Detecting Quality Manipulation Corruption in Scoring Auctions: A Structural Approach*

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Abstract

In a procurement scoring auction, bids have multiple attributes. The buyer may lack the expertise to evaluate quality of these bids and have to hire a procurement agency to help. Corruption via quality manipulation arises when the procurement agency is bribed to misreport the true quality of a corrupted firm. This causes systematic distortion of bids and such distortion is testable. We propose three tests to detect corruption using a based on structural estimation within a scoring auction framework. They are applied to a server room procurement auction data set.

Keywords: Scoring Auction, Quality Manipulation Corruption, Structural Estimation, Corruption Detection.

JEL codes: C1, D44, H57, L40, L74.

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1 Introduction

This paper serves two purposes: it develops a structural estimation method empirical scoring auction data set and propose three tests of quality manipulation corruption.

In a real world procurement auction, the target item is typically of differential quality with multi-dimensional attributes. For example, in procurement of a construction plan, its design, material, equipment, delivery date, safety, service, and maintenance are important attributes that need to be written in the contract at the moment of transaction. In this environment, a *bid* consists of both price and many other non-price attributes (*quality*, henceforth). A winning bid turns into a contract or a guideline of writing a detailed contract. Table 12 and Figure 15 show that a bid in a multi-attribute auction is fairly complicated and difficult to evaluate. Literature starts extending the empirical analysis of auction data to multi-attribute auction has been growing (see a review in Section 1.1). If the auction has a pre-announced scoring rule specifying how bids are evaluated, it is called a *scoring auction*. If the buyer does not specifically announce how the winner is selected, it is called a *beauty contest* or a *design-build auction*. In this paper, we will only study scoring auctions. We assume the scoring rule and relevant scores of each bid are observable to researchers.

When quality is involved in an auction, the subjectivity and complexity of quality evaluation bring in new challenges in designing and conducting procurement auctions. Figure 1 shows three roles in our analysis: the buyer (she), the procurement agency (he), and a group of supplying firms (them/it) . In almost all public procurements, the *buyer* is not an expert in the industry and lack the expertise to evaluate quality of submitted bids, so she will hire some mediators between her and supplying firms. These mediators may include the buyer's representatives with industrial experiences (street-level bureaucrat), professional procurement agency company, auctioneer (house), committee of industrial experts. In this paper, we abstract multiple layers of mediators into one layer, called the *procurement agency*. Because the procurement agency is given some discretion in evaluating quality, he can exert it to seek bribery from a favored firm. If corruption indeed occurs, then the quality evaluation report is manipulated. Hence, we call this problem *quality manipulation corruption*. It is an intrinsic problem in the procurement process when quality is both important and requires special expertise to evaluate.

Quality manipulation corruption is a prominent issue in procurement both in public and private sector, especially in developing countries. Existing studies of corruption in auctions focus on bidding rings among bidders or cheatings between bidder and auctioneer (defined later in Section 1.2). Bidding rings and cheatings suppress competition and causes monetary loss for the buyer. However, corruption leads to another perceivable consequence: inferiority of quality ("jerry-built" projects). Take the bridge construction as example: Ji and Fu (2010) find that there are a total of 85 major bridge collapse accidents[1] in China between 2000 and 2009. 40 cases among them are caused by

[1] Causes at least one death or over one million CNY economic loss. The number does not include collapses caused by earthquake.

Figure 1: The Agency Structure of Procurement Auction

Buyer (she)	sets scoring rule
$\Downarrow$	
Procurement Agency (he): Street-level bureaucrat / Auctioneer/house / Committee of experts	evaluates bids
$\Downarrow$	
Firms (sellers, they/it)	submit bids

frauds in planning and construction. In a 2012 media report titled “Chinese-style of bridge collapse”,[2] government officials and industry experts conclude three major reasons: (i) construction plan fails to meet industrial regulation, (ii) construction is of low quality, and (iii) lack of necessary maintenance. No buyer will buy a bridge if she knows it will collapse in the near future. The discrepancy between quality written on the winning bid and quality actually delivered is usually caused by corruption in the procurement auction process.

The theoretical model in this paper is based upon existing framework of scoring auctions, mainly developed by Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). Our contribution is three-folds:

(1) We show that equilibrium costs and social surplus of each firm can be nonparametrically identified and structurally estimated from a quasilinear scoring auction data set. Our method does not a require parametric cost function because we consider *pseudotypes* (Asker and Cantillon (2008), defined later in this paper). Our method holds under multi-dimensionality of quality and private information environments.

(2) We introduce quality manipulation corruption into the scoring auction model and characterize the systematic distortion of bidding behaviors. These model implications and structural estimation method are put together to construct three tests of quality manipulation corruption. These tests are based on usual scoring auction record data and do not require rich firm covariates and identities of corrupted firms.

(3) The structural estimation method and corruption detection tests are applied to a scoring auction data set of server room procurement.[3] The data and empirical results provide several pieces of empirical evidence supporting the theoretical model. We show that projects with a high quality weight result in higher payoffs (rents) for both buyer and firms, but is also subject to a higher risk of corruption. Corruption is also more likely to happen at projects with higher engineer’s estimated

[2] http://club.kdnet.net/dispbbs.asp?page=1&boardid=89&id=8581905

[3] Server room is an indoor place designed to contain machines of data storage, servers, and large computers.

costs.

1.1 Literature Review of Scoring Auction

When the object of procurement is of differential quality, a scoring auction is commonly used. Its advantage is proven both by theory and its popularity. In practice, each bidder is asked to submit one bid that combines price and quality (attributes). The contract is awarded to the bidder that receives the highest score based on a pre-announced scoring rule. The seminal paper by Che (1993) provides the framework for a scoring auction. He derives the equilibrium of scoring auction under quasilinear scoring rule and shows that firm's quality and price choice can be separated under certain conditions. He shows that both first-score auctions (FSA) and second-score auctions (SSA) implement the optimal mechanism and yield the same expected utility to the buyer.

Asker and Cantillon (2008) introduce multi-dimensionality of private information and quality attributes to Che's model. They characterize the equilibrium and expected score equivalence of FSA and SSA. In addition, they show that a scoring auction with a quasilinear scoring rule dominates other alternatives including beauty contests[4], menu auctions[5], and price-only auctions with a minimum quality threshold. In Branco (1997), costs of different firms have a common component and thus are correlated. In this case, an optimal contract cannot be implemented by first or second-score auctions, but instead requires a two stage auction: first select a firm through an auction, then readjust the level of quality via bilateral bargaining. David et al. (2006) and Chen-Ritzo et al. (2005) provide experimental evidence indicating that scoring auctions dominate traditional price-only ones. Wang and Liu (2014), Dastidar (2014), and Hanazono et al. (2015) extend the model to a non-quasilinear scoring rule environment. Among these papers, Hanazono et al. (2015) consider the most general setting that includes price-quality ratio, fixed price best proposal, and a convex scoring rule. They characterize the equilibrium of FSA and SSA and show that their expected score rankings depends on the curvature of the induced utility of firms.

In general, it is difficult to characterize the optimal mechanism and scoring auction implementation when the environment is complicated. David et al. (2006) characterize an optimal scoring rule within the class of weighted criteria rules with restriction of additively separability of attributes on both value and cost. Asker and Cantillon (2010) find the optimal mechanism in a specific environment where firm types are two binary random variables. They show that a scoring auction yields a performance closed to that optimal mechanism numerically. Nishimura (2015) show that implementation of the optimal mechanism via a scoring rule requires substantial cost complementarity between quality attributes. In other words, the widely used linear weighted scoring rule is suboptimal because there is not enough complementarity between attributes to provide the correct incentive.

[4] In a beauty contest or design built auction, there is no pre-announced scoring rule. The buyer does not select winner solely on price, but also on their submitted quality attributes and observed characteristics like reputation.

[5] In a menu auction, bidders are allow to submit multiple price quality combination bids (instead of only one in usual scoring auction). The buyer will then determine the winner and the item on its menu.

Concerning quality manipulation corruption, Celentani and Ganuza (2002) introduce an endogenous corruption relation forming process based on the scoring auction model of Che (1993). They allow the corrupted firm to win for sure once the procurement agency accept a bribe. Their model focuses on the formation of a corruption side contract and show how an increase competition may not reduce corruption. Burguet and Che (2004) consider a Bertrand-style environment of two firms with complete information and endogenous corruption relation. Two firms involve in both bribery competition and market competition. Because the weaker firm can spend all its resources on one side of market competition and bribery competition, the efficient firm cannot guarantee winning the contract. Huang and Xia (2015) consider a similar environment with exogenous corruption relation and focus on the buyer's optimal scoring rule under corruption. The scoring rule affects the relative magnitude of technological advantage and corruption advantage, which determines the auction outcome. In such an environment, the dominance of a scoring auction as shown in Asker and Cantillon (2008) disappears. A price-only auction with a mandatory minimum quality could be better in some cases.

There is a growing literature on the empirical analysis of scoring auction data.[6] The strategy space of each bidder is expanded from one-dimensional price to multi-dimensional quality attributes, so scoring auction data can potentially answer a richer set of questions. Lewis and Bajari (2011) explore a highway contract data set from California generated from an "A+B auction", where bids are evaluated on both price and time of delivery. They show that by introducing time incentive, the overall gain in social welfare is significant. Bajari et al. (2014) analyze another highway contract data set where each bid consists of a list of unit prices. These unit prices are multiplied by quantities estimated by engineers to determine which bid has the lowest cost. Their analysis focuses on the *ex post* adjustment of final payment and how firms strategically reflect potential adaption costs in their bids. Krasnokutskaya et al. (2011) study data from online programming service market. They provide an identification and estimation strategy for data that features both auction and differential product discrete choice. Koning and Van de Meerendonk (2014) study data from service provider procurement auction under weighted scoring rule. They explore how variation of weights on different components affect bids and the procurement outcome.

Nakabayashi and Hirose (2015) study a similar scoring auction data set similar to the one in this paper. They provide identification and structural estimation results based on the theoretical model of Hanazono et al. (2015) under a general scoring rule. They assume a parametric cost function that is common knowledge except for L parameters as bidder's private information. The identification is based on an invertibility condition of the set of best response functions that define the equilibrium, which itself depends on the parametric assumption. In our analysis, we consider only the class of quasilinear scoring rules, but our whole analysis is nonparametric.

[6] Takahashi (2014), Yoganarasimhan (2013), and Yoganarasimhan (2015) study data of beauty contest auction. We won't go into detail in this paper.

1.2 Detecting Corruption in Auction

In the *Handbook of Procurement* (edited by (Dimitri et al., 2006)), Lengwiler and Wolfstetter (2006) point out procurement auction participants may suppress competition by four major forms of collusion or corruption. In the literature, *collusion* usually refers to a *bidding ring* or a *cartel*, where a group of bidders coordinate their bids to increase price. In an efficient cartel, the *cartel leader* (the one with lowest cost or winners of an internal per-auction knockout) compete against other bidders, while the other cartel members submit *phantom bids*, which are high bids that will not compete against the cartel leader. There is a body of literature on bidding rings both theoretically (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)) and empirically (e.g. Pesendorfer (2000), Bajari and Ye (2003), and Asker (2010)). Unlike collusion among bidders, *corruption* refers to the auctioneer (who runs the auction) twisting the auction rule in exchange for bribes. It can take three major forms: (i) *bid rigging* (*bid revision* or "*magic number" cheating*), meaning that the auctioneer allows a favored bidder to adjust his bid after receiving information about rival bids (e.g. Compte et al. (2005) and Burguet and Perry (2009)). (ii) *Bid orchestration*, meaning that the auctioneer serves as a "ring manager" of a collusive cartel and coordinates their bids. (iii) *Distortion of quality ranking,* which is called *quality manipulation corruption* in this paper, meaning that the bid evaluation committee is bribed to submit biased quality scores. (e.g. Celentani and Ganuza (2002) and Burguet and Che (2004)).

In this section, we briefly review existing empirical works on corruption in auctions and its detection. We focus on a relative small number of papers to sketch their key insights. For a more comprehensive reviews including the theoretical side literature, readers can consult other surveys, for example Harrington (2008) and John Asker's note.

Porter and Zona (1993) is one of the earliest works on collusion detection. They study bid rigging in procurement auctions of Long Island highway construction contracts. Because some bidders are of relative large size and interact with each other in sequence of auctions, they are able to coordinate as a cartel. They estimate parameters of a linear bidding function and a logistic bid ranking model. Because the model can be estimated from using either the whole sample or only winning bids, two sets of parameter estimate shall be equal in a competitive environment. But when there is bid rigging, the ranking of bids will not fully reflect the economic factors of bidders, leading to different estimates.

Colluding bidders' bidding behavior can be studied with reduced-form models when detailed data of cartel members identities and characteristics are available from legal investigation by antitrust authorities. Porter and Zona (1999) analyze data from school milk contract auctions in Ohio, where a group of firms in Cincinnati were convicted for colluding. The bidding behavior of cartel members is compared to a controlled group. They show that collusion raised market prices by 6.5% on average. Pesendorfer (2000) also analyzes data from school milk contract auctions, where some firms in Florida and Texas were found colluding. He considers the effect of both bid rigging and market spiting. He estimates the coefficients of a reduced-form bidding function regression using three sub-samples: low cartel bids plus all non-cartel bids, low cartel bids, and all non-cartel bids. A Chow test for equality of coefficients is applied to show that the cartel firms bid less aggressively than non-cartel firms. Feinstein et al. (1985) point out that a cartel may seek not only a higher winning bid, but also collectively use bids to pass false information to the buyer to avoid a "ratchet effect".[7] It happens when the buyer uses past information to form an expectation and estimation for future auctions. Feinstein et al. (1985) found empirical evidence by data of collusion in the construction of North Carolina highways.

However, if the data does not provide exact identities of the cartel and non-cartel bidders, then the method above cannot be implemented (unless one runs regression on each possible partitions of the cartel and non-cartel bidders). In addition, the data may not be rich in bidder's characteristics. Harrington (2008) points out that an abnormally high profit margin is not evidence of collusion, but evidence of market power. According to Baldwin et al. (1997), there are three (non-mutually exclusive) ways to explain a high profit margin: collusion, demand side factors, and supply side factors. The supply side can be captured by auction-specific covariates describing the object or contract for sale. To identify collusion, researchers also need to control demand side factors by bidder-specific covariates. To encounter these data limitations, researchers start using structural model to detect collusion.

Bajari and Ye (2003) construct their test based on two key different model implications by competition and collusion model: conditional independence and exchangeability of bids. If bidders are competitive, bids must be independent controlling for all information on costs that are publicly observed (under IPV framework). But if there is a cartel, their bids may be correlated and such correlation can be detected. Moreover, a competitive bidder's bid depends on other bidders' economic factors but not their identities, so exchanging other bidders' characteristics shall not change the distribution of competitive bidder's bid. In a regression specification, if one regresses bidder i's bid on the covariates of bidder j and k (with other controls), then these two coefficients should be equal. An F-test can be used to check this exchangeability restriction. Identities of potential cartel members can be found by testing each pair of bidders. In addition, Bayesian estimation of a structural model provides exact likelihood of the data coming a collusion model.

Aryal and Gabrielli (2013) take a full structural approach to test collusion based on an estimation method in Guerre et al. (2000). For the same set of bids data, two sets of costs are structurally estimated from a competitive model and a collusion model, denoted as $\left\{\hat{c}^A\right\}$ and $\left\{\hat{c}^B\right\}$ respectively. Because collusion lowers competition, for the same bid b, it implies $c^A(b) \geq c^B(b)$. Detecting collusion boils down to testing for first-order stochastic dominance of two cost distributions recovered from two models.

Besides bidding ring, Ingraham (2005) studies the corruption between auctioneer and bidder. His model is based on a bid revision model in Compte et al. (2005). The auctioneer let the corrupted firm observe others' bids before submitted its. When the corrupted firm's cost is lower than the

[7] see Freixas et al. (1985)

lowest bid of other firms, it will submit a bid that barely wins the contract. As a result, the difference between the lowest and second lowest bid is smaller than a usual competitive sample. This is a testable implication

All works mentioned above are based on first-price sealed-bid auction. Collusion can be a more prominent problem in open auctions where tacit collusion is easier. Athey et al. (2011) study a timber auction data set with two auction formats (sealed-bid and open) and two sets of bidders (mills and loggers). They assume mills are potential cartel and use the sealed-bid auction as benchmark to evaluate the competitiveness of behaviors under the open auction. Bajari and Yeo (2009) studies collusion in FCC spectrum auction and Klemperer (2002) in telecoms license auction. Marmer et al. (2014) recently provide tool to identify collusion in English auction.

Some other empirical works are based on some unique feature of their data set. Asker and Cantillon (2010) study internal knockout auction data from a cartel of stamp dealers. They test the theory of internal organization of bidding rings and measure ring members' benefit from colluding. Tran (2009) uses internal bribery data of a company to compare corruption under different auction format. Kawai and Nakabayashi (2014) study an auction data set from Japanese government procurements. Because the reserve price is secret, observation of bids may consist of several rounds and the ranking of bidders across rounds can be used to detect collusion.

