Detecting Quality Manipulation Corruption in Scoring Auctions: A Structural Approach*

Yangguang Huang†

Department of Economics
University of Washington

November 2015

Abstract

Scoring auctions are widely used to support the procurement of items that differ in quality. These auctions are particularly susceptible for corruption because the quality assessment usually requires special expertise that the buyer does not possess, which necessitates the participation of a skilled intermediary agent to evaluate quality. Corruption via quality manipulation arises when the agent is bribed to elevate quality score of a seller. It causes a systematic distortion of bids and such distortion is testable. This paper proposes a structural estimation method of scoring auction data and three tests for detecting quality manipulation. We apply them to study a series of server room scoring auctions in China. We find empirical evidence for the primary implications of the theoretical model and some signs of corruption in sub-samples with high quality weight scoring rules and large engineer's estimated costs.

Keywords: Scoring Auction, Structural Estimation, Quality Manipulation, Corruption Detection.

JEL codes: C1, D44, H57, L40, L74.

*I would like to thank Yanqin Fan, Xu Tan, Quan Wen, Ming He, Jenny Ho, Carlos Manzanares, Xuetao Shi, and seminar participants at University of Washington for their comments and suggestions. The latest version of the paper can be found at the author's website: https://sites.google.com/site/sunnyelan/.

†University of Washington, Box 353330, Seattle, WA 98195. Email: huangyg@uw.edu. Tel: (206)354-5536.

1 Introduction

This paper serves two purposes: it develops a structural estimation method to study scoring auction data and propose three tests of detecting corruption via quality manipulation. The target items in procurement auctions are typically of differential quality with multi-dimensional attributes. For example, in procurement of a construction project, the design, materials, equipments, delivery date, safety, service, and maintenance are important attributes that need to be written in the contract at the moment of transaction. In this environment, a multi-dimensional *bid* consisting of a listed price and other non-price quality attributes, usually written as a proposal. Depending on the rules of the auction, the winning bid either obligates the bidder contractually or serves as a guideline for writing a detailed contract. If the auction has a pre-announced scoring rule specifying how bids are evaluated, it is called a *scoring auction*. If the auction does not specifically announce how the winner is selected before firms submitting their bids, it is called a *beauty contest* or a *design-build auction*. In this paper, we will only study scoring auctions, where the scoring rules and relevant scores of all bids are observable to researchers.

When quality assessment is involved in the procurement auction process, its complexity and subjectivity make scoring auction particularly susceptible for corruption. This is primarily because, in a typical procurement, the buyer is not an expert in the industry and lacks the expertise to evaluate quality of bids submitted by supplying firms. In some cases, at the time of determining the auction winner, the quality is unobservable to the buyer. In other cases, even after the item or project is delivered, the quality in unverifiable. Then the buyer is forced to hire mediators who have ability to evaluate the quality of offered items. In real practice, these mediators may include a street-level bureaucrat with industrial experiences, a professional procurement agency company, an auctioneer, and a committee of industrial experts. In this paper, we abstract multiple layers of mediators into one layer, called the *agent*. Corruption would not be an issue if the buyer could observe quality directly without leaving any discretion to the agent. But because the agent is given some discretion in evaluating quality, he can exert it to seek bribes from firms. In particular, the agent may raise the quality assessment of the corrupted firm. Because the process of quality evaluation is both complex and subjective, to some extent, such manipulation may not trigger investigation. If this kind of corruption indeed occurs, then the quality evaluation report is manipulated. Hence, we call this problem *quality manipulation*.

Quality manipulation is a prominent issue in procurement both in the public sector and private sectors, especially in developing countries. Existing studies of corruption in auctions focus on either bidding rings among bidders, or bid revision cheatings between the auctioneer and a bidder (defined later in Section 1.2). Bidding rings and bid revision cheatings suppress competition and causes monetary loss for the buyers, but they ignore another perceivable consequence of corruption: inferiority of quality. Take the bridge construction as example: Ji and Fu (2010) found that there were a total of 85 major bridge collapse accidents in China between 2000 and 2009. Forty cases among them were later convicted to be caused by corruption during the procurements. In a 2012 media report titled "Chinese-style of bridge collapse",[1] government officials and industry experts concluded three main frauds causing bridge collapse accidents: (i) the construction design proposal failed to meet industrial regulation, (ii) the construction was carried out at low quality, and (iii) the finished bridge lacks necessary maintenance. No buyer will purchase a bridge if she knows it will collapse in the near future. Quality manipulation results in the discrepancy between the quality written on the winning bid and the quality actually delivered, which can lead to deadly tragedies.

Current solutions for detecting corruption in auctions focus on auctions where price is the only corruptible outcome variable, and these tests are not applicable to auction settings where quality evaluation is involved in the auction process and can be manipulated. We introduce quality to procurement auctions based on the theoretical works on scoring auctions, mainly by Che (1993), Asker and Cantillon (2008), and Hanazono et al. (2015). Our contribution is three-fold:.

First, we show that equilibrium cost and total social surplus of each firm can be nonparametrically identified and structurally estimated from standard data of scoring auctions. The equilibrium of scoring auction derived in Che (1993) is not directly applicable because it predicts that the auction winner's always submits the highest quality. In reality, the winning bid could be high quality and expensive, or low quality and cheap. To accommodate the model prediction and the data, we allow each firm's type to be multi-dimensional. On the other hand, the result in Asker and Cantillon (2008) allows us to transform the problem into bidding according to a one-dimensional pseudotype, which avoids the complexity of multiple private information. Our identification and estimation do not require a parametric cost function and hold under environments with multiple quality attributes.

Second, we introduce quality manipulation into the scoring auction model and characterize the systematic distortion of corrupted firm's bidding behaviors. The corruption model implication and structural estimation method are put together to construct three tests of detecting corruption via quality manipulation. A corrupted firm with elevated quality will bid more aggressively compared to a competitive counterpart. Based on one auction outcome, one cannot tell whether the aggressive bidding is due to corruption or the efficiency of the firm. But with a large sample of auctions, the abnormally aggressive bids by corrupted firms will reject the competitive bidding model null hypotheses of our tests. The novel feature of these tests is that they are based on data of standard scoring auction records. Performing them require neither prior knowledge of identities of the corrupted firms, nor repeated observation of bids from the same set of firms, nor rich firm-specific covariates.

Third, we apply the structural estimation method and corruption detection tests to a series of scoring auctions of server room construction procurements.[2] The data and estimation results provide empirical evidence for the three key implications of theoretical model. Che (1993) predict

[1] http://club.kdnet.net/dispbbs.asp?page=1&boardid=89&id=8581905

[2] Server room is an indoor place designed to contain machines of data storage, servers, and large computers.

each firm choose their quality maximizing the total social surplus given its cost function and the scoring rule. We show that the choice of quality is indeed separable from the choice of score because quality choice is uncorrelated to the number of bidders. We find that the project procured by a scoring rule with high quality weight tends to result in higher payoffs for both buyer and firms, but it is also subject to a higher risk of corruption. Corruption is also more likely to happen at projects with high engineer's estimated costs.

1.1 Literature Review of Scoring Auction

When the procurement target items are of differential quality, scoring auctions are commonly used. The advantage of scoring auction is proven both by theory and its popularity. In practice, each bidder is asked to submit one bid as a price-quality combination. The contract is awarded to the bidder that receives the highest score based on a pre-announced scoring rule. By specifying a transparent scoring rule, firms are able to compute the monetary value of supplying at each quality level and submit proposals desirable to the buyer. In the seminal paper by Che (1993), he derives the equilibrium of scoring auction under quasilinear scoring rule and shows that firm's quality and price choice can be separated. He shows that both first-score auctions (FSA) and second-score auctions (SSA) implement the optimal mechanism and yield the same expected utility to the buyer.

Asker and Cantillon (2008) introduce multi-dimensionality of private information and quality attributes to Che's model. They characterize the equilibrium and expected score equivalence of FSA and SSA. In addition, they show that a scoring auction with a quasilinear scoring rule dominates other alternative procurement schemes including beauty contests, menu auctions[3], and price-only auctions with minimum quality standards. In Branco (1997), costs of different firms have a common component and thus are correlated. In this case, an optimal contract cannot be implemented by first or second-score auctions, but instead requires a two stage mechanism: first select a firm through an auction, then readjust the level of quality via bilateral bargaining. David et al. (2006) and Chen-Ritzo et al. (2005) provide experimental evidence indicating that scoring auctions dominate traditional price-only ones. Wang and Liu (2014), Dastidar (2014), and Hanazono et al. (2015) extend the model to non-quasilinear scoring rule environments. Among these papers, Hanazono et al. (2015) consider the most general setting that covers price-quality ratio, fixed price best proposal, and convex scoring rules. They characterize the equilibrium of FSA and SSA and show that their expected score rankings depends on the curvature of the induced utility of firms.

In general, it is difficult to characterize the optimal mechanism and its implementation by a scoring auction when the environment is complicated. David et al. (2006) characterize an optimal scoring rule within the class of weighted criteria rules with restriction of additively separability of attributes on both preference and cost. Asker and Cantillon (2010) find the optimal mechanism in a specific environment where firm's types are two binary distributed random variables. They show that a scoring auction yields a performance closed to that optimal mechanism numerically.

[3]In a menu auction, bidders are allow to submit multiple price-quality combination bids, instead of only one in scoring auctions. The buyer will then determine the winner and the item on its menu.

Nishimura (2015) show that implementation of the optimal mechanism via a scoring rule requires substantial cost complementarity among quality attributes. In other words, the widely used linear weighted scoring rule is suboptimal because it does not exhibit enough complementarity among attributes to provide the correct incentive.

Concerning quality manipulation, Celentani and Ganuza (2002) introduce an endogenous corruption relation forming process to the scoring auction model of Che (1993). They allow the corrupted firm to win for sure once the agent accept a bribe. Their model focuses on the formation of the corruption side contract and show how increasing competition may not reduce corruption. Burguet and Che (2004) consider a Bertrand-style environment of two firms with complete information. The endogenous formation of corruption relation is a bribery competition. Because the weaker firm can spend all its resources on either bribery competition or bidding competition, the efficient firm cannot guarantee winning the contract, which causes efficiency loss. Huang and Xia (2015) consider a similar environment with exogenous corruption relation and focus on the buyer's optimal scoring rule under corruption. The scoring rule affects the relative magnitude of the efficient firm's technological advantage and the corrupted firm's corruption advantage, which further determines the auction outcome. In such an environment, the superiority of scoring auction shown in Asker and Cantillon (2008) disappears. A price-only auction with minimum quality standards may be better.

There is a growing literature on the empirical analysis of scoring auctions. In a scoring auction data set, each bid consists of a price and a number of quality attributes. It can potentially answer richer questions than price-only auction data. Lewis and Bajari (2011) explore a highway procurement data set from California generated from "A+B auctions", where bids are evaluated on both price and time of delivery. They show that by introducing time incentive, the overall gain in social welfare is significant. Bajari et al. (2014) analyze another highway procurement data set where bids observations consist of the complete list of unit prices. These unit prices are multiplied by quantities estimated by engineers to determine which bid has the lowest cost. Their analysis focuses on the *ex post* adjustment of final payments and how firms strategically reflect potential adaption costs in their bids. Krasnokutskaya et al. (2011) study data from online programming service market. They provide an identification and estimation strategy for data that features both auction and discrete choice. Koning and Van de Meerendonk (2014) study data from welfare service provider procurement auctions under weighted scoring rule. They explore how variation of weights on different components affect bids and procurement outcomes. Nakabayashi and Hirose (2015) study a Japanese scoring auction data set similar to the one in this paper. They provide identification and structural estimation results based on a parametric cost function. This cost function is common knowledge to all except for several parameters as bidder's private information. The identification is based on invertibility conditions of the system of equations obtained by best responses in choosing the multi-dimensional bid. In our analysis, we consider only the class of quasilinear scoring rules, but our cost function is nonparametric.

1.2 Detecting Corruption in Auction

In the *Handbook of Procurement* (edited by Dimitri et al. 2006), Lengwiler and Wolfstetter (2006) point out procurement auction participants may suppress competition by four major forms of collusion or corruption. In the literature, *collusion* usually refers to a *bidding ring* or a *cartel*, where a group of bidders coordinate their bids to suppress rivalry and capture some of the rents that otherwise would be transferred to the buyer. In an efficient cartel, the *cartel leader* (the one with the lowest cost or the winner of an internal pre-auction knockout) is the only serious bidder, while the other cartel members submit high *phony bids*. There is a body of literature on bidding rings both theoretically (e.g. Graham et al. (1990), McAfee and McMillan (1992), and Hendricks et al. (2008)) and empirically (e.g. Pesendorfer (2000), Bajari and Ye (2003), and Asker (2010)). *Corruption* usually refers to the auctioneer (who runs the auction) twisting the auction rule in exchange for bribes. It can take three major forms: (i) *bid revision* (*bid rigging* or "*magic number" cheating*), meaning that the auctioneer allows a favored bidder to adjust his bid after receiving information about rival bids (e.g. Compte et al. (2005) and Burguet and Perry (2009)). (ii) *Bid orchestration*, meaning that the auctioneer serves as the ring manager of a collusive cartel and coordinates their bids.[4] (iii) *Quality manipulation* (or *distortion of quality ranking*), meaning that the agent of bid evaluation is bribed to submit biased quality scores (e.g. Celentani and Ganuza (2002) and Burguet and Che (2004)).

In this section, we briefly review existing empirical works on corruption in auctions and its detection. We focus on a relative small number of papers and only sketch their key insights. For a more comprehensive reviews including the theoretical side literature, readers can consult other surveys like Harrington (2008) and John Asker's note.

Porter and Zona (1993) is one of the earliest works on collusion detection. They study bidding rings in procurement auctions of Long Island highway construction contracts. Because some bidders are of relative large size and interact with each other in a sequence of auctions, they are able to coordinate as a cartel. They estimate parameters of a linear bid function and a logistic bid ranking model. Because the model can be estimated from using either the whole sample or only winning bids, two sets of parameter estimate shall be equal in a competitive environment. But when there is a bidding ring, the ranking of bids will not fully reflect the economic factors of bidders, leading to different estimates.

Colluding bidders' behaviors can be studied and tested by reduced-form models when detailed data of cartel members identities and characteristics are available from records of investigation by antitrust authorities. Porter and Zona (1999) analyze data from school milk contract auctions in Ohio, where a group of firms in Cincinnati were convicted for colluding. The bidding behavior of cartel members is compared to a controlled group. They show that collusion raised market prices by 6.5% on average. Pesendorfer (2000) also analyzes data from school milk contract auctions, where some firms in Florida and Texas were found colluding. He considers the effect of both bid

[4] Just how rotten? (http://www.economist.com/node/3308447), *The Economists*, October 21, 2004

rigging and market splitting. He estimates the coefficients of reduced-form bid function regressions using three sub-samples: low cartel bids plus all non-cartel bids, low cartel bids, and all non-cartel bids. A Chow test for equality of coefficients shows that the cartel firms bid less aggressively than non-cartel firms. Feinstein et al. (1985) point out that a cartel may seek not only a higher winning bid, but also collectively use bids to pass false information to the buyer to avoid a "ratchet effect" (Freixas et al. 1985). It happens when the buyer uses past information to form expectations of future auctions. Feinstein et al. (1985) found empirical evidence from data of convicted collusion cases in North Carolina highways procurements.

However, if the data does not provide exact identities of the cartel and non-cartel bidders, the methods above cannot be implemented (unless one runs regression on all possible partitions of the cartel and non-cartel bidders). In addition, the data may not be rich in bidder's characteristics. Harrington (2008) points out that an abnormally high profit margin is not the evidence of collusion, but the evidence of market power. According to Baldwin et al. (1997), there are three (non-mutually exclusive) ways to explain a high profit margin: collusion, demand side factors, and supply side factors. The supply side can be captured by auction-specific covariates describing the object. To identify collusion, researchers need to control demand side factors by enough bidder-specific covariates. To encounter these data limitations, researchers start using structural model to detect collusion.

Bajari and Ye (2003) construct their test based on two distinct model implications of the competition and the collusion model: conditional independence and exchangeability of bids. If bidders are competitive, bids must be independent controlling for all publicly observable information on costs under IPV framework. But if there is a cartel, their bids may be correlated and such correlation can be detected. Moreover, a competitive bidder's bid shall not depend on other bidder's identities, so exchanging other bidders' characteristics shall not change the distribution of competitive bidder's bid. In a regression specification, if one regresses bidder i's bid on the covariates of bidder j and k (with other controls), then these two coefficients should be equal. An F-test can be used to check this exchangeability restriction. Identities of potential cartel members can be found by testing each pair of bidders. In addition, Bayesian estimation of the structural model provides the likelihood of the data coming from the collusion model. Aryal and Gabrielli (2013) take a full structural approach to test collusion based on the estimation method of auction data in Guerre et al. (2000). For the same set of bids data, two sets of costs are structurally estimated by assuming the competitive model and the collusion model, denoted as $\left\{\hat{c}^A\right\}$ and $\left\{\hat{c}^B\right\}$ respectively. Because collusion lowers competition, for the same bid b, it implies $\hat{c}^A(b) \geq \hat{c}^B(b)$. Detecting collusion boils down to testing for first-order stochastic dominance of two cost distributions recovered from two models.

Besides bidding rings, Ingraham (2005) studies the corruption between the auctioneer and a bidder. His model is based on the bid revision model in Compte et al. (2005). The auctioneer let the corrupted firm observe others' bids before submitted its. When the corrupted firm's cost is lower than the lowest bid of other firms, it will submit a bid that barely wins the contract. As a result, the difference between the lowest and second lowest bid is smaller than a usual competitive sample. This is a testable model implication

All works mentioned above are based on first-price sealed-bid auction. Collusion can be a more prominent problem in open auctions where tacit collusion is easier. Athey et al. (2011) study a timber auction data set with two auction formats (sealed-bid and open) and two sets of bidders (mills and loggers). They assume mills are potential cartel and use the sealed-bid auction as benchmark to evaluate whether bids in open auctions satisfy the competitive hypothesis. Bajari and Yeo (2009) studies collusion in FCC spectrum auction and Klemperer (2002) in telecoms license auction. Some other empirical works are based on the unique features of their data set. Asker and Cantillon (2010) study internal knockout auction from side-transfer data of a stamp dealers cartel. They test the theory of internal organization of bidding rings and measure ring members' benefit from colluding. Tran (2009) uses internal bribery data of a company to compare corruption under two different auction formats. Kawai and Nakabayashi (2014) study an auction data set from Japanese government procurements. Because the reserve price is secret, observation of bids may consist of multiple rounds and the ranking of bidders across rounds can be used to detect collusion.

In summary, to detect collusion, researchers need to derive some key model implications distinguishing the competition model and the collusion model, and then test which model the data supports. Hence, all these collusion detection methods suffer from some common problems: (i) When the null hypothesis of the competitive model is rejected, it is hard to tell whether the reason is collusion or model mis-specification (See Figure 6). (ii) If corrupted bidders coordinate their bids in a sophisticated way, the recorded bids can pass nearly all these tests. It is called "beating a test of collusion" , discussed in Harrington (2008). (iii) Nearly all these tests rely on repeated observations of bids from the set of potential corrupted bidders. Dynamic interaction between bidders are very informative of whether they are competing or colluding. But one implicit assumption made here is that the identities of cartel and non-cartel members do not change across auctions.

Our tests are subject to problem (i) as others, but suffer less from problem (ii) and (iii). The quality manipulation problem usually only happens to one bidder. If the agent and corrupted firm wants to avoid being detected, they must reduce the manipulation power. Therefore, beating our tests will directly restrict corruption. Besides that, our tests are also useful for antitrust authorities because they requires only standard auction data. In particular, we do not need a prespecified set of suspicious corrupted bidders, identities of bidders, or repeated bidding behaviors of bidders across auctions. Our tests can be perform with very little or even no bidder-specific covariates.

The rest of the paper is organized as follows. We present the theoretical model of scoring auction and quality manipulation in Section 2. The identification and structural estimation of scoring auction model are shown in Section 3.1. Section 3.2 provides three corruption detection tests and a Monte Carlo experiment. In Section 4, we apply the estimation and collusion detection method to a server room procurement auction data set. Section 5 concludes.

2 Model

2.1 Benchmark Model

A *buyer* (she) seeks procurement of a project that can be delivered at various level of quality $\mathbf{q} \in \mathbb{R}^L_+$. The buyer faces a price-quality tradeoff between cheap-low-quality and expensive-high-quality projects.[5] Setting up a scoring rule $S(p, \mathbf{q}) : \mathbb{R}^{L+1}_+ \to \mathbb{R}_+$ reflects the buyer's willingness-to-pay for procuring the project at a higher quality. The scoring rule ranks different price-quality combinations and provide supplying firms incentives to submit desirable project proposals. Che (1993) shows that the buyer will under-report her preference on quality in the optimal scoring rule, but if she lacks commitment power, the only feasible scoring rule is the one that reflects her true preference. In this paper, because researchers usually only observe the score but not buyer's "payoff" empirically, we put aside the buyer's optimal scoring rule design problem and simply treat $S(p, \mathbf{q})$ as her objective function. We focus on the firm's equilibrium bidding behavior and implication of quality manipulation with the goal of conducting an empirical study.

After a scoring rule $S(p, \mathbf{q})$ is announced, suppose there are n symmetric risk neutral supplying *firms* (they/it) enter the auction exogenously, indexed by $i = 1, 2, ..., n$. A generic firm i's type (private information) is a vector of efficiency parameter $\theta \in \mathbb{R}^M$, drawn independently from an identical distribution F. F is absolutely continuous and has density $f = F'$ with support $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}^M$. Firm i with type θ_i pays a cost $C(\mathbf{q}, \theta_i)$ if it delivers the project with quality $\mathbf{q}$. Provided the scoring rule, each firm submits its sealed bid as a price-quality combination. If the firm wins the contract with bid $(p, \mathbf{q})$, its payoff is $\pi(p, \mathbf{q}; \theta_i) = p - C(\mathbf{q}, \theta_i)$. Firm's payoff is normalized to zero if it does not win the contract. Submitted bids are evaluated according to $S(p, \mathbf{q})$ and the firm with the highest score wins the contract. We only consider first-score auctions (FSA) and independent private information framework in this paper.

When the buyer does not possess the expertise to evaluate quality of submitted bids, she hires an *agent* (he) to evaluate them. In real procurement practice, there may be several intermediary agents between the buyer and the firms, including a professional procurement agency company, a street level bureaucrat, an auctioneer, and a quality evaluation committee of experts. We abstract them into one agent. In the benchmark model, the agent is honest and report the true quality of proposals of submitted bids. Because both the firms and the agent are experts in the industry, they should agree on how to evaluate quality. Therefore, except for some tiny uncertainty, firms are effectively choosing quality evaluation scores of their proposals and the agent verifies these quality scores for the buyer.

We restrict our attention to cost functions and scoring rules satisfying the following two assumptions throughout the paper.

Assumption CF (cost function): For all $\mathbf{q} \in \mathbb{R}^L$, $C(\mathbf{q}, \theta)$ is continuous, $C(\mathbf{q}, \theta) \geq 0$, $\frac{\partial C(\mathbf{q}, \theta)}{\partial \mathbf{q}} \gg 0$, and $\frac{\partial^2 C(\mathbf{q}, \theta)}{\partial \mathbf{q}' \partial \mathbf{q}}$ is positive definite.

[5] Dini et al. (2006) provides a practical survey on selecting scoring rules.

Assumption QL (quasi-linear scoring rule): The scoring auction uses a quasilinear scoring rule $S(p, \mathbf{q}) = V(\mathbf{q}) - p$, where $V(\mathbf{q})$ is increasing, continuously differentiable, and weakly concave.[6]

With these two assumption, we have the following lemma:

Lemma 1: *Consider a quasilinear scoring rule $S(p, \mathbf{q}) = V(\mathbf{q}) - p$, with $\mathbf{q} \in \mathbb{R}_+^L$ and $L > 1$. For a firm with cost function $C(\mathbf{q}, \theta)$, it is equivalent to consider it bids according to a transformed cost function $\tilde{C}(v, \theta)$, which is defined by the minimization problem:*

$$\tilde{C}(v, \theta) \equiv \min_{\text{s.t. } V(\mathbf{q})=v} C(\mathbf{q}, \theta)$$

$\tilde{C}(v, \theta)$ is single-valued, continuous, increasing, and strictly convex in v.

Given Lemma 1, there is no loss of generality to reduce the dimensionality of quality attributes to one. For notational convenience, we will consider the cost function and the scoring rule as $C(q, \theta)$ and $S(p, q)$ respectively, where q is one-dimensional. The firm's profit maximization problem is

$$\max_{p,q} [p - C(q, \theta)] \Pr(\text{win}|S(p, q)). \tag{1}$$

As shown in Che (1993), the quality choice can be separated from score or price choice. Each firm will choose its quality according to

$$q(\theta) = \arg\max_q V(q) - C(q, \theta). \tag{2}$$

The proof of (2) will be show under a more general setting at Theorem 2. Assumption CF and QL ensure $q(\theta)$ to be a single-valued continuous function by the Maximum Theorem (Berge, 1963). The firm's problem (1) is equivalent to a two-step optimization problem where the firm first chooses its score s, then choose a (p, q) combination to fulfill that score.

$$\begin{aligned}
(1) \quad &\Leftrightarrow \quad \max_s \left\{ \max_{(p,q) \text{ s.t. } S(p,q)=s} [p - C(q, \theta)] \Pr(\text{win}|s) \right\} \\
&\Leftrightarrow \quad \max_s \left\{ \max_q [V(q) - s - C(q, \theta)] \Pr(\text{win}|s) \right\} \\
&\Leftrightarrow \quad \max_s \left\{ [V(q(\theta)) - C(q(\theta), \theta) - s] \Pr(\text{win}|s) \right\}. \qquad (3)
\end{aligned}$$

At the second and third step, we plug in $p = V(q) - S(p, q)$ and (2) respectively. Following Asker and Cantillon (2008), we define the *pseudotype*[7] of a firm as the value function

$$K(\theta) \quad \equiv \quad \max_q V(q) - C(q, \theta) = V(q(\theta)) - C(q(\theta), \theta), \tag{4}$$

[6] Any scoring rule that is additively seprarable in price, including the commonly used linear weighted factor rule, can be transformed into a quasi-linear one. However, a quality-price ratio scoring rule cannot be transformed into an equivalent QL rule, analyzed in Hanazono et al. (2015).

[7] It is called effective cost in Hanazono et al. (2015) and productive potential in Che (1993).

which is the *total social surplus* of a firm (if $S(p,q)$ represents the true payoff of the buyer). Again by the Maximum Theorem, $K(\theta)$ is a single-valued continuous function. The distribution of pseudotype K can be obtained from the (joint) distribution of θ by probability transformation formula:

$$F_K(k) = \Pr(K(\theta) \le k) = \Pr(\theta \in D_{\{\theta:\, K(\theta)\le k\}}) = \int_{\theta \in D} f(\theta) d\theta. \tag{5}$$

Denote $\underline{k} = \min_{\theta\in[\underline{\theta},\overline{\theta}]}\{K(\theta), 0\}$ and $\overline{k} = \max_{\theta\in[\underline{\theta},\overline{\theta}]}\{K(\theta), 0\}$. We assume $\underline{k} \ge 0$ so that the least efficient firm participates. The support of pseudotype is $[\underline{k}, \overline{k}] \subset \mathbb{R}_+$. Asker and Cantillon (2008) show that pseudotypes are sufficient statistics to describe the equilibrium of scoring auctions under quasi-linear scoring rules. Therefore, instead of dealing with the multi-dimensional type θ, it is equivalent to consider that firms draw their one-dimensional pseudotype k from distribution F_K. Problem (3) can then be further rewritten as if the firm is selecting its score based on its pseudotype:

$$\max_s (k - s) \Pr(\text{win}|s). \tag{6}$$

Directly from Asker and Cantillon (2008), we have the following theorem and two corollaries:

Theorem 1: *Every equilibrium in a scoring auction where firms bid on the basis of their types θ, is type-wise outcome equivalent to an equilibrium in the scoring auction where firms are constrained to bid only on the basis of their pseudotypes $k = K(\theta)$. Firm with type θ and pseudotype $k = K(\theta)$ bids its quality according to* (2) *and score according to*

$$s(k) = k - \frac{\int_{\underline{k}}^{k} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}. \tag{7}$$

The corresponding price is $p(\theta) = V(q(\theta)) - s(K(\theta))$.

Corollary 1: *The conditional expectation of the winner's score equals to the strongest rival's pseudotype, i.e., $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$.*

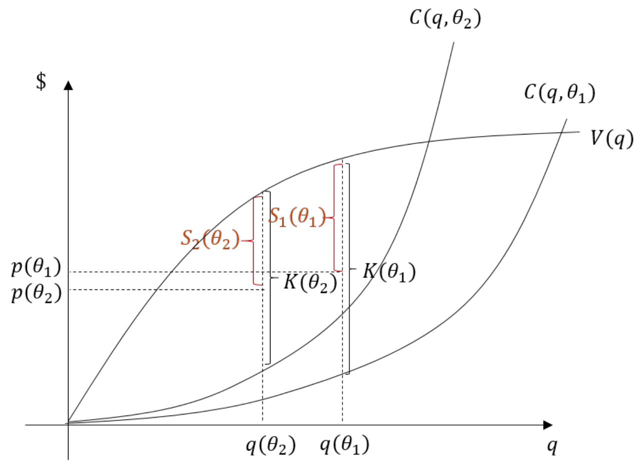
Corollary 2: *The buyer receives a higher expected utility in a scoring auction than a price-only auction with minimum quality standards.*

Corollary 1 is the expected utility equivalence of FSA and SSA, which is similar to the revenue equivalence principle in Vickrey (1961). Corollary 2 is describing the superiority of scoring auctions. Throughout the paper, $X_{(j:n)}$ denotes the jth highest order statistic from an i.i.d. sample of size n from distribution F_X. The distribution function of order statistic $X_{(j:n)}$ is denoted as $F_X^{(j:n)}$. If θ is one-dimensional, the equilibrium in Theorem 1 reduced to the one in Che (1993):

Lemma 2: *When θ is one-dimensional and $C_\theta > 0$, $C_{q\theta} > 0$, there is a symmetric Bayesian Nash equilibrium of a first-score auction where each firm with type θ submits its bid as $q(\theta) = \arg\max_q V(q) - C(q,\theta)$ and $p(\theta) = C(q(\theta),\theta) + \int_{\theta}^{\overline{\theta}} C_\theta(q(t),t)[1-F(t)]^{n-1}dt/[1-F(\theta)]^{n-1}$.*

However, using a one dimensional θ implies the monotonicity property of $q(\theta)$. The existence of the equilibrium requires the assumption $C_{q\theta} > 0$ and Topkis (1978) theorem immediately implies that $q'(\theta) < 0$. To see this, because $q(\theta)$ satisfies first-order condition $V_q(q) - C_q(q,\theta) = 0$, by implicit function theorem, $q'(\theta) = C_{q\theta}/(V_{qq} - C_{qq}) > 0$ under assumption CF, QL and $C_{q\theta} > 0$. This monotonicity feature implies that the most efficient firm with lowest θ always wins by submitting the highest quality. It does not fit real world data because some contracts are awarded to firms offering cheap-low-quality bids. Therefore, we drop the assumption $C_{q\theta} > 0$ and assume θ to be at least two dimensional. In this way, we relaxes the monotonicity property of quality, which is shown later in the Monte Carlo Example.

Figure 1: Illustration of the Equilibrium of Scoring Auction



$\theta_1 < \theta_2 \Leftrightarrow q(\theta_1) > q(\theta_2) \Leftrightarrow S(\theta_1) > S(\theta_2)$

In summary, the equilibrium of a competitive scoring auction has three implications that can be tested empirically. First, in choosing the quality, firm only consider the scoring rule and its cost function, but not the competition environment. So the number of bidders in the auction shall not affect the choice of quality, but affect the choice of score or price. Second, according to (2), a higher slope of $V(\cdot)$ induces firms to bid higher quality. Lastly, because $V(\cdot)$ is the upper bound of pseudotype (total social surplus), firms endogenously choosing high quality in general have higher pseudotypes and win the contract. Therefore, in a scoring auction, the competition of firms is mainly reflected on the quality dimension instead of undercutting each other by price. Although the quality choice is not monotone in pseudotype, we expect to see a majority of winning bids are of the high quality and relative high price, instead of cheap-low-quality ones. In the empirical application, we provide evidence for each of these three model implications.

2.2 Quality Manipulation

Aforementioned, the complexity and subjectivity features of quality evaluation in scoring auction brings in the problem of quality manipulation. Assume that the agent randomly matches with one firm and forms a *corruption relation*. This relation can be the result of a bribery side-contract, a long term relationship, favoritism, or other reasons. According to Lengwiler and Wolfstetter (2006), by approaching only one bidder, the auctioneer minimize the number of side-contracts and thus the risk of detection. Large coalitions are of course possible, but the detection risk obviously increases with the number of people who know about the corruption. For simplicity, we assume that the agent matches with each firm with equal probability.[8]

Following Burguet and Che (2004), we assume that the agent can manipulate the evaluation of quality by raising the corrupted firm's quality score by $m > 0$. It means that if the corrupted firm submits a bid (p, q), the score is elevated from $S(p, q)$ to $S(p, q+m)$. This parameter m is called the agent's *manipulation power*. The interpretation of m can be (i) the quality score of the corrupted firm is raised; (ii) the actual delivered quality is lower than the one written on the proposal; or (iii) the evaluation method is twisted to give an advantage to the corrupted firm. The magnitude of manipulation power is determined by the discretion given to the agent and the nature of the industry. It is restricted by the extent of not being suspicious and not triggering investigation. For example, in the procurement of a bridge, the agent may claim that the corrupted firm's bridge can serve 30 years while the actual building code is designed for only 25 years. However, he will not say the bridge will last 100 years because it would be very suspicious. The manipulation power is assumed to be an constant number known by the agent and the corrupted firm, but not others. Empirically, researchers don't observe how much quality is manipulated and the magnitude of manipulation can vary across auctions.

The timeline of the auction with corruption is as follows. The buyer announces a scoring rule and hires the agent. A number of firms enter the auction exogenously and draw their private information θ from F. The agent then randomly matches with one firm and offers him a side contract that raises the firm's quality score by m in exchange for a bribe. The firm decides whether to accept this side contract or not. Then every firm submits a sealed-bid simultaneously as a price-quality combination. If the matched firm accepts the side contract, his quality will be raised by m. The auction outcome is then revealed and the firm with highest score wins the contract.

We skip a detailed model of the endogenous formation process of the corruption relation. We assume the agent is an expert in this industry and is able to design a bribery side contract that the matched firm will accept. For example, if the agent learns θ of the matched firm, he can make a take-it-or-leave-it offer, asking for a bribe slightly less than the difference between the expected pay-off of being corrupted and not. Our simple model is enough from an empirical point of view, because variables directly related to corruption are usually unobservable (e.g. side payments, identities of corrupted firms, amounts of quality distortion). Writing a complicated model of quality manipulation usually ends up with the same qualitative prediction. We further impose an assumption on other firms' knowledge about the existence of the corruption relation.

[8]In Huang and Xia (2015), the probability of each firm being corrupted is explicitly modeled. We can relax this equal probability assumption. However, if inefficient firms are being corrupted with significant probabilities and thus the winning bids are mostly not corrupted, then power of our tests will be very weak.

Assumption UA: The buyer and other uncorrupted firms are unaware of the existence of the corruption relation.

There are both realistic and technical reasons for this assumption. In reality, if either the buyer or some other firms notice the existence of corruption, they will report it to the antitrust authority because corruption directly hurts their interests. The agent and the corrupted firm will control the scope of quality manipulation so that it does not trigger investigation. Moreover, for technical reason, with incomplete information on costs, adding another layer of incomplete information brings in mixed strategies and the equilibrium becomes both complicated and uninformative (see Huang and Xia (2015)). Therefore, assumption UA is widely used in the literature of bidding rings (e.g. Porter and Zona (1993), and Aryal and Gabrielli (2013)) and bid revisions (e.g. Burguet and Perry (2009)). An alternative way to circumvent the problem is assuming complete information on corruption relation. For example, most bidding ring literature assume both cartel members and non-cartel members know identities of colluding firms (e.g. McAfee and McMillan (1992), Bajari and Ye (2003), and Athey et al. (2011)). The bidders then have two types and the auction is asymmetric with type-specific bidding functions. The qualitative prediction of assuming complete information is usually similar to assuming UA.

Given assumption UA, all uncorrupted firms follow the same strategy as in Theorem 1. The corrupted firm, once matched with the agent, solves a modified problem:

$$\max_{p,q} \left[p - C(q-m,\theta)\right] \Pr\left(\text{win}|S(q,p)\right).$$

The equilibrium bidding strategy is summarized as the following theorem.

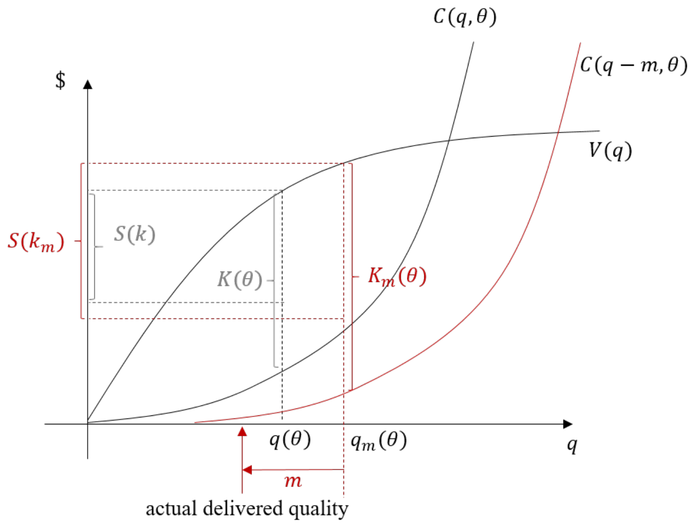
Theorem 2: *Under assumption CF, QL, and UA, the corrupted firm bids*

$$\begin{aligned} q_m(\theta) &= \arg\max_q V(q) - C(q-m,\theta), && (8)\\ p_m(\theta) &= V(q_m(\theta)) - s(K_m(\theta)), && (9)\\ s(k_m) &= \frac{k_m - \int_{\underline{k}}^{k_m}[F_K(t)]^{n-1}dt}{[F_K(k_m)]^{n-1}} \end{aligned}$$

where $k_m = K_m(\theta) \equiv \max_q V(q) - C(q-m,\theta)$ is the corrupted firm's pseudotype. Compared to an uncorrupted firm with the same type, the corrupted firm has a higher pseudotype, bids a higher quality, and reaches a higher score. All three effects magnify as m increases.

Therefore, the corrupted firm will bid *more aggressively* compared to a competitive firm of the same type. Because the corrupted firm has a large winning probability, it causes a systematic distributional shift of the winning bid. It is the key factor that allows us to construct corruption detection tests. Note that the "more aggressive" prediction is different from the implication of bidding ring models. When an auction involves a bidding ring, both the ring leader and other members bid less aggressively to suppress competition. But with quality manipulation, the corrupted firm pays a lower cost with help from the agent. As a result, the corrupted firm will bid more aggressively to increase its chance of winning the contract.

Figure 2: Illustration of the Equilibrium with Corruption



3 Econometrics of Scoring Auctions and Corruption Detection

The key of detecting quality manipulation corruption lies in checking abnormally aggressive bidding behaviors of corrupted firms. It is impossible to distinguish normally competitive bidding behavior and abnormally predatory bidding behavior by a single observation because the manipulation power (m) in unknown. But when the sample size gets large, the consistent pattern of aggressive winning bids can be captured by statistical tests. In this section, we propose three tests and provide a Monte Carlo example.

3.1 Structural Estimation

We first present the identification and structural estimation of the scoring auction model. Consider a sample of T independent and repeated scoring auctions of the same industry with the same scoring rule.[9] For scoring auction t, assume researchers observe the number of firms n_t, some auction-specific covariates z_t (of dimension d), bids of each firm $\{p_{it}, q_{it1}, q_{it2}, ..., q_{itL}\}_{i=1}^{n_t}$ (of dimension $L+1$) and scores $s_{it} = V(q_{it1}, q_{it2}, ..., q_{itL}) - p_{it}$. We set aside endogenous entry and reserve price issues in this paper. By result in Theorem 1, the identification result can be established by the method in Guerre et al. (2000).

Theorem 3: *Under assumption QL and CF, pseudotypes and equilibrium costs of firms are non-parametrically identified.*

[9] We will discuss variations of scoring rules later in the empirical application section.

Proof: Because $G_S(s) = \Pr(S \leq s) = \Pr(K \leq k) = F_K(k)$ and $g_S(s) = f_K(k)/s'(k)$, by (21), pseudotype k is identified from the observation of scores via

$$k = s(k) + s'(k)\frac{F_K(k)}{(n-1)f_K(k)} = s + \frac{G_S(s)}{(n-1)g_S(s)}, \tag{10}$$

The equilibrium cost is then identified by the definition of pseudotype,

$$C(q(\theta), \theta) = V(q(\theta)) - k = p(\theta) - \frac{G_S(s)}{(n-1)g_S(s)}. \tag{11}$$

Q.E.D.

Given observations of n_t, z_t, and s_{it}, the conditional distribution function and density of score can be estimated by kernel estimators,

$$\begin{aligned}
\hat{G}_S(s|n_t, z_t) &= \frac{1}{Th_1h_2^d}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\mathbb{I}(s \leq s_{it})\kappa_G\left(\frac{n-n_t}{h_1}, \frac{z-z_t}{h_2}\right), \\
\hat{g}_S(s|n_t, z_t) &= \frac{1}{Th_3h_4h_5^d}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\kappa_g\left(\frac{s-s_{it}}{h_3}, \frac{n-n_t}{h_4}, \frac{z-z_t}{h_5}\right).
\end{aligned}$$

We use Gaussian kernels and select bandwidths by least-square cross validation throughout this paper. Pseudotypes and equilibrium costs at corresponding quality are estimated by

$$\hat{k}_{it} = s_{it} + \frac{\hat{G}_S(s_{it}|n_t, z_t)}{(n_t-1)\hat{g}_S(s_{it}|n_t, z_t)}, \tag{12}$$

$$\hat{c}_{it} = V(q_{it}) - \hat{k}_{it}. \tag{13}$$

Equation (12) shows that the firm chooses a score s as the portion delivered to the buyer from total social surplus k. The second term is the firm's *rent*, reflecting its competitive advantage and information rent. Notice that in a price-only auction, quality of the target item is fixed, so the model primitive is a cost distribution. In a scoring auction, the model primitive is the cost function defined on the domain of quality attributes. Costs estimated via (13) are not randomly drawn from a fixed cost distribution, but rather chosen by firms.

Monte Carlo Example

Suppose the scoring rule is $S(q, p) = 2q - p$ and cost function is $C(q, \theta) = \theta_0 + q^2/\theta_1$. Each firm draws its two-dimensional type $(\theta_0, \theta_1) \equiv \theta$ independently from Uniform[0,1] and Uniform[1,2], respectively. Assume θ_0 and θ_1 are independent, so their joint density equals to one on the support $[0, 1] \times [1, 2]$. By Theorem 1, the optimal quality choice of a firm with type θ is $q(\theta) = \theta_1$ and its pseudotype is

$$K(\theta) = V(q(\theta)) - C(q(\theta), \theta)) = 2\theta_1 - \theta_0 - \frac{\theta_1^2}{\theta_1} = \theta_1 - \theta_0.$$

The support of k is $[0, 2]$. By (5) and the derivation in the appendix, the distribution of pseudotype follows CDF:

$$F_K(k) = \Pr(\theta_1 - \theta_0 < k) = \begin{cases} \frac{k^2}{2}, & \text{for } k \in [0, 1], \\ 1 - \frac{(2-k)^2}{2}, & \text{for } k \in (1, 2]. \end{cases} \tag{14}$$

Notice that, by allowing types to be two-dimensional, firm who submits a high equilibrium quality does not necessarily has a high pseudotype. For example, when firm 1 is type $(0.5, 1.5)$ and firm 2 is type $(0.1, 1.2)$, firm 1 will produce at $q = 1.5$ and have pseudotype $k = 1$; firm 2 will produce at a lower level $q = 1.2$ but have a higher pseudotype $k = 1.1$. The numbers of firms n are randomly draws from 3 to 20 with equal probability. Using (7), we can generate a simulated data set and apply our estimator (12) and (13), as illustrated in Figure 3. The estimation is based on 1000 auctions.

In this example, if researchers know the parametric form of the cost function, he can identify two structural parameters by conditions of optimal quality and score choice: $\theta_1 = q$ and $\theta_0 = q - s - G(s|n)/\left[(n-1)g(s|n)\right]$. In general, as long as $K(\theta)$ is monotone in θ under the parametric assumption, θ is identified. In application to an actual data set, determining the parametric family of cost function is usually difficult.

Notice that, the identification result in Theorem 3 is established in a competitive bidding environment. If there is corruption, the manipulation power is unobservable and may vary across auctions. From a single observation, a researcher cannot conclude whether a high pseudotype is due to a real competitive advantage or a manipulated quality. In the example, for some $m > 0$,

$$q_m(\theta) = \arg\max_q \left\{ 2q - \theta_0 - \frac{(q-m)^2}{\theta_1} \right\} = \theta_1 + m = q(\theta) + m, \tag{15}$$

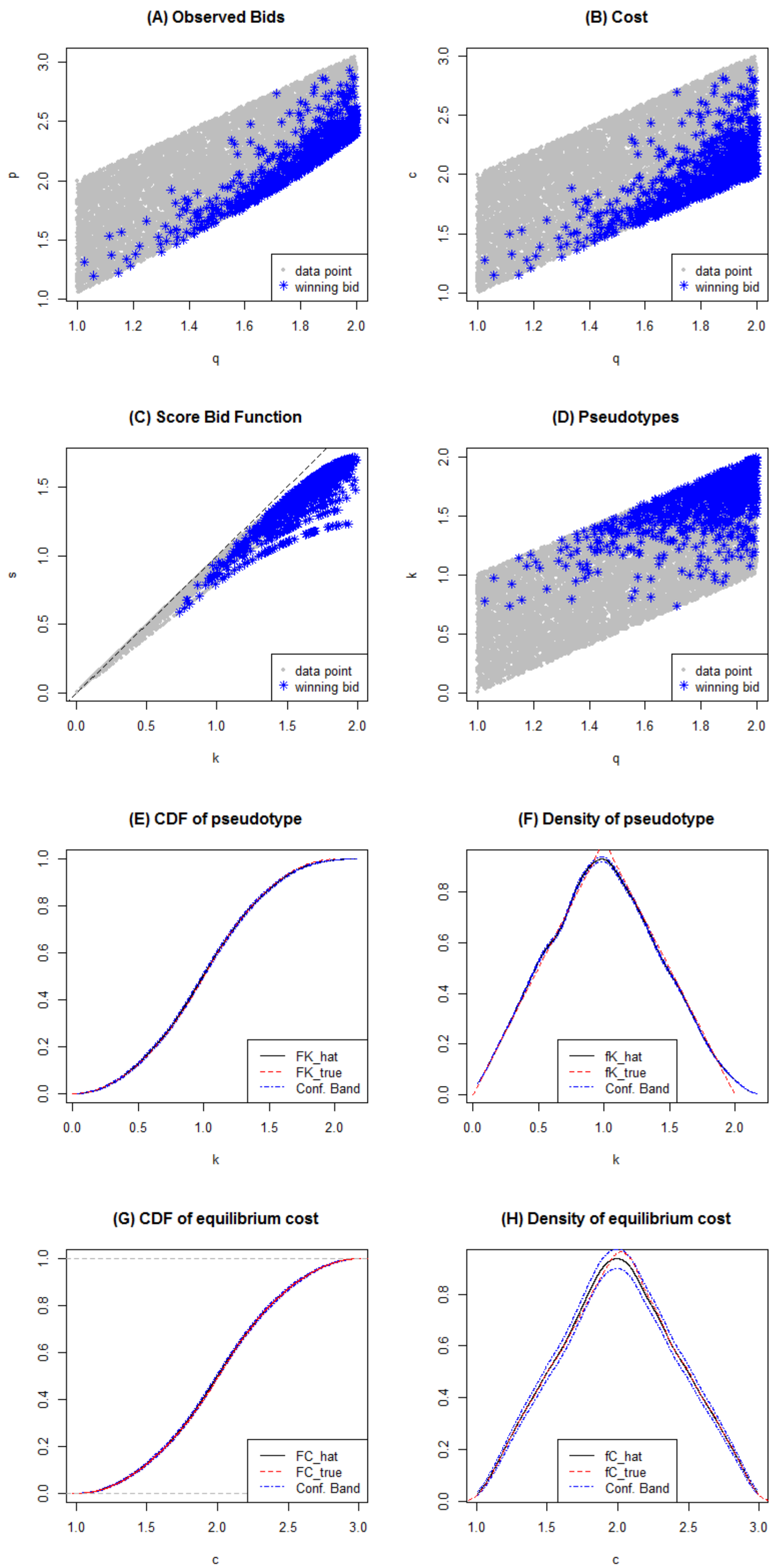
$$K_m(\theta) = 2(\theta_1 + m) - \theta_0 - \theta_1 = \theta_1 - \theta_0 + 2m = K(\theta) + 2m. \tag{16}$$

Therefore one cannot separately identify k and m. Although it is not an identified model, the systematic distortion of submitted bids can be captured with a large sample.

3.2 Corruption Detection Tests

The basic intuition of our corruption detection tests is capturing the consistent abnormally aggressive bidding behaviors of winning bids. Corruption distorts only the corrupted firm's bid, while all other bids remain competitive. The distorted bid is the winning bid with a large probability. Hence, even we don't observe the identities of corrupted bidders, we can test for the existence of systematic deviations from competitive bidding behaviors by comparing the winning bids and other bids. We propose three tests in this section. For each test, the null hypothesis is that the data

Figure 3: Illustration of Data and Estimation



are generated from the competitive model, i.e. H_0: $m = 0$. It is tested against the alternative hypothesis that the data are generated from the corruption model, i.e. H_1: $m > 0$. Test I and II can be performed on one sample of auctions from the same agent. Test III can only be performed on two or more sub-samples with difference in their agents or other aspects. These tests are illustrated in a Monte Carlo example and later applied to a real procurement data set. We denote the observed highest score or pseudotype of each auction by subscript "win". The observed second highest and third highest ones are denoted by subscript "rival" and "third", respectively.

Test I

Among the $n-1$ rivals, the *strongest rival* has pseudotype $k_{(1:n-1)}$, therefore the winning probability of a firm with pseudotype k is $\Pr\left(k > k_{(1:n-1)}\right) = F_K^{(1:n-1)}(k)$. The corrupted firm wins with probability $\Pr\left(k_m > k_{(1:n-1)}\right) = F_K^{(1:n-1)}(k_m)$ and appears to be the strongest rival with probability $\Pr\left(k_{(2:n-1)} < k_m < k_{(1:n-1)}\right) = F_K^{(2:n-1)}(k_m) - F_K^{(1:n-1)}(k_m)$. Therefore, the observed winning score

$$s_{win} = \begin{cases} s(k_m), & \text{with prob. } F_K^{(1:n-1)}(k_m), \\ s(k_{(1:n-1)}), & \text{with prob. } 1 - F_K^{(1:n-1)}(k_m), \end{cases}$$

and the strongest rival's score

$$s_{rival} = \begin{cases} s(k_m), & \text{with prob. } F_K^{(2:n-1)}(k_m) - F_K^{(1:n-1)}(k_m), \\ s(k_{(1:n-1)}), & \text{with prob. } F_K^{(1:n-1)}(k_m), \\ s(k_{(2:n-1)}), & \text{with prob. } 1 - F_K^{(2:n-1)}(k_m). \end{cases}$$

By Theorem 2, for any $m > 0$, $E\left[s_{win}\right] > E\left[s(k_{(1:n)})\right]$ and $E\left[s_{rival}\right] < E\left[s(k_{(2:n)})\right]$. Then, by Corollary 1, $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$. Hence,

$$\forall m > 0, \quad E\left[s_{win}\right] > E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right] > E\left[s(k_{(1:n)})\right].$$

When $m = 0$, we have $E\left[s_{win}\right] = E\left[k_{rival}\right]$, because $s_{win} = s(k_{(1:n)})$ and $k_{rival} = k_{(2:n)}$ in the equilibrium of the competitive model. The corruption detection problem becomes testing

$$H_0 : E\left[s_{win}\right] = E\left[k_{rival}\right], \quad \text{vs.} \quad H_1 : E\left[s_{win}\right] > E\left[k_{rival}\right].$$

We use the Welch's t-test with test statistic

$$\mathcal{T}^I = \frac{T^{-1}\sum_{t=1}^{T} s_{win,t} - T^{-1}\sum_{t=1}^{T} \hat{k}_{rival,t}}{\sqrt{T^{-1}var(s_{win,t}) + T^{-1}var(\hat{k}_{rival,t})}}, \tag{17}$$

where $\hat{k}_{rival,t}$ are estimated from (12). Lucking-Reiley (1999) also uses t-test for revenue equivalence but their samples are generated from different auction formats. For our application, we are studying one sample of the same auction format, therefore we use a bootstrap critical value to account for the correlation between scores and estimated pseudotypes.

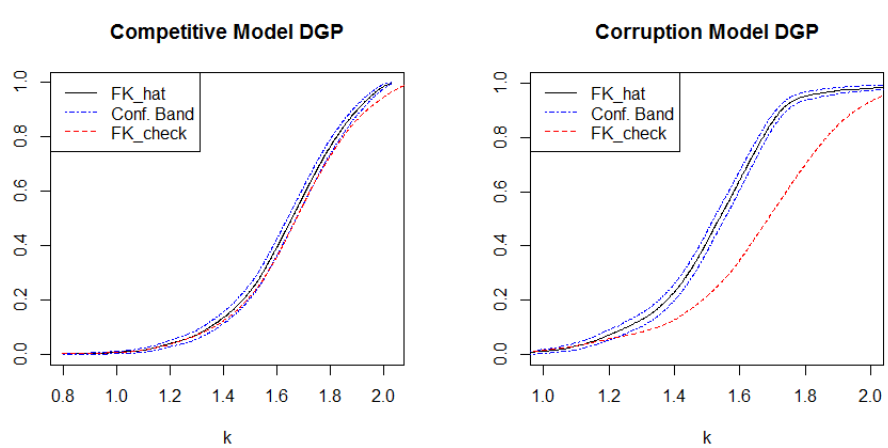
Test II

For auctions with symmetric independent private value bidders, Athey and Haile (2002) show that the underlying value distribution is nonparametrically identified even when only one bid of each auction (an order statistic) is observed. When there is no corruption, the estimates of pseudotype distribution from all bids and from only the winning bids should be the same except for some statistical errors. When there is corruption, the winning bids are distorted and the two methods will result in statistically different estimates.

Practically, we construct the test by comparing two empirical CDFs of pseudotypes of winners from two estimation methods. By using all bids, pseudotypes of all firms, $\{\hat{k}_{1t}, \cdots, \hat{k}_{n_t t}\}$, can be estimated via (12). Denote the pseudotype corresponding to the winning bid as $\hat{k}_{win,t}$ and its empirical CDF $\hat{F}_K^{win}(k) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\hat{k}_{win,t} \leq k\right)$. By using only winning bids, these winning scores have distribution function $G_W(s_{win}|n) = G_S^{(1:n)}(s_{win}) = [G_S(s_{win}|n)]^n$ and density $g_W(s_{win}|n) = n\left[G_S(s_{win}|n)\right]^{n-1} g_S(s_{win}|n)$. By replacing relevant terms in (10), the winners' pseudotypes are identified by $k_{win} = s_{win} + nG_W(s_{win}|n)/(n-1)g_W(s_{win}|n)$. The underlying pseudotype of each winning bid can then be estimated, denoted as $\check{k}_{win}$. The empirical CDF of $\check{k}_{win}$ is $\check{F}_K^{win}(k) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}\left(\check{k}_{win,t} \leq k\right)$. The corruption detection problem becomes testing

$$H_0: \forall k \in [\underline{k}, \overline{k}],\ \hat{F}_K^{win}(k) = \check{F}_K^{win}(k), \quad \text{vs.} \quad H_1: \exists k \in [\underline{k}, \overline{k}],\ \hat{F}_K^{win}(k) \leq \check{F}_K^{win}(k).$$

Figure 4: Illustration of Test II



The natural option is Kolmogorov–Smirnov (KS) test. The test is one-sided because the aggressive scores in the corruption model results in higher estimate of k. The KS test statistic is

$$\begin{aligned}\mathcal{T}^{II} &= \sup_{k\in[\underline{k},\overline{k}]}\left|\hat{F}_K^{win}(k)-\check{F}_K^{win}(k)\right| \\ &= \sup_{k\in[\underline{k},\overline{k}]}\left|\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\hat{k}_{win,t}\leq k\right)-\frac{1}{T}\sum_{t=1}^{T}\mathbb{I}\left(\check{k}_{win,t}\leq k\right)\right|.\end{aligned}$$

Similar to test I, there is dependence between the two sets of estimated pseudotypes of the winners, so we use bootstrap critical values. Test II is illustrated in Figure 4.

Test III

Inspired by Ingraham (2005), test III is based on the following Markovian property of the conditional distribution of order statistics (see the proof in Arnold et al. (1992)):

Lemma 3: *Denote the first spacing of the highest two order statistics as $X_{12}=X_{(1:n)}-X_{(2:n)}$. Its conditional distribution only depends on the third order statistic, that is*

$$f_{X_{12}}(x_{12}|X_{(3:n)}=x_3)=f_{X_{12}}(x_{12}|X_{(3:n)}=x_3,X_{(4:n)}=x_4,\cdots,X_{(n:n)}=x_n).$$

Test III is easy to implement but needs at least two sub-samples. Suppose the observed auctions can be divided into two (or several) sub-samples that are differed in their procurement agencies or other aspects. Let D_τ be dummy variable of sub-sample τ. Consider the regression

$$(\hat{k}_{win,t}-\hat{k}_{rival,t})=\beta_0+\beta_1\hat{k}_{third,t}+\beta_2D_{\tau,t}+\beta_3z_t+\epsilon_t,$$

where z_t controls for other auction-specific covariates. In a competitive auction, $\hat{k}_{win}$, $\hat{k}_{rival}$, and $\hat{k}_{third}$ coincide with $k_{(1:n)}$, $k_{(2:n)}$, and $k_{(3:n)}$. According to Lemma 3, the conditional distribution of the first spacing of pseudotypes, $k_{12}=k_{(1:n)}-k_{(2:n)}$, is the same across auctions if we control the third highest order pseudotype $k_{(3:n)}$. Therefore the conditional means of two sub-samples are equal if $m=0$. We can apply a standard t-test for $H_0:\beta_2=0$ versus $H_1:\beta_2\neq 0$ with test statistic $\mathcal{T}^{III}=\hat{\beta}_2/SE(\hat{\beta}_2)$. We can also skip the first stage estimation of pseudotypes and directly use score data to perform the test by the regression

$$(s_{win,t}-s_{rival,t})=\beta_0+\beta_1s_{third,t}+\beta_2D_{\tau,t}+\beta_3z_t+\beta_4n_t+\epsilon_t,$$

If $\hat{\beta}_2$ is significantly greater than 0, the gap between the winner and the strongest rival is larger in the $D_\tau=1$ sub-sample, which implies a higher likelihood of the sub-sample is subject to corruption.

Monte Carlo Example (Continue)

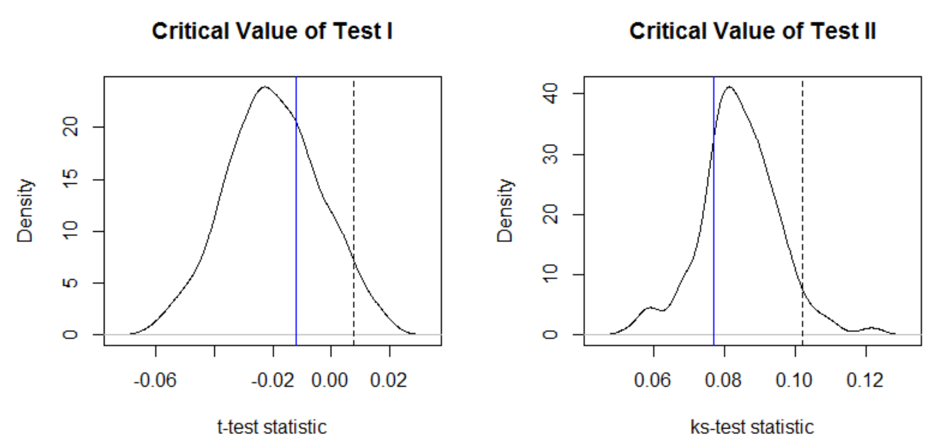
We continue with the previous example to illustrate the corruption detection tests. To find the distribution of test statistics under the null, m can be an unknown positive number. But to study the powers of these tests, we let m to be a known fixed number across observations. We generate $B = 199$ samples under the null hypothesis ($m = 0$) and compute test statistics for each sample, $\{\mathcal{T}_b^j\}_{b=1}^B$, $j = I$ and II.[10] Setting the significance level at 5%, the relevant *bootstrap critical value* of the test, $CV(\mathcal{T}^j)$, is the 190th highest among these test statistics (since $(B+1)\times(1-0.05) = 190$). In both diagrams of Figure 5, the blue line and the black dashed line denote the test statistic and the bootstrap critical value respectively, while the black curve represents the density of 199 bootstrap test statistics.

We explore the powers of these tests under three alternative hypotheses by taking m equals 0.2, 1, and 2, shown in Table 1. A randomly selected corrupted firm will choose a higher quality and have a higher pseudotype according to (15) and (16) respectively. The *bootstrap power of the test* is defined and computed by

$$\text{power} \quad = \quad 1 - \Pr(\text{accept } H_0|H_1 \text{ is true}) = 1 - \frac{1}{B}\sum_{b=1}^{B} \mathbb{I}\left(\mathcal{T}_b^j \leq CV(\mathcal{T}^j)\right).$$

The Monte Carlo results show that as the manipulation power m and the sample size T increase, the powers of all three tests improve. The power of test I is relatively weak compared to test II and III, especially in the case of small m.

Figure 5: Distribution of Test Statistics Under the Null and Bootstrap Critical Values



[10]To check the validity of bootstrap, we use the data generating process to repeatedly generate data sets and construct the compare the distribution of test statistic. It is use to compare with the bootstrap distribution of the test statistics. They are similar.

Table 1: Power of the Tests

Manipulation Power	Test	Sample Size 200	500	1000
$m = 0.2$	I	0.2462	0.2362	0.2613
	II	0.9246	0.9347	0.9497
	III	0.8241	0.7889	0.8291
Corrupted firm wins with probability 0.2348.				
$m = 1$	I	0.3869	0.4925	0.5528
	II	0.9749	0.9648	0.9849
	III	0.9246	0.9397	0.9598
Corrupted firm wins with probability 0.4596.				
$m = 2$	I	0.9347	0.9648	0.9749
	II	0.9899	0.9950	0.9950
	III	0.9950	0.9950	1.0000
Corrupted firm wins with probability 0.9618				

Note: For test III, one half of the sample is generated under the null, the other half under the alternative.

Discussion

Compared to most existing collusion detection tests, our tests require less data, so it can be performed on a lot of procurement auction data sets. Existing tests generally require identities of bidders, (rich) bidder-specific covariates, repeated observation from the same set of bidders in several auctions. Some of these tests requires exact identities of (suspected) colluding bidders, for example Porter and Zona (1993), Pesendorfer (2000), and Athey et al. (2011). Some tests, like Bajari and Ye (2003), can be conducted without identities of the corrupted firms, but need to be run on each combination of bidder pairs. Some tests are constructed upon repeated observations from the same set of bidders, which reveal the systematic difference between colluding bidders and competing bidders. Our tests do not require any of these data and hence can be performed before the case-by-case antitrust investigation.

Moreover, with different sub-samples, our tests do not need to specify a prior on which sub-sample is more likely to be corrupted. (For example, Athey et al. (2011) assumes that the sample from open auctions are collusive and sealed-bid auctions are competitive.) Test I and II can be performed on each of the sub-sample and compare their likelihoods of corruption by p-values. Test III estimates a "fixed effect" for each sub-sample by regression and can rank their likelihoods of being corrupted. However, because these tests are constructed on fairly limited sample information, there are several shortcomings:

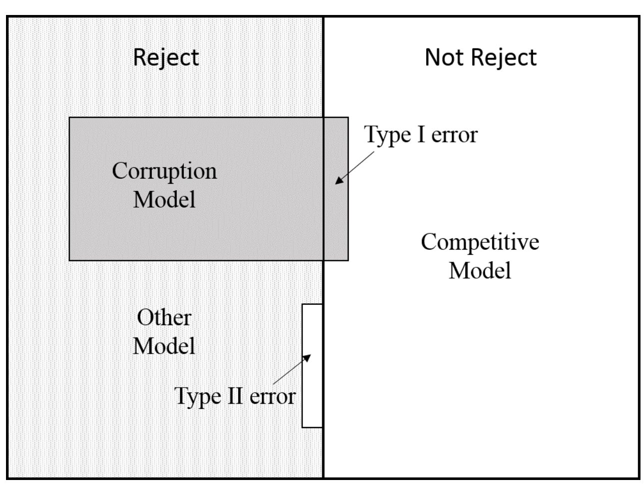
(1) Our test statistics involve two-step estimation based on pseudotypes. Because estimated pseudotypes are correlated, asymptotic distribution under the null is hard to derive analytically. We therefore use bootstrap critical values to make rejection decisions. Researchers start developing

inference and tests based on one-step estimation of auction data. For example, Liu and Luo (2014) propose a test of exogenous entry based on empirical quantile of bids, which circumvents the correlation issue of estimated pseudo-values. However, because bids depend on the number of bidders (n), one cannot pool data from auctions with different n together, but needs to conduct the test separately on sub-samples according to n. In our application, there is a great deal of variation of n (see Figure 7), so we choose to take the two-step approach.

(2) The powers of our tests are also very difficult to be studied analytically. First, they depend on the manipulation power m, which is unobservable and may vary across auctions. Second, m cannot be estimated even if we assume it follows a parametric distribution. The model is not identified under the alternative hypothesis because the corrupted firm is not always the winner. So the manipulation power cannot be recovered without knowing the exact identities of the corrupted firms. In other words, the corrupted firm's bid and other bids are not generated from the same data generation process, and we don't know which bid comes from the corrupted firm. These complications restrict us from studying the powers of the tests rigorously. A desirable data set to study corruption should include some *ex post* information of convicted corruption records. With identities of corrupted firms, then it is possible to identify the corruption model. Researchers can then study the powers and their "in-sample" prediction correctness of these tests. We don't have such a data set for now and the main contribution of these tests are their *ex ante* applicable feature in corruption detection.

(3) Figure 6 illustrates a common problem of our tests and most collusion detection tests in the literature. When the data does not reject the null, it supports that the data rationalizes the competitive model. But when the data rejects the competitive model, it cannot distinguish whether the reason is corruption or model mis-specification. For example, rejecting test I can be due to any reason related to failures of expected score equivalence, like bidder's risk aversion. The one-sided tests in test I and test II alleviate this problem: if we find that the winning bid is not aggressive but conservative, we do not reject the null.

Figure 6: Interpretation of Test Results



4 Empirical Application

4.1 Data and Server Room Construction Industry

Our data comes from two major procurement platforms: Guangzhou Public Resource Trading Center and Public Resources Trading Center in Guangdong Province.[11] Nearly all procurements conducted in these two trading centers are sealed-bid scoring auctions due to both legal requirements and their economic advantages. The Chinese Law of Tender[12] requires government related projects with values over certain thresholds to go through the open tender process coordinated by these trading centers. The law also provides guidelines to forming tender evaluation committees, selecting industrial experts, designing of scoring rules, and the detailed process of auction. Besides public sector, private sector buyers also use these two trading centers frequently because trading centers have connections to a large pool of industrial experts that perform bid evaluations.

The data set covers a series of procurement auctions of server room construction projects. Server room (or data center) is an indoor place designed for containing machines like data storage, servers, and large computers. Evaluating quality of a server room construction proposal needs specific expertise. To ensure reliability and safety, the construction of server rooms have detailed technical requirements on various aspects like temperature, humidity, electricity supply, fire control, etc.. Each bid contains a full construction proposal and a itemized price list. Firm's reputation, experience, certificate, and financial status need to be considered in the bids evaluation. Therefore, compared to lands or cargo, server room construction procurements are subject to higher risks of quality manipulation.

During the two year period (01/01/2012 to 12/31/2013) of our data set, there are total 2147 observed projects. On average, 8.8 bidders enter and submit valid bids for each project. The summation of engineer's estimated costs of all observed projects is over 10 billion CNY (1.6 billion USD). Hence the industry is both large and has enough observations for structural estimations. For each project, our information includes its engineer's estimated cost, number of bidders, factor weights, city, the buyer's name, and the winning firm's identity. On bid level, we observe factor scores of each bid. Table 2 and Figure 7 summarize the data set. All price data are in units of 1,000 CNY. Transformed quality and transformed score are defined later in this section.

Several remarks about the data set:

(1) Our data set contains much less firm-specific covariates than those in Porter and Zona (1993) and Bajari and Ye (2003). Among the 1046 winning firms, 451 firms win only one contract. Therefore, tracking firm's bidding history to construct variables like "backlog", "capacity", or "utilization rate" is not practical.[13] In addition, we do not observe the identities of all losing firms, so we cannot construct explanatory variables like rival firm's distance or rival capacity.

[11]Website: http://gzggzy.cn/, http://www.bcmegp.com/. Starting November 2009, these two major procurement platforms publicly announced auction results of all government related projects.

[12]Law of the People's Republic of China on Tenders and Bids (click this link for its full article in English).

[13]See Porter and Zona (1993), Section IV for definitions of backlog, capacity, and utilization rate.

Table 2: Descriptive Statistics of the Data

Variables	Obs.	Mean	SD	Min.	Max
Project-specific					
Engineer's estimated cost, p_0	2,147	5,049.90	1,478.49	835	13,239
Weight on tech. factor, w_p	2,147	0.4958	0.0627	0.4	0.55
Weight on price factor, w_q	2,147	0.4042	0.0627	0.35	0.5
Weight on business factor, w_r	2,147	0.1	0	0.1	0.1
Number of firms, n	2,147	8.832	3.857	3	36
Winning score, s	2,147	78.550	6.068	52.49	95.13
Project city	2,147	(21 cities in Guangdong province)			
Bid-specific					
Price factor score, s_p	18,963	69.80	10.43	1.0881	100
Tech. factor score, s_q	18,963	60.47	28.55	0	100
Business factor score, s_r	18,963	72.30	10.15	29	100
Price, p	18,963	4,162.96	1,843.70	363.3	18,417.38
Savings rate, $\rho = \frac{p_0 - p}{p_0}$	18,963	0.1980	0.1044	-0.4891	0.50
Weighted score, s	18,963	66.50	11.10	26.98	95.1337
Transformed quality, $\tilde{q}$	18,963	8,260.86	2,915.77	1,140.24	29,135.76
Transformed score, $\tilde{s}$	18,963	4,097.89	1,481.16	712.09	11,559.27

Figure 7: Visualization of Some Variables

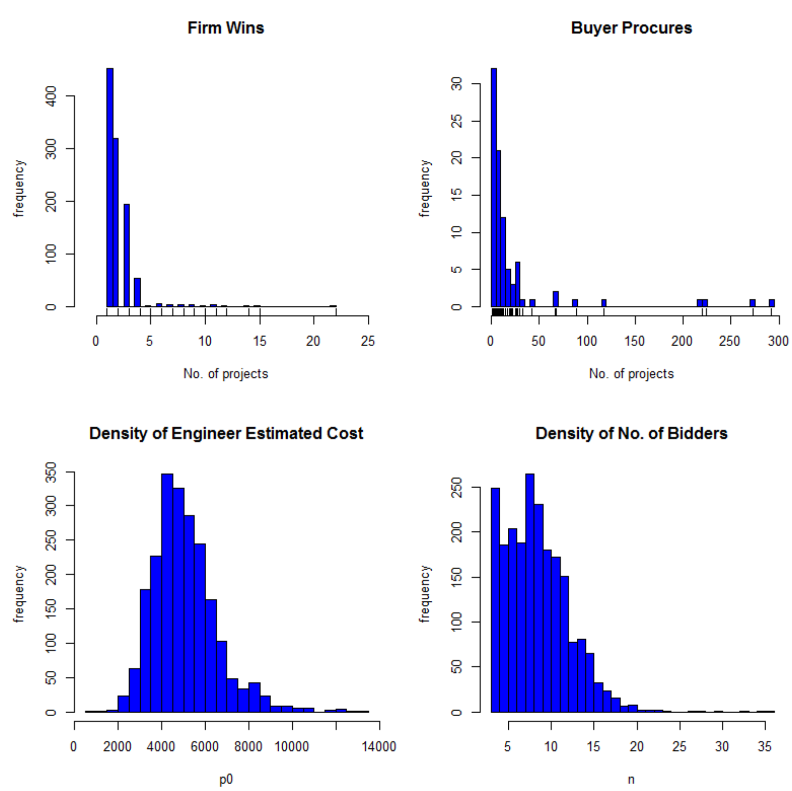


Table 3: Summary of Market Structure

	Market Share		Number of Projects			
	Mean	H.H. index	Mean	SD	Min	Max
Firms	0.0956%	0.0015	2.0525	1.6044	1	22
Total= 1046						
No. of firm wins one project= 451						
Buyers	1.1236%	0.0659	24.1236	53.4929	1	292
Total= 89						
No. of buyer procures one project= 10						
No. of buyer procures less than 10 project= 50						

Index	No. of Project	Market Share	Total Value	Share of Value
Top five firms:				
1	22	0.0102	94,641	0.0087
2	15	0.0070	69,990	0.0065
3	14	0.0065	63,264	0.0058
4	14	0.0065	72,496	0.0067
5	12	0.0056	64,504	0.0059
Top five buyers:				
1	292	0.1360	1,463,947	0.1350
2	273	0.1272	1,370,988	0.1264
3	224	0.1043	1,154,855	0.1065
4	220	0.1025	1,110,387	0.1024
5	117	0.05449	579,093	0.05341
Procurement platforms (agents):				
1	1,466	0.6828	7,374,657	0.6802
2	681	0.3172	3,467,485	0.3198

(2) The market structure of this industry is relative simple. There is a large number of supplying firms and no buyers or firms dominates the industry. Table 3 show that the largest firm only takes a 1% market share. Also, because server room project design and construction costs are not much affected by their geographic location, combining data from different cities is reasonable. Moreover, subcontracting is common in this industry, a firm's distance to the project is less important when most components of the project are carried out by subcontractors. These features support the independent private information setting of our model.

(3) The scoring rule of this data set is relative easy to analyze. The business factor weights is constant at 0.1 across all projects and (w_p, w_q) combination takes only five pairs of values.[14] Hence, the variation of the scoring rule can be controlled by w_q. The price factor evaluation rule is consistent and not interdependent. In our sample, the engineer's estimated costs and prices of the

[14]The law of tender requires all construction projects shall economic factor weight $w_p \geq 0.4$.

bid are transferred into a 100 point price factor score by formula (18) below. We will now discuss the scoring rule in details.

All projects in our data set use the most popular "comprehensively evaluation method" in China. It is simply a linear weighted scoring rule consisting of three components: an economic factor, a technical factor, and a business factor. The e*conomic factor* (s_p) evaluates the price of the bid.[15] Denote the *engineer's estimated cost* as p_0, if a company submits price p in his bid, his economic factor score is computed by

$$s_p = \begin{cases} 0, & p > \frac{3}{2}p_0, \\ 100\left(\frac{1}{2}+\rho\right) = 100\left(\frac{1}{2}+\frac{p_0-p}{p_0}\right), & \frac{1}{2}p_0 \le p \le \frac{3}{2}p_0 \ , \\ 100, & p < \frac{1}{2}p_0. \end{cases} \tag{18}$$

$\rho = \frac{p_0-p}{p_0}$ is called the *savings rate* of that bid. The *technical factor* (s_q) evaluates quality of the construction proposal including the design, building standard, equipment, server machines, follow-up service, warranty, delivery date, payment condition, insurance, etc.. The *business factor* (s_r) evaluates the firm's reputation, experience, risk of default, risk of bankruptcy etc.. Technical and business factors are evaluated by a committee of experts.[16] Each bid receives three 100 points scores on three factors, and then a grand score is computed via

$$S(s_p, s_q, s_r) = w_p s_p + w_q s_q + w_r s_r, \tag{19}$$

where weights, w_p, w_q, and w_r, add up to one. The firm who receives the highest 100-scale grand score wins the contract. Because firms are experts of the industry and may have repeated interactions with the agents, they understand the score evaluation process. Therefore, they are effectively selecting technical factor scores by submitting corresponding construction proposals. Business factor scores are also endogenously chosen by firms because firms can hire experienced engineers, acquire relevant certificates, form bidding consortia, and allocate more financial resources to raise s_r.

The linear weighted scoring rule can be transformed into a quasilinear one that reflects the same preference of the buyer. Define the *transformed score*, $\tilde{s} \equiv \frac{p_0}{100w_p}\left(S(s_p, s_q, s_r) - 50w_p\right)$ and the *transformed quality*, $\tilde{q} \equiv \frac{p_0}{100w_p}\left(100w_p + w_q s_q + w_r s_r\right)$, then (19) can be transformed into

$$\tilde{S}(\tilde{q}, p) = \tilde{q} - p. \tag{20}$$

Because the transformation is monotone, firm's winning probability and bidding strategy are not affected. We assume the cost of supplying the transformed quality $\tilde{q}$ satisfies assumption CF. Then

[15] As mentioned in Bajari et al. (2014), in reality price evaluation is not just "the low the better". A highly unbalanced bid (extreme itemized price) or a bid lower than cost could be penalized or rejected.

[16] The law of tender require the committee shall contains five or more members (odd number). There is one representative from the buyer. All the other members are either randomly selected from the pool of experts connected to the procurement agency company.

the environment satisfies the condition of Lemma 1, which allows us to use the single dimensional quality index $\tilde{q}$ for structural estimation.

Both the original scoring rule (19) and the transformed one (20) reflect the buyer's willingness-to-pay for a higher quality. The scoring rule determines the *monetary equivalent for quality.*[17] If the firm raises the price by 1 CNY, then the 100-scale grand score will reduce by $100\frac{w_p}{p_0}$. To remain at the same payoff, the buyer needs to be compensated by a higher quality, which requires $w_q \Delta s_q + w_r \Delta s_r = 100\frac{w_p}{p_0}$. We add a boundary condition that the buyer receives zero payoff from a contract with $s_q = 0$, $s_r = 0$, and $p = p_0$. Therefore, both scoring rules (19) and (20) have are the same monetary equivalent for quality and the same boundary condition. Given (20), $\tilde{q}$ represents the benefit of the buyer from a project delivered at (s_q, s_r) , and $\tilde{s}$ as the buyer's *payoff* after compensating the winning firm p. Because the transformed quality and score are anchored on price that has real monetary interpretation, they can be compared across auctions.

4.2 Reduced-form Estimation

We have three main findings in the following reduced-form empirical study:

(1) We test two implications of the theoretical model. First, a higher quality weight (lower price weight) shall induce firms to submit bids with higher quality and higher grand score. Second, firm's choice of quality and price are separated under additively separable scoring rule. By using the *original strategy space* (s_p, s_q, s_r) as the dependent variable, we do not find robust evidence. But we find the evidence supporting both model implications by using the *transformed strategy space* ($\tilde{q}$,$\tilde{s}$), which in turn justifies our use of the transformed strategy space for structural estimations and corruption detection tests.

(2) We tests for unobserved heterogeneity of projects with respect to fringe/non-fringe firms, fringe/non-fringe buyers, and two agents. We do not find strong evidence of unobserved heterogeneity among these projects.

(3) Based on the transformed strategy space, we find that projects with high engineer's estimated costs end up with winning contracts of both high quality scores and high prices. Projects with low engineer's estimated costs induce more competition on price and end up with higher savings rates (lower markups).

Consider the reduced-form regression model:

$$Y_t = \alpha_0 + \alpha_1 p_{0,t} + \alpha_2 n_t + \alpha_3 w_{q,t} + \alpha_4 D_{\text{fringe.firm},t} + \alpha_5 D_{\text{fringe.buyer},t} + D_{\text{agency2},t} + \epsilon_t,$$

where Y_t stands for the dependent variable. The main independent project-specific covariates are engineer's estimated cost and the number of bidders. On bids level, we only observed three factor scores that are endogenously chosen by firms. Price, grand score, transformed quality, and transformed score are all functionally correlated with factor scores. The data also lacks the losing firm's identities, so there is no explanatory variable on bid level. Therefore, we estimate reduced-form

[17] The definition of monetary equivalent for quality reflected by a scoring rule can be found in Dini et al. (2006).

models on project level with only winning bids. Because there are lots of firms or buyers that only appear in one project, we do not include firm or project fixed effects in the model. Instead, we add two indicators for fringe firms and fringe buyers: $D_{\text{fringe.firm}} = 1$ if the winning firm is fringe (wins only one project) and $D_{\text{fringe.buyer}} = 1$ if buyer is fringe (procures less than 10 projects). D_{agency2} is the indicator for if the project is process by agent 2.

Figure (8) and Table 4 summarize the estimation results. Based on the coefficients of technical factor weight, a higher w_q results in a higher technical score, a higher price, and a lower savings rate. Therefore, if a buyer wants to procure the project at a higher quality, the cost will also increase significantly. In other word, firms ask for higher markups in high technical weight procurements. More entry of the procurement auction increases competition and results in positive effects on all five dependent variables. In addition, diagram (A), (B) and regression (I), (II) show that for variations of engineer's estimated costs does not technical scores and business scores if the winning bids. None of these regressions shows significant differences between the large and fringe firms (buyers). The two agents also appear to be similar.

These regressions with the dependent variable within the observed strategy space have one major drawback: scores on the 100-scale are intangible concepts and hard to compare across auctions. Receiving the same 100 point technical scores may mean completely different things for two projects. We also find that increasing w_q does not significantly increase grand scores in regression (IV), which is not consistent with theoretical model prediction. In addition, the goodness-of-fit, measured by R^2, are relatively low except for regression (III).

Nevertheless, we can consider the transformed strategy space with $\tilde{q}$ and $\tilde{s}$. These two variables are directly related to price and thus can be compared across auctions. Table 5 displays estimation results using $\tilde{q}$ and $\tilde{s}$ as dependent variables. For all six regressions, their R^2s improve and show significantly positive coefficient estimates of w_q. We can interpret that by increasing the technical factor weight for 5%, it induces the project to be delivered at a higher quality and higher score to the buyer. The estimated average buyer's payoff increment ranges from 800,858 to 1,044,340 CNY (129,171 to 168,442 USD).

Concerning the coefficient estimates of regressor n, increasing the number of bidders has no significant effect on $\tilde{q}$, but has significant positive effects on $\tilde{s}$. It provides the important evidence supporting a theoretical model implication: firms choose their quality level based on their own social surplus maximization problem (equation (2) and (8)), hence n does not affect their choice of $\tilde{q}$. Because this property of independent quality choices relies on fairly weak assumption, confirming it in empirical study also supports the validity of our strategy space transformation.

In addition, we also observe some meaningful patterns across winning bids. In Figure 9 diagram (A) and (B), we plot the density of savings rate and transformed quality respectively. The black curve represents the density of all observed bids while the red dashed curve represent the density of only winning bids. These two density diagrams show that winning bids have consistent pattern of both higher quality and higher price, compared to other bids. Table 6 shows that 74.24% of winning bids have the highest transformed quality in that auction. On the other hand, only 4.01% of winning bids have the highest savings rate in that auction. On average, winning bids ask for higher prices (a lower savings rate) than losing bids. Among the 2147 auction we observed, 145 projects ended up with negative savings rates, meaning that the winning contracts have prices higher than the engineer's estimated costs. All these 145 negative savings rate auctions occur at projects with engineer's estimated costs higher than 5,565 thousand CNY. Hence, a high p_0 project is more likely to be awarded to a high quality and high price bidder, illustrated in Figure 10. So we see high quality and high price contracts concentrating at high p_0 projects, where there are more room for quality manipulation. These signs of corruption motivate the corruption detection tests below.

Figure 8: Illustration of Winning Bids in Observed Strategy Space

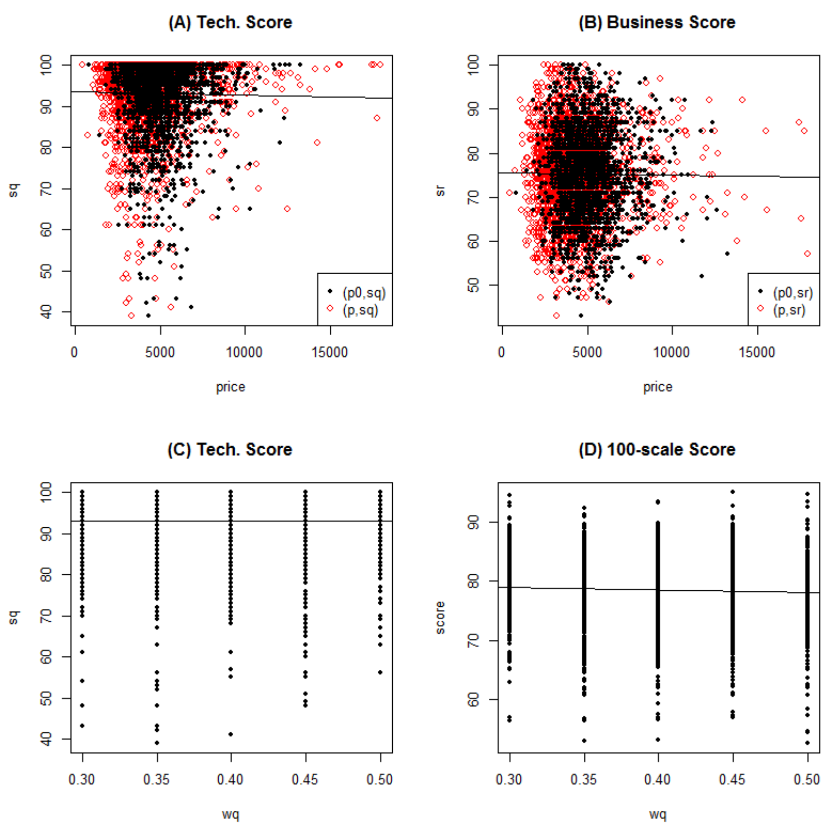


Table 4: Reduced-form Regressions in Observed Strategy Space

	(I)	(II)	(III)	(IV)	(V)
Dep.Var.	s_q	s_r	p	s	ρ
p_0	0.0000	0.0000	1.288**		
	(0.0001)	(0.0001)	(0.0050)		
n	0.7447**	0.1236*	4.341*	0.3809**	0.0013*
	(0.048)	(0.057)	(1.99)	(0.034)	(0.0006)
w_q	11.17**	-4.95	3294.64**	1.53	-0.5758**
	(2.92)	(3.51)	(122.03)	(2.09)	(0.0377)
$D_{\text{fringe.firm}}$	0.1008	-0.5357	-28.67	-0.5463	-0.0093
	(0.4378)	(0.5265)	(18.30)	(0.3127)	(0.0056)
$D_{\text{fringe.buyer}}$	1.0405	-0.5909	-4.416	0.7195	0.0064
	(0.5998)	(0.7212)	(25.07)	(0.4285)	(0.0077)
D_{agency2}	-0.0694	0.3887	-30.78	-0.1587	-0.0034
	(0.3826)	(0.4601)	(16.0)	(0.2734)	(0.0049)
Constant	81.83**	76.48**	-3511.04**	74.66**	0.3927**
	(1.50)	(1.81)	(62.87)	(0.98)	(0.0176)
R^2	0.1050	0.0050	0.9687	0.0595	0.1123
Obs	2147	2147	2147	2147	2147

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

Figure 9: Illustration of Winning Bids in Transformed Strategy Space

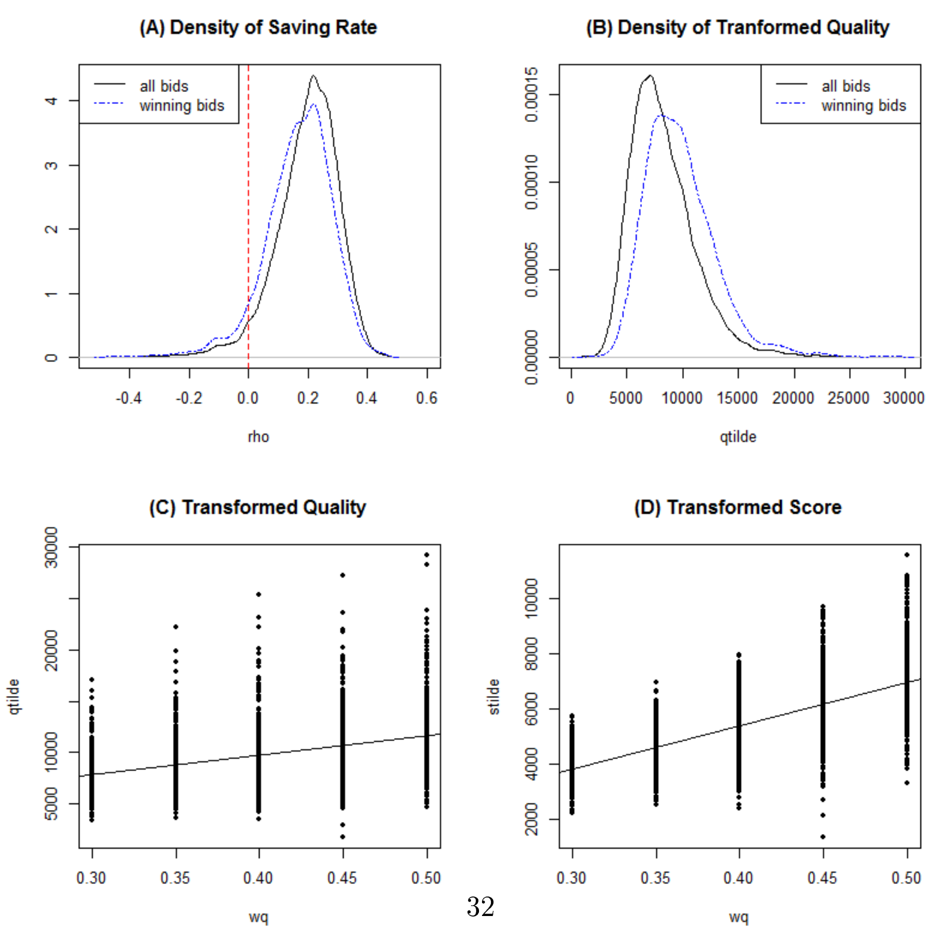


Table 5: Regression in Transformed Strategy Space

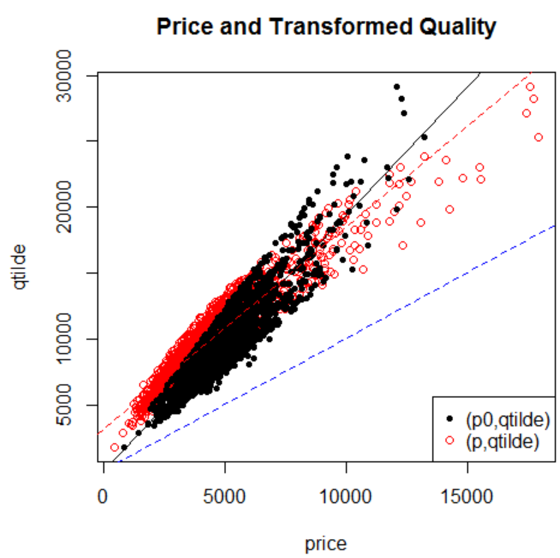
	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var.	Transformed Quality $\tilde{q}$			Transformed Score $\tilde{s}$		
Data	All	$p_0 < 5,565$	$p_0 \geq 5,565$	All	$p_0 < 5,565$	$p_0 \geq 5,565$
n	3.05	-0.85	47.56	20.15**	16.38**	39.08**
	(16.90)	(11.44)	(25.71)	(6.38)	(5.71)	(7.01)
w_q	19,007.23**	16,876.99**	26,367.82**	16,017.17**	14,326.23**	20,886.79**
	(1,038.50)	(690.20)	(1,652.03)	(391.94)	(344.24)	(450.61)
$D_{\text{fringe.firm}}$	200.15	186.62	-69.02	69.71	80.97	-41.14
	(155.65)	(103.21)	(248.59)	(58.75)	(51.48)	(67.81)
$D_{\text{fringe.buyer}}$	-158.57	-80.29	-560.93	-24.58	-21.19	-131.02
	(213.30)	(141.10)	(342.64)	(80.50)	(70.37)	(93.46)
D_{agency2}	121.59	124.31	-242.29	70.75	61.07	-28.83
	(136.08)	(89.99)	(218.49)	(51.36)	(44.88)	(59.60)
Constant	2,021.96**	1,552.37**	2,431.78**	-1,234.99**	-987.43**	-2,068.26**
	(485.21)	(325.33)	(756.52)	(183.13)	(162.26)	(206.35)
R^2	0.1421	0.2943	0.3066	0.4446	0.5378	0.7863
Obs	2147	1551	596	2147	1551	596

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

Table 6: Pattern of Winning Bids

	Mean of All Bids	Mean of Winning Bids	Highest in the Auction		Lowest in the Auction	
			Number	Percentage	Number	Percentage
s_q	60.48	93.05	1582	73.68%	4	0.19%
s_r	72.30	75.32	455	21.19%	193	8.99%
ρ	0.1980	0.1691	86	4.01%	530	24.69%
$\tilde{q}$	8260.86	4746.27	1594	74.24%	3	0.14%

Figure 10: Illustration of Winning Bids in Transformed Strategy Space (Continue)



4.3 Structural Estimation and Corruption Detection Tests

Lacking bid-specific covariates, project level regressions put aside information in all losing bids because they are endogenous, but structural estimation can draw information from all bids. The pattern of both winning bids and losing bids together reveal whether the bidding behaviors are competitive. In the transformed strategy space, the structural estimation and corruption detection tests developed in Section 3.1 and 3.2 can be directly applied. Varying the scoring rule affects the distribution of pseudotype, therefore we consider sub-samples according to technical weights and agents. For each sub-sample, we apply formula (12) to structurally estimate pseudotypes. The estimation results are reported in Table 7 and Figure 11.

Recall that $\hat{k}$ represents the total social surplus of each firm producing at its efficient level, and $\tilde{s}$ represents how much of the social surplus is harvested by the buyer. Their difference, $\hat{k} - \tilde{s}$, is the *estimated rent* retained by the firm. Table 8 compares performance of two agents. The projects processed by two agents are similar in their observed characteristics, but we find that in general, firms bid in agent 1 gets higher rent than agency 2. Specifically, overall, firms at agent 1 ask for 63,030 CNY (10,166 USD) more rent compared to agency 2. But if we consider only winning bids, at each sub-sample, winning firms at agent 1 do not earns significantly more rent than those at agency 2.

Consider comparative statics on technical factor (quality) weights: a higher w_q in general leads to both higher transformed scores and higher rents, benefiting both parties. Quality weights reflect the buyers' willingness-to-pay for high quality projects, while the supplying firms only care monetary compensations. Serving buyers with higher willingness-to-pay naturally lead to higher payoffs for both sides. The theoretical model (Corollary 2), reduced-form, and structural estimation results are consistent in this prediction.

Aforementioned, a high rent alone is not the sign of corruption. To test for quality manipulation, we need to explore the consistently suspicious patterns of relationship among bids revealed in a large sample. We apply the three tests proposed in Section 3.2. Table 9 and Figure 12 show results of test I and II. For test I, there are a total five sub-samples rejecting the competitive model. In general, they happen at high w_q auctions. For test II, none of sub-samples rejects the competitive model. For test III, we consider six regression models shown in Table 10 and find only one coefficient of D_{agency2} being significant. Regression (VI) is run on the sub-sample with high engineer's estimated costs. It implies the first spacing of transformed scores is larger at agent 2, which is the sign of aggressive bidding behavior. Since we also find that rent at agent 2 is generally lower, the reality could be that firms are earning their rent under the table by delivering low quality projects.

In summary, a majority of the data set passes our corruption detection tests. Recall Figure 6 these failures of rejection support the theoretical prediction of the competitive model, which makes the structural estimation trustworthy for this data set. For some sub-samples of the data set, we find signs of quality manipulation. The data patterns shown in Figure 10 and results of corruption detection tests suggest that antitrust authorities should spend more investigation resources on projects with high technical weights and high engineer's estimated costs, especially on those processed by agent 2.

It is worth mentioning that high technical weight and estimated cost are proxy for complexity of the project. Bajari and Tadelis (2001) and Tadelis (2012) compare auction and negotiation at different levels of project complexity. Complexity may potentially jeopardize the advantage of competitive tendering because the uncertainty in project design stage may lead to costly renegotiation of *ex post* adjustment. The buyer may choose a bilateral negotiation with a reputable supplier, because in the negotiation, the reputable firm can help design the complex project and save the *ex post* adaption cost. In a scoring auction, quality and design of the project are chosen by firms, so it reaps benefit from both price-only auctions and negotiation. However, all these cross-procurement-scheme comparisons are not robust if quality is not perfectly observable and/or verifiable at the moment of transaction. The corruption problem analyzed in this paper not only affects the optimal scoring rule and auction format, but also optimal procurement scheme.

At the end, we want to point out that our nonparametric approach is based on pseudotypes and the distribution of pseudotype changes as scoring rule varies. A fully nonparametric method is may not be applicable when there is a great deal of variation in scoring rules across auctions and the sample size is relatively small. In this case, a parametric or semi-parametric approach shall be adopted. Once a parametric cost function is specified, the optimal quality and scores can be expressed as a system of equation of parameters. Nakabayashi and Hirose (2015) show the exact conditions that ensure the data on quality and score can reverse-engineer the parameters.

Table 9: Results of Test I and II

Sub-sample		Test I			Test II		
w_q	Agency	Test Stat.	BT c.v.	BT p.v	Test Stat.	BT c.v.	BT p.v
0.3	1	-6.7284	-4.0347	0.5400	0.2422	0.2870	0.6600
	2	-3.3526	-5.0863	**0.0000**	0.2644	0.3563	0.8350
0.35	1	-6.2109	-4.9973	0.2800	0.2131	0.2623	0.7000
	2	-5.0055	-4.6895	0.1050	0.2836	0.3209	0.3800
0.4	1	-7.3851	-5.8942	0.2600	0.2396	0.2656	0.4750
	2	-4.7708	-5.6530	**0.0000**	0.2513	0.2923	0.5850
0.45	1	-5.3859	-5.8718	**0.0000**	0.2260	0.2604	0.6500
	2	-5.5837	-5.8815	**0.0150**	0.2330	0.2784	0.6700
0.5	1	-5.3204	-5.2688	0.0700	0.2067	0.2500	0.5900
	2	-4.4933	-4.8648	**0.0100**	0.2584	0.3258	0.5300

Note: BT c.v. and BT p.v stand for "bootstrap critical value " at 0.05 significance level and "bootstrap p-value" respectively. They are computed based on 199 bootstrap samples at project level. Bold numbers indicate rejection of the null.

Figure 11: Illustration of Structural Estimation Result

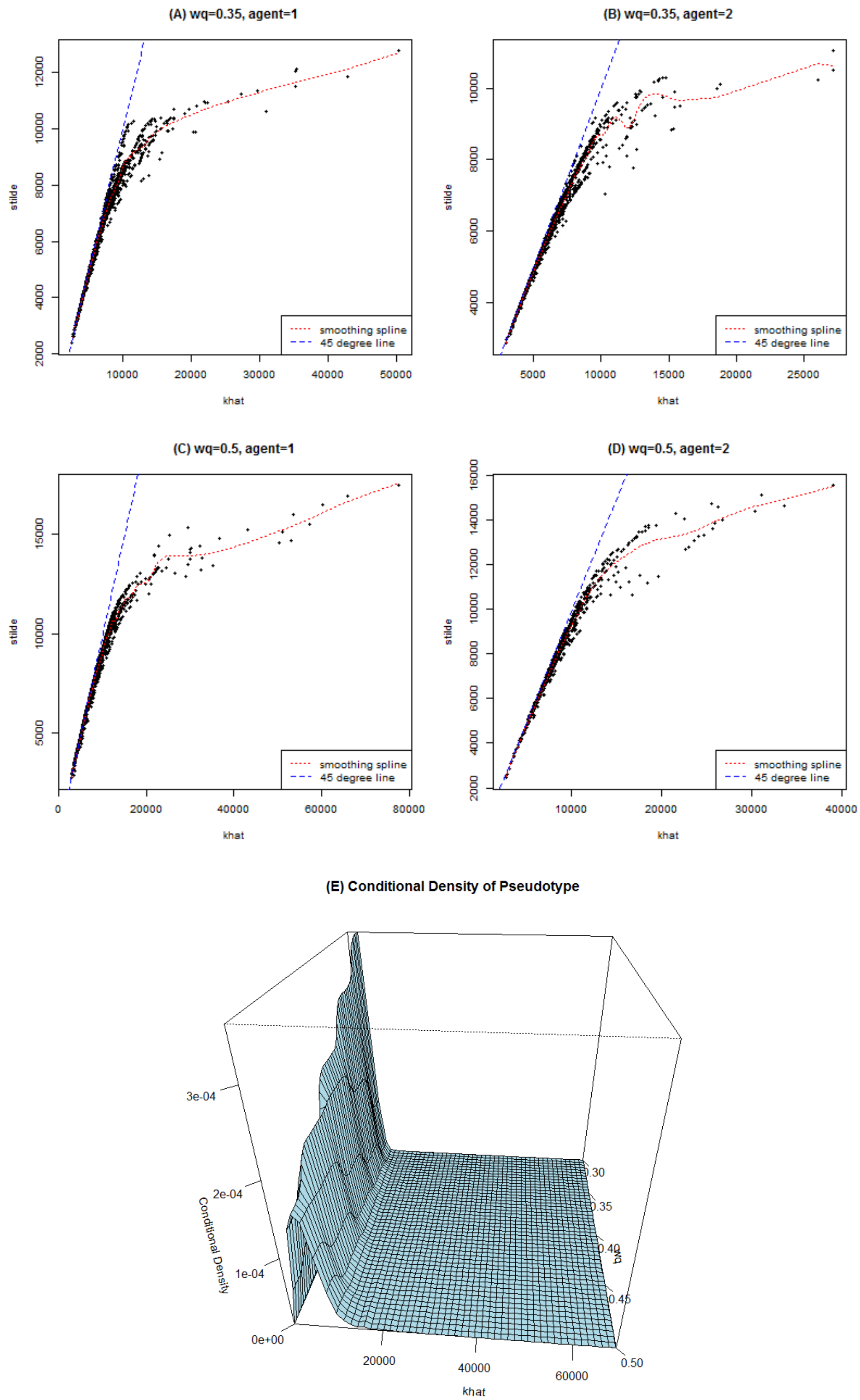


Table 7: Structural Estimation Results

Sub-sample w_q	Agency	No. of Projects	No. of Bids		Mean	SD	Min	Max
0.3	1	223	2384	$\hat{k}$:	3,358.97	1,292.52	1,156.09	27,485.10
				$\tilde{s}$:	3,126.94	797.31	1,135.51	5,747.27
0.3	2	87	913	$\hat{k}$:	3,389.06	1,088.43	1,308.24	10,132.73
				$\tilde{s}$:	3,173.24	811.30	1,264.24	5,407.96
0.35	1	244	2266	$\hat{k}$:	3,913.30	1,750.54	891.22	35,700.95
				$\tilde{s}$:	3,546.35	1,032.03	821.27	6,945.76
0.35	2	134	1228	$\hat{k}$:	3,942.98	1,473.08	1,540.13	19,494.30
				$\tilde{s}$:	3,617.00	1,014.27	1,494.09	6,303.84
0.4	1	384	3437	$\hat{k}$:	4,451.63	2,250.47	1,253.65	56,690.62
				$\tilde{s}$:	3,980.14	1,242.36	1,175.52	7,971.81
0.4	2	195	1687	$\hat{k}$:	4,448.67	1,805.02	1,052.06	22,355.00
				$\tilde{s}$:	4,053.84	1,227.13	1,015.22	7,568.83
0.45	1	407	3335	$\hat{k}$:	5,342.17	3,181.92	773.96	68,737.29
				$\tilde{s}$:	4,691.41	1,515.09	712.09	9,711.02
0.45	2	176	1510	$\hat{k}$:	5,131.95	2,360.78	1,603.51	28,208.93
				$\tilde{s}$:	4,585.34	1,505.13	1,431.86	9,324.88
0.5	1	208	1567	$\hat{k}$:	6,054.45	3,463.00	1,724.88	58,419.10
				$\tilde{s}$:	5,224.02	1,870.11	1,620.04	11,559.27
0.5	2	89	636	$\hat{k}$:	6,474.78	3,054.39	1,699.51	28,500.12
				$\tilde{s}$:	5,667.42	1,894.45	1,539.81	10,811.87

Figure 12: Result of Test I and Test II

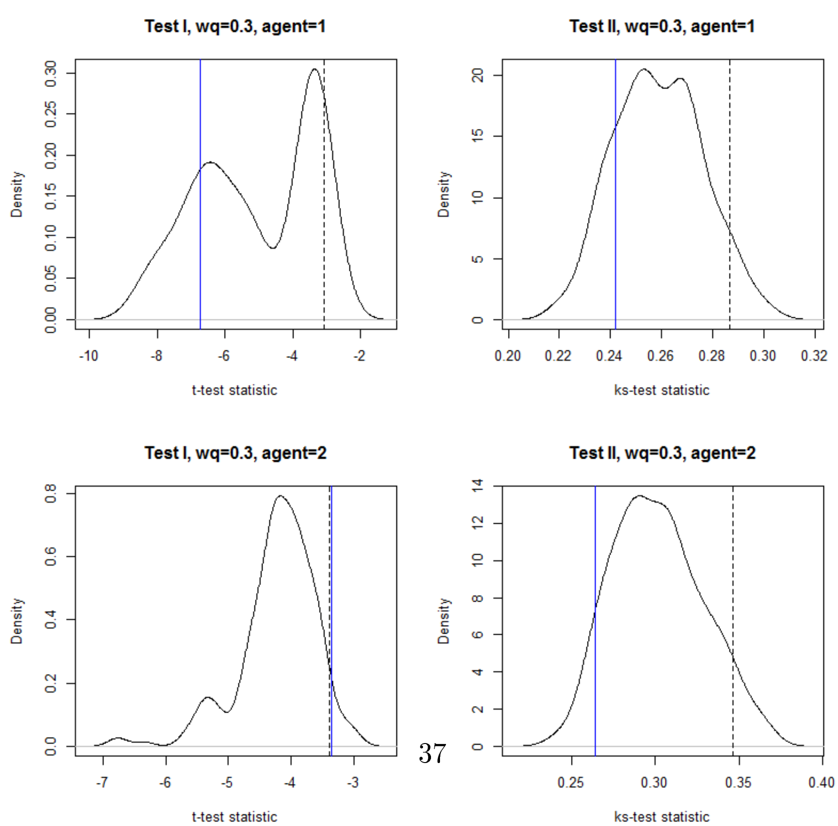


Table 8: Comparison of Two Agents

	Pro. Agency 1		Pro. Agency 2		t-test of Equal Mean	
	Mean	SD	Mean	SD	Statistic	p-value
n	8.860	3.937	8.772	3.681	0.5029	0.6151
p_0	5030.46	1515.45	5091.75	1395.77	-0.9212	0.3571
$\tilde{s}$	5435.89	1489.60	5484.52	1470.62	-0.7101	0.4778
	Estimated Rent $(\hat{k} - \tilde{s})$ of All Bids					
$w_q = 0.3$	232.03	749.33	215.82	380.90	0.8162	0.4144
$w_q = 0.35$	366.95	1,014.52	325.98	664.14	1.4365	0.1509
$w_q = 0.4$	471.49	1,396.39	394.82	873.68	2.4007	**0.0164**
$w_q = 0.45$	650.76	2,261.30	546.60	1,191.91	2.0940	**0.0363**
$w_q = 0.5$	830.42	2,089.09	807.36	1,551.44	0.2846	0.7760
Overall	498.63	1,634.84	435.60	985.48	3.2844	**0.0010**
	Estimated Rent $(\hat{k} - \tilde{s})$ of Winning Bids					
$w_q = 0.3$	816.54	2,212.67	646.67	821.04	0.9857	0.3251
$w_q = 0.35$	1,239.56	2,667.93	967.06	1,640.22	1.2279	0.2203
$w_q = 0.4$	1,534.06	3,529.86	1,208.71	2,164.06	1.3692	0.1715
$w_q = 0.45$	2,006.02	5,095.38	1,632.98	2,743.71	1.1428	0.2536
$w_q = 0.5$	2,228.15	4,393.84	2,183.71	3,214.99	0.0972	0.9226
Overall	1,605.41	3,914.21	1,326.43	2,330.57	2.0551	**0.0400**

Note: Bold numbers indicate rejection of the null at 0.05 significance level.

Table 10: Results of Test III

	(I)	(II)	(III)	(IV)	(V)	(VI)
Dep.Var	First Spacing of $\hat{k}$			First Spacing of $\tilde{s}$		
Data	All	$p_0 < 5565$	$p_0 \geq 5565$	All	$p_0 < 5565$	$p_0 \geq 5565$
D_{agency2}	-39.78	3.372	-245.88	20.45	4.750	103.25*
	(110.36)	(29.02)	(591.77)	(16.84)	(18.05)	(46.37)
3rd order	0.3082**	-0.0863**	0.0568	-0.0506**	-0.0922**	-0.2180**
statistic	(0.0296)	(0.0129)	(0.1194)	(0.0072)	(0.0098)	(0.0246)
w_q	-1,120.81	3,472.47**	5,910.49	1,813.97**	2,298.35**	4,947.44**
	(929.72)	(268.23)	(5274.83)	(160.63)	(180.07)	(578.31)
n				-24.62**	-17.96**	-9.485
				(2.199)	(2.432)	(6.388)
Constant	-236.29	-547.17**	187.46	64.50	-56.40	-207.70
	(337.84)	(88.85)	(1857.27)	(60.86)	(66.11)	(175.94)
R^2	0.0556	0.0986	0.0089	0.1588	0.1875	0.2875
Obs	2147	1551	596	2147	1551	596

Note: Significance levels are denoted by asterisks (* $p < 0.05$, ** $p < 0.01$).

5 Conclusion

We conclude by reviewing main results with some policy implications. In this paper, we develop a structural estimation method and three corruption detection tests of scoring auctions. They are built upon fairly standard data of procurement auctions and can be applied to data from a wide range of industry with enough observations. The estimation method and corruption detection tests in this paper complement the theoretical side of the optimal procurement problem. By using historical data, they provide quantitative prediction of the effect of varying the scoring rule and the risk of quality manipulation. These predictions are particularly useful for designing desirable procurement schemes. Therefore, recording and aggregating procurement auction data are valuable for improving procurement outcome and identifying corruption.

We applied the method to a data set of server room procurement auctions. The data patterns and estimation results provide evidence for the theoretical scoring auction model. First, under additively separable scoring rules, the choice of quality can be separated from the choice of price and score. The reduced-form estimation shows that qualities are not affected by the number of competing bidders, but scores are. Second, with competition on both price and quality, firms mainly compete on offering high quality and expensive contracts. In the data set, over 70% of winning bids have the highest quality, but only about 4% of winning bids have the lowest price. Therefore, a reliable quality evaluation procedure is very important in keeping the auction efficient.

We also explore the effect of varying quality weight. The theoretical model predicts that a higher weight on quality induces firms to submit bids at higher quality and score, which is confirmed by estimation results in the transformed strategy space. The structural estimation results show that projects procured with higher quality weights result in both higher payoffs for the buyers and the winning firms. However, the buyer is restricted in picking the quality weight because the scoring rule must reflect her willingness-to-pay of higher quality. The theoretical model of scoring auctions shows that the buyer will not over-state its preferences on quality,[18] instead, the optimal scoring rule "shade" buyer's preference on quality to avoid giving up too much rent to the efficient firm. Besides the shading for optimal screening, does a higher quality weight gives more room for quality manipulation? As Lengwiler and Wolfstetter (2006) suggested, when quality scores are problematic due to the possibility of corruption, the quality weight shall be reduced. Therefore, in designing the scoring rule, the buyers need to balance the efficiency and the risk of quality manipulation. We run the three corruption detection tests proposed in this paper and find that, in general, the data set passes our tests. But there are some signs of corruption in sub-samples with higher quality weights and higher engineer's estimated costs.

Our corruption detection tests are *ex ante* in the sense that they can label identities of corrupted firms, projects, and agents without specifying a prior of suspects. For future research, one important complement is an *ex post* study of corruption behaviors by data from convicted corruption cases,

[18] Huang and Xia (2015) shows that the buyer may over-state it to fight against quality manipulation.

for example, from investigation reports from collapsed bridges. Then researchers can study the "in-sample" property of these corruption detection tests (e.g. Bajari and Ye (2003)) and even the internal organization of corrupted agents (e.g. Asker (2010)). In this way, historical auction data, antitrust records, and economic analysis can together construct stronger tools for antitrust purposes.

Appendix

Proof of Lemma 1: For the minimization problem $\min_{\mathbf{q}} C(\mathbf{q},\theta)$ such that $V(\mathbf{q})=v$, the Lagrangian expression is

$$\mathcal{L}=C(\mathbf{q},\theta)-\lambda(V(\mathbf{q})-v).$$

The first-order condition yields a system of these $L+1$ equations of λ and $\mathbf{q}$

$$\begin{cases}\frac{\partial}{\partial \mathbf{q}}C(\mathbf{q},\theta)-\lambda\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})=\mathbf{0}_{L\times 1}, \\ V(\mathbf{q})-v=0\end{cases}.$$

By assumption CF, $C(\cdot,\theta)$ is strictly convex in $\mathbf{q}$, matrix $\frac{\partial^2 C(\mathbf{q},\theta)}{\partial \mathbf{q}'\partial \mathbf{q}}$ is positive definite. By assumption QL, $V(\mathbf{q})$ is weakly concave, $\frac{\partial^2 V(\mathbf{q})}{\partial \mathbf{q}'\partial \mathbf{q}}$ is negative semi-definite, there is a unique solution to this system equations, denoted as $\mathbf{q}(v|\theta)$ and $\lambda(v|\theta)$. The value function of the minimization problem is $\tilde{C}(v,\theta)=C(\mathbf{q}(v|\theta),\theta)$. By the Maximum Theorem (Berge, 1963), it is single-valued and continuous in v.

By envelop theorem, the value function satisfies $\tilde{C}_v=\lambda$. Plug $\mathbf{q}(v|\theta)$ into the constraint $V\left(\mathbf{q}(v|\theta)\right)=v$. Differentiate with respect to v implies $\frac{\partial}{\partial \mathbf{q}'}V\left(\mathbf{q}(v|\theta)\right)\mathbf{q}_v=1$. Therefore, $\frac{\partial}{\partial \mathbf{q}}C(\mathbf{q}(v|\theta),\theta)-\lambda(v|\theta)\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q}(v|\theta))=\mathbf{0}$ implies $\lambda(v|\theta)=\frac{\partial}{\partial \mathbf{q}}C\left(\mathbf{q}(v|\theta),\theta\right)/\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q}(v|\theta))=\frac{\partial}{\partial \mathbf{q}}C(\mathbf{q}(v|\theta),\theta)\mathbf{q}_v=\tilde{C}_v>0$.

To show $\tilde{C}_{vv}>0$ is equivalent to show $\lambda_v(v|\theta)>0$. Differentiate the first-order condition above with respect to v:

$$\begin{cases}\frac{\partial}{\partial v}(\frac{\partial \mathcal{L}}{\partial \mathbf{q}})=\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}C\left(\mathbf{q},\theta\right)\mathbf{q}_v-\lambda\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}V(\mathbf{q})\mathbf{q}_v-\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})\lambda_v(v|\theta)=0 \\ \frac{\partial}{\partial v}(\frac{\partial \mathcal{L}}{\partial \lambda})=\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})\mathbf{q}_v-1=0\end{cases}$$

The first L equation yields

$$\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})\lambda_v(v|\theta)=\left[\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}C\left(\mathbf{q},\theta\right)-\lambda\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}V(\mathbf{q})\right]\mathbf{q}_v.$$

Premultiply by $\mathbf{q}_v^T$,

$$\mathbf{q}_v^T\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})\lambda_v(v|\theta)=\mathbf{q}_v^T\left[\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}C\left(\mathbf{q},\theta\right)-\lambda\frac{\partial^2}{\partial \mathbf{q}'\partial \mathbf{q}}V(\mathbf{q})\right]\mathbf{q}_v,$$

because $\frac{\partial}{\partial \mathbf{q}}V(\mathbf{q})\mathbf{q}_v = 1$ and $\frac{\partial^2 \mathcal{L}}{\partial \mathbf{q}' \partial \mathbf{q}} = \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q}, \theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q})$ is positive definite (PD), we have

$$\lambda_v(v|\theta) = \mathbf{q}_v^T \underbrace{\left[\frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} C(\mathbf{q}, \theta) - \lambda \frac{\partial^2}{\partial \mathbf{q}' \partial \mathbf{q}} V(\mathbf{q})\right]}_{PD} \mathbf{q}_v > 0$$

Therefore, $\tilde{C}_v = \lambda(v|\theta) > 0$ and $\tilde{C}_{vv} = \lambda_v(v|\theta) > 0$. *Q.E.D.*

Proof of Theorem 1: (2) holds as as a special case by taking $m = 0$ in the proof of Theorem 2 below. Problem (6) is a standard first-price auction problem in IPV environment. The existence and uniqueness of a symmetric monotone Bayesian Nash equilibrium $s(\cdot)$ is established in the literature (see Maskin and Riley (1985)). When all other firms is following $s(\cdot)$, a generic firm solves $\max_s (k-s)\Pr(\text{win}|s) = (k-s)[F_K(s^{-1}(s))]^{n-1}$. The first-order condition yields $(k-s)(n-1)[F_K(s^{-1}(s))]^{n-2} f_K(s^{-1}(s)) \frac{ds^{-1}(s)}{ds} - [F_K(s^{-1}(s))]^{n-1} = 0$. At the symmetric equilibrium, we have differential equation

$$s(k)(n-1)[F_K(k)]^{n-2} f_K(k) + s'(k)[F_K(k)]^{n-1} = k(n-1)[F_K(k)]^{n-2} f_K(k). \tag{21}$$

$$\Leftrightarrow \frac{d\left(s(k)[F_K(k)]^{n-1}\right)}{dk} = k(n-1)[F_K(k)]^{n-2} f_K(k).$$

Integrate on both side with boundary condition $s(\underline{k}) = 0$,

$$s(k) = \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2} f_K(t)dt}{[F_K(k)]^{n-1}} = k - \frac{\int_{\underline{k}}^{k} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}}.$$

The last equality is obtained via integration by parts. The equilibrium price can be computed by $p(\theta) = V(q(\theta)) - s(K(\theta))$. *Q.E.D.*

Note that when θ is one-dimensional and $C_\theta < 0$, it reduces to (**??**) in Che (1993). By envelop theorem, from the value function $K(\theta) = \max V(q) - C(q,\theta)$, we have $K'(\theta) = C_\theta(q(\theta), \theta) < 0$. The lowest type $K(\overline{\theta}) = \min K(\theta) = \underline{k}$. $[1 - F(\theta)]^{n-1} = [\Pr(\Theta > \theta)]^{n-1} = [\Pr(K(\Theta) < K(\theta))]^{n-1} = [F_K(k)]^{n-1}$. Let $k = K(\theta)$, $dk = K'(\theta)d\theta = C_\theta(q(\theta), \theta)d\theta$,

$$\begin{aligned} s(K(\theta)) &= K(\theta) - \frac{\int_{\underline{k}}^{K(\theta)} [F_K(t)]^{n-1} dt}{[F_K(k)]^{n-1}} \\ &= K(\theta) - \frac{\int_{\theta}^{\overline{\theta}} [1 - F(\tau)]^{n-1} K'(\tau) d\tau}{[1 - F(\theta)]^{n-1}} \\ &= K(\theta) - \frac{\int_{\theta}^{\overline{\theta}} [1 - F(\tau)]^{n-1} C_\theta(q(\tau), \tau) d\tau}{[1 - F(\theta)]^{n-1}}. \end{aligned}$$

Hence, $p(\theta) = V(q(\theta)) - s(K(\theta)) = C(q(\theta), \theta) + \int_{\theta}^{\overline{\theta}} C_\theta(q(\tau), \tau) \frac{[1-F(\tau)]^{n-1}}{} d\tau / [1 - F(\theta)]^{n-1}$.

Proof of Corollary 1: $k_{(1:n-1)}$ has distribution function $F_K^{(1:n-1)}(t) = [F_K(t)]^{n-1}$, and density

$f_K^{(1:n-1)}(t) = (n-1)[F_K(t)]^{n-2} f_K(t)$. If the winner has pseudotype k, the conditional expectation of the highest rival's pseudotype is

$$E[k_{(1:n-1)}|k_{(1:n-1)} < k] = \frac{\int_{\underline{k}}^{k} t(n-1)[F_K(t)]^{n-2} f_K(t)dt}{[F_K(k)]^{n-1}} = s(k),$$

which is equal to the score the winner will bid. At the equilibrium, the winner has pseudotype being the highest order statistic $k_{(1:n)}$, while the second highest bidder has pseudotype $k_{(2:n)}$, hence $E\left[s(k_{(1:n)})\right] = E\left[k_{(2:n)}\right]$. *Q.E.D.*

Proof of Corollary 2: Asker and Cantillon (2008) shows a straightforward proof. Suppose the minimum quality standard is set at q and the scoring rule represents the buyer's true preference. By Corollary 1, Expected utility of price-only auction is

$$\begin{aligned} V(q) - E\left[C(q, \theta_{(n-1:n)})\right] &= E\left[(V(q) - C(q,\theta))_{(2:n)}\right] \\ &\leq E\left[\max_q (V(q) - C(q,\theta))_{(2:n)}\right] = E\left[k_{(2:n)}\right], \end{aligned}$$

which is the expected utility in scoring auction. *Q.E.D.*

Proof of Theorem 2: (1) Quality

Suppose the corrupted firm with type θ bids (p', q') at some $q' \neq q_m$, we can show that by choosing q_m, the corrupted firm can always find a price p_m that yields a higher payoff upon winning. Let $p_m = V(q_m) - V(q') + p'$, then (p', q') and (p_m, q_m) have the same score because $S(q', p') = V(q') - p' = V(q_m) - p_m = S(q_m, p_m) = s$. These two bids has the same expected payoff $\Pr(\text{win}|s)$. Their expected payoffs satisfies

$$\begin{aligned} \pi(p_m, q_m) - \pi(p', q') &= \left[p_m - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\ &= \left[V(q_m) - V(q') + p' - C(q_m - m, \theta) - p' + C(q' - m, \theta)\right] \Pr(\text{win}|s) \\ &= \left[V(q_m) + C(q_m - m, \theta) - \left(V(q') - C(q' - m, \theta)\right)\right] \Pr(\text{win}|s) > 0, \end{aligned}$$

because q_m is chosen by (8). The scoring rule being quasilinearity (additively separable) is essential for this result to hold.

(2) Score and price

Under assumption UA, all other firms pick their score according to (7), so the corrupted firm's pick its core according to

$$\max_{s_m} (k_m - s_m) \Pr(\text{win}|s_m) = (k_m - s_m)\left[F_K(s^{-1}(s_m))\right]^{n-1}.$$

Following the same step in getting (7), the corrupted firm choose its score according to $s(k_m) = k_m - \int_{\underline{k}}^{k_m} [F_K(t)]^{n-1} dt / [F_K(k_m)]^{n-1}$. The corresponding price is $p_m(\theta) = V(q_m(\theta)) - s(K_m(\theta)) =$

$C(q_m(\theta) - m, \theta) + \int_{\underline{k}}^{K_m(\theta)} [F_K(t)]^{n-1} dt / [F_K(K_m(\theta))]^{n-1}$.

(3) For any $m > 0$, at the equilibrium, $q_m(\theta) > q(\theta)$, $K_m(\theta) > K(\theta)$, and $s(K_m(\theta)) > s(K(\theta))$.

The unique solution of quality choice of (2) and (8) are both determined by their first-order conditions. Suppose $\tilde{q}$ solves $V_q(q) = C_q(q, \theta)$. Because $C_{qq} > 0$, the cost function has increasing slope, $V_q(\tilde{q}) = C_q(\tilde{q}, \theta) > C_q(\tilde{q} - m, \theta)$. By assumption QL, $V_{qq} \leq 0$, the solution to $V_q(q) = C_q(q - m, \theta)$ must be strictly larger than $\tilde{q}$, therefore $q_m(\theta) > q(\theta)$.

The other two are straight-forward. Because $C_q > 0$, $C(q - m, \theta) < C(q, \theta)$ for all q and θ, $K_m(\theta) = \max_q V(q) - C(q - m, \theta) > \max_q V(q) - C(q, \theta) = K(\theta)$. The equilibrium score bidding function $s(\cdot)$ is increasing, hence $s(K_m(\theta)) > s(K(\theta))$. It is obvious that all three effects magnify as m increases. *Q.E.D.*

Derivation of $F_K(\cdot)$ in the Monte Carlo Example

θ_0, θ_1 are jointly uniformly distributed with density equals 1 at the area $[0, 1] \times [1, 2]$.

$$F_K(k) = \Pr(K(\theta_0, \theta_1) < k) = \Pr(\theta_1 - \theta_0 < k) = \Pr(\theta_1 < k + \theta_0).$$

With help of Figure 13, when $k \in [0, 1]$,

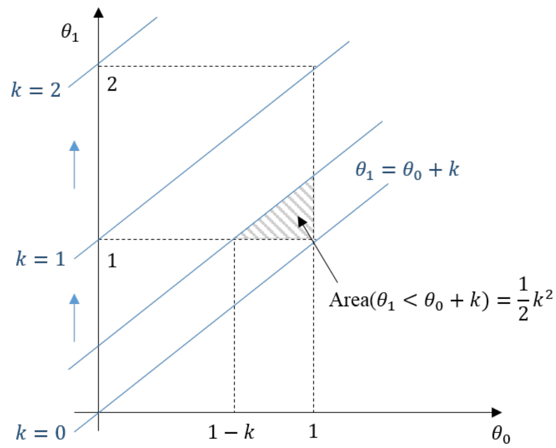
$$\begin{aligned} F_K(k) &= \int_{1-k}^{1} \int_{1}^{\theta_0 + k} 1 d\theta_1 d\theta_0 = \int_{1-k}^{1} (\theta_0 + k - 1)\, d\theta_0 \\ &= \left[\frac{1}{2}\theta_0^2 + (k-1)\theta_0\right]_{1-k}^{1} = \frac{1}{2} + (k-1) - \frac{1}{2}(1-k)^2 - (k-1)(1-k) = \frac{k^2}{2}. \end{aligned}$$

When $k \in (1, 2]$,

$$\begin{aligned} F_K(k) &= 1 - \int_{0}^{2-k} \int_{\theta_0 + k}^{2} 1 d\theta_1 d\theta_0 = \int_{0}^{2-k} (2 - \theta_0 - k)\, d\theta_0 \\ &= \left[(2-k)\theta_0 - \frac{1}{2}\theta_0^2\right]_{0}^{2-k} = 1 - (2-k)^2 + \frac{1}{2}(2-k)^2 = 1 - \frac{(2-k)^2}{2}. \end{aligned}$$

We therefore get $F_K(\cdot)$ in (14).

Figure 13: Derivation of $F_K(\cdot)$



A sample bid[19]

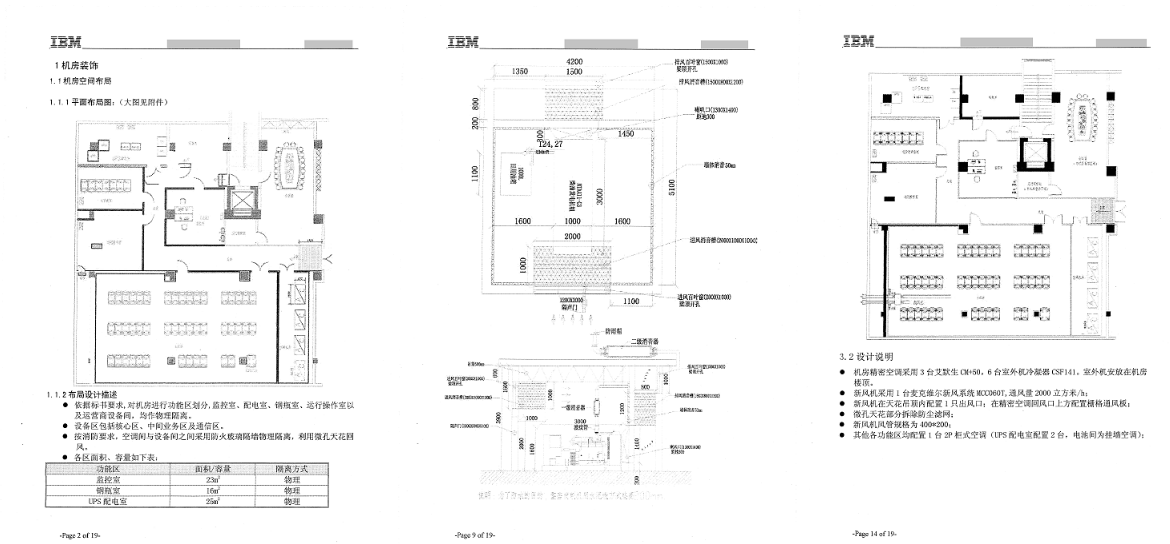
This is a bid of a server room construction project. The buyer is Bank of Dongguan, a regional bank centered at Dongguan, Guangdong province, China. The firm is IBM Engineering Technology (Shanghai) Co., Ltd.. The bid consists of a construction proposal and a detail list of items and their costs. The construction proposal is a 19-page document including standard of construction, condition of delivery, delivery date, equipment purchase plan, payment plan etc. Some selected pages are shown in Figure 14. The itemized price list is a 11-page spreadsheet. Table 11 shows its major categories, categorical prices, and total price (3,630,000 CNY).

Table 11: Summary of the Itemized Price List

Category	Price (CNY)	No. of Items
Data center room renovation	924,295	17
Main power distribution system	108,185	11
Auxiliary power distribution system	176,830	14
Uninterrupted power supply (UPS) system	913,680	13
Generators and environmental engineering	413,050	14
Air conditioning	99,170	11
Precision air conditioning	528,570	2
Cabinets and cabling system	242,230	9
Lightning protection	23,820	3
Room monitoring	185,120	43
Room bridging	15,050	4
Total	3,630,000	141

[19]The author receives authorization to disclose the document for non-profit academic research purpose. The original document is in Chinese. All technical details are remain confidential and the relevant copyrights are owned by Bank of Dongguan and IBM Engineering Technology (Shanghai) Co., Ltd. The author declare that he has no relevant or material financial interests that relate to the research described in this paper.

Figure 14: Selected Pages of the Construction Proposal



References

Arnold, B. C., N. Balakrishnan, and H. N. Nagaraja (1992). *A first course in order statistics*, Volume 54. Siam.

Aryal, G. and M. F. Gabrielli (2013). Testing for collusion in asymmetric first-price auctions. *International Journal of Industrial Organization 31*(1), 26–35.

Asker, J. (2010). A study of the internal organization of a bidding cartel. *The American Economic Review 100*(3), 724–762.

Asker, J. and E. Cantillon (2008). Properties of scoring auctions. *The RAND Journal of Economics 39*(1), 69–85.

Asker, J. and E. Cantillon (2010). Procurement when price and quality matter. *The Rand journal of economics 41*(1), 1–34.

Athey, S. and P. A. Haile (2002). Identification of standard auction models. *Econometrica 70*(6), 2107–2140.

Athey, S., J. Levin, and E. Seira (2011). Comparing open and sealed bid auctions: Evidence from timber auctions*. *The Quarterly Journal of Economics 126*(1), 207–257.

Bajari, P., S. Houghton, and S. Tadelis (2014). Bidding for incomplete contracts: an empirical analysis of adaptation costs. *The American Economic Review 104*(4), 1288–1319.

Bajari, P. and S. Tadelis (2001). Incentives versus transaction costs: A theory of procurement contracts. *RAND Journal of Economics*, 387–407.

Bajari, P. and L. Ye (2003). Deciding between competition and collusion. *Review of Economics and Statistics 85*(4), 971–989.

Bajari, P. and J. Yeo (2009). Auction design and tacit collusion in fcc spectrum auctions. *Information Economics and Policy 21*(2), 90–100.

Baldwin, L. H., R. C. Marshall, and J.-F. Richard (1997). Bidder collusion at forest service timber sales. *Journal of Political Economy 105*(4), 657–699.

Berge, C. (1963). Topological spaces (oliver and boyd, edinburgh and london). *1st English edition*.

Branco, F. (1997). The design of multidimensional auctions. *The RAND Journal of Economics*, 63–81.

Burguet, R. and Y.-K. Che (2004). Competitive procurement with corruption. *RAND Journal of Economics*, 50–68.

Burguet, R. and M. K. Perry (2009). Preferred suppliers in auction markets. *The RAND Journal of Economics 40*(2), 283–295.

Celentani, M. and J.-J. Ganuza (2002). Corruption and competition in procurement. *European Economic Review 46*(7), 1273–1303.

Che, Y.-K. (1993). Design competition through multidimensional auctions. *The RAND Journal of Economics*, 668–680.

Chen-Ritzo, C.-H., T. P. Harrison, A. M. Kwasnica, and D. J. Thomas (2005). Better, faster, cheaper: An experimental analysis of a multiattribute reverse auction mechanism with restricted information feedback. *Management Science 51*(12), 1753–1762.

Compte, O., A. Lambert-Mogiliansky, and T. Verdier (2005). Corruption and competition in procurement auctions. *Rand Journal of Economics*, 1–15.

Dastidar, K. G. (2014). Scoring auctions with non-quasilinear scoring rules.

David, E., R. Azoulay-Schwartz, and S. Kraus (2006). Bidding in sealed-bid and english multi-attribute auctions. *Decision Support Systems 42*(2), 527–556.

Dimitri, N., G. Piga, and G. Spagnolo (2006). *Handbook of procurement*. Cambridge University Press.

Dini, F., R. Pacini, and T. Valletti (2006). Scoring rules. *Handbook of procurement*, 293–321.

Feinstein, J. S., M. K. Block, and F. C. Nold (1985). Asymmetric information and collusive behavior in auction markets. *The American Economic Review*, 441–460.

Freixas, X., R. Guesnerie, and J. Tirole (1985). Planning under incomplete information and the ratchet effect. *The review of economic studies*, 173–191.

Graham, D. A., R. C. Marshall, and J.-F. Richard (1990). Differential payments within a bidder coalition and the shapley value. *The American Economic Review*, 493–510.

Guerre, E., I. Perrigne, and Q. Vuong (2000). Optimal nonparametric estimation of first-price auctions. *Econometrica 68*(3), 525–574.

Hanazono, M., J. Nakabayashi, and M. Tsuruoka (2015). Procurement auctions with general price-quality evaluation.

Harrington, J. E. (2008). Detecting cartels. *Handbook of antitrust economics 213*, 245.

Hendricks, K., R. Porter, and G. Tan (2008). Bidding rings and the winner's curse. *The RAND Journal of Economics 39*(4), 1018–1041.

Huang, Y. and J. Xia (2015). Procurement auction with corruption by quality manipulation. *working paper*.

Ingraham, A. T. (2005). A test for collusion between a bidder and an auctioneer in sealed-bid auctions. *Contributions in Economic Analysis & Policy 4*(1).

Ji, B. and Z. Fu (2010). Analysis of chinese bridge collapse accident cause in recent years. *China Civil Engineering Journal 43*, 495–498.

Kawai, K. and J. Nakabayashi (2014). Detecting large-scale collusion in procurement auctions. *Available at SSRN 2467175*.

Klemperer, P. (2002). How (not) to run auctions: The european 3g telecom auctions. *European Economic Review 46*(4), 829–845.

Koning, P. and A. Van de Meerendonk (2014). The impact of scoring weights on price and quality outcomes: An application to the procurement of welfare-to-work contracts. *European Economic Review 71*, 1–14.

Krasnokutskaya, E., K. Song, and X. Tang (2011). Multi-attribute auctions with unobserved heterogeneity in supplier qualities and buyer tastes.

Lengwiler, Y. and E. Wolfstetter (2006). Corruption in procurement auctions. *Available at SSRN 874705*.

Lewis, G. and P. Bajari (2011). Procurement contracting with time incentives: Theory and evidence. *The Quarterly Journal of Economics 126*(3), 1173–1211.

Liu, N. and Y. Luo (2014). A nonparametric test of exogenous participation in first-price auctions. Technical report.

Lucking-Reiley, D. (1999). Using field experiments to test equivalence between auction formats: Magic on the internet. *American Economic Review*, 1063–1080.

Maskin, E. S. and J. G. Riley (1985). Auction theory with private values. *The American Economic Review*, 150–155.

McAfee, R. P. and J. McMillan (1992). Bidding rings. *The American Economic Review*, 579–599.

Nakabayashi, J. and Y. Hirose (2015). Structural estimation of the scoring auction model. *Unpublished paper. ISER, Osaka University, Japan.*

Nishimura, T. (2015). Optimal design of scoring auctions with multidimensional quality. *Review of Economic Design 19*(2), 117–143.

Pesendorfer, M. (2000). A study of collusion in first-price auctions. *The Review of Economic Studies 67*(3), 381–411.

Porter, R. H. and J. D. Zona (1993). Detection of bid rigging in procurement auctions. *Journal of political economy*, 518–538.

Porter, R. H. and J. D. Zona (1999). Ohio school milk markets: An analysis of bidding. *The Rand Journal of Economics*, 263–288.

Tadelis, S. (2012). Public procurement design: Lessons from the private sector. *International Journal of Industrial Organization 30*(3), 297–302.

Topkis, D. M. (1978). Minimizing a submodular function on a lattice. *Operations research 26*(2), 305–321.

Tran, A. (2009). Can procurement auctions reduce corruption? evidence from the internal records of a bribe-paying firm. Technical report, working paper.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance 16*(1), 8–37.

Wang, M. and S. Liu (2014). Equilibrium bids in practical multi-attribute auctions. *Economics Letters 123*(3), 352–355.