-k}} \times \left[\frac{2k-1}{2k-2} - \frac{k}{2k-2} (t\tau\xi)^{k-1} - \frac{1}{2} (t\tau\xi)^k \right] \\
&+ \frac{\xi^k}{1 + (t\tau)^{-k}} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right) \\
&= \underbrace{\frac{1}{1 + (t\tau)^{-k}} \times \frac{2k-1}{2k-2}}_{\text{weighted expected markup in domestic}} \\
&+ \underbrace{\frac{1 - (t\tau\xi)^k}{1 + (t\tau)^{-k}} \times \frac{1}{t^{k-1}\tau^k} \left\{ \frac{1}{2} \left[1 - (t\tau\xi)^k \right] + \frac{k}{2k-2} \left[1 - (t\tau\xi)^{k-1} \right] \right\}}_{\text{weighted expected markup from foreign exporters}} \\
&+ \underbrace{\frac{\xi^k}{1 + (t\tau)^{-k}} \times \left(\frac{k}{2k-2} \frac{1}{\varphi\xi} + \frac{1}{2} \right)}_{\text{weighted expected markup from foreign FDI}}
\end{aligned}
$$

A.7 Proof of Proposition 2

Proof: Based on equation (27), the social welfare can be rewritten as follow:

$$
\mathbb{U}^H + \mathbb{U}^F = I^H + I^F + \frac{\alpha - c_D^H}{2\eta} \left(\alpha - \frac{k+1}{k+2} c_D^H \right) + \frac{\alpha - c_D^F}{2\eta} \left(\alpha - \frac{k+1}{k+2} c_D^F \right)
$$

Since consumer receive income from wage (which is equal to 1) and tariff revenue, so the above equation can be rewritten as:

$$\mathbb{U}^H + \mathbb{U}^F = 2 + \left(t^H - 1\right) IM^H + \left(t^F - 1\right) IM^F$$
$$+ \frac{\alpha - c_D^H}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^H\right) + \frac{\alpha - c_D^F}{2\eta}\left(\alpha - \frac{k+1}{k+2}c_D^F\right)$$

To see the welfare implication of free trade, I evaluate the first order condition of the above expression with respect to tariff under symmetry when $t^H = t^F = 1$. Since symmetry implies $IM^F = IM^H, \partial IM^F/\partial t^H = \partial IM^H/\partial t^F, c_D^F = c_D^H, \partial c_D^F/\partial t^H = \partial c_D^H/\partial t^F$, therefore :

$$\frac{\partial\left(\mathbb{U}^H + \mathbb{U}^F\right)}{\partial t}|_{t^H=t^F=1} = \underbrace{(t-1)\left(\frac{\partial IM^H}{\partial t^H} + \frac{\partial IM^F}{\partial t^H}\right)}_{\circledast} + IM^H$$
$$+ \frac{2(k+1)c_D - (2k+3)\alpha}{2\eta(k+2)}\left(\frac{\partial c_D^H}{\partial t^H} + \frac{\partial c_D^F}{\partial t^H}\right)$$

Notice, when $t^H = t^F = 1$, $\circledast = 0$. Based on equation (26) and (28),

$$IM^H|_{t^H=1} = N_E|_{t^H=1} \times \frac{(c_D)^{k+2}\tau^{-k}}{2\gamma(k+2)(c_M)^k}|_{t=1}$$
$$= \frac{2(c_M)^k(k+1)\gamma\left(1-\tau^{-k}\right)}{\eta\left(1-\tau^{-2k}\right)}\frac{(c_D)^{k+2}\tau^{-k}}{2\gamma(k+2)(c_M)^k}|_{t=1}$$
$$= \frac{\tau^{-k}(k+1)}{\eta\left(1+\tau^{-k}\right)(k+2)}(\alpha - c_D)c_D|_{t=1}$$

Based on the definition of Φ^H and Φ^F, it is straightforward to show

$$\Phi_1 + \Phi_2 = \tau^{-k}$$

Based on the proof of Lemma 2, it is straightforward to show

$$\frac{\partial c_D^H}{\partial t^H} + \frac{\partial c_D^F}{\partial t^H}|_{t=1} = \frac{-(k+1)c_D^H\tau^{-2k}}{(k+2)\left(1-\tau^{-2k}\right)} + \frac{c_D^F}{k+2}\frac{-(k+1)\tau^{-k}}{\left(1-\tau^{-2k}\right)}$$
$$= \frac{\tau^{-k}(k+1)}{\left(1+\tau^{-k}\right)(k+2)}c_D|_{t=1}$$

