Credit Supply and Asset Market Volatility Under Value-at-Risk-Based Capital Requirements*

(Job Market Paper)

Alex Hubbard†

University of Washington

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Abstract

Following the 2008 financial crisis, concerns arose about the possible procyclical effects of risk-sensitive capital requirements that rely on Value-at-Risk (VaR) for determining the market risk capital charge. To analyze this issue, I modify the monopolistically competitive banking sector developed by Gerali et al. (2010) to include a tractable method for banks to adjust their trading book position while also accounting for changes in VaR-based capital requirements and marked-to-market balance sheets in a fully dynamic general equilibrium model. The model is calibrated to U.S. data and estimated with Bayesian techniques to pin down the dynamics. The results suggest that VaR-based capital requirements creates a link between financial asset markets and credit markets when financial institutions manage a portfolio of trading securities. This type of regulation can create spillover effects from financial asset price and volatility shocks into credit markets as banks adjust their balance sheets to comply with capital standards.

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†Correspondence: Department of Economics, University of Washington, Savery Hall, Box 353330, Seattle, WA 98195, U.S.A.; e-mail: ajhubb@uw.edu; URL: https://sites.google.com/site/hubbardalex/.
1 Introduction

The 2008 financial crisis has provided a boon for new research topics in macroeconomics, but more importantly, significant empirical evidence against the Modigliani-Miller (1958) theorem. This theorem suggests that banks’ capital structure is irrelevant to the real economy if financial markets are perfectly competitive; however, evidence in favor of imperfect competition in the banking sector contradicts this theorem implying that financial factors, such as leverage, are important for the analysis of business cycle dynamics.

A major contributing factor to the financial crisis was the inability to correctly price the risk of asset-backed securities (ABS) (Wickens (2011), Gennaioli, Shleifer, and Vishny (2013)). ABS’s were designed to create a category of safe assets by pooling loans and taking advantage of diversification benefits. Because many ABS’s were considered nearly riskless and investors are highly interested in riskless debt (Bernanke et al. (2011)), ABS’s were often used by banks as collateral to raise short term debt via repurchase agreements (repos), a contract in which the borrowing party sells a security for cash with the promise to buy it back in the future at an agreed upon price. When more loans defaulted than was expected, a large adverse shock to the price of ABS’s occurred along with uncertainty about the value of the collateral underlying many of these assets. This made it difficult for banks to refinance short term debt when repo markets froze as a result (Archarya and Mora (2011), Gorton and Metrick (2012)). The crisis did not stop there, however. Troubles in the ABS markets spilled over in to other markets, including equities markets, causing asset prices to fall and volatility to spike. Ultimately, banks lost a significant amount of equity capital, leaving many highly under capitalized and on the brink of failure.

During the aftermath of the financial crisis, bank capital regulations came under scrutiny. Risk-sensitive capital regulations as suggested by the Basel Committee on Banking Supervision and implemented by the Federal Reserve in the United States require banks to hold capital against three types of risk: credit risk, market risk, and operational risk. Credit risk applies to default risk on loans, market risk applies to price and interest rate volatility associated with trading securities, and operational risk applies to losses that may be sustained from failed internal processes. The intention of capital requirements is to ensure banks’ ability to honor debt repayment in the face of operating losses and continue to conduct business into the foreseeable future. However, because capital requirements are risk-sensitive, concerns arose about their potential procyclical, amplifying effects, notably with market risk capital requirements.\footnote{See “Fundamental Review of the Trading Book” by the Bank for International Settlements (2012).} As part of market risk capital regulations, banks are required to hold capital against the Value-at-Risk (VaR) of their portfolio of trading securities, which may consist of those ABS’s that became so problematic during the crisis, or equity securities among others. VaR is a statistical measure aimed at quantifying the largest loss a trader can expect with a given level of confidence. It was created as a response to the 1987 market crash and has been a risk management standard in the financial.
industry since the 1990's before being adopted as regulation in 1996. It is typically calculated using a minimum of one year of historical data with a 10-day holding period. But because VaR is calculated with such an arguably short time interval, it is essentially a local measure of risk. This opens the door for perceived risk to vary with trading activity over the business cycle. When asset prices are expected to decline and markets become more volatile, which typically coincide at the onset of a downturn, banks’ financial assets appear riskier. VaR will increase and banks will be required to hold more capital, which could lead to or amplify a credit contraction as banks adjust their balance sheets to comply with the higher capital requirements. Another concern about risk-sensitive capital requirements as it applies to market risk is that all banks under this type of regulation may act uniformly to market conditions causing asset prices to be more volatile.

While each of the three types of risk is important for financial operations, this paper will focus solely on market risk because of the concerns noted by the Basel Committee. If those concerns are legitimate, then high volatility episodes that occurred after risk-sensitive capital regulations were instituted, like those that occurred during the Enron scandal plus dot com bubble burst in 2001 and the 2008 financial crisis, could have contributed negatively to credit availability. Thus, this paper is motivated to answer the question of why changes in financial market conditions, namely prices and volatility, can affect credit market conditions even though the two markets may seem disconnected. To do so, I modify the monopolistically competitive banking sector developed by Gerali, Neri, Sessa, and Signoretti (2010) and propose VaR-based capital regulations and marked-to-market balance sheets to be the link that propagates financial market shocks to credit markets and the real economy. The model is calibrated to U.S. data over the period 1997:Q2-2007:Q4 and includes a number of shocks to facilitate Bayesian estimation of the model to pin down the dynamics.

The major contribution of this paper identifies Value-at-Risk of the trading portfolio as part of bank balance sheets to be a supply side factor in credit market dynamics when used as a risk management technique or as part of risk-sensitive capital regulations. The main result from the impulse response analysis suggests that asset price and volatility shocks can be transmitted through the financial sector to the real economy as they interact with banks’ VaR measure. The model is also able to replicate four empirical correlations between financial factors and macroeconomic activity. It captures the correlations of financial institution leverage with asset market volatility, leverage with loans, trading portfolio size with volatility, and leverage with aggregate investment. Most importantly, the model is able to capture the procyclicality of financial institution leverage with respect to asset market volatility and aggregate investment.

The rest of this paper is organized as follows: section 2 reviews some empirical facts about financial business cycles, section 3 reviews other related literature on the financial accelerator and relevant macroeconomic models with a financial sector. Sections 4 and 5 presents the model, section 6 discusses the calibration and Bayesian estimation of the model, and section 7 presents the results from impulse responses to an asset market price
2 Empirical Relevance of Market Volatility in Business Cycles

In this section, I document some empirical evidence regarding the cyclical nature of the financial sector, asset market volatility, and their connection to the real economy for which the model constructed in this paper will help explain. Adrian and Shin (2010) and Adrian, Moench, and Shin (2010) note some empirical facts about financial sector leverage adjustments and their role in affecting macroeconomic dynamics. They show that there is a strong positive relationship between changes in asset prices, leverage, and balance sheet size. When balance sheets are continuously marked-to-market, changes in asset prices have an immediate impact on bank capital. Financial intermediaries are not passive, and adjust balance sheets in such a way that leverage, defined as the ratio of total assets over equity, is generally procyclical. The evidence suggests that U.S. chartered financial institutions target a fixed leverage ratio implying active balance sheet management; however, security broker dealers – financial intermediaries including investment banks that operate primarily in capital markets – display even stronger balance sheet management through procyclical leverage choices. Adrian and Shin (2013) suggest that these financial institutions manage leverage in the short-term to maintain a constant VaR-equity ratio, even as market conditions deteriorate, by financing changes in asset growth with changes in debt.

Leverage is inversely related to total assets. When asset prices rise, net worth increases, and leverage falls. For leverage to remain constant or increase, banks must expand their assets through security purchases or loan creation to use up excess capital. Financing these moves is often done through short term debt issuance or repurchase agreements (repos) which increases leverage. When this process works in reverse and an excess supply of securities is not met with sufficient demand, it puts downward pressure on prices adding feedback to the leverage process by weakening balance sheets (Adrian and Shin 2010). As financial intermediary balance sheets become weaker, macroeconomic activity diminishes (Adrian, Moench, and Shin (2010)). In fact, regulatory risk models actually dictate active management of their Value-at-Risk (VaR) through balance sheet adjustments. Adrian and Shin (2010) use a panel regression to show that lagged VaR is negatively related to leverage. They find that when banks face the possibility of losing more at the same confidence level, banks adjust their balance sheet to reduce leverage.

These observations can be seen in figures 2.1 and 2.2 which span the time period 1990:Q1-2014:Q4. Figure 2.1 displays the “volatility paradox” coined by Brunnermeir and Sannikov (2014) in which low levels of risk tends to lead to higher leverage. Looking at two different types of financial institutions, U.S. chartered financial institutions and security broker dealers, the former has maintained a fairly stable leverage ratio that has been declining since 1990. Security broker dealers, however, have had a pattern of increasing
leverage in low volatility periods measured by the Chicago Board of Exchange Volatility Index (VIX). This happened in the U.S. in the 1990’s and early 2000’s. There are two noted exceptions to this: 1997-98 and 2008-09. 1997-98 corresponds to the Asian and Russian financial crises and 2008-09 corresponds to the most recent financial crisis. One aspect common to both periods was an increased level of market volatility and perceived risk. When financial institutions calculate their VaR, they are required to do so using a minimum of the past 250 trading days of price movements. When this local measure of volatility is low, perceived risk measured by VaR tends to be low. The correlation coefficient between leverage for security broker dealers and the VIX is -0.33 for this period, suggesting a negative relationship between leverage and volatility.\(^2\)

One asset class subject to market volatility that affects financial institutions’ VaR is corporate equities. While equities were not at the cause of the recent financial crisis, these markets were affected after ABS markets froze. Figure 2.2 shows security broker dealers’ corporate equity holdings normalized by the S&P 500 price began increasing around 1999 when volatility was relatively high but continued to increase in the early 2000’s as market

\(^2\)The correlation coefficient is calculated using HP-filtered data except for the VIX which is demeaned. The Pearson’s product-moment correlation test suggests that the correlation is significant at the 99% level.
volatility dropped. U.S. chartered financial institutions also exhibit this pattern but not nearly to the same degree. Security broker dealers slightly decreased their holdings of these securities around the time of the 1997-98 crises. However, both types of financial institutions decreased their equity holdings much more when volatility and risk spiked in 2008-09 during the financial crisis. Because VaR-based risk-sensitive capital requirements increase with risk, banks are required to hold more capital during high volatility episodes. To meet the higher capital requirements, banks can raise more capital or lower their VaR by selling risky securities. The correlation coefficient over the period 1990:Q1-2014:Q4 between corporate equity holdings and the VIX for U.S. chartered financial institutions and security broker dealers is -0.56 and -0.24 respectively. This suggests that financial institutions subject to risk-based capital requirements buy assets when their perceived risk is low and sell the same assets when their perceived risk is high. During the upward phase of the leverage cycle, security broker dealers increased their holdings of corporate equities, but did the opposite during the deleveraging phase as they removed risk from their balance sheets. The correlation coefficient between security broker dealer leverage and corporate
Two other critical observations about financial business cycles were made by Jordà, Schularick, and Taylor (2013). They study the role of credit in business cycles and find two key facts by exploiting a panel data set over 14 countries and 140 years. One is that financial crisis recessions tend to be more costly than other types of recessions. And two, more credit-intensive expansions tend to be followed by much deeper recessions and more prolonged recoveries. Highly leveraged expansion periods appear to be associated with slower credit, investment, and output growth following the bust as households, firms, and financial institutions are forced to deleverage. These periods are also accompanied with deflationary pressures making it more costly for debtors to repay as the real value of debt rises: i.e. Fisherian debt-deflation mechanism. Adrian, Colla, and Shin (2012) discuss the importance of the supply side of credit to firms and argue that a supply shock to credit intermediation was much more important during the 2008 financial crisis than a demand shock. Their findings indicate that credit supply shocks are a key driver of financial cycles. As securitization has become more prominent within the financial sector, security broker dealers have begun to play a more central role in credit intermediation as they are often the market-making institutions for such products. In the sense that leverage indicates willingness to take on risk, security broker dealer leverage has become a good indicator for credit availability and macroeconomic activity. Thus, financial factors are hugely important for explaining the modern business cycle, so models of the macroeconomy need to be supplemented with them.

Figure 2.3 displays the connection between financial intermediary leverage and real activity: i.e. credit and investment. Commercial and industrial (C&I) loans, investment, and security broker dealer leverage all exhibit procyclicality. However, market volatility is countercyclical as the correlation coefficient between the VIX and investment is -0.38. When market volatility and perceived risk was low in the mid-1990’s, and mid-2000’s security broker dealer leverage was increasing along with C&I loans and investment as the two have a 0.14 correlation coefficient. However, when market volatility increased around 1997-98 and 2008, security broker dealer leverage fell. Investment and C&I loans did not begin declining until leverage bottomed out in 2001 but declined almost simultaneously with leverage around 2008. Because volatility affects banks' VaR, capital requirements, and leverage, volatility may also impact credit availability and investment since security broker dealer leverage appears to be a good predictor of macroeconomic activity. The correlation coefficient between security broker dealer leverage and C&I loans and security

\(^3\)All correlation coefficients are calculated using HP-filtered data except for the VIX which is demeaned. The Pearson’s product-moment correlation test suggests that both correlations are significant at the 95% level.
broker dealer leverage and investment is 0.18 and 0.68 respectively. The model I develop in this paper is able to replicate these observations about financial business cycles. Namely, higher market volatility and perceived risk is correlated with deleveraging, a reduction in risky security holdings as dictated by VaR-based capital regulations, and a reduction in credit availability and investment.