In summary, to detect collusion, one needs to derive key features that are unique to a competition model and a collusion model, and then test which model the data supports. Hence, all these collusion detection methods suffer from some common problems:

(i) When the null hypothesis of a competitive model is rejected, it is hard to distinguish whether the deviation comes from collusion, market power, or just misspecification of the model (See Figure 8).

(ii) If corrupted bidders coordinate their bids in a sophisticated way, the recorded bids can pass nearly all these tests. See detail in Harrington (2008), section "Beating a test of collusion."

(iii) Nearly all these tests rely on repeated observations of bids from the set of potential corrupted bidders. Dynamic interaction between bidders are very informative of whether they are competitive or colluding. One one implicit assumption made in most papers is that the identities of cartel and non-cartel members are prespecified and do not change across auctions.

Our tests suffer from problem (i) as others, but suffer less from problem (ii) and (iii). The quality manipulation problem usually only happens to one bidder. If the procurement agency and corrupted firm wants to avoid being detected, they must reduce the scope of corruption. Therefore, beating our tests will directly restrict the corruption. Besides that, our tests are also useful for antitrust authorities because they requires only standard auction data. In particular, we don't need a prespecified set of suspicious corrupted bidders, identities of bidders, and repeated bidding behaviors of bidders across auctions. Our tests can be perform with very little or even no bidder-specific covariates. At the end of introduction, we want to point out that all these collusion detection methods are complements rather than substitutes to case by case investigation.

The rest of the paper is organized as follows. We present the theoretical model of scoring auction and quality manipulation corruption in Section 2. We show the identification and structural estimation of scoring auction model in Section 3.1. Section 3.2 provides three corruption detection tests and a Monte Carlo experiment. In Section 4, we apply the estimation and collusion detection method to a server procurement auction data set. Section 5 concludes.

2 Theoretical Model

A buyer (she) seeks procurement of a project which can be delivered at various level of quality $q \in \mathbb{R}_+^L$. q can be a single-dimensional quality index or multi-dimensional quality attributes. Before the auction, the buyer announces a scoring rule $S(p,q) : \mathbb{R}_+^{L+1} \to \mathbb{R}$. Suppose there are n symmetric risk neutral firms (they/it) that enter the scoring auction (exogenously), indexed by $i = 1, 2, ..., n$. Provided the scoring rule, each firm submits its sealed bid as a combination of quality and price, i.e. (p_i, q_i). These n bids are then evaluated according to $S(p,q)$ and the firm with the highest score wins the contract. It delivers the project at quality q and is compensated by p. We only consider first-score auctions (FSA) and independent private information framework in this paper. The quality manipulation issue kicks in when q is not directly observable by the buyer, and she has to hire a *procurement agency* (he) to evaluate quality score of bids.

A generic firm i's type (private information) is an (vector of) efficiency parameter θ, drawn independently from an identical distribution F. F is absolutely continuous and it has density $f = F'$ with support $[\underline{\theta}, \overline{\theta}]$. Firm i with type θ_i pays a cost $C(q, \theta_i)$ if it delivers the project with quality q. If the firm wins the contract with bid (p,q), its payoff is $\pi(p,q;\theta_i) = p - C(q,\theta_i)$. Firm's payoff is normalized to zero if it does not win the contract. We assume the cost function satisfies the following assumption:

Assumption CF (Cost function): $C(q,\theta)$ is continuous in q. For any q, $C(q,\theta) > 0$, $C_q(q,\theta) > 0$, $C_{qq}(q,\theta) > 0$.[8]

Two remarks on the model setup:

(1) In this paper, we will put aside a buyer's optimal scoring rule design problem and simply and treat $S(p,q)$ as her objective function. We focus on a firm's equilibrium bidding behavior and the issue of quality manipulation corruption with the goal of conducting an empirical study on bids data. As we can only observe the score but not buyer's "payoff", it is nearly impossible to infer a buyer's true preference from an empirical point of view. In addition, Che (1993) shows that if the buyer lacks commitment power, the only feasible scoring rule is one that reflects the her preference.

(2) Concerning the dimensionality issue, the dimension of quality attributes can be reduced to one under certain restriction, which we will show in Lemma 1. This justifies that we do not need to

[8]Compared to previous literature, we relax the assumption on the sign of C_θ and $C_{q\theta}$.

dig into a detailed record of quality attributes, but can use quality index data for our an empirical analysis. For private information, we allow for multi-dimensionality. In Che (1993), the equilibrium of scoring auction implies the most efficient firm wins the contract by submitting the highest quality in the equilibrium. By allowing private information be (at least) two dimensional, this undesirable property disappears. See details below.

2.1 Equilibrium under Quasilinear Scoring Rule

To pave the way for analyzing quality manipulation and empirical study, we review the equilibrium of a scoring auction under a quasilinear (QL) scoring rule. More details can be found in Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). Consider the scoring rule satisfies the following restriction:

Assumption QL (quasi-linear scoring rule): The scoring auction uses a quasilinear scoring rule $S(p,q) = V(q) - p$. $V(q)$ is increasing, continuously differentiable and weakly concave.

We restrict our attention to QL scoring rule for exposition clarity and empirical application purpose. One commonly used scoring rule is a linear weighted summation of factors, which can be transformed into QL class.[9] With assumption CF and QL, we have the following lemma:

Lemma 1: *Consider a quasilinear scoring rule $S(p,q) = V(q) - p$, with $q \in \mathbb{R}_+^L$ and $L > 1$. The cost function of L quality attributes $C(q,\theta)$ can be transformed into a function of one dimensional quality index $C(v,\theta)$, $v \in \mathbb{R}$, which is also continuous, strictly increasing and convex in v.*

Lemma 1 shows that the problem of L-dimensional quality attributes can be reduced to one-dimensional quality index without loss of generality. For the rest theoretical analysis, we will take q to be one-dimensional. With a pre-announced scoring rule $S(p,q)$, the firm's problem is to select a price-quality combination to maximize its expected payoff:

$$\max_{p,q} \left[p - C(q,\theta)\right] \Pr\left(\text{win}|S(p,q)\right). \tag{1}$$

We start describing the equilibrium bidding strategy by the following result in Che (1993).

Lemma 2: *Under assumption CF and QL, when θ is one dimension and $C_\theta > 0$, $C_{q\theta} > 0$, there is a symmetric Bayesian Nash equilibrium of a first-score auction where each firm with type θ submit its bid as*

$$q(\theta) = \arg\max_q V(q) - C(q,\theta), \tag{2}$$

$$p(\theta) = C(q(\theta),\theta) + \int_\theta^{\overline{\theta}} C_\theta(q(t),t)\frac{[1-F(t)]^{n-1}}{[1-F(\theta)]^{n-1}}dt. \tag{3}$$

The key finding of this equilibrium is that quality choice $q(\theta)$ is separated from price choice and

[9] However, a quality-price ratio scoring rule cannot be transformed into an equivalent QL rule, see Hanazono et al. (2015) for an analysis.

each firm will choose the quality that maximizes social surplus.[10] However, $q(\theta)$ is not suitable for direct empirical application because of its monotonicity property. The existence of the equilibrium requires the assumption $C_{q\theta} > 0$. Topkis (1978) theorem immediately implies that $q'(\theta) < 0$.[11] It means that the most efficient firm with lowest θ wins by submitting the highest quality. This prediction obviously does not fit real world data because some contracts are won by low quality but cheap firm. If we drop assumption $C_{q\theta} > 0$ and allow θ to be at least two dimensional, the problem disappears (see Example 1).

Under multi-dimensional private information, assumption CF and QL implies quality choice following (2), and $q(\theta)$ is a single-valued continuous function by Berge's Maximum Theorem. The firm's problem (1) is equivalent to a two-step optimization problem where the firm first chooses score s, then choose a p, q combination to fulfill that score. Because $p = V(q) - S(p,q)$,

$$\begin{aligned} (1) \quad &\Leftrightarrow \quad \max_s \left\{ \max_{(p,q) \text{ s.t. } S(p,q)=s} [p - C(q,\theta)] \Pr(\text{win}|s) \right\} \\ &\Leftrightarrow \quad \max_s \left\{ \max_q [V(q) - s - C(q,\theta)] \Pr(\text{win}|s) \right\} \\ \text{plug in (2)} \quad &\Leftrightarrow \quad \max_s \{ [V(q(\theta)) - C(q(\theta),\theta) - s] \Pr(\text{win}|s) \}. \end{aligned} \tag{4}$$

Following Asker and Cantillon (2008), we define the *pseudotype*[12] of a firm as the value function

$$K(\theta) \quad \equiv \quad \max_q V(q) - C(q,\theta) = V(q(\theta)) - C(q(\theta),\theta). \tag{5}$$

Again by Berge's Maximum Theorem, $K(\theta)$ is a single-valued continuous function. The distribution of pseudotype K can be obtained from the (joint) distribution of θ by the transformation formula:

$$F_K(k) = \Pr(K(\theta) \leq k) = \Pr(\theta \in D_{\{\theta:\, K(\theta) \leq k\}}) = \int_{\theta \in D} f(\theta) d\theta. \tag{6}$$

Denote $\underline{k} = \min_{\theta \in [\underline{\theta},\overline{\theta}]} \{K(\theta), 0\}$ and $\overline{k} = \max_{\theta \in [\underline{\theta},\overline{\theta}]} \{K(\theta), 0\}$ and we assume the least efficient firm participates, i.e. $\underline{k} \geq 0$. The support of pseudotype is $[\underline{k}, \overline{k}]$. According to Asker and Cantillon (2008), pseudotypes are sufficient statistics to describe the equilibrium of scoring auction (under QL). Instead of drawing multi-dimensional type θ from a joint distribution F, firms can draw their one-dimensional type k from distribution F_K. Problem (4) can then be rewritten as if the firm is

[10] See Hanazono et al. (2015) for detailed analysis when quality and price choice are not separable.

[11] $q(\theta)$ satisfies FOC $V_q(q) - C_q(q,\theta) = 0$. By implicit function theorem, $q'(\theta) = C_{q\theta}/(V_{qq} - C_{qq})$. Assumption CF and QL require $V_{qq} \leq 0$ and $C_{qq} > 0$, hence adding assumption $C_{q\theta} > 0$ implies that quality choice is monotonically decreasing in θ.

[12] It is called effective cost in Hanazono et al. (2015) and productive potential in Che (1993).

selecting its score based on its pseudotype:

$$\max_s (k - s)\Pr(\text{win}|s). \tag{7}$$

Theorem 1: *Every equilibrium in the scoring auction is type-wise outcome equivalent to an equilibrium in the scoring auction where suppliers are constrained to bid only on the basis of their pseudotypes. Firm with type θ (pseudotype $k = K(\theta)$) bids quality according to (2) and score*

$$s(k) = k - \frac{\int_{\underline{k}}^{k} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}. \tag{8}$$

The relevant price is $p(\theta) = V(q(\theta)) - s(K(\theta))$.

Throughout the paper, we use $X_{(j:n)}$ to denote the jth *highest* order statistic from an i.i.d. sample of size n from distribution F_X. The distribution function of order statistic $X_{(j:n)}$ is denoted as $F_X^{(j:n)}$.[13] We have the following Corollary similar to the revenue equivalence theorem in Vickrey (1961) and Myerson (1981):

Corollary 1: *The conditional expectation of winner's score equals to the highest rival's pseudotype, i.e., $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$.*

In summary, the bidding behavior of the competitive scoring auction model has three implications that can be tested empirically. First, competition among firms are mainly on qualities. Efficient firms usually submit bid with high quality and high price to win the contract. Second, a higher slope of $V(\cdot)$ induces higher quality according to (2). Lastly, separation of quality and score choice implies that the number of bidders shall not affect choice of quality, but affect choice of score.

2.2 Quality Manipulation Corruption

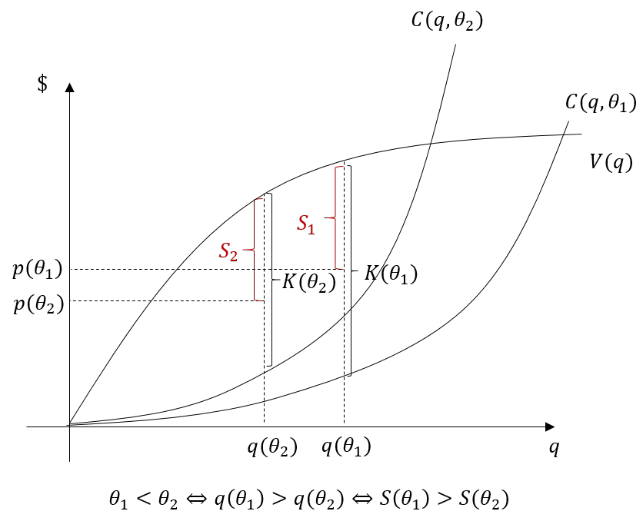
Aforementioned, complexity and subjectivity features of quality evaluation in scoring auction brings in the intrinsic agency problem of quality manipulation corruption. Assume that the procurement agency (randomly) matches with one firm and forms a *corruption relation.* This relation can be the result of a long term relationship, favoritism due to some exogenous reason,[14] bribery side-contract, or other reasons. We assume that the corrupted firm's quality score is raised by some positive number m, which is a similar setting in Burguet and Che (2004). It means that if the firm submits a bid (p, q), the score is changed to $S(p, q + m)$ instead of $S(p, q)$. The interpretation of m can be (i) the quality score of corrupted firm is raised, (ii) its actual delivered quality is not as high, or (iii) the auction rule is biased to give an advantage to the corrupted firm.

The discretion (or error allowance) given to the procurement agency determines the magnitude of m. Indeed, this discretionary power is restricted by specific properties of the industry, if the

[13] David and Nagaraja (2007) and Arnold et al. (1992) are two major references we used for order statistics in this paper.

[14] For example, in Branco (1994), a domestic firm is favored.

Figure 2: Illustration of the Equilibrium with Corruption



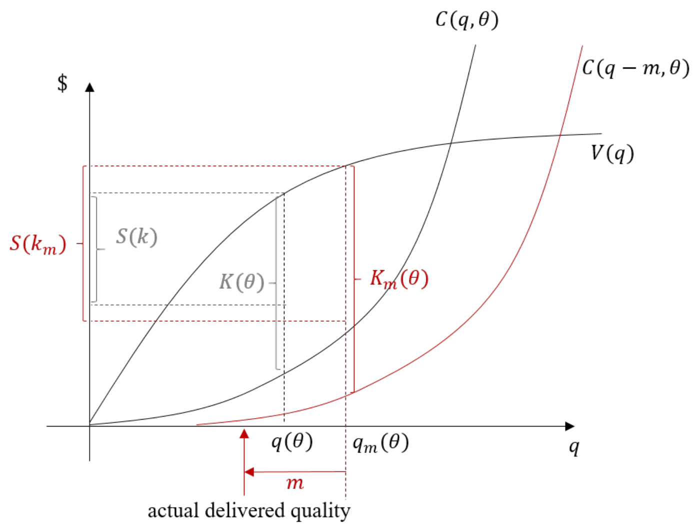
procurement agency doesn't want to trigger investigation. For example, in the procurement of a bridge, the procurement agency may claim that the corrupted firm's bridge can serve 30 years while the actual building code is designed for 25 years. However, he will not say the bridge will last over 100 years because it would look suspicious. Hence m is a number measuring *the scope of corruption*.

The timeline of the game is now as follows. The buyer announces a scoring rule and hires the procurement agency. A group of firms enter the auction and draw their private information θ from F. The procurement agency then randomly matches with one firm and offers him to raise his quality score by m in exchange for a bribe. The firm decides whether to accept this offer or not. Then all firm submit a sealed-bid simultaneously as a price-quality combination. If the matched firm accept the offer, his quality will be raised by m. The auction outcome is then revealed and the firm with highest score wins the contract.

We skip a detailed model of the endogenous formation process of the corruption relation. We assume the procurement agency is an expert in this industry and is able to design a bribery side contract that the matched firm will accept. For example, if the procurement agency knows θ of the matched firm, he can make a take-it-or-leave-it offer, asking for a bribery slightly less than the difference between the expected payoff of being corrupted and not. Our simple model is enough from an empirical point of view, because variables directly related to corruption are usually unobservable in most data sets (e.g. bribery side payment, identities of corrupted firms, amount of quality distortion). Writing a more complicated model of quality manipulation corruption usually ends up with the same qualitative prediction. We further impose an assumption on other firms' knowledge about the essence of corruption relation.

Assumption UA: The buyer and the other uncorrupted firms are unaware of the existence of corruption. Because there is incomplete information on costs, adding another layer of incomplete information brings in mixed strategies and the equilibrium become both complicated and uninfor-

Figure 3: Illustration of the Equilibrium with Corruption



mative (see Huang and Xia (2015)). Assumption UA is widely used in auction collusion literature, both in the analysis of bidding rings (e.g. Porter and Zona (1993), and Aryal and Gabrielli (2013)) and bid revision corruption (e.g. Burguet and Perry (2009)). An alternative way to circumvent the problem is assuming complete information on the presence of collusion. For example, most bidding ring literature assume both cartel members and non-cartel members know identities of colluding firms (e.g. McAfee and McMillan (1992), Bajari and Ye (2003), and Athey et al. (2011)). The bidders then have two types and the auction is asymmetric with a type specific bidding function, but the qualitative prediction of assuming complete information is similar.

Given assumption UA, all uncorrupted firms follow the same strategy as in Theorem 1. The corrupted firm, once matched with the procurement agency, solves a modified problem:

$$\max_{p,q} \left[p - C(q-m,\theta)\right] \Pr\left(\text{win}|S(q,p)\right).$$

The equilibrium bidding strategy is summarized as the following theorem.

Theorem 2: *Under QL scoring rule, the corrupted firm bids according to*

$$q_m(\theta) = \arg\max_q V(q) - C(q-m,\theta), \tag{9}$$

$$p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)), \tag{10}$$

where $k_m = K_m(\theta) \equiv \max_q V(q) - C(q-m,\theta)$ *is the corrupted firm's pseudotype and* $s(k_m) = k_m - \frac{\int_{\underline{k}}^{k_m}[F_K(t)]^{n-1}dt}{[F_K(k_m)]^{n-1}}$. *Compared to an uncorrupted firm with the same type, a corrupted firm has a higher pseudotype and will bid a higher quality and a higher score. (The prediction on price is ambiguous). All three effects magnify as* m *increases.*

Therefore, the corrupted firm will bid *more aggressively* compared to a competitive firm of the same type. Because the corrupted firm has a large winning probability, it causes a systematic distributional change of the winning bid. It is the key factor which allows us to construct corruption detection tests. Note that the "more aggressive" prediction is different from the implication of bidding ring models. When an auction involves a bidding ring, both the ring leader and phantom bidders bid less aggressively to suppress competition. But with quality manipulation, the corrupted firm pays a lower cost with help from the procurement agency. As a result, the corrupted firm can bid more aggressively to win the contract.

3 Econometrics of Scoring Auction and Corruption Detection

The key of corruption detection lies on checking abnormal aggressive bidding behaviors of corrupted firms. It is impossible to distinguish normal competitive bidding and abnormal predatory bidding behavior on a single observation because the scope of manipulation (m) in unknown. But when the sample size gets large and m is not too small, the consistent pattern of an aggressive winning bid can be captured by statistical tests. In this Section, we provide three tests and show a Monte Carlo example. For identification and estimation of scoring auction model, we use tools provided Guerre et al. (2000), Paarsch and Hong (2006), and Athey and Haile (2007). In constructing tests for corruption, we use some results from Lucking-Reiley (1999), Athey and Haile (2002), and Ingraham (2005).

3.1 Structural Estimation

We first present the identification and the structural estimation of a scoring auction model. Consider a sample of T independent and repeated scoring auctions within the same industry under the same scoring rule (We will discuss variations of scoring rules later in the empirical application section). For scoring auction t, assume researchers observe the number of firms n_t, some auction-specific covariates z_t (of dimension d), bids of each firm $\{p_{it}, q_{it1}, q_{it2}, ..., q_{itL}\}_{i=1}^{n_t}$ (of dimension $L+1$) and their score $s_{it} = V(q_{it1}, q_{it2}, ..., q_{itL}) - p_{it}$. We set aside endogenous entry and reserve price/score issue in this paper. By result in Theorem 1, identification can be established straightforward by standard result in Guerre et al. (2000).

Theorem 3: *Under assumption QL and CF, pseudotypes and equilibrium costs of firms are nonparametrically identified.*

Proof: Because $G_S(s) = \Pr(S \leq s) = \Pr(K \leq k) = F_K(k)$, $g_S(s) = f_K(k)/s'(k)$, by (24), pseudotype k is identified from the observation of scores and its distribution via

$$k \quad = \quad s(k) + s'(k)\frac{F_K(k)}{(n-1)f_K(k)} = s + \frac{G_S(s)}{(n-1)g_S(s)}, \tag{11}$$

The equilibrium cost is then identified by definition of the pseudotype,

$$C(q(\theta),\theta) \quad = \quad V(q(\theta)) - k = p(\theta) - \frac{G_S(s)}{(n-1)g_S(s)}. \tag{12}$$

Q.E.D.

Given the number of firms, auction-specific covariates[15], and bids, the conditional distribution function and density of score can be estimated by kernel estimators,

$$\begin{aligned}
\hat{G}_S(s|n,z) &= \frac{1}{Th^2}\sum_{t=1}^{T}\frac{1}{n}\sum_{i=1}^{n}\mathbb{I}(s \le s_{it})\kappa_G\left(\frac{n-n_t}{h},\frac{z-z_{it}}{h}\right),\\
\hat{g}_S(s|n,z) &= \frac{1}{Th_1h_2^d}\sum_{t=1}^{T}\frac{1}{n}\sum_{i=1}^{n}\kappa_g\left(\frac{s-s_{it}}{h_1},\frac{n-n_t}{h_2},\frac{z-z_{it}}{h_2}\right).
\end{aligned}$$

As standard to this literature, we use Gaussian kernel and pick the bandwidth by least-square cross validation throughout this paper. Pseudotypes and equilibrium costs (at corresponding quality) are estimated by

$$\hat{k}_{it} \quad = \quad s_{it} + \frac{\hat{G}_S(s|n,z)}{(n-1)\hat{g}_S(s|n,z)}, \tag{13}$$

$$\hat{c}_{it} \quad = \quad V(q_{it}) - \hat{k}_{it}. \tag{14}$$

Some intuition of (13): pseudotype is the total social surplus of a firm. In the auction, the firm chooses a score s as the portion delivered to the buyer. The second term the firm's *rent*, reflecting its competitive advantage and information rent. Notice that our model is a variation of a standard first-price auction of contracts. In a standard model, quality is fixed, so the model primitive is a cost distribution . In a scoring auction, the model primitive is a cost function defined on the domain of quality attributes. Costs estimated via (14) are not randomly drawn from a fixed cost distribution, but rather chosen by firms.

Nakabayashi and Hirose (2015) consider a similar problem under a general scoring rule but with parametric assumption of the cost function. We keep a quasilinear scoring rule but allow the cost function to be fully nonparametric. If a parametric cost function $C(q,\theta)$ is assumed, it is possible to identity θ in the equilibrium condition, as we shown in the Example 1 below.

Monte Carlo Example

The scoring auction uses a linear scoring rule $S(q,p) = 2q - p$. Each firm draws its two dimensional type $\theta = (\theta_0, \theta_1)$ independently from Uniform[0,1] and Uniform[1,2] respectively. Assume θ_0 and θ_1 are independent, so their joint density equals to one on the support. Each firm then has its cost function as

[15] If there are lots of auction-specific covariates, a similar semi-parametric estimator can be used.

$$C(q,\theta) = \theta_0 + \frac{q^2}{\theta_1}. \tag{15}$$

By Theorem 1, the optimal quality choice of a firm with type θ is $q(\theta) = \theta_1$ and its pseudotype is

$$K(\theta) = V(q(\theta)) - C(q(\theta),\theta)) = 2\theta_1 - \theta_0 - \frac{\theta_1^2}{\theta_1} = \theta_1 - \theta_0.$$

The support of k is $[0,2]$. By (6), the distribution function of pseudotype is

$$F_K(k) = \Pr(\theta_1 - \theta_0 < k) = \begin{cases} \frac{k^2}{2}, & \text{for } k \in [0,1], \\ 1 - \frac{(2-k)^2}{2}, & \text{for } k \in (1,2]. \end{cases} \tag{16}$$

Notice that, by allowing two dimensional types, firm who submits a high equilibrium quality does not necessarily have a high pseudotype. For example, when firm 1 is of type $\theta = (0.5, 1.5)$ and firm 2 is of type $\theta = (0.1, 1.2)$, firm 1 will produce at $q = 1.5$ and have pseudotype $k = 1$; firm 2 will produce at a lower level $q = 1.2$ but have a higher pseudotype $k = 1.1$. We allow number of firms n to be randomly draws from 3 to 20. Using (8), we can generate a simulated data set and apply our estimator (13) and (12), as illustrated in Figure 4 and 5. The estimation is based on 1000 auctions.

In this example, if researchers know the parametric form of the cost function, he can identify two structural parameters by conditions of optimal quality and score choice: $\theta_1 = q$ and $\theta_0 = q - s - \frac{G(s|n)}{(n-1)g(s|n)}$. In general, as long as $K(\theta)$ is monotone in θ under the parametric assumption, θ is identified. Under a non-additively separable scoring rule, the bidder's pseudotype does not summarize its true type θ (see detail in Hanazono et al. (2015)). In application to an actual data set, restricting the parametric family of cost function reduces the credibility of the estimation.

Notice that, the identification result in Theorem 3 is established in a competitive bidding environment. If there is corruption, the scope of corruption is unobservable and may vary across auctions. From a single observation, a researcher cannot conclude weather a high pseudotype is due to a real competitive advantage or manipulated quality. In the example, for some $m > 0$,

$$q_m(\theta) = \arg\max_q \left\{ 2q - \theta_0 - \frac{(q-m)^2}{\theta_1} \right\} = \theta_1 + m = q(\theta) + m, \tag{17}$$

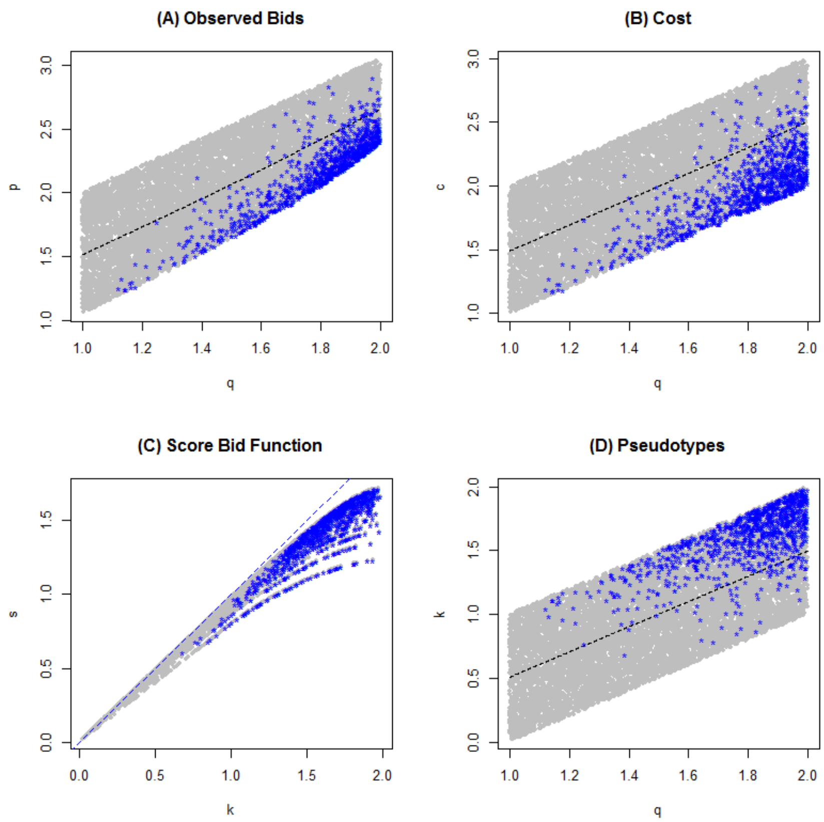
$$K_m(\theta) = 2(\theta_1 + m) - \theta_0 - \theta_1 = \theta_1 - \theta_0 + 2m = K(\theta) + 2m. \tag{18}$$

Therefore one cannot separately identify k and m. Although it is not a identified model, the systematic distortion of submitted bids can be captured with a large enough sample.

3.2 Corruption Detection Tests

The basic intuition of our corruption detection tests is to capture abnormal aggressive bidding behavior. Corruption distorts only the corrupted firm's bid, while all other bids remain competitive. The distorted bid is the winning bid with a large probability. Hence, even we don't know the identity

Figure 4: Illustration of the Data

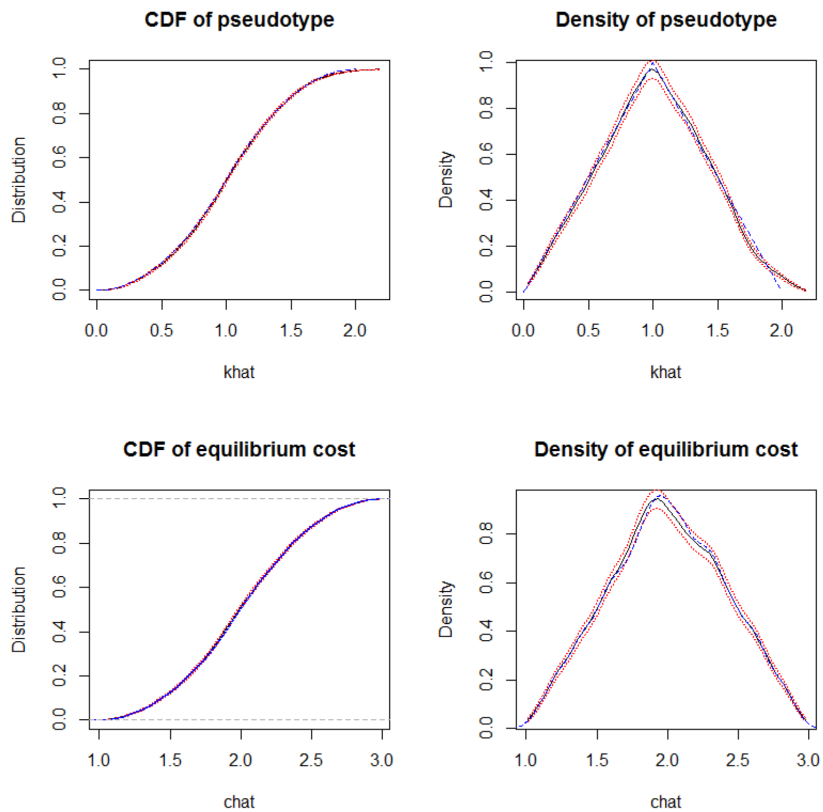


Note: In each diagram, the grey points represent bid level data, and the black dashed curve is a smoothing spline; (see Green and Silverman (1993) and Hastie and Tibshirani (1990) for reference of smoothing spline); blue stars (*) denote winning bids.

of the corrupted bidder, we can test for systematic deviation from competitive bidding behavior by comparing winning bids and other bids. Test I and II can be performed on a sample from one procurement agency, while test III can only be performed on sample from two or more procurement agencies. These tests are illustrated in a Monte Carlo example and later applied in an empirical example.

For all these three tests, the null hypothesis is that the data is generated from a *competitive model*, H_0: $m = 0$. It is tested against the alternative hypothesis that the data is generated from a *corruption model*, H_1: $m > 0$. To construct the test statistic and its distribution under the null, m can be an unknown positive number. But to find the power of the test in the Monte Carlo example, we let m to be a known fixed number across observation. We denote the observed highest score/pseudotype by subscript "win" and denote the observed second highest score/pseudotype by subscript "rival", referred as *the (strongest) rival*. (The third highest score or pseudotype is denoted by subscript "third".)

Figure 5: Estimation of Pseudotype and Cost



Note: For all nonparametric estimators, we select the bandwidth by least-square cross validation . The blue dashed lines denote the "true" distribution, while the black lines denote the estimated one. The red dotted lines denote point-wise confidence band for two standard errors.

Test I

We have shown in Theorem 2, compared to a competitive firm with the same θ, the pseudotype and winning probability of a corrupted firm both increases, i.e., $K_m(\theta) > K(\theta)$ and $\Pr\left(K_m(\theta) > k_{(1:n-1)}\right) > \Pr\left(K(\theta) > k_{(1:n-1)}\right)$. Among the $n-1$ rivals, the strongest one is of pseudotype $k_{(1:n-1)}$. For any type of corrupted firm θ, the corrupted firm wins with probability $\Pr\left(K_m(\theta) > k_{(1:n-1)}\right) = \Pr\left(k_m > k_{(1:n-1)}\right) = F_K^{(1:n-1)}(k_m)$, which is increasing in m. The corrupted firm appears to be the strongest rival with probability $\Pr\left(k_{(2:n-1)} < k_m < k_{(1:n-1)}\right) = F_K^{(2:n-1)}(k_m) - F_K^{(1:n-1)}(k_m)$. Therefore, the observed winning score

$$s_{win} = \begin{cases} s(k_m), & \text{with prob } F_K^{(1:n-1)}(k_m), \\ s(k_{(1:n-1)}), & \text{with prob } 1 - F_K^{(1:n-1)}(k_m), \end{cases}$$

and strongest rival's score

$$s_{rival} = \begin{cases} s(k_m), & \text{with prob } F_K^{(2:n-1)}(k_m) - F_K^{(1:n-1)}(k_m), \\ s(k_{(1:n-1)}), & \text{with prob } F_K^{(1:n-1)}(k_m), \\ s(k_{(2:n-1)}), & \text{with prob } 1 - F_K^{(2:n-1)}(k_m). \end{cases}$$

By Theorem 2, for any $m > 0$, $E[s_{win}] > E[s(k_{(1:n)})]$ and $E[s_{rival}] < E[s(k_{(2:n)})]$. By formula (13), the estimate of the strongest rival's pseudotype $\hat{k}_{rival}$ also decreases. By Corollary 1, $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$. Hence, for any $m > 0$, we have

$$E[s_{win}] > E[s(k_{(1:n)})] = E[k_{(2:n)}] > E[k_{rival}].$$

But in competitive auction, by corollary 1, we have $E\left[s_{win}\right] = E\left[k_{rival}\right]$ because $s_{win} = s(k_{(1:n)})$ and $k_{rival} = k_{(2:n)}$ in the equilibrium. The test of corruption becomes testing

$$\begin{aligned} & H_0: \quad E\left[s_{win}\right] = E\left[k_{rival}\right], \\ \text{v.s.} \quad & H_1: \quad E\left[s_{win}\right] > E\left[k_{rival}\right]. \end{aligned}$$

Lucking-Reiley (1999) uses t-test for revenue equivalence of data generated from different auction formats. We also apply t-test here but with a bootstrap corrected critical value. The Welch's t-test statistic is

$$T^I = \frac{\frac{1}{T}\sum_{t=1}^{T} s_{win,t} - \frac{1}{T}\sum_{t=1}^{T} \hat{k}_{t,rival}}{\sqrt{\frac{var(s_{win})}{T} + \frac{var(\hat{k}_{rival})}{T}}}, \tag{19}$$

where $\hat{k}_{t,rival}$ are estimated from (13).[16]

Test II

In the symmetric IPV model, Athey and Haile (2002) show that the underlying value distribution is nonparametrically identified even when only one bid per auction (an order statistic) is observed. When there is no corruption, using all bids and using only the winning bid should result in the same pseudotype distribution estimates except for statistical error. When there is corruption, the winning bid is distorted with a large probability. In this case, two methods will result in statistically different estimates. Practically, we construct the test by comparing two empirical CDFs of pseudotypes of winners from two estimation methods.

[16] A more sophisticated test is *Wilcoxon signed-rank test* (Wilcoxon (1945)) for paired samples without normality assumption. The Wilcoxon signed-rank test statistic is $W = \frac{1}{T}\sum_t^T \left\{\text{sgn}(s_{t,win} - \hat{k}_{t,rival}) \times R_t\right\}$, where $\text{sgn}(\cdot)$ is the sign function. The variable R_t is constructed by sorting absolute difference $|s_{t,win} - \hat{k}_{t,rival}|$ of all pairs in the sample. The smallest receives $R_t = 1$, second smallest $R_t = 2$, and so forth. Ties receive a rank equal to the average of the ranks they span. However, computation of critical value of the rank test involves permutation, which is similar to bootstrap. We cannot use bootstrap to further correct its critical value, so we choose not to use it. Interested readers can inquire the author for code that illustrates the problem of using bootstrap on the rank test.

By using all bids, pseudotype estimate of each bid $\{\hat{k}_{1t}, \cdots, \hat{k}_{n_t t}\}$ can be estimated via (13). Denote the pseudotype corresponding to the winning bid as $\hat{k}_{win}$ and its empirical CDF $\hat{F}_K^{win}(k) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\hat{k}_{win} \leq k\right)$. By using only winning bids, these winning scores have distribution function $G_W(s_{win}|n) = G_S^{(1:n)}(s_{win}) = [G_S(s_{win}|n)]^n$ and density $g_W(s_{win}|n) = n\,[G_S(s_{win}|n)]^{n-1}\, g_S(s_{win}|n)$. By replacing relevant terms in (11), winning pseudotypes are identified via

$$k_{win} = s_{win} + \frac{nG_W(s_{win}|n)}{(n-1)g_W(s_{win}|n)}. \tag{20}$$

The underlying pseudotype of each winning bid can then be estimated, denoted as $\check{k}_{win}$. The empirical CDF of $\check{k}_{win}$ is $\check{F}_K^{win}(k) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\check{k}_{win} \leq k\right)$. The corruption detection problem becomes testing

$$\begin{aligned} & H_0: \quad \forall k \in [\underline{k}, \overline{k}],\ \hat{F}_K^{win}(k) = \check{F}_K^{win}(k), \\ \text{v.s.} \quad & H_1: \quad \exists k \in [\underline{k}, \overline{k}],\ \hat{F}_K^{win}(k) \leq \check{F}_K^{win}(k). \end{aligned}$$

The natural option is Kolmogorov–Smirnov (KS) test,[17] whose test statistic is

$$\begin{aligned} T^{II} &= \sup_{k\in[\underline{k},\overline{k}]} \left|\hat{F}_K^{win}(k) - \check{F}_K^{win}(k)\right| \\ &= \sup_{k\in[\underline{k},\overline{k}]} \left|\frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\hat{k}_{win} \leq k\right) - \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\check{k}_{win} \leq k\right)\right|. \end{aligned}$$

Under the competitive model, because $\hat{F}_K^{win}(k) = \hat{F}_K^{(1:n)}(k) = \left[\hat{F}_K(k)\right]$, the value of T^{II} will be small. Similar to test I, there is dependence between the two sets of estimated pseudotypes of winners, so we use bootstrap corrected critical values. Test II is illustrated in Figure 6.

Test III

Test III is inspired by Ingraham (2005). It is based on the following Markovian property of conditional distribution of order statistics.

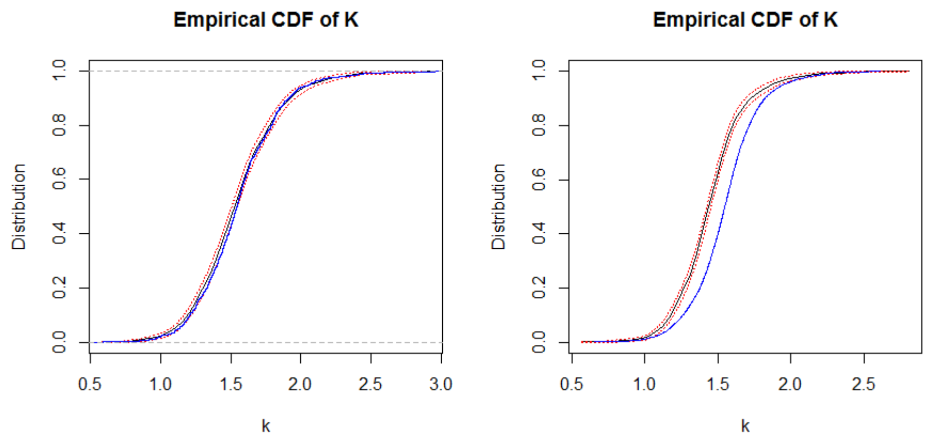
Lemma 3 (Arnold et al. (1992)): Denote the *first spacing* of (the two highest) order statistics as $X_{12} = X_{(1:n)} - X_{(2:n)}$. It has conditional distribution only depends on the third order statistic (Markovian property), that is

$$f_{X_{12}}(x_{12}|X_{(3:n)} = x_3) = f_{X_{12}}(x_{12}|X_{(3:n)} = x_3, X_{(4:n)} = x_4, \cdots, X_{(n:n)} = x_n).$$

Test III is easy to implement but needs at least two sub-samples. Suppose the observed auctions

[17] The test is one-sided because under the alternative, the aggressive scores in corruption model results in higher estimate of k.

Figure 6: Illustration of Test II



Note: In each diagram, the black curve is the empirical CDF estimated from all bids; the blue curve is the empirical CDF estimated from only winning bids. The left-hand-side diagram represents estimation result from a competitive data set, the right-hand-side diagram represents one under corruption.

can be divided into two (or several) sub-samples according to different procurement agencies.[18] Let D_τ be dummy variable of sub-sample τ. Consider the following regression model:

$$(\hat{k}_{win} - \hat{k}_{rival}) = \beta_0 + \beta_1 \hat{k}_{third} + \beta_2 D_\tau + \beta_3 z + \epsilon,$$

where z controls for other auction-specific covariates. In a competitive auction, $\hat{k}_{win}$, $\hat{k}_{rival}$, $\hat{k}_{third}$ coincide with $k_{(1:n)}$, $k_{(2:n)}$, $k_{(3:n)}$. According to Lemma 3, the conditional distribution of first spacing of pseudotype, $k_{12} = k_{(1:n)} - k_{(2:n)}$, is the same across auctions if we control the third highest order statistic $k_{(3:n)}$. Therefore the conditional means of two sub-samples are equal if $m = 0$. We can apply a standard t-test for $H_0 : \beta_2 = 0$ versus $H_1 : \beta_2 \neq 0$, whose test statistic $T^{III} = \hat{\beta}_2 / se(\hat{\beta}_2)$. We can also directly use score data to perform the test with a regression

$$(s_{win} - s_{rival}) = \beta_0 + \beta_1 s_{third} + \beta_2 D_\tau + \beta_3 z + \beta_4 n + \epsilon,$$

which skips the first stage structural estimation of pseudotypes. If $\hat{\beta}_2$ is significantly greater than 0, it suggests that the gap between the winner and the rival is larger in the $D_\tau = 1$ group, which implies a higher likelihood of corruption.

[18] In the empirical example below, we only use sub-sample defined by procurement agency. Sub-sample division can also be based on locations, auction format, private or public sector buyer, etc.

Table 1: Power of Tests

Scope of quality manipulation	Test	Number of observed auction $T = 200$	$T = 500$	$T = 1000$
	I	0.2462	0.2362	0.2613
$m = 0.2$	II	0.9246	0.9347	0.9497
	III	0.8241	0.7889	0.8291
Corrupted firm wins with probability 0.2348.				
	I	0.3869	0.4925	0.5528
$m = 1$	II	0.9749	0.9648	0.9849
	III	0.9246	0.9397	0.9598
Corrupted firm wins with probability 0.4596.				
	I	0.9347	0.9648	0.9749
$m = 2$	II	0.9899	0.9950	0.9950
	III	0.9950	0.9950	1.0000
Corrupted firm wins with probability 0.9618				

Monte Carlo Example (Continue)

We continue to use the setting in the previous example to show corruption detection tests. We generate $B = 199$ samples under the null hypothesis ($m = 0$) and compute test statistics for each sample, $\{T_b^j\}_{b=1}^B$, $j = I, II$.[19] Setting the significance level at 5%, the relevant *bootstrap critical value of the test*, $CV(T^j)$, is the 190th highest of these test statistics (since $(B+1) \times (1 - 0.05) = 190$), illustrated in Figure 7.

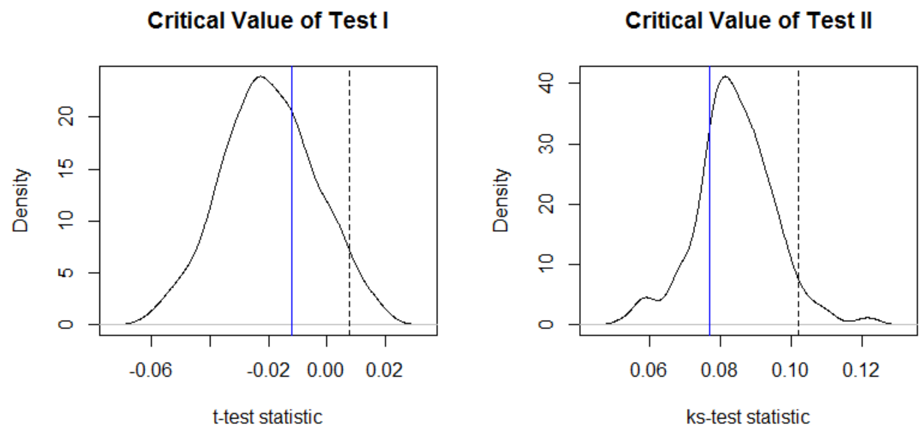
We explore power of these tests under three different alternative hypotheses as shown in Table 1. The data under the alternative is generated by taking m equals 0.2, 1, and 2. A randomly selected corrupted firm will produce at a higher quality and have a higher pseudotype according to (17) and (18) respectively. The *bootstrap power of the test* is defined and computed via

$$\text{power} \quad = \quad 1 - \Pr(\text{accept } H_0 | H_1 \text{ is true}) = 1 - \frac{1}{B} \sum_{b=1}^{B} \mathbb{I}\left(T_b^j \leq CV(T^j)\right).$$

For test III, we let half of the sample generated under the alternative. The Monte Carlo results show that as the scope of corruption m and number of observed auction T increases, the power of the test improves. The power of test I is relatively weak compared to test II and III, especially in the case where m is low.

[19] In Monte Carlo simulation, we can use the same data generating process to generate B data sets of 1000 observations. It provides an accurate distribution of test statistic and critical value, which can be used to check validity of bootstrap.

Figure 7: Distribution of Test Statistics Under the Null and Bootstrap Critical Values



Note: The black curve represents the density of 199 bootstrap test statistic. The black dashed line denotes the bootstrap critical value. The blue line denotes the test statistic of a competitive data set.

Discussion

The major advantage of our tests is that they require less data than most existing collusion detection tests. Hence, it can be performed on a lot of procurement auction data sets. Existing tests generally require bidder's identity, (rich) bidder-specific covariates, repeated observation of bidders in several auctions. Some of these tests requires exact identities of (suspected) colluding bidders, for example Porter and Zona (1993), Pesendorfer (2000), and Athey et al. (2011). Some tests, like Bajari and Ye (2003), can be conducted without identities of corrupted firms, but need to be run on each combination of bidder pairs. Repeated observation of the same set of bidders tracks dynamic of bidding behaviors, which is important to reveal the systematic difference from the colluding bidders and competitive bidders. Our tests do not require any of these data and hence can be performed *ex ante* before case by case antitrust investigation.

Moreover, with different sub-samples, our tests do not require a prior on which sample is more likely to be corrupted. (For example, Athey et al. (2011) assumes that the sample from open auctions are collusive while sealed-bid auctions are competitive). Test I and II can be performed on each of the sub-sample and compare their relative likelihoods of corruption by p-values. Test III estimates a "fixed effect" to each sub-sample by regression and can rank their likelihoods of being corrupted.

However, because these tests are constructed for fairly limited sample information, there are several potential shortcomings:

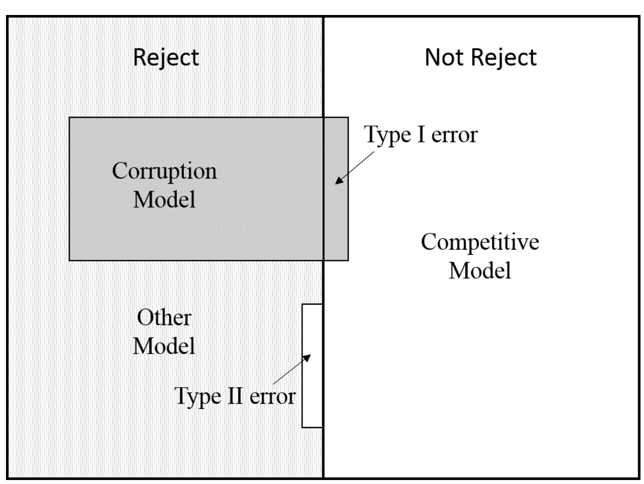
(1) Our test statistics are computed by two-step estimation based on pseudotypes estimated via (13). Because estimated pseudotypes are correlated, asymptotic distribution under the null cannot be derived analytically. We therefore use bootstrap critical values to make rejection decision. Econometricians start developing inference and tests on auction model based on one-step estimation.

For example, Liu and Luo (2014) propose a test of exogenous entry whose test statistic is based on quantile estimates of bids, instead of estimated values. One disadvantage of bids-based tests is bids depend on number of bidders (n), therefore one cannot pool bids data from auction with different n together, but needs to separate observed auction by n. Therefore, there is a tradeoff between using more data and applying a test with better asymptotic property. In our case, there is usually a great deal of variation of n (see Figure 9), we take the path of using more data.

(2) The power of our tests is also very difficult to be studied analytically. First, power depends on the scope of corruption (m), but m is unobservable and may vary across auction. Hence, we cannot set a simple alternative of a fixed m. Second, m cannot be estimated even we give a parametric assumption of its distribution. The model is not identified under the alternative hypothesis mainly because the corrupted firm is not always the winner. So the scope of corruption cannot be recovered without strong assumption like the corrupted firm always win. In other words, the corrupted firm's bid and other bids are not generated from the same data generation process, and we don't know which bid comes from the corrupted firm. All these complication restrict us from studying power of the test rigorously. A desirable data set to study corruption should include some *ex post* information of convicted corruption records. With identities of corrupted firms, then it is possible to identify the corruption model. Researchers can then study the power of the tests and its "in-sample" prediction correctness. We don't have such a data set for now and the main contribution of these tests are their *ex ante* feature in corruption detection.

(3) Figure 8 illustrates a common shortcoming of our tests and most collusion detection tests in the literature. When the data does not reject the competitive model (null hypothesis), it provide evidence that the data rationalizes the competitive model. But when the data rejects the competitive model, it cannot distinguish whether the reason is corruption or model mis-specification. For example, rejecting test I can be due to any reason related to expected score equivalence failures, like bidder's risk aversion. The one-sided tests in test I and test II alleviate this problem: if we find that the winning bid is not aggressive but conservative, we do not reject the null.

Figure 8: Test Result Interpretation



4 Empirical Application

4.1 Data and Server Room Construction Industry

Our scoring auction data set comes from two major procurement agencies: Guangzhou Public Resource Trading Center[20] and Public Resources Trading Center in Guangdong Province[21]. Nearly all procurements conducted on these two trading centers are sealed-bid scoring auctions due to both legal requirements and their economic advantage. It also provides guidelines to forming tender evaluation committees, selecting industrial experts, designing of scoring rules, and the detailed process of auction. Starting November 2009, these two major procurement platforms publicly announced auction results of all government related projects.[22] The Chinese Law of Tender[23] requires government related projects with values over a certain threshold to go through the open tender process coordinated by these trading centers. Besides public sector, private sector buyers also use these two trading centers frequently because trading center have connections to a large pool of industrial experts that perform bid evaluations.

In this paper, we focus on procurement auctions of a particular industry: server room construction projects. Server room is an indoor place designed to contain machines of data storage, servers, and large computers. During the two year period (01/01/2012 to 12/31/2013) of our data set, there are total 2147 projects procured via auction. There are 8.8 bids on average of each auction. The summation of engineer's estimated costs of all observed projects is over 10 billion CNY (1.6 billion USD).[24] Hence the industry is both large in size and has enough observations for structural estimations. For each project, our information includes its engineer's estimated cost, number of bidders, weights, city, identities of the buyer and the winning firm. On bid level, we observe factor score records of each bid. Table 2, 3, 4, and Figure 9 summarize the data set. All price data are in units of 1000 CNY. $\tilde{q}$ and $\tilde{s}$ are defined later in this section.

Several remarks about the data set:

(1) The market structure is relative simple. First, there is a large number of supplying firms and no buyers or firms dominates the industry. Table 3 shows that the largest firm only has a 1% market share. Second, because server room project designs and construction costs are not much affected by their geographic location, combining data from different cities is reasonable. Third, subcontracting is common in this industry, a firm's distance to the project is less important when most components of the project are carried out by subcontractors.[25] These features support the independent private information setting of our model.

(2) Quality evaluation of a server room construction plan needs specific expertise. To ensure reliability and safety, the construction of server rooms have specific technological requirements on

[20] http://gzggzy.cn/.
[21] http://www.bcmegp.com/
[22] http://gzggzy.cn/cms/wz/view/.
[23] Law of the People's Republic of China on Tenders and Bids (click link for full article in English).
[24] In 2014, the GDP of Guangdong province is 6,779 trillion CNY (1.104 trillion USD).
[25] Distance source: http://www.distancecalculator.net/

various aspects like temperature, humidity, electricity supply, fire control, etc.. Each bid contains a full construction plan and a itemized price list. Firm's reputation, experience, certificate, and size is also considered in the bid evaluation. In the Appendix, we provide description of a sample bid. Giving a 100 technological factor score on the construction plan is both difficult and subjective. Therefore, compared to land or cargo, server room construction is subject to higher risk and a larger scope of quality manipulation corruption.

(3) Our data set contains much less firm level observations than those in Porter and Zona (1993) or Bajari and Ye (2003). Among the 1046 winning firms, 451 firms win only one contract. Therefore, tracking firm's bidding history to construct variables like "backlog", "capacity", or "utilization rate " is impossible.[26] Moreover, we do not observe the identities of all losing firms, so we cannot construct explanatory variables like rival firm's distance or rival capacity. Therefore, empirical analysis and collusion detection methods in Porter and Zona (1993) and Bajari and Ye (2003) are not implementable in our data set.

(4) The scoring rule of this data set is relative easy to analyze. The business factor weights is constant at 0.1 across all project. The (w_p, w_q) combination takes a total of five sets of values with $w_p + w_q = 0.9$.[27] Hence, variation of the scoring rule can be controlled by including w_q in explanatory variable. Because the slope of scoring rule affects the distribution of pseudotypes, we also control w_q in the structural estimation. The price factor evaluation rule is consistent and not interdependent. In our sample, the engineer's estimated costs and prices of the bid are transferred into a 100 point price factor score by formula (21) below.[28] The linear weighted scoring rule can thus be transformed into a quasilinear scoring rule for further analysis. We will now discuss the details.

[26] See Porter and Zona (1993), Section IV for definitions.

[27] The law of tender requires all construction projects shall economic factor weight $w_p \geq 0.4$.

[28] An interdependent price score evaluation formula $s_p = \left[1 + \alpha\left(\frac{p_{best} - p}{p_{best}}\right)\right] \times 100$. p_{best} is not the minimum, but computed by a formula taking in all submitted bids, then subtract it by 10% to 15%. and the existing scoring auction analysis have studied this kind of scoring rule. Actually, quality factor is also evaluating in an interdependent way.

Table 2: Descriptive Statistics of the Data

Variables	Obs.	Mean	SD	Min.	Max
Project-specific					
Engineer's estimated cost, p_0	2,147	5,049.90	1,478.49	835	13,239
Weight on tech. factor, w_p	2,147	0.4958	0.0627	0.4	0.55
Weight on price factor, w_q	2,147	0.4042	0.0627	0.35	0.5
Weight on business factor, w_r	2,147	0.1	0	0.1	0.1
Number of firms, n	2,147	8.8323	3.8566	3	36
Winning score, s	2,147	78.5502	6.0677	52.4942	95.1337
Project city	2,147	(21 cities in Guangdong province)			
Bid-specific					
Price factor score, s_p	18,963	69.8007	10.4362	1.0881	100
Tech. factor score, s_q	18,963	60.4788	28.5503	0	100
Business factor score, s_r	18,963	72.3002	10.1547	29	100
Price, p	18,963	4,162.96	1,843.70	363.3	18,417.38
Savings rate, $\rho = \frac{p_0 - p}{p_0}$	18,963	0.198	0.1044	-0.4891	0.50
Weighted score, s	18,963	66.4975	11.1019	26.9841	95.1337
Transformed quality, $\tilde{q}$	18,963	8260.86	2915.77	1140.24	29135.76
Transformed score, $\tilde{s}$	18,963	4097.89	1481.16	712.09	11559.27

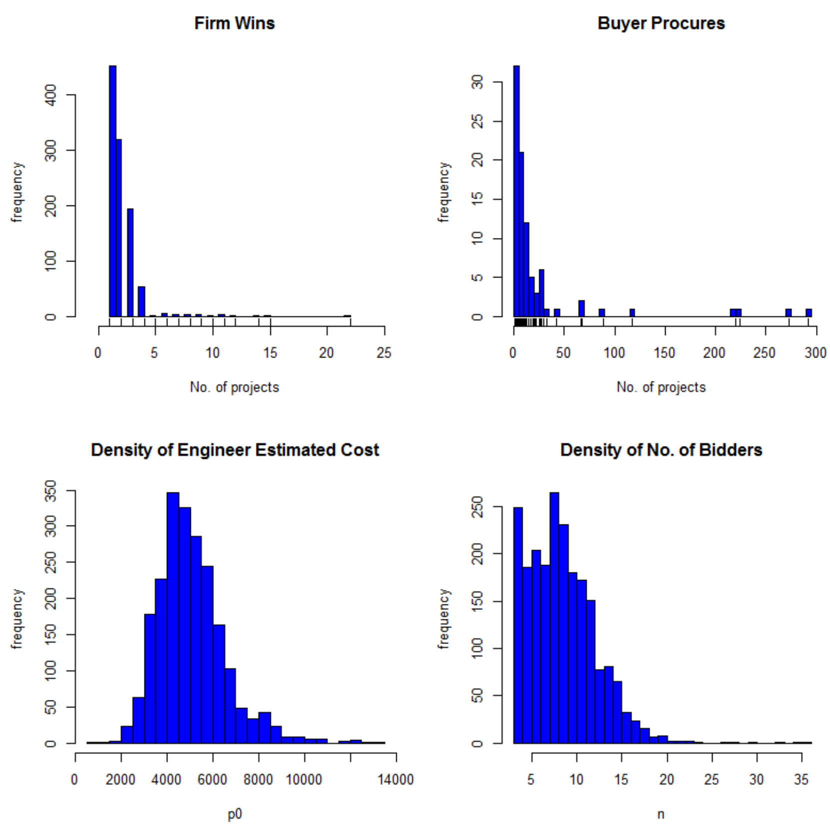
Table 3: Summary of Market Structure

	Market Share		Number of Projects			
	Mean	H.H. index	Mean	SD	Min	Max
Firms	0.0956%	0.0015	2.0525	1.6044	1	22
Total= 1046						
No. of firm wins one project= 451						
Buyers	1.1236%	0.0659	24.1236	53.4929	1	292
Total= 89						
No. of buyer procures one project= 10						
No. of buyer procures less than 10 project= 50						

Table 4: Summary of Market Structure (Continue)

Index	No. of Project	Market Share	Total Value	Share of Value
Top five firms:				
1	22	0.0102	94,641	0.0087
2	15	0.0070	69,990	0.0065
3	14	0.0065	63,264	0.0058
4	14	0.0065	72,496	0.0067
5	12	0.0056	64,504	0.0059
Top five buyers:				
1	292	0.1360	1,463,947	0.1350
2	273	0.1272	1,370,988	0.1264
3	224	0.1043	1,154,855	0.1065
4	220	0.1025	1,110,387	0.1024
5	117	0.05449	579,093	0.05341
Procurement agencies:				
1	1,466	0.6828	7,374,657	0.6802
2	681	0.3172	3,467,485	0.3198

Figure 9: Visualization of Some Variables



All projects in our data set use the most common "comprehensively evaluation method" in China. It is simply a linear weighted scoring rule consisting of three components: an economic factor, a technological factor, and a business factor. The e*conomic factor* (s_p) evaluates the price of the bid.[29] Denote the *engineer's estimated cost* as p_0, if a company submits price p in his bid, his price score is computed by ($\rho = \frac{p_0 - p}{p_0}$ is called the *savings rate* of that bid)

$$s_p = \begin{cases} 0, & p > \frac{3}{2}p_0, \\ \left(\frac{1}{2} + \frac{p_0 - p}{p_0}\right) \times 100, & \frac{1}{2}p_0 \le p \le \frac{3}{2}p_0 , \\ 100, & p < \frac{1}{2}p_0. \end{cases} \tag{21}$$

Th *technological factor* (s_q) evaluates quality of the construction plan including the design, building standard, equipment, server machine, follow-up service, warranty, delivery date, payment condition, insurance, etc.. The *business factor* (s_r) evaluates the firm's reputation, experience, risk of default, risk of bankruptcy etc.. Technological and business factors are evaluated by a tender evaluation committee.[30] Each bid receives three 100 points scores on three factors, and then a grand score is computed via

$$S(s_p, s_q, s_r) = w_p s_p + w_q s_q + w_r s_r, \tag{22}$$

where weights w_p, w_q, w_r add up to one. The firm with the bid receiving the highest 100-scale grand score wins the contract. The linear weighted scoring rule can be transferred into a quasilinear one by redefining quality and score. Let the *transformed score*, $\tilde{s} = \frac{p_0}{100 w_p}\left(S(s_p, s_q, s_r) - 50 w_p\right)$ and the *transformed quality*, $\tilde{q} = \frac{p_0}{100 w_p}\left(100 w_p + w_q s_q + w_r s_r\right)$, then (22) can be transformed as

$$\tilde{S}(\tilde{q}, p) = \tilde{q} - p. \tag{23}$$

Because it is a monotonic transformation of the original 100-scale score, firm's winning probability and bidding strategy are not affected. We make the following assumption:

Assumption TP (true preference): scoring rule (23) represents the preference of the buyer.

$\tilde{q}$ represents the buyer's benefit from the project delivered at s_q and s_r, while $\tilde{s}$ the *payoff* after paying the winning firm p. The buyer's choice of weights in (22) reflect its preference of substitution on price and quality. In the original scoring rule, if price is raised by 1 CNY, then the 100-scale grand score reduced by $100\frac{w_p}{p_0}$ (if the reduction is not at the boundary). To retain the same level of payoff, the buyer need to be compensated on higher quality, which requires $w_q \Delta s_q + w_r \Delta s_r = 100\frac{w_p}{p_0}$. If we also add a boundary condition: the buyer receives zero payoff from a contract with $s_q = 0$, $s_r = 0$ and p_0. Then scoring rule (23) satisfies both the substitution condition and the boundary

[29] As mentioned in Bajari et al. (2014), in reality price evaluation is not just "the low the better". A highly unbalanced bid (extreme itemized price) or a bid lower than cost could be penalized or rejected.

[30] The law of tender require the committee shall contains five or more members (odd number). There is one representative from the buyer. All the other members are either randomly selected from the pool of experts of the procurement agency.

condition to reflect the true preference of the buyer.

In addition, Che (1993) points out that if the buyer lacks commitment power, the only scoring rule she can commit to is the one that represents her true preference. If the buyer has commitment power and chooses the optimal mechanism, the optimal scoring rule would "handicap" the efficient firm by under-reporting buyer's preference on quality. If we assume the buyer is optimally choosing the scoring rule, the estimated payoff of the buyer is the lower bound of her true one. Assumption TP allow us to compare payoffs across auctions.

We also assume a convexity condition of cost:

Assumption CF': the cost function of supplying transformed quality satisfies assumption CF (continuous and convex in $\tilde{q}$).

Now, both the scoring rule and cost satisfy condition of Lemma 1. It allow us to use the single dimensional quality index $\tilde{q}$ in a structural estimation. Due to both restriction on data availability and exposition clarity, we skip the analysis of endogenous entry issues in this paper.[31]

4.2 Reduced-form Estimation

In the following empirical exercise, we provide some descriptive graphs and estimation results based on some reduced-form models. There are three main findings:

(1) We test two implications of the theoretical model. First, a higher quality weight (lower price weight) shall induce firms to submit bids with higher quality and higher grand score. Second, firm's choice of quality and price are separated under additively separable scoring rule. We don't find evidence in the reduced-form regression by using the *original strategy space* (s_p, s_q, s_r) as the dependent variable. But we find robust evidence for both model implications by using the *transformed strategy space* $(\tilde{q}$,$\tilde{s})$. It justifies our use of transformed strategy space for structural estimations and corruption detection tests.

(2) We tests for unobserved heterogeneity of projects with respect to fringe/non-fringe firms, fringe/non-fringe buyers, and two procurement agencies. We find no evidence of unobserved heterogeneity of these projects.

(3) Based on the transformed strategy space, we find that projects with high engineer's estimated costs end up with winning contracts of both a high quality scores and a high prices. Projects with low engineer's estimated costs induce more competition on price and end up with positive savings rates.

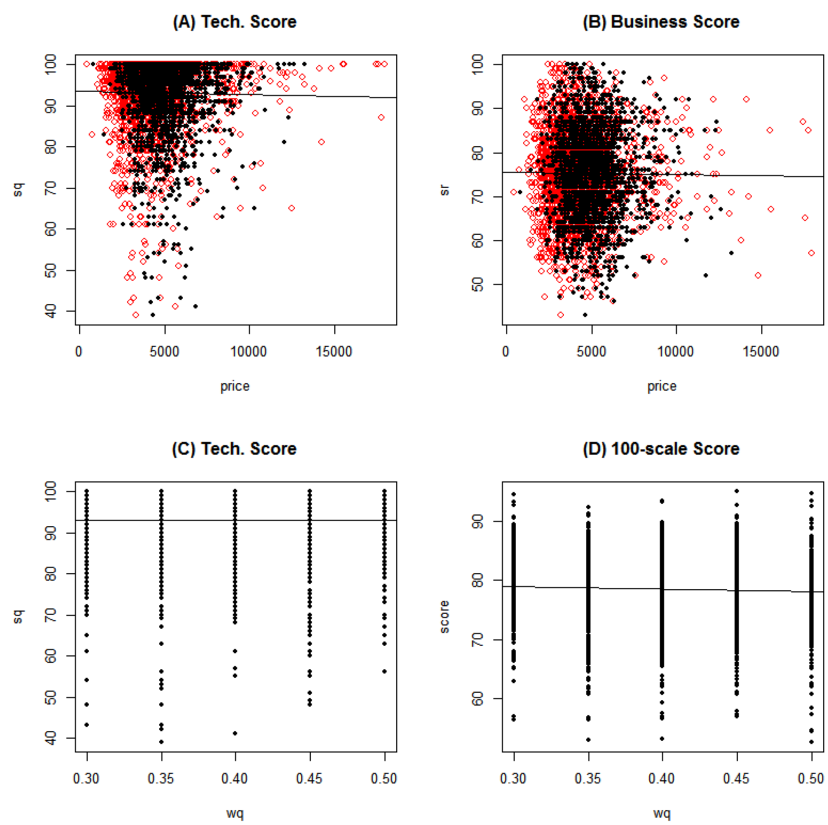
The basic regression model is

[31] Empirical analysis of auction with entry usually requires parametric assumption of private information distribution and enough auction-specific covariates. See Li and Zheng (2009) for a analysis of procurement data focusing on entry issues.

$$
\begin{aligned}
Y_t &= \alpha_0 + \alpha_1 p_{0,t} + \alpha_2 n_t + \alpha_3 w_{q,t} \\
&\quad + \alpha_4 D_{\text{fringe.firm},t} + \alpha_5 D_{\text{fringe.buyer},t} + D_{\text{agency2},t} + \epsilon_t.
\end{aligned}
$$

where Y_t stands for the dependent variable. The main independent project-specific covariates are engineer's estimated cost and the number of bidders. Because on bids level, we only observed three factor scores that are endogenously chosen by firms. Price, grand score, transformed quality and score are all functionally correlated with factor scores. The data also lacks the losing firm's identities, so there is no explanatory variable on bid level. Therefore, we estimate reduced-form models *on project level* with only winning bids. Because there are lots of firms or buyers that only appear in one project, we do not include firm or project fixed effect in the model. Instead, we add two indicators for fringe firms and fringe buyers: $D_{\text{fringe.firm}} = 1$ if the winning firm is fringe (wins only one project), zero otherwise; $D_{\text{fringe.buyer}} = 1$ if buyer is fringe (procures less than 10 projects). D_{agency2} is the indicator for if the project is process by a procurement agency 2.

Figure 10: Illustration of Winning Bids in Observed Strategy Space



Note: For clarity, we only plot winning bids in these diagrams. In diagram (A) and (B), black points represent the x-axis variable and p_0; red circles represent the same x-axis variable and submitted price.

Table 5: Reduced-form Regressions in Observed Strategy Space

	(I)	(II)	(III)	(IV)	(V)
Dep.Var.	s_q	s_r	p	s	ρ
p_0	0.0000	0.0000	1.288**		
	(0.0001)	(0.0001)	(0.0050)		
n	0.7447**	0.1236*	4.341*	0.3809**	0.0013*
	(0.048)	(0.057)	(1.99)	(0.034)	(0.0006)
w_q	11.17**	-4.95	3294.64**	1.53	-0.5758**
	(2.92)	(3.51)	(122.03)	(2.09)	(0.0377)
$D_{\text{fringe.firm}}$	0.1008	-0.5357	-28.67	-0.5463	-0.0093
	(0.4378)	(0.5265)	(18.30)	(0.3127)	(0.0056)
$D_{\text{fringe.buyer}}$	1.0405	-0.5909	-4.416	0.7195	0.0064
	(0.5998)	(0.7212)	(25.07)	(0.4285)	(0.0077)
D_{agency2}	-0.0694	0.3887	-30.78	-0.1587	-0.0034
	(0.3826)	(0.4601)	(16.0)	(0.2734)	(0.0049)
Constant	81.83**	76.48**	-3511.04**	74.66**	0.3927**
	(1.50)	(1.81)	(62.87)	(0.98)	(0.0176)
R^2	0.1050	0.0050	0.9687	0.0595	0.1123
Obs	2147	2147	2147	2147	2147

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

Figure (10) and Table 5 summarize the estimation results. The main finding concerns the technological factor weight. A higher w_q results in a higher technological score, higher price, and lower savings rate. Therefore, if a buyer wants to procure the project at a higher quality, the cost will also increase significantly. In other word, firms ask for a higher markup in a high technological weight procurement. More entry of the procurement auction increases competition and results in positive effects on all five dependent variables. In addition, diagram (A), (B) and regression (I), (II) show that for projects with different engineer's estimated costs, the technological score and business score does not show significant difference. None of these regressions shows significant differences between the firm or buyer being fringe or not. The two procurement agencies also appear to be similar.

These regressions with the dependent variable as the observed strategy space[32] have one major drawback: scores on the 100-scale are intangible concepts and hard to compare across auction. Receiving the same 100 point technological scores may mean completely different things for two projects. We also find that increasing w_q does not significantly increase grand scores in regression (IV), which is not consistent with theoretical model prediction. In addition, the goodness-of-fit, measured by R^2, are relatively low, except for model (III). Although there are significant variation on engineer's estimated costs, it is not reflected in model using s_q and s_r as dependent variable. On

[32]The original strategy space is the entire construction plan and list of itemized price, illustrated in the Appendix, the sample bid. Mapping them into 100-scale score may involves some uncertainty, which is beyond the scope of this paper.

the other hand, if one uses the grand score and savings rate as dependent variable, we lose the most informative variable as regressor p_0 as it is functionally correlated with s and ρ.

Nevertheless, we can consider the transformed strategy space with $\tilde{q}$ and $\tilde{s}$. These two variables are directly related to price and thus can be compared across auctions. Table 7 shows regression models using $\tilde{q}$ and $\tilde{s}$ as dependent variables. The goodness-of-fit improves and all six regressions show significantly positive coefficient estimates of w_q. The interpretation is that by increasing the technological factor weight for 5%, it induces the project to be delivered at a higher quality and higher score to the buyer. The estimated average buyer's payoff increment ranges from 800,858 to 1,044,340 CNY (129,171 to 168,442 USD).

The coefficient estimates of regressor n bear some more important information: increasing the number of bidders has no significant effect on $\tilde{q}$, but has significant positive effects on $\tilde{s}$. It provides evidence supporting the theoretical model: firms choose their quality level based on their own social surplus maximization problem (equation (2) and (9)), hence n does not affect their choice of $\tilde{q}$. Because this property of independent quality choices relies on fairly weak assumption, confirming it by empirical evidence also supports the validity of our strategy space transformation.

In addition, we also observe some meaningful patterns across winning bids. In Figure 11 diagram (A) and (B), we plot the density of savings rate and transformed quality respectively. The black curve represents density from all observed bids while the red dashed curve represent density from only winning bids. These two density diagrams show that winning bids have consistent pattern of both higher quality and higher price, compared to other bids. Table 6 shows that 74.24% of winning bids have the highest transformed quality in that auction. On the other hand, only 4.01% of winning bids have the highest savings rate in that auction. On average, winning bids ask for higher prices than losing bids (the average savings rate is lower). Among the 2147 auction we observed, 145 projects ended up with *negative* savings rates on the contract where prices submitted by winning firms are higher than the engineer's estimated costs. Moreover, all these 145 negative savings rate auctions occur at projects with engineer's estimated costs higher than 5,565 thousand CNY. Hence, a high p_0 project is more likely to be awarded to a high quality and high price bidder. illustrated in Figure 11 diagram (E).

In a price-only procurement auction market, competition is solely on price, which results in low markup (high savings rate). But in a scoring auction environment, both theoretical model and our empirical results find that competition is mainly on the quality dimension. Inducing competition on quality could be beneficial to the buyer. Compared to price-only procurement with a fixed quality standard, scoring auction allows quality to be a choice variable and hence increases both buyer's payoff and the entire social surplus of the project.[33] However, these high quality and high price contracts give more room for quality manipulation corruption, especially when p_0 is high. These

[33] Asker and Cantillon (2008) shows a straightforward proof. Suppose the minimum quality is set at q and the scoring rule represents the buyer's true preference. By corollary 1 in this paper: Experected payoff in price only auction $= V(q) - E\left[C(q, \theta_{(n-1:n)})\right] = E\left[(V(q) - C(q,\theta))_{(2:n)}\right] \leq E\left[\max_q (V(q) - C(q,\theta))_{(2:n)}\right] = E\left[k_{(2:n)}\right] =$ Expected payoff in scoring auction.

signs of corruption motivate the corruption detection tests below.

Table 6: Pattern of Winning Bids

	Mean of All Bids	Mean of Winning Bids	Highest in the Auction Number	Percentage	Lowest in the Auction Number	Percentage
s_q	60.48	93.05	1582	73.68%	4	0.19%
s_r	72.30	75.32	455	21.19%	193	8.99%
ρ	0.1980	0.1691	86	4.01%	530	24.69%
$\tilde{q}$	8260.86	4746.27	1594	74.24%	3	0.14%

Table 7: Regression in Transformed Strategy Space

	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var.	Transformed Quality $\tilde{q}$			Transformed Score $\tilde{s}$		
Data	All	$p_0 < 5,565$	$p_0 \geq 5,565$	All	$p_0 < 5,565$	$p_0 \geq 5,565$
n	3.05	-0.85	47.56	20.15**	16.38**	39.08**
	(16.90)	(11.44)	(25.71)	(6.38)	(5.71)	(7.01)
w_q	19,007.23**	16,876.99**	26,367.82**	16,017.17**	14,326.23**	20,886.79**
	(1,038.50)	(690.20)	(1,652.03)	(391.94)	(344.24)	(450.61)
$D_{\text{fringe.firm}}$	200.15	186.62	-69.02	69.71	80.97	-41.14
	(155.65)	(103.21)	(248.59)	(58.75)	(51.48)	(67.81)
$D_{\text{fringe.buyer}}$	-158.57	-80.29	-560.93	-24.58	-21.19	-131.02
	(213.30)	(141.10)	(342.64)	(80.50)	(70.37)	(93.46)
D_{agency2}	121.59	124.31	-242.29	70.75	61.07	-28.83
	(136.08)	(89.99)	(218.49)	(51.36)	(44.88)	(59.60)
Constant	2,021.96**	1,552.37**	2,431.78**	-1,234.99**	-987.43**	-2,068.26**
	(485.21)	(325.33)	(756.52)	(183.13)	(162.26)	(206.35)
R^2	0.1421	0.2943	0.3066	0.4446	0.5378	0.7863
Obs	2147	1551	596	2147	1551	596

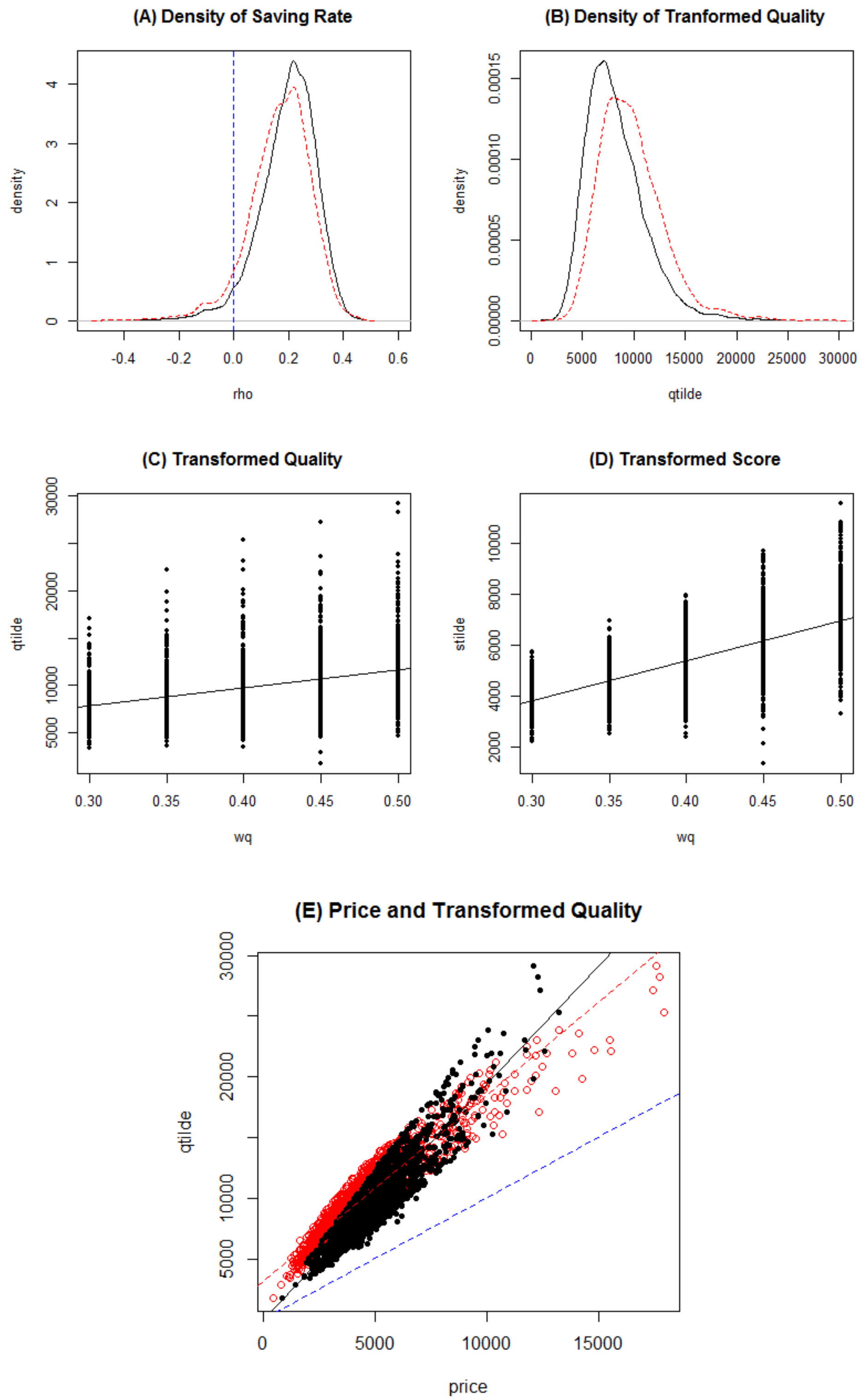
Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

4.3 Structural Estimation and Corruption Detection Tests

Because the data set lacks bid-specific covariates, reduced-form regression cannot be performed on bid level. Project level regressions put aside all losing bids as they are endogenous variables, but structural estimation can draw information from all bids. The pattern of both winning bids and losing bids together reveal whether the bidding behavior is competitive. Therefore, although our structural estimation is based on more assumptions, it is able to use more data and yield more results.

In the transformed strategy space, the structural estimation and corruption detection method developed in Section 3.1 and 3.2 can be directly applied. Varying the scoring rule affects the

Figure 11: Illustration of Winning Bids in Transformed Strategy Space



Note: In diagram (A) and (B), the black curve represents density of all bids; the red dashed curve represent density of winning bids. In diagram (E), black points represent observed $(\tilde{q}, p_0)$; red circles represent $(\tilde{q}, p)$; the blue dashed line is the 45 degree line.

distribution of pseudotype, therefore, we divide the data into sub-samples for different w_q. We also perform estimation separately for two procurement agencies. For each sub-sample, we apply formula (13) to structurally estimate pseudotypes. The estimation results are reported in Table 8 and Figure 12. Notice that $\hat{k}$ represents the total social surplus of each firm producing at its efficient level; $\tilde{s}$ represents how much of the social surplus is harvested by the buyer. Their difference, $\hat{k} - \tilde{s}$, is the *estimated rent* retained by the firm. A desirable scoring auction shall encourage competition and lead to higher surplus to the buyers and less rent to the firms (referred to as *performance* hereafter).

Table 9 compares performance of two procurement agencies. The projects processed by two procurement agencies are similar in their observed characteristics, but we find that in general, firms bid in procurement agency 1 receives gets higher rent than agency 2. Specifically, overall, firms at procurement agency 1 ask for 63,030 CNY (10,166 USD) more rent compared to agency 2. Nevertheless, if we consider only winning bids, at each sub-sample, firms win contract at procurement agency 1 do not earns significantly more rent than agency 2. If we compare "vertically" on technological factor weights, a higher w_q in general leads to both higher transformed scores and higher rents, benefiting both parties. Quality weights reflect the buyers' willingness-to-pay for high quality projects, while the supplying firms only care monetary compensations. Serving buyers with higher willingness-to-pay naturally lead to higher payoffs for both sides. Prediction of the theoretical model and estimation results are therefore consistent.

Aforementioned, a high rent alone is not the sign of corruption. To check whether there are signs of quality manipulation corruption, we need to explore the consistently suspicious patterns of relationship among bids revealed in large sample. We apply three tests proposed in Section 3.2 based on structural estimation outputs. Table 10 and Figure 13 show results of test I and II.

For test I, we find that there are a total five sub-samples that reject the competitive bidding model. In general, they happen at high w_q auctions. For test II, none of sub-samples rejects the competitive model, according to bootstrap critical value. Notice that, for both tests, the original p-value and bootstrap p-value give opposite conclusions of the same hypothesis test. Because the correlation of observations, we should draw a conclusion based on bootstrap critical value.

For test III, we consider six regression models shown in Table 11 and find that only one coefficient of D_{agency2} is significant. Regression (VI) is run on the sub-sample with high engineer's estimated costs. It implies the first spacing of transformed score is larger at procurement agency 2, which is the sign of aggressive bidding behavior. Since we also find that rent at procurement agency 2 is generally lower, the reality could be that firms are earning their rent under the table by delivering at low quality. Hence, antitrust authorities should spend more investigation resources on projects processed by procurement agency 2 with high engineer's estimated costs.

As a summary, a majority of the data set passes our corruption detection tests. The data set supports the theoretical prediction of a competitive model. Recall Figure 8, these failures of rejection of corruption detection tests provide evidence that the whole competitive scoring auction model is valid. The structural estimation is thus trustworthy for this data set. For some sub-

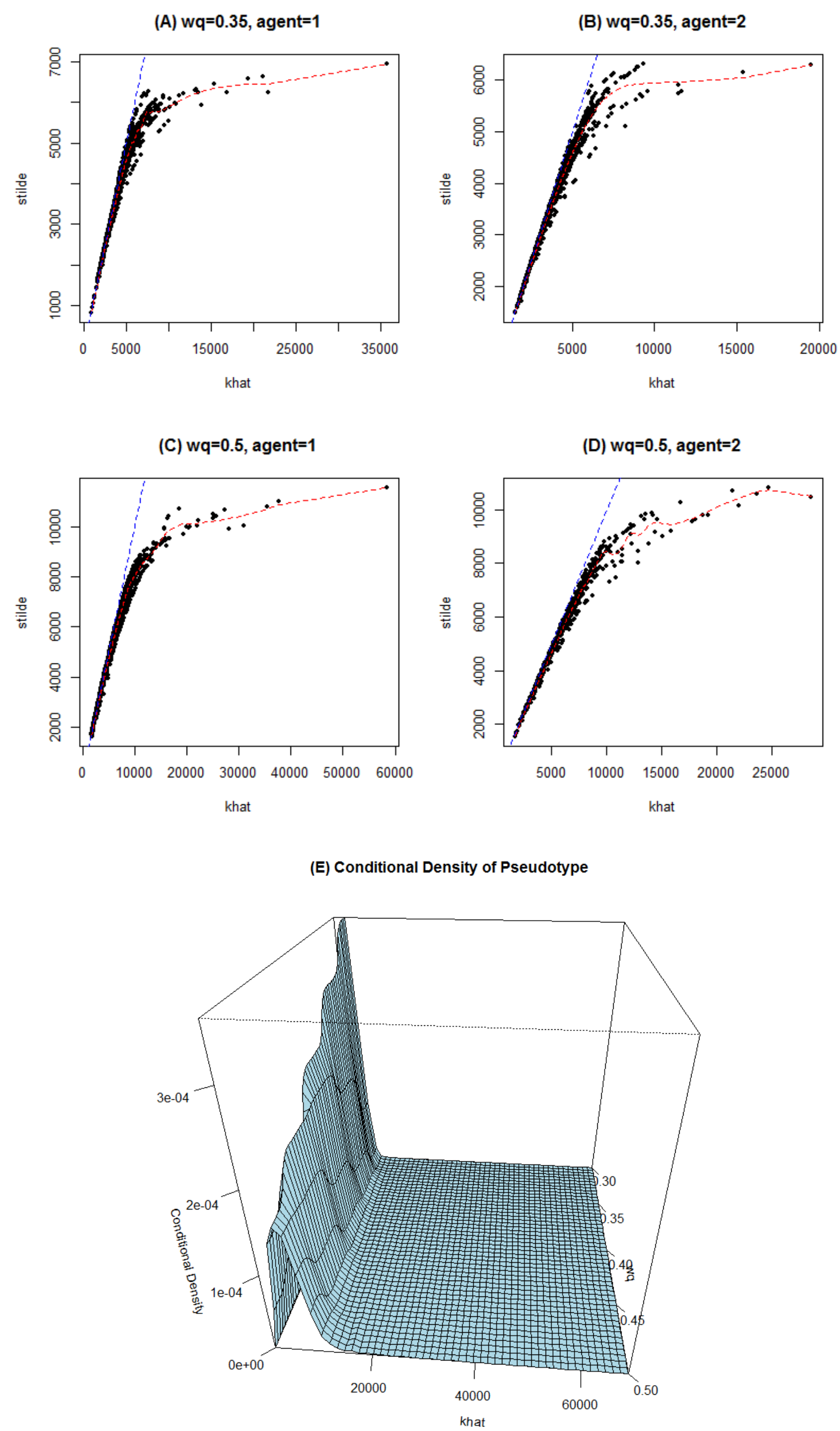
Table 8: Structural Estimation Results

Sub-sample w_q	Agency	No. of Projects	No. of Bids		Mean	SD	Min	Max
0.3	1	223	2384	$\hat{k}$:	3,358.97	1,292.52	1,156.09	27,485.10
				$\tilde{s}$:	3,126.94	797.31	1,135.51	5,747.27
0.3	2	87	913	$\hat{k}$:	3,389.06	1,088.43	1,308.24	10,132.73
				$\tilde{s}$:	3,173.24	811.30	1,264.24	5,407.96
0.35	1	244	2266	$\hat{k}$:	3,913.30	1,750.54	891.22	35,700.95
				$\tilde{s}$:	3,546.35	1,032.03	821.27	6,945.76
0.35	2	134	1228	$\hat{k}$:	3,942.98	1,473.08	1,540.13	19,494.30
				$\tilde{s}$:	3,617.00	1,014.27	1,494.09	6,303.84
0.4	1	384	3437	$\hat{k}$:	4,451.63	2,250.47	1,253.65	56,690.62
				$\tilde{s}$:	3,980.14	1,242.36	1,175.52	7,971.81
0.4	2	195	1687	$\hat{k}$:	4,448.67	1,805.02	1,052.06	22,355.00
				$\tilde{s}$:	4,053.84	1,227.13	1,015.22	7,568.83
0.45	1	407	3335	$\hat{k}$:	5,342.17	3,181.92	773.96	68,737.29
				$\tilde{s}$:	4,691.41	1,515.09	712.09	9,711.02
0.45	2	176	1510	$\hat{k}$:	5,131.95	2,360.78	1,603.51	28,208.93
				$\tilde{s}$:	4,585.34	1,505.13	1,431.86	9,324.88
0.5	1	208	1567	$\hat{k}$:	6,054.45	3,463.00	1,724.88	58,419.10
				$\tilde{s}$:	5,224.02	1,870.11	1,620.04	11,559.27
0.5	2	89	636	$\hat{k}$:	6,474.78	3,054.39	1,699.51	28,500.12
				$\tilde{s}$:	5,667.42	1,894.45	1,539.81	10,811.87

samples of the data set, we find signs of quality manipulation corruption. The data patterns (Figure 11 diagram (E)) and corruption detection tests results suggest that antitrust investigation should focus on projects with high technological weights and high engineer's estimated costs, especially those processed by procurement agency 2.

It is worth mentioning that high technological weight and estimated cost are proxy for complexity of the project. Bajari and Tadelis (2001) and Tadelis (2012) compare auction and negotiation at different level of complexity. Complexity and high uncertainty may potentially jeopardize the advantage of auction due to costly renegotiation for *ex post* adjustment. Find a reputable supplier and use a cost-plus contract under negotiation can be a better option. In their analysis, auctions are price-only and quality of the projects are fixed by the buyer. Because negotiations bring in reputable firms that can help design the complex project and thus save *ex post* adaption cost. In a scoring auction, quality and design becomes the choice variable of the firm, so it reaps benefit from both price-only auction and negotiation. However, all these cross-procurement-scheme comparison are not robust when quality is not perfectly observable *ex ante* or verifiable *ex post*. The principal-agent problem analyzed in this paper or uncertainty problem studied in Takahashi (2014) not only affects optimal scoring rule or auction format, but also optimal procurement scheme.

Figure 12: Illustration of Structural Estimation Result



Note: Diagram (A) to (D) are estimated pseudotypes and related transformed scores, $(\hat{k}, \tilde{s})$. The blue line is the 45 degree line. The red dashed line is a smoothed spline. Diagram (E) shows the kernel smoothed conditional density $f(\hat{k}|w_q)$

Table 9: Comparison of Two Procurement Agency

	Pro. Agency 1		Pro. Agency 2		t-test of Equal Mean	
	Mean	SD	Mean	SD	Statistic	p-value
n	8.860	3.937	8.772	3.681	0.5029	0.6151
p_0	5030.46	1515.45	5091.75	1395.77	-0.9212	0.3571
$\tilde{s}$	5435.89	1489.60	5484.52	1470.62	-0.7101	0.4778
	Estimated Rent ($\hat{k} - \tilde{s}$) of All Bids					
$w_q = 0.3$	232.03	749.33	215.82	380.90	0.8162	0.4144
$w_q = 0.35$	366.95	1,014.52	325.98	664.14	1.4365	0.1509
$w_q = 0.4$	471.49	1,396.39	394.82	873.68	2.4007	**0.0164**
$w_q = 0.45$	650.76	2,261.30	546.60	1,191.91	2.0940	**0.0363**
$w_q = 0.5$	830.42	2,089.09	807.36	1,551.44	0.2846	0.7760
Overall	498.63	1,634.84	435.60	985.48	3.2844	**0.0010**
	Estimated Rent ($\hat{k} - \tilde{s}$) of Winning Bids					
$w_q = 0.3$	816.54	2,212.67	646.67	821.04	0.9857	0.3251
$w_q = 0.35$	1,239.56	2,667.93	967.06	1,640.22	1.2279	0.2203
$w_q = 0.4$	1,534.06	3,529.86	1,208.71	2,164.06	1.3692	0.1715
$w_q = 0.45$	2,006.02	5,095.38	1,632.98	2,743.71	1.1428	0.2536
$w_q = 0.5$	2,228.15	4,393.84	2,183.71	3,214.99	0.0972	0.9226
Overall	1,605.41	3,914.21	1,326.43	2,330.57	2.0551	**0.0400**

Note: Bold numbers indicate rejection of the null at 0.05 significance level.

Table 10: Results of Test I and II

Sub-sample		Test I				Test II			
w_q	Agency	Test Stat.	p-value	BT c.v.	BT p.v	Test Stat.	p-value	BT c.v.	BT p.v
0.3	1	-6.7284	1.0000	-4.0347	0.5400	0.2422	**0.0000**	0.2870	0.6600
	2	-3.3526	0.9994	-5.0863	**0.0000**	0.2644	**0.0023**	0.3563	0.8350
0.35	1	-6.2109	1.0000	-4.9973	0.2800	0.2131	**0.0000**	0.2623	0.7000
	2	-5.0055	1.0000	-4.6895	0.1050	0.2836	**0.0000**	0.3209	0.3800
0.4	1	-7.3851	1.0000	-5.8942	0.2600	0.2396	**0.0000**	0.2656	0.4750
	2	-4.7708	1.0000	-5.6530	**0.0000**	0.2513	**0.0000**	0.2923	0.5850
0.45	1	-5.3859	1.0000	-5.8718	**0.0000**	0.2260	**0.0000**	0.2604	0.6500
	2	-5.5837	1.0000	-5.8815	**0.0150**	0.2330	**0.0001**	0.2784	0.6700
0.5	1	-5.3204	1.0000	-5.2688	0.0700	0.2067	**0.0001**	0.2500	0.5900
	2	-4.4933	1.0000	-4.8648	**0.0100**	0.2584	**0.0026**	0.3258	0.5300

Note: BT c.v. and BT p.v stand for "bootstrap critical value " at 0.05 significance level and "bootstrap p-value" respectively. They are computed based on 199 bootstrap samples at project level.
Bold numbers indicate rejection of the null.

Figure 13: Result of Test I and Test II

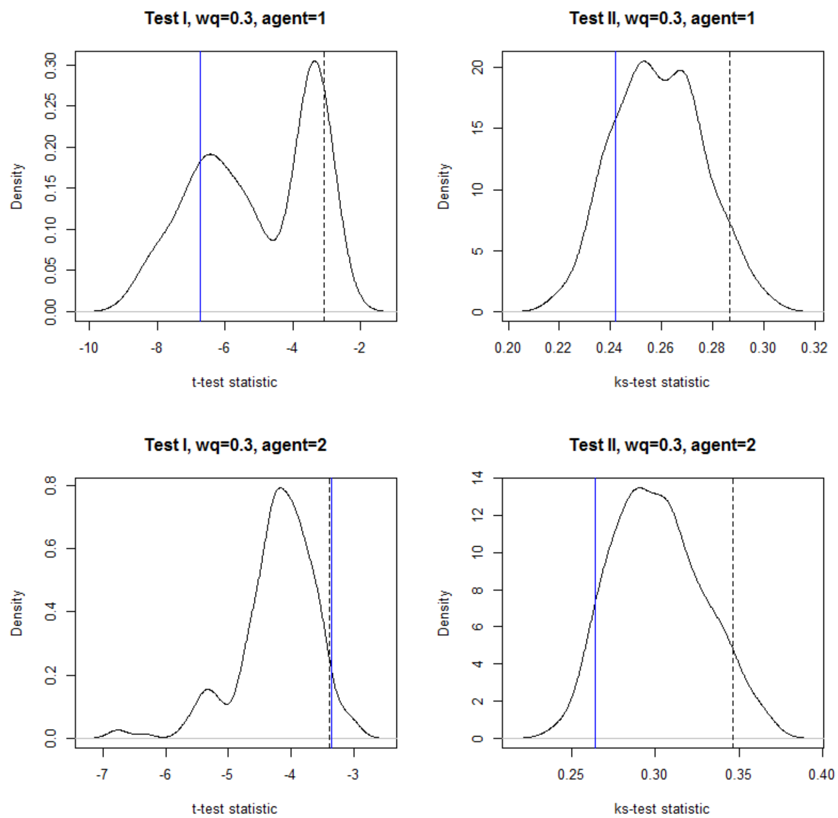


Table 11: Results of Test III

	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var	First Spacing of $\hat{k}$			First Spacing of $\tilde{s}$		
Data	All	$p_0 < 5565$	$p_0 \geq 5565$	All	$p_0 < 5565$	$p_0 \geq 5565$
D_{agency2}	-39.78	3.372	-245.88	20.45	4.750	103.25*
	(110.36)	(29.02)	(591.77)	(16.84)	(18.05)	(46.37)
3rd order	0.3082**	-0.0863**	0.0568	-0.0506**	-0.0922**	-0.2180**
statistic	(0.0296)	(0.0129)	(0.1194)	(0.0072)	(0.0098)	(0.0246)
w_q	-1,120.81	3,472.47**	5,910.49	1,813.97**	2,298.35**	4,947.44**
	(929.72)	(268.23)	(5274.83)	(160.63)	(180.07)	(578.31)
n				-24.62**	-17.96**	-9.485
				(2.199)	(2.432)	(6.388)
Constant	-236.29	-547.17**	187.46	64.50	-56.40	-207.70
	(337.84)	(88.85)	(1857.27)	(60.86)	(66.11)	(175.94)
R^2	0.0556	0.0986	0.0089	0.1588	0.1875	0.2875
Obs	2147	1551	596	2147	1551	596

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

5 Conclusion

We conclude the paper by reviewing results from a policy point of view and proposing some directions for future research.

We develop a theoretical and structural estimation framework that can be applied to a wide range of procurement auction data sets. Our method can be applied to scoring auction data sets with the most widely used linear weighted factor scoring rule. The total social surpluses (pseudotypes), buyer's payoff, and firms' rents can be structural estimated from the data. The effect of varying scoring rules on auction performance can be quantitatively predicted. It provides empirical tools that use past auction data to design an optimal scoring rule for procurement in the future.

We empirically analyze a data set of server room procurement auction. We show how to transform the 100-scale observed strategy space into an equivalent strategy space that is quantitatively related to the buyer's payoff. The data pattern and estimation results provide evidence of the theoretical scoring auction model on two model implications. First, under additively separable scoring rule, the choice of quality can be separated from choice of price/score. The reduced-form estimation shows that the selected qualities are not affected by the number of competing bidders, but selected scores are affected. Second, with competition on both price and quality, firms mainly compete on offering high quality and expensive contracts. In the data set, over 70% of winning bids have the highest quality, but only about 4% of winning bids have the lowest price. Therefore, a reliable quality evaluation procedure is very important in keeping the auction efficient.

We also explore the effect of varying technological weight (quality weight). The theoretical model predicts that higher weight on quality induces firms to submit bids at higher quality and score, which is confirmed by the data at transformed strategy space. The structural estimation results show that projects procured with higher quality weights benefit result in both higher payoff for the buyers and more rents for the winning firms. The buyers are restricted in picking the quality weight because it must reflect its willingness-to-pay of higher quality. The theoretical model of scoring auctions shows that the buyer will not over-state its preferences on quality,[34] instead, the optimal scoring rule "shade" buyer's preference on quality to avoid giving up too much rent to the efficient firm.

Moreover, a higher quality weight also gives more room for quality manipulation. Lengwiler and Wolfstetter (2006) suggest reducing quality weight to account for the less reliable quality score due to the possibility of corruption. We run the three corruption detection tests proposed in this paper. We apply find some evidence of corruption in procurements with higher quality weights and higher engineer's estimated costs. Therefore, in designing the scoring rule, the buyers need to balance its efficiency and risk of quality manipulation corruption. In general, most sub-samples pass our tests and are consistent to competitive model. These failures of rejecting the null provide positive evidence of validity of structural estimation based on the competitive model.

[34] Huang and Xia (2015) shows that the buyer may over-state it to fight against quality manipulation in certain situation.

Our empirical analysis method and corruption detection tests only require fairly standard data recorded from procurement auctions. In principle, they can be applied to any industry with enough observations. We show that with corruption, the model is not fully identified as one cannot observe the side-payment and the actual scope of corruption, but tests can be perform on auction data sets. If the scope of corruption is large (and sample size is big), powers of these tests improve. Therefore, recording and aggregating procurement auction data is valuable for both in designing optimal procurement scheme and identifying corrupt behavior.

There are several limitation that can potentially jeopardize our results. Some of them are worthy for further research in the future:

(1) Our conjecture of the form of corruption is based on intuition and industry experience. Besides quality manipulation, the procurement auction may involve in other form of corruption, like bidding ring and bid revision. In the industry, there is another prominent form of corruption called "*attaching*". It means that a firm with a bad reputation or less experience uses another firm's name to submit its bid, so it can receive a higher business factor score. These different forms of corruption are not mutually exclusive, and they have different implications as to how they distort bidding behaviors. Therefore, identifying the exact form of corruption may be even more important than detecting the existence of corruption. Co-existence of several forms of corruption will further complicate the analysis.

(2) Our estimation and tests are based on pseudotypes. The distribution of pseudotype changes as scoring rule varies. Hence, our analysis is restricted when there is a great deal of variation in scoring rules across auctions and they cannot be easily categorized. In this case, especially when the sample size is small, a parametric or semi-parametric approach shall replace our nonparametric approach. Once a parametric cost function is specified, the optimal choice of quality and scores can be expressed, then the data on quality and score can reverse-engineer the parameters.

(3) Due to the limitation of data and modeling capacity, we consider a relatively simple model. We make assumptions on risk neutrality, independent private information, exogenous entry, and convexity of cost function. Bajari and Tadelis (2001) and Bajari et al. (2014) point out that in such a complicated bidding environment, strategic unbalanced bidding and adaption cost are also important issues. We also let the buyer and procurement agency to be passive in this paper. These factors could be valuable extensions for future research.

(4) With *ex post* observation of convicted corruption firms (for example, from investigation reports from collapsed bridges), construct a data set with identities of corrupted firm is possible. Then researchers can study the "in-sample" property of these corruption detection tests. A more sophisticated model can also be identified and estimated. Using historical auction data and antitrust records together with economic model could potentially construct stronger tools for antitrust purposes.

Appendix

Proof of Lemma 1: Define $C(v, \theta)$ as the value function of the minimization problem:

$$C(v, \theta) = \min_q C(q, \theta) \text{ s.t } V(q) = v.$$

Define a quality index v of $q \in \mathbb{R}^L$ by $v \equiv V(q)$. Under assumption QL and CF, $C(v, \theta)$ is single-valued, strictly increasing, convex, and continuous function. The Lagrangian expression of the minimization problem is

$$\mathcal{L} = C(q, \theta) - \lambda(V(q) - v).$$

When $v > 0$, $q \gg 0$, first-order condition yields

$$\begin{cases} C_q(q|\theta) - \lambda V_q(q) = \mathbf{0} \in \mathbb{R}^L, \\ V(q) - v = 0 \in \mathbb{R}. \end{cases} .$$

By assumption CF, C is strictly convex in q, C_{qq} is positive definite. By assumption QL, $V(q)$ is weakly concave, $V_{qq} \leq 0$. So there is a unique solution to the system of these $L+1$ linear equations, denoted as $q(v|\theta)$ and $\lambda(v|\theta)$. The solution correspondence of the minimization problem is the value function $C(v, \theta) = C(q(v|\theta), \theta)$.

By envelop theorem, the value function satisfies $C_v = \lambda$. $V(q) - v = 0$ implies $V_q(q(v|\theta))q_v(v|\theta) = 1$. Therefore, $C_q(q|\theta) - \lambda V_q(q) = 0$ implies $\lambda = C_q(q|\theta)/V_q(q) = C_q(q|\theta)q_v(v|\theta) = C_v > 0$.

To further show $C_{vv} > 0$, differentiate FOC above with respect to v:

$$\frac{\partial}{\partial v}(\frac{\partial \mathcal{L}}{\partial q}) = \begin{pmatrix} C_{qq}(q|\theta) - \lambda V_{qq}(q) & -V_q^T \\ V_q & 0 \end{pmatrix} \begin{pmatrix} q_v \\ \lambda_v \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\Rightarrow V_q^T \lambda_v = (C_{qq}(q|\theta) - \lambda V_{qq}(q))q_v.$$

Premultiply by q_v^T, because $V_q(q(v|\theta))q_v(v|\theta) = 1$ and $\frac{\partial^2 \mathcal{L}}{\partial q^2} = C_{qq} - \lambda V_{qq}$ is positive definite (PD) for minimization, we have

$$\underbrace{q_v^T V_q^T}_{=1} \lambda_v = q_v^T \underbrace{(C_{qq} - \lambda V_{qq})}_{PD} q_v > 0$$

Therefore, we can transform the cost function by $C(q(v|\theta), \theta) = C(v, \theta)$, which satisfies property $C_v = \lambda(v|\theta) > 0$ and $C_{vv} = \lambda_v(v|\theta) > 0$. *Q.E.D.*

Proof of Theorem 1: (2) holds as as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following $s(\cdot)$, a generic firm solves $\max_s(k-s)\Pr(\text{win}|s) = (k-s)[F_K(s^{-1}(s))]^{n-1}$. The first-order condition yields $(k-s)(n-1)[F_K(s^{-1}(s))]^{n-2}f_K(s^{-1}(s))\frac{ds^{-1}(s)}{ds} - [F_K(s^{-1}(s))]^{n-1} = 0$. At the symmetric equilibrium, we have differential equation

$$s(k)(n-1)[F_K(k)]^{n-2}f_K(k) + s'(k)[F_K(k)]^{n-1} = k(n-1)[F_K(k)]^{n-2}f_K(k). \tag{24}$$

$$\Leftrightarrow \frac{d\left(s(k)[F_K(k)]^{n-1}\right)}{dk} = k(n-1)[F_K(k)]^{n-2}f_K(k).$$

Integrate on both side with boundary condition $s(\underline{k}) = 0$,

$$s(k) \quad = \quad \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2}f_K(t)dt}{[F_K(k)]^{n-1}} = k - \frac{\int_{\underline{k}}^{k}[F_K(t)]^{n-1}dt}{[F_K(k)]^{n-1}}.$$

The last equality is obtained via integration by parts. The equilibrium price can be computed by $p(\theta) = V(q(\theta)) - s(K(\theta))$. *Q.E.D.*

Note that when θ is one-dimensional and $C_\theta < 0$, it reduces to (3) in Che (1993). By envelop theorem, from the value function $K(\theta) = \max V(q) - C(q,\theta)$, we have $K'(\theta) = C_\theta(q(\theta),\theta) < 0$. The lowest type $K(\overline{\theta}) = \min K(\theta) = \underline{k}$. $[1-F(\theta)]^{n-1} = [\Pr(\Theta > \theta)]^{n-1} = [\Pr(K(\Theta) < K(\theta))]^{n-1} = [F_K(k)]^{n-1}$. Let $k = K(\theta)$, $dk = K'(\theta)d\theta = C_\theta(q(\theta),\theta)d\theta$,

$$\begin{aligned} s(K(\theta)) &= K(\theta) - \frac{\int_{\underline{k}}^{K(\theta)}[F_K(t)]^{n-1}dt}{[F_K(k)]^{n-1}} \\ &= K(\theta) - \frac{\int_{\theta}^{\overline{\theta}}[1-F(\tau)]^{n-1}K'(\tau)d\tau}{[1-F(\theta)]^{n-1}} \\ &= K(\theta) - \frac{\int_{\theta}^{\overline{\theta}}[1-F(\tau)]^{n-1}C_\theta(q(\tau),\tau)d\tau}{[1-F(\theta)]^{n-1}}. \end{aligned}$$

Hence, $p(\theta) = V(q(\theta)) - s(K(\theta)) = C(q(\theta),\theta) + \int_{\theta}^{\overline{\theta}} C_\theta(q(\tau),\tau)\frac{[1-F(\tau)]^{n-1}}{}d\tau/[1-F(\theta)]^{n-1}$.

Proof of Corollary 1: $k_{(1:n-1)}$ has distribution function $F_K^{(1:n-1)}(t) = [F_K(t)]^{n-1}$, and density $f_K^{(1:n-1)}(t) = (n-1)[F_K(t)]^{n-2}f_K(t)$. If the winner has pseudotype k, the conditional expectation of the highest rival's pseudotype is

$$E[k_{(1:n-1)}|k_{(1:n-1)} < k] = \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2}f_K(t)dt}{[F_K(k)]^{n-1}} = s(k),$$

which is equal to the score the winner will bid. At the equilibrium, the winner has pseudotype being the highest order statistic $k_{(1:n)}$, while the second highest bidder has pseudotype $k_{(2:n)}$, hence $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$. *Q.E.D.*

Proof of Theorem 2: (1) Quality

Suppose the corrupted firm with type θ bids (p',q') at some $q' \neq q_m$, we can show that by choosing

q_m, the corrupted firm can always find a price p_m that yields a higher payoff upon winning. Let $p_m = V(q_m) - V(q') + p'$, then (p', q') and (p_m, q_m) have the same score because $S(q', p') = V(q') - p' = V(q_m) - p_m = S(q_m, p_m) = s$. These two bids has the same expected payoff $\Pr(\text{win}|s)$. Their expected payoffs satisfies

$$
\begin{aligned}
\pi(p_m, q_m) - \pi(p', q') &= \left[p_m - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\
&= \left[V(q_m) - V(q') + p' - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\
&= \left[V(q_m) + C(q_m - m, \theta) - \left(V(q') - C(q' - m, \theta)\right)\right] \Pr(\text{win}|s) > 0,
\end{aligned}
$$

because q_m is chosen by (9). The scoring rule being quasilinearity (additively separable) is essential for this result to hold.

(2) Score and price

Under assumption UA, all other firms pick their score according to (8), so the corrupted firm's pick its core according to

$$
\max_{s_m} (k_m - s_m) \Pr(\text{win}|s_m) = (k_m - s_m) \left[F_K(s^{-1}(s_m))\right]^{n-1}.
$$

Following the same step in getting (8), the corrupted firm choose its score according to $s(k_m) = k_m - \int_{\underline{k}}^{k_m} [F_K(t)]^{n-1} dt / [F_K(k_m)]^{n-1}$. The corresponding price is $p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)) = C(q_m(\theta) - m, \theta) + \int_{\underline{k}}^{K_m(\theta)} [F_K(t)]^{n-1} dt / [F_K(K_m(\theta))]^{n-1}$.

(3) For any $m > 0$, at the equilibrium, $q_m(\theta) > q(\theta)$, $K_m(\theta) > K(\theta)$, and $s(K_m(\theta)) > s(K(\theta))$.

The unique solution of quality choice of (2) and (9) are both determined by their first-order conditions. Suppose $\tilde{q}$ solves $V_q(q) = C_q(q, \theta)$. Because $C_{qq} > 0$, the cost function has increasing slope, $V_q(\tilde{q}) = C_q(\tilde{q}, \theta) > C_q(\tilde{q} - m, \theta)$. By assumption QL, $V_{qq} \leq 0$, the solution to $V_q(q) = C_q(q - m, \theta)$ must be strictly larger than $\tilde{q}$, therefore $q_m(\theta) > q(\theta)$.

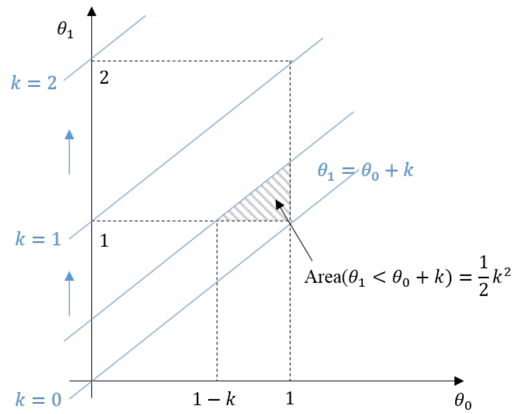
The other two are straight-forward. Because $C_q > 0$, $C(q - m, \theta) < C(q, \theta)$ for all q and θ, $K_m(\theta) = \max_q V(q) - C(q - m, \theta) > \max_q V(q) - C(q, \theta) = K(\theta)$. The equilibrium score bidding function $s(\cdot)$ is increasing, hence $s(K_m(\theta)) > s(K(\theta))$. It is obvious that all three effects magnify as m increases. *Q.E.D.*

Derivation of $F_K(\cdot)$ in the Monte Carlo Example

θ_0, θ_1 are jointly uniformly distributed with density equals 1 at the area $[0, 1] \times [1, 2]$.

$$
F_K(k) = \Pr(K(\theta_0, \theta_1) < k) = \Pr(\theta_1 - \theta_0 < k) = \Pr(\theta_1 < k + \theta_0).
$$

Figure 14: Derivation of $F_K(\cdot)$



With help of Figure 14, when $k \in [0, 1]$,

$$\begin{aligned} F_K(k) &= \int_{1-k}^{1} \int_{1}^{\theta_0+k} 1 d\theta_1 d\theta_0 = \int_{1-k}^{1} (\theta_0 + k - 1)\, d\theta_0 \\ &= \left[\frac{1}{2}\theta_0^2 + (k-1)\theta_0\right]_{1-k}^{1} = \frac{1}{2} + (k-1) - \frac{1}{2}(1-k)^2 - (k-1)(1-k) = \frac{k^2}{2}. \end{aligned}$$

When $k \in (1, 2]$,

$$\begin{aligned} F_K(k) &= 1 - \int_{0}^{2-k} \int_{\theta_0+k}^{2} 1 d\theta_1 d\theta_0 = \int_{0}^{2-k} (2 - \theta_0 - k)\, d\theta_0 \\ &= \left[(2-k)\theta_0 - \frac{1}{2}\theta_0^2\right]_{0}^{2-k} = 1 - (2-k)^2 + \frac{1}{2}(2-k)^2 = 1 - \frac{(2-k)^2}{2}. \end{aligned}$$

We therefore get $F_K(\cdot)$ in (16).

A sample bid[35]

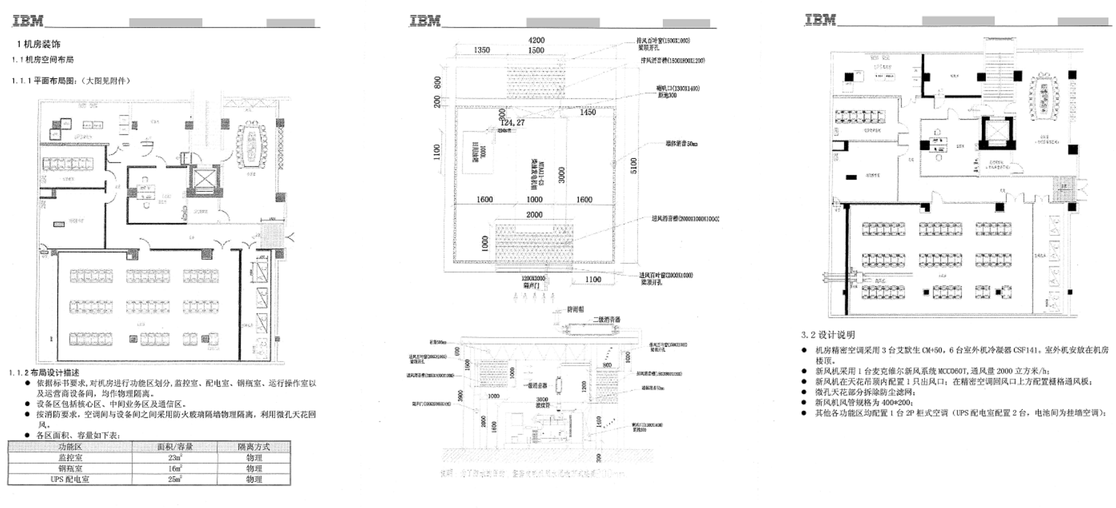
This is a bid of a server room construction project. The buyer is Bank of Dongguan, a regional bank centered at Dongguan, Guangdong province, China. The firm is IBM Engineering Technology (Shanghai) Co., Ltd.. The bid consists of a construction plan and a detail list of items and their costs. The construction plan is a 19-page document including standard of construction, condition of delivery, delivery date, equipment purchase plan, payment plan etc. Some selected pages are shown in Figure 15. The itemized price list is a 11-page spreadsheet. Table 12 shows its major categories, categorical prices, and total price (3,630,000 CNY).

[35]The author receives authorization to disclose the document for non-profit academic research purpose. The original document is in Chinese. All technological details are remain confidential and the relevant copyrights are owned by Bank of Dongguan and IBM Engineering Technology (Shanghai) Co., Ltd. The author declare that he has no relevant or material financial interests that relate to the research described in this paper.

Table 12: Summary of the Itemized Price List

Category	Price (CNY)	No. of Items
Data center room renovation	924,295	17
Main power distribution system	108,185	11
Auxiliary power distribution system	176,830	14
Uninterrupted power supply (UPS) system	913,680	13
Generators and environmental engineering	413,050	14
Air conditioning	99,170	11
Precision air conditioning	528,570	2
Cabinets and cabling system	242,230	9
Lightning protection	23,820	3
Room monitoring	185,120	43
Room bridging	15,050	4
Total	3,630,000	141

Figure 15: Selected Pages of the Construction Plan



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