Therefore, the original first order condition can be rewritten as

$$\begin{aligned}
\frac{\partial\left(\mathbb{U}^{H}+\mathbb{U}^{F}\right)}{\partial t}|_{t^{H}=t^{F}=1} &= \frac{\tau^{-k}(k+1)}{\eta\left(1+\tau^{-k}\right)(k+2)}\left(\alpha-c_{D}\right)c_{D}|_{t=1} \\
&+\frac{2(k+1)c_{D}-(2k+3)\alpha}{2\eta(k+2)}\frac{\tau^{-k}(k+1)}{\left(1+\tau^{-k}\right)(k+2)}c_{D}|_{t=1} \\
&= \frac{\tau^{-k}(k+1)c_{D}}{2\eta(k+2)^{2}\left(1+\tau^{-k}\right)}\left[2(k+2)\left(\alpha-c_{D}\right)+2(k+1)c_{D}-(2k+3)\alpha\right] \\
&= \frac{\tau^{-k}(k+1)c_{D}}{2\eta(k+2)^{2}\left(1+\tau^{-k}\right)}\left(\alpha-2c_{D}\right)|_{t=1}
\end{aligned}$$

Define $\tilde{c}_D \equiv c_D|_{t=1}$, then this completes the proof of proposition 2. □

A.8 Proof of Second-Best Social Planner Problem

Based on the definition of average consumer surplus, it can be rewritten in terms of $\tilde{c}_D$:

$$\text{Avg. CS} \equiv \frac{\gamma}{2}\int_{0}^{\tilde{c}_{D}}\left(q_{D}(c)\right)^{2}dG(c)=\frac{\left(\tilde{c}_{D}\right)^{k+2}}{4\gamma c_{M}^{k}(k+1)(k+2)}>0$$

Based on equation (26), under symmetry and $t=1$,

$$N_{E}=\frac{2\gamma c_{M}^{k}(k+1)\left(\alpha-\tilde{c}_{D}\right)}{\eta\left(\tilde{c}_{D}\right)^{k+1}\left(1+\tau^{-k}\right)}$$

Then, the variety effect can be defined as the difference between consumer surplus and the sum of average surplus at each variety:

$$\begin{aligned}
\text{VE} &\equiv \text{CS} - N_E \times \text{Avg. CS} \\
&= \frac{\alpha-\tilde{c}_{D}}{2\eta}\left(\alpha-\frac{k+1}{k+2}\tilde{c}_{D}\right)-\frac{2\gamma c_{M}^{k}(k+1)\left(\alpha-\tilde{c}_{D}\right)}{\eta\left(\tilde{c}_{D}\right)^{k+1}\left(1+\tau^{-k}\right)}\times\frac{\left(\tilde{c}_{D}\right)^{k+2}}{4\gamma c_{M}^{k}(k+1)(k+2)} \\
&= \frac{\left(\alpha-\tilde{c}_{D}\right)}{2\eta}\left[\alpha-\frac{(k+1)\left(1+\tau^{-k}\right)+1}{(k+2)\left(1+\tau^{-k}\right)}\tilde{c}_{D}\right]
\end{aligned}$$

The expected profit of a firm can be derived from equation (20):

$$\begin{aligned}\bar{\pi} &= \int_0^{\tilde{c}_D} \pi_D(c)\, dG(c) + \int_0^{\tilde{c}_X} \pi_X(c)\, dG(c) \\ &= \frac{(\tilde{c}_D)^{k+2}}{2\gamma c_M^k (k+1)(k+2)} + \frac{\tau^2 (\tilde{c}_X)^{k+2}}{2\gamma c_M^k (k+1)(k+2)} = \frac{(\tilde{c}_D)^{k+2}\left(1+\tau^{-k}\right)}{2\gamma c_M^k (k+1)(k+2)}\end{aligned}$$