3 Other Related Literature

The importance of the balance sheet strength in propagating shocks has been known for quite some time. The seminal paper by Bernanke and Gertler (1989) was one of the first to understand how borrower balance sheets affect output dynamics, and was later implemented in a business cycle model in Bernanke, Gertler, and Gilchrist (1999), which became known as the financial accelerator (Bernanke, Gertler, Gilchrist (1996)). The idea of the financial accelerator is that changes in the market for credit amplify and increase

\footnote{All correlation coefficients are calculated using HP-filtered data except for the VIX which is demeaned. The Pearson’s product-moment correlation test suggests that all correlations are significant at the 99% level except for the correlations between C&I loans and investment and between security broker dealer leverage and C&I loans, which are not statistically significant at 95% confidence level.}
propagate shocks to real economy through endogenous developments in the external finance premium: i.e. the difference between the cost of external funding and the opportunity cost of using internal funds for financing investment decisions. The external finance premium has an inverse relationship with borrowers net worth. When net worth is high, borrowers become closer to fully collateralizing external funds implying less risk for the lender. When net worth is low, borrowers cannot fully collateralize external funds, implying more risk for the lender and a larger premium. Because borrower net worth is procyclical, the external finance premium exacerbates shocks to borrower net worth affecting investment, consumption, and output.

This mechanism was studied further by Kiyotaki and Moore (1997) who use limited-liability collateral constraints to generate a transmission mechanism based on the relationship between credit limits and asset prices. With this, they show that persistent shocks can amplify and spill over to the rest of the economy. Lenders cannot force borrowers to repay unless loans are secured with durable assets which take on a dual role. Durable assets, such as capital, are factors in production and also serve as collateral for loans. When shocks hit borrowing constrained firms, they have to cut their demand for capital as their net worth falls, causing a fall in capital prices and further tightening borrowing constraints. This mechanism builds on the Bernanke Gertler external finance premium and shows that shocks to net worth act through changes in the value of borrowers assets in a forward looking manner.

The introduction of the Basel II risk-sensitive capital adequacy framework introduced concerns that the financial sector could provide substantial financial accelerator type effects on the real economy, some of which are studied by Darraqu Parie és, Sørensen, and Rodriguez-Palenzuela (2011). They use the banking sector in a DSGE model developed by Gerali et al. 2010 (discussed in further detail below) to show that these regulations are a cause for concern. Using the credit risk requirements of Basel II and a quadratic adjustment cost applied to the risk-weighted capital-asset ratio, they show that risk-sensitive capital requirements imply a higher volatility to output growth and inflation. While increasing credit risk played a major factor in loan contraction during the 2008-09 crisis, especially with regards to mortgage finance, securities markets experienced extreme volatility and asset price declines as well. Since banks hold securities for profit and balance sheet management reasons, and because many loans were held as asset-backed securities rather than on the loan book, their analysis needs to be supplemented with market risk considerations. This is one area I contribute to the literature.

One paper that attempts to tackle Value-at-Risk in a general equilibrium model is Danielsson, Shin, and Zigrand (2004) who develop a framework where only traders exist and are subject to a VaR constraint for portfolio decisions. They investigate the model’s outcome when traders treat market risk as exogenous: i.e. they do not take into account the effects of market participant behavior on asset prices. Traders in the model use VaR to restrict their portfolio choice and forecast risk in a backward-looking manner as is standard under regulatory risk practices that require VaR to be calculated using historical
data. They show that in their standard asset pricing framework, when the VaR constraint holds, it behaves like an increase in risk aversion. The degree to which the constraint binds is determined by market outcomes, effectively making risk aversion time-varying. Using a VaR constraint gives the model the ability to determine the steady state portfolio size when agents maximize a risk-neutral objective function, which would not otherwise be possible. Their main results are that prices are lower on average with the VaR constraint but are also more volatile. What is left out of this analysis is the role that bank balance sheets play in a macroeconomic setting when subject to a VaR constraint. This is another area my model makes a contribution.

The model I use to analyze the procyclical effects of Value-at-Risk to financial shocks essentially builds on the work of Gerali et al. (2010). They build a financial sector into a DSGE model and include a number of different channels for shocks to propagate through including the interest rate channel, a nominal debt channel, a collateral channel, and an asset price channel, all of which have been shown to amplify technology shocks compared to frictionless financial models. The financial sector developed in their paper is monopolistically competitive, which allows banks to set interest rates that adjust sluggishly to changes in the central bank policy rate. Because banks are price setters due to credit market power, the resulting loan rate markup amplifies changes in monetary policy for borrowers. However, the resulting deposit rate markdown dampens changes in the policy rate for depositors. The presence of credit market power and interest rate frictions alters the pass-through of policy rate changes, dampening the effect compared to the cases where interest rates are fully flexible and a model with perfectly competitive banks. They also introduce capital requirements in such a way that changes in leverage will either amplify or dampen changes in monetary policy. If leverage increases with the policy rate, then the transmission of shocks will be amplified and vice versa. Overall, they find that due to the presence of interest rate frictions, credit market power dampens the effect of monetary policy and technology shocks to real variables. Their model does not qualitatively change the response of the main macroeconomic variables of interest compared to more standard New Keynesian models. Therefore, this makes for a good model in which to embed risk-based capital regulations.

Their model has also become fairly influential in the macro-financial literature and has proven to be quite flexible. It has been used in or influenced models to study macroprudential policies, monetary policy transmission, capital requirements, real exchange rate dynamics, financial integration, maturity transformation, financial frictions, yield curve dynamics, loan defaults, sovereign debt defaults, liquidity constraints, and financial intermediation in small open economies.

Gerali et al. (2010) note that their model omits some elements of the 2008 financial crisis, including the increase in risk and freeze up of financial asset markets. This is the omission I tackle in this paper. I contribute to the financial accelerator and macro-financial literature by analyzing the procyclical effects of VaR-based capital requirements in response to asset market price and volatility shocks from the supply side of credit. While the
potential procyclical concerns of VaR-based capital requirements have been noted in other studies, to the best of my knowledge, this is the first paper to illustrate this concern within a fully dynamic general equilibrium model. I implement risk-weighted capital requirements in the way Darracq Pariès, Sørensen, and Rodriguez-Palenzuela (2011) do within the Gerali et al. (2010) framework. However, as they study the effects of credit risk capital regulations on business cycle fluctuations, I study the effects of market risk capital regulations. I also make use of a VaR constraint similar to the one used in Adrian and Shin (2011) and Danielsson, Shin, and Zigrand (2004) financial trader models that dictate how traders manage their portfolio decision in response to asset price changes. The combination of the two allows me to effectively analyze the effect of asset price and volatility shocks on financial business cycles in a realistic way.

4 The Supply and Demand Sectors of the Model Economy

The model economy is a fairly standard New Keynesian model augmented with a financial sector. It consists of two agents who differ only in their degrees of patience: households and entrepreneurs. The assumption that the discount factor for households (\( \beta_H \)) is higher than that for entrepreneurs (\( \beta_E \)) ensures that households are more patient than entrepreneurs and will choose to save while entrepreneurs will choose to borrow.

Households consume, supply differentiated labor, set their wage, and save using a portfolio of bank deposits and equity securities. Entrepreneurs consume and produce a homogeneous intermediate good using household labor and capital. Capital is purchased from perfectly competitive capital goods producers and financed with collateralized bank loans. The intermediate good is then sold to monopolistically retailers who costlessly differentiate it, set the retail price, and sell it to households as the final consumption good. Capital goods producers are included to derive a market price of capital.

The main financial assets, one period deposits and loans, are supplied by monopolistically competitive banks.\(^5\) This allows banks to set interest rates in order to maximize profits. Heterogeneity in the rate of time preference between households and entrepreneurs will ensure the flow of funds through the financial sector from depositor households to borrower entrepreneurs. Households face no financial constraints but when financing capital purchases, entrepreneurs are constrained by the future value of undepreciated capital according to a Kiyotaki and Moore (1997) collateral constraint. This assumption is meant to be consistent with some empirical evidence that suggests that firm balance sheet conditions are important for investment decisions and credit availability.\(^6\)

\(^5\)I do not model an endogenous motive for the existence of banks. However, the model does assume that there is some form of market imperfection, such as asymmetric information or monitoring costs, that prevents households from directly lending to entrepreneurs. Banks solve this issue by specializing in credit monitoring and pooling funds that reduces the cost of supplying credit. The existence of banks in this model is based on these assumptions.

\(^6\)See Kiyotaki and Moore (1997) for further discussion.
Apart from deposits and loans, households and banks participate in a secondary market for risky corporate equities, which is the new asset introduced into the model. Households purchase a portfolio of equities to help smooth intertemporal consumption by equating the rates of return on deposits, which return interest payments, and equities, which return dividend payments out of retailer profits. On the other side of this market, banks purchase a portfolio of equities to maximize profits subject to a Value-at-Risk constraint that explicitly takes into account price volatility and limits the amount of risk they can hold on their balance sheet.

The economy is also subject to number of nominal frictions that are used to generate persistence observed in the data. Nominal rigidities are an important foundation for New Keynesian models as it allows for the transmission of monetary policy shocks and is a tractable way to improve the model performance relative to the data. Retailers and households are responsible for setting the consumption good price and the nominal wage subject to quadratic adjustment costs. Since retail interest rates are essentially another nominal goods price, banks are also subject to a quadratic adjustment cost on interest rate setting. Some authors have shown wage rigidities to be more important than price rigidities. Christiano, Eichenbaum, and Evans (2005) find wage rigidities to be crucial to the model’s performance, whereas price rigidities play a much smaller role. Price frictions alone cannot generate enough persistence in output unless price contracts are assumed to be extremely long. However, their model with only wage frictions does not have this problem. It is also an important feature of Gerali et al. (2010) where the estimated wage adjustment cost parameter is about three-and-half times larger than the price adjustment cost parameter. It is also important to note that the use of Rotemberg-type quadratic adjustment costs to achieve nominal rigidities is not microfounded and is somewhat of an ad hoc assumption. However, it is no more ad hoc than the Calvo-type price adjustment mechanism, and there is a relationship between the two. I also do not impose nominal rigidities a priori but will estimate all adjustment cost parameters to match the persistence that best fits the data.

In the following sections, I describe the set up of the entrepreneurs, households, and financial sector. I leave the rest of the model, which is more or less standard, to be described in detail in appendix A.

4.1 Entrepreneurs

I first start with the description of supply side of economy as it will inform the setup of the household problem. There is a continuum of measure one of entrepreneurs indexed by $i$ that maximize utility by choosing the final goods consumption bundle $c_{E}^{F}(i)$, loans $b_{t}(i)$ costing the interest rate $r_{t}^{b}$, labor input $l_{t}(i)$ costing the real wage rate $w_{t}$, and capital input $k_{t}(i)$ costing the price $q_{t}^{k}$. Utility from consumption is assumed to depend on deviations from lagged external group specific habits in consumption, where $h$ represents the
degree of habit formation.\(^7\) Labor and capital are combined to produce a homogeneous intermediate good, \(y_t\), using a Cobb-Douglas technology production function where total factor productivity \((A^E_t)\) is assumed to be an exogenously given stochastic process.

Entrepreneurs are financially constrained and must borrow from banks in order to finance capital purchases. They can only borrow a fraction (the loan-to-value ratio, \(M^E_t\)) of the expected future value of their undepreciated capital stock when loans come due in the next period. \(M^E_t\) will also be modeled as an exogenously given stochastic process. It is assumed that entrepreneurs always repay in full so the model remains in a neighborhood of the steady state and the collateral constraint always binds.\(^8\) Capital is assumed to depreciate at rate \(\delta\) and entrepreneurs resell undepreciated capital to capital goods producers at the end of each period.

The entrepreneur problem is then to maximize utility:\(^9\)

\[
\max_{\{c^E_t(i), b_t(i), k_t(i), l_t(i)\}} E_0 \sum_{t=0}^\infty \beta^E_t \left[ \left(1 - h\right)^\gamma \frac{c^E_t(i) - h c^E_{t-1}(i)^{1-\gamma}}{1 - \gamma} \right]
\]

Subject to the budget constraint, collateral constraint\(^10\), and production function:

\[
c^E_t(i) + w_t l_t(i) + \frac{(1 + r^b_t) b_{t-1}(i)}{\pi_t} + q^k_t k_t(i) \leq \frac{y_t(i)}{x_t} + b_t(i) + q^k_t (1 - \delta) k_{t-1}(i) \tag{4.1}
\]

\[
(1 + r^b_t) b_t(i) \leq M^E_t E_t \left[ q^k_{t+1} \pi_{t+1} k_t(i)(1 - \delta) \right] \tag{4.2}
\]

\[
y_t(i) = A^E_t k_{t-1}(i)^\alpha l_t(i)^{1-\alpha} \tag{4.3}
\]

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\(^7\)Habit formation introduces non-separability of preferences over time. An increase in current consumption lowers the marginal utility of consumption and increases it in the next period, implying households would prefer to consume more tomorrow when consumption increases today. In the business cycle literature, habit formation is used to capture the hump-shaped response of consumption and will tend to smooth consumption. Multiplying by \((1 - h)^\gamma\) offsets the impact of habit formation on the steady state marginal utility of consumption.

\(^8\)To assume the collateral constraint always binds near the steady state, the size of shocks must be “sufficiently small”.

\(^9\)The specification of CRRA utility implies that consumption increases in the same proportion as income and that the intertemporal elasticity of substitution will be equal to the inverse of the relative risk aversion parameter \(\gamma\). Thus, the risk aversion and intertemporal elasticity of substitution will be constant with respect to the level of wealth and consumption. The intertemporal elasticity of substitution is defined as \(-U'(c_t)/(c_tU''(c_t)) = 1/RRA = 1/\gamma\).

\(^10\)The implicit assumption behind the collateral constraint is banks cannot force entrepreneurs to foreclose on output, so debt must be secured for repossession in the case of default. If entrepreneurs choose to default, banks know they may be able to renegotiate debt down to the value of capital, so banks formulated the collateral constraint as an expected break-even condition.
where \( \pi_t = P_t / P_{t-1} \) is the gross inflation rate and \( x_t = P_t / P_t^W \) is the markup of the retail price over the wholesale price.\(^{11}\)

This setup includes three channels for shocks to propagate through. The first, since debt is defined in nominal terms, is the nominal debt channel. When interest and debt payments are denominated in nominal terms, changes in inflation effectively redistribute wealth between borrowers and lenders. The incorporation of the collateral constraint links entrepreneur balance sheets to credit and makes capital play a dual role: it is a factor in production and collateral for loans. The collateral constraint also introduces two other transmission channels. The second channel is the collateral channel whereby changes in the interest rate affect the shadow value of borrowing. The third is an asset price effect in which changes in the capital price affects the value of collateral entrepreneurs can borrow against. When credit limits and asset prices interact, shocks can become persistent, amplify, and spillover to other sectors as a result of the collateral constraint.

Once the intermediate output is produced, entrepreneurs sell it to a measure one of monopolistically competitive retailers indexed by \( j \) at the wholesale price \( P_t^W \). Retailers then costlessly differentiate it and sell it to households at the retail price \( P_t \). Retail profits, written without subscripts \( j \), will be:

\[
\Pi^R_t = y_t \left[ 1 - \frac{1}{x_t} - \frac{\kappa_p}{2} (\pi_t - \pi_{t-1} \pi^{1-\epsilon})^2 \right] \tag{4.4}
\]

where \( y_t \) is the amount of the intermediate good a retailer buys from entrepreneurs and \( x_t \) is the markup a retailer applies to the final good. \( (\kappa_p/2)(\pi_t - \pi_{t-1} \pi^{1-\epsilon})^2 \) is the adjustment cost on price setting.

### 4.2 Households

There is a continuum of measure one of households indexed by \( i \) that maximize utility by choosing deviations of their final goods consumption bundle \( c^H_t(i) \) from lagged external group habits, deposits \( d_t(i) \) that pay the interest rate \( r_t \), and equity shares \( s^H_{j,t}(i) \) at price \( q^e_{j,t}(i) \). Each retailer \( j \) pays an exogenous fraction \( \delta^e_j \) of retail profits \( \Pi^R_{j,t} \) as dividends to holders of its equity shares \( s_{j,t} \). I will assume that it is costly for households to manage an equity portfolio of size \( s^H_t(i) = \int_j s^H_{j,t}(i) dj \) in that a fee, \( F s^H_t(i) \), is paid to the financial sector as revenue for providing this service. This fee will prove to be important once I discuss the financial sector in section 5. Its inclusion will allow for the arbitrage conditions for households and banks to hold simultaneously. I will assume that retailers will not be issuing new equity shares or buying back any existing shares, so all transactions happen in the secondary market.

\(^{11}\)Debt is denominated in nominal terms to ensure the inclusion of the gross inflation variable, \( \pi_t \). If instead debt were denominated in real terms, critical nominal price and wage frictions would not play a role in the model.
Households earn income by providing differentiated labor services, \( l_t(i) \), that pays the individual real wage rate \( w_t(i) \). Each households \( i \) also owns one retailer \( j \) so that profits net of dividend payments are paid to households as a lump sum. The household problem is then to:

\[
\max_{\{c_t^H(i), d_t(i), s_t^H(i)\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ c_t^e(1-h)\gamma \frac{(c_t^H(i) - hc_t^{H-1})^{1-\gamma}}{1-\gamma} - \frac{l_t(i)^{1+\phi}}{1+\phi} \right]
\]

Subject to the budget constraint:

\[
c_t^H(i) + d_t(i) + \int \frac{q_{j,t}^e s_{j,t}^H(i)}{\pi_t} dj \leq \frac{w_t l_t(i)}{\pi_t} + \frac{(1+r_{t-1}^d)l_{t-1}(i)}{\pi_t} + \int \left( q_{j,t}^e(i) + \frac{\delta_j^e \Pi_t^{R,j,t-1}}{\pi_t} - F \right) s_{j,t-1}^H dj + (1-\delta_j^e)\Pi_t^{R,j,t}
\]

where \( c_t^e \) represents a consumption demand shock. Households are assumed to be passive in equity purchases in that financial wealth does not directly enter the utility function. Debt is again denominated in nominal terms for depositors so the nominal debt channel also affects households.

To illustrate why the portfolio management fee is important when I examine the bank’s problem, we will need the first order conditions for deposits and equity shares from households’ first order conditions:

\[
[s_t^H]: q_t^e = \beta_E t \left( q_{t+1}^e + \frac{\delta_E \Pi_t^{R}}{\pi_{t+1}} - F \right) + \epsilon_t^q \tag{4.6}
\]

\[
[d_t]: \lambda_t^H = \beta_E t \left[ \frac{\lambda_{t+1}^H}{\pi_{t+1}} \left( 1 + r_{t+1}^d \right) \right]
\]

written without subscripts \( i \) or \( j \). \( \lambda_t^H \) is the Lagrange multiplier on the budget constraint and \( \epsilon_t^q \) is a shock to the equity price. In steady state, these arbitrage conditions require that the rates of return from the two assets be equalized. Since banks will also purchase a portfolio of equity securities, the portfolio management fee will be needed to equalize the steady state rates of return for banks and ensure that the Value-at-Risk constraint binds at the steady state.

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12 The specification of the disutility of labor implies that \( \phi \) is the inverse of the Frisch elasticity of labor supply and that it is constant with respect to the real wage rate holding the marginal utility of wealth constant. The Frisch elasticity of labor is defined as \( U''(l_t)/(l_t U''(l_t)) = 1/\phi \) and measures the substitution effect of a change in the wage rate on the labor supply.
If we rewrite the first order condition for household equity holdings we get:

\[
q_t^e = \sum_{k=0}^{\infty} E_t \left[ \beta_H^{k+1} \frac{\lambda_{t+1+k}}{\lambda_t} \left( \frac{\delta^{\prime} \Pi_{t+k}^R}{\pi_{t+1+k}} - F \right) \right] + \lim_{T \to \infty} \beta_H^T E_t \left[ \frac{\lambda_{t+1+T}}{\lambda_t} q_{t+T}^e \right] \sum_{k=0}^{\infty} E_t \left[ \beta_H^{k} \frac{\lambda_{t+k}}{\lambda_t} \epsilon_{t+k}^e \right]
\]

The first term on the right hand side represents the “fundamental value” equal to the sum of all expected future dividend payments net of the portfolio management fee. The second term is the traditional transversality condition that must go to zero as \( T \to \infty \). The third term on the right hand side is an addition to the transversality condition representing a “transitory” shock to the equity price. Thus, the equity price shock can be thought of as a shock that temporarily sets the price above or below its “fundamental value” similar to a pure expectations or “bubble” shock.

5 The Financial Sector of the Model Economy

The financial sector is modeled after Gerali et al. (2010) which offers a way to study credit intermediation from the supply side. Banks will play a key role in the model economy as they act as an intermediary for all financial transactions and are assumed to be monopolistically competitive. This allows banks to set and adjust interest rate spreads over the course of the business cycle. Theoretical reasons for the existence of market power in the banking industry range from the presence of switching costs, asymmetric information, menu costs for opening accounts, regulatory restrictions, market contestability, and customer relationships. The degree to which these affect market power does influence interest rate spreads.\(^{13}\)

The modeled financial sector consists of a measure one of banks with three branches: two retail branches and a wholesale branch. The retail branches include a deposit branch and a loan branch. The financial sector sector will be responsible for transforming household deposits into loans to entrepreneurs and it will also help households manage a portfolio of equities. The branch of the bank that helps manage the household equity portfolio will receive the fee, \( F_{t+1}^H(i) \), but will incur a cost exactly equal to this so that it does not affect bank profits or their decision problem. This setup allows for an interbank market to be modeled, a market that is very important in the credit intermediation process. Shocks in one part of the financial sector can often affect other parts of it through interbank lending and changing interest rate spreads, affecting borrowing costs and available credit.

The deposit branch will be responsible for collecting debt from households by setting the deposit rate and channeling funds to the wholesale branch through the first part of

the interbank market as wholesale deposits. The wholesale branch then pipes wholesale deposits to the loan branch as wholesale loans through the second part of the interbank market. Finally, the loan branch takes wholesale loans and provides credit to entrepreneurs by setting the loan rate. By funneling wholesale funds through the interbank market, the wholesale branch is in charge of managing the capital (equity) position of the entire bank. It chooses how much debt to take on, how much credit to make available, and how much to invest in a portfolio of risky equities subject to a balance sheet constraint and regulatory risk-weighted capital requirements. Risk-weighted capital requirements are instituted via quadratic adjustment costs on the risk-weighted capital-asset ratio, where the target ratio is a regulator determined value. The trading desk of the wholesale branch is then subject to a Value-at-Risk (VaR) constraint designed to limit the amount of risk it can hold on its balance sheet and be consistent with these capital requirements.

As mentioned earlier, quadratic adjustment costs on interest rate setting in the loan and deposit branch optimization will be used to capture persistence in interest rate markups (or markdowns) from some base interest rate observed in the data, which tends to be the federal funds rate in the U.S. Theoretical and empirical reasons for interest rate frictions include switching costs, menu costs, maintaining customer relations, adverse selection, uncertainty about future monetary policy actions, and monopolistic competition. Although, the practice of indexing deposit and loan rates to some current market rate may make interest rate setting more flexible.\textsuperscript{14}

A quadratic adjustment cost on banks’ risk-weighted capital-asset ratio is also used in the wholesale branch optimization to be consistent with capital regulations and capture another empirical observation that commercial banks set a target leverage ratio but also allows for leverage to be procyclical as is observed of security broker dealers (Adrian and Shin (2010)). This mechanism also captures the behavior that financial institutions tend to target a fixed VaR-equity ratio by financing asset growth with debt (Adrian and Shin (2013)). In addition, others have found that firms that target a leverage ratio and the speed at which they converge to it depends on a number of factors including whether they are under- or over-leveraged, cash flows, access to capital markets, size, and the institutional environment in which they operate in. Firms that are larger, more financially constrained, under-leveraged, well-capitalized, under heavy-regulations, or operating in less developed institutions are all found to adjust leverage more slowly than their counterparts.\textsuperscript{15}

The addition of risky equity shares as part of the bank’s balance sheet in conjunction with risk-weighted capital requirements is the main contribution I make to the literature.

\textsuperscript{14}For further discussion on interest rate setting frictions see Berger and Hannan (1991), Berger and Udel (1992), Calem, Gordy, and Mester (2006), de Bondt, Mojon, Valla (2005), Driscoll and Judson (2013), Kok Sørensen and Werner (2006), Gropp, Kok Sørensen, and Lichtenberger (2007), Nakajima and Teranishi (2009), Adrian and Shin (2013).

\textsuperscript{15}For further discussion on leverage frictions see Warr, Elliot, Koeter, and Öztekin (2012), Faulkender, Flannery, Hankins, Smith (2012), and Flannery and Öztekin (2012), and Berger, DeYoung, Flannery, Lee, and Öztekin (2008).
Adding this asset allows me to study how asset price and volatility shocks transmit from the financial sector to the real economy. This transmission mechanism requires both marked-to-market balance sheets and a VaR constraint that is consistent with risk-weighted capital requirements. The transmission mechanism connects financial assets to the real economy because equity price and volatility shocks affect regulatory bank capital. Ensuing balance sheet adjustments alters the flow of funds through the interbank market by affecting the interbank rate spread and thereby changing the amount of debt banks are willing to take on or the amount of credit to make available.

5.1 Retail Branches

I will start the description of the financial sector with a brief overview of the deposit and loan branches, which are identical to those used by Gerali et. al (2010). The description of these two branches will help inform the derivation of the Value-at-Risk constraint and the wholesale branch problem.

5.1.1 Deposit Branch

There is a measure one of monopolistically competitive deposit branches indexed by $i$ that collect deposits from households $d_t(i)$ paying the deposit rate $r^d_t(i)$ and remits them to the wholesale branch sector as wholesale deposits $D_t(i)$ at the central bank policy rate $r^{cb}_t$. It is implicitly assumed that banks can borrow directly from the central bank at this rate and is used to close the model. Deposit branches maximize period profits subject to adjustment costs on interest rate setting ($\kappa_d$) and a Dixit-Stiglitz type CES demand curve for deposits. The interest-rate elasticity for deposits, $\kappa^d_t$, is assumed to be time-varying and modeled as an exogenous stochastic process. Since deposit branches only have one source of revenue, wholesale deposits must be equal to household deposits: $D_t(i) = d_t(i)$.

The maximization program for the deposit branch can be written as:

$$\max_{\{r^d_t(i)\}} E_0 \sum_{t=0}^{\infty} \lambda^H_t \beta^H_t \left[ r^{cb}_t D_t(i) - r^d_t(i) d_t(i) - \frac{\kappa_d}{2} \left( \frac{r^d_t(i)}{r^d_{t-1}(i)} - 1 \right)^2 r^d_t(i) d_t(i) \right]$$

subject to the deposits demand curve:

$$d_t(i) = \left( \frac{r^d_t(i)}{r^d_t} \right)^{-\kappa^d_t} d_t$$

Under completely flexible rates, the deposit rate is determined by:

$$r^d_t = \frac{\kappa^d_t}{\epsilon^d_t - 1} r^{cb}_t$$

---

$^{16}$The use of $\lambda^H_t \beta^H_t$ as the discount factor for banks connects the bank’s rate of time preference to the household’s through the Euler equation: $1 = \beta H E_t \left[ (\lambda^H_{t+1}/\lambda^H_t)((1 + r^d_t)/\pi_{t+1}) \right]$. It essentially sets the intertemporal discount factor for banks to be the deposit rate.
where $\epsilon_t^d / (\epsilon_t^d - 1)$ is the markdown of the deposit rate compared to the central bank policy rate.

### 5.1.2 Loan Branch

There is a measure one of monopolistically competitive loan branches indexed by $i$ that takes wholesale funding $B_t(i)$ from the wholesale sector at the interbank rate $r_t^{ib}$ and issues loans $b_t(i)$ to entrepreneurs at the loan rate $r_t^b(i)$ to maximize period profits subject to adjustment costs on interest rate setting ($\kappa_b$) and a Dixit-Stiglitz type CES demand curve for loans. The interest-rate elasticity for loans, $\epsilon_t^b$, is also assumed to be time-varying and modeled as an exogenous stochastic process. Since loan branches only have one source of revenue they will issue all its wholesale borrowing as loans so that $B_t(i) = b_t(i)$.

This maximization program for the loan branch can be written as:

$$\max_{\{r_t^b(i)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_t^H \left[ r_t^b(i)b_t(i) - r_t^{ib}B_t(i) - \frac{\kappa_b}{2} \left( \frac{r_t^b(i)}{r_{t-1}^b(i)} - 1 \right) ^2 r_t^b(i)b_t \right]$$

subject to the loan demand curve:

$$b_t(i) = \left( \frac{r_t^b(i)}{r_t^{ib}} \right)^{-\epsilon_t^b} b_t$$

Under completely flexible rates, the loan rate is determined by:

$$r_t^b = \frac{\epsilon_t^b}{\epsilon_t^b - 1} r_t^{ib}$$

where $\epsilon_t^b / (\epsilon_t^b - 1)$ is the markup of the loan rate compared to the interbank rate.

At this point it is important to note that the use of the CES demand curve for deposits and loans may be an unrealistic assumption. Since the U.S. financial sector contains a few very large banks among other smaller more regional banks, an oligopoly structure may be more realistic. However, the monopolistic competition framework provides a convenient way to capture the existence of market power in the financial sector and generate non-zero steady state interest rate spreads.

### 5.2 Background on Value-at-Risk and Risk-Weighted Capital Regulations

Two new features will be added to the wholesale branch compared to the one used by Gerali et al (2010). The first is the inclusion of risk-weighted assets, and the second is the trading desk that is subject a Value-at-Risk constraint. However, before the wholesale branch problem can be fully defined, some background on Value-at-Risk and risk-weighted capital regulations is necessary.
5.2.1 Value-at-Risk

Value-at-Risk originated as a concept during a research effort within J.P. Morgan in the late 1980’s. It was spearheaded by chairman Dennis Weatherstone and research chief Till Guldimann to quantify and manage potential risks as a response to the 1987 stock market crash. The concept became part of Weatherstone’s “4:15 report”, which gave him an estimated measure of risks comparable to profit and loss aggregated from all trading desks just 15 minutes after the market closed. VaR provided Weatherstone with information on trading activities he had not known previously and used it to make judgments about how to adjust the firm’s future trading positions. This research group was later spun off and became known as RiskMetrics™. It began providing consultative services on the VaR method, eventually leading to its adoption into financial risk management in the 1990’s and by the Basel Committee on Banking Supervision in 1996 as suggested regulation. VaR then gained widespread acceptance after the collapse of Long Term Capital Management (LTCM), a hedge fund that employed Nobel Prize winners Myron Scholes and Robert Merton, in 1998 after the Asian and Russian financial crises of 1997-98. LTCM employed VaR as a risk management technique, so its failure naturally sparked many narratives as to why. However, more emphasis was put into LTCM’s misuse of VaR rather than to any of VaR’s shortcomings at the time.\(^{17}\)

At the most basic level, VaR is a statistical concept. The idea behind it is to give a statistical measure about how much one can expect to lose from market fluctuations with a certain degree of confidence over a given time period. Expected losses give an idea about how much capital the portfolio manager must hold to cover losses from market volatility so the organization can remain a going concern into the foreseeable future. Mathematically, Value-at-Risk for a given confidence level \(\alpha\) is defined as:

\[
VaR_\alpha = \{v : Pr(x \leq v) = 1 - \alpha\}
\]

for \(0 \leq \alpha \leq 1\). So, if the distribution of price changes is continuous and denoted \(f(x)\), then \(VaR_\alpha\) is the quantile that solves:

\[
1 - \alpha = \int_{-\infty}^{VaR_\alpha} f(x)dx
\]

When \(f(x)\) is assumed to be normal, \(VaR_\alpha\) can be written as:

\[
VaR_\alpha = \mu - Z_\alpha \sigma
\]

where \(\mu\) is the expected value, \(\sigma\) is the standard deviation, and \(Z_\alpha\) is a constant that depends on the confidence level \(\alpha\). At a 99% confidence level, \(VaR_{99\%}\) says that there is only a 1% chance that losses will be larger than \(\mu - Z_\alpha \sigma\) over the specified time period.

\(^{17}\text{See The Black Swan: The Impact of the Highly Improbable by Nassim Nicholas Taleb.}\)
VaR is really better thought of as possible losses from “normal” market conditions and has been known to have a fairly poor performance during times of financial stress. Thus, it is important to distinguish between “normal” and “crisis” periods and remember that VaR is still a very limited risk measure. It is especially limited when thought of in terms of a normal distribution when fluctuations in stock returns have been proven to be non-normal. Price fluctuations tend to appear normally distributed in times of calm markets but become more non-normally distributed depending on the severity of market stress. VaR also has nothing to say about risks in the “tail” of the distribution: i.e. how bad things can get if losses exceed the VaR level. Consequently, it can be a dangerous risk measure if misinterpreted as “the worst possible loss” rather than as its strict definition.

Conditional Value-at-Risk (CVaR) or Expected Shortfall (ES) is an alternative measure that aims to capture tail risks, and its benefits over VaR have been well documented.\(^\text{18}\) Most notably, VaR has been proven not to be a coherent risk measure: i.e. there are cases when the sum of the VaR’s of two portfolios considered separately (the concept Weatherstone used to aggregate risks across trading desks) can be lower than the VaR of the combined portfolio violating the diversification principle that a well-diversified portfolio carries lower risk. CVaR, however, is coherent and does not suffer from this problem.

Financial regulators are currently contemplating moving market risk regulations from VaR to CVaR, but it looks as though they will adopt a 97.5% confidence level instead of the 99% confidence level used for VaR\(^\text{19}\). The difference between VaR and CVaR at these confidence levels will be negligible since I will assume equity price fluctuations to be normally distributed and will reduce the problem down to a representative equity price. These two measures of risk are essentially the same in this case and removes many of the benefits of CVaR over VaR. CVaR is likely to have different properties compared to VaR in more micro-level scenarios when multiple assets are included in the portfolio and asset price movements are not normally distributed. However, this paper focuses on the macroeconomic implications of risk-based capital requirements and not about the differences between these two risk measures. All that is needed to study this is a risk measure that depends on volatility. VaR accomplishes exactly this in very simple manner. Macroprudential policies of CVaR over VaR are left for further study.

One further note about risk management systems that rely on statistical measures of risk is that they treat volatility as exogenous. This neglects any behavior that market participants may have on market prices which could create extra feedback (Danielsson, Shin, and Zigrand (2004)). Relying on these types of risk practices will always be imperfect since they do not account for this pecuniary externality.

\(^{18}\)See Rockafellar and Uryasev (2000, 2002).

5.2.2 Risk-Weighted Capital Regulations

The Federal Reserve requires banks to calculate the denominator of their risk-based capital-asset ratio as the sum of its risk-adjusted assets, or risk-weighted assets for short. Assets are converted to risk-weighted assets by multiplying the measure of risk for each asset by the inverse of the target risk-weighted capital-asset ratio. The Federal Reserve now considers the minimum risk-weighted capital-asset ratio to be 8% but could also include an extra 2.5% capital conservation buffer so that the minimum is 10.5%.

Market risk is defined as losses to trading positions that could result from market movements that affect interest rates, credit spreads, equity prices, exchange rates, or commodity prices. A trading position is defined as “a position that is held by a bank for the purpose of short-term resale or with the intent of benefiting from actual or expected short-term price movements or to lock in arbitrage profits.” In what follows, I will focus solely on market risk applied to equity prices. The specific market risk capital rule states that “a bank’s VaR-based capital charge be equal to the greater of 1) the previous day’s VaR-based measure or 2) the average of the daily VaR-based measures for each of the preceding 60 days of multiplied by 3 or a higher factor based on the back-testing of the bank’s modeling of its VaR.” Banks are allowed to use internal models to calculate its VaR, which is done with a 10-day holding period and one-tail 99% confidence level estimated from a period of at least one year’s worth of historical data. The model I estimate here is done using data at a quarterly frequency so VaR will be calculated according to quarterly price fluctuations. Because VaR is calculated in this manner, it is essentially a local measure of risk and has the potential to have procyclical and amplifying effects on bank balance sheets and the real economy since asset markets are generally more stable in expansions but more volatile in downturns. Plus, internal VaR models have the potential to be very similar across banks with the standardization of the methodology, creating very little model risk diversification. This could cause banks to act similarly to changes in asset market volatility and amplify effects on the real economy.

The calculation of the risk-weighted capital-asset ratio in the model is as follows. Loans are assumed not to be risky in the model; however, banks will still be required to hold capital against them. The capital charge on loans at steady state will be the target risk-weighted capital-asset ratio multiplied by the size of the loan book:

\[ K^{b,B} = v_b B \]

The capital charge on the equity portfolio (trading book) at steady state is calculated using

\[ K^{b,B} = v_B B \]

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20See Federal Register Vol. 77 No. 169 Part V (2012): “Risk Based Capital Guidelines: Market Risk.” The total market risk capital charge is equal to the sum of the VaR-based capital charge, stressed VaR-based capital charge, specific risk capital charge, incremental risk capital requirement, comprehensive risk capital requirement, and capital charge for de minimis exposures. The only capital charged considered in this paper is the VaR-based capital charge.
the market risk capital rule described above:

\[ K^{b,pf} = M \cdot VaR \]

Total bank capital at steady state is then the sum of the two capital charges:

\[ K^b = K^{b,B} + K^{b,pf} = v_b B + M \cdot VaR \]

Risk-weighted assets at any time \( t \) must be:

\[ RW A_t = B_t + \frac{M}{v_b} \cdot VaR_t \tag{5.1} \]

to be consistent with the steady state risk-weighted capital-asset ratio being equal to the target:

\[ \frac{K^b_t}{RW A_t} = v_b \tag{5.2} \]

Bank leverage is related to the inverse of the risk-weighted capital-asset ratio and will be defined using Adrian and Shin’s definition as total assets-over-equity:

\[ L_t = \frac{B_t + q^e_t s^b_t}{K^b_t} = \frac{B_t + q^e_t s^b_t}{B_t + q^e_t s^b_t - D_t} \tag{5.3} \]

To see how leverage changes with asset prices, take the partial derivative of \( L_t \) with respect to \( q^e_t \):

\[ \frac{\partial L_t}{\partial q^e_t} = - \frac{D_t s^b_t}{(K^b_t)^2} < 0 \tag{5.4} \]

Thus, asset price increases will cause leverage to fall, ignoring any balance sheet adjustments banks may make. If bank leverage remains constant or increases in response to asset price increases, this is a signal of active balance sheet management observed of both U.S. chartered financial institutions and security broker dealers.

### 5.3 Wholesale Branch

The wholesale branch is perfectly competitive and manages the capital position of its combined bank: deposit branch \( i \), loan branch \( i \), and wholesale branch \( i \). It combines

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\( M \) can be thought as a safety factor. If the holding period is 10 days and the confidence level is 99%, then a loss greater than VaR will occur once every 4 years. Arguably, financial stress events would happen far too often under this scenario. Thus, multiplying by \( M \) should lessen the likelihood of such losses. \( M \) can also be thought of as increasing the time horizon for the holding period or the confidence level.
bank capital $K_t^b$ (or net worth) with wholesale deposits $D_t$, which it gets from the deposit branch sector, to make wholesale loans $B_t$, which it lends to the loan branch sector. It also purchases a portfolio of equity shares $s_{j,t}^b$. Wholesale deposits are charged at the central bank policy rate $r_t^c$, while interbank loans are charged at the interbank rate $r_t^b$. The central bank policy rate can be thought of as the Federal Reserve’s target federal funds rate, while the interbank rate can be thought of as the effective federal funds rate.

From the perspective of the wholesale branch, bank capital and wholesale deposits are perfect substitutes as sources of funds. Gerali et al. (2010) use a quadratic adjustment cost on the capital-asset ratio to pin down the steady state choice of bank capital. In their model, the capital-asset ratio consists of loans and bank capital only and is not risk-weighted since risk is absent from the model. I build on this concept by making capital requirements risk-weighted according to current regulatory capital requirements set forth by the Federal Reserve and Basel Committee on Banking Supervision in conjunction with a Value-at-Risk constraint. The target risk-weighted capital-asset ratio will be regulator determined and denoted $v_b$. The VaR constraint is used to pin down the size of banks’ trading books, but it also pins down the steady state level of bank capital adding an extra degree of freedom to the steady state calibration. The quadratic adjustment cost on the risk-weighted capital-asset ratio will link risk-weighted balance sheet conditions and the interbank market.

The addition of these two elements allow me to analyze the contribution of banks’ trading books on the dynamics of the model economy through the interaction with the risk-weighted capital-asset ratio. This mechanism also helps capture the trade-off involved with managing bank resources. If the capital-asset ratio is too high, banks may be able to earn higher profits by reducing it, or if the capital-asset ratio is too low, banks may face punishment from regulators.

The maximization program for the wholesale branch is to maximize period profits:

$$\max_{\{B_t, D_t, s_{j,t}^b\}} r_t^b B_t - r_t^b D_t + \int_j E_t \left[ q_{j,t+1}^e \pi_{t+1} - q_{j,t}^e + \delta_{j,t}^r \Pi_{j,t}^R \right] s_{j,t}^b \, dq_{j,t} - \frac{\kappa K_t^b}{2} \left( \frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b$$

(5.5)

subject to the balance sheet identity:

$$B_t + \int_j q_{j,t}^e s_{j,t}^b \, dq_{j,t} = D_t + K_t^b$$

(5.6)

where the index $j$ denotes the equity price and shares from retailer $j$. $RWA_t$ stands for risk-weighted assets and is defined as in the previous section.

The problem for this branch can be boiled down to how much to invest in riskless loans and a risky portfolio of equities given its deposits and capital level. The first step is to
substitute the balance sheet identity in for $D_t$ to get:

$$\max_{(B_t, s^b_{t,j})} \left( r_{t}^{ib} - r_{t}^{cb} \right) B_t + \int_J E_t \left[ q_{j,t+1}^{\pi} - q_{j,t}^{\pi} + \delta_j^{\text{R}} q_{j,t}^{\pi} - r_{t}^{cb} q_{j,t}^{\pi} \right] s^b_{j,t} dj$$

$$\downarrow r_{t}^{ib} \uparrow \kappa K_t^b \left( \frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b + \kappa K_t^b K_t^b$$

The wholesale branch will be assumed to make two decisions. The first is to choose the size of its loan book ($B_t$). The second is to choose the size of its trading book ($s^b_{t,j}$) given market prices subject to risk limits set by wholesale branch management. This means that only the trading desk, and hence the portfolio first order condition, will be subject to the VaR constraint.\textsuperscript{22}

### 5.3.1 Loan Book

The first order condition for wholesale loans is:

$$[B_t, D_t] : r_{t}^{ib} = r_{t}^{cb} - \kappa K_t^b \left( \frac{K_t^b}{RWA_t} - v_b \right) \left( \frac{K_t^b}{RWA_t} \right)^2 \quad (5.7)$$

Equation 5.7 determines the interbank rate spread over the central bank policy rate and is similar to Gerali et al. (2010) with the exception of risk-weighted assets. The appearance of risk-weighted assets in this condition captures the idea that financial institutions take into account borrower balance sheet conditions as well as its own balance sheet condition when lending in the interbank market. In steady state, the interbank rate spread over the central bank policy rate will be zero. If the risk-weighted capital-asset ratio falls below the target, the interbank rate spread, $r_{t}^{ib} - r_{t}^{cb}$, will increase as bank’s balance sheets appear riskier when they are under-capitalized. One way this happens is with an increase in asset market volatility that raises the portfolio variance. When asset market volatility increases, then before any other balance sheet adjustments occur, $RWA$ increase as VaR increases, lowering the risk-weighted capital-asset ratio below the target $v_b$.

Again, it is worth noting that the use of adjustment costs to derive the interbank rate condition is ad hoc. Although, it could be seen as a shorthand to a more formal microfounded approach. For example, Adrian and Shin (2013) set up a contracting problem between a bank and creditor to derive a rationale for the behavior of banks that can be described with VaR constraints. Their solution endogenously solves for bank leverage, asset size, and the repo interest rate while determining the conditions that give rise to VaR as a contracting outcome.

\textsuperscript{22}This assumption prevents the Lagrange multiplier from appearing in the first order condition for wholesale loans and deposits. This ensures that the steady state target rate equals the steady state interbank rate.
With the appropriate capital allocated for wholesale loans determined by equation 5.7, wholesale branch management then sets the rest of the capital aside for the trading desk to invest in a portfolio of risky equities subject to a Value-at-Risk constraint.

5.3.2 Value-at-Risk Constraint

In what follows is where the major contribution of this paper is made. I will derive a VaR constraint similar to those used by Adrian and Shin (2011), Adrian, Colla, and Shin (2012), and Danielsson, Shin, and Zigrand (2004) but embed it into a fully dynamic general equilibrium model that will dictate how banks manage their trading books in response to equity price and asset market volatility shocks.

Banks’ equity holdings are inherently risky due to price fluctuations from market activity. Because the equity price is determined in general equilibrium, the model implies a variance for the equity price, which can be derived from the first order Taylor approximation of the equity price condition 4.6 around the steady state:

\[
q^e_t - q^e = \frac{q^e}{\chi H} (\lambda t + 1 - \lambda H) - \frac{q^e}{\chi H} (\lambda t - \lambda H) + \beta H (q^e_{t+1} - q^e) + \beta H \delta H (\Pi^R - \Pi^R)
\]

\[
- \beta H \delta \Pi^R (\pi_{t+1} - \pi) + \epsilon^q_t
\]

Squaring both sides and taking the expectation produces the variance of \(q^e_t\) as a function of other variances and covariances:

\[
\sigma_{\epsilon, q}^2 \approx \left( \frac{(\beta H \delta H)^2}{1 - \beta_H^2} \right) \sigma_{\Pi}^2 + \left( \frac{(\beta H \delta H \Pi^R)^2}{1 - \beta_H^2} \right) \sigma_{\Pi}^2 + \sigma_{\epsilon}^2 + 2 \left( \frac{\beta_H^2 \delta H}{1 - \beta_H^2} \right) \cdot Cov(q^e, \Pi^R)
\]

\[
- 2 \left( \frac{\beta_H^2 \delta H \Pi^R}{1 - \beta_H^2} \right) \cdot Cov(q^e, \Pi^R) - 2 \left( \frac{(\beta H \delta H)^2 \Pi^R}{1 - \beta_H^2} \right) \cdot Cov(\Pi^R, \Pi)
\]

where \(\epsilon^t\) is assumed to be independent of all other variables, and variables without a subscript \(t\) represent steady state values. Covariances can be solved for in a similar fashion. Since the model is stationary, this variance (and all other covariances and variances) will be constant at any point in time unless there is a shock to \(\sigma_{\epsilon}^2\). Once the complete model is linearized, all endogenous variances and covariances can be solved for and \(\sigma_{\epsilon}^2\) can be determined. The equity price shock, \(\epsilon^t\), then creates excess volatility in the equity price beyond what the fundamental value is responsible for and can be broken down into two terms: the fundamental variance and the transitory variance represented by \(\sigma_{\epsilon}^2\). Therefore, banks have an endogenous motive to make equity purchases subject to a VaR constraint.

---

23 Instead of squaring both sides, multiply both sides by \((x_t - x)^2\) and then take expectations to get the covariance between any variable \(x_t\) and \(q^e_t\).

24 I do not endogenize the variance of the representative equity price since I am interested in shocking this variable and I do not believe much would be gained from doing so. However, if one were to do so, shocking \(\sigma_{\epsilon}^2\) could create extra feedback effects increasing \(\sigma_{\epsilon, q}^2\) more than the shock itself.
in accordance with regulatory risk practices and financial regulators also have motive to institute risk-weighted capital requirements to ensure bank soundness.

This idea can be illustrated in figure 5.1. If each period $t$ is one quarter of a year, then there are higher frequency price movements that happen in the secondary market between periods. The equity price $q^e_t$, containing both the fundamental and transitory components, can be thought of at the weekly frequency (solid black line), while the fundamental value can be thought of as the quarterly average (dashed blue line). Asset market volatility shocks then can be imagined to come from exogenous market activity not explained by the fundamental component of the equity price.

![Figure 5.1: S&P 500](image)

To see how the VaR constraint can be applied to the model at hand, first combine the profits from all three branches:

$$
\Pi^b_t = r^b_t b_t + \sum_j E_t \left[ q^e_{j,t} \pi_{t+1} - q^e_{j,t} + \delta^c_j \Pi^{R}_t \right] s^b_j d^b_j - r^d_t d^b_t - Adj^B_t
$$

where $Adj^B_t$ contains all bank adjustment cost terms. Bank capital will be assumed to be accumulated out of retained earnings according to:

$$
K^b_{t+1} = (1 - \delta^b_t) K^b_t + \Pi^b_t + \epsilon^b_t
$$

and is assumed to depreciate at a constant rate $\delta^b$ meant to capture the idea that it is costly to manage the capital position of the bank. Without this assumption, bank’s can accumulate an infinite amount of capital and become self financing.\(^{25}\) $\epsilon^b_t$ represents a

\(^{25}\)One other method that has been used in the literature to avoid this issue is to assume that banks are finitely lived and exit the market with some exogenously given probability. I choose to follow Gerali et al. (2010) and use the bank capital depreciation method for its simplicity and intuitive interpretation.
shock to the bank’s ability to retain earnings and can be thought of as unforeseen internal losses from operational risk if the shock is negative.

In order for banks to remain in operation, a bank’s net worth must remain positive:

$$K^b_{t+1} \geq 0 \implies \Pi^b_t \geq -(1 - \delta^b)K^b_t$$

Combine bank profits (5.8) with this last constraint and subtract expected losses from loans ($v_BB_t$) and from holding a portfolio of risky equities, $Z\sigma_{t,pf}$. Finally, trading profits and equities risk need to be multiplied by $M$ to arrive at the VaR constraint consistent with the risk-weighted capital-asset ratio:

$$\begin{align*}
\rho_{ji} & = \rho_{ji} \cdot \sigma_i \cdot \sigma_j \cdot s^b_{j,t} \cdot s^b_{i,t} \\
\sigma^2_{t,pf} & = \int \int \rho_{ji} \sigma_i \cdot \sigma_j \cdot s^b_{j,t} \cdot s^b_{i,t} \cdot \mathrm{d}j
\end{align*}$$

where $\rho_{ji}$ is the correlation between equities $i$ and $j$ and $\sigma_j$ is the standard deviation of equity price $j$. $\int \delta^R_j \Pi^R_j s^b_{j,t} \mathrm{d}j$ are the “riskless” profits from dividend payments and $M \cdot VaR_t$ is the expected loss on the portfolio consistent with the risk-weighted capital requirements in steady state. These regulations effectively reduce the amount of risk that financial institutions are allowed to take for any level of capital. Or equivalently, they require financial institutions to hold more capital for any level of risk.

For simplicity, equity price fluctuations are assumed to be normally distributed, so VaR and portfolio variance can be written as above. Under this specification, VaR is positive when trading losses are expected. $Z$ is a constant determined by the relevant confidence level $\alpha$ which is set to 99% according to regulatory risk practices. This implies that $Z = 2.3264$.

Combining bank profits (5.8), bank capital accumulation (5.9), and the VaR constraint and then evaluating the result at the steady state, reveals:

$$K^b \geq v_BB + M \cdot VaR$$

showing that bank capital in steady state must be at least as large as the total capital charge. The VaR constraint will be assumed to hold with equality in the region near the steady state so $K^b = v_BB + M \cdot VaR$ in steady state, which is consistent with the risk-weighted capital-asset ratio.
5.3.3 Trading Book

With the VaR constraint now derived, the trading desk optimization of the wholesale branch can be defined. The trading desk’s problem is to maximize period profits (5.5) subject to the VaR constraint and the balance sheet identity (5.6). Each equity share choice of the trading desk depends on the idiosyncratic characteristics of each equity security ($\delta^e_i$, $q^c_{jt}$, $\sigma_j$, $\rho_{ji}$). To simplify the analysis, I will assume a symmetric equilibrium for all sectors of the economy resulting in conditions:

$$
\delta^e_i = \delta^e_j = \delta^e \implies q^c_{it} = q^c_{jt} = q^c_t
$$

$$
\sigma_i = \sigma_j = \sigma, \rho_{ij} = \rho_{ik} = \rho \implies s^b_{it} = s^b_{jt} = s^b_t
$$

thus abstracting from portfolio shuffling. This assumption reduces the problem to a representative equity security corresponding to a market index (i.e. S&P 500) and allows wholesale bank profits and the VaR constraint to be written more succinctly as:

$$
\max_{\{B_t, s^b_{jt}\}} \left( r^{ib}_t - r^{cb}_t \right) B_t + E_t \left[ q^c_{t+1} \pi_{t+1} - q^c_t + \delta^e \Pi^R_t - r^{cb}_t q^c_t \right] s^b_t - \frac{K^b_t}{RWA_t} - v_t \right)^2 K^b_t + r^{cb}_t K^b_t
$$

$$
= r^{b}_t b_t - v_t B_t - M \cdot VaR_t - r^{d}_t d_t + \delta^e \Pi^R_t s^b_t - Adj^B_t \geq -(1 - \delta^b) K^b_t
$$

$$
VaR_t = E_t \left[ q^c_t - q^c_{t+1} \pi_{t+1} \right] s^b_t + Z\sigma_{t,pf}
$$

$$
\sigma_{t,pf} = \sigma_t s^b_t (1 + 2\rho)^{1/2}
$$

where $\rho$ will be set to 0 for convenience. The volatility term $\sigma_t$ that shows up in the VaR equation (5.11) represents the standard deviation of the representative equity price and will be assumed to change only if there is a shock to transitory volatility. Assuming a symmetric equilibrium in the wholesale market of the financial sector implies that I am essentially modeling an exaggerated case in which there is zero diversification of internal VaR-models across banks. The consequence of this is that all banks will react identically to changes in asset market volatility, which is one of the concerns about VaR-based capital regulations, and is not that unrealistic of an assumption.

The first order condition for the bank’s trading book size is:

$$
[s^b_t] : E_t \left[ q^c_{t+1} \pi_{t+1} - q^c_t + \delta^e \Pi^R_t \right] - r^{cb}_t q^c_t - \frac{\partial Adj^B_t}{\partial s^b_t} + \lambda_t \left( M \cdot E_t \left[ q^c_t - q^c_{t+1} \pi_{t+1} \right] + Z\sigma_t + \delta^e \Pi^R_t - \frac{\partial Adj^B_t}{\partial s^b_t} \right) = 0
$$

29
\[
\frac{\partial \text{Adj} K_b}{\partial s_t} = -\kappa K_b \left( \frac{K_b}{RW A_t} - v_b \right) \left( \frac{K_b}{RW A_t} \right)^2 \frac{M}{v_b} \left( E_t \left[ q_t - q_{t+1} \pi_{t+1} \right] + Z \sigma_t \right)
\]

where \( \lambda_t^b \) is the Lagrange multiplier on the VaR constraint. This shows that the first order condition for the bank’s portfolio size does not determine the size of the trading book in steady state. Instead, the trading book size is determined by the VaR constraint, which will be assumed to hold in the neighborhood of the steady state.

Rearranging this first order condition for \( \lambda_t^b \) yields:

\[
\lambda_t^b = \frac{E_t \left[ q_t^e \pi_{t+1} - q_t^e + \delta \Pi_t^R \right] - r_t^b q_t^e - \frac{\partial \text{Adj} K_b}{\partial s_t}}{M \left( E_t \left[ q_t - q_{t+1} \pi_{t+1} \right] + Z \sigma_t \right) - \delta \Pi_t^R + \frac{\partial \text{Adj} K_b}{\partial s_t}} \quad (5.14)
\]

\( \lambda_t^b \) is essentially the marginal change of profit with respect to bank capital and will always be positive given the assumption that the VaR constraint will hold in the neighborhood of the steady state. To see how profit changes with volatility, take the partial derivative of \( \lambda_t^b \) with respect to \( \sigma_t \) around the steady state:

\[
\frac{\partial \lambda_t^b}{\partial \sigma_t} \approx \frac{(r_t^b q_t^e - M Z \sigma) \frac{\partial^2 \text{Adj} K_b}{\partial s_t^b \partial \sigma_t} - M Z q_t^e (r_t^b - r_t^{cb})}{(M Z \sigma - \delta \Pi_t^R)^2} < 0
\]

\[
\frac{\partial^2 \text{Adj} K_b}{\partial s_t^b \partial \sigma_t} \approx \kappa K_b \left( \frac{K_b^3}{RW A_t^4} \right) \left( \frac{M Z}{v_b} \right) \sigma > 0
\]

where \( M Z \sigma > r_t^{cb} q_t^e \) and \( r_t^b > r_t^{cb} \) under the steady state conditions which will be discussed in the following section. So, when asset market volatility increases, higher capital is required under risk-weighted capital regulations, and \( \lambda_t^b \) declines reflecting the loss in profit. Bank’s will need to adjust their balance sheets to meet capital requirements and could sell risky securities, decrease lending, or both. With lower market volatility, less bank capital is required under risk-weighted capital regulations and bank’s will search for more profitable, and possibly risky, uses of its excess capital.

5.3.4 Binding VaR Constraint

Because the first order condition for the portfolio size does not determine the size of the trading book in steady state, the VaR constraint will. This means that the VaR constraint needs to bind in steady state. To ensure this, first note that the expected profits from investing in a portfolio of equities is \( E_t \left[ q_t^e \pi_{t+1} - q_t^e + \delta \Pi_t^R - q_t^e r_t^{cb} \right] s_t^b \). The wholesale branch will only have incentive to invest in a portfolio of equities in steady state if \( \delta \Pi_t^R \geq q_t^e r_t^{cb} \).
Now, examining the household’s first order conditions (4.6 and 4.7). In steady state, these equations reduce to:

\[
[d]: 1 = \beta_H(1 + r^d) \\
[s^H]: q^e = \frac{\beta_H}{1 - \beta_H}(\delta^e \Pi^R - F) = \frac{\delta^e \Pi^R - F}{r^d}
\]

If the portfolio management fee \( F \) were 0, this would imply that \( r^d q^e = \delta^e \Pi^R \), but it is required that \( \delta^e \Pi^R \geq q^e r^{cb} \). However, with \( F = 0 \), the household arbitrage condition would prevent the wholesale branch from having an incentive to invest in risky equities. Since \( r^d < r^{cb} \) if the deposit branch is to have positive profits, then \( \delta^e \Pi^R = r^d q^e < r^{cb} q^e \) violating the necessary condition.

This is where the portfolio management fee becomes important. With \( F > 0 \), it is possible to obtain a steady state calibration with \( \delta^e \Pi^R \geq q^e r^{cb} \). I calibrate \( F \) so that the dividend yield equals the return to lending \( \left( \delta^e \Pi^R / q^e = r^b \right) \). This way, the wholesale branch has an incentive to invest in a portfolio of risky equities in steady state, but it provides no added benefit over lending to entrepreneurs for the bank as a whole. The VaR constraint will bind in steady state making for effective regulation in limiting the size of banks’ balance sheet risk. Setting \( F = q^e(r^b - r^d) \) where \( q^e \) is the steady state equity price, achieves the desired result.26

Figure 5.2 illustrates the bank’s problem of allocating its assets between loans to entrepreneurs and investing in a portfolio of risky equities subject to the VaR constraint given the level of deposits. \( \Pi^w \) (the green line) represents wholesale bank profits, \( \Pi^b \) (the blue line) is total bank profits (5.8), and \( C \) (the red line) is the effect of the constraint. The equation for \( C \) is derived by rearranging the VaR constraint (5.10) to look like \( \Pi^b \geq v_b B + MZs^b - (1 - \delta^b)K^b \). The equations are then written as a function of the portfolio size by substituting the balance sheet identity into the relevant equations for \( B \) and then noting that in steady state \( K^b = v_B B + MZs^b, \delta^e \Pi^R = r^b q^e \), and \( r^{cb} = r^b \).

The diagram shows that the equations will increase in the portfolio size only if \( \sigma / q^e > v_b / (MZ) \), and the constraint will increase at a faster rate only if \( \delta^b > r^b \). If these do not hold, both the constraint and bank profits will be decreasing or non-intersecting and the constraint will not bind.

The VaR constraint is satisfied with portfolio holdings left of the vertical dotted line and above the constraint represented by the grey shaded region. Point 1 denotes the bank’s equilibrium portfolio size, and point 2 denotes wholesale bank profits at the equilibrium portfolio size. As a result of the separation of decisions among the three branches, the wholesale branch has an incentive to increase its equity holdings with less benefit to overall profits and with the added cost of extra balance sheet risk. The VaR constraint then

---

26Setting \( \delta^e \Pi^R = q^e r^{cb} \) so that the wholesale branch is indifferent between lending and investing in a portfolio of risky equities in steady state would set \( \lambda^h = 0 \) in steady state, implying that the VaR constraint is not binding.
effectively limits the risk the bank is able to hold on its books and prevents one branch of
the bank from endangering the operations of the entire bank. This is potentially a concern
for financial institutions with a separation-of-decisions structure similar to this model. The
trading desk of an institution like this may not take into account its decisions on the entire
institution and could take on excess risk, putting the financial stability of the institution
in question. Capital regulations, like the one considered in this paper, should be designed
with this mind.

With the VaR constraint (5.10) now shown to bind, rearranging it to get the bank’s
demand function for the trading book size results in:

\[
s_b^t = \frac{(r^b_t - r^d_t - v_b) D_t + (1 + r^b_t - \delta^b - v_b) K_t^b}{\pi_t + (r^b_t - v_b) q_t^c - \delta^c \Pi_t^R}
\]

(5.15)
after substituting in the balance sheet identity (5.6) for \(B_t\). From this, it can be seen
that the portfolio size depends positively on the loan rate spread \((r^b_t - r^d_t)\), the return to
bank capital \((1 + r^b_t - \delta^b - v_b)\), and dividend payments \((\delta^c \Pi_t^R)\). More importantly, higher
volatility \((\sigma_t)\) and tighter market risk regulations \((M, Z)\) reduce the size of the bank’s
trading book. Higher asset market volatility and expected losses can lead to sell-offs from
VaR constrained institutions. Flooding the market with these securities could lower asset
prices and further impair balance sheets: a type of pecuniary externality. If liquidity for
these risky assets dries up, the effect can be magnified. Since households are assumed to be passive in the equities market in this model, the equities market will be very liquid as household portfolio demand will be determined from the market clearing condition for equity securities.

The structure of price change expectations \(E_t [q_t^e - q_{t+1}^e \pi_{t+1}]\) will also be important to the portfolio decision. Rational expectations, random walk expectations, and backward-looking expectations will have very different consequences. If rational expectations are assumed and the equity price falls, then banks will believe correctly that the equity price will rise in the future as it returns to its steady state value. Banks will increase portfolio holdings to take advantage of trading profits from rising future prices. However, if expectations are backward-looking, banks could have the opposite response. If random walk expectations are assumed, then expected price changes are always zero and price changes will not affect the portfolio decision.

6 Calibration and Estimation

6.1 Data and Methodology

The model is log-linearized around the steady and the resulting state-space is used to compute the likelihood function. I use Bayesian techniques to estimate the parameters that affect the dynamics of the model which include the standard deviations and autoregressive coefficients of the shock processes as well as all adjustment costs and the parameters of the Taylor rule of monetary policy. All shocks \(\epsilon_t^x\) are assumed to follow AR(1) processes of the form \(\epsilon_t^x = (1 - \rho_x)\epsilon_{t-1}^x + \rho_x \epsilon_t + e_t^x\). Estimation of the posterior distribution is done using the Metropolis-Hastings algorithm similar to Smets and Wouters (2007).\(^{27}\) The observables are real output, real consumption, real investment, real deposits, real loans to entrepreneurs, and the real equity price as well as inflation, wage inflation, the interbank policy rate, the deposit rate, and the entrepreneur loan rate.

All macroeconomic data are taken from the St. Louis Federal Reserve Economic Data (FRED) over the time period 1997:Q2 to 2007:Q4 and include seasonally adjusted gross domestic product, personal consumption expenditures, gross fixed capital formation, commercial and industrial loans, total savings deposits at all depository institutions, the consumer price index, nonfarm business sector compensation per hour, the effective federal funds rate, the M2 OWN rate, and the weighted-average effective loan rate of all commercial and industrial loans. The weighted average rate on commercial and industrial loans limits the time frame since its first observation is 1997:Q2. I include data up until 2007:Q4 to try to incorporate as much information as possible while this is also near the end of the time frame that a Taylor rule can approximate movements in the federal funds rate reasonably well. Financial data regarding the S&P 500 is taken from Robert Shiller’s Irrational

\(^{27}\)Calculation of the steady-state and Bayesian estimation are computed using Dynare 4.4.3 in conjunction with Matlab R2015a.
Figure 6.1: Observable Data

Note: Real variables are logged and detrended using the HP filter with smoothing parameter set at 1,600 as suggested by Ravn-Uhlig (2002) except for the VIX which is logged and demeaned. All rates are expressed on quarterly basis and demeaned.

Exuberance data set which includes the historical S&P 500 price as well as dividends and earnings data. All nominal data are converted into billions of US dollars and deflated by the consumer price index to convert them into real terms. Variables that exhibit trends are logged and detrended using the Hodrick-Prescott filter with smoothing parameter set at 1600, following the Ravn-Uhlig (2002) suggestion, while all annual rates are demeaned and converted into quarterly rates. More information about the data can be found in appendix A.

6.2 Calibrated Parameters

The set of calibrated parameters includes the household and entrepreneur discount factors ($\beta_H, \beta_E$), the coefficient of relative risk aversion ($\gamma$), the inverse Frisch elasticity of labor supply ($\phi$), the steady state values of all price elasticities ($\epsilon^d, \epsilon^b, \epsilon^y, \epsilon^l$), the depreciation rates of capital and bank capital ($\delta, \delta^b$), the target risk-weighted capital-asset ratio ($v_b$), the dividend rate ($\delta^c$), the steady state loan-to-value ratio for entrepreneurs ($M^E$), and the share of output paid to capital ($\alpha$). These are summarized in table 6.1. I set the household discount rate, $\beta_H$, to 0.995 in order to obtain a steady-state deposit rate equal to the mean of the M2 OWN rate over the sample period which is 2.02% on an
annual basis. This also implies that the steady-state deposit rate elasticity, $\epsilon^d$, must be set to -1.21. To set the steady-state entrepreneur loan rate, the loan rate elasticity, $\epsilon^b$, is set to 3.03 to match the mean of the weighted-average rate on commercial and industrial loans over the sample period which is about 5.52%. To ensure a borrowing motive for entrepreneurs, I follow Gerali et al. (2010) and set the discount factor for entrepreneurs, $\beta_E$, to 0.975, which is also in the range suggested by Iacoviello (2005) and Iacoviello and Neri (2010).

Table 6.1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_H$</td>
<td>Household Discount Factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$\beta_E$</td>
<td>Entrepreneur Discount Factor</td>
<td>0.975</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of Relative Risk Aversion</td>
<td>1.38</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse Frisch Elasticity of Labor Supply</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Capital’s Share of Output</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate of Physical Capital</td>
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</tr>
<tr>
<td>$\delta^b$</td>
<td>Bank Capital Management Cost</td>
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</tr>
<tr>
<td>$\delta^e$</td>
<td>Dividend Rate</td>
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</tr>
<tr>
<td>$\epsilon^y$</td>
<td>Price Elasticity</td>
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</tr>
<tr>
<td>$\epsilon^l$</td>
<td>Wage Elasticity</td>
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<tr>
<td>$\epsilon^d$</td>
<td>Deposit Rate Elasticity</td>
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</tr>
<tr>
<td>$\epsilon^b$</td>
<td>Loan Rate Elasticity</td>
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<tr>
<td>$M^E$</td>
<td>Firm LTV</td>
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</tr>
<tr>
<td>$v_b$</td>
<td>Target RW-CAR</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Volatility</td>
<td>0.67</td>
</tr>
</tbody>
</table>

The consumption price elasticity, $\epsilon^y$, is set to 6 to deliver a steady state markup of 20%, and the wage elasticity, $\epsilon^l$, is set at 5 to get a steady state markup of 25% following Gerali et al. (2010). Capital’s share of income in the production function, $\alpha$, is set to 0.25. The values of the coefficient of relative risk aversion ($\gamma$), and the inverse Frisch elasticity of labor supply ($\phi$) are taken to be the posterior means of Smets and Wouters (2007) estimates which are 1.38 and 1.83 respectively. Gandelman and Hernández-Murillo (2015) estimate the coefficient of relative risk aversion for multiple countries using GMM techniques and find it to be 1.39 for the U.S. Using data from the Board of Governors of the Federal Reserve System on the ratio of total liabilities to total assets for nonfinancial corporate businesses, I infer the loan-to-value ratio for entrepreneurs, $M^E$, to be 0.46.

The capital depreciation rate, $\delta$, is set according to what is standard in the business cycle literature at 0.025. Gerali et al. (2010) set the depreciation rate on bank capital, $\delta^b$, to ensure the steady-state capital-asset ratio is exactly equal to the target, $v_b$. However, the
presence of the VaR constraint ensures that the risk-weighted capital-asset ratio exactly equals the target in steady state, so there is more freedom to set $\delta^b$. I set the depreciation of bank capital to match the ratio of loans-to-assets for U.S. chartered commercial banks which is about 0.67. This ensures that the bank's trading book in the model is not given too much weight on the bank balance sheet. As a result, $\delta^b$ is set to 0.04.

In regards to the financial parameters (the target risk-weighted capital-asset ratio ($v_b$), volatility ($\sigma$) and the dividend rate ($\delta^u$)), the target risk-weighted capital-asset ratio is set to 0.105. This corresponds to the minimum risk-weighted capital-asset ratio of 8% suggested by the Federal Reserve plus the 2.5% capital conservation buffer. The standard deviation of the representative equity price is calibrated so that $\sigma/q$ equals the standard deviation of the quarterly percentage change of the S&amp;P 500 price. This results in $\sigma$ being about 8% of the steady state equity price with a value of 0.67. And finally, the dividend rate is determined by the ratio of dividends-per-share over earnings-per-share which is about 0.51.

### 6.3 Prior Distributions

The set of estimated parameters includes all adjustment costs ($\kappa_i, \kappa_p, \kappa_w, \kappa_{Kb}, \kappa_b, \kappa_d$), parameters for the Taylor rule of monetary policy ($\phi_R, \phi_\pi, \phi_y$), the inflation and wage indexation parameters ($\iota_p, \iota_w$), the degree of consumption habit formation ($h$), and the standard error and autoregressive coefficients of all shocks ($\xi_t^E, \xi_t^r, \xi_t^K, \xi_t^b, \xi_t^d, \xi_t^l, \xi_t^i, \xi_t^Kb, A_t^E, M_t^E, \sigma_t$). Prior distributions are chosen to be similar to Gerali et al. (2010) and Smets and Wouters (2007) which can be found in tables 6.2 and 6.3. For the autoregressive coefficients on the shock processes and the degree of habit formation, I use a beta distribution with a prior mean of 0.5 and standard deviation of 0.15 so as to not falsely identify persistence based on prior specification. The inverse gamma distribution with a mean of 0.01 and standard deviation of 0.1 is chosen for the standard error of all shocks.

The parameters in the Taylor rule for monetary policy, $\phi_R, \phi_\pi,$ and $\phi_y,$ are given prior means of 0.75, 2.0, and 0.1 with standard deviations 0.1, 0.5, and 0.15 respectively. The prior distribution for $\phi_\pi$ is chosen to be a gamma distribution, for $\phi_R,$ a beta distribution, and for $\phi_y,$ a normal distribution. The priors for the inflation and wage inflation indexation parameters, $\iota_p$ and $\iota_w,$ are chosen to be beta distributions centered at 0.5 with a standard deviation of 0.15 following Smets and Wouters (2007).

Prior distributions for adjustment cost parameters are chosen to be gamma distributions following Gerali et al. (2010). For the investment adjustment cost, $\kappa_i$ is set with a prior mean of 4 and standard deviation of 1.0 according to Smets and Wouters (2007). The adjustment cost parameters for price and wage setting, $\kappa_p$ and $\kappa_w,$ are set with a prior mean of 50 and standard deviation of 20 to be fairly uninformative. According to Keen and Wang (2005) who attempt to relate Rotemberg adjustment costs to Calvo-type frictions suggest that $\kappa_p$ depends on the steady state markup and percent of firms reoptimizing prices in a given period. With a steady state markup of 20% as calibrated in this model, they
suggest that it be somewhere between 0 and 100 if 20% or more of firms are reoptimizing in a given period. The mean of interest rate setting costs, $\kappa_d$ and $\kappa_b$, are chosen to be 5 with standard deviations of 2.5, and the mean of the capital-asset ratio cost, $\kappa_{KB}$, is set to 10 with a standard deviation of 5 to also be fairly uninformative and similar to Gerali et al. (2010).

### 6.4 Posterior Estimation

Results from the posterior estimation can be found in tables 6.2 and 6.3. Draws from the posterior distribution for all estimated parameters are obtained using the Markov Chain Monte Carlo method of the Metropolis-Hastings algorithm where the scale factor is calibrated to achieve an acceptance rate of about 33%.\(^{28}\) I use sixteen parallel chains with length 100,000 each. Convergence is assessed using diagnostics suggested by Brooks and Gelman (1998), which can be found in appendix C.\(^{29}\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_w$</td>
<td>Wage Adj. Cost</td>
<td>Gamma 50 20</td>
<td>12.0676 33.4580 31.9991 54.3123 15.3928</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>Price Adj. Cost</td>
<td>Gamma 50 20</td>
<td>21.150 23.9495 22.9599 35.6001 7.4613</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>Investment Adj. Cost</td>
<td>Gamma 4 1</td>
<td>7.8569 9.9354 9.8792 12.0151 1.3309</td>
</tr>
<tr>
<td>$\kappa_{KB}$</td>
<td>RW-CAR Adj. Cost</td>
<td>Gamma 10 5</td>
<td>1.6616 7.2969 6.0991 12.5668 3.7449</td>
</tr>
<tr>
<td>$\kappa_d$</td>
<td>Deposit Rate Adj. Cost</td>
<td>Gamma 5 2.5</td>
<td>0.1251 1.1263 0.8916 2.2248 0.9235</td>
</tr>
<tr>
<td>$\kappa_b$</td>
<td>Loan Rate Adj. Cost</td>
<td>Gamma 5 2.5</td>
<td>0.1011 0.6331 0.4543 1.1256 0.8450</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Taylor Rule Coef. on Inflation</td>
<td>Gamma 2 0.5</td>
<td>2.2712 2.9284 2.8878 3.5414 0.4022</td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>Taylor Rule Coef. on Lagged Policy Rate</td>
<td>Beta 0.75 0.1</td>
<td>0.8805 0.8926 0.8943 0.9254 0.0201</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Taylor Rule Coef. on Lagged Output</td>
<td>Normal 0.1 0.15</td>
<td>0.1513 0.3756 0.3756 0.6035 0.1398</td>
</tr>
<tr>
<td>$\psi_p$</td>
<td>Price Inflation Indexation</td>
<td>Beta 0.5 0.15</td>
<td>0.0924 0.2678 0.2541 0.4312 0.1085</td>
</tr>
<tr>
<td>$\psi_i$</td>
<td>Wage Inflation Indexation</td>
<td>Beta 0.5 0.15</td>
<td>0.2526 0.4859 0.4843 0.7234 0.1401</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Consumption Habits</td>
<td>Beta 0.5 0.15</td>
<td>0.4309 0.5978 0.6112 0.7700 0.1002</td>
</tr>
</tbody>
</table>

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.

Most shocks exhibit persistence above 0.5. The shocks with AR(1) coefficients with persistence below 0.5 include wage elasticity, investment shock, and equity price shock. The two inflation indexation parameters are very similar to the estimates found by Smets and Wouters (2007). However, neither the wage or price inflation indexation parameter exhibits persistence with estimated posterior medians of 0.48 and 25 respectively. That inflation persistence is estimated to be so low is not necessarily surprising given that the gross inflation rate in figure 6.1 appears strongly mean reverting. The degree of consumption habit formation is fairly strong with an estimated posterior median of 0.61. For monetary policy, there appears to be a high degree of persistence as $\phi_R$ has a posterior median of 0.25.

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\(^{28}\)In Dynare 4.4.3, this is done using a combination of the mode compute 6 and 9 options in the estimation command.

\(^{29}\)These are automatically provided from the estimation command in Dynare 4.4.3. Convergence is determined if the pooled draws from all chains converges with the draws from within individual chains and settles around a particular value.
0.89. The posterior median of the monetary policy response to inflation, $\phi_\pi$, is estimated to be 2.89 while the response to output fluctuations, $\phi_y$, is estimated to be 0.38.

Table 6.3: Prior and Posterior Distributions of Shock Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{AE}$</td>
<td>Technology</td>
<td>Beta 0.01 0.1</td>
<td>Distribution Mean Std. Dev. 5% Mean Median 95% Std. Dev.</td>
</tr>
<tr>
<td>$e_r$</td>
<td>Monetary Policy</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0125 0.0165 0.0163 0.0203 0.0024</td>
</tr>
<tr>
<td>$e_c$</td>
<td>Consumption Preference</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0015 0.0019 0.0019 0.0023 0.0002</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Investment Efficiency</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0233 0.0325 0.0318 0.0416 0.0058</td>
</tr>
<tr>
<td>$e_p$</td>
<td>Price Elasticity</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0145 0.0205 0.0200 0.0264 0.0036</td>
</tr>
<tr>
<td>$e$</td>
<td>Wage Elasticity</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.2151 0.3902 0.3770 0.5584 0.1047</td>
</tr>
<tr>
<td>$e_d$</td>
<td>Deposit Rate Elasticity</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.8364 1.8648 1.7395 2.8464 0.7390</td>
</tr>
<tr>
<td>$e_b$</td>
<td>Loan Rate Elasticity</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0108 0.0144 0.0141 0.0180 0.0022</td>
</tr>
<tr>
<td>$e_Kb$</td>
<td>Bank Capital</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0578 0.0957 0.0896 0.1333 0.0310</td>
</tr>
<tr>
<td>$e^\sigma$</td>
<td>Volatility</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0735 0.1092 0.1012 0.1405 0.0402</td>
</tr>
<tr>
<td>$e^\tau$</td>
<td>Equity Price</td>
<td>Inverse Gamma 0.01 0.1</td>
<td>0.0217 0.0273 0.0270 0.0330 0.0035</td>
</tr>
</tbody>
</table>

Estimation of the financial adjustment cost parameters suggest a very little frictions at the aggregate level for both deposit and loan rate setting. The posterior medians of $\kappa_d$ and $\kappa_b$ are estimated to be 0.89 and 0.45 respectively. This is not necessarily surprising since both rates appear to move closely with the federal funds rate as can be seen in figure 6.1. The posterior median of adjustment costs on the risk-weighted capital-asset ratio, $\kappa_{Kb}$, is much higher than that of interest rate setting costs at 6.7. However, this could be due to weak identification.\footnote{Identification strength for an estimated parameter is assessed using Dynare’s identification command (figure C.3 in appendix C) and by the movement of the posterior distribution away from the prior distribution.} Identification analysis did indicate that $\kappa_{Kb}$ was identified but that it has the weakest identification of all the adjustment cost parameters. Figure 6.2 shows the posterior distribution is close to the prior distribution. After experimenting with smaller and larger prior means for $\kappa_{Kb}$, leads one to see that the log posterior likelihood is relatively flat for this parameter. The posterior mean moved to the original estimated mean when the prior distribution was set at 15 but moved lower when set at 5. The log
data density at the original posterior estimation was 1439 but was slightly lower at 1438 when the prior mean was set at 5 or 15. Therefore, setting $\kappa_{Kb}$ at 6.7 seems to best fit the data.

Figure 6.2: Posterior Distribution of Structural Parameters

Investment adjustment costs have an estimated median of about 9.88, somewhat higher than Smets and Wouters (2007) estimate. And as for nominal rigidities on price and wage setting, wage frictions are estimated to be stronger than price frictions. $\kappa_w$ has an estimated posterior median of 31.1 whereas $\kappa_p$ has an estimated posterior median of 22.96. The findings that wage rigidities are larger that of price rigidities is consistent with results found in the models mentioned in section 4.

7 Performance and Simulations

7.1 Performance Relative to the Data

The performance of the model relative to the data is assessed from the matrix of four target correlations discussed in the empirical section between 1990:Q1-2014:Q4. These include the correlations of security broker dealer leverage with investment, leverage with commercial and industrial loans, leverage with market volatility represented by the VIX, and between volatility with the size of security broker dealers’ corporate equities portfolio.
Table 7.1: Correlations: Model vs. Data

<table>
<thead>
<tr>
<th></th>
<th>Leverage</th>
<th>Investment</th>
<th>Loans</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Leverage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.44</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loans</td>
<td>0.64</td>
<td>0.18</td>
<td>0.72</td>
<td>0.14</td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.20</td>
<td>-0.33</td>
<td>-0.01</td>
<td>-0.38</td>
</tr>
<tr>
<td>Portfolio Size</td>
<td>-0.60</td>
<td>0.68</td>
<td>-0.42</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: Data correlations are HP-filtered between 1990:Q1-2014:Q4.

Overall, the model matches the target correlations well, getting the sign right for all four correlations. Importantly, the model captures the procyclicality of leverage documented by Adrian and Shin (2010) evidenced by the positive correlation between leverage and investment. However, the model does not quite capture the magnitude of this relationship, nor does it quite capture the magnitude of the relationship between leverage and volatility or leverage and investment. The model is able to fully capture the relationship between volatility and portfolio size. This suggests that the Value-at-Risk constraint is able to capture the relationship between the leverage, volatility, and portfolio size with investment. This will be explored further in this section.

The model overshoots the relationship between leverage and loans and between loans and investment by a good amount. This is likely due to the fact the model only provides entrepreneurs with one source of outside funding: bank loans. The model does not allow entrepreneurs to raise funds via capital markets, which is an important source of funding for many firms. In fact, Adrian, Colla, and Shin (2012) note that firms which had access to capital markets were able to make up a large portion of the decline in lending by issuing corporate debt following the 2008 financial crisis. The model falls short in two other areas as well, namely the correlations of leverage with portfolio size and portfolio size with investment. In these two cases, the model produces correlations with the opposite signs as those observed in the data. This may be due to oversimplified bank balance sheets in the model as they only have two assets and one source of debt to manage leverage. In the model, deposits are the only source of funding since short term debt like repos, which is an important source of funding for financial institutions, are not modeled.

7.2 Simulations

To study the dynamics of the linearized model to financial shocks, I focus on impulse responses to unanticipated one standard deviation shocks to $\epsilon_t$ and $\sigma_t$ with parameter
values set at their posterior median.\textsuperscript{31} The goal is to assess how marked-to-market balance sheets and VaR-based capital requirements transmit financial shocks to the real economy.

Three different types of asset price expectations are considered to evaluate the properties of the model: rational expectations, backward-looking expectations, and random walk expectations. Rational expectations in the model are specified exactly as $E_t [q_{t+1}^e \pi_{t+1} - q_t^e]$. Random walk expectations sets $E_t [q_{t+1}^e \pi_{t+1} - q_t^e] = 0$. For backward-looking expectations, a best fit ARIMA model of the S&P 500 suggests forecasting $q_{t+1}^e \pi_{t+1}$ with an ARIMA(2,0,0).\textsuperscript{32}

7.2.1 Transitory Asset Price Shock

The transmission of a transitory asset price shock is studied by analyzing the impulse response of a negative unanticipated one standard deviation shock to $e_t^q$ and is shown in figure 7.1. The black line with dot markers plots the impulses responses for the benchmark rational expectations model. The blue line with “o” markers plots the impulse responses with random walk expectations, and the red line with “x” markers plots the impulse responses with backward-looking expectations.

Upon impact of the shock, the equity price falls, impairing bank balance sheets by reducing bank capital, and increasing leverage as the value of bank assets declines. In all three expectation cases, Value-at-Risk (VaR) falls along with risk-weighted assets (RWA), putting upward pressure on the risk-weighted capital-asset ratio (RW-CAR). However, because bank capital also falls, an overall decline in the RW-CAR occurs, putting upward pressure on the interbank and loan rates.

Under rational expectations, banks believe correctly that the asset price will increase in the future as it returns to its steady state value. Banks increase the size of their trading book to take advantage of future trading profits as evidenced by the increase in the VaR constraint multiplier that represents the marginal change in profit from having to hold another unit of capital. Households respond by selling their equity shares to banks and switch their savings channel towards deposits. With more funds available to banks in the form of cheaper debt, banks have more funds available to lend. This process is consistent with security broker dealers financing asset growth with debt (Adrian, Moench, and Shin (2010)). Under the other two expectation cases, banks’ choose to shrink the size of their trading book, with the portfolio size shrinking the most under backward-looking expectations. Households now respond by buying equity shares from banks and switch

\textsuperscript{31}Simulations are computed under the assumption that the expected value of all future shocks is zero. Since the model stochastic and log-linearized around the steady state, agents will behave as if the value of future shocks is zero due to certainty equivalence. This also implies that perturbation methods are valid only in a neighborhood of the steady state. As shocks become too large, the linear approximation becomes less accurate and the conditions that ensure that all constraints bind may no longer hold.

\textsuperscript{32}The best fit ARIMA model is estimated using the “auto.arima” function available in the “forecast” package for R. The estimated autoregressive coefficients are 1.103 and -0.207 for the first and second lags respectively.
their savings mechanism away from deposits. With less debt available and at a higher cost than the rational expectations case, loans do not increase as much since some of that loss is offset by selling equities. Compared to the rational expectations case, the response of all the major macroeconomic variables is muted.

In all three cases, households increase consumption as a result of equity sales and are able to work less. Entrepreneurs are forced to substitute out of labor and increase the use of capital in production. Because output was assumed to be more labor intensive, a decline in output follows the decline in labor. The central bank responds to the fall in output with a decrease in the policy rate that outweighs balance sheet effects in the interbank market. This decrease in the policy rate lowers the deposit rate, but depositors do not receive the full effect due to the presence of the markdown over the policy rate. The policy rate cut
also lowers the interbank rate and loan rate, but the fall in the loan rate is amplified due to the markup over the interbank rate. The net effect is an decrease in the intermediation spread \((r_t^b - r_t^d)\). Credit increases and firms choose to invest more as borrowing becomes cheaper.

The model’s response to a negative equity price shock captures all four target correlations, but only under the rational expectations case where a decline in the equity price results in an increase in trading book size, leverage, loans, and investment. The fact that the trading book sized increases with a decline in the equity price runs counterintuitive to Adrian and Shin (2010) findings in which security broker dealer leverage increases with asset prices. When bank expectations about future asset prices is specified as a random walk or backward-looking, the trading book sized decreases with the equity price. However, this only dampens the increase in leverage, loans, and investment when a decrease might be expected.

7.2.2 Transitory Volatility Shock

The transmission of a transitory volatility shock is studied by analyzing the impulse response of a positive unanticipated one standard deviation shock to \(\sigma_t\) and is shown in figure 7.2. The rational expectations case is the only case presented here, because the other two cases did not significantly alter the responses.

The volatility shock originates in the financial system through the VaR constraint and risk-weighted capital requirements. Upon impact of the shock, the VaR constraint multiplier decreases representing the loss in profit as banks are forced to hold more capital. The increase in volatility initially increases VaR and RWA, decreasing the RW-CAR and putting upward pressure on the interbank and loan rates. However, banks decrease the size of their trading book according to their demand function (5.15), which outweighs the effect of the increase in volatility on the interbank rate. The net effect is for VaR and RWA to decrease while the RW-CAR increases, leaving the interbank rate nearly unchanged in the first period.

As banks decrease the size of their trading book, households play the role of backstop and purchase the excess supply. In order for the households to be able to do this and minimize losses to consumption, the equity price falls and households switch their main savings channel away from deposits and towards equities. Declining asset prices results in a loss in bank capital, dampening the rise in the RW-CAR. The loss in deposits reduces the funds available to banks, so loans and investment decrease as well. Banks deleverage as the loss in deposits outweighs the effects of reducing the trading book size and falling asset prices.

With loans falling, firms decrease the use of capital in production and increase the use of labor. Households choose to work more as their income from deposits and purchases of equity shares reduces consumption. Again, because output was assumed to be more labor intensive, an increase in output follows the increase in labor. The central bank then
Note: Impulse responses are in percent deviation from steady state values.

responds with an increase in the policy rate, driving up both the deposit and loan rates. The markdown of the deposit rate prevents depositors from receiving the full benefit of a policy rate increase, while the markup of the loan rate amplifies the increase in the policy rate to borrowers. The net effect is an increase in the intermediation spread and banks choose to lend less. With borrowing becoming more expensive, investment declines further as time progresses.

The model’s response to a positive volatility shock captures all four target correlations. A positive volatility shock initiated a decline in banks’ trading book size and leverage followed by decreased lending and investment. This shock also generates a feedback effect as asset prices decline in order for households to take up the slack in the equities market, resulting in a loss in banks’ equity capital. Although the overall model does not capture
the correlation between trading book size and leverage observed in the data, the model’s response to this shock does show banks’ portfolio size and leverage decreasing together. Comparing the results of a volatility shock to that of an equity price shock, unexpected changes in volatility are most responsible for banks’ procyclical leverage behavior in this model while unexpected changes in asset prices dampen banks’ balance sheet response to volatility shocks.

7.2.3 What’s Driving the Dynamics?

Now that the impulse responses to both an equity price and volatility shock have been analyzed, it is important to understand what features are driving the dynamics. Specifically, how are these two financial asset shocks being transmitted to credit markets? Are the dynamics driven by the nominal debt channel, the collateral constraint, or constraints placed on bank balance sheets? Most importantly, because the risk-weighted capital-asset ratio and the VaR constraint are closely linked, are the dynamics driven by the adjustment cost on the risk-weighted capital-asset ratio that affects the interbank-policy rate spread or the VaR constraint? To answer this, I compare the impulse responses of the baseline model described in this paper to the model responses after progressively shutting down each feature. The left side of figure 7.3 plots selected responses to a positive volatility shock, and the right side plots the selected responses to a negative asset price shock. The black line with dot markers plots the baseline model under rational expectations and the red line with “o” markers plots the response when the adjustment cost on the risk-weighted capital-asset ratio is turned off ($\kappa_{Kb} = 0$). The blue line with “x” markers plots the model response after the nominal debt channel is turned off and all debt is denominated in real terms (RD). Finally, the green line with square markers plots the model’s response after the collateral constraint has been turned off (No CC).

I focus my attention on the responses of consumption, loans, and investment as the responses of the other variables are not significantly affected. First, the effect of the adjustment cost on the risk-weighted capital-asset ratio has minimal effect on the model’s dynamics. The impulse responses between the $\kappa_{Kb} = 0$ version and the baseline model is negligible. This is at least in part due to the small estimated value of this parameter in the baseline model. The nominal debt channel appears to have a strong effect on macroeconomic activity in the model. When debt is denominated in nominal terms and interest rates are set by rate setting banks, it has an overall dampening effect on macroeconomic activity and also increases the persistence of the shocks. Removing the collateral constraint alone has very little effect compared to the baseline model; however, its interaction with real

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33The effect of price, wage, and interest rate rigidities are left out of the analysis. The literature on price and wage rigidities have shown them to be important modeling devices that help capture features in the data. The effect of interest rate rigidities are left out of the analysis as Gerali et al. (2010) discuss their effects in detail. Briefly, interest rate frictions are found to dampen the economy’s response to monetary policy shocks. The interest rate frictions in this model were estimated to be quite small and are unlikely to have a significant effect on the model economy.

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The collateral constraint amplifies the effect on the model’s responses when debt is denominated in real terms. When the collateral constraint is removed and debt is in real terms, the response of macroeconomic activity in the model is dampened and persistence is reduced.

The most important result from this exercise comes from analyzing the differences between the model with and without the risk-weighted capital-asset adjustment cost. Because the model’s responses are nearly identical in both cases, the dynamics of the model in response to financial shocks are driven by the VaR constraint and not the adjustment cost. The fact that the VaR constraint was able to generate the dynamics that accounted for the target correlations without relying on the ad hoc feature that is the adjustment cost is important. The microfoundations of the adjustment cost are weak at best, whereas financial institutions and capital regulations actually utilize VaR as a risk management technique.
8 Conclusion

This paper developed a DSGE model to study the role of marked-to-market balance sheets and Value-at-Risk-based capital requirements in response to two specific shocks that have been observed to have potentially important effects on credit supply: asset price and volatility shocks. Including VaR-based capital regulations in a DSGE framework was able to capture four important correlations observed of financial institution leverage with asset market volatility, leverage with loans, trading portfolio size with volatility, and leverage with aggregate investment. Most importantly, the model was able to capture the procyclicality of financial institution leverage with respect to asset market volatility and aggregate investment. These results contribute to the growing literature on the importance of financial factors in business cycles by identifying the VaR of banks’ trading book to be a link between financial markets and credit markets, suggesting that VaR-based capital requirements are an important driver of credit supply and financial cycles.

The model was estimated using Bayesian techniques to U.S. data over the period 1997:Q2-2007:Q4. I used a Value-at-Risk constraint that accounts for VaR-based capital regulations in conjunction with an adjustment cost on the target risk-weighted capital-asset ratio as a mechanism to transmit asset market shocks to the real economy in a fully dynamic general equilibrium model that included a monopolistically competitive financial sector with financial and nominal frictions. The use of the adjustment cost on the risk-weighted capital-asset ratio was included to be consistent with the observation that banks tend to target a constant VaR-equity ratio while also allