Detecting Quality Manipulation Corruption in Scoring Auctions: A Structural Approach

YANGGUANG HUANG†

Department of Economics
University of Washington

Oct 2015

Abstract

Scoring auctions are widely used in procurement of differential quality objects, which ask bidders submit bids as price-quality combinations. The buyer may lack the expertise to evaluate quality of these bids and have to hire a procurement agency to help. Corruption via quality manipulation arises when the procurement agency is bribed to elevate quality score of a corrupted firm. It causes a systematic distortion of bids and such distortion is testable. We propose three tests to detect corruption based on structural estimation of scoring auction data. They are applied to a server room procurement auction data set.

Keywords: Scoring Auction, Structural Estimation, Quality Manipulation Corruption, Corruption Detection.

JEL codes: C1, D44, H57, L40, L74.

*I would like to thank Yanqin Fan, Xu Tan, Quan Wen, and fellow students Ming He, Jenny Ho, Xuetao Shi, and seminar participants at University of Washington for their comments and suggestions. The latest version of the paper can be found at the author’s website: https://sites.google.com/site/sunnyelan/.

†Email: huangyg@uw.edu. Tel: (206)354-5536. Address: University of Washington, Box 353330, Seattle, WA 98195.
1 Introduction

This paper serves two purposes: it develops a structural estimation method of scoring auction data and propose three tests of detecting quality manipulation corruption. In real world procurement auctions, the objects are typically of differential quality with multi-dimensional attributes. For example, in procurement of a construction project, the design, materials, equipments, delivery date, safety, service, and maintenance are important attributes that need to be written in the contract at the moment of transaction. In this environment, a bid consists of both price and many other non-price quality attributes. A winning bid turns into a contract or a guideline of writing a detailed contract. Table[1] and Figure[14] show that a bid in a multi-attribute auction is fairly complicated and difficult to evaluate. If the auction has a pre-announced scoring rule specifying how bids are evaluated, it is called a scoring auction. If the auction does not specifically announce how the winner is selected before firms submitting their bids, it is called a beauty contest or a design-build auction. In this paper, we will only study scoring auctions, where the scoring rules and relevant scores of all bids are observable to researchers.

The complexity and subjectivity of quality evaluation bring in new challenges in conducting procurement auctions. Figure[1] shows three main roles in a procurement: the buyer (she), the procurement agency (he), and a group of supplying firms (them/it). In a typical procurement, the buyer is not an expert in the industry and lacks the expertise to evaluate quality of submitted bids, so she will hire some mediators between her and supplying firms. These mediators may include the buyer’s representatives with industrial experiences (street-level bureaucrat), professional procurement agency company, auctioneer (house), and a committee of industrial experts. In this paper, we abstract multiple layers of mediators into one layer, called the procurement agency. Because the procurement agency is given some discretion in evaluating quality, he can exert it to seek bribes from firms. If corruption indeed occurs, then the quality evaluation report is manipulated. Hence, we call this problem quality manipulation corruption. Such problem is intrinsic in the procurement process when quality is both important and requiring special expertise to evaluate.

Quality manipulation corruption is a prominent issue in procurement both in public and private sectors, especially in developing countries. Existing studies of corruption in auctions focus on bidding rings among bidders or cheatings between bidder and auctioneer (defined later in Section 1.2). Bidding rings and cheatings suppress competition and causes monetary loss for the buyers. However, corruption leads to another perceivable consequence: inferiority of quality (‘‘jerry-built’’ projects). Take the bridge construction as example: Ji and Fu (2010) find that there are a total of 85 major bridge collapse accidents in China between 2000 and 2009. 40 cases among them are caused by frauds in planning and construction. In a 2012 media report titled ‘‘Chinese-style of bridge collapse’’ government officials and industry experts conclude three major reasons: (i) construction plan fails to meet industrial regulation, (ii) construction is of low quality, and (iii) lack of necessary
maintenance. No buyer will purchase a bridge if she knows it will collapse in the near future. The discrepancy between the quality written on the winning bid and the quality actually delivered is usually caused by corruption in the procurement auction process.

To study the role of quality and corruption in procurements, we construct the theoretical model based upon existing framework of scoring auctions, mainly developed by Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). Our contribution is three-folds:

(1) We show that equilibrium cost and social surplus of each firm can be nonparametrically identified and structurally estimated from standard data of scoring auctions. Our method does not require parametric cost function and hold under environments with multiple quality attributes and multi-dimensional private information.

(2) We introduce quality manipulation corruption into the scoring auction model and characterize the systematic distortion of corrupted firm's bidding behaviors. The corruption model implication and structural estimation method are put together to construct three tests of detecting quality manipulation corruption. These tests are based on usual scoring auction data and do not require rich firm-specific covariates and identities of corrupted firms.

(3) The structural estimation method and corruption detection tests are applied to a scoring auction data set of server room procurements. The data and estimation results provide empirical evidence supporting the theoretical model. We show that projects with a high quality weight results in higher payoffs (rents) for both buyer and firms, but is also subject to a higher risk of corruption. Corruption is also more likely to happen at projects with higher engineer’s estimated costs.

1.1 Literature Review of Scoring Auction

When the procurement objects are of differential quality, scoring auctions are commonly used. The advantage of scoring auctions is proven both by theory and its popularity. In practice, each bidder is asked to submit one bid as a price-quality combination. The contract is awarded to the bidder that
receives the highest score based on a pre-announced scoring rule. The seminal paper by Che (1993) provides the benchmark model of scoring auctions. He derives the equilibrium of scoring auction under quasilinear scoring rule and shows that firm’s quality and price choice can be separated. He shows that both first-score auctions (FSA) and second-score auctions (SSA) implement the optimal mechanism and yield the same expected utility to the buyer.

Asker and Cantillon (2008) introduce multi-dimensionality of private information and quality attributes to Che’s model. They characterize the equilibrium and expected score equivalence of FSA and SSA. In addition, they show that a scoring auction with a quasilinear scoring rule dominates other alternatives including beauty contests, menu auctions and price-only auctions with a minimum quality threshold. In Branco (1997), costs of different firms have a common component and thus are correlated. In this case, an optimal contract cannot be implemented by first or second-score auctions, but instead requires a two stage mechanism: first select a firm through an auction, then readjust the level of quality via bilateral bargaining. David et al. (2006) and Chen-Ritzo et al. (2005) provide experimental evidence indicating that scoring auctions dominate traditional price-only ones. Wang and Liu (2014), Dastidar (2014), and Hanazono et al. (2015) extend the model to non-quasilinear scoring rule environments. Among these papers, Hanazono et al. (2015) consider the most general setting that covers price-quality ratio, fixed price best proposal, and convex scoring rules. They characterize the equilibrium of FSA and SSA and show that their expected score rankings depends on the curvature of the induced utility of firms.

In general, it is difficult to characterize the optimal mechanism and scoring auction implementation when the environment is complicated. David et al. (2006) characterize an optimal scoring rule within the class of weighted criteria rules with restriction of additively separability of attributes on both preference and cost. Asker and Cantillon (2010) find the optimal mechanism in a specific environment where firm types are two binary random variables. They show that a scoring auction yields a performance closed to that optimal mechanism numerically. Nishimura (2015) show that implementation of the optimal mechanism via a scoring rule requires substantial cost complementarity among quality attributes. In other words, the widely used linear weighted scoring rule is suboptimal because it does not exhibit enough complementarity among attributes to provide the correct incentive.

Concerning quality manipulation corruption, Celentani and Ganuza (2002) introduce an endogenous corruption relation forming process to the scoring auction model of Che (1993). They allow the corrupted firm to win for sure once the procurement agency accept a bribe. Their model focuses on the formation of the corruption side contract and show how increasing competition may not reduce corruption. Burguet and Che (2004) consider a Bertrand-style environment of two firms with complete information and endogenous formation of corruption relation. Two firms involve in both

---

3 In a beauty contest or design built auction, there is no pre-announced scoring rule. The buyer does not select winner based on her preference over all submitted bids.

4 In a menu auction, bidders are allow to submit multiple price-quality combination bids, instead of only one in scoring auctions. The buyer will then determine the winner and the item on its menu.
bribery competition and market competition. Because the weaker firm can spend all its resources on either market competition or bribery competition, the efficient firm cannot guarantee winning the contract. [Huang and Xia (2015)] consider a similar environment with exogenous corruption relation and focus on the buyer’s optimal scoring rule under corruption. The scoring rule affects the relative magnitude of the efficient firm’s technological advantage and the corrupted firm’s corruption advantage, which further determines the auction outcome. In such an environment, the dominance of a scoring auction as shown in [Asker and Cantillon (2008)] disappears. A price-only auction with a minimum quality threshold could be better.

There is a growing literature on the empirical analysis of scoring auctions. In a scoring auction data set, each bid consists of price and a number of quality attributes, it can potentially answer richer questions than price-only auction data. [Lewis and Bajari (2011)] explore a highway procurement data set from California generated from “A+B auctions”, where bids are evaluated on both price and time of delivery. They show that by introducing time incentive, the overall gain in social welfare is significant. [Bajari et al. (2014)] analyze another highway procurement data set where bids observations consist of the complete list of unit prices. These unit prices are multiplied by quantities estimated by engineers to determine which bid has the lowest cost. Their analysis focuses on the ex post adjustment of final payments and how firms strategically reflect potential adaption costs in their bids. [Krasnokutskaya et al. (2011)] study data from online programming service market. They provide an identification and estimation strategy for data that features both auction and discrete choice. [Koning and Van de Meerendonk (2014)] study data from welfare service provider procurement auctions under weighted scoring rule. They explore how variation of weights on different components affect bids and procurement outcomes. [Nakabayashi and Hirose (2015)] study a Japanese scoring auction data set similar to the one in this paper. They provide identification and structural estimation results based on the theoretical model of [Hanazono et al. (2015)] under a general scoring rule. They assume a parametric cost function that is common knowledge except for \( L \) parameters as bidder’s private information. The identification is based on invertibility conditions of the system of equations obtained by best responses. In our analysis, we consider only the class of quasilinear scoring rules, but our whole analysis is nonparametric.

### 1.2 Detecting Corruption in Auction

In the *Handbook of Procurement* (edited by Dimitri et al. (2006), Lengwiler and Wolfstetter (2006) point out procurement auction participants may suppress competition by four major forms of collusion or corruption. In the literature, *collusion* usually refers to a *bidding ring* or a *cartel*, where a group of bidders coordinate their bids to increase price. In an efficient cartel, the *cartel leader* (the one with the lowest cost or the winner of an internal per-auction knockout) is the only serious bidder, while the other cartel members submit high *phony bids*. There is a body of literature on bidding rings both theoretically (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)) and empirically (e.g. Pesendorfer (2000), Bajari and Ye (2003), and Asker
Corruption usually refers to the auctioneer (who runs the auction) twisting the auction rule in exchange for bribes. It can take three major forms: (i) **bid rigging** (bid revision or "magic number" cheating), meaning that the auctioneer allows a favored bidder to adjust his bid after receiving information about rival bids (e.g. Compte et al. (2005) and Burguet and Perry (2009)). (ii) **Bid orchestration**, meaning that the auctioneer serves as the ring manager of a collusive cartel and coordinates their bids. (iii) **Distortion of quality ranking**, which is called quality manipulation corruption in this paper, meaning that the bid evaluation committee is bribed to submit biased quality scores (e.g. Celentani and Ganuza (2002) and Burguet and Che (2004)).

In this section, we briefly review existing empirical works on corruption in auctions and its detection. We focus on a relative small number of papers to sketch their key insights. For a more comprehensive reviews including the theoretical side literature, readers can consult other surveys, for example Harrington (2008) and John Asker’s note.

Porter and Zona (1993) is one of the earliest works on collusion detection. They study bidding rings in procurement auctions of Long Island highway construction contracts. Because some bidders are of relative large size and interact with each other in a sequence of auctions, they are able to coordinate as a cartel. They estimate parameters of a linear bid function and a logistic bid ranking model. Because the model can be estimated from using either the whole sample or only winning bids, two sets of parameter estimate shall be equal in a competitive environment. But when there is a bidding ring, the ranking of bids will not fully reflect the economic factors of bidders, leading to different estimates.

Colluding bidders’ behaviors can be studied and tested by reduced-form models when detailed data of cartel members identities and characteristics are available from records of investigation by antitrust authorities. Porter and Zona (1999) analyze data from school milk contract auctions in Ohio, where a group of firms in Cincinnati were convicted for colluding. The bidding behavior of cartel members is compared to a controlled group. They show that collusion raised market prices by 6.5% on average. Pesendorfer (2000) also analyzes data from school milk contract auctions, where some firms in Florida and Texas were found colluding. He considers the effect of both bid rigging and market splitting. He estimates the coefficients of reduced-form bid function regressions using three sub-samples: low cartel bids plus all non-cartel bids, low cartel bids, and all non-cartel bids. A Chow test for equality of coefficients shows that the cartel firms bid less aggressively than non-cartel firms. Feinstein et al. (1985) point out that a cartel may seek not only a higher winning bid, but also collectively use bids to pass false information to the buyer to avoid a “ratchet effect” (Freixas et al. 1985). It happens when the buyer uses past information to form expectations of future auctions. Feinstein et al. (1985) found empirical evidence by data from convicted collusion cases in North Carolina highways procurements.