Notice when $t = 1, c_{FDI} = 0$ and $\tilde{c}_X = \tilde{c}_D/\tau$. With all these components and the fact that $\alpha > \tilde{c}_D$, equation (32) can now be properly signed:

$$\begin{aligned}\text{Avg. CS} &= \frac{(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)} > 0 \\ N_E \frac{\partial \text{Avg. CS}}{\partial N_E} &= \frac{(\alpha - \tilde{c}_D)(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)\left[k\tilde{c}_D - \alpha(k+1)\right]} < 0 \\ \frac{\partial \text{VE}}{\partial N_E} &= \frac{(\tilde{c}_D)^{k+2}\left\{2\left[(k+1)\left(1+\tau^{-k}\right)+1\right](\alpha - \tilde{c}_D) + \tau^{-k}\alpha\right\}}{4\gamma c_M^k (k+1)(k+2)\left[\alpha(k+1) - k\tilde{c}_D\right]} > 0 \\ N_E \frac{\partial \bar{\pi}}{\partial N_E} &= \frac{(\tilde{c}_D)^{k+2}\left(1+\tau^{-k}\right)(\alpha - \tilde{c}_D)}{4\gamma c_M^k (k+1)\left[k\tilde{c}_D - (k+1)\alpha\right]} < 0\end{aligned}$$

Therefore, the externality of entry equals to

$$\begin{aligned}&\text{Avg. CS} + N_E \frac{\partial \text{Avg. CS}}{\partial N_E} + \frac{\partial \text{VE}}{\partial N_E} + N_E \frac{\partial \bar{\pi}}{\partial N_E} \\ =&\frac{\left(1+\tau^{-k}\right)(\tilde{c}_D)^{k+2}}{4\gamma c_M^k (k+1)(k+2)\underbrace{\left[k\tilde{c}_D - (k+1)\alpha\right]}_{<0}} \times (\alpha - 2\tilde{c}_D)\end{aligned}$$

Therefore, the externality will be negative if $\alpha > 2\tilde{c}_D$, will be positive if $\alpha < 2\tilde{c}_D$. $\square$

A.9 Proof of Proposition 3

Proof: First, it is straightforward to show that $\partial IM^H / \partial t^H < 0$, this is due to tariff-jumping FDI. Now compare equation (34) and (36)

$$t_S^H - 1 = \frac{\frac{\partial CS^H}{\partial t^H} + IM^H + \frac{\partial CS^F}{\partial t^H} + \left(t^F - 1\right) \times \frac{\partial IM^F}{\partial t^H}}{-\frac{\partial IM^H}{\partial t^H}}$$

$$t_N^H - 1 = \frac{\frac{\partial CS^H}{\partial t^H} + IM^H}{-\frac{\partial IM^H}{\partial t^H}}$$

It's straightforward to show that when $\alpha > 2\tilde{c}_D$:

$$\frac{\partial CS^F}{\partial t^H} = \underbrace{\frac{\partial CS^F}{\partial c_D^F}}_{<0} \times \underbrace{\frac{\partial c_D^F}{\partial t^H}}_{>0} < 0$$

$$\begin{aligned} \left(t^F - 1\right) \times \frac{\partial IM^F}{\partial t^H} =& \frac{t^F\left(t^F - 1\right)\left(\tau^F\right)^2}{4\gamma\left(k+2\right)\left(c_M\right)^k}\left[2\left(\frac{1}{t^F\tau^F}\right)^{k+2}\right. \\ &\left. - \frac{k+2}{\left(t^F\tau^F\right)^2}\left(\xi^F\right)^k + k\left(\xi^F\right)^{k+2}\right]\underbrace{\frac{\partial N_E^H\left(c_D^F\right)^{k+2}}{\partial t^H}}_{>0} > 0 \end{aligned}$$

The first item indicates the distortion on F's consumption generated by t^H, the second item indicates the distortion on F's tariff revenue generated by t^H. It's straightforward to show the distortion on consumption is larger than the distortion on tariff revenue. Therefore the sum of these two items is negative, indicating the numerator of the first equation is smaller than that of the second equation, it must be the case that $t_S^H < t_N^H$. Hence the Nash tariff is higher than the socially optimal tariff. $\square$

A.10 Social Optimum

The social planner solves the following problem

$$\max_{\{N_E^H, q_0^H, q_i^H, N_E^F, q_0^F, q_i^F\}} \mathbb{W}$$

$$\begin{aligned} \text{s.t. } q_0^H + q_0^F + f\left(N_E^H + N_E^F\right) + N_E^H \int_0^{c_M} \left[c q_D^H(c) + \tau^F c q_X^H(c) + \varphi^F c q_{FDI}^H(c)\right] dG(c) \\ + N_E^F \int_0^{c_M} \left[c q_D^F(c) + \tau^H c q_X^F(c) + \varphi^H c q_{FDI}^F(c)\right] dG(c) = 2 + \bar{q}_0^H + \bar{q}_0^F \end{aligned}$$

Notice, $\mathbb{W} \equiv \mathbb{U}^H + \mathbb{U}^F$ and since labor has been normalized to 1, $\mathbb{U}^H$ is defined as follow

$$\begin{aligned}\mathbb{U}^H \equiv\ & q_0 + \alpha N_E^H \left\{ \int \left[q_D^H(c) + q_X^H(c) + q_{FDI}^H(c) \right] dG(c) \right\} \\ & - \frac{\gamma}{2} \left\{ N_E^H \int \left(q_D^H(c) \right)^2 dG(c) + N_E^F \int \left[\left(q_X^F(c) \right)^2 + \left(q_{FDI}^F(c) \right)^2 \right] dG(c) \right\} \\ & - \frac{\eta}{2} \left\{ N_E^H \int q_D^H(c)\, dG(c) + N_E^F \int \left[q_X^F(c) + q_{FDI}^F(c) \right] dG(c) \right\}\end{aligned}$$

The first order conditions with respect to q_D, q_X, q_{FDI} deliver the following results

$$\begin{aligned} q_D^H(c) &= \frac{c_D^{HO} - c}{\gamma}, c_D^{HO} = \alpha - \eta Q^{HO} \\ q_X^H(c) &= \frac{c_X^{HO} - c}{\gamma/\tau^F}, c_X^{HO} = \frac{\alpha - \eta Q^{FO}}{\tau^F} \\ q_{FDI}^H(c) &= \frac{c_{FDI}^{HO} - c}{\gamma/\varphi^F}, c_{FDI}^{HO} = \frac{\alpha - \eta Q^{FO}}{\varphi^F} \end{aligned}$$

The first order condition with respect to N_E deliver the following results

$$Q^{HO} = \frac{N^{HO} + 2N^{FO}}{\gamma + \eta\left(N^{HO} + 2N^{FO}\right)} \left(\alpha - \frac{k}{k+1} \left[\frac{N^{HO} + \left(\tau^H/\varphi^H\right)^{k+1} N^{FO}}{N^{HO} + 2N^{FO}} \right] c_D^{HO} \right)$$

Combine the corresponding results for the foreign country, it's straightforward to obtain the cutoff level under the planner's problem

$$c_D^{HO} = \left[\gamma (k+1)(k+2) f c_M^k \frac{1 - O_F}{1 - O_F O_H} \right]^{\frac{1}{k+2}}$$

where $O_F \equiv \left(\varphi^F\right)^{-k} + \frac{(k+1)(k+2)}{2} \left[\left(\tau^F\right)^{-k} - \left(\varphi^F\right)^{-k} \right] - k(k+2)\tau^F \left[\left(\tau^F\right)^{-(k+1)} - \left(\varphi^F\right)^{-(k+1)} \right] + \frac{k(k+1)\left(\tau^F\right)^2}{2} \left[\left(\tau^F\right)^{-(k+2)} - \left(\varphi^F\right)^{-(k+2)} \right]$. All the rest of the equilibrium variables, such as N_E^{HO}, N^{HO} etc, can be expressed as a function of c_D^{HO} and other parameters. Compare the domestic cutoff between social optimum and market outcome,

$$\left(\frac{c_D^{HM}}{c_D^{HO}} \right)^{k+2} = \frac{2}{\frac{1 - O_F}{1 - O_F O_H} / \frac{1 - \Phi_X^F - \Phi_{FDI}^F}{1 - \left(\Phi_X^F + \Phi_{FDI}^F\right)\left(\Phi_X^H + \Phi_{FDI}^H\right)}}$$

The term in the numerator of the above expression is defined as $\triangle$.

A.11 Proof of Covariance Term

Once again, to simplify the analysis, I imposed symmetry. It's clear from equation (26) that

$$N_E = \frac{2\gamma c_M^k (k+1)(\alpha - c_D)}{\eta (c_D)^{k+1} (1 + t^{-k}\tau^{-k})}$$

where $c_D = c_D^H = c_D^F$. Given equation (7), (11) and (16), and the following expression for the mass of firms:

$$\begin{aligned}
N_D = N_E \times G(c_D) &= \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \\
N_X = N_E \times [G(c_X) - G(c_{FDI})] &= \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \left[(t\tau)^{-k} - \xi^k\right] \\
N_{FDI} = N_E \times G(c_{FDI}) &= \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \xi^k
\end{aligned}$$

the covariance term can be derived as follow:

$$\begin{aligned}
\textbf{cov}\left(m^i(\omega), \frac{dl^i(\omega)}{L^j}\right) &= N_D \int_0^{c_D} p_D(c)\, d[q_D(c)] \frac{dG(c)}{G(c_D)} + N_X \int_{c_{FDI}}^{c_X} p_X(c)\, d[q_X(c)] \frac{dG(c)}{G(c_X)} \\
&+ N_{FDI} \int_0^{c_{FDI}} p_{FDI}(c)\, d[q_{FDI}(c)] \frac{dG(c)}{G(c_{FDI})} \\
&= \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \int_0^{c_D} \frac{k dc_D (c_D + c) c^{k-1}}{4\gamma c_D^k} dc \\
&+ \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \left[(t\tau)^{-k} - \xi^k\right] \int_{c_{FDI}}^{c_X} \frac{t^2\tau^2 k dc_X (c_X + c) c^{k-1}}{4\gamma c_X^k} dc \\
&+ \frac{2\gamma (k+1)(\alpha - c_D)}{\eta (1 + t^{-k}\tau^{-k}) c_D} \xi^k \int_0^{c_{FDI}} \frac{k dc_D (c_D + \varphi c) c^{k-1}}{4\gamma c_F^k} dc \\
&= \frac{(\alpha - c_D) dc_D}{2\eta (1 + t^{-k}\tau^{-k})} \left\{2k + 1 + \left[(t\tau)^{-k} - \xi^k\right] \right. \\
&\times \left. \left[2k + 1 - k(1 - t\tau\xi)(t\tau\xi)^k - (t\tau\xi)^k\right] + \xi^k (k + k\varphi + 1)\right\}
\end{aligned}$$

When $\varphi > t\tau$, it is straightforward to show that $t\tau\xi < 1$. Therefore, the covariance term can be rewritten as

$$\text{cov}\left(m^{i}(\omega), \frac{dl^{i}(\omega)}{L^{j}}\right) = \frac{(\alpha - c_D)\, dc_D}{2\eta\left(1 + t^{-k}\tau^{-k}\right)} \left\{ 2k + 1 + (t\tau)^{-k} \underbrace{\left[1 - (t\tau\xi)^{k}\right]}_{>0} \right.$$
$$\left. \times \left[k + k\,(t\tau\xi)^{k+1} + \underbrace{k - k\,(t\tau\xi)^{k}}_{>0} + \underbrace{1 - (t\tau\xi)^{k}}_{>0}\right] + \xi^{k}\,(k + k\varphi + 1) \right\}$$

Notice, under symmetry

$$\begin{aligned}
\frac{dc_D}{dt} &= \frac{d}{dt}\left[\frac{\gamma\phi}{1 + \Phi_1 + \Phi_2}\right]^{\frac{1}{k+2}} \\
&= \frac{1}{k+2}\left[\frac{\gamma\phi}{1 + \Phi_1 + \Phi_2}\right]^{-\frac{k+1}{k+2}} \times \frac{-\gamma\phi}{\left(1 + \Phi_1 + \Phi_2\right)^{2}} \frac{d\left(\Phi_1 + \Phi_2\right)}{dt} \\
&= -\frac{1}{k+2} \times \frac{c_D}{1 + \Phi_1 + \Phi_2} \times \frac{d\left(\Phi_1 + \Phi_2\right)}{dt}
\end{aligned}$$

According to Appendix A.2, $d\left(\Phi_1 + \Phi_2\right)/dt < 0$, hence $dc_D/dt > 0$, hence the covariance term is positive for $dt > 0$. □