Housing market cycles, productivity growth, and household debt*

Job market paper
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Abstract

Housing market crashes are associated with household deleveraging and a very persistent decline in economic activity in an unbalanced panel of 50 countries. The persistence of the output response is driven by a slowdown in productivity growth and capital accumulation and is increasing in the amount of preexisting household debt. To interpret these stylized facts, I construct a two-agent (borrower-saver) dynamic general equilibrium model with occasionally binding collateral constraint tied to housing equity. Productivity grows endogenously in the model through forward-looking innovation investment. When the preexisting level of debt is sufficiently high, negative housing demand shocks cause collateral constraint to bind and trigger deleveraging. Endogenous slowdown in TFP growth emerges as one of the adjustment margins during this process, prolonging the real effects of a crisis. The initial shock is amplified by a negative feedback loop between deleveraging, borrowers’ housing wealth and growth. I use the calibrated model to draw implications for macroeconomic policy during episodes of deleveraging.

JEL Codes: E32, E44, G01, O42, R21

Keywords: Business Cycles; Endogenous Growth; Financial Crises; Housing; Collateral Constraints; Occasionally Binding Constraints.

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1 Introduction

Recoveries from financial crises tend to be slow and incomplete (e.g. Cerra and Saxena 2008, Reinhart and Rogoff 2009). This became especially evident in the years after the global financial crisis of 2007-2008 as growth in many affected economies remained slow considerably longer than forecasters anticipated (figure 1). Popular explanations of the lackluster rebound from the crisis involve shortfalls in aggregate demand and prolonged private deleveraging (e.g. Blanchard et al. 2015, Lo and Rogoff 2015, Anzoategui et al. 2019). In particular, the Global Financial Crisis marked the end of the global household debt cycle that accompanied a rapid increase in housing prices worldwide (figure 2). The collapse of housing markets during the crisis had significant negative effects on the economy, including large drops in consumption, employment, and young-firm activity (Mian et al. 2013, Mian and Sufi 2014, Davis and Haltiwanger 2019).

This paper contributes to the debate on the causes of persistent effects of financial crises by focusing on housing market boom-and-bust cycles. The sheer size of the housing and mortgage markets makes their interplay with the macroeconomy important. Housing accounts for a large percentage of households wealth and most household debt in advanced countries is mortgage debt.

I first provide new cross-country evidence on the dynamics of recessions and subsequent recoveries associated with housing market crashes. Using the local projections method developed by Jordà (2005), I estimate responses of key aggregate variables to turning points in housing market cycles conditioning on a broad set of controls. Three conclusions emerge. First, an average housing market crash is associated with household — but not firm — deleveraging. Second, the associated decline in economic activity is sizable and persistent. For example, I find that a 10% decline in house price is associated with a 2% decrease in per-capita GDP and a 2.8% decrease in per-capita consumption during the crash. The decrease in output and consumption remain at -0.7% and -1%, respectively, fifteen years after the beginning of the crash. A growth accounting decomposition suggests that the persistent negative effect on the level of output is largely driven by a medium-run decline in the economy’s capital stock and utilization-adjusted total factor productivity (TFP). Third, rapid accumulation of household debt during the boom phase of a housing market cycle predicts a deeper recession and a shallower recovery after the

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1 Countries with the largest increases in household debt and housing prices in the years leading to the crisis tended to experience the biggest declines in consumption and growth once the cycle reversed (IMF 2012, Glick and Lansing 2010).

2 In 2011 the median share of mortgages in household credit was about 70 percent across mostly developed countries, according to Cerutti et al. (2015). Among the G7 economies in 2010, the share of housing wealth in the total national wealth was in the range of 20-50%, according to Piketty and Zucman (2014).
crash. A variety of checks confirms that the conclusions are robust across estimation strategies and samples. In particular, the results are not driven by the global financial crisis or other systemic financial crises that sometimes coincide with housing market crashes.

The results of the cross-country analysis are further corroborated by evidence from US states and metropolitan statistical areas. I document a significant negative association between the magnitude of the housing market crash at the onset the Great Recession and the cumulative productivity growth since the pre-crisis peak. To alleviate endogeneity concerns, I adopt two instrumental variables proposed in the literature: the Saiz (2010) housing supply elasticity index, and the Guren et al. (2018) regional sensitivity instrument. Depending on the specification, the elasticity of 2007-2017 labor productivity growth to the decline in house prices during the crash falls in the range of 0.11 to 0.25.

In the second part of the paper, I interpret the empirical observations through the lens of a quantitative dynamic general equilibrium model, which I use to explore the channels through which the crisis propagated and to perform counterfactuals. The model combines elements from the literature on deleveraging with borrower-saver heterogeneity (Eggertsson and Krugman 2012), the literature on the role of collateral constraints tied to housing wealth (Iacoviello 2005, Guerrieri and Iacoviello 2017), and the literature on endogenous growth (Romer 1990, Comin and Gertler 2006). In particular, the model features representative borrower and saver households, occasionally binding collateral constraints tied to housing equity, endogenous growth driven by the introduction of new products, and nominal rigidity. Monetary policy conducted through interest rate setting subject to a zero-lower-bound constraint.

My modeling strategy is guided by the empirical evidence. First, the persistent decline in utilization-adjusted TFP in the aftermath of housing market crashes motivates the inclusion of the endogenous growth mechanism. Second, the evidence of household deleveraging and a significant interaction between preexisting household debt and the house price decline motivates my focus on household mortgage debt. In my model credit constraints affect agents who supply, rather than demand, labor and capital. As in standard quantitative macro models, inclusion of physical capital as a factor of production, subject to endogenous utilization, improves the ability of the framework to replicate the data. Finally, I abstract from the role of housing in production and the construction sector of the economy, assuming a fixed housing supply. As documented by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by fluctuations in prices of residential land, of which there is ultimately a limited supply, and not by the price of structures.

Within this framework, I explore the aggregate effects of negative housing wealth shocks.
Motivated by evidence from the existing literature, I resort to a housing preference (demand) shock as a source of exogenous variation in the price of housing. Liu et al. (2013) identified this shock as the one that drives most of the observed fluctuations in land prices and as important for generating empirically relevant comovement between land prices and investment. Furthermore, the estimates of Guerrieri and Iacoviello (2017) suggest that about 70% of the US consumption decline during the Great Recession can be attributed to housing demand shocks. More recently, Kaplan et al. (2019) argue that demand-side factors, but not changes in credit conditions, were the dominant force behind the U.S. housing market boom-and-bust cycle around the Great Recession.

When the preexisting level of debt is sufficiently high, negative demand shocks trigger the collateral constraint and cause deleveraging: credit-constrained households must reduce their spending to satisfy a lower borrowing limit. Under nominal rigidity in the short run, deleveraging leads to a sharp demand-driven contraction. However, over time, deleveraging affects the economy’s potential output as it slows the pace of capital and firm-creation investment. The shock thus acts like an aggregate demand shock in the short run, and like an aggregate supply shock in the medium run.

The calibrated model is successful in accounting for the empirical comovement between aggregate variables associated with housing market crashes. I calibrate parameters of the mortgage market and the balanced growth path (BGP) productivity growth to an average across economies in the cross-country panel under study. In addition, I directly use the empirical impulse responses to discipline several quantitative parameters of the model through impulse-response matching. The negative feedback loop between deleveraging, borrowers net worth, and growth appears to be strong enough to explain the entirety of the estimated aggregate dynamics associated with housing market crashes. The endogenous growth mechanism embedded in the model is key to its success by generating the empirically-relevant persistence in the responses of capital and TFP.

Specifically, I identify and illustrate four key channels of shock propagation that shape the general equilibrium response to housing demand shocks. Under nominal rigidity, borrowers deleveraging results in a demand-driven recession in the short run as the real interest rate does not adjust enough to cause savers to pick up the slack. I refer to this as the aggregate demand channel. This is especially pronounced when monetary policy is constrained by the zero lower bound, which significantly amplifies the effect of the shock. This short-run effect has the potential to leave deep scars on the level of economic activity through the productivity growth channel. The basic insight is that producer entry and product introduction is a form of investment, which responds to current and expected market conditions just like investment
in physical capital does. Hence, changes in aggregate demand and credit availability affect entry and productivity growth. This is especially true for large recessions that occur in a high-leverage environment. Finally, two additional channels amplify the above effects. The first is a negative feedback loop between deleveraging and borrowers housing wealth: Fisherian debt deflation channel. The initial negative shock causes the collateral constraint to bind and trigger deleveraging. The resulting weak demand then exacerbates the damage to borrowers balance sheets and causes further deleveraging. The second is a negative feedback loop between future expected growth and current consumption: the expected income growth channel. Downward revisions in growth expectations weigh down current demand, which in turn further suppresses growth.

I conclude my study by employing the calibrated model to perform several policy counterfactuals. First, I ask how sensitive is the aggregate welfare cost of the shock to the stance of monetary policy. Both in the baseline model and in the counterfactual where without endogenous variations in growth, the welfare cost of the shock is most responsive to the strength of policy reaction to cyclical changes in output. Strong inflation targeting, in contrast, does little or nothing to improve welfare. In fact, for some combination of parameter values stronger reaction to inflation is welfare-reducing. The comparison to the counterfactual where endogenous variations in growth are shut down, shows that the endogenous productivity growth mechanism warrants stronger focus of monetary policy on the short-run output stabilization.

Turning to fiscal policy, I explore the effects of a policy that redistributes from savers to borrowers. In a narrow view, this policy can be considered as a debt forgiveness program. More broadly, it can be interpreted as a budget-neutral policy that shifts the tax burden from savers to borrowers. I demonstrate that this policy is effective in alleviating the negative feedback loop between deleveraging, asset prices, and productivity growth.

1.1 Contribution to the existing literature

My cross-country analysis adds to the existing literature on the real effects of financial and asset market cycles. The empirical strategy I follow is closest to Jordà et al. (2015), who studied the short-run output dynamics following housing and equity-market crashes in a panel of developed countries. However, my analysis importantly differs in its scope and focus. I significantly expand the number of events in the study and explore the dynamics of a broader set of macroeconomic variables. More substantively, I focus on the persistent dynamics of...
aggregate variables and identify their potential drivers. Using the harmonized cross-country data on household and corporate debt, I emphasize the role of household indebtedness.

Furthermore, my empirical analysis is also related to the existing literature that demonstrated that the deterioration of household balance sheets during the 2006-09 housing market collapse in the US played a significant role in the sharp decline in employment and consumption (Mian et al. 2013, Mian and Sufi 2014). The evidence from US metropolitan statistical areas that I present suggests that the implications of the housing market’s crash for the US economy extend far beyond the contemporaneous effect; rather, the legacy of the crash lingers to this day.

Finally, this paper also relates to the broader literature that explores the nexus between the housing market dynamics and business dynamism. Recent contributions provide evidence of a causal relationship between homeowner housing wealth and young-firm activity (Davis and Haltiwanger 2019), as well as on the probability of homeowners becoming entrepreneurs (Corradin and Popov 2015, Schmalz et al. 2017). According to such work, a significant fall in housing prices causes a slowdown in startup activity. Periods when entry is especially weak consequently result in a missing generation of firms, which may have a very persistent effect on output and measured productivity (Gourio et al. 2016).

This paper theoretically relates to several bodies of existing literature. Similar to the literature on deleveraging crises pioneered by Eggertsson and Krugman (2012), recession in my model is a result of a reduction in the borrowing capacity of debtor households. Differently from this literature, however, the borrowing limit is not exogenous, but it is tied to the borrowers’ housing wealth determined in general equilibrium. This feature connect the paper to the body of literature on the macroeconomic effects of home equity-based borrowing started by Iacoviello (2005). The implications of treating the borrowing limit as endogenously determined are far from trivial. This approach allows the model to account for the amplification effect through the two-way interactions between deleveraging, borrower housing wealth, and economic activity. This effect is state-dependent, shaped by the policy response, and very important quantitatively.

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4 Andersen et al. (2014) and Bunn et al. (2015) identify similar patterns in the data from Denmark and U.K. respectively. Overall, changes in household debt correlate stronger with growth than changes in corporate debt, both in developed and developing countries, as documented by Bahadir and Gumus 2016 and Mian et al. 2017.

5 To put this statement in context, the U.S. establishment entry rate plummeted by 26% between 2006 and 2009, according to the CENSUS Business Dynamics Statistics.

6 See also Benigno and Romei (2014), Benigno et al. (2016b), Guerrieri and Lorenzoni (2017), and Korinek and Simsek (2016).

7 Other important contributions include Ferrero (2015), Jensen et al. (2019), Justiniano et al. (2015), Liu et al. (2013), Liu et al. (2016), Midrigan and Philippon (2016), Iacoviello and Minetti (2006), and Iacoviello and Neri (2010).
Empirically, episodes of household deleveraging during housing market crashes are associated with very persistent declines in economic activity. I interpret this observation building on insights from the literature on the interconnectedness between business cycles growth. Much of the recent theoretical literature on this topic builds on the seminal contribution of Comin and Gertler (2006). In particular, particularly large contractions and slow recoveries in many countries after the Global Financial Crisis have motivated research on the role endogenous growth and financial shocks in generating such persistence. So far, the existing literature to date has either focused on financial frictions that directly affect financing of innovations (Queralto 2019, Guerron-Quintana and Jinnai 2018, Ikeda and Kurozumi 2018) or remained agnostic about the source of the financial shock all together, treating it as exogenous (Anzoategui et al. 2019). I contribute to this debate by investigating the persistent effects of household deleveraging generated by negative housing demand shocks.

I approach the issue of persistent effects of business-cycle fluctuations with a particular focus on the house equity-based borrowing. This is complementary to the alternative mechanisms proposed in the literature. Persistent effects of temporary shocks may also stem from the labor market dynamics (Blanchard and Summers 1987, Acharya et al. 2018); purely from self-fulfilling expectations of low growth (Benigno and Fornaro 2018). A number of papers emphasize the role of firm dynamics. Ates and Saffie (2016) document that firms born during the credit shortage are fewer, but more productive. Schmitz (2017) focuses on how tight financial conditions cause small and young innovating firms reduce their R&D resulting in R&D misallocation.

Finally, this work also contributes to the literature on the non-linear effects of occasionally binding constraints (OBCs). The closest reference is Guerrieri and Iacoviello (2017), who show that a model with an OBCs tied to housing wealth makes it possible to account for the asymmetry in the link between housing prices and consumption growth during the latest

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9 See later contributions by Bianchi et al. (2019), Comin et al. (2014), Correa-López and de Blas (2018), Croce et al. (2012), Garga and Singh (2018), Gornemann (2015), Gornemann et al. (2018), Holden (2016), Moran and Queralto (2018), Queralto (2019). Recently, endogenous growth mechanisms have also been used in the finance literature. By generating a small but persistent endogenous productivity component, this feature has been shown to improve the asset-pricing implications of dynamic general equilibrium models, see Bocola and Gornemann 2013, Gavazzoni and Santacreu 2015, Guerrón-Quintana et al. 2019, and Kung and Schmid 2015 among others.

10 Fernald et al. (2017) and Gordon (2015) point out that that the productivity growth slowdown in the U.S. economy that followed the crisis has started before the Great Recession and argue that the dynamics in the recent years is the continuation of a secular trend. Similar observations have been made about European countries. In this context, the question is not why there seems to be a secular decline in output growth, but whether the global financial crisis has accelerated an existing trend.

11 See also Garcia-Macia (2015), Knowles (2018), and Kozlowski et al. (2019).
housing market cycle in the U.S.\footnote{OBCs are also a central ingredient of small open economy models designed to study sudden stops and macroprudential policy, see Akinci and Chahrour (2018), Benigno et al. (2016a), Bianchi and Mendoza (2018), Korinek (2018), and Mendoza (2010). The models in these papers account for financial crises — periods when credit constraints of borrowers bind — as rate events nested within the regular business cycle.}

Outline. The rest of the paper is organized as follows. Section 2 discusses the empirical evidence on short-run and persistent effects of housing crashes. Section 3 presents the model. Section 4 discusses calibration and the ability of the model to account for the empirical evidence. Section 5 explores the key mechanisms driving the dynamics. Section 6 draws implications for monetary and fiscal policy. Section 7 concludes.

2 Evidence on the effects of housing market crashes

In this section, I first provide a comprehensive cross-country account of dynamics of recessions and subsequent associated with housing market crashes. I then show that the conclusions of the cross-country analysis are consistent with the evidence from US Metropolitan Statistical Areas in the aftermath of the Great Recession.

2.1 Housing market cycles worldwide

On average, what is the dynamics of recessions and subsequent recoveries associated with a housing market crash? In this section, I explore a cross-country panel of real house price indexes, identify turning points of housing market cycles and estimate the associated dynamics of a number of macroeconomic indicators, controlling for the magnitude of the price decline and a rich set of macroeconomic indicators.

My empirical approach is similar to the event analysis of Jordà et al. (2015).\footnote{See also Cerra and Saxena (2008), Leigh et al. (2017), Jordà et al. (2013).} I find studying large and sudden house price declines that were preceded by explosive growth appealing for several reasons. First, as emphasized by Guerrieri and Iacoviello (2017), the relation between house prices and economic activity can be highly asymmetrical and state-dependent due to the presence of occasionally binding collateral constraints tied to housing wealth. This evidence motivates my focus on periods when household financial frictions, which are at the center of my interest, matter.\footnote{On average, the studied events are preceded by accelerated credit growth, both household and firm. See summary statistics in table 7 for details.} In addition, during the studied periods the house price dynamics is more
likely to be an important independent factor affecting the macroeconomy and is less likely to be subject to reverse causality from macroeconomic fundamentals to housing prices. Nevertheless, one should be cautious to place a strong causal interpretation on the results of this section. Rather, they illustrate the equilibrium comovement between variables of interest during periods of house price decline. In the second part of the paper, I develop a dynamics general equilibrium model that allows to sheds some light on the mechanisms and gauge how much of the empirical comovements between variables can be accounted for by the house price decline alone.

2.1.1 Defining housing market boom-and-bust cycles

I start with constructing a database of turning points of housing market cycles, which I then use to study the associated dynamics of macroeconomic variables. For that purpose, I collect a comprehensive cross-country data set of aggregate real housing price indexes. It combines series from BIS, the Dallas FED, OECD, and Jorda et al. (2016) Macrohistory databases. The resulting unbalanced panel consists of annual observations covering 50 countries from 1950 to 2017.

For each country, I determine periods of sudden and large declines in house prices. My strategy consists of two steps. First, the aggregate housing price index needs to be sufficiently elevated relative to the long-run trend defined using a one-sided HP filter. Second, the price index needs to sharply fall from the peak. As a rule of thumb, I use the threshold of at least 10% decline in the first three years after the peak, although the results are robust to choosing a different threshold. To put this threshold in perspective, the US aggregate real housing price index fell by around 14% during the first three years from the peak in 2006. This second step intends to filter out “soft landing” situations when the rapid house price growth was not followed by an equally rapid correction. Figure 17 in appendix A presents an example of this definition applied to the UK, the US, Sweden, and Denmark. The UK, for instance, has experienced 3 major housing market cycles in postwar history with turning points in 1973, 1989 and 2007.

The above procedure identifies 63 events in total, 39 of which happened before the Global Financial Crisis; 43 events have taken place in Europe, 6 in Americas, 12 in Asia and Oceania, and 2 in Africa. On average, the price decline continues for 5 years and reaches the magnitude of -31% with the two-thirds of the price decline occurring in the first three years. Table 1 lists all identified events along with some descriptive statistics. To estimate the responses of macro variables to a housing market crash, I construct a variable \( \Delta_3p_{crash}^{i,t} \) that marks the three-year

\[ \Delta_3p_{crash}^{i,t} \]

15 I use the smoothing parameter of \( 400000/4^4 \) for annual observations for this procedure to be symmetric to the definition of credit cycles by the BIS.
price decline from the peak of the identified housing market boom-and-bust cycles in country $i$. I use this variable as an exogenous perturbation for the impulse response estimations in the next section.

2.1.2 Impulse responses to a housing market crash by local projections

I ask: (1) how long-lasting is the effect of recessions associated with housing market crashes on the level of output, consumption, and investment? (2) What factors drive the observed dynamics of output? (3) Does household and/or corporate pre-existing indebtedness amplify this dynamics? To address these questions, I complement the cross-country housing price data with information on the dynamics of key macro-economic variables, such as output, consumption, investment, household/corporate debt, and measures of productivity. Table 7 in appendix A includes the full list of variables with summary statistics and data sources.

The dynamic responses of variables are estimated by local projections (Jordà 2005). This method involves running a series of regressions of a variable of interest on the lagged shock, which contrasts approaches that rely on estimating an autoregressive model. This model-free approach is attractive because of its flexibility. However, as noted by Ramey (2012), this flexibility comes at a price of efficiency. Responses at each horizon are estimated independently from the rest, which may result in wider confidence bounds and erratic oscillations of impulse responses.\textsuperscript{16} This feature should be kept in mind when interpreting the results.

The baseline specification is the following dynamic cross-country panel:

$$
\Delta_{h} y_{i,t+h} = \alpha_{i}^{h} + \alpha_{t}^{h} + \delta_{t} + \beta^{h} \Delta_{3} p_{i,t}^{\text{crash}} + X_{i,t} \Gamma^{h} + \varepsilon_{i,t}^{h}
$$

The dependent variable is the country $i$’s $h$-period log difference of the response variable: $\Delta_{h} y_{i,t+h} = \log(Y_{i,t+h}) - \log(Y_{i,t})$; and the perturbation variable is the three-year house price decline $\Delta_{3} p_{i,t}^{\text{crash}}$ during the identified events of interest. Estimating this relation at different horizons $h$ produces a set of coefficients $\{\beta^{h}\}_{h=1:H}$ that can be interpreted as an $H$-period response of the variable $Y$ to a 1\% house price decline, conditional on the rich set of controls described below.

The baseline specification includes a rich set of controls and fixed effects. In particular, I include country fixed effects ($\alpha_{i}^{h}$) to control for time-invariant country-level characteristics; year fixed effects ($\alpha_{t}^{h}$) to control for common shocks such as the Global Financial Crisis; and

\textsuperscript{16} Barnichon and Brownlees (2019) suggest improving the precision of local-projection impulse responses using a method based on penalized B-splines. The use of this method would not change the main conclusions of the analysis and is beyond the scope of the paper.
country time trends ($\delta t$). A variety of other country-specific factors is accounted for with a set of macroeconomic controls $X_{i,t}$. It includes values at the peak of a housing market cycle and one lag of (1) the response-variable growth rate; (2) real per-capita investment growth, (3) GDP-deflator inflation rate, (4) net exports to GDP ratio, (5) real house price growth rate. In addition, to control for the possible effect of an exchange-rate regime, I include fixed exchange rate indicators from Ilzetzki et al. (2019). Many but not all housing market crashes coincide with broad financial crises. I include Laeven and Valencia (2013) indicators of systemic banking and currency crisis to account for that. Finally, I use investment to GDP ratio to control for the overall investment intensity of an economy.

On average, how different the path of the economy is, conditional on the set of controls described above, if there is a housing marker crash? Figure 3 presents the resulting impulse responses. Several key observations emerge. First, the studied events are associated with a rapid house price decline lasts for the first 3 year and is associated with a broad decline in the economic activity. At the though in year 3 the 1.3% decline in house price is associated with a decline in consumption larger than output (-0.2% and -0.28% respectively) and a -0.6% decline in investment. Second, the decline in the economic activity is associated with households deleveraging: the household debt-to-GDP gap falls by about 0.2 per 1.3% decline in house prices. The non-financial corporate debt burden falls as well but the change is less significant and the magnitude is twice as low.

The negative effect on the levels of output and consumption is very persistent and lasts longer than conventional business cycles. My results suggest that the sluggish dynamics of capital accumulation and productivity growth is likely to be responsible. Figure 3 also shows impulse responses of capital, labor, and measures of productivity. Responses of the capital stock, total factor productivity, and labor productivity are very persistent. Employment-to-population ratio, on the other hand, exhibits a stronger mean reversion dynamics.

Note that I use utilization-adjusted total factor productivity. The persistent response of productivity then is unlikely to be driven by time-varying factor utilization. Appendix D.1 provides details on the utilization adjustment procedure, which follows Imbs (1999). This method is an alternative to the approach of Basu et al. (2006) that utilizes industry-level data to also control for nonconstant returns to scale and aggregation effects. Unfortunately, cross-country data limitations do not allow to account for these factors.

Appendix B gathers some additional empirical results. Figure 20 illustrates that housing

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17 Comin and Gertler (2006) refer to the frequencies from 2 to 32 quarters as the business cycle component.
18 See other applications of this method in Taylor et al. (2019) and Levchenko and Pandalai-Nayar (2018).
market crashes are associated with a short-run improvement in the trade balance, driven primarily by a decline in imports; and a mild inflationary effect. The effect on the real exchange rate and the GDP-deflator inflation rate is ambiguous.

2.1.3 The role of credit imbalances

Rapid growth of household – but not corporate – debt during the boom is associated with a deeper recession during the crash. This result is complimentary to the one of Jordà et al. (2015) who show that recession associated with housing market crashes tend to be deeper when preceded by below-average bank lending growth. I revisit this question using the novel IMF Global Debt database that contains separate data on household and non-financial corporate debt.\(^{19}\) I use this data to calculate household and non-financial corporate debt-to-GDP gaps, denoted $\hat{B}_{HH}$ and $\hat{B}_F$ respectively. Similarly to output gaps, this variable is defined as a deviation of the debt-to-GDP ratio from its long-run trend, defined using the one-sided HP filter.\(^ {20}\) In my analysis, I use debt-to-GDP gaps as a measure credit intensity of expansions preceding housing market crashes relative to the secular trend of financialization. Figure 19 shows an example of U.S. household and non-financial corporate debt-to-GDP gaps along with the cyclical dynamics of the aggregate house price. As this visual suggests, the cyclical dynamics of the US housing market correlates more closely with the dynamics of household, rather than corporate, indebtedness.

I extend the baseline specification by adding interactions between the price decline and the debt-to-GDP gaps to formally test whether preexisting credit imbalances affect the transmission of house price declines:\(^ {21}\)

$$\Delta y_{h,t+h} = \alpha_{h}^{i} + \alpha_{t}^{i} + \delta_{t}^{i} + (\beta^{h} + \beta_{HH}^{h} \hat{B}_{i,t}^{HH} + \beta_{F}^{h} \hat{B}_{i,t}^{F}) \Delta p_{i,t}^{crash} + \hat{B}_{i,t}^{HH} + \hat{B}_{i,t}^{F} + X_{i,t} \Gamma^{h} + \varepsilon_{i,t}^{h} \quad (2)$$

To assess the relative role of household and corporate debt, I then calculate marginal effects of the house price decline at different values of household and corporate debt-to-GDP gaps. For illustrative purposes I consider the following three scenarios when credit gaps take values of 0% or 10%: $(\hat{B}_{HH}, \hat{B}_F) = \{(0, 0); (0.1, 0); (0.0, 1)\}$. Figure 4 shows the corresponding conditional impulse responses for output, consumption, and employment. The main message from this

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\(^ {19}\) See Mbaye et al. 2018 for the database description. Compared to the existing counterparts, the database is more than double in the cross-sectional dimension and employs a broader and more consistent definition of debt across countries and time.

\(^ {20}\) Following the BIS definition the smoothing parameter is set to $4 \cdot 10^5 / 4^4$. This variable has recently become popular among policymakers as a reduced-form way to capture credit imbalances. For instance, it is used in the Basel III regulatory framework as a guide for setting countercyclical capital buffers.

\(^ {21}\) Debt data is available for only 50 events in the sample.
result is the following. The same house price decline is associated with a deeper recession and a slower recovery when household — but not corporate — debt-to-GDP gap is high. In other words, a rapid preceding expansion of household debt tends to exacerbate the effect of housing market crashes.\textsuperscript{22}

\subsection*{2.1.4 Robustness}

The results of the cross-country analysis are robust to alternative empirical specifications. Figure 23 in appendix B presents results of several checks. I estimate the following alternatives in addition to the baseline specification: (1) excluding post-2007 observations; (2) including 4 lags of controls instead of 2; (3) excluding macro controls; (4) excluding macro controls and year fixed effects. I specifically want to point out that the baseline results are not simply driven by the events associated with the Global Financial Crisis: the estimated dynamics of variables is very similar for the sample that excludes years from 2007 onward. These checks also make clear that the large set of macroeconomic controls and year fixed effects mop up a sizable amount of variables response. The indicator of systemic banking and currency crises plays a particularly prominent role in this respect. The results are then not likely to be driven by financial crises that sometimes coincide with housing market crashes.

Two additional checks included in appendix B relate to the evidence on the interaction between the house price decline and the private indebtedness. Figure 21 shows that the conclusions regarding the role of households debt relative to corporate debt are robust to excluding years from 2007 onward. Figure 22 also shows conditional impulse responses from the specification that excludes country-specific trends.

\subsection*{2.2 Evidence from U.S. states and Metropolitan Statistical Areas}

The cross-country evidence of the decline in productivity growth associated with housing market crashes is consistent with the labor productivity dynamics across U.S. states and Metropolitan Statistical Areas (MSAs). Figure 5 shows a negative correlation between the magnitude of the housing market crash (peak to trough) and the cumulative growth of real GDP per worker from 2007 to 2017 across states and MSAs. Of course, one should be cautious to interpret this result causally. Unobserved factors may have been driving both housing prices and productivity growth.

\textsuperscript{22} This evidence is consistent with the results of Mian et al. (2017) who uncover the (unconditional) detrimental effect of increases in the household debt on growth and employment in the medium run.
I adopt two instrumental variables proposed in the literature. The first is the Saiz (2010) housing supply elasticity index, which captures geographical and regulatory constraints to construction. This variable has been argued to generate variation in house price growth uncorrelated with factors that might otherwise be driving the housing market dynamics. The use of housing supply elasticity instrument, although common in the literature,\textsuperscript{23} has been a subject of controversy lately, see, for instance, a discussion in Davidoff (2013). As an additional check I use the regional sensitivity instrument developed by Guren et al. (2018). This instrument exploits the fact that house prices in some cities are systematically more sensitive to regional housing cycles than in others. Figure 24 in appendix B plots first-stage relationships. In addition to the house price decline I also employ the housing net worth shock constructed by Mian and Sufi (2011). This variable measures the percentage decline in household net worth due to the house price decline from 2006 to 2009. Note the non-linear relationship between the variables of interest and the Saiz (2010) housing supply elasticity: the relationship is stronger for low-elasticity areas. Following Kaplan et al. (2016), in addition to the linear first-stage specification, given its poor fit, I consider a quadratic polynomial in the housing supply elasticity index as an instrument.

Table 2 reports the results. I obtain elasticities of the cumulative labor productivity growth (2007-17) to the decline in house prices (2007-12) of 0.16 using OLS; 0.11/0.13 using the housing supply elasticity index as an instrument with a linear/quadratic first stage; and 0.25 using the regional sensitivity instrument. To put it differently, these estimates suggest that a 10% house price shock during the latest crash slowed down labor productivity growth by 1.1-2.5% in the decade from 2007, depending on the preferred specification. For comparison, the ten-year elasticity of the labor productivity to the house price decline in the cross country study of the previous section is 0.06.

3 Model

In this section, I develop the dynamic general equilibrium model that I will use for the exercises in the remainder of the paper. The core of the model is a Woodford (2003) cashless economy with capital accumulation and monetary policy conducted through interest rate setting. This framework is extended along two dimensions. First, instead of a representative household the model features two agents: borrowers and savers (Eggertsson and Krugman\textsuperscript{23})

\textsuperscript{23} See Mian et al. 2013, Mian and Sufi 2014, Giroud and Mueller 2017, Davis and Haltiwanger 2019, among others
Both saver and borrower households supply labor, trade risk-free bonds, and hold housing, which generates a utility flow. Borrowers are subject to an occasionally binding collateral constraint tied to their housing wealth (Kiyotaki and Moore 1997, Iacoviello 2005). Borrowers also have an access to investment opportunities: they accumulate capital and finance firm creation. As suggested by Eggertsson and Krugman (2012), borrowers should not necessarily be interpreted as ‘liquidity-constrained poor”, as is common in the literature. Instead, they can be broadly interpreted as those who have an access to investment opportunities and are in need of external funding, which is constrained by the debt limit. Second, the model features productivity growth through expanding variety of intermediate products, broadly interpretable as horizontally differentiated innovations (Romer 1990, Comin and Gertler 2006).

Figure 6 presents a flow chart of the model that summarizes its key participants and their interactions.

### 3.1 Households

There are two types of households: savers and borrowers denoted by a superscript $H \in \{S, B\}$. A common way in the literature to motivate borrowing and lending is to assume that savers are more patient than borrowers $\beta_S > \beta_B$.25

Each household gains utility from consumption and the stock of housing in its possession, and disutility from labor: $U(C_t^H, L_t^H) + G(h_t^H)$. I assume Greenwood et al. (1988) period utility function in consumption and labor: $U(C_t^H, L_t^H) = \left( \frac{C_t^H - \Upsilon_t \left( \frac{L_t^H}{1+\epsilon_L} \right)^{1+\epsilon_L}}{1+\epsilon_L} \right)^{(1-\sigma)}$, where the aggregate consumption is a CES basket of differentiate retail goods $C_t^H = \left( \int_0^1 C_t^H(j) \frac{n-1}{\pi} dj \right)$, and the two parameters, $\sigma$ and $\epsilon_L$, are the inverse elasticity of intertemporal substitution and the inverse Frisch elasticity of labor supply respectively. The GHH preference abstracts from the wealth effect in labor supply, this assumption allows to avoid a counterfactual dynamics of borrowers labor supply during periods when they are credit-constrained. As in Queralto (2019), the disutility of labor is governed by the following process:26

$$\Upsilon_t = \Upsilon_{t-1}^{\rho_Y} N_t^{1-\rho_Y}$$

---

24 This variety-based approach differs from the quality ladder growth models of Grossman and Helpman (1991) and Aghion and Howitt (1992), where endogenous growth takes a form of repeated quality improvements over the pre-fixed number of varieties. However, as Grossman and Helpman (1991) note, these two frameworks result in very similar reduced forms.

25 An alternative but conceptually similar way to ensure that borrowers do not self-finance in the long run is to assume that they are finitely-lived. See, for example, the formulation of bankers problem in Gertler and Karadi (2011).

26 Jaimovich and Rebelo (2009) suggest a similar preference that allows to parameterize the short-run wealth effect on the labor supply.
The parameter $\rho_T$ determines the responsiveness of disutility of labor to changes in productivity growth. This formulation insures that the BGP with constant hours exists, but the medium-run swings in growth do not excessively affect labor supply.\footnote{One way to interpret this feature of the preference is as follows. As noted by Benhabib et al. (1991), the GHH preference can be interpreted as a reduced form of an economy with home production. The disutility of work then consists of the forgone output in home production. This disutility increases as productivity improvements in the formal sector spill over to the home production. However, to the extent this process takes time, disutility of labor exhibits inertia.}

As common in the literature, utility from housing is assumed to be separable from consumption and labor. I assume that it takes the standard CRRA form

$$G(h^H_t) = \kappa_t \vartheta_t (h^H_t)^{1-\epsilon_h} - 1,$$

where $\epsilon_h$ is the inverse elasticity of housing demand; $\kappa_t$ is the weight of housing in the total period utility, which is allowed to exhibit trend to ensure that the marginal rate of substitution between consumption and housing is constant on the balanced growth path; and $\vartheta_t$ is a housing preference shock.

### 3.1.1 Saver households

Saver households supply labor $L^S_t$; earn wage $W_t$; consume the aggregate basket of goods $C^S_t$; trade nominal risk-free bonds $B^S_{t+1}$; and adjust their housing stock $h^S_t$ at a price $P^h_t$ per unit. The representative household maximizes its expected discounted lifetime utility subject to the budget constraint:

$$\max \left\{ C^S_t + \frac{P^h_t (h^S_t - h^S_{t-1}) + (1 + r_{t-1}) B^S_{t+1}}{P_t} \right\}$$

subject to

$$C^S_t + \frac{P^h_t (h^S_t - h^S_{t-1}) + (1 + r_{t-1}) B^S_{t+1}}{P_t} = W_t L^S_t + \frac{B^S_{t+1}}{P_t}$$

From now on, let $\lambda^H_t$ denote the household $H$’s Lagrange multiplier with respect to the budget constraint, and $\Lambda^H_{t+1} = \frac{\lambda^H_{t+1}}{\lambda^H_t}$ denote the households stochastic discount factor. The intertemporal optimality condition, labor supply, and housing demand implied by the saver household’s optimization problem are as follows:

$$\mathbb{E}_t \left( \Lambda^S_{t,t+1} \frac{1 + r_t}{P_{t+1}} \right) = 1$$

$$W_t = \Upsilon_t (L^S_t)^{\epsilon_L}$$

$$P^h_t = \mathbb{E}_t \left( \Lambda^S_{t,t+1} P^h_{t+1} \right) + \frac{\kappa_t \vartheta_t (h^S_t)^{-\epsilon_h}}{\lambda^S_t}$$
of utility from housing expressed in the units of the consumption good.

3.1.2 Borrower households

Similarly to savers, borrower households supply labor, consume, trade nominal risk-free bonds, and demand housing. Moreover, they have an access to investment opportunities. They accumulate capital and, as in Bilbiie et al. (2012), hold shares in a mutual fund of intermediate firms. In each period $t$ the household buys $\iota_{t+1}$ shares in a mutual fund of $N_t + N_{e,t}$ firms (already operating and new entrants) at the price $v_t$ per share; receive the dividend income from currently owned firms $d_t$, as well as the return on the shares purchased in the previous period. Finally, as Greenwood et al. (1988) I endogenize capital utilization by assuming that the depreciation rate of capital is an increasing function of utilization. This quantitative feature is important for capturing the short-run dynamics of measured TFP.

The amount these households can borrow from savers ($B_t^B$) is subject to a Kiyotaki and Moore (1997) type occasionally binding collateral constraint: the household cannot borrow more than a fraction $m$ of the current value of its housing stock $P^h_t h_t^B$. Note that I do not impose that the household is constrained per se. Whether the collateral constraint binds or not would be determined in equilibrium depending on the state of the economy. As in Guerrieri and Iacoviello (2017), the borrowing limit does not reset within one period, parameter $\rho_B$ governs the degree of its persistence. This quantitative feature allows to capture the empirically-relevant gradual adjustment of the borrowing capacity in response to changes in borrowers housing wealth.

The full program of borrowers takes the following form:

$$\max_{\{C_t^B, L_t^B, h_t^B, B_t^B\}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j B \left( U(C_t^B, L_t^B) + \kappa_t \delta G(h_t^B) \right) \text{ s.t.}$$

28 Following the existing literature, I do no explicitly model the origins of this constraint. A natural interpretation, however, should be that due to the imperfect enforceability of contracts, the ability of households to borrow is bounded by a fraction of the value of their collateral assets that can be seized by creditors in a case of default. The parameter $m$ can be narrowly interpreted as the maximum loan-to-value (LTV) ratio. It can also be linked to the degree of country’s financial markets development, as suggested by some of the existing literature.

29 A natural interpretation of this feature is the implicit existence of multi-period credit contracts, as in Kydland et al. (2016).
\begin{align*}
C_t^B + I_t + P_t^h(h_t^B - h_{t-1}^B) + (1 + r_{t-1}) \frac{B_t^B}{P_t} + \iota_{t+1} v_t(N_t + N_{e,t}) = \\
(1 + r_t - \rho_B) m P_t^h h_t^B + \delta_t(v_t + \delta_t) N_t + W_t L_t^B + R_t^K u_t K_t + \frac{B_{t+1}^B}{P_t} + \text{div}_t \\
B_{t+1}^B \leq \rho_B \frac{B_t^B}{P_{t-1}} + (1 - \rho_B) m P_t^h h_t^B \\
K_{t+1} = (1 - \delta_K(u_t)) K_t + (1 - AC_{t,t}) I_t \\
\delta_K(u_t) = \delta_K + c_1(u_t - 1) + \frac{c_2}{2} (u_t - 1)^2
\end{align*}

Let $\chi_t$ be the Lagrange multiplier with respect to the borrowing constraint (9). The intertemporal optimality condition and the complimentary slackness condition are then:

\begin{align*}
E_t \left( \Lambda_{B_{t+1}}^B \frac{1 + r_t - \rho_B \chi_{t+1}}{\Pi_{t+1}} \right) = 1 - \chi_t \\
\left( \frac{B_{t+1}^B}{P_t} - \rho_B \frac{B_t^B}{P_{t-1}} - (1 - \rho_B) m P_t^h h_t^B \right) \chi_t = 0, \quad \chi_t \geq 0
\end{align*}

When the collateral constraint binds ($\chi_t > 0$) it creates an endogenous wedge between the real interest rate and the borrowers intertemporal marginal rate of substitution in consumption. In other words, the consumption path of the credit-constrained borrowers deviates from the one predicted by the real interest rate dynamics.

Next, borrowers housing demand is:

\begin{align*}
P_t^h = E_t \left( \Lambda_{t+1}^B P_t^h \right) + \kappa_t \theta_t \frac{(h_t^B)^{-\epsilon_h}}{\lambda_t^B} + \chi_t m P_t^h
\end{align*}

The expression is symmetric to savers housing demand, equation (7), except for the last term. Its interpretation is the following: the direct effect of borrowers being credit-constrained ($\chi_t > 0$) is that they value a marginal unit of housing more since it has an additional benefit of relaxing their borrowing limit. However, the general equilibrium effect of the binding borrowing constraint on their housing demand is the opposite. I postpone the detailed discussion of this effect until section (5.3).

The first order condition with respect to share holdings $\iota_{t+1}$ implies the following expression:

\begin{align*}
v_t = (1 - \delta_N) E_t \left( \Lambda_{t+1}^B (d_{t+1} + v_{t+1}) \right),
\end{align*}

When iterated forward, it produces the standard firm value equation: $v_t = E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}^B (1 - \delta_N)^j d_{t+j}$. The present firm value equals to its expected discounted profit stream, accounting for the fact that each period a firm faces an exogenous probability of exiting the market $\delta_N$, as
discussed further.

The optimality conditions for accumulation and utilization of capital are standard:

\[
q_t = \mathbb{E}_t \left( A_{t,t+1}^B \left( (1 - \delta_{K,t}) q_{t+1} + R^K_t \right) \right) \tag{15}
\]

\[
q_t = 1 + q_t(AC_{I,t} + AC'_{I,t} I_t) - \mathbb{E}_t \left( A_{t,t+1}^B q_{t+1} AC'_{I,t+1} I_{t+1} \right) \tag{16}
\]

\[
R^K_t = c_1 + c_2 (u_t - 1) \tag{17}
\]

where \( q_t \) is Tobin’s q: the Lagrange multiplier with respect to the capital law of motion (10). Capital investment is assumed to be subject to a standard quadratic adjustment cost \( AC_{I,t} = \frac{\psi_K}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \), where \( \psi_K \) governs the size of the adjustment cost and \( g \) in the BGP growth rate.

Finally, the labor supply condition is symmetric to the one of savers:

\[
W_t = \Upsilon_t (L^B_t)^{\epsilon_L} \tag{18}
\]

### 3.2 Production

The production structure of the economy consists of two upstream and two downstream sectors. The upstream sectors are the production sector that employs labor, capital and a basket of intermediate good, and a sector of intermediate-good suppliers. The downstream sectors are the wholesale and retail sectors that differentiate the production-sector good and distribute it to final consumers. Moreover, a sector of innovators invents blueprints of intermediate goods. Nonrivalry of ideas generated by innovators is the source of endogenous growth in the economy. Given the focus of the paper, I abstract from the role of housing in production.

#### 3.2.1 Production sector

The production sector is populated by perfectly competitive firms that employ homogeneous labor supplied by both households, \( L_t = L^S_t + L^B_t \), effective capital, \( \bar{K}_t = u_t K_t \), and a CES basket of intermediate products with elasticity of substitution \( \frac{\nu}{\nu-1} \), \( X_t = \left[ \int N_t x_t(\omega)^{\frac{1}{\nu}} d\omega \right]^\nu \). Positive externalities in the innovation sector, as discussed further, cause the mass of intermediate products, \( N_t \), to expand over time. This brings about efficiency gains to diversity implied by the CES aggregator and increases the measured TFP. As in Comin and Gertler (2006), the aggregate production function takes the following form:

\[
F_t = Z_t \left( \bar{K}_t^\alpha L_t^{1-\alpha} \right)^{1-\xi} X_t^\xi = Z_t \left( \bar{K}_t^\alpha L_t^{1-\alpha} \right)^{1-\xi} \left( \int N_t x_t(\omega)^{\frac{1}{\nu}} d\omega \right)^\nu \xi
\]

19
Given input prices, the representative firm maximizes its extended profit stream (expressed in units of the consumption good):

$$\max_{\{x_{t+j},L_{t+j},K_{t+j}\}_{j=0}^\infty} \mathbb{E}_t \sum_{j=0}^\infty \Lambda^B_{t,j} \left[ p^F_{t+j} F_{t+j} - R^K_{t+j} \tilde{K}_{t+j} - W_{t+j} L_{t+j} - \int_0^{N_t} p^x_{t+j} (\omega) x_{t+j}(\omega) d\omega \right]$$

The problem implies the following input demands:

$$W_t = p^F_t (1 - \alpha) (1 - \xi) \frac{F_t}{L_t}$$  \hspace{1cm} (19)  

$$R^K_t = p^F_t \alpha (1 - \xi) \frac{F_t}{K_t}$$  \hspace{1cm} (20)  

$$p^x_t (\omega) = p^F_t \xi \frac{F_t}{X_t} x_t(\omega)^{1-\nu}$$

where $p^F_t = \frac{P^F_t}{P_t}$ and $p^x_t (\omega) = \frac{P^x_t (\omega)}{P_t}$ are relative prices of the production-sector and the intermediate-sector goods respectively.

### 3.2.2 Intermediate-good sector

The intermediate sector is populated by a mass $[0, N_t]$ of monopolistically competitive firms, each operating a roundabout technology that requires $A^{-1}$ units of the domestic good to produce a unit of the intermediate good. One should not take this setup literally. The correct interpretation of this formal description is that the forgone final good is never manufactured. The resources that would have been used to produce the forgone output are used instead to manufacture intermediate goods.

Each intermediate-sector firm maximizes its real profit subject to the production sector demand:

$$\max_{p^x_t (\omega)} \left[ (p^x_t (\omega) - A^{-1}) x_t(\omega) \right] \text{ s.t. } p^x_t (\omega) = p^F_t \xi \frac{F_t}{X_t} x_t(\omega)^{1-\nu}$$

In a symmetric equilibrium the optimal quantity of the intermediate good ($x_t$), the firm’s profit ($d_t$), and its relative price ($p^x_t$) are the following:

$$x_t = \left( \frac{A^\xi}{\nu} \right)^{\nu-1} \left( p^F_t Z_t \right)^{\nu-1} N_t^{\frac{\nu-1}{\nu}} \tilde{K}_t^\alpha L_t^{1-\alpha}$$  \hspace{1cm} (21)  

$$d_t = \frac{\nu - 1}{\nu} p^x_t x_t = \frac{\nu - 1}{A} x_t$$  \hspace{1cm} (22)  

$$p^x_t = \nu A^{-1}$$

Positive profit in this sector motivates entry. To open a firm, an entrepreneur needs to pay an
sunk entry cost that consists of the cost of buying a blueprint of a new product from innovators at a price $p^b$. New firms finance entry by selling shares of their equity to entrepreneurs. Free entry pins down the equilibrium value of an intermediate firm, which should be equal to the entry cost: $v_t = p^b$.

3.2.3 Wholesalers

Each monopolistically competitive wholesale firm $j \in (0, 1)$ purchases homogeneous production-sector good $F_t$ at the price $P^F_t$ and produces a differentiated variety sold to retailers. Following Rotemberg (1982), wholesale-good prices are sticky due to the presence of a quadratic price adjustment cost $AC_{p,t} = \psi_p \left( \frac{P_t(j)}{P_{t-1}(j) \Pi} - 1 \right)^2 Y_t$, where parameter $\psi_p \geq 0$ governs the strength of nominal rigidity and $\Pi$ is the steady-state inflation rate.

Each wholesaler sets the price $P_t(j)$ to maximize the future expected profit stream $d^w_t(j)$, subject to the production function, the retailers demand, and the price adjustment cost:

$$
\max_{\{P_t(j)_{t+k}\}_{k=0}^{\infty}} \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_t B_{t,t+k} d^w_t(j) = \mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_t B_{t,t+k} \left[ \frac{P_{t+k}(j)}{P_t} Y_{t+k}(j) - \frac{P_{t+k}^F}{P_t} F_{t+k}(j) - AC_{p,k}(j) - \Gamma \right], \quad \text{s.t}
$$

$$
Y_t(j) = F_t(j)
$$

$$
Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t
$$

$$
AC_{p,t}(j) = \psi_p \left( \frac{P_t(j)}{P_{t-1}(j) \Pi} - 1 \right)^2 Y_t
$$

The problem implies that the optimal price is a time-varying markup $\mu_t$ over the price of the production-sector good $P^F_t$:

$$
P_t(j) = \mu_t P^F_t
$$

$$
\mu_t = \frac{\eta}{(\eta - 1) + \psi_p \frac{\Pi}{\Pi} \left( \frac{\Pi}{\Pi} - 1 \right) - \psi_p \mathbb{E}_t \Lambda_t B_{t,t+1} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \frac{Y_{t+1}}{Y_t}}
$$

When prices are flexible, $\psi_p = 0$, the markup is constant over the business cycle $\mu = \frac{\eta}{\eta - 1}$. Given the optimal price choice, the real period profit of a wholesaler $j$ is $d^w_t(j) = \left( 1 - \frac{1}{\mu_t} \right) Y_t(j) - AC_{p,t}(j) - \Gamma$. To ensure zero steady state profit in this sector and rule out entry, I assume that production involves a fixed cost $\Gamma = \frac{1}{\eta} Y$. 

21
3.2.4 Retailers

Firms in the retail sector are perfectly competitive and demand varieties of a wholesale good $Y_t(j)$ to produce the final consumption-investment good. The final good is a CES aggregate of wholesale varieties $Y_t = \left( \int_0^1 Y_t(j) \frac{1}{\eta} dj \right)^{-\eta}$. The corresponding aggregate price index and demands are standard: $P_t = \left( \int_0^1 P_t(j) \frac{1}{1-\eta} dj \right)^{1-\eta}$ and $Y_t^d(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t$.

3.2.5 Innovators

The sector of innovators involves inventing blueprints for new types of intermediate goods. The sector is populated with the unbounded mass of potential innovators. Let $S_t$ be the total innovation spending and $\phi^i_t$ be the innovators’ individual productivity parameter. The individual production function blueprints of intermediate goods is then $N_{i,t} = \phi^i_t S_t^i$. In aggregate, however, the technology coefficient $\phi_t$ depends on the existing stock of knowledge, measured by the number of existing intermediate goods, $N_t$. As in Romer (1990), this knowledge spillover externality is responsible for the existence of the balanced growth path in the model. Moreover, in line with Comin and Gertler (2006), I include congestion externally $N^\rho_t S_t^{1-\rho}$ that allows to control for the aggregate elasticity of blueprints output with respect to innovation spending. The resulting aggregate innovators productivity is:

$$\phi_t = \phi \frac{N_t}{N^\rho_t S_t^{1-\rho}},$$

(24)

where $S_t = \int_i S_t^i di$. The aggregate production function of innovators is then $N_{et} = \phi N_t \left( \frac{S_t^i}{N_t} \right)^\rho$. Investors maximize their expected profit stream subject to a quadratic adjustment costs in innovation spending:

$$\max_{\{S_{t+j}^i\}_{j=0}^\infty} \mathbb{E}_t \sum_{j=0}^\infty \Lambda_{t,t+j}^B \left( p_{t+j}^b \phi_{t+j} S_{t+j}^i - (1 + AC_{S,t+j}) S_{t+j}^i \right)$$

The first-order condition is:

$$\phi_t p_t^b = 1 + AC_{S,t} + AC'_{S,t} S_t^i - \mathbb{E}_t \left( \Lambda_{t,t+1}^E AC'_{S,t+1} S_{t+1}^i \right)$$

(25)

I assume the standard quadratic adjustment cost $AC_{S,t} = \frac{\psi_g}{2} \left( \frac{S_t^i}{S_t^{1-g}} - 1 \right)^2$, where $g$ is the growth rate of the economy on the balanced growth path.

As in Bilbiie et al. (2012), I there is a time-to-build lag: newly invented blueprints are
adopted with a one-period lag.\textsuperscript{30} At each period existing varieties of the intermediate good face a constant probability of becoming obsolete $\delta N$. The resulting law of motion for the total number of intermediate-good varieties is $N_{t+1} = (1 - \delta N)(N_t + N_{et})$, where $N_{et} = \int_i N_{et}di$.

The positive knowledge spillover externality in the innovation sector gives rise to variety-driven endogenous growth, which rate equals to:

$$g_{t+1} = \frac{N_{t+1}}{N_t} = (1 - \delta N) \left( 1 + \phi \left( \frac{S_t}{N_t} \right)^{\phi} \right)$$

(26)

The endogenous growth rate $g_t$ varies over the business cycle depending on the level of innovation spending $S_t$, which is determined in general equilibrium.

### 3.3 Monetary policy

Monetary policy is conducted through interest rate setting. The policy rate $r_t$ is governed by a Taylor-type rule that allows for an occasionally binding zero lower bound constraint:

$$1 + r_t = \max \left[ 0; (1 + r_{t-1})^{\rho_r} \left( 1 + r \right) \left( \frac{y^{GDP}_t}{y^{GDP}_t} \phi_Y \left( \frac{\Pi_t}{\Pi} \right)^{\phi_{\Pi}} \right)^{1 - \rho_r} \tilde{r}_t \right]$$

(27)

Parameters $\phi_Y \geq 0$ and $\phi_{\Pi} > 0$ govern the policy response to changes in GDP and inflation relative to their steady-state levels; $\rho_r \in [0, 1)$ determines the degree of interest rate smoothing; $\tilde{r}_t$ it an exogenous disturbance to the policy rate. Note that the above rule responds to changes in the detrended output, $y^{GDP}_t = \frac{Y^{GDP}_t}{N_t}$. In other words, the monetary authority does not respond to endogenous changes in the productivity growth, which is consistent with how central banks conduct monetary policy.

### 3.4 Symmetric equilibrium

In a symmetric equilibrium all retailers are alike, so $P_t(j) = P_t$ and $Y_t(j) = Y_t = F_t$, $\forall j$, and the relative price of the production-sector is the inverse of the wholesale markup $p^F_t = \frac{1}{\mu_t}$. Similarly, a representative borrower owns all intermediate firms $\iota_{t+1} = \iota_t = 1$, each of which is alike $x_t(\omega) = x_t$, $p^x_t(\omega) = p^x_t$, $d_t(\omega) = d_t$, $\forall \omega$.

Since some output is used by the intermediate sector, the correct measure of real GDP in

\textsuperscript{30} I abstract from endogenous adoption of technologies as in Comin and Gertler (2006), Anzoategui et al. (2019), and Correa-López and de Blas (2018), among others. The inclusion of this feature would allow to capture the cyclicity of adoption, but would not alter the main conclusions.
the model economy is the retail-sector output net intermediate consumption. Real GDP then equals to the sum of the total consumption, capital investment, firm creation spending (sunk entry cost), and the total adjustment cost spending:

$$Y_t^{GDP} = Y_t - N_t \frac{x_t}{A} = C_t^S + C_t^B + I_t + e_t N_{e,t} + AC_{p,t} + AC_{S,t} + AC_{I,t}$$

Equilibrium choices of intermediate producers along with other optimality conditions allow to express the production-sector output as follows:

$$Y_t = N_t^{\xi(\nu-1)} t^{\xi(1-\nu)} Z_t^{\nu} \mu_t^{\xi} \bar{K}_t^\alpha \bar{L}_t^{1-\alpha}$$

This expression makes it clear that the following condition on structural parameters needs to be satisfied for growth to take a labor-augmenting form: $\frac{\xi(\nu-1)}{1-\xi} = 1 - \alpha$. In this case, real GDP in the model economy simplifies to $Y_t^{GDP} = \Omega_t Z_t^{\nu} \bar{K}_t^\alpha (N_t L_t)^{1-\alpha}$, where $\Omega_t = \left( \frac{A \epsilon}{\nu \mu} \right)^{\xi(1-\nu)} - \left( \frac{A \epsilon}{\nu \mu} \right)^{\nu(1-\xi)}$. Now, define the Solow residual as $TPF_t = Y_t^{GDP} / (K_t^\alpha L_t^{1-\alpha})$. The following model-consistent output decomposition then holds true (expressed in log differences):

$$\Delta Y_t^{GDP} = \Delta TPF_t + \alpha \Delta K_t + (1-\alpha) \Delta L_t$$

$$\Delta TPF_t = \Delta \Omega_t + \alpha \Delta u_t + (1-\xi)^{-1} \Delta Z_t + (1-\alpha) \Delta N_t$$

The measured TFP in the model is driven by four components. The first one relates to the fact that the wholesale-sector markup distorts the quantity of the intermediate good produced; the second component is the short-run variations in the measured TFP due to time-varying capital utilization; the third is a stationary TFP shock. These three terms drive stationary fluctuations in the measured TFP. The last term is the innovation effect stemming from accumulation of the stock of intermediate goods, which drives fluctuations in the trend growth.

Finally, the credit, housing, and labor markets clear:

$$B^B + B^S = 0$$  \hspace{1cm} (28)

$$h_t^B + h_t^S = 1$$  \hspace{1cm} (29)

$$L_t^B + L_t^S = L_t$$  \hspace{1cm} (30)

Equilibrium definition: equations (5-32) determine 28 endogenous variables ($c_t^B$, $c_t^S$, $b_t^B$, $b_t^{S+1}$, $\delta_t$, $x_t$, $L_t^S$, $L_t^B$, $L_t$, $v_t$, $w_t$, $R_t^K$, $k_{t+1}$, $i_t$, $q_t$, $s_t$, $\phi_t$, $d_t$, $g_{t+1}$, $u_t$, $\delta_{K,t}$) as a function of endogenous states ($b_t^B$, $b_t^S$, $v_{t-1}$, $k_t$, $i_{t-1}$, $r_{t-1}$, $h_t^B$, $h_t^S$, $s_t^{L-1}$, $g_t$) and exogenous states ($\tilde{o}_t$, $Z_t$, $\tilde{r}_t$). Table 3 lists all equilibrium conditions of the model expressed in terms of stationary lower-case variables that remain constant on the balanced growth path, e.g. $y_t = \frac{Y_t}{N_t}$, $i_t = \frac{I_t}{N_t}$, and $c^B = \frac{C^B}{N_t}$.  

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4 Calibration and simulation

4.1 Calibrated parameters

The model is calibrated at quarterly frequency. To the extent possible, the parameter choice is informed either directly by the data or by the existing estimates in the literature.

I first describe the structural parameters of the household sector. I set the relative risk aversion to $\sigma = 2$, as common in the literature. I calibrate innovators productivity $\phi$ to match the 0.8% annual TFP growth rate on the BGP, an average across countries in the empirical exercise, i.e. $g = 1.008^{1/4}$. The steady-state real interest rate is then pinned down by the savers discount rate as follows: $R = \frac{1+r}{\Pi} = \frac{g}{\beta}$. I chose $\beta^S$ to replicate the steady-state annual real interest rate of 4%, which implies $\beta^S = 0.9968$. The borrowers discount factor should be lower than the savers discount factor. Under this assumption, the collateral constraint binds in the deterministic steady state as impatient households choose to borrow as much as possible. The difference between the two discount factors determines the steady-state shadow value of the borrowing constraint as follows: $\chi = \frac{\beta^S - \beta^B}{\beta^S} \left( 1 - \frac{\rho_B \beta^B}{\Pi g^\sigma} \right)$; it also determines how often the collateral constraint binds over the business cycle. I choose $\beta^B = 0.9963$ so that the borrowers are only slightly more impatient than savers and hence become credit-constrained only occasionally.

I assume the Frisch elasticity of labor supply of $\frac{1}{\epsilon_L} = 4$, consistent with the King and Rebelo (1999) calibration. The price elasticity of housing demand choice is based on the results from Hanushek and Quigley (1980) who provide estimates of this elasticity in the range of -0.2 to -0.9 in the long-run and around -0.1 in the short-run (within a year). Given my focus on short-and medium-run fluctuations I assume relatively inelastic demand and set $\epsilon_h = 5$. Calibration of the following two parameters relies on the estimates from Warnock and Warnock (2008). I set the loan-to-value ratio (LTV henceforth) to $m = 0.75$, which is close to an average LTV ratio across European countries. The steady-state weight of housing in the utility function $\kappa$ is calibrated to set the steady-state mortgage debt-to-GDP ratio to 55%, an average for 23 developed countries over 2001-2005.

I now turn to the structural parameters of the production side of the economy. The capital share is set to the average value across the sample of studied events $\alpha = 0.4$ (Penn World Table v. 9.1 estimates). The share of intermediate goods is set to $\xi = 0.5$, consistent with the existing literature. I set the elasticity of substitution between intermediate inputs then is pinned down by the BGP requirement $\frac{\xi (\nu - 1)}{1 - \xi} = 1 - \alpha$, which implies $\nu = 1.6$, as in Comin.
and Gertler (2006) who motivate the low value of this parameter by the specialized nature of intermediate products. Next, I set the elasticity of substitution between varieties of the final investment-consumption good to $\eta = 11$ implying a steady-state markup of 10%, a conventional choice in the literature. The parameter of the quadratic price adjustment cost is set to $\psi_p = 120$ to replicate, in a linearized setting, the slope of the Phillips curve derived using Calvo stickiness with an average price duration of about a year, which is close to direct estimate of Galí and Gertler (1999).

To normalize the steady-state capital utilization to $1$, I set $c_1 = \frac{\beta_S}{\beta_B} R - 1 - \delta_K$. I set the value of quarterly capital depreciation standard in the literature $\delta_K = 0.025$. Following Bilbiie et al. (2012), I set the intermediate firm exit rate $\delta_N = 0.025$. This value is based on the Bernard et al. (2010) estimate of the minimum production destruction rate, measured as a market share. It is also consistent with the Caballero and Jaffe (1993) estimate of technological obsolescence rate. As in Comin and Gertler (2006), the elasticity of innovators output to expenditure is set to $\rho = 0.8$. The steady-state aggregate productivity level $Z$ is chosen to normalize the steady-state GDP to unity.

Finally, I assume 2% steady-state annual inflation, and choose the conventional parameters of the Taylor rule: $\phi_Y = 0.25$, $\phi_\pi = 1.5$, $\rho_r = 0.7$.\textsuperscript{31} Table 4 summarizes parameters of the baseline model.

### 4.2 Structural shock and solution method

I use the structural model described in the previous section to explore the mechanism and draw policy implications of a negative shock to housing prices. This scenario is simulated as a result of a negative housing preference (demand) shock $\varepsilon_t^\theta$ that affects the housing preference parameter governed by a standard AR(1) process $\ln(\vartheta_t) = (1 - \rho_\vartheta) \ln(\vartheta) + \rho_\vartheta \ln(\vartheta_{t-1}) + \varepsilon_t^\theta$. I resort to this shock as an exogenous disturbance to the house price based on the evidence from the existing literature. In particular, Liu et al. (2013) identify this shock as the one that drives most of the fluctuations in the U.S. land prices and is crucial for generating the empirical comovement between land prices and investment. Later estimates of Guerrieri and Iacoviello (2017) suggest that about 70% of the consumption decline during the Great Recession in the U.S. can be traced back to housing demand shocks.\textsuperscript{32} This shock should be treated as reflecting not only pure changes in the taste for housing but also other unmodeled factors that shift

\textsuperscript{31} See Carare and Tchaidze (2005) for a summary of Taylor rule estimates for the U.S. economy.

\textsuperscript{32} See also the study of sources and consequences of US housing market fluctuations in Iacoviello and Neri (2010).
housing demand. In the context of housing market crashes the former can be interpreted as a
sudden realization that the explosive housing market dynamics is unsustainable and the asset is
overvalued, akin to the work of Minsky (1986) that got back in vogue since the Global Financial
Crisis. Other factors central for housing demand and about which the model is silent include
mortgage credit availability.

The model features two sources of non-linearities: the zero lower bound constraint on the
policy rate and the collateral constraint. The presence of these two occasionally binding con-
straint (OBC henceforth) poses a computational difficulty since the model cannot be solved
using standard perturbation methods. One way to tackle this issue is to resort to policy/value
function iteration or other global solution methods that allow to fully account for the non-
linearities and precautionary behavior linked to the possibility that the constraint may become
binding in the future. However, these methods are computationally demanding and are not
easily scalable due to the curse of dimensionality. The simplest alternative solution was intro-
duced by Guerrieri and Iacoviello (2015) and involves using a piecewise-linear solution. This
method builds on the insight that occasionally binding constraints can be handled as different
regimes of the same model. Under one regime, the occasionally binding constraint is slack.
Under the other regime, the same constraint is binding. The piecewise linear solution method
involves linking the first-order approximation of the model around the same point under each
regime. However, just like any linear solution, this method does not allow to capture the effects
of uncertainty and so to account for precautionary behavior. I use a similar approach devel-
oped by Holden (2019) and implemented as an extension to Dynare: DynareOBC.33 Unlike the
Guerrieri and Iacoviello (2015) method, its compatible with higher-order approximations and
by integrating over future uncertainty allows to capture some of the precautionary behavior.

4.3 Accounting for the empirical evidence

4.3.1 IRF matching estimation

I first investigate the ability the model to account for the empirical evidence from section 2.
For that purpose, I conduct a “crisis” experiment by hitting the model economy with a series
of negative housing preference shocks $\varepsilon^h_t$ calibrated to match the empirical dynamics of the
aggregate housing price index. Autocorrelation of the shock process is set to 0.978, close to the
estimate of Iacoviello and Neri (2010) who find housing preference shocks to be very persistent.

Given the sequence of negative housing demand shocks, I estimate the remaining five param-

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33 Available at https://github.com/tholden/dynareOBC
eters of the model $P = \{\rho_b, \psi_N, \psi_K, \rho_\Upsilon, c_2\}$ to minimize the distance between empirical (local-projection) and theoretical (model-based) impulse responses, as in Christiano et al. (2005). These are parameters that govern the borrowing limit inertia ($\rho_b$), innovation spending adjustment cost ($\psi_N$), investment adjustment cost ($\psi_K$), the disutility of labor inertia ($\rho_\Upsilon$), and the responsiveness of capital depreciation to utilization ($c_2$). Before proceeding I want to emphasize that the overall quantitative predictions of the model are robust to the choice of the above parameters.

Formally, the problem is to minimize the weighted distance between the two IRFs:

$$\min_P \left( \Sigma^{DSGE}(P) - \Sigma^{LP} \right) \Omega^{-1} \left( \Sigma^{DSGE}(P) - \Sigma^{LP} \right)^{\prime},$$

where $\Sigma^{DSGE}(P)$ denotes the mapping between the estimated parameters of the model and the theoretical impulse responses; $\Sigma^{LP}$ is a vector of empirical impulse responses; and $\Omega$ is the weighing matrix. As common in the literature, the weighting matrix includes standard deviations of the empirical impulse responses on the main diagonal, thus putting a larger weight on matching empirical impulse responses that are estimated with more precision. For estimation, I use empirical impulse responses of output, consumption, investment, capital, labor, TFP, and labor productivity. I exclude the IRF of the household debt-to-GDP ratio because of the problematic mapping between this variable in the model and the data. Firstly, this ratio in the data includes all household debt, not only mortgage debt. Moreover, it combines debt held by all households, not only those who do not hold enough liquid assets and are likely to become credit-constrained, the group I focus on in my analysis. Since the model is calibrated at the quarterly frequency, I average theoretical impulse responses to make them comparable with empirical responses of annual frequency.

The estimation results in the following parameter values. The borrowing limit inertia $\rho_b = 0.65$, which is close to the estimate of Guerrieri and Iacoviello (2017); adjustment costs parameters for capital and innovation spending are $\psi_N = 0$ and $\psi_K = 1$ respectively. The low value of the capital utilization parameter $c_2 = 0.05$ suggests that this component of the model is important for capturing the short-run dynamics of the Solow residual. Finally, disutility of labor exhibits a significant degree of inertia, $\rho_\Upsilon = 0.974$, implying that in the short run the wealth effect in labor supply is weak.

\footnote{For the purpose of estimation I consider the sequence of shocks that hits the economy at the deterministic steady state at which the collateral constraint binds. In the later sections I explicitly explore the role of non-linearities induced by the occasionally binding collateral constant.}
4.3.2 Impulse responses

Overall, the model accounts for the empirical evidence well. Immediate results of IRF matching are presented on Figure 7. Two things are worth noting. First, the model predicts a somewhat weaker response of investment and capital than in the data. The reason is straightforward: the theoretical framework abstracts from any frictions that pertain capital accumulation directly and are likely to amplify the response. Lastly, the short-run response in labor is stronger than in the data. One possible reason being that the empirical response of labor is likely to be underestimated, since it does not account for changes in working hours.

Figure 9 shows a broader set of impulse responses associated with the IRF-matching experiment. Given the perfectly inelastic housing supply, a sequence of negative demand shocks $\vartheta_t$ leads to a sharp decrease in the equilibrium house price. As a result, borrowers housing wealth falls reducing their borrowing capacity. Note that borrowers housing wealth falls by more than the house price. Although the housing preference shock is aggregate, in general equilibrium the housing stock is reallocated towards savers. This effect additionally contributes to the worsening of borrowers balance sheets, see section 5.3 for a detailed discussion.

When borrowers housing wealth falls and the collateral constraint binds, they are forced to reduce spending to meet the lower debt limit.\textsuperscript{35} Under nominal rigidity, the reduction in spending leads to a demand-driven recession in the short run. Judging by the dynamics of debt and the borrowing constraint multiplier, the active phase of deleveraging lasts for about 20 quarters. It leaves, however, a long-lasting scarring effect on the level of consumption and output due to its detrimental effect on the pace of innovation and capital accumulation. Figure 8 presents the decomposition of output and TFP dynamics. Consistent with the empirical evidence, the decline in the economic activity at medium-run horizons is driven largely by the negative effect on the level of capital stock and TFP. The model also allows to shed some light on the relative contributions of factors driving measured TFP. A significant part of a short-run response of the measured TFP, more than a half during the first year, is driven by a decrease in capital utilization and the markup distortion.

The shock is inflationary in the medium run. Although the inflation rate falls on impact, in about 8 quarters, as the acute phase of deleveraging is over, it persistently overshoots the steady-state level. Intuitively, this medium-run inflationary effect is driven by a persistent decrease in the capital stock and TFP, both of which push marginal costs up. The effect disappears if one shuts down the endogenous response of TFP and capital by setting the respective adjustment

\textsuperscript{35} In general, borrowers also deleverage by supplying more labor, but the assumption of GHH utility eliminates this effect.
costs to arbitrary high values.

The role of household indebtedness. The model is successful in accounting for the empirical interaction between household indebtedness and house price declines outlined in section 2.1.3. Figure 10 presents impulse responses conditional of different values of household debt-to-GDP and loan-to-value (LTV) ratios. Consistent with the evidence, higher initial household debt-to-GDP ratio magnifies the response of an economy to a negative housing price shock, at both short- and medium-run horizons. Perhaps more surprisingly, increasing the LTV ratio — holding the debt-to-GDP ratio the same — mostly plays the short-run amplification role. Figure 25 in appendix C presents a detailed view of the welfare cost associated with the shock both in the baseline case and the “no growth” counterfactual.\footnote{Please refer to appendix D.5 for the discussion of welfare calculation.} By generating the persistent effect of the shock — as well as amplifying the on-impact effect as discussed further — the endogenous growth mechanism magnifies the welfare cost of the shock approximately by a factor of 4 to 6. The welfare loss seems to be slightly more sensitive to increasing debt levels, rather than LTV ratios. Overall, these results suggest that countries with deeper credit markets are more susceptible to deleveraging shocks.

The role of occasionally binding collateral constraint. The occasionally binding collateral constraint is one of the two sources of non-linearities in the model. It causes the effects of housing demand shocks to be state- and sign-dependent. To illustrate this property, Figure 11 compares the responses to a large negative and positive housing demand shock. The stark difference between the two scenarios is driven by the fact that the collateral constraint amplifies the effects of negative shocks. Positive shocks, on the other hand, cause the constraint to become slack and irrelevant. The occasionally binding collateral constraint generates asymmetry in the link between house prices, economic activity and growth. This state- and sign-dependent dynamics was the reason why in the cross-country empirical exercise of section 2 I focused on periods of large and rapid declines in house prices. Collateral constraints tied to housing wealth are more likely to play a prominent role for macroeconomic dynamics during these periods.

In what follows, I focus on the case of negative shocks that cause the collateral constraint to bind. The complementary paper Brizhatyuk (2018) focuses on the role of the occasionally binding collateral constraints in generating asymmetric and state-dependent fluctuations in growth. The second source of non-linearities, the zero lower bound on nominal interest rate, is discussed in section 5.1.
5 Exploring the mechanism

I identify and illustrate four channels that shape the general equilibrium response to negative housing demand shocks. For illustrative purposes throughout this section I consider a one-time 10% negative housing preference shock. The calibration is the same as previously described, unless otherwise is noted.

5.1 Aggregate demand channel

Under nominal rigidity borrowers deleveraging results in a demand-driven recession in the short run: the aggregate demand channel. When a negative housing preference shock causes the collateral constraint (9) to bind forcing borrowers to reduce spending to meet the lower credit limit. It might be tempting to think that the aggregate implications of this reduction in borrowing are of a second-order importance. After all, in a closed economy context debt is money we owe to ourselves, so the implications of deleveraging may be mostly redistributional. However, the downward revision in the borrowing limit causes a temporary decrease in the natural rate, determined by the dynamics of the savers stochastic discount factor in the flexible price economy. Under nominal rigidity, the real interest rate may fail to adjust accordingly allow savers pick up most of the slack.

Doing away with nominal rigidity significantly reduces the effect of the shock. Panel (a) of figure 12 shows responses to a negative 10% housing preference shock of a baseline economy and the flexible-price counterfactual, achieved by setting the price adjustment cost to zero ($\psi_p = 0$). When prices are flexible, the real interest rate fully adjusts. As a result, consumption of savers increases on impact fully offsetting the fall in consumption of credit-constrained borrowers. The mild negative effect on the economy that remains is driven by supply-side factors. Even in a flexible-price economy, credit-constrained borrowers still decrease investment and innovation spending. The associated decrease in the marginal product of labor then lowers households labor supply.

Binding zero lower bound on the policy rate (ZLB henceforth) amplifies the effect of this channel by orders of magnitude. To illustrate it, I proceed as follows. I first append a risk-premium shock $a_t$ to the savers intertemporal optimality condition (5): $E_t \left( \Lambda_S^{\frac{1+r_{t+1}}{1+r_{t+1}}} \right) = 1 + a_t$. 

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As common in the literature, I use this shock to simulate a situation when the ZBL binds.\footnote{See Eggertsson et al. (2003), Eggertsson (2008), and Christiano et al. (2016), among others. This shock can be interpreted as a reduced-form way to capture the temporary increase in the agents desire to save. As Fisher (2015) has shown, it can also be interpreted as a structural shock to the demand for safe and liquid assets. To a first-order approximation, this shock is isomorphic to a savers discount factor shock.} A temporary increase in $a_t$ reduces the natural rate and causes the policy rate to fall accordingly. I then calculate the effect of a negative housing preference shock contingent on the ZLB binding for the first 4 quarters.\footnote{Formally, the effect of a shock conditional on binding ZLB is calculated as a difference between responses of variables in the following two simulations. (1) Baseline: a savers discount factor shock at $t = 0$ that causes the ZLB bind for a chosen number of periods. (2) Counterfactual: a savers discount factor shock at $t = 0$ that causes the ZLB bind for a chosen number of periods and a shock of interest at $t = 1$.} Panel (b) of figure 12 presents the result. Binding ZLB amplifies the effect of the shock manifold. The amplification is driven by a larger decrease in the aggregate demand when the policy rate is constrained by the ZLB. This is most clear in the real interest rate dynamics: it increases during the the first periods when the ZLB binds exacerbating the slump. This aggregate demand channel is reminiscent of the Eggertsson and Krugman (2012) discussion of the implications of household deleveraging under nominal rigidity and monetary policy constraints.

### 5.2 Productivity growth channel

The endogenous slowdown in productivity growth, the **productivity growth channel**, emerges as an additional margin of adjustment to the shock due to a combination of two forces. Forward-looking innovation spending falls as returns on it are temporary lower. Moreover, the consumption-smoothing motive of credit-constrained borrowers make them reduce investment by more than consumption when deleveraging. For facilitate illustration, throughout this section I abstract from adjustment costs in innovation spending by setting $\psi_N = 0$.

I first illustrate the contribution of this channel by shutting down endogenous fluctuations in TFP growth. Panel (a) of figure figure 13 presents counterfactual impulse responses resulting from setting the innovation spending adjustment cost to an arbitrary high value ($\psi_S = 10^5$). This exercise shuts down permanent changes in the level of TFP and capital, which is cointegrated with it. As a result, consumption and output recover quickly and fully. Moreover, shutting down the endogenous growth mechanism lowers the medium-run inflationary effect of the shock. Finally, the on-impact effect of the shock is muted. This effect has to do with the fact that changes in expected income growth affect present consumption. I discuss this channel further in section 5.4.

What is the relative size of factors that cause a fall in productivity growth after a negative
housing preference shock? Recall that entry of intermediate firms — and ultimately innovation spending — is financed by selling equity to households. To illustrate the equity market dynamics, I linearise the relevant equilibrium conditions to get the model-consistent linear equity supply and demand curves. Please refer to appendix D.3 for derivations. The result is as follows:

**Equity supply:**

\[ v_t = (1 - \rho)s_t \]  

(31)

**Equity demand:**

\[ v_t = \mathbb{E}_t \left( (A_v d_{t+1} + A_v^2 v_{t+1} - R_{t+1}) - A_v^3 \chi_t \right) \]  

(32)

Given general equilibrium outcomes, these curves determine the current-period equity price \( v_t \) and the amount of innovation spending financed \( s_t \), in percentage deviations from the deterministic steady state. In the absence of innovation spending adjustment costs the supply curve does not shift. The equity demand shifts due to a combination of two factors: changes in returns from firm ownership, which consist of future expected stream of profit. Moreover, since these are borrowers who have an access to the equity market, their investment decisions are affected when they are credit-constrained (\( \chi_t > 0 \)).

Panel (b) of figure figure 13 shows the equity market dynamics consistent with the simulation on the panel (a). As a result of a negative housing preference shock, the equity market equilibrium moves from the steady state at point A to an equilibrium with lower innovation spending as equity demand falls at \( t = 1 \) (point B). How much of this demand shift is due to the binding collateral constraint effect, \( \chi_t > 0 \)? The dashed line plots the equity demand curve at \( t = 1 \) excluding, in a partial equilibrium sense, the collateral effect. About a quarter of the shift in the demand curve appears to be due to the collateral effect, the rest of it is driven by demand factors.

### 5.3 Fisherian debt deflation channel

Two more channels amplify the initial effect of the shock. First is the negative feedback loop between deleveraging and borrowers housing wealth: *Fisherian debt deflation.*\(^{39}\) At the core of this channel is the pecuniary externality: credit-constrained borrowers do not internalize the effect of their individual spending reduction on aggregate housing wealth that ultimately affects their borrowing capacity. The initial deleveraging then exacerbates the damage to borrowers balance sheets and causes further deleveraging. I provide a glimpse into the strength of this

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\(^{39}\) Lately, this channel has been most widely discussed in the literature on emerging market crises, see for instance Mendoza (2010). However, it is standard for any model where the borrowing capacity is linked to the relative price of collateral, see the original discussion in Fisher (1933).
channel with two experiments. To simplify linearization discussed below throughout this section I abstract from the borrowing limit inertia and set $\rho_B = 0$.

Although the housing preference shock is aggregate, binding collateral constraint drives a sizable asymmetry in the dynamics of borrowers and savers housing demand. Similarly to the previous section, I use linearized equilibrium conditions to illustrate this point. Please refer to appendix D.2 for details. Let the san-serif font denote percentage deviations of variables from the deterministic steady state. The resulting linear housing demand curves are as follows:

Savers demand: $p^h_t = A^S_{h1} h^B_t + A^S_{h2} \hat{\vartheta}_t - A^S_{h3} E_t (p^h_{t+1} + g_{t+1} - R_{t+1})$  

Borrowers demand: $p^h_t = -A^B_{h1} h^B_t + A^B_{h2} \hat{\vartheta}_t - A^B_{h3} E_t (p^h_{t+1} + g_{t+1} - R_{t+1}) - A^B_{h4} \chi_t$  

These model-consistent linear demand curves determine the aggregate real house price $p^h_t$ and the allocation of housing towards borrowers $h^B_t$ given the aggregate housing preference shock $\vartheta_t$ and general equilibrium outcomes (expressed in percentage deviations from the deterministic steady state). What causes these demand curves to shift? Common factors are the aggregate preference shock and changes in the expected next-period return. Demands are also affected by household-specific wealth effects: when the shadow value of consumption is high, the valuation of housing in consumption units is low and vice versa. What drives the asymmetry between borrowers and savers housing demands is the collateral effect: the effect of a binding collateral constraint ($\chi_t > 0$). Appendix D.2 shows that for plausible parameter values the collateral effect is negative ($A^B_{h4} > 0$). When negative shocks cause the constraint to bind, borrowers decrease their housing demand more than savers.

The negative equilibrium effect of binding collateral constraint on borrowers housing wealth is large and it significantly exacerbates deleveraging. Panel (b) of figure 14 plots the model-consistent demand curves implied by equations (33). The left panel shows the initial steady-state housing market equilibrium at $t = 0$ (point A). The right panel present an on-impact response of the housing market to an aggregate negative 10% housing preference shock. The shock pushes both demand curves down and causes the equilibrium price to sharply fall (point B). Importantly, the borrowers housing demand falls by more. As a result, borrowers housing wealth suffers not only because the house price falls, but also because the housing stock is being redistributed towards savers. The lion’s share of this outcome is driven the collateral effect. The dashed line shows, in a partial equilibrium sense, the shift of borrowers demand curve excluding the collateral effect (point C).

Panel (a) of figure 14 shows the contrafactual dynamics of the economy where the effect of housing reallocation towards savers on the borrowing limit is shut down. In other words,
equation (9) is modified as follows: \( \frac{B_t}{P_t} \leq \rho_B m P_t^B h^B \), where \( h^B \) is steady-state housing of borrowers. This exercise allows to mute some — but not all — of the effect of the Fisherian debt deflation channel. As the resulting impulse responses show, the reallocation of housing that arises endogenously as a result of borrowers deleveraging plays a quantitatively significant role in shaping the overall response of the economy.

5.4 Expected income growth channel

The second amplification force is the expected income growth channel: lower expected growth decreases current spending through intertemporal substitution. As such, the endogenous productivity growth mechanism is not only responsible for the persistent effect of deleveraging, but also amplifies the short-run response.

As before, for illustrative purposes I linearise and aggregate intertemporal optimality conditions of borrowers and savers expressed in stationary (lower-case) variables, please refer to D.4 for derivation. For tractability, I abstract from the borrowing limit inertia (\( \rho_B = 0 \)). This results in the following expression for the expected growth rate of the aggregate marginal utility of consumption:

\[
E_t g_{t+1} = -E_t R_{t+1} + \sigma E_t g_{t+1} - A \lambda \chi_t
\]

According to this expression, the expected growth rate \( (E_t g_{t+1}) \) has a positive direct effect on the aggregate marginal utility of consumption growth \( (E_t g_{t+1}) \). The magnitude of this effect is governed by the elasticity of intertemporal substitution. When growth is expected to be slower, the marginal utility of consumption today increases (and the consumption level today decreases). However, this direct effect of the expected growth rate in principle may be offset by general equilibrium effects that work through the expected real interest rate \( (E_t R_{t+1}) \) and the tightness of the collateral constraint \( \chi_t \).

Given the baseline calibration, there is a negative general equilibrium relation between expected income growth and current consumption. In particular, I compare the dynamics of the marginal utility of consumption growth rate in the baseline relative to the counterfactual where the endogenous growth response is shut down by setting a very high value of innovation adjustment cost \( (\psi_N = 10^5) \). This experiment allows to single out the effect of endogenous growth on the cyclical consumption dynamics. Panel (a) of figure 15 plots the short run response of the detrended variables in the two cases. When productivity growth is allowed to respond to cyclical conditions, the on-impact fall in the cyclical component of consumption is almost 30% larger. Panel (b) illustrates why. It employs equation (34) to plot the difference between the
baseline and the counterfactual. The left panel shows the relative dynamics of the marginal utility of consumption growth. The right panel plots separately factors contributing to this dynamics. Although the real interest rate falls by more, it does not fall enough to compensate for the effect of a decrease in the expected income growth, which drags the growth of marginal utility of consumption down. Relative to the other two factors, tightness of the borrowing constraint does not change much between the baseline and the counterfactual scenario.

6 Policy scenarios

I conclude highlighting important implications for monetary and fiscal policy.

Monetary policy. Housing demand shocks — and more broadly shocks that manifest in deleveraging — warrant stronger focus of monetary policy on output stabilization. As section 5.1 illustrates, in the short run housing demand shocks propagate similarly to aggregate demand shocks. Naturally, their effects (at both short- and long-run horizons) are shaped by monetary policy. My question is of positive nature: I do not attempt to access optimal monetary policy. Rather, taking that the policy rate follows the Taylor rule as given, I want to access the sensitivity of the welfare cost of the shock to parameters of the rule. Please refer to appendix D.5 for the discussion of welfare calculations.

I simulate responses to a 10% negative housing demand shock for various values of the policy rate response to inflation ($\phi_\pi$) and cyclical variations of output ($\phi_Y$). Moreover, I compare the results of the baseline model to the ones of the alternative where the endogenous growth mechanism is shut down (by setting adjustment cost $\psi_N = 10^5$). Figure 16 illustrates the results. Two things stand out. First, in both cases the welfare cost of the shock is strictly decreasing in the strength of policy response to output. Stronger response to inflation is not welfare-improving. Under certain combination of parameters it may even exacerbate the welfare loss. Moreover, the endogenous growth mechanism magnifies the effect of the shock and emphasizes the importance of output stabilization relative to inflation stabilization.40

Fiscal policy. How effective fiscal policy can be in mitigating the damage to borrowers balance sheets and resulting deleveraging due to a decline in house prices? I offer a glimpse into this question by studying the following policy. Suppose the government finances its expenditure $G_t$ by levying lump-sum taxes of savers and borrowers, $T_t^S$ and $T_t^B$ respectively. The policy then

40 This is broadly consistent with results of Garga and Singh (2018) and Ikeda and Kurozumi (2018) who study optimal monetary policy in business cycles models with endogenous growth.
involves shifting the tax burden from borrowers to savers by $\Delta_t$. This can be interpreted as a
debt relief program or, more generally, as a revenue-neutral tax reform that benefits borrowers.
The government budget constraint and the modified budget constraints of savers and borrowers
take the following form:

$$G_t = T_t^B + T_t^S$$

$$C_t^S + P_t^h(h_t^S - h_{t-1}^S) + (1 + r_{t-1}) \frac{B_t^S}{P_t} = W_tL_t^S + \frac{B_{t+1}^S}{P_t} - (T_t^S + \Delta_t)$$

$$C_t^B + I_t + P_t^h(h_t^B - h_{t-1}^B) + (1 + r_{t-1}) \frac{B_t^B}{P_t} + v_{t+1}v_t(N_t + N_{e,t}) =
\nu_t(v_t + d_t)N_t + W_tL_t^B + R_t^K u_tK_t + \frac{B_{t+1}^B}{P_t} + div_t - (T_t^S - \Delta_t)$$

Debt relief measures implemented amid the crisis are very effective in alleviating both immediate and persistent effects of deleveraging. Figure 17 presents baseline IRF-matching simulation along with the policy intervention counterfactual. The policy is a transfer $\Delta_t$ equivalent to a 0.25% of borrowers’ steady-state debt burden. As in Guerrieri and Iacoviello (2017) the transfer is governed by an AR(1) process with a persistence coefficient equal to 0.5. The success of the policy is based on two factors. First, in the economy where borrowers are credit-constrained Ricardian equivalence no longer holds. As a result, a transfer towards borrowers is expansionary. Moreover, this policy strikes at the heart of amplification mechanisms that operate in the economy and are responsible for the lion’s share of the resulting negative effect. The policy effectively alleviates the negative feedback loop between borrowers deleveraging, their housing wealth, and growth expectations (the debt-deflation channel and the expected income channel). Panel (b) shows the net effect on the policy and clearly indicates that the debt write-down relaxes the borrowers’ credit constraint. As such, it offsets the immediate decrease in consumption and improves the medium-run trajectory of the economy by decreasing the decline in innovation and capital investment: the two components of the aggregate demand that are affected by deleveraging the most.

41The result is consistent with empirical estimates of Auclert et al. (2019) who document the large and highly persistent effect of U.S. debt forgiveness measures on non-tradable employment. However, it is important to keep in mind that the ultimate effect of such measures depends on their pass-through to households. See Piskorski and Seru (2018) for the discussion of the role of housing market and housing finance rigidities.
7 Conclusion

Why recoveries from some recessions are particularly slow and incomplete? I contribute to this debate by studying the effects of housing market boom-and-bust cycles. First, using cross-country data, I explore the dynamics of recessions and recoveries associated with housing markets crashes. Such events are robustly associated a decline in consumption, output, and utilization-adjusted TFP that lasts longer than regular business-cycle fluctuations. Next, I built a dynamics general equilibrium model with borrower and saver households, occasionally binding collateral constraints tied to housing wealth, endogenous growth through forward-looking investment, and nominal rigidity to study the channels through which declines in house prices affect the macroeconomy. The model successfully accounts for the empirical comovements between variables; highlights the importance of endogenous innovation in generating persistent effect of the shock; and illustrates the key amplification mechanisms. I then use the model to study several policy scenarios. House price shocks that trigger delivering warrant stronger focus of monetary policy on output stabilization, and even more so in the presence of endogenous response in the productivity growth. Fiscal intervention that alleviates the debt burden of borrowers is effective in offsetting a large fraction of the shock when implemented during the crisis.

This paper opens a number of promising avenues for future research. The presented U.S. state-level and MSA-level evidence have illustrated the persistent regional divergence that can occur when member states are subject to asset market boom-and-bust cycles of different intensity. Thinking beyond the U.S. economy, a similar pattern has been even more vivid in the Eurozone in recent years. An interesting question then is how regional and supraregional policies can be designed and coordinated to alleviate this problem. A two-country open-economy extension of the theoretical framework of this paper would be suitable to explore this question. In terms of policy, the present work focused on a simple scenario of a transfer from savers to borrowers. A study of a broader set of more realistic fiscal policy measures, such as tax-and/or borrowing-financed increase in government spending, is warranted. I am working on the extensions to address these issues.
Figure 1: US real GDP: actual vs vintages of growth projections [cited on page 2]

Note: vintages of growth projections are from CBO
Figure 2: Housing market boom and bust cycle across countries, 2000-2018 [cited on page 2]

Note: real housing prices are from various sources, see description in the text; household debt to GDP ratios are from IMF Global Debt Database; real per-capita GDP growth is from World Bank.
Table 1: Identified housing market crashes [cited on page 9]

<table>
<thead>
<tr>
<th>Country code</th>
<th>Peak</th>
<th>Trough</th>
<th>First Peak to trough</th>
<th>Country code</th>
<th>Peak</th>
<th>Trough</th>
<th>First Peak to trough</th>
</tr>
</thead>
<tbody>
<tr>
<td>BGR</td>
<td>2008</td>
<td>2013</td>
<td>-39.1%</td>
<td>JPN</td>
<td>1974</td>
<td>1977</td>
<td>-23.3%</td>
</tr>
<tr>
<td>BRA</td>
<td>2014</td>
<td>2017</td>
<td>-15.6%</td>
<td>JPN</td>
<td>1991</td>
<td>2012</td>
<td>-13.2%</td>
</tr>
<tr>
<td>CHE</td>
<td>1990</td>
<td>2000</td>
<td>-20.0%</td>
<td>LVA</td>
<td>2007</td>
<td>2010</td>
<td>-47.0%</td>
</tr>
<tr>
<td>CHE</td>
<td>1959</td>
<td>1961</td>
<td>-12.4%</td>
<td>MYS</td>
<td>1997</td>
<td>1999</td>
<td>-14.8%</td>
</tr>
<tr>
<td>COL</td>
<td>1995</td>
<td>2003</td>
<td>-14.4%</td>
<td>NLD</td>
<td>1978</td>
<td>1985</td>
<td>-34.0%</td>
</tr>
<tr>
<td>CZE</td>
<td>2008</td>
<td>2013</td>
<td>-15.4%</td>
<td>NLD</td>
<td>2008</td>
<td>2013</td>
<td>-11.5%</td>
</tr>
<tr>
<td>DNK</td>
<td>1979</td>
<td>1982</td>
<td>-33.6%</td>
<td>NZL</td>
<td>1974</td>
<td>1980</td>
<td>-18.4%</td>
</tr>
<tr>
<td>DNK</td>
<td>1986</td>
<td>1993</td>
<td>-18.4%</td>
<td>NZL</td>
<td>2007</td>
<td>2009</td>
<td>-11.4%</td>
</tr>
<tr>
<td>ESP</td>
<td>2007</td>
<td>2014</td>
<td>-14.5%</td>
<td>PHL</td>
<td>1996</td>
<td>2004</td>
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<tr>
<td>EST</td>
<td>2007</td>
<td>2009</td>
<td>-51.0%</td>
<td>POL</td>
<td>2010</td>
<td>2013</td>
<td>-15.6%</td>
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<tr>
<td>FIN</td>
<td>1974</td>
<td>1979</td>
<td>-24.7%</td>
<td>PRT</td>
<td>1992</td>
<td>1996</td>
<td>-10.7%</td>
</tr>
<tr>
<td>FIN</td>
<td>1989</td>
<td>1993</td>
<td>-42.2%</td>
<td>RUS</td>
<td>2008</td>
<td>2011</td>
<td>-33.0%</td>
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<tr>
<td>FRA</td>
<td>1980</td>
<td>1985</td>
<td>-11.3%</td>
<td>SGP</td>
<td>1983</td>
<td>1986</td>
<td>-31.4%</td>
</tr>
<tr>
<td>GBR</td>
<td>1973</td>
<td>1977</td>
<td>-23.5%</td>
<td>SGP</td>
<td>1996</td>
<td>1998</td>
<td>-32.2%</td>
</tr>
<tr>
<td>GBR</td>
<td>1989</td>
<td>1996</td>
<td>-21.9%</td>
<td>SRB</td>
<td>2010</td>
<td>2013</td>
<td>-29.4%</td>
</tr>
<tr>
<td>GBR</td>
<td>2007</td>
<td>2012</td>
<td>-16.0%</td>
<td>SVK</td>
<td>2008</td>
<td>2012</td>
<td>-21.4%</td>
</tr>
<tr>
<td>GRC</td>
<td>2007</td>
<td>2017</td>
<td>-15.1%</td>
<td>SVN</td>
<td>2011</td>
<td>2014</td>
<td>-20.6%</td>
</tr>
<tr>
<td>HKG</td>
<td>1981</td>
<td>1984</td>
<td>-46.8%</td>
<td>SWE</td>
<td>1979</td>
<td>1985</td>
<td>-26.3%</td>
</tr>
<tr>
<td>HKG</td>
<td>1997</td>
<td>2003</td>
<td>-42.2%</td>
<td>SWE</td>
<td>1990</td>
<td>1993</td>
<td>-30.4%</td>
</tr>
<tr>
<td>HRV</td>
<td>1999</td>
<td>2002</td>
<td>-14.3%</td>
<td>THA</td>
<td>2006</td>
<td>2009</td>
<td>-30.1%</td>
</tr>
<tr>
<td>HRV</td>
<td>2009</td>
<td>2015</td>
<td>-18.6%</td>
<td>USA</td>
<td>2006</td>
<td>2012</td>
<td>-14.2%</td>
</tr>
<tr>
<td>HUN</td>
<td>2006</td>
<td>2013</td>
<td>-16.7%</td>
<td>ZAF</td>
<td>1984</td>
<td>1987</td>
<td>-39.4%</td>
</tr>
<tr>
<td>IRL</td>
<td>2006</td>
<td>2012</td>
<td>-30.4%</td>
<td>ZAF</td>
<td>2007</td>
<td>2012</td>
<td>-16.1%</td>
</tr>
<tr>
<td>ISL</td>
<td>2007</td>
<td>2010</td>
<td>-32.3%</td>
<td>Median</td>
<td>5 years</td>
<td>-0.6%</td>
<td>30.0%</td>
</tr>
</tbody>
</table>

Note: unbalanced panel of 50 countries, 1950-2017. Housing market boom-and-bust cycles are identified in 43 countries. The sample consists 63 events: 39 before 2006 and 24 during/after the GFC.

Housing market bubbles are defined as periods when the aggregate housing price index (1) deviates from the long-run trend by more than one standard deviation and (2) declines of at least 10% within the first three years from the peak.
Figure 3: Impulse responses to a housing market crash, baseline [cited on page 11]

Note: cross-country panel estimation, controlling for country/time fixed effects, country-level trends, and macroeconomic conditions.

Variables: (1) Real per-capita GDP; (2) Real per-capita consumption; (3) Real per-capita investment; (4) Real house price index; (5) Household debt-to-GDP gap; (6) Corporate non-financial debt-to-GDP gap; (7) Employment-to-population ratio; (8) Real per-capita capital stock; (9) Total factor productivity, utilization-adjusted.

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 90% confidence intervals. Standard errors are clustered at the country level.
Figure 4: Impulse responses to a housing market crash, the role of household and corporate debt [cited on page 12]

Note: cross-country panel estimation, controlling for country/time fixed effects, country-level trends, and macroeconomic conditions. Interactions between house price decline and household/corporate debt-to-GDP gaps at the peak of housing market cycle. Conditional IRFs:
- Gray: zero household and non-financial corporate debt-to-GDP gaps
- Blue (top panel): 10% household debt-to-GDP gap and zero non-financial corporate debt-to-GDP gap.
- Purple (bottom panel): 10% non-financial corporate debt-to-GDP gap and zero households debt-to-GDP gap

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 95% confidence intervals. Standard errors are clustered at the country level.
Figure 5: Housing market crash magnitude vs labor productivity growth [cited on page 13]

Table 2: Elasticity of labor productivity growth (2007-17) [cited on page 14]

<table>
<thead>
<tr>
<th></th>
<th>States + DC</th>
<th>MSAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing price growth</td>
<td>(1)</td>
<td>(6)</td>
</tr>
<tr>
<td>2007-2012</td>
<td>OLS</td>
<td>OLS</td>
</tr>
<tr>
<td></td>
<td>0.34**</td>
<td>0.14***</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Mian and Sufi (2014)</td>
<td>(2)</td>
<td>(7)</td>
</tr>
<tr>
<td>∆ housing net worth</td>
<td>OLS IV 1</td>
<td>IV 1</td>
</tr>
<tr>
<td></td>
<td>0.16***</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>∆ housing net worth</td>
<td>OLS IV 2</td>
<td>IV 2</td>
</tr>
<tr>
<td></td>
<td>0.11*</td>
<td>0.32**</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>∆ housing net worth</td>
<td>OLS IV 3</td>
<td>IV 3</td>
</tr>
<tr>
<td></td>
<td>0.13**</td>
<td>0.54***</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.09)</td>
</tr>
</tbody>
</table>

Robust st. errors in parentheses, * p < 0.10, ** p < 0.05, *** p < 0.01

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. Housing price growth is based on the Federal Housing Finance Agency all-transactions house price indexes.

IV 1: Saiz (2010) housing supply elasticity instrument, linear first stage;
IV 2: Saiz (2010) housing supply elasticity instrument, quadratic first stage;
IV 3: Guren et al. (2018) regional sensitivity instrument.
Figure 6: Baseline model flow chart [cited on page 15]
Table 3: Model summary (stationary variables) [cited on page 24]

1. Final good market clearing
\[ y_t - r_t^e = c_t^S + c_t^B + i_t + (f^e + p_t^B)n_{e,t} + AC_{S,t} + AC_{p,t} + AC_{t,t} \]

2. Savers budget constraint
\[ c_t^S + p_t^B(h_t^S - h_t^{S,t-1}) + \frac{1 + \gamma_t - \beta}{\Pi_t} \frac{b_t}{g_t} = w_t L_t + b_{t+1} \]

3. Intermediate good output
\[ x_t = \left( \frac{\Delta x_t}{\nu} \right)^{1 - \alpha} \tilde{k}_t L_t^{1 - \alpha} \]

4-5. Savers/borrowers Euler equation
\[ \mathbb{E}_t \left( A_t \left( 1 + \frac{1 + \gamma_t}{\Pi_t} \right) \right) = 1, \quad \mathbb{E}_t \left( \left( \frac{1 + \gamma_t - \beta}{\Pi_t} \right)^{1 + \gamma_t} \right) = 1 - \chi_t \]

6. Collateral constraint
\[ (b_t^S - \rho B_{t+1} \frac{b_t}{g_t} - (1 - \rho_B)mp_t h_t^S)\chi_t = 0, \quad \chi_t \geq 0 \]

7. Credit market clearing
\[ b_t^S + b_t^C = 0 \]

8-9. Savers/borrowers labor supply
\[ w_t = v_t (L_t^H)^{\epsilon}, \quad H \in \{S, B\} \]

10. Disutility of labor
\[ v_t = \left( \frac{v_{t-1}}{g_t} \right)^{\epsilon} \]

11. Labor demand
\[ w_t = \frac{1}{\nu} (1 - \alpha) (1 - \xi) \frac{w_t}{L_t} \]

12. Labor market clearing
\[ L_t = L_t^B + L_t^S \]

13. Capital supply
\[ q_t = \mathbb{E}_t \left( A_{t+1}^B (1 - \delta_{K,t}) q_{t+1} + R_{t+1}^K \right) \]

14. Tobin’s q
\[ q_t = 1 + q_t \left( AC_{t,t} + AC_{t,t}^r i_t \right) - \mathbb{E}_t \left( \left( A_{t+1}^B q_{t+1} + AC_{t,t+1}^r i_{t+1} \right) \right) \]

15. Capital law of motion
\[ k_{t+1} = \left( 1 - \delta_{K,t} \right) k_t + \left( 1 - AC_{t,t} \right) i_t \]

16-17. Capital utilization
\[ R_t^K = c_t + c_t^2 (u_t - 1), \quad \delta_{K,t} = \delta_t = \delta_t + c_t^2 (u_t - 1) + \frac{\epsilon_t}{u} (u_t - 1)^2 \]

18. Capital demand
\[ R_t^K = \frac{1}{\nu} (1 - \xi) \frac{w_t}{L_t} \]

19. Savers housing demand
\[ p_t^S = \mathbb{E}_t \left( A_{t+1}^S p_{t+1} q_{t+1} + \lambda_{t} \frac{h_t^S}{h_t^S} \right) \]

20. Borrowers housing demand
\[ p_t^h = \mathbb{E}_t \left( A_{t+1}^S p_{t+1} q_{t+1} + \lambda_{t} \frac{h_t^S}{h_t^S} \right) + \chi_t m p_t^h \]

21. Housing market clearing
\[ h_t^S + h_t^S = 1 \]

22. Equity demand
\[ v_t = (1 - \delta_N) \mathbb{E}_t \left( A_{t+1}^B (d_{t+1} + v_{t+1}) \right) \]

23. Equity supply (free entry)
\[ \phi_t v_t = 1 + AC_{S,t} + AC_{S,t}^r s_t - \mathbb{E}_t \left( A_{t+1}^B AC_{S,t+1}^r s_{t+1} \right) \]

24. Innovators productivity
\[ \phi_t = \phi s_t^{\eta_{t-1}} \]

25. Intermediate firms profit
\[ d_t = \frac{\nu_{t-1}}{L_t} x_t \]

26. Growth rate
\[ g_t = (1 - \delta_N) \left( 1 + \phi s_t^\eta \right) \]

27. Markup
\[ \mu_t = \frac{\eta_t}{(\eta_t - 1) + \psi_p \frac{\mu_t}{\Pi_t} (\frac{\Pi_t}{\Pi_t - 1}) - \psi_p \mathbb{E}_t A_{t+1}^B \left( \frac{h_{t+1} - h_t}{h_t} \right) \frac{g_{t+1}}{g_t} \}

28. Taylor rule
\[ 1 + r_t = \max \left\{ 0, (1 + r_{t-1})^{\rho_r} \left( 1 + r \right) \left( \frac{\nu_t^{GDP}}{\nu_t^{OPT}} \right)^{\phi_Y} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_{\pi}} \right\}^{1-\rho_r} u_t \]

Final output
\[ y_t = \left( \frac{\Delta y_t}{\nu} \right)^{1 - \alpha} \tilde{k}_t^{\alpha} \tilde{L}_t^{1 - \alpha}, \quad \tilde{k}_t = u_t k_t \]

Stochastic discount factors
\[ A_t^{H,t+1} = \beta_H \frac{\lambda_{t+1}^{H,t+1} g_{t+1}^{-\sigma}}{\Lambda_t^{H,t+1}}, \quad \lambda_t^{H} = \left( c_t^H - v_t \left( \frac{L_t^H 1 + L_t}{1 + L_t} \right)^{-\sigma} \right), \quad H \in \{S, B\} \]

Note: for variables that exhibit tred growth, lower-case variables denote stationary counterparts of the original variables, i.e. \( c_t = \tilde{c}_t^{\bar{\pi}}. \)
### Table 4: Structural parameters [cited on page 26]

**Calibrated parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_S$</td>
<td>Savers discount factor</td>
<td>0.9968</td>
<td>4% annual real interest rate</td>
</tr>
<tr>
<td>$\beta_B$</td>
<td>Borrowers discount factor</td>
<td>0.9963</td>
<td>$\beta_B = \beta_S - \varepsilon$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Relative risk aversion</td>
<td>2</td>
<td>Conventional</td>
</tr>
<tr>
<td>$1/\epsilon_L$</td>
<td>Frisch elasticity of labor supply</td>
<td>0.25</td>
<td>King and Rebelo (1999)</td>
</tr>
<tr>
<td>$1/\epsilon_h$</td>
<td>Price elasticity of housing demand</td>
<td>0.2</td>
<td>Hanushek and Quigley (1980)</td>
</tr>
<tr>
<td>$m$</td>
<td>Max leverage</td>
<td>0.75</td>
<td>Warnock and Warnock (2008)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.4</td>
<td>Data median, PWT 9.1</td>
</tr>
<tr>
<td>$\nu/(\nu-1)$</td>
<td>Intermediate-good elasticity of subst.</td>
<td>1.6</td>
<td>BGP requirement $\xi(\nu-1) = 1 - \alpha$</td>
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<tr>
<td>$\eta$</td>
<td>Retail-good elasticity of subst.</td>
<td>11</td>
<td>10% steady-state markup</td>
</tr>
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<td>$\xi$</td>
<td>Intermediate good share</td>
<td>0.5</td>
<td>Comin and Gertler (2006)</td>
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<tr>
<td>$1/A$</td>
<td>Intermediate sector marginal cost</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Innovation output elasticity</td>
<td>0.8</td>
<td>Comin and Gertler (2006)</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>Steady state capital depreciation</td>
<td>0.025</td>
<td>Conventional</td>
</tr>
<tr>
<td>$\delta_N$</td>
<td>Intermediate sector exit rate</td>
<td>0.025</td>
<td>Bilbie et al. (2012)</td>
</tr>
<tr>
<td>$\phi_y : \phi_\pi : \rho_T$</td>
<td>Taylor rule parameters</td>
<td>0.25; 1.5; 0.7</td>
<td>Carare and Tchaidze (2005)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Innovators productivity</td>
<td>0.11</td>
<td>Annual TFP growth = 0.8% (data median, PWT 9.1)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Share of housing in utility</td>
<td>0.03</td>
<td>Mortgage debt to GDP = 0.55, Warnock and Warnock (2008)</td>
</tr>
<tr>
<td>$\bar{Z}$</td>
<td>Final sector productivity</td>
<td>1.74</td>
<td>Normalization, $Y_{GDP} = 1$</td>
</tr>
<tr>
<td>$\psi_p$</td>
<td>Price adjustment cost</td>
<td>120</td>
<td>4-quarter average Calvo price ridigity equivalent</td>
</tr>
</tbody>
</table>

**Estimated parameters:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_b$</td>
<td>Borrowing limit inertia</td>
<td>0.65</td>
<td>IRF matching</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>Disutility of labor inertia</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>$\psi_K$</td>
<td>Investment adjustment cost</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\psi_N$</td>
<td>Innovation adjustment cost</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$c_2$</td>
<td>Capital utilization responsiveness</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>
Figure 7: Baseline simulation: target empirical responses vs model-implied responses (IRF matching) [cited on page 29]

Note: (1) Data: baseline local projection responses to a housing market crash; (2) Model: responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline.
Figure 8: Baseline simulation, model-implied output and TFP decompositions [cited on page 29]

Note: model-based decompositions

Left panel, growth accounting: \( \Delta Y_t = \Delta TFP_t + \alpha \Delta K_t + (1 - \alpha) \Delta L_t \)

Right panel, Solow residual: \( \Delta TPF_t = \Delta \Omega_t + \alpha \Delta u_t + (1 - \xi)^{-1} \Delta Z_t + (1 - \alpha) \Delta N_t \)
Figure 9: Baseline simulation, more variables [cited on page 29]

Note: impulse responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline. Inflation rate and the policy rate are annualized.
Figure 10: Baseline simulation, the role of household indebtedness [cited on page 30]

(a) Various debt to GDP ratios

(b) Various loan-to-value ratios

Note: impulse responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline. Inflation rate and the policy rate are annualized. Counterfactual responses to the baseline on figure 8.
Figure 11: Negative vs positive shock [cited on page 30]

[To be updated]

Note: impulse responses to a large negative and positive housing preference shock. Inflation rate and the policy rate are annualized.
Figure 12: 10% negative housing demand shock, the aggregate demand channel [cited on page 32]

(a) The effect of price rigidity

(b) The effect of the binding zero lower bound

Note: impulse responses to a one-time 10% negative housing demand shock, baseline calibration. Panel (a): flexible price counterfactual by setting $\psi_p = 0$. Panel (b): net effect of the housing preference shock contingent on a savers discount factor shock that makes the zero lower bound bind for the first 4 quarters, see details in the text.
Figure 13: 10% negative housing demand shock, endogenous growth channel [cited on page 33]

(a) No endogenous response of productivity growth

(b) Model-consistent equity market equilibrium

Note: responses to a one-time 10% negative housing demand shock, baseline calibration, except $\rho_B = 0$. Panel (a): no growth response counterfactual by setting $\psi_N = 10^3$. Panel (b): intermediate-firm equity market dynamics consistent with the simulation on the top panel, see details in the text.
Figure 14: 10% negative housing demand shock, Fisherian debt deflation channel [cited on page 34]

(a) No housing reallocation effect in the borrowing constraint

(b) Model-consistent housing market equilibrium

Note: impulse responses to a one-time 10% negative housing demand shock, baseline calibration except $\rho_B = 0$. Panel (a): counterfactual by fixing the housing quantity in the borrowing limit $\frac{B_t^B}{P_t} \leq \rho_B m t^B h^B$. Panel (b): housing market dynamics consistent with the simulation on the top panel, see details in the text.
Figure 15: 10% negative housing demand shock, expected income growth channel [cited on page 35]

(a) Responses of stationary (detrended) variables

(b) Marginal utility of consumption growth, the difference between the baseline and the “no growth response” scenarios

Note: impulse responses to a one-time 10% negative housing demand shock, baseline calibration except $\rho_B = 0$. Panel (a) shows the dynamics of detrended variables (excluding the direct effect of growth) in the baseline economy and and the counterfactual economy with no endogenous growth response. Panel (b) shows the difference between the growth of marginal utility of consumption in the baseline and the counterfactual: $\Delta E_t g_{t+1} = -\Delta E_t R_{t+1} + \sigma \Delta E_t g_{t+1} - A_\lambda \Delta X_t$. Please refer to appendix D.4 for details.
Figure 16: 10% negative housing demand shock, welfare loss under different Taylor rule parameters, % of the steady-state welfare [cited on page 36]

Note: 10% negative housing demand shock, lifetime utility loss relative to the steady-state under different values of parameters that govern the policy rate reaction to output ($\phi_Y$) and inflation ($\phi_{\Pi}$). In all cases the interest rate inertia is set to $\rho_r = 0.7$. Dashed lines mark baseline values of parameters.
Figure 17: Baseline simulation, debt relief policy [cited on page 37]

(a) Baseline simulation vs policy

(b) Net effect of the policy

Note: impulse responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline. Inflation rate and the policy rate are annualized. Counterfactual responses to the baseline on figure 8. Debt relief policy consists of a temporary transfer from savers to borrowers equivalent to a 0.25% of borrowers steady-state debt burden implemented at the time of the shock.
References


66
Appendix to “Housing market cycles, productivity, and household debt”

Dmitry Brizhatyuk*

Abstract

The appendix gathers supplementary materials to Brizhatyuk (2019)

*Department of Economics, University of Washington, Seattle, WA 98195, USA. E-mail: dbrizh@uw.edu
A Data appendix

World Bank, 1960-2018  data.worldbank.org/indicator
GDP per capita, constant LCU  NY.GDP.PCAP.KN
Households and NPISHs final consumption expenditure, constant LCU,  NE.CON.PRVT.KN
Gross fixed capital formation, constant LCU  NE.GDI.FTOT.KN
GDP deflator  NY.GDP.DEFL.ZS

Penn World Table version 9.1, 1950-2017  rug.nl/ggdc/productivity/pwt
Output-side real GDP at chained PPPs (in mil. 2011 USD)  rgdpo
Population (in millions)  pop
Number of persons engaged (in millions)  emp
TFP at constant national prices (2011=1)  rtfpna
Capital stock at constant 2011 national prices (in mil. 2011 USD)  rnna
Share of merchandise exports at current PPPs  csh_x
Share of merchandise imports at current PPPs  csh_m
Share of labour compensation in GDP at current national prices  labsh

Household debt, loans and debt securities, percent of GDP
Nonfinancial corporate debt, loans and debt securities, percent of GDP

Jordà-Schularick-Taylor Macrohistory Database  http://www.macrohistory.net/data
House prices (nominal index, 1990=100)  hpnom
Total loans to households (nominal, local currency)  thh
Total loans to business (nominal, local currency)  tbus
Consumer prices (index, 1990=100)  cpi

Aggregate real housing price indexes, other sources
BIS real residential property indices  bis.org/statistics/pp_selected.htm
Dallas FED International House Price Database  dallasfed.org/institute/houseprice
OECD real house price indices  stats.oecd.org/Index.aspx?DataSetCode=HOUSE_PRICES
Laevan and Valencia (2013)  
[sites.google.com/site/laevenl/codes]

Systemic Banking Crises Database

Ilzetzki et. al. (2019)  
[carmenreinhart.com/data/browse-by-topic/topics/11]

Exchange rate regime classification

Bruegel  
[bruegel.org/publications/datasets]

Real effective exchange rates

<table>
<thead>
<tr>
<th>Table 5: Country sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARG</td>
</tr>
<tr>
<td>AUS</td>
</tr>
<tr>
<td>AUT</td>
</tr>
<tr>
<td>BEL</td>
</tr>
<tr>
<td>BGR</td>
</tr>
</tbody>
</table>

*Note:* List of 50 countries with available aggregate housing price indices, ISO 3166-1 alpha-3 codes. Countries for which no housing bubbles have been identified are highlighted.

<table>
<thead>
<tr>
<th>Table 6: Frequency of housing market crashes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1961</td>
</tr>
<tr>
<td>1965</td>
</tr>
<tr>
<td>1974</td>
</tr>
<tr>
<td>1975</td>
</tr>
<tr>
<td>1979</td>
</tr>
<tr>
<td>1980</td>
</tr>
<tr>
<td>1981</td>
</tr>
<tr>
<td>1982</td>
</tr>
<tr>
<td>1984</td>
</tr>
</tbody>
</table>

Total: 63
Table 7: Summary statistics [cited on page 8 and 10]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>∆Y</td>
<td>2485</td>
<td>0.025</td>
<td>0.025</td>
<td>0.034</td>
</tr>
<tr>
<td>2.</td>
<td>∆C</td>
<td>2230</td>
<td>0.032</td>
<td>0.032</td>
<td>0.038</td>
</tr>
<tr>
<td>3.</td>
<td>∆I</td>
<td>2370</td>
<td>0.037</td>
<td>0.041</td>
<td>0.103</td>
</tr>
<tr>
<td>4.</td>
<td>∆L</td>
<td>2890</td>
<td>0.002</td>
<td>0.002</td>
<td>0.019</td>
</tr>
<tr>
<td>5.</td>
<td>∆K</td>
<td>2690</td>
<td>0.043</td>
<td>0.039</td>
<td>0.031</td>
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<tr>
<td>6.</td>
<td>∆TFP</td>
<td>2690</td>
<td>0.008</td>
<td>0.008</td>
<td>0.028</td>
</tr>
<tr>
<td>7.</td>
<td>∆Y/L</td>
<td>2890</td>
<td>0.023</td>
<td>0.022</td>
<td>0.036</td>
</tr>
<tr>
<td>8.</td>
<td>∆P_{housing}</td>
<td>1822</td>
<td>0.022</td>
<td>0.019</td>
<td>0.092</td>
</tr>
<tr>
<td>9.</td>
<td>\pi_{GDP deflator}</td>
<td>2485</td>
<td>0.106</td>
<td>0.044</td>
<td>0.290</td>
</tr>
<tr>
<td>10.</td>
<td>NX/Y</td>
<td>2940</td>
<td>-0.033</td>
<td>-0.021</td>
<td>0.108</td>
</tr>
<tr>
<td>11.</td>
<td>∆B_{HH}</td>
<td>1655</td>
<td>0.040</td>
<td>0.028</td>
<td>0.110</td>
</tr>
<tr>
<td>12.</td>
<td>∆B_{F}</td>
<td>1637</td>
<td>0.017</td>
<td>0.016</td>
<td>0.094</td>
</tr>
</tbody>
</table>

Credit-to-GDP gaps:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.</td>
<td>\hat{B}_{HH}</td>
<td>1608</td>
<td>0.029</td>
<td>0.009</td>
<td>0.162</td>
</tr>
<tr>
<td>14.</td>
<td>\hat{B}_{F}</td>
<td>1590</td>
<td>0.020</td>
<td>0.007</td>
<td>0.124</td>
</tr>
</tbody>
</table>

Conditional on the housing market crash at \( t \), (\( \mathbb{1}_{t\text{,crash}} = 1 \)):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.</td>
<td>∆<em>{3}P</em>{housing,t+3}</td>
<td>63</td>
<td>-0.242</td>
<td>-0.216</td>
<td>0.106</td>
</tr>
<tr>
<td>16.</td>
<td>\hat{B}_{t,HH}</td>
<td>50</td>
<td>3.600</td>
<td>3.955</td>
<td>4.381</td>
</tr>
<tr>
<td>17.</td>
<td>\hat{B}_{t,F}</td>
<td>50</td>
<td>5.821</td>
<td>5.472</td>
<td>8.572</td>
</tr>
</tbody>
</table>

Note: unbalanced panel of 50 countries, 1950-2018. (1) Real GDP per capita; (2) Real consumption expenditure per capita; (3) Real investment per capita; (4) Employment-to-population ratio; (5) Real capital stock per-capita; (6) TFP index at constant national prices; (7) Real output per worker; (8) National real housing price index; (9) GDP-deflator inflation rate; (10) Net exports to GDP ratio;

(11) Household debt, loans and debt securities, % of GDP; (12) Nonfinancial corporate debt, loans and debt securities, % of GDP. Where applicable, the IMF Global Debt Database data is extended for the earlier years by the banking lending data from the Jordà-Schularick-Taylor Macrohistory Database. (13) Household debt-to-GDP gap; (14) Firm debt-to-GDP gap;

Conditional on the housing market crash at \( t \): (15) Real housing price index decline in the first 3 years from the peak; (16) Household debt-to-GDP gap at the peak of the housing market cycle; (17) Firm debt-to-GDP gap at the peak of the housing market cycle.
Figure 18: Housing market boom-and-bust cycles definition [cited on page 9]

Note: definition similar to Jordà et al. (2015). Blue shaded areas correspond to a 1 st. dev. bound around the one-sided HP trend (smoothing parameter = 400000/4).

Housing market crashes are defined as periods when (1) the aggregate housing price index deviates from the long-run trend by more than one standard deviation (marked by gray shaded areas) and (2) exhibit the price decline of at least 10% within the first three years from the peak.
Figure 19: U.S. household and non-financial corporate debt to GDP ratios [cited on page 12]

Note: a debt-to-GDP gap is defined as a deviation of the debt-to-GDP ratio from the the long-run HP trend (smoothing parameter of $10^5/4^3$ for annual observations). The data on household and non-financial corporate debt comes from the IMF Global Debt Database and includes both loans and debt securities.
B Additional empirical results

Figure 20:
Impulse responses to a housing market crash, baseline, additional results [cited on page 11]

Note: cross-country panel estimation, controlling for country/time fixed effects, country-level trends, and macroeconomic conditions.

Variables: (1) Trade balance to GDP; (2) Exports to GDP; (3) Imports to GDP; (4) Real exchange rate index; (5) GDP deflator index; (6) Real GDP per worker.

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 90% confidence intervals. Standard errors are clustered at the country level.
Figure 21: Impulse responses to a housing market crash, the role of household and corporate debt (pre-2007 sample) [cited on page 13]

Note: cross-country panel estimation, controlling for country-level fixed effects and macroeconomic conditions and debt-to-GDP gaps at the peak of the housing market cycle; LSDV estimator.

IRFs conditional on household and corporate debt-to-GDP gaps at the peak of a housing market cycle:
- Gray: zero household and non-financial corporate debt-to-GDP gaps
- Blue: 5% household debt-to-GDP gap and zero non-financial corporate debt-to-GDP gap.

Shaded areas correspond to 95% confidence intervals. Standard errors are clustered at the country level.
Figure 22: Impulse responses to a housing market crash, the role of household and corporate debt (no country trends) [cited on page 13]

Note: cross-country panel estimation, controlling for country-level fixed effects and macroeconomic conditions and debt-to-GDP gaps at the peak of the housing market cycle; LSDV estimator.

IRFs conditional on household and corporate debt-to-GDP gaps at the peak of a housing market cycle:
- Gray: zero household and non-financial corporate debt-to-GDP gaps
- Blue: 5% household debt-to-GDP gap and zero non-financial corporate debt-to-GDP gap.

Shaded areas correspond to 95% confidence intervals. Standard errors are clustered at the country level.
Figure 23: Impulse responses to a housing market crash, robustness [cited on page 13]

Note: cross-country panel estimation, the baseline specification (1) controls for country/time fixed effects, country-level trends, and macroeconomic conditions. Alternatives: (2) Baseline, excluding observations from 2007; (3) 4 lags of macro controls instead of 2; (4) No macro controls; (5) No macro controls and year fixed effects.

Responses are estimated by local projections and are expressed in log deviations times 100 (see details in the text). Shaded areas correspond to 90% confidence intervals of the baseline IRFs. Standard errors are clustered at the country level.
Figure 24: MSA-level evidence, first stage regressions [cited on page 14]

(a) Saiz (2010) housing supply elasticity instrument
(linear and quadratic first-stage regressions)

(b) Guren et al. (2018) regional sensitivity instrument

Note: labor productivity is defined as real GDP per worker. State and MSA output and employment data is from BEA. Housing price growth is based on the Federal Housing Finance Agency all-transactions house price indexes. Housing net worth shock is from Mian and Sufi (2014).
C Additional simulation results

Figure 25: Baseline simulation, welfare loss under different levels of debt-to-GDP and loan-to-value ratios [cited on page 30]

Note: responses to a series of negative housing preference shocks calibrated to match the empirical real house price decline. Dashed lines mark baseline values of debt-to-GDP and LTV ratios.
D Derivations

D.1 Utilization-adjusted TFP

I follow an approach from Imbs (1999) which employs a partial-equilibrium version of a model of Burnside and Eichenbaum (1996). The aggregate production function is assumed to be constant returns to scale in effective capital and labor services:

\[ Y_t = Z_t(u_t K_t)^\alpha (e_t L_t)^{1-\alpha} = Z_t u_t^\alpha e_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} \]

where \( u_t \) is capital utilization rate and \( e_t \) is labor effort. Capital utilization is endogenized by assuming that it affects capital depreciation: \( \delta_t = \delta u_t^\phi \). Firms labor is assumed to be predetermined within one period, while the labor effort \( e_t \) can be adjusted instantaneously against wage changes. The firm’s period optimization problem then can be written as follows:

\[
\max_{K_t, u_t, e_t} \left[ Z_t(u_t K_t)^\alpha (e_t L_t)^{1-\alpha} - w(e_t) L_t - (r_t + \delta u_t^\phi) K_t \right]
\]

which yields the following first-order conditions:

\[
\begin{align*}
\alpha \frac{Y_t}{K_t} &= r_t + \delta u_t^\phi \\
\alpha \frac{Y_t}{u_t} &= \delta \phi u_t^{\phi-1} K_t \\
(1-\alpha) \frac{Y_t}{e_t} &= w'(e_t)L_t
\end{align*}
\]

Combining equations (38) and (39), and that at the steady state \( u = 1 \) yields:

\[
u_t = \left( \frac{Y_t/K_t}{Y/K} \right)^{\frac{\delta}{r+\delta}} \tag{41}\]

where \( Y/K \) is the steady state capital-to-output ratio.

Turning to households, they solve the following optimization problem, which assumes that the two margins of labor enter the utility function separately:

\[
\max_{\{c_t, l_t, e_t\}_{t=0}^\infty} \mathbb{E}_t \sum_{j=0}^\infty \beta^j \left( \ln(C_t) - \frac{L_t^{1+\epsilon}}{1+\epsilon} - \frac{e_t^{1+\psi}}{1+\psi} \right) \quad \text{s.t.} \quad C_t \leq w(e_t)L_t
\]
The first-order condition with respect to labor effort is the following:

$$w'(e_t) = \frac{e_t^\psi C_t}{L_t}$$  \hspace{1cm} (42)

Combining equations (42) and (40), and assuming the steady state effort $e = 1$ yields:

$$e_t = \left( \frac{Y_t/C_t}{Y/C} \right)^\frac{1}{1+\psi}$$  \hspace{1cm} (43)

Equations (41) and (43) are used to construct measures of capital and labor utilization. The steady-state values of output, consumption, and capital are determined using a one-sided HP filter with a high smoothing parameter ($400000/4^4$). I set the parameter that governs elasticity of effort with respect to wage to the average value across OECD countries according to Imbs (1999): $\psi = 0.1$, although the results are robust to different values of this parameter. I set the two remaining parameters to $r = 0.04$ and $\delta = 0.1$, standard values in the RBC literature (annual calibration). The total utilization component of the Solow residual then equals to $u_t^{\alpha_t} e_t^{1-\alpha_t}$, where I use the time-varying labor share from Feenstra et al. (2015).

As a validation exercise, I compare the resulting changes in factor utilization for the U.S. with the widely-used measure based on Basu et al. (2006) methodology. The two series exhibit very strong correlation.

Figure 26: U.S. factor utilization

Note: annual changes in U.S. factor utilization according to (1) Imbs (1999) methodology (author’s calculations) and (2) Basu et al. (2006) methodology (annual data from https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tpf). Correlation between the two measures = 0.82
D.2 Housing market equilibrium

For illustrative purposes, I linearise equilibrium conditions of the housing market around the deterministic steady state at which the collateral constraint binds.

From borrowers and savers first order conditions with respect to their housing holdings follow housing demand functions (written in terms of stationary variables):

\[
\begin{align*}
    p_t^h &= \mathbb{E}_t \left( \Lambda_{t,T+1}^S p_{t+1}^h g_{t+1} \right) + \kappa \vartheta_t \frac{\left( h_t^S \right)^{-\alpha_h}}{\lambda_t^S} \\
    p_t^h &= \mathbb{E}_t \left( \Lambda_{t,T+1}^B p_{t+1}^h g_{t+1} \right) + \kappa \vartheta_t \frac{\left( h_t^B \right)^{-\alpha_h}}{\lambda_t^B} + \chi_m p_t^h
\end{align*}
\]

where \( p_t^h = \frac{P_t^h}{N_t^h} \) and \( \bar{\lambda}_t^h = \lambda_t^h N_t^\sigma \).

For illustrative purposes, I linearize these demands around the deterministic steady state where the collateral constraint binds using sans-serif font to denote percentage deviations from the steady state (i.e. \( x_t = \frac{x_t - x_t}{x_t} \)). Note that the housing market clearing condition \( h_t^S + h_t^B = 1 \) implies \( h_t^S = -\frac{\beta}{h_t^S} h_t^B \). The linearized demand functions then take the following form:

\[
\begin{align*}
    p_t^h &= \Lambda^S g \mathbb{E}_t (\Lambda_{t,t+1}^S + p_{t+1}^h + g_{t+1}) + \frac{\kappa}{\lambda^S p^h(h^S)^{\alpha_h}} \left( \vartheta_t - \bar{\lambda}_{t+1}^S + \epsilon_h h_t^B \right) \\
    p_t^h &= \Lambda^B g \mathbb{E}_t (\Lambda_{t,t+1}^B + p_{t+1}^h + g_{t+1}) + \frac{\kappa}{\lambda^B p^h(h^B)^{\alpha_h}} \left( \vartheta_t - \bar{\lambda}_{t+1}^B - \epsilon_h h_t^B \right) + \chi m (\chi_t + p_t^h)
\end{align*}
\]

Given other general equilibrium outcomes, these two demand curves determine deviations from the deterministic steady state of the current housing price \( p_t^h \) and its quantity held by borrower households \( h_t^B \).

Hosing demands can be further simplified as follows. For tractability, I abstract from the borrowing limit inertia assuming \( \rho_b = 0 \). Denote the gross real interest rate \( R_t = \frac{1 + R_{t-1}}{1 + \beta} \), then the linearized intertemporal optimality conditions for borrowers and savers are \( \mathbb{E}_t \Lambda_{t,t+1}^B = -\mathbb{E}_t R_{t+1} - \frac{\chi}{1 - \chi} \chi_t \) and \( \mathbb{E}_t \Lambda_{t,t+1}^S = -\mathbb{E}_t R_{t+1} \) respectively. The steady-state shadow value of the collateral constraint is \( \chi = \frac{\beta^S - \beta^B}{\beta^H} \), the savers stochastic discount factor is \( \Lambda^S = R^{-1} \), and the borrowers stochastic discount factor is \( \Lambda^B = (1 - \chi) R^{-1} \). All combined, the final expressions are:

\[
\begin{align*}
    p_t^h &= A_{h,1}^B h_t^B + A_{h,2}^S (\vartheta_t - \bar{\lambda}_{t+1}^S) + A_{h,3}^S \mathbb{E}_t (p_{t+1}^h + g_{t+1} - R_{t+1}) \\
    p_t^h &= -A_{h,1}^B h_t^B + A_{h,2}^B (\vartheta_t - \bar{\lambda}_{t+1}^B) + A_{h,3}^B \mathbb{E}_t (p_{t+1}^h + g_{t+1} - R_{t+1}) - A_{h,4}^B \chi_t
\end{align*}
\]
where the positive constants are $A_{h1}^S = \epsilon_h A_{h2}^B$, $A_{h2}^S = \frac{\kappa}{\epsilon_h p^b(h)}$, $A_{h3}^S = \frac{g}{R}$, and $A_{h4}^B = \epsilon_h A_{B1}^B$, $A_{B1}^B = \frac{\kappa}{R}$. The dynamics of the equity market that finances innovation in determined by households demand, equation (14) and the blueprint price $p_b^t$, determined by equation (25), along with the free-entry condition that equalizes the firm values and the blueprint price (entry cost): $v_t = p_b^t$.

$$v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_t + v_{t+1}) \right)$$

$$v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_t + v_{t+1}) \right)$$

$$v_t = \phi^{-1} s_t^{1-\rho}$$

To simplify derivations, abstract from adjustment costs in innovation spending ($\psi_N = 0$) and borrowing constraint inertia ($\rho_B = 0$). Using the definition of the time-varying innovators productivity $\phi_t$, equation (24), the system can be simplified as follows:

$$v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_t + v_{t+1}) \right)$$

Taking into account that $\Lambda_{t,t+1}^B = -\mathbb{E}_t R_{t+1} - \frac{\chi}{1-\chi} \xi_t$, the linearized counterparts of the above conditions are:

$$v_t = \mathbb{E}_t (A_{v1} d_{t+1} + A_{v2} v_{t+1} - R_{t+1}) - A_{v3} \chi_t$$

$$v_t = (1 - \rho) s_t$$

where $A_{v1} = \frac{d}{v+d}$, $A_{v2} = \frac{v}{v+d}$, and $A_{v3} = \frac{\chi}{1-\chi}$.

D.3 Equity market equilibrium

For illustrative purposes, I linearise equilibrium conditions of the equity market around the deterministic steady state at which the collateral constraint binds.

The dynamics of the equity market that finances innovation in determined by households demand, equation (14) and the blueprint price $p_b^t$, determined by equation (25), along with the free-entry condition that equalizes the firm values and the blueprint price (entry cost): $v_t = p_b^t$.

$$v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_t + v_{t+1}) \right)$$

$$v_t = (1 - \delta_N) \mathbb{E}_t \left( \Lambda_{t,t+1}^B (d_t + v_{t+1}) \right)$$

$$v_t = \phi^{-1} s_t^{1-\rho}$$

Taking into account that $\Lambda_{t,t+1}^B = -\mathbb{E}_t R_{t+1} - \frac{\chi}{1-\chi} \xi_t$, the linearized counterparts of the above conditions are:

$$v_t = \mathbb{E}_t (A_{v1} d_{t+1} + A_{v2} v_{t+1} - R_{t+1}) - A_{v3} \chi_t$$

$$v_t = (1 - \rho) s_t$$

where $A_{v1} = \frac{d}{v+d}$, $A_{v2} = \frac{v}{v+d}$, and $A_{v3} = \frac{\chi}{1-\chi}$.
D.4 Aggregate marginal utility of consumption growth

Start with borrower and saver Euler, equations (5) and (12), expressed in stationary (lower-case) variables. For tractability, I abstract from the borrowing limit inertia setting $\rho_B = 0$:

$$
\mathbb{E}_t \left( \beta_S (\lambda^S_{t+1}/\bar{\lambda}^S) g_{t+1}^\sigma R_{t+1} \right) = 1
$$

$$
\mathbb{E}_t \left( \beta_B (\lambda^B_{t+1}/\bar{\lambda}^B) g_{t+1}^{-\sigma} R_{t+1} \right) = 1 - \chi t
$$

Linearizing the above equilibrium conditions delivers:

$$
-\mathbb{E}_t (\tilde{\lambda}^S_{t+1} - \bar{\lambda}^S) = \mathbb{E}_t R_{t+1} - \sigma \mathbb{E}_t g_{t+1}
$$

$$
-\mathbb{E}_t (\tilde{\lambda}^B_{t+1} - \bar{\lambda}^B) = \mathbb{E}_t R_{t+1} - \sigma \mathbb{E}_t g_{t+1} + \frac{\chi}{1 - \chi} X_t
$$

Next, define the aggregate marginal utility of consumption as $\tilde{\lambda}_t = \frac{\tilde{\lambda}^S_t + \tilde{\lambda}^B_t}{2}$, which yields $\bar{\lambda} \tilde{\lambda}_t = \bar{\lambda}^S \tilde{\lambda}_t^S + \bar{\lambda}^B \tilde{\lambda}_t^B$ when linearized, to perform aggregation:

$$
\mathbb{E}_t g_{\lambda t+1} = \mathbb{E}_t (\tilde{\lambda}_{t+1} - \bar{\lambda}_t) = -\mathbb{E}_t R_{t+1} + \sigma \mathbb{E}_t g_{t+1} - A_\lambda \chi t
$$  (48)

where $A_\lambda = \frac{\tilde{\lambda}^B \chi}{\lambda (1 - \chi)}$. Note that the households utility function is not separable in consumption and labor: $\tilde{\lambda}_t^H = \left( c^H - v_t \left( \frac{(L^H)^{1+\epsilon_L}}{1+\epsilon_L} \right) \right)^{-\sigma}$, so $\bar{\lambda}_t^H = -A_{\lambda H} c^H + A_{\lambda H}^H \left( \varphi_t + (1 + \epsilon_L) L_t^H \right)$, $H \in \{S, B\}$, where $A_{\lambda H} = \sigma \left( c^H - v_t \left( \frac{(L^H)^{1+\epsilon_L}}{1+\epsilon_L} \right) \right)^{1-\sigma}$ and $A_{\lambda H}^H = v_t \left( \frac{(L^H)^{1+\epsilon_L}}{1+\epsilon_L} \right) A_{\lambda H}^H$.

D.5 Aggregate welfare function

Recall that lifetime utility of a household $H \in \{S, B\}$ is the following:

$$
\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j_t \left( U(C^S_{t+j}, L^H_{t+j}) + \kappa_t \varphi_t G(h^H_{t+j}) \right),
$$

where

$$
U(C^H_t, L^H_t) = \left( C^H_t - \gamma \frac{(L^H_t)^{1+\epsilon_L}}{1+\epsilon_L} \right)^{1-\sigma} - 1 / (1 - \sigma)
$$

$$
G(h^H_t) = \kappa_t \varphi_t \frac{(h^H_t)^{1-\epsilon_h} - 1}{1 - \epsilon_h}
$$

Period utilities can be rewritten in terms of stationary variables and the stock of knowledge that yields $U(c^H_t, L^H_t) = N_t^{1-\sigma} \left( c^H_t - v_t \left( \frac{(L^H_t)^{1+\epsilon_L}}{1+\epsilon_L} \right)^{1-\sigma} - 1 \right) / (1 - \sigma)$ and $G(h^H_t) = N_t^{1-\sigma} \kappa_t \varphi_t \frac{(h^H_t)^{1-\epsilon_h} - 1}{1 - \epsilon_h}$. 17
Lifetime utility of each household then can be expressed recursively in terms of stationary variables and growth rates:

$$W_H^t = \left( \left( c_H^t - \psi_t \frac{(L_H^t)^{1+\epsilon_L}}{1+\epsilon_L} \right)^{1-\sigma} - 1 \right) \right) / (1-\sigma) + \kappa h \frac{(h_H^t)^{1-\epsilon_h} - 1}{1-\epsilon_h} + \beta W_H^t \delta_{t+1}^{1-\sigma} \tag{49}$$

Finally, the aggregate welfare then is the weighted sum of welfare of savers and borrowers $W_t = \gamma_B W_B^t + \gamma_S W_S^t$. The baseline case assumes that each of the types of households is of the same mass: $\gamma_B = \gamma_S = 0.5$. I use the second-order approximation of the model to calculate lifetime utility of households under different policy scenarios.