However, if the data does not provide exact identities of the cartel and non-cartel bidders, the methods above cannot be implemented (unless one runs regression on all possible partitions of the

---

cartel and non-cartel bidders). In addition, the data may not be rich in bidder’s characteristics. Harrington (2008) points out that an abnormally high profit margin is not the evidence of collusion, but the evidence of market power. According to Baldwin et al. (1997), there are three (non-mutually exclusive) ways to explain a high profit margin: collusion, demand side factors, and supply side factors. The supply side can be captured by auction-specific covariates describing the object. To identify collusion, researchers need to control demand side factors by enough bidder-specific covariates. To encounter these data limitations, researchers start using structural model to detect collusion.

Bajari and Ye (2003) construct their test based on two distinct model implications of the competition and the collusion model: conditional independence and exchangeability of bids. If bidders are competitive, bids must be independent controlling for all publicly observable information on costs under IPV framework. But if there is a cartel, their bids may be correlated and such correlation can be detected. Moreover, a competitive bidder’s bid shall not depend on other bidder’s identities, so exchanging other bidders’ characteristics shall not change the distribution of competitive bidder’s bid. In a regression specification, if one regresses bidder $i$’s bid on the covariates of bidder $j$ and $k$ (with other controls), then these two coefficients should be equal. An F-test can be used to check this exchangeability restriction. Identities of potential cartel members can be found by testing each pair of bidders. In addition, Bayesian estimation of the structural model provides the likelihood of the data coming from the collusion model. Aryal and Gabrielli (2013) take a full structural approach to test collusion based on an estimation method in Guerre et al. (2000). For the same set of bids data, two sets of costs are structurally estimated from the competitive model and the collusion model, denoted as $\{\hat{c}^A\}$ and $\{\hat{c}^B\}$ respectively. Because collusion lowers competition, for the same bid $b$, it implies $c^A(b) \geq c^B(b)$. Detecting collusion boils down to testing for first-order stochastic dominance of two cost distributions recovered from two models.

Besides bidding rings, Ingraham (2005) studies the corruption between auctioneer and bidder. His model is based on a bid revision model in Compte et al. (2005). The auctioneer let the corrupted firm observe others’ bids before submitted its. When the corrupted firm’s cost is lower than the lowest bid of other firms, it will submit a bid that barely wins the contract. As a result, the difference between the lowest and second lowest bid is smaller than a usual competitive sample. This is a testable implication

All works mentioned above are based on first-price sealed-bid auction. Collusion can be a more prominent problem in open auctions where tacit collusion is easier. Athey et al. (2011) study a timber auction data set with two auction formats (sealed-bid and open) and two sets of bidders (mills and loggers). They assume mills are potential cartel and use the sealed-bid auction as benchmark to evaluate whether bids in open auctions satisfy the competitive hypothesis. Bajari and Ye (2009) studies collusion in FCC spectrum auction and Klemperer (2002) in telecoms license auction. Some other empirical works are based on the unique features of their data set. Asker and Cantillon (2010) study internal knockout auction from side-transfer data of a stamp dealers cartel. They
test the theory of internal organization of bidding rings and measure ring members’ benefit from colluding. Tran (2009) uses internal bribery data of a company to compare corruption under two different auction format. Kawai and Nakabayashi (2014) study an auction data set from Japanese government procurements. Because the reserve price is secret, observation of bids may consist of multiple rounds and the ranking of bidders across rounds can be used to detect collusion.

In summary, to detect collusion, one needs to derive key features that are distinct to the competition model and the collusion model, and then test which model the data supports. Hence, all these collusion detection methods suffer from some common problems: (i) When the null hypothesis of the competitive model is rejected, it is hard to distinguish whether the reason is collusion or model mis-specification (See Figure 7). (ii) If corrupted bidders coordinate their bids in a sophisticated way, the recorded bids can pass nearly all these tests. It is called “beating a test of collusion”, which is discussed in Harrington (2008). (iii) Nearly all these tests rely on repeated observations of bids from the set of potential corrupted bidders. Dynamic interaction between bidders are very informative of whether they are competing or colluding. But one implicit assumption is made: the identities of cartel and non-cartel members do not change across auctions.

Our tests are subject to problem (i) as others, but suffer less from problem (ii) and (iii). The quality manipulation problem usually only happens to one bidder. If the procurement agency and corrupted firm wants to avoid being detected, they must reduce the scope of corruption. Therefore, beating our tests will directly restrict corruption. Besides that, our tests are also useful for antitrust authorities because they requires only standard auction data. In particular, we do not need a prespecified set of suspicious corrupted bidders, identities of bidders, or repeated bidding behaviors of bidders across auctions. Our tests can be perform with very little or even no bidder-specific covariates.

The rest of the paper is organized as follows. We present the theoretical model of scoring auction and quality manipulation corruption in Section 2. The identification and structural estimation of scoring auction model are shown in Section 3.1. Section 3.2 provides three corruption detection tests and a Monte Carlo experiment. In Section 4, we apply the estimation and collusion detection method to a server procurement auction data set. Section 5 concludes.

2 Theoretical Model

A buyer (she) seeks procurement of a project which can be delivered at various level of quality \( q \in \mathbb{R}^L \). She announces a scoring rule \( S(p,q) : \mathbb{R}^{L+1} \to \mathbb{R} \) and invite firms to submit bids. Suppose there are \( n \) symmetric risk neutral firms (they/it) enter the scoring auction, indexed by \( i = 1, 2, \ldots, n \). A generic firm \( i \)'s type (private information) is a vector of efficiency parameter \( \theta \), drawn independently from an identical distribution \( F \). \( F \) is absolutely continuous and has density
f = F' with support [θ, ̄θ]. Firm i with type θ_i pays a cost C(q, θ_i) if it delivers the project with quality q. If the firm wins the contract with bid (p, q), its payoff is π(p, q; θ_i) = p − C(q, θ_i). Firm's payoff is normalized to zero if it does not win the contract. We assume the cost function satisfies the following assumption:

**Assumption CF (Cost function):** For all q ∈ [q, q'], C(q, θ) is continuous in q, C(q, θ) ≥ 0, C_q(q, θ) > 0, C_qq(q, θ) > 0.  

Provided the scoring rule, each firm submits its sealed bid as a price-quality combination, i.e. (p_i, q_i). These n bids are then evaluated according to S(p, q) and the firm with the highest score wins the contract. We only consider first-score auctions (FSA) and independent private information framework in this paper. The quality manipulation issue kicks in when the buyer does not directly observe q, and she hires a procurement agency (he) to evaluate quality of bids.

In this paper, we will put aside a buyer’s optimal scoring rule design problem and simply and treat S(p, q) as her objective function. We focus on a firm’s equilibrium bidding behavior and the issue of quality manipulation corruption with the goal of conducting an empirical study on bids data. As we can only observe the score but not buyer’s “payoff”, it is nearly impossible to infer a buyer’s true preference from an empirical point of view. In addition, Che [1993] shows that if the buyer lacks commitment power, the only feasible scoring rule is one that reflects her preference.

### 2.1 Equilibrium under Quasilinear Scoring Rule

To pave the way for analyzing quality manipulation corruption and empirical study, we review the equilibrium of a scoring auction under a quasilinear (QL) scoring rule in Che [1993], Askar and Cantillon [2008], and Hanazono et al. [2015]. We restrict our attention to scoring rules satisfying the following restriction:

**Assumption QL (quasi-linear scoring rule):** The scoring auction uses a quasilinear scoring rule S(p, q) = V(q) − p, where V(q) is increasing, continuously differentiable and weakly concave.

The commonly used linear weighted factor scoring rule can be transformed into QL class. However, a quality-price ratio scoring rule cannot be transformed into an equivalent QL rule, analyzed in Hanazono et al. [2015]. With assumption CF and QL, we have the following lemma:

**Lemma 1:** Consider a quasilinear scoring rule S(p, q) = V(q) − p, with q ∈ ℝ_+^L and L > 1. The cost function of L quality attributes C(q, θ) can be transformed into a function of one-dimensional quality index C(v, θ), v ∈ ℝ, which is also continuous, strictly increasing and convex in v.

Lemma 1 shows that the dimension quality attributes can be reduced one quality index without loss of generality. For the rest theoretical analysis, we will take q to be one-dimensional. Provided the scoring rule S(p, q), the firm’s problem is

---

6 Compared to previous literature, we relax the assumption on the signs of C_θ and C_qθ.
\[
\max_{p,q} [p - C(q, \theta)] \Pr (\text{win}|S(p, q)).
\]

We start describing the equilibrium bidding strategy by the following result in Che (1993):

**Lemma 2:** Under assumption CF and QL, when \( \theta \) is one-dimensional and \( C_{\theta} > 0, C_{q\theta} > 0 \), there is a symmetric Bayesian Nash equilibrium of a first-score auction where each firm with type \( \theta \) submits its bid as

\[
q(\theta) = \arg\max_q V(q) - C(q, \theta),
\]

\[
p(\theta) = C(q(\theta), \theta) + \int_{q}^{\theta} C_{\theta}(q(t), t) \left[ \frac{1 - F(t)^{n-1}}{1 - F(\theta)^{n-1}} \right] dt.
\]

The key feature of this equilibrium is that the quality choice \( q(\theta) \) is separated from the price choice and each firm will choose the quality that maximizes the social surplus. However, \( q(\theta) \) is not suitable for direct empirical application because of its monotonicity property. The existence of the equilibrium requires the assumption \( C_{q\theta} > 0 \). Topkis (1978) theorem immediately implies that \( q'(\theta) < 0 \). It means that the most efficient firm with lowest \( \theta \) wins by submitting the highest quality. This prediction does not fit real world data because some contracts are awarded to low quality and low price firms. If we drop assumption \( C_{q\theta} > 0 \) and allow \( \theta \) to be at least two dimensional, the problem disappears (see the Monte Carlo Example).

Under multi-dimensional private information, assumption CF and QL ensure \( q(\theta) \) being a single-valued continuous function by the Maximum Theorem (Berge, 1963). The firm’s problem (1) is equivalent to a two-step optimization problem where the firm first chooses its score \( s \), then choose a \((p, q)\) combination to fulfill that score. Because \( p = V(q) - S(p, q) \),

\[
\begin{align*}
(1) & \iff \max_s \left\{ \max_{(p, q) \text{ s.t. } S(p, q) = s} [p - C(q, \theta)] \Pr (\text{win}|s) \right\} \\
& \iff \max_s \left\{ \max_q [V(q) - s - C(q, \theta)] \Pr (\text{win}|s) \right\} \\
& \iff \max_s \{[V(q(\theta)) - C(q(\theta), \theta) - s] \Pr (\text{win}|s) \}. 
\end{align*}
\]

Following Asker and Cantillon (2008), we define the *pseudotype* of a firm as the value function

\[
K(\theta) \equiv \max_q V(q) - C(q, \theta) = V(q(\theta)) - C(q(\theta), \theta).
\]

\(^7\) \(q(\theta)\) satisfies FOC \( V_q(q) - C_q(q, \theta) = 0 \). By implicit function theorem, \( q'(\theta) = C_{q\theta}/(V_{qq} - C_{qq}) \). Assumption CF and QL require \( V_{qq} \leq 0 \) and \( C_{qq} > 0 \), hence adding assumption \( C_{q\theta} > 0 \) implies that quality choice is monotonically decreasing in \( \theta \).

\(^8\) It is called effective cost in Hanazono et al. (2015) and productive potential in Che (1993).
Again by the Maximum Theorem, \( K(\theta) \) is a single-valued continuous function. The distribution of pseudotype \( K \) can be obtained from the (joint) distribution of \( \theta \) by

\[
F_K(k) = \Pr(K(\theta) \leq k) = \Pr(\theta \in D_{\theta:K(\theta) \leq k}) = \int_{\theta \in D} f(\theta) d\theta.
\]  

(6)

Denote \( k = \min_{\theta \in [\theta, \bar{\theta}]} \{ K(\theta), 0 \} \) and \( \bar{k} = \max_{\theta \in [\theta, \bar{\theta}]} \{ K(\theta), 0 \} \). We assume \( k \geq 0 \) so that the least efficient firm participates. The support of pseudotype is \([k, \bar{k}]\). According to \textit{Asker and Cantillon} (2008), pseudotypes are sufficient statistics to describe the equilibrium of scoring auctions under QL. Therefore, instead of drawing multi-dimensional type \( \theta \) from a joint distribution \( F \), firms can draw their one-dimensional type \( k \) from distribution \( F_K \). Problem (4) can then be rewritten as if the firm is selecting its score based on its pseudotype:

\[
\max_s (k - s) \Pr(\text{win}|s).
\]  

(7)

Directly from \textit{Asker and Cantillon} (2008), we have the following Theorem and two corollaries:

**Theorem 1:** Given QL, every equilibrium in a scoring auction is type-wise outcome equivalent to an equilibrium in the scoring auction where firms are constrained to bid only on the basis of their pseudotypes. Firm with type \( \theta \) and pseudotype \( k = K(\theta) \) bids its quality as \( s \) and score according to

\[
s(k) = k - \frac{\int_k^k [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}.
\]  

(8)

The relevant price is \( p(\theta) = V(q(\theta)) - s(K(\theta)) \).

**Corollary 1:** The conditional expectation of the winner’s score equals to the strongest rival’s pseudotype, i.e., \( E[s(k_{(1:n)})] = E[k_{(2:n)}] \).

**Corollary 2:** The buyer receives a higher expected utility in a scoring auction than a price-only auction with minimum quality.

Corollary 1 is the expected utility equivalence of FSA and SSA, which is similar to the revenue equivalence theorem in \textit{Vickrey} (1961) and \textit{Myerson} (1981). Corollary 2 is about the superiority of scoring auctions. Throughout the paper, \( X_{(j:n)} \) denotes the \( j \)th highest order statistic from an i.i.d. sample of size \( n \) from distribution \( F_X \). The distribution function of order statistic \( X_{(j:n)} \) is denoted as \( F_X^{(j:n)} \).

In summary, the bidding behavior of the competitive scoring auction model has three implications that can be tested empirically. First, competition among firms are mainly on quality. The Efficient firm usually submit bids with high quality and high price to win the contract. Second, according to (2), a higher slope of \( V(\cdot) \) induces higher quality. Lastly, separation of quality and score choice implies that the number of bidders shall not affect the choice of quality, but affect the choice of score or price.
2.2 Quality Manipulation Corruption

Aforementioned, the complexity and subjectivity features of quality evaluation in scoring auction brings in quality manipulation corruption. Assume that the procurement agency randomly matches with one firm and forms a corruption relation. This relation can be the result of bribery a side-contract, a long term relationship, favoritism, or other reasons. As Lengwiler and Wolfstetter (2006) point out, by approaching only one bidder, the auctioneer minimize the number of side-contracts and thus the risk of detection. Large coalitions are of course possible, but the detection risk obviously increases with the number of people who know about the corruption. We follow Burguet and Che (2004) assuming that the corrupted firm’s quality score is raised by $m > 0$ and call it the scope of corruption. It means that if the corrupted firm submits a bid $(p, q)$, the score is elevated from $S(p, q)$ to $S(p, q + m)$. The interpretation of $m$ can be (i) the quality score of the corrupted firm is raised, (ii) the actual delivered quality is lower than the one written on the bid, or (iii) the evaluation method is twisted to give an advantage to the corrupted firm.

The discretionary power (or error allowance) given to the procurement agency determines the scope of corruption. Indeed, this discretionary power is restricted by the nature of the industry and the risk of triggering investigation. For example, in the procurement of a bridge, the procurement agency may claim that the corrupted firm’s bridge can serve 30 years while the actual building code is designed for only 25 years. However, he will not say the bridge will last 100 years because it would be very suspicious.

The timeline of the auction with corruption is as follows. The buyer announces a scoring rule and hires the procurement agency. A number of firms enter the auction and draw their private information $\theta$ from $F$. The procurement agency then randomly matches with one firm and offers him to raise his quality score by $m$ in exchange for a bribe. The firm decides whether to accept this.
offer or not. Then every firm submits a sealed-bid simultaneously as a price-quality combination. If the matched firm accept the offer, his quality will be raised by \( m \). The auction outcome is then revealed and the firm with highest score wins the contract.

We skip a detailed model of the endogenous formation process of the corruption relation. We assume the procurement agency is an expert in this industry and is able to design a bribery side contract that the matched firm will accept. For example, if the procurement agency knows \( \theta \) of the matched firm, he can make a take-it-or-leave-it offer, asking for a bribe slightly less than the difference between the expected payoff of being corrupted and not. Our simple model is enough from an empirical point of view, because variables directly related to corruption are usually unobservable (e.g. side payments, identities of corrupted firms, amounts of quality distortion). Writing a complicated model of quality manipulation corruption usually ends up with the same qualitative prediction. We further impose an assumption on other firms’ knowledge about the existence of the corruption relation.

**Assumption UA:** The buyer and other uncorrupted firms are unaware of the existence of the corruption relation.

There are both realistic and technical reasons for this assumption. In reality, if either the buyer or some other firms notice the existence of corruption, they will report it to the antitrust authority because corruption directly hurts their interests. The procurement agency and the corrupted firm will control the scope of corruption so that it does not trigger investigation. Moreover, for technical reason, with incomplete information on costs, adding another layer of incomplete information brings in mixed strategies and the equilibrium becomes both complicated and uninformative (see Huang and Xia (2015)). Therefore, assumption UA is widely used in the literature of bidding rings (e.g. Porter and Zona (1993), and Aryal and Gabrielli (2013)) and bid revisions (e.g. Burguet and Perry (2009)). An alternative way to circumvent the problem is assuming complete information on corruption relation. For example, most bidding ring literature assume both cartel members and non-cartel members know identities of colluding firms (e.g. McAfee and McMillan (1992), Bajari and Ye (2003), and Athey et al. (2011)). The bidders then have two types and the auction is asymmetric with type-specific bidding functions. The qualitative prediction of assuming complete information is usually similar to assuming UA.

Given assumption UA, all uncorrupted firms follow the same strategy as in Theorem 1. The corrupted firm, once matched with the procurement agency, solves a modified problem:

\[
\max_{p,q} \left[ p - C(q - m, \theta) \right] \Pr (\text{win} | S(q, p)) .
\]

The equilibrium bidding strategy is summarized as the following theorem.
Theorem 2: Under assumption CF, QL, and UA, the corrupted firm bids

\[
q_m(\theta) = \arg \max_q \argmax V(q) - C(q - m, \theta),
\]

\[
p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)),
\]

\[
s(k_m) = \frac{k_m - \int_k^{k_m} [F_K(t)]^{n-1} dt}{[F_K(k_m)]^{n-1}}
\]

where \(k_m = K_m(\theta) = \max_q V(q) - C(q - m, \theta)\) is the corrupted firm’s pseudotype. Compared to an uncorrupted firm with the same type, the corrupted firm has a higher pseudotype and will bid a higher quality and a higher score. All three effects magnify as \(m\) increases. (The prediction on price is ambiguous).

Therefore, the corrupted firm will bid more aggressively compared to a competitive firm of the same type. Because the corrupted firm has a large winning probability, it causes a systematic distributional change of the winning bid. It is the key factor that allows us to construct corruption detection tests. Note that the “more aggressive” prediction is different from the implication of bidding ring models. When an auction involves a bidding ring, both the ring leader and other members bid less aggressively to suppress competition. But with quality manipulation, the corrupted firm pays a lower cost with help from the procurement agency. As a result, the corrupted firm will bid more aggressively to win the contract.

Figure 3: Illustration of the Equilibrium with Corruption

3 Econometrics of Scoring Auctions and Corruption Detection

The key of corruption detection lies on checking abnormal aggressive bidding behaviors of corrupted firms. It is impossible to distinguish normal competitive bidding and abnormal predatory
bidding behavior by a single observation because the scope of manipulation (m) in unknown. But when the sample size gets large, the consistent pattern of aggressive winning bids can be captured by statistical tests. In this section, we propose three tests and provide a Monte Carlo example.

3.1 Structural Estimation

We first present the identification and structural estimation of the scoring auction model. Consider a sample of \( T \) independent and repeated scoring auctions of the same industry with the same scoring rule. For scoring auction \( t \), assume researchers observe the number of firms \( n_t \), some auction-specific covariates \( z_t \) (of dimension \( d \)), bids of each firm \( \{p_{it}, q_{it1}, q_{it2}, ..., q_{itL}\}_{i=1}^{n_t} \) (of dimension \( L + 1 \)) and scores \( s_{it} = V(q_{it1}, q_{it2}, ..., q_{itL}) - p_{it} \). We set aside endogenous entry and reserve price issues in this paper. By result in Theorem 1, the identification result can be established by the method in Guerre et al. (2000).

**Theorem 3:** Under assumption QL and CF, pseudotypes and equilibrium costs of firms are non-parametrically identified.

**Proof:** Because \( G_S(s) = \Pr(S \leq s) = \Pr(K \leq k) = F_K(k) \) and \( g_S(s) = f_K(k)/s'(k) \), by \( (22) \), pseudotype \( k \) is identified from the observation of scores via

\[
k = s(k) + s'(k) \frac{F_K(k)}{(n-1)f_K(k)} = s + \frac{G_S(s)}{(n-1)g_S(s)}, \tag{11}
\]

The equilibrium cost is then identified by the definition of pseudotype,

\[
C(q(\theta), \theta) = V(q(\theta)) - k = p(\theta) - \frac{G_S(s)}{(n-1)g_S(s)}. \tag{12}
\]

Q.E.D.

Given observations of \( n_t, z_t, \) and \( s_{it} \), the conditional distribution function and density of score can be estimated by kernel estimators,

\[
\hat{G}_S(s|n_t, z_t) = \frac{1}{Th_1h_2^2} \sum_{t=1}^{T} \frac{1}{n_t} \sum_{i=1}^{n_t} \mathbb{I}(s \leq s_{it}) \kappa_G \left( \frac{n - n_t}{h_1}, \frac{z - z_t}{h_2} \right),
\]

\[
\hat{g}_S(s|n_t, z_t) = \frac{1}{Th_3h_4h_5^2} \sum_{t=1}^{T} \frac{1}{n_t} \sum_{i=1}^{n_t} \kappa_g \left( \frac{s - s_{it}}{h_3}, \frac{n - n_t}{h_4}, \frac{z - z_t}{h_5} \right).
\]

We use Gaussian kernels and select bandwidths by least-square cross validation throughout this

\[9\] We will discuss variations of scoring rules later in the empirical application section.
Paper. Pseudotypes and equilibrium costs at corresponding quality are estimated by
\[ \hat{k}_{it} = s_{it} + \frac{\hat{G}_S(s_{it}|n_t, z_t)}{(n_t - 1)\hat{g}_S(s_{it}|n_t, z_t)}, \]
\[ \hat{c}_{it} = V(q_{it}) - \hat{k}_{it}. \]  
(13)
(14)

Equation (13) shows that the firm chooses a score \( s \) as the portion delivered to the buyer from total social surplus \( k \). The second term is the firm’s rent, reflecting its competitive advantage and information rent. Notice that in a price-only auction, quality of the object is fixed, so the model primitive is a cost distribution. In a scoring auction, the model primitive is a cost function defined on the domain of quality attributes. Costs estimated via (14) are not randomly drawn from a fixed cost distribution, but rather chosen by firms.

**Monte Carlo Example**

Consider the scoring rule is \( S(q, p) = 2q - p \) and cost function is \( C(q, \theta) = \theta_0 + q^2/\theta_1 \). Each firm draws its two-dimensional type \((\theta_0, \theta_1) \equiv \theta \) independently from Uniform[0,1] and Uniform[1,2], respectively. Assume \( \theta_0 \) and \( \theta_1 \) are independent, so their joint density equals to one on the support \([0,1] \times [1,2] \). By Theorem 1, the optimal quality choice of a firm with type \( \theta \) is \( q(\theta) = \theta_1 \) and its pseudotype is
\[ K(\theta) = V(q(\theta)) - C(q(\theta), \theta)) = 2\theta_1 - \theta_0 - \frac{\theta_1^2}{\theta_1} = \theta_1 - \theta_0. \]
The support of \( k \) is \([0,2] \). By (6) and the derivation in the appendix, the distribution function of pseudotype is
\[ F_K(k) = \Pr(\theta_1 - \theta_0 < k) = \begin{cases} \frac{k^2}{2}, & \text{for } k \in [0,1], \\ 1 - \frac{(2-k)^2}{2}, & \text{for } k \in (1,2]. \end{cases} \]  
(15)

Notice that, by allowing two-dimensional types, firm who submits a high equilibrium quality does not necessarily have a high pseudotype. For example, when firm 1 is type \((0.5,1.5) \) and firm 2 is type \((0.1,1.2) \), firm 1 will produce at \( q = 1.5 \) and have pseudotype \( k = 1 \); firm 2 will produce at a lower level \( q = 1.2 \) but have a higher pseudotype \( k = 1.1 \). The numbers of firms \( n \) are randomly draws from 3 to 20 with equal probability. Using (8), we can generate a simulated data set and apply our estimator (13) and (12), as illustrated in Figure 4. The estimation is based on 1000 auctions.

In this example, if researchers know the parametric form of the cost function, he can identify two structural parameters by conditions of optimal quality and score choice: \( \theta_1 = q \) and \( \theta_0 = q - s - \frac{G(s|n)}{(n-1)g(s|n)} \). In general, as long as \( K(\theta) \) is monotone in \( \theta \) under the parametric assumption, \( \theta \) is identified. In application to an actual data set, determining the parametric family of cost function is usually difficult.

Notice that, the identification result in Theorem 3 is established in a competitive bidding environment. If there is corruption, the scope of corruption is unobservable and may vary across
auctions. From a single observation, a researcher cannot conclude whether a high pseudotype is due to a real competitive advantage or a manipulated quality. In the example, for some \( m > 0 \),

\[
q_m(\theta) = \arg \max_q \left\{ 2q - \theta_0 - \frac{(q - m)^2}{\theta_1} \right\} = \theta_1 + m = q(\theta) + m, \tag{16}
\]

\[
K_m(\theta) = 2(\theta_1 + m) - \theta_0 - \theta_1 = \theta_1 - \theta_0 + 2m = K(\theta) + 2m. \tag{17}
\]

Therefore one cannot separately identify \( k \) and \( m \). Although it is not an identified model, the systematic distortion of submitted bids can be captured with a large sample.

### 3.2 Corruption Detection Tests

The basic intuition of our corruption detection tests is capturing the consistent abnormal aggressive bidding behaviors of winning bids. Corruption distorts only the corrupted firm’s bid, while all other bids remain competitive. The distorted bid is the winning bid with a large probability. Hence, even we don’t observe the identities of corrupted bidders, we can test for the existence of systematic deviations from competitive bidding behaviors by comparing the winning bids and other bids. We propose three tests in this section. For each test, the null hypothesis is that the data are generated from the competitive model, i.e. \( H_0: m = 0 \). It is tested against the alternative hypothesis that the data are generated from the corruption model, i.e. \( H_1: m > 0 \). Test I and II can be performed on one sample of auctions from the same procurement agency. Test III can only be performed on two or more sub-samples with difference in their procurement agencies or other aspects. These tests are illustrated in a Monte Carlo example and later applied to a real procurement data set. We denote the observed highest score or pseudotype of each auction by subscript “win”. The observed second highest and third highest ones are denoted by subscript “rival” and “third”, respectively.

**Test I**

Among the \( n-1 \) rivals, the strongest rival has pseudotype \( k_{1:n-1} \), therefore the winning probability of a firm with pseudotype \( k \) is \( \Pr(k > k_{1:n-1}) = F^{(1:n-1)}_K(k) \). The corrupted firm wins with probability \( \Pr(k_m > k_{1:n-1}) = F^{(1:n-1)}_K(k_m) \) and appears to be the strongest rival with probability

\[
\Pr(k_{2:n-1} < k_m < k_{1:n-1}) = F^{(2:n-1)}_K(k_m) - F^{(1:n-1)}_K(k_m).
\]

Therefore, the observed winning score

\[
s_{\text{win}} = \begin{cases} 
  s(k_m), & \text{with prob } F^{(1:n-1)}_K(k_m), \\
  s(k_{1:n-1}), & \text{with prob } 1 - F^{(1:n-1)}_K(k_m),
\end{cases}
\]

and the strongest rival’s score

\[
s_{\text{rival}} = \begin{cases} 
  s(k_m), & \text{with prob } F^{(2:n-1)}_K(k_m) - F^{(1:n-1)}_K(k_m), \\
  s(k_{1:n-1}), & \text{with prob } F^{(1:n-1)}_K(k_m), \\
  s(k_{2:n-1}), & \text{with prob } 1 - F^{(2:n-1)}_K(k_m).
\end{cases}
\]
Figure 4: Illustration of Data and Estimation

Note: In each diagram, the grey points represent bid level data, and the black dashed curve is a smoothing spline. For all estimation illustration, the blue dashed lines denote the true distribution, while the black lines denote the estimated one. The red dotted lines denote point-wise confidence band for two standard errors.
By Theorem 2, for any \( m > 0 \), \( E[s_{\text{win}}] > E[s(k_{(1:m)})] \) and \( E[s_{\text{rival}}] < E[s(k_{(2:n)})] \). Then, by Corollary 1, \( E[s(k_{(1:m)})] = E[k_{(2:n)}] \). Hence,

\[
\forall m > 0, \ E[s_{\text{win}}] > E[s(k_{(1:m)})] = E[k_{(2:n)}] > E[s(k_{(1:m)})].
\]

When \( m = 0 \), we have \( E[s_{\text{win}}] = E[k_{\text{rival}}] \) because \( s_{\text{win}} = s(k_{(1:n)}) \) and \( k_{\text{rival}} = k_{(2:n)} \) in the equilibrium of the competitive model. The corruption detection problem becomes testing

\[
H_0: E[s_{\text{win}}] = E[k_{\text{rival}}], \quad \text{v.s.} \quad H_1: E[s_{\text{win}}] > E[k_{\text{rival}}].
\]

We use the Welch’s \( t \)-test with test statistic

\[
T^* = \frac{T^{-1}\sum_{t=1}^{T} s_{\text{win},t} - T^{-1}\sum_{t=1}^{T} \hat{k}_{t,\text{rival}}}{\sqrt{T^{-1}\text{var}(s_{\text{win},t}) + T^{-1}\text{var}(\hat{k}_{t,\text{rival}})}},
\]

(18)

where \( \hat{k}_{t,\text{rival}} \) are estimated from \( \text{(13)} \). Lucking-Reiley (1999) also uses \( t \)-test for revenue equivalence but their samples are generated from different auction formats. For our application, we are studying one sample of the same auction format, therefore we use a bootstrap critical value to account for the correlation between scores and estimated pseudotypes.

**Test II**

For auctions with symmetric independent private value bidders, Athey and Haile (2002) show that the underlying value distribution is nonparametrically identified even when only one bid of each auction (an order statistic) is observed. When there is no corruption, the estimates of pseudotype distribution from all bids and from only the winning bids should be the same except for some statistical errors. When there is corruption, the winning bids are distorted and the two methods will result in statistically different estimates.

Practically, we construct the test by comparing two empirical CDFs of pseudotypes of winners from two estimation methods. By using all bids, pseudotypes of all firms, \( \{\hat{k}_{1t}, \ldots, \hat{k}_{n_{\text{rival}}}\} \), can be estimated via \( \text{(13)} \). Denote the pseudotype corresponding to the winning bid as \( \hat{k}_{\text{win}} \) and its empirical CDF \( \hat{F}^\text{win}_K(k) = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}(\hat{k}_{\text{win}} \leq k) \). By using only winning bids, these winning scores have distribution function \( G_W(s_{\text{win}}|n) = G_S^{(1:n)}(s_{\text{win}}) = [G_S(s_{\text{win}}|n)]^n \) and density \( g_W(s_{\text{win}}|n) = n[G_S(s_{\text{win}}|n)]^{n-1} g_S(s_{\text{win}}|n) \). By replacing relevant terms in \( \text{(11)} \), the winners’ pseudotypes are identified by \( k_{\text{win}} = s_{\text{win}} + nG_W(s_{\text{win}}|n)/(n-1)g_W(s_{\text{win}}|n) \). The underlying pseudotype of each winning bid can then be estimated, denoted as \( \hat{k}_{\text{win}} \). The empirical CDF of \( \hat{k}_{\text{win}} \) is \( \hat{F}^\text{win}_K(k) = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}(\hat{k}_{\text{win}} \leq k) \). The corruption detection problem becomes testing

\[
H_0: \forall k \in [k_1, k_2], \ \hat{F}^\text{win}_K(k) = \hat{F}^\text{win}_K(k), \quad \text{v.s.} \quad H_1: \exists k \in [k_1, k_2], \ \hat{F}^\text{win}_K(k) \leq \hat{F}^\text{win}_K(k).
\]
The natural option is Kolmogorov–Smirnov (KS) test. The test is one-sided because the aggressive scores in the corruption model results in higher estimate of \( k \). The KS test statistic is

\[
T^{I11} = \sup_{k \in [k, \bar{k}]} \left| \hat{F}^{\text{win}}_K(k) - \hat{F}^\text{win}_K(k) \right|
\]

\[
= \sup_{k \in [k, \bar{k}]} \left| \frac{1}{T} \sum_{t=1}^{T} I(\hat{k}_{\text{win},t} \leq k) - \frac{1}{T} \sum_{t=1}^{T} I(\hat{k}_{\text{win},t} \leq k) \right|
\]

Similar to test I, there is dependence between the two sets of estimated pseudotypes of the winners, so we use bootstrap critical values. Test II is illustrated in Figure 5.

**Figure 5: Illustration of Test II**

Note: In each diagram, the black curve is the empirical CDF estimated from all bids; the blue curve is the empirical CDF estimated from only winning bids. The left-hand-side diagram represents estimation result from a competitive data set, the right-hand-side diagram represents one under corruption.

**Test III**

Inspired by Ingraham (2005), test III is based on the following Markovian property of the conditional distribution of order statistics (see proof in Arnold et al. (1992)):

**Lemma 3:** Denote the first spacing of the highest two order statistics as \( X_{12} = X_{(1:n)} - X_{(2:n)} \). Its conditional distribution only depends on the third order statistic, that is

\[
f_{X_{12}}(x_{12} | X_{(3:n)} = x_3) = f_{X_{12}}(x_{12} | X_{(3:n)} = x_3, X_{(4:n)} = x_4, \ldots, X_{(n:n)} = x_n).
\]

Test III is easy to implement but needs at least two sub-samples. Suppose the observed auctions can be divided into two (or several) sub-samples that are differed in their procurement agencies or
other aspects. Let $D_\tau$ be dummy variable of sub-sample $\tau$. Consider the regression

$$(\hat{k}_{\text{win},t} - \hat{k}_{\text{rival},t}) = \beta_0 + \beta_1 \hat{k}_{\text{third},t} + \beta_2 D_\tau,t + \beta_3 z_t + \epsilon_t,$$

where $z_t$ controls for other auction-specific covariates. In a competitive auction, $\hat{k}_{\text{win}}$, $\hat{k}_{\text{rival}}$, and $\hat{k}_{\text{third}}$ coincide with $k_{(1:n)}$, $k_{(2:n)}$, and $k_{(3:n)}$. According to Lemma 3, the conditional distribution of the first spacing of pseudotypes, $k_{12} = k_{(1:n)} - k_{(2:n)}$, is the same across auctions if we control the third highest order pseudotype $k_{(3:n)}$. Therefore the conditional means of two sub-samples are equal if $m = 0$. We can apply a standard $t$-test for $H_0 : \beta_2 = 0$ versus $H_1 : \beta_2 \neq 0$ with test statistic $T_{III} = \hat{\beta}_2 / SE(\hat{\beta}_2)$. We can also skip the first stage estimation of pseudotypes and directly use score data to perform the test by the regression

$$(s_{\text{win},t} - s_{\text{rival},t}) = \beta_0 + \beta_1 s_{\text{third},t} + \beta_2 D_\tau,t + \beta_3 z_t + \beta_4 n_t + \epsilon_t,$$

If $\hat{\beta}_2$ is significantly greater than 0, the gap between the winner and the strongest rival is larger in the $D_\tau = 1$ sub-sample, which implies a higher likelihood of the sub-sample is subject to corruption.

**Monte Carlo Example (Continue)**

We continue with the previous example to illustrate the corruption detection tests. To find the distribution of test statistics under the null, $m$ can be an unknown positive number. But to study the powers of these tests, we let $m$ to be a known fixed number across observations. We generate $B = 199$ samples under the null hypothesis ($m = 0$) and compute test statistics for each sample, $\{T^j_b\}_{b=1}^B, j = I$ and $II^{10}$. Setting the significance level at 5%, the relevant bootstrap critical value of the test, $CV(T^j)$, is the 190th highest among these test statistics (since $(B + 1) \times (1 - 0.05) = 190$), illustrated in Figure 6.

We explore the powers of these tests under three alternative hypotheses by taking $m$ equals 0.2, 1, and 2, shown in Table 1. A randomly selected corrupted firm will choose a higher quality and have a higher pseudotype according to (16) and (17) respectively. The bootstrap power of the test is defined and computed by

$$\text{power} = 1 - \Pr(\text{accept } H_0 | H_1 \text{ is true}) = 1 - \frac{1}{B} \sum_{b=1}^B \mathbb{I} \left( T^j_b \leq CV(T^j) \right).$$

The Monte Carlo results show that as the scope of corruption $m$ and the sample size $T$ increase, the powers of all three tests improve. The power of test I is relatively weak compared to test II and III, especially in the case of small $m$.

---

To check the validity of bootstrap, we use the data generating process to repeatedly generate data sets and construct the compare the distribution of test statistic. It is use to compare with the bootstrap distribution of test statistic. They are similar.
Figure 6: Distribution of Test Statistics Under the Null and Bootstrap Critical Values

Note: The black curve represents the density of 199 bootstrap test statistic. The black dashed line denotes the bootstrap critical value. The blue line denotes the test statistic of a competitive data set.

Table 1: Power of the Tests

<table>
<thead>
<tr>
<th>Scope of Sample Size</th>
<th>Quality Manipulation</th>
<th>Test</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>II</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td></td>
<td>III</td>
<td>1000</td>
</tr>
</tbody>
</table>

Corrupted firm wins with probability 0.2348.

|                      | m = 0.2              | I    | 200         | 0.2362 | 0.4925 | 0.5528 |
|                      |                      | II   | 500         | 0.9347 | 0.9648 | 0.9849 |
|                      |                      | III  | 1000        | 0.7889 | 0.9397 | 0.9598 |

Corrupted firm wins with probability 0.4596.

|                      | m = 1                | I    | 200         | 0.9347 | 0.9648 | 0.9749 |
|                      |                      | II   | 500         | 0.9899 | 0.9950 | 0.9950 |
|                      |                      | III  | 1000        | 0.9950 | 0.9950 | 1.0000 |

Corrupted firm wins with probability 0.9618

Note: For test III, one half of the sample is generated under the null, the other half under the alternative.
Discussion

Compared to most existing collusion detection tests, our tests require less data, so it can be performed on a lot of procurement auction data sets. Existing tests generally require identities of bidders, (rich) bidder-specific covariates, repeated observation of bidders in several auctions. Some of these tests require exact identities of (suspected) colluding bidders, for example Porter and Zona (1993), Pesendorfer (2000), and Athey et al. (2011). Some tests, like Bajari and Ye (2003), can be conducted without identities of the corrupted firms, but need to be run on each combination of bidder pairs. Some tests are constructed upon repeated observations of the same set of bidders, which reveal the systematic difference between colluding bidders and competing bidders. Our tests do not require any of these data and hence can be performed before the case-by-case antitrust investigation.

Moreover, with different sub-samples, our tests do not need to specify a prior on which sub-sample is more likely to be corrupted. (For example, Athey et al. (2011) assumes that the sample from open auctions are collusive and sealed-bid auctions are competitive.) Test I and II can be performed on each of the sub-sample and compare their likelihoods of corruption by p-values. Test III estimates a “fixed effect” for each sub-sample by regression and can rank their likelihoods of being corrupted. However, because these tests are constructed on fairly limited sample information, there are several shortcomings:

1. Our test statistics involve two-step estimation based on pseudotypes. Because estimated pseudotypes are correlated, asymptotic distribution under the null is hard to derive analytically. We therefore use bootstrap critical values to make rejection decisions. Researchers start developing inference and tests based on one-step estimation of auction data. For example, Liu and Luo (2014) propose a test of exogenous entry based on empirical quantile of bids, which circumvents the correlation issue of estimated pseudo-values. However, because bids depend on the number of bidders \( n \), one cannot pool data from auctions with different \( n \) together, but needs to conduct the test separately on sub-samples according to \( n \). In our application, there is a great deal of variation of \( n \) (see Figure 8), so we choose to take the two-step path.

2. The powers of our tests are also very difficult to be studied analytically. First, they depend on the scope of corruption \( m \), but \( m \) is unobservable and may vary across auctions. Second, \( m \) cannot be estimated even if we assume it follows a parametric distribution. The model is not identified under the alternative hypothesis because the corrupted firm is not always the winner. So the scope of corruption cannot be recovered without knowing the exact identities of the corrupted firms. In other words, the corrupted firm’s bid and other bids are not generated from the same data generation process, and we don’t know which bid comes from the corrupted firm. These complications restrict us from studying the powers of the tests rigorously. A desirable data set to study corruption should include some \textit{ex post} information of convicted corruption records. With identities of corrupted firms, then it is possible to identify the corruption model. Researchers can then study the powers and their “in-sample” prediction correctness of these tests. We don’t have
such a data set for now and the main contribution of these tests are their *ex ante* applicable feature in corruption detection.

(3) Figure 7 illustrates a common problem of our tests and most collusion detection tests in the literature. When the data do not reject the null, it supports that the data rationalizes the competitive model. But when the data rejects the competitive model, it cannot distinguish whether the reason is corruption or model misspecification. For example, rejecting test I can be due to any reason related to failures of expected score equivalence, like bidder's risk aversion. The one-sided tests in test I and test II alleviate this problem: if we find that the winning bid is not aggressive but conservative, we do not reject the null.

![Figure 7: Interpretation of Test Results](image)

4 Empirical Application

4.1 Data and Server Room Construction Industry

Our scoring auction data set comes from two major procurement agencies: Guangzhou Public Resource Trading Center ([http://gzggzy.cn/](http://gzggzy.cn/)) and Public Resources Trading Center in Guangdong Province ([http://www.bcmegp.com/](http://www.bcmegp.com/)). Starting November 2009, these two major procurement platforms publicly announced auction results of all government related projects. Nearly all procurements conducted in these two trading centers are sealed-bid scoring auctions due to both legal requirements and their economic advantages. The Chinese Law of Tender requires government related projects with values over certain thresholds to go through the open tender process coordinated by these trading centers. The law also provides guidelines to forming tender evaluation committees, selecting industrial experts, designing of scoring rules, and the detailed process of auction. Besides public sector, private sector buyers also use these two trading centers frequently because trading centers have connections to a large pool of industrial experts that perform bid evaluations.

\[\text{Law of the People's Republic of China on Tenders and Bids (click this link for its full article in English).}\]
The data we used are procurement auctions of server room construction projects. Server room is an indoor place designed for containing machines like data storage, servers, and large computers. Evaluating quality of a server room construction plan needs specific expertise. To ensure reliability and safety, the construction of server rooms have detailed technological requirements on various aspects like temperature, humidity, electricity supply, fire control, etc. Each bid contains a full construction plan and a itemized price list. Firm’s reputation, experience, certificate, and financial status need to be considered in the bids evaluation. Therefore, compared to lands or cargo, server room construction procurements are subject to higher risks and larger scopes of quality manipulation corruption.

During the two year period (01/01/2012 to 12/31/2013) of our data set, there are total 2147 observed projects. On average, 8.8 bidders enter and submit valid bids for each project. The summation of engineer’s estimated costs of all observed projects is over 10 billion CNY (1.6 billion USD). Hence the industry is both large and has enough observations for structural estimations. For each project, our information includes its engineer’s estimated cost, number of bidders, factor weights city, the buyer’s name, and the winning firm’s identity. On bid level, we observe factor scores of each bid. Table 2 and Figure 8 summarize the data set. All price data are in units of 1000 CNY. $\tilde{q}$ and $\tilde{s}$ are defined later in this section.

<table>
<thead>
<tr>
<th>Table 2: Descriptive Statistics of the Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variables</strong></td>
</tr>
<tr>
<td><strong>Project-specific</strong></td>
</tr>
<tr>
<td>Engineer’s estimated cost, $p_0$</td>
</tr>
<tr>
<td>Weight on tech. factor, $w_p$</td>
</tr>
<tr>
<td>Weight on price factor, $w_q$</td>
</tr>
<tr>
<td>Weight on business factor, $w_r$</td>
</tr>
<tr>
<td>Number of firms, $n$</td>
</tr>
<tr>
<td>Winning score, $s$</td>
</tr>
<tr>
<td>Project city</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Bid-specific</strong></td>
</tr>
<tr>
<td>Price factor score, $s_p$</td>
</tr>
<tr>
<td>Tech. factor score, $s_q$</td>
</tr>
<tr>
<td>Business factor score, $s_r$</td>
</tr>
<tr>
<td>Price, $p$</td>
</tr>
<tr>
<td>Savings rate, $\rho = \frac{p_0 - p}{p_0}$</td>
</tr>
<tr>
<td>Weighted score, $s$</td>
</tr>
<tr>
<td>Transformed quality, $\tilde{q}$</td>
</tr>
<tr>
<td>Transformed score, $\tilde{s}$</td>
</tr>
</tbody>
</table>

Several remarks about the data set:

and Bajari and Ye (2003). Among the 1046 winning firms, 451 firms win only one contract. Therefore, tracking firm’s bidding history to construct variables like “backlog”, “capacity”, or “utilization rate” is not practical. In addition, we do not observe the identities of all losing firms, so we cannot construct explanatory variables like rival firm’s distance or rival capacity.

(2) The market structure of this industry is relatively simple. There is a large number of supplying firms and no buyers or firms dominates the industry. Table 3 shows that the largest firm only takes a 1% market share. Also, because server room project design and construction costs are not much affected by their geographic location, combining data from different cities is reasonable. Moreover, subcontracting is common in this industry, a firm’s distance to the project is less important when most components of the project are carried out by subcontractors. These features support the independent private information setting of our model.

Table 3: Summary of Market Structure

<table>
<thead>
<tr>
<th></th>
<th>Market Share</th>
<th>Number of Projects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean H.H. index</td>
<td>Mean SD Min Max</td>
</tr>
<tr>
<td>Firms</td>
<td>0.0956% 0.0015</td>
<td>2.0525 1.6044 1 22</td>
</tr>
<tr>
<td>Total = 1046</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of firm wins one project = 451</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buyers</td>
<td>1.1236% 0.0659</td>
<td>24.1236 53.4929 1 292</td>
</tr>
<tr>
<td>Total = 89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of buyer procures one project = 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of buyer procures less than 10 project = 50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Index</th>
<th>No. of Project</th>
<th>Market Share</th>
<th>Total Value</th>
<th>Share of Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top five firms:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>22</td>
<td>0.0102</td>
<td>94,641</td>
<td>0.0087</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>0.0070</td>
<td>69,990</td>
<td>0.0065</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>0.0065</td>
<td>63,264</td>
<td>0.0058</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>0.0065</td>
<td>72,496</td>
<td>0.0067</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>0.0056</td>
<td>64,504</td>
<td>0.0059</td>
</tr>
</tbody>
</table>

| Top five buyers: |                |              |             |                |
| 1     | 292            | 0.1360       | 1,463,947   | 0.1350         |
| 2     | 273            | 0.1272       | 1,370,988   | 0.1264         |
| 3     | 224            | 0.1043       | 1,154,855   | 0.1065         |
| 4     | 220            | 0.1025       | 1,110,387   | 0.1024         |
| 5     | 117            | 0.05449      | 579,093     | 0.05341        |

| Procurement agencies: |                |              |             |                |
| 1     | 1,466          | 0.6828       | 7,374,657   | 0.6802         |
| 2     | 681            | 0.3172       | 3,467,485   | 0.3198         |

See Porter and Zona (1993), Section IV for definitions of backlog, capacity, and utilization rate.
The scoring rule of this data set is relatively easy to analyze. The business factor weights is constant at 0.1 across all projects and \((w_p, w_q)\) combination takes only five pairs of values.\(^{13}\) Hence, the variation of the scoring rule can be controlled by \(w_q\). The price factor evaluation rule is consistent and not interdependent. In our sample, the engineer’s estimated costs and prices of the bid are transferred into a 100 point price factor score by formula (19) below. We will now discuss the scoring rule in detail.

All projects in our data set use the most popular “comprehensively evaluation method” in China. It is simply a linear weighted scoring rule consisting of three components: an economic factor, a technological factor, and a business factor. The economic factor \((s_p)\) evaluates the price of the bid.\(^{14}\) Denote the engineer’s estimated cost as \(p_0\), if a company submits price \(p\) in his bid, his price score is computed by

\[
s_p = \begin{cases} 
0, & p > \frac{3}{2}p_0, \\
\left(\frac{1}{2} + \frac{p_0 - p}{p_0}\right) \times 100, & \frac{1}{2}p_0 \leq p \leq \frac{3}{2}p_0, \\
100, & p < \frac{1}{2}p_0.
\end{cases}
\]  

\(^{13}\)The law of tender requires all construction projects shall economic factor weight \(w_p \geq 0.4\).

\(^{14}\)As mentioned in [Bajari et al. 2014], in reality price evaluation is not just “the low the better”. A highly unbalanced bid (extreme itemized price) or a bid lower than cost could be penalized or rejected.
\[ \rho = \frac{p_0 - p}{p_0} \] is called the savings rate of that bid. The technological factor \((s_q)\) evaluates quality of the construction plan including the design, building standard, equipment, server machines, follow-up service, warranty, delivery date, payment condition, insurance, etc.. The business factor \((s_r)\) evaluates the firm’s reputation, experience, risk of default, risk of bankruptcy etc.. Technological and business factors are evaluated by a committee of experts. Each bid receives three 100 points scores on three factors, and then a grand score is computed via

\[ S(s_p, s_q, s_r) = w_p s_p + w_q s_q + w_r s_r, \quad (20) \]

where weights, \(w_p, w_q,\) and \(w_r,\) add up to one. The firm who receives the highest 100-scale grand score wins the contract. Because firms are experts of the industry and may have repeated interactions with the procurement agencies, they understand and score evaluation process. Therefore, they are effectively selecting technological factor scores by submitting corresponding construction plans. Business factor scores are also endogenously chosen by firms because firms can hire experienced engineers, acquire relevant certificates, and allocate more financial resources to raise \(s_r.\)

The linear weighted scoring rule can be transformed into a quasilinear one by redefining quality and score. Define the transformed score, \(\tilde{s} = \frac{p_0}{100w_p} (S(s_p, s_q, s_r) - 50w_p)\) and the transformed quality, \(\tilde{q} = \frac{p_0}{100w_p} (100w_p + w_q s_q + w_r s_r),\) then (20) can be transformed into

\[ \tilde{S}(\tilde{q}, p) = \tilde{q} - p. \quad (21) \]

Because the transformation is monotone, firm’s winning probability and bidding strategy are not affected. We assume the cost of supplying the transformed quality \(\tilde{q}\) satisfies assumption CF. Then the environment satisfies the condition of Lemma 1, which allows us to use the single dimensional quality index \(\tilde{q}\) for structural estimation.

Similar to the theoretical model, we take (21) as the objective function of the buyer that reflects her true preference. The original scoring rule (20) reflects how much the buyer is willing to substitute paying a higher price for a higher quality. If the firm raises the price by 1 CNY, then the 100-scale grand score will reduce by \(100 \frac{w_p}{p_0}.\) To remain at the same payoff, the buyer needs to be compensated by a higher quality, which requires \(w_q \Delta s_q + w_r \Delta s_r = 100 \frac{w_p}{p_0}.\) We add a boundary condition that the buyer receives zero payoff from a contract with \(s_q = 0, s_r = 0\) and \(p = p_0.\) The transformed scoring rule (21) satisfies both the substitution condition and the boundary condition reflecting the true preference of the buyer. Therefore, we take \(\tilde{q}\) as the buyer’s benefit from a project delivered at \(s_q\) and \(s_r,\) and \(\tilde{s}\) as the payoff after paying the winning firm \(p.\) Because the transformed quality and score are anchored on price that has real monetary interpretation, they can be compared across auctions.

\cite{15} The law of tender require the committee shall contain five or more members (odd number). There is one representative from the buyer. All the other members are either randomly selected from the pool of experts of the procurement agency.
4.2 Reduced-form Estimation

We have three main findings in the following reduced-form empirical study:

(1) We test two implications of the theoretical model. First, a higher quality weight (lower price weight) shall induce firms to submit bids with higher quality and higher grand score. Second, firm’s choice of quality and price are separated under additively separable scoring rule. By using the original strategy space \((s_p, s_q, s_r)\) as the dependent variable, we do not find robust evidence. But we find the evidence supporting both model implications by using the transformed strategy space \((\tilde{q}, \tilde{s})\), which in turn justifies our use of the transformed strategy space for structural estimations and corruption detection tests.

(2) We tests for unobserved heterogeneity of projects with respect to fringe/non-fringe firms, fringe/non-fringe buyers, and two procurement agencies. We do not find strong evidence of unobserved heterogeneity among these projects.

(3) Based on the transformed strategy space, we find that projects with high engineer’s estimated costs end up with winning contracts of both high quality scores and high prices. Projects with low engineer’s estimated costs induce more competition on price and end up with higher savings rates (lower markups).

Figure 9: Illustration of Winning Bids in Observed Strategy Space

Note: In diagram (A) and (B), black points represent the \(p_0\); red circles represent submitted prices.
Consider the reduced-form regression model

\[ Y_t = \alpha_0 + \alpha_1 p_{0,t} + \alpha_2 n_{t} + \alpha_3 w_{q,t} + \alpha_4 D_{\text{fringe.firm},t} + \alpha_5 D_{\text{fringe.buyer},t} + D_{\text{agency2},t} + \epsilon_t, \]

where \( Y_t \) stands for the dependent variable. The main independent project-specific covariates are engineer’s estimated cost and the number of bidders. On bids level, we only observed three factor scores that are endogenously chosen by firms. Price, grand score, transformed quality, and transformed score are all functionally correlated with factor scores. The data also lacks the losing firm’s identities, so there is no explanatory variable on bid level. Therefore, we estimate reduced-form models on project level with only winning bids. Because there are lots of firms or buyers that only appear in one project, we do not include firm or project fixed effects in the model. Instead, we add two indicators for fringe firms and fringe buyers: \( D_{\text{fringe.firm}} = 1 \) if the winning firm is fringe (wins only one project) and \( D_{\text{fringe.buyer}} = 1 \) if buyer is fringe (procures less than 10 projects). \( D_{\text{agency2}} \) is the indicator for if the project is process by a procurement agency 2.

Table 4: Reduced-form Regressions in Observed Strategy Space

<table>
<thead>
<tr>
<th>Dep.Var.</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
<th>(V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_0 )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.288**</td>
<td>( p_s )</td>
<td>( p_q )</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0050)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>0.7447**</td>
<td>0.1236*</td>
<td>4.341*</td>
<td>0.3809**</td>
<td>0.0013*</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.057)</td>
<td>(1.99)</td>
<td>(0.034)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>( w_q )</td>
<td>11.17**</td>
<td>-4.95</td>
<td>3294.64**</td>
<td>1.53</td>
<td>-0.5758**</td>
</tr>
<tr>
<td></td>
<td>(2.92)</td>
<td>(3.51)</td>
<td>(122.03)</td>
<td>(2.09)</td>
<td>(0.0377)</td>
</tr>
<tr>
<td>( D_{\text{fringe.firm}} )</td>
<td>0.1008</td>
<td>-0.5357</td>
<td>-28.67</td>
<td>-0.5463</td>
<td>-0.0093</td>
</tr>
<tr>
<td></td>
<td>(0.4378)</td>
<td>(0.5265)</td>
<td>(18.30)</td>
<td>(0.3127)</td>
<td>(0.0056)</td>
</tr>
<tr>
<td>( D_{\text{fringe.buyer}} )</td>
<td>1.0405</td>
<td>-0.5909</td>
<td>-4.416</td>
<td>0.7195</td>
<td>0.0064</td>
</tr>
<tr>
<td></td>
<td>(0.5998)</td>
<td>(0.7212)</td>
<td>(25.07)</td>
<td>(0.4285)</td>
<td>(0.0077)</td>
</tr>
<tr>
<td>( D_{\text{agency2}} )</td>
<td>-0.0694</td>
<td>0.3887</td>
<td>-30.78</td>
<td>-0.1587</td>
<td>-0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.3826)</td>
<td>(0.4601)</td>
<td>(16.0)</td>
<td>(0.2734)</td>
<td>(0.0049)</td>
</tr>
<tr>
<td>Constant</td>
<td>81.83**</td>
<td>76.48**</td>
<td>-3511.04**</td>
<td>74.66**</td>
<td>0.3927**</td>
</tr>
<tr>
<td></td>
<td>(1.50)</td>
<td>(1.81)</td>
<td>(62.87)</td>
<td>(0.98)</td>
<td>(0.0176)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.1050</td>
<td>0.0050</td>
<td>0.9687</td>
<td>0.0595</td>
<td>0.1123</td>
</tr>
<tr>
<td>Obs</td>
<td>2147</td>
<td>2147</td>
<td>2147</td>
<td>2147</td>
<td>2147</td>
</tr>
</tbody>
</table>

Note: Significance levels are denoted by asterisks (* \( p < 0.05 \), ** \( p < 0.01 \)).

Figure (9) and Table 4 summarize the estimation results. Based on the coefficients of technological factor weight, a higher \( w_q \) results in a higher technological score, a higher price, and a lower savings rate. Therefore, if a buyer wants to procure the project at a higher quality, the cost will also increase significantly. In other word, firms ask for higher markups in high technological weight procurements. More entry of the procurement auction increases competition and results in positive effects on all five dependent variables. In addition, diagram (A), (B) and regression (I), (II) show that for variations of engineer’s estimated costs does not technological scores and business scores if the winning bids. None of these regressions shows significant differences between the large and
fringe firms (buyers). The two procurement agencies also appear to be similar.

These regressions with the dependent variable within the observed strategy space have one major drawback: scores on the 100-scale are intangible concepts and hard to compare across auctions. Receiving the same 100 point technological scores may mean completely different things for two projects. We also find that increasing $w_q$ does not significantly increase grand scores in regression (IV), which is not consistent with theoretical model prediction. In addition, the goodness-of-fit, measured by $R^2$, are relatively low except for regression (III).

Nevertheless, we can consider the transformed strategy space with $\tilde{q}$ and $\tilde{s}$. These two variables are directly related to price and thus can be compared across auctions. Table 5 displays estimation results using $\tilde{q}$ and $\tilde{s}$ as dependent variables. For all six regressions, their $R^2$s improve and show significantly positive coefficient estimates of $w_q$. We can interpret that by increasing the technological factor weight for 5%, it induces the project to be delivered at a higher quality and higher score to the buyer. The estimated average buyer’s payoff increment ranges from 800,858 to 1,044,340 CNY (129,171 to 168,442 USD).

Concerning the coefficient estimates of regressor $n$, increasing the number of bidders has no significant effect on $\tilde{q}$, but has significant positive effects on $\tilde{s}$. It provides the important evidence supporting the theoretical model: firms choose their quality level based on their own social surplus maximization problem (equation (2) and (9)), hence $n$ does not affect their choice of $\tilde{q}$. Because this property of independent quality choices relies on fairly weak assumption, confirming it in empirical study also supports the validity of our strategy space transformation.

Table 5: Regression in Transformed Strategy Space

<table>
<thead>
<tr>
<th>Dep.Var.</th>
<th>(I) Transformed Quality $\tilde{q}$</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV) Transformed Score $\tilde{s}$</th>
<th>(V)</th>
<th>(VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>All $p_0 &lt; 5,565$</td>
<td>$p_0 \geq 5,565$</td>
<td>All $p_0 &lt; 5,565$</td>
<td>$p_0 \geq 5,565$</td>
<td>All $p_0 &lt; 5,565$</td>
<td>$p_0 \geq 5,565$</td>
</tr>
<tr>
<td>$n$</td>
<td>3.05</td>
<td>-0.85</td>
<td>47.56</td>
<td>20.15**</td>
<td>16.38**</td>
<td>39.08**</td>
</tr>
<tr>
<td></td>
<td>(16.90)</td>
<td>(11.44)</td>
<td>(25.71)</td>
<td>(6.38)</td>
<td>(5.71)</td>
<td>(7.01)</td>
</tr>
<tr>
<td>$w_q$</td>
<td>19,007.23**</td>
<td>16,876.99**</td>
<td>26,367.82**</td>
<td>16,017.17**</td>
<td>14,326.23**</td>
<td>20,886.79**</td>
</tr>
<tr>
<td></td>
<td>(1,038.50)</td>
<td>(690.20)</td>
<td>(1,552.03)</td>
<td>(391.94)</td>
<td>(344.24)</td>
<td>(450.61)</td>
</tr>
<tr>
<td>$D_{fringe.firm}$</td>
<td>200.15</td>
<td>186.62</td>
<td>-69.02</td>
<td>69.71</td>
<td>80.97</td>
<td>-41.14</td>
</tr>
<tr>
<td></td>
<td>(155.65)</td>
<td>(103.21)</td>
<td>(248.59)</td>
<td>(58.75)</td>
<td>(51.48)</td>
<td>(67.81)</td>
</tr>
<tr>
<td>$D_{fringe.buyer}$</td>
<td>-158.57</td>
<td>-80.29</td>
<td>-560.93</td>
<td>-24.58</td>
<td>-21.19</td>
<td>-131.02</td>
</tr>
<tr>
<td></td>
<td>(213.30)</td>
<td>(141.10)</td>
<td>(342.64)</td>
<td>(80.50)</td>
<td>(70.37)</td>
<td>(93.46)</td>
</tr>
<tr>
<td>$D_{agency2}$</td>
<td>121.59</td>
<td>124.31</td>
<td>-242.29</td>
<td>70.75</td>
<td>61.07</td>
<td>-28.83</td>
</tr>
<tr>
<td></td>
<td>(136.08)</td>
<td>(89.99)</td>
<td>(218.49)</td>
<td>(51.36)</td>
<td>(44.88)</td>
<td>(59.60)</td>
</tr>
<tr>
<td>Constant</td>
<td>2,021.96**</td>
<td>1,552.37**</td>
<td>2,431.78**</td>
<td>-1,234.99**</td>
<td>-987.43**</td>
<td>-2,068.26**</td>
</tr>
<tr>
<td></td>
<td>(485.21)</td>
<td>(325.33)</td>
<td>(756.52)</td>
<td>(183.13)</td>
<td>(162.26)</td>
<td>(206.35)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.1421</td>
<td>0.2943</td>
<td>0.3066</td>
<td>0.4446</td>
<td>0.5378</td>
<td>0.7863</td>
</tr>
<tr>
<td>Obs</td>
<td>2147</td>
<td>1551</td>
<td>596</td>
<td>2147</td>
<td>1551</td>
<td>596</td>
</tr>
</tbody>
</table>

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).
Table 6: Pattern of Winning Bids

<table>
<thead>
<tr>
<th>Mean of All Bids</th>
<th>Mean of Winning Bids</th>
<th>Highest in the Auction</th>
<th>Lowest in the Auction</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_q</td>
<td>60.48</td>
<td>93.05</td>
<td>1582</td>
</tr>
<tr>
<td>s_r</td>
<td>72.30</td>
<td>75.32</td>
<td>455</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.1980</td>
<td>0.1691</td>
<td>86</td>
</tr>
<tr>
<td>(\tilde{q})</td>
<td>8260.86</td>
<td>4746.27</td>
<td>1594</td>
</tr>
</tbody>
</table>

In addition, we also observe some meaningful patterns across winning bids. In Figure 10 diagram (A) and (B), we plot the density of savings rate and transformed quality respectively. The black curve represents the density of all observed bids while the red dashed curve represent the density of only winning bids. These two density diagrams show that winning bids have consistent pattern of both higher quality and higher price, compared to other bids. Table 6 shows that 74.24\% of winning bids have the highest transformed quality in that auction. On the other hand, only 4.01\% of winning bids have the highest savings rate in that auction. On average, winning bids ask for higher prices (a lower savings rate) than losing bids. Among the 2147 auction we observed, 145 projects ended up with negative savings rates, meaning that the winning contracts have prices higher than the engineer’s estimated costs. All these 145 negative savings rate auctions occur at projects with engineer’s estimated costs higher than 5,565 thousand CNY. Hence, a high \(p_0\) project is more likely to be awarded to a high quality and high price bidder, illustrated in Figure 10 diagram (E). So we see high quality and high price contracts concentrating at high \(p_0\) projects, where there are more room for quality manipulation corruption. These signs of corruption motivate the corruption detection tests below.

4.3 Structural Estimation and Corruption Detection Tests

Lacking bid-specific covariates, project level regressions put aside information in all losing bids because they are endogenous, but structural estimation can draw information from all bids. The pattern of both winning bids and losing bids together reveal whether the bidding behaviors are competitive. In the transformed strategy space, the structural estimation and corruption detection tests developed in Section 3.1 and 3.2 can be directly applied. Varying the scoring rule affects the distribution of pseudotype, therefore we consider sub-samples according to technological weights and procurement agencies. For each sub-sample, we apply formula \(13\) to structurally estimate pseudotypes. The estimation results are reported in Table 7 and Figure 11.

Recall that \(\hat{k}\) represents the total social surplus of each firm producing at its efficient level, and \(\tilde{s}\) represents how much of the social surplus is harvested by the buyer. Their difference, \(\hat{k} - \tilde{s}\), is the estimated rent retained by the firm. Table 8 compares performance of two procurement agencies. The projects processed by two procurement agencies are similar in their observed characteristics,
Figure 10: Illustration of Winning Bids in Transformed Strategy Space

Note: In diagram (A) and (B), the black curve represents density of all bids; the red dashed curve represent density of winning bids. In diagram (E), black points represent observed $(\tilde{q}, p_0)$; red circles represent $(\tilde{q}, p)$; the blue dashed line is the 45 degree line.
but we find that in general, firms bid in procurement agency 1 receives gets higher rent than agency 2. Specifically, overall, firms at procurement agency 1 ask for 63,030 CNY (10,166 USD) more rent compared to agency 2. But if we consider only winning bids, at each sub-sample, winning firms at procurement agency 1 do not earn significantly more rent than those at agency 2.

We can also compare “vertically” on technological factor weights. A higher $w_q$ in general leads to both higher transformed scores and higher rents, benefiting both parties. Quality weights reflect the buyers’ willingness-to-pay for high quality projects, while the supplying firms only care monetary compensations. Serving buyers with higher willingness-to-pay naturally lead to higher payoffs for both sides. The theoretical model (Corollary 2), reduced-form, and structural estimation results are consistent in this prediction.

### Table 7: Structural Estimation Results

<table>
<thead>
<tr>
<th>Sub-sample $w_q$</th>
<th>Agency No. of Projects</th>
<th>No. of Bids</th>
<th>Mean $\hat{k}$</th>
<th>SD $\hat{s}$</th>
<th>Min $\hat{s}$</th>
<th>Max $\hat{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>1</td>
<td>223</td>
<td>2384</td>
<td>$3,358.97$</td>
<td>$1,292.52$</td>
<td>$1,156.09$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $3,126.94$</td>
<td>$797.31$</td>
<td>$1,135.31$</td>
<td>$5,747.27$</td>
</tr>
<tr>
<td>0.3</td>
<td>2</td>
<td>87</td>
<td>913</td>
<td>$3,389.06$</td>
<td>$1,088.43$</td>
<td>$1,308.24$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $3,173.24$</td>
<td>$811.30$</td>
<td>$1,264.24$</td>
<td>$5,407.96$</td>
</tr>
<tr>
<td>0.35</td>
<td>1</td>
<td>244</td>
<td>2266</td>
<td>$3,913.30$</td>
<td>$1,750.54$</td>
<td>$891.22$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $3,546.35$</td>
<td>$1,032.03$</td>
<td>$821.27$</td>
<td>$6,945.76$</td>
</tr>
<tr>
<td>0.35</td>
<td>2</td>
<td>134</td>
<td>1228</td>
<td>$3,942.98$</td>
<td>$1,473.08$</td>
<td>$1,540.13$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $3,617.00$</td>
<td>$1,014.27$</td>
<td>$1,494.09$</td>
<td>$6,303.84$</td>
</tr>
<tr>
<td>0.4</td>
<td>1</td>
<td>384</td>
<td>3437</td>
<td>$4,451.63$</td>
<td>$2,250.47$</td>
<td>$1,253.65$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $3,980.14$</td>
<td>$1,424.36$</td>
<td>$1,175.52$</td>
<td>$7,971.81$</td>
</tr>
<tr>
<td>0.4</td>
<td>2</td>
<td>195</td>
<td>1687</td>
<td>$4,448.67$</td>
<td>$1,805.02$</td>
<td>$1,052.06$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $4,053.84$</td>
<td>$1,227.13$</td>
<td>$1,015.22$</td>
<td>$7,568.83$</td>
</tr>
<tr>
<td>0.45</td>
<td>1</td>
<td>407</td>
<td>3335</td>
<td>$5,342.17$</td>
<td>$3,181.92$</td>
<td>$773.96$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $4,691.41$</td>
<td>$1,515.09$</td>
<td>$712.09$</td>
<td>$9,711.02$</td>
</tr>
<tr>
<td>0.45</td>
<td>2</td>
<td>176</td>
<td>1510</td>
<td>$5,131.95$</td>
<td>$2,360.78$</td>
<td>$1,603.51$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $4,585.34$</td>
<td>$1,505.13$</td>
<td>$1,431.86$</td>
<td>$9,324.88$</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>208</td>
<td>1567</td>
<td>$6,054.45$</td>
<td>$3,463.00$</td>
<td>$1,724.88$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $5,224.02$</td>
<td>$1,870.11$</td>
<td>$1,620.04$</td>
<td>$11,559.27$</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>89</td>
<td>636</td>
<td>$6,474.78$</td>
<td>$3,054.39$</td>
<td>$1,699.51$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\hat{s}$: $5,667.42$</td>
<td>$1,894.45$</td>
<td>$1,539.81$</td>
<td>$10,811.87$</td>
</tr>
</tbody>
</table>

Aforementioned, a high rent alone is not the sign of corruption. To test for quality manipulation corruption, we need to explore the consistently suspicious patterns of relationship among bids revealed in a large sample. We apply the three tests proposed in Section 3.2. Table 9 and Figure 12 show results of test I and II. For test I, there are a total five sub-samples rejecting the competitive model. In general, they happen at high $w_q$ auctions. For test II, none of sub-samples rejects the competitive model. Notice that, for both tests, the original $p$-values and bootstrap $p$-values provide
Table 8: Comparison of Two Procurement Agency

<table>
<thead>
<tr>
<th></th>
<th>Pro. Agency 1</th>
<th>Pro. Agency 2</th>
<th>t-test of Equal Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
</tr>
<tr>
<td>( n )</td>
<td>8.860</td>
<td>3.937</td>
<td>8.772</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>5030.46</td>
<td>1515.45</td>
<td>5091.77</td>
</tr>
<tr>
<td>( \hat{s} )</td>
<td>5435.89</td>
<td>1489.60</td>
<td>5484.52</td>
</tr>
</tbody>
</table>

Estimated Rent (\( \hat{k} - \hat{s} \)) of All Bids

\[ w_q = 0.3 \]
\begin{align*}
232.03 & \quad 749.33 \\
215.82 & \quad 380.90 \\
0.8162 & \quad 0.4144
\end{align*}

\[ w_q = 0.35 \]
\begin{align*}
366.95 & \quad 1,014.52 \\
325.98 & \quad 664.14 \\
1.4365 & \quad 0.1509
\end{align*}

\[ w_q = 0.4 \]
\begin{align*}
471.49 & \quad 1,396.39 \\
394.82 & \quad 873.68 \\
2.4007 & \quad 0.0164
\end{align*}

\[ w_q = 0.45 \]
\begin{align*}
650.76 & \quad 2,261.30 \\
546.60 & \quad 1,191.91 \\
2.0940 & \quad 0.0363
\end{align*}

\[ w_q = 0.5 \]
\begin{align*}
830.42 & \quad 2,089.09 \\
807.36 & \quad 1,551.44 \\
0.2846 & \quad 0.7760
\end{align*}

Overall
\begin{align*}
498.63 & \quad 1,634.84 \\
435.60 & \quad 985.48 \\
3.2844 & \quad 0.0010
\end{align*}

Estimated Rent (\( \hat{k} - \hat{s} \)) of Winning Bids

\[ w_q = 0.3 \]
\begin{align*}
816.54 & \quad 2,212.67 \\
646.67 & \quad 821.04 \\
0.9857 & \quad 0.3251
\end{align*}

\[ w_q = 0.35 \]
\begin{align*}
1,239.56 & \quad 2,667.93 \\
967.06 & \quad 1,640.22 \\
1.2279 & \quad 0.2203
\end{align*}

\[ w_q = 0.4 \]
\begin{align*}
1,534.06 & \quad 3,529.86 \\
1,208.71 & \quad 2,164.06 \\
1.3692 & \quad 0.1715
\end{align*}

\[ w_q = 0.45 \]
\begin{align*}
2,006.02 & \quad 5,095.38 \\
1,632.98 & \quad 2,743.71 \\
1.1428 & \quad 0.2536
\end{align*}

\[ w_q = 0.5 \]
\begin{align*}
2,228.15 & \quad 4,393.84 \\
2,183.71 & \quad 3,214.99 \\
0.0972 & \quad 0.9226
\end{align*}

Overall
\begin{align*}
1,605.41 & \quad 3,914.21 \\
1,326.43 & \quad 2,330.57 \\
2.0551 & \quad 0.0400
\end{align*}

Note: Bold numbers indicate rejection of the null at 0.05 significance level.

The opposite conclusions of the same hypothesis tests. Because the correlation among observations, we should draw conclusions based on bootstrap critical values. For test III, we consider six regression models shown in Table 10 and find only one coefficient of \( D_{\text{agency2}} \) being significant. Regression (VI) is run on the sub-sample with high engineer’s estimated costs. It implies the first spacing of transformed scores is larger at procurement agency 2, which is the sign of aggressive bidding behavior. Since we also find that rent at procurement agency 2 is generally lower, the reality could be that firms are earning their rent under the table by delivering low quality projects.

In summary, a majority of the data set passes our corruption detection tests. Recall Figure 7 these failures of rejection support the theoretical prediction of the competitive model, which makes the structural estimation trustworthy for this data set. For some sub-samples of the data set, we find signs of quality manipulation corruption. The data patterns shown in Figure 10 diagram (E) and results of corruption detection tests suggest that antitrust authorities should spend more investigation resources on projects with high technological weights and high engineer’s estimated costs, especially on those processed by procurement agency 2.

It is worth mentioning that high technological weight and estimated cost are proxy for complexity of the project. Bajari and Tadelis (2001) and Tadelis (2012) compare auction and negotiation at different level of project complexity. Complexity may potentially jeopardize the advantage of
auctions because it brings in uncertainty in project design and costly renegotiation of *ex post* adjustment. The buyer may choose a bilateral negotiation with a reputable supplier, because in the negotiation, the reputable firm can help design the complex project and save the *ex post* adaption cost. In a scoring auction, quality and design of the project are chosen by firms, so it reaps benefit from both price-only auctions and negotiation. However, all these cross-procurement-scheme comparisons are not robust if quality is not perfectly observable and/or verifiable at the moment of transaction. The corruption problem analyzed in this paper not only affects the optimal scoring rule and auction format, but also optimal procurement scheme.

At the end, we want to point out that our nonparametric approach is based on pseudotypes and the distribution of pseudotype changes as scoring rule varies. A fully nonparametric method is may not be applicable when there is a great deal of variation in scoring rules across auctions and the sample size is relatively small. In this case, a parametric or semi-parametric approach shall be adopted. Once a parametric cost function is specified, the optimal quality and scores can be expressed as a system of equation of parameters. Nakabayashi and Hirose (2015) show the exact conditions that ensure the data on quality and score can reverse-engineer the parameters.

Table 9: Results of Test I and II

<table>
<thead>
<tr>
<th>w_q</th>
<th>Sub-sample</th>
<th>Test I</th>
<th>Test II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agency</td>
<td>Test Stat.</td>
<td>p-value</td>
<td>BT c.v.</td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>-6.7284</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-3.3526</td>
<td>0.9994</td>
</tr>
<tr>
<td>0.35</td>
<td>1</td>
<td>-6.2109</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-5.0055</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.4</td>
<td>1</td>
<td>-7.3851</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-4.7708</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.45</td>
<td>1</td>
<td>-5.3859</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-5.5837</td>
<td>1.0000</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>-5.3204</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-4.4933</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: BT c.v. and BT p.v stand for “bootstrap critical value” at 0.05 significance level and “bootstrap p-value” respectively. They are computed based on 199 bootstrap samples at project level. Bold numbers indicate rejection of the null.
Note: Diagram (A) to (D) are estimated pseudotypes and related transformed scores, \((\hat{k}, \hat{s})\). The blue line is the 45 degree line. The red dashed line is a smoothed spline. Diagram (E) shows the kernel smoothed conditional density \(f(\hat{k}|w_q)\).
Figure 12: Result of Test I and Test II

![Graphs for Test I and Test II]

Table 10: Results of Test III

<table>
<thead>
<tr>
<th>Dep.Var</th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
<th>(V)</th>
<th>(VI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>All</td>
<td>$p_0 &lt; 5565$</td>
<td>$p_0 \geq 5565$</td>
<td>All</td>
<td>$p_0 &lt; 5565$</td>
<td>$p_0 \geq 5565$</td>
</tr>
<tr>
<td>$D_{agency2}$</td>
<td>-39.78</td>
<td>3.372</td>
<td>-245.88</td>
<td>20.45</td>
<td>4.750</td>
<td>103.25*</td>
</tr>
<tr>
<td></td>
<td>(110.36)</td>
<td>(29.02)</td>
<td>(591.77)</td>
<td>(16.84)</td>
<td>(18.05)</td>
<td>(46.37)</td>
</tr>
<tr>
<td>3rd order</td>
<td>0.3082**</td>
<td>-0.0863**</td>
<td>0.0568</td>
<td>-0.0506**</td>
<td>-0.0922**</td>
<td>-0.2180**</td>
</tr>
<tr>
<td></td>
<td>(0.0296)</td>
<td>(0.0129)</td>
<td>(0.1194)</td>
<td>(0.0072)</td>
<td>(0.0098)</td>
<td>(0.0246)</td>
</tr>
<tr>
<td>$w_q$</td>
<td>-1,120.81</td>
<td>3,472.47**</td>
<td>5,910.49</td>
<td>1,813.97**</td>
<td>2,298.35**</td>
<td>4,947.44**</td>
</tr>
<tr>
<td></td>
<td>(929.72)</td>
<td>(268.23)</td>
<td>(5274.83)</td>
<td>(160.63)</td>
<td>(180.07)</td>
<td>(578.31)</td>
</tr>
<tr>
<td>Constant</td>
<td>-236.29</td>
<td>-547.17**</td>
<td>187.46</td>
<td>64.50</td>
<td>-56.40</td>
<td>-207.70</td>
</tr>
<tr>
<td></td>
<td>(337.84)</td>
<td>(88.85)</td>
<td>(1857.27)</td>
<td>(60.86)</td>
<td>(66.11)</td>
<td>(175.94)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.0556</td>
<td>0.0986</td>
<td>0.0089</td>
<td>0.1588</td>
<td>0.1875</td>
<td>0.2875</td>
</tr>
<tr>
<td>Obs</td>
<td>2147</td>
<td>1551</td>
<td>396</td>
<td>2147</td>
<td>1551</td>
<td>596</td>
</tr>
</tbody>
</table>

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).
5 Conclusion

We conclude by reviewing main results with policy implications. In this paper, we develop a structural estimation method and three corruption detection tests of scoring auctions. They are built upon fairly standard data of procurement auctions and can be applied to data from a wide range of industry with enough observations. The estimation method and corruption detection tests in this paper complement the theoretical side of the optimal procurement problem. By using historical data, they provide quantitative prediction of the effect of varying the scoring rule and the risk of quality manipulation corruption. These predictions are particularly useful for designing desirable procurement schemes. Therefore, recording and aggregating procurement auction data are valuable for improving procurement outcome and identifying corruption.

We applied the method to a data set of server room procurement auctions. The data patterns and estimation results provide evidence of the theoretical scoring auction model. First, under additively separable scoring rules, the choice of quality can be separated from the choice of price and score. The reduced-form estimation shows that qualities are not affected by the number of competing bidders, but scores are. Second, with competition on both price and quality, firms mainly compete on offering high quality and expensive contracts. In the data set, over 70% of winning bids have the highest quality, but only about 4% of winning bids have the lowest price. Therefore, a reliable quality evaluation procedure is very important in keeping the auction efficient.

We also explore the effect of varying quality weight. The theoretical model predicts that a higher weight on quality induces firms to submit bids at higher quality and score, which is confirmed by estimation results in the transformed strategy space. The structural estimation results show that projects procured with higher quality weights result in both higher payoffs for the buyers and the winning firms. However, the buyer is restricted in picking the quality weight because the scoring rule must reflect her willingness-to-pay of higher quality. The theoretical model of scoring auctions shows that the buyer will not over-state its preferences on quality\footnote{Huang and Xia (2015) shows that the buyer may over-state it to fight against quality manipulation.} instead, the optimal scoring rule “shade” buyer’s preference on quality to avoid giving up too much rent to the efficient firm. Besides the shading for optimal screening, does a higher quality weight gives more room for quality manipulation? As Lengwiler and Wolfstetter (2006) suggested, when quality scores are problematic due to the possibility of corruption, the quality weight shall be reduced. Therefore, in designing the scoring rule, the buyers need to balance the efficiency and the risk of quality manipulation corruption. We run the three corruption detection tests proposed in this paper and find that, in general, the data set passes our tests. But there are some signs of corruption in sub-samples with higher quality weights and higher engineer’s estimated costs.

Our corruption detection tests are ex ante in the sense that they can label identities of corrupted firms, projects, and procurement agencies without specifying a prior of suspects. For future research, one important complement is an ex post study of corruption behaviors by data from convicted...
corruption cases, for example, from investigation reports from collapsed bridges. Then researchers can study the “in-sample” property of these corruption detection tests (e.g. Bajari and Ye (2003)) and even the internal organization of corrupted agents (e.g. Asker (2010)). In this way, historical auction data, antitrust records, and economic analysis can together construct stronger tools for antitrust purposes.

Appendix

Proof of Lemma 1: Define $C(v, \theta)$ as the value function of the minimization problem:

$$C(v, \theta) = \min_q C(q, \theta) \text{ s.t } V(q) = v.$$ 

Define the quality index $v$ of $q \in \mathbb{R}^L$ by $v \equiv V(q)$. Under assumption QL and CF, $C(v, \theta)$ is single-valued, strictly increasing, convex, and continuous function. The Lagrangian expression of the minimization problem is

$$\mathcal{L} = C(q, \theta) - \lambda(V(q) - v).$$

When $v > 0$, the first-order condition yields

$$\begin{cases} C_q(q|\theta) - \lambda V_q(q) = 0 \in \mathbb{R}^L, \\ V(q) - v = 0 \in \mathbb{R}. \end{cases}$$

By assumption CF, $C(\cdot, \theta)$ is strictly convex in $q$, matrix $C_{qq}$ is positive definite. By assumption QL, $V(q)$ is weakly concave, $V_{qq} \leq 0$. So there is a unique solution to the system of these $L + 1$ linear equations, denoted as $q(v|\theta)$ and $\lambda(v|\theta)$. The solution correspondence of the minimization problem is the value function $C(v, \theta) = C(q(v|\theta), \theta)$.

By envelop theorem, the value function satisfies $C_v = \lambda$. $V(q) - v = 0$ implies $V_q(q(v|\theta))q_v(v|\theta) = 1$. Therefore, $C_q(q|\theta) - \lambda V_q(q) = 0$ implies $\lambda = C_q(q|\theta)/V_q(q) = C_q(q|\theta)q_v(v|\theta) = C_v > 0$.

To further show $C_{vv} > 0$, differentiate FOC above with respect to $v$:

$$\frac{\partial}{\partial v} \left( \frac{\partial \mathcal{L}}{\partial q} \right) = \begin{pmatrix} C_{qq}(q|\theta) - \lambda V_{qq}(q) & -V_q^T \\ V_q & 0 \end{pmatrix} \begin{pmatrix} q_v \\ \lambda_v \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\Rightarrow V_q^T \lambda_v = (C_{qq}(q|\theta) - \lambda V_{qq}(q))q_v.$$

Premultiply by $q_v^T$, because $V_q(q(v|\theta))q_v(v|\theta) = 1$ and $\frac{\partial^2 \mathcal{L}}{\partial q \partial v} = C_{qq} - \lambda V_{qq}$ is positive definite (PD) for minimization, we have

$$q_v^T V_q^T \lambda_v = q_v^T (C_{qq} - \lambda V_{qq}) q_v > 0$$

PD
Therefore, we can transform the cost function by $C(q(v|\theta), \theta) = C(v, \theta)$, which satisfies property $C_v = \lambda(v|\theta) > 0$ and $C_{vv} = \lambda_v(v|\theta) > 0$. 

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

**Proof of Theorem 1**: (2) holds as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.

Problem (7) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following and uniqueness of a symmetric monotone Bayesian Nash equilibrium below. Problem (7) is a standard first-price auction problem in IPV environment. The existence.
which is equal to the score the winner will bid. At the equilibrium, the winner has pseudotype being the highest order statistic \(k_{(1:n)}\), while the second highest bidder has pseudotype \(k_{(2:n)}\), hence 
\[
E[s(k_{(1:n)})] = E[k_{(2:n)}].
\]

**Proof of Corollary 2:** Asker and Cantillon (2008) shows a straightforward proof. Suppose the minimum quality is set at \(q\) and the scoring rule represents the buyer’s true preference. By Corollary 1, Expected utility of price-only auction is 
\[
V(q) - E[C(q, \theta_{(n-1:n)})] = E[(V(q) - C(q, \theta))(2:n)]
\]
\[
\leq E\left[\max_q(V(q) - C(q, \theta))(2:n)\right] = E[k_{(2:n)}],
\]
which is the expected utility in scoring auction. 

**Proof of Theorem 2:** (1) Quality
Suppose the corrupted firm with type \(\theta\) bids \((p', q')\) at some \(q' \neq q_m\), we can show that by choosing \(q_m\), the corrupted firm can always find a price \(p_m\) that yields a higher payoff upon winning. Let 
\[
p_m = V(q_m) - V(q') + p',
\]
then \((p', q')\) and \((p_m, q_m)\) have the same score because 
\[
S(q', p') = V(q') - p' = V(q_m) - p_m = S(q_m, p_m) = s.
\]
These two bids have the same expected payoff \(Pr(\text{win} \mid s)\). Their expected payoffs satisfies
\[
\pi(p_m, q_m) - \pi(p', q') = [p_m - C(q_m - m, \theta) - p' + C(q' - m, \theta)] Pr(\text{win} \mid s)
\]
\[
= [V(q_m) - V(q') + p' - C(q_m - m, \theta) - p' + C(q' - m, \theta)] Pr(\text{win} \mid s)
\]
\[
= [V(q_m) + C(q_m - m, \theta) - (V(q') - C(q' - m, \theta))] Pr(\text{win} \mid s) > 0,
\]
because \(q_m\) is chosen by (9). The scoring rule being quasilinearity (additively separable) is essential for this result to hold.

(2) Score and price
Under assumption UA, all other firms pick their score according to (8), so the corrupted firm’s pick its core according to
\[
\max_{s_m}(k_m - s_m) Pr(\text{win} \mid s_m) = (k_m - s_m) \left[F_K(s^{-1}(s_m))\right]^{n-1}.
\]
Following the same step in getting (8), the corrupted firm choose its score according to 
\[
s(k_m) = k_m - \int_k^{k_m} [F_K(t)]^{n-1} dt / [F_K(k_m)]^{n-1}.\]
The corresponding price is 
\[
p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)) = C(q_m(\theta) - m, \theta) + \int_k^{K_m(\theta)} [F_K(t)]^{n-1} dt / [F_K(K_m(\theta))]^{n-1}.
\]
(3) For any \(m > 0\), at the equilibrium, 
\[
q_m(\theta) > q(\theta), K_m(\theta) > K(\theta),\text{ and } s(K_m(\theta)) > s(K(\theta)).
\]
The unique solution of quality choice of (2) and (9) are both determined by their first-order conditions. Suppose \(\tilde{q}\) solves \(V_\theta(q) = C_\theta(q, \theta)\). Because \(C_\theta > 0\), the cost function has increasing slope, \(V_\theta(\tilde{q}) = C_\theta(\tilde{q}, \theta) > C_\theta(\tilde{q} - m, \theta)\). By assumption QL, \(V_{qq} \leq 0\), the solution to \(V_\theta(q) =
\[ C_q(q - m, \theta) \text{ must be strictly larger than } \bar{q}, \text{ therefore } q_m(\theta) > q(\theta). \]

The other two are straight-forward. Because \( C_q > 0 \), \( C(q - m, \theta) < C(q, \theta) \) for all \( q \) and \( \theta \), \( K_m(\theta) = \max_q V(q) - C(q - m, \theta) > \max_q V(q) - C(q, \theta) = K(\theta) \). The equilibrium score bidding function \( s(\cdot) \) is increasing, hence \( s(K_m(\theta)) > s(K(\theta)) \). It is obvious that all three effects magnify as \( m \) increases. \( Q.E.D. \)

**Derivation of \( F_K(\cdot) \) in the Monte Carlo Example**

\( \theta_0, \theta_1 \) are jointly uniformly distributed with density equals 1 at the area \([0,1] \times [1,2]\).

\[ F_K(k) = \Pr(K(\theta_0, \theta_1) < k) = \Pr(\theta_1 - \theta_0 < k) = \Pr(\theta_1 < k + \theta_0). \]

With help of Figure 13, when \( k \in [0,1] \),

\[ F_K(k) = \int_{1-k}^1 \int_{1}^{\theta_0+k} 1 d\theta_1 d\theta_0 = \int_{1-k}^1 (\theta_0 + k - 1) d\theta_0 \]
\[ = \left[ \frac{1}{2} \theta_0^2 + (k - 1) \theta_0 \right]_{1-k}^1 = \frac{1}{2} + (k - 1) - \frac{1}{2}(1-k)^2 - (k - 1)(1-k) = \frac{k^2}{2}. \]

When \( k \in (1,2] \),

\[ F_K(k) = 1 - \int_{0}^{2-k} \int_{\theta_0+k}^{2} 1 d\theta_1 d\theta_0 = \int_{0}^{2-k} (2 - \theta_0 - k) d\theta_0 \]
\[ = \left[ (2 - k)\theta_0 - \frac{1}{2} \theta_0^2 \right]_{0}^{2-k} = 1 - (2-k)^2 + \frac{1}{2}(2-k)^2 = 1 - \frac{(2-k)^2}{2}. \]

We therefore get \( F_K(\cdot) \) in (15).

Figure 13: Derivation of \( F_K(\cdot) \)
A sample bid

This is a bid of a server room construction project. The buyer is Bank of Dongguan, a regional bank centered at Dongguan, Guangdong province, China. The firm is IBM Engineering Technology (Shanghai) Co., Ltd.. The bid consists of a construction plan and a detail list of items and their costs. The construction plan is a 19-page document including standard of construction, condition of delivery, delivery date, equipment purchase plan, payment plan etc. Some selected pages are shown in Figure 14. The itemized price list is a 11-page spreadsheet. Table 11 shows its major categories, categorical prices, and total price (3,630,000 CNY).

<table>
<thead>
<tr>
<th>Category</th>
<th>Price (CNY)</th>
<th>No. of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data center room renovation</td>
<td>924,295</td>
<td>17</td>
</tr>
<tr>
<td>Main power distribution system</td>
<td>108,185</td>
<td>11</td>
</tr>
<tr>
<td>Auxiliary power distribution system</td>
<td>176,830</td>
<td>14</td>
</tr>
<tr>
<td>Uninterrupted power supply (UPS) system</td>
<td>913,680</td>
<td>13</td>
</tr>
<tr>
<td>Generators and environmental engineering</td>
<td>413,050</td>
<td>14</td>
</tr>
<tr>
<td>Air conditioning</td>
<td>99,170</td>
<td>11</td>
</tr>
<tr>
<td>Precision air conditioning</td>
<td>528,570</td>
<td>2</td>
</tr>
<tr>
<td>Cabinets and cabling system</td>
<td>242,230</td>
<td>9</td>
</tr>
<tr>
<td>Lightning protection</td>
<td>23,820</td>
<td>3</td>
</tr>
<tr>
<td>Room monitoring</td>
<td>185,120</td>
<td>43</td>
</tr>
<tr>
<td>Room bridging</td>
<td>15,050</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3,630,000</strong></td>
<td><strong>141</strong></td>
</tr>
</tbody>
</table>

The author receives authorization to disclose the document for non-profit academic research purpose. The original document is in Chinese. All technological details are remain confidential and the relevant copyrights are owned by Bank of Dongguan and IBM Engineering Technology (Shanghai) Co., Ltd. The author declare that he has no relevant or material financial interests that relate to the research described in this paper.
Figure 14: Selected Pages of the Construction Plan

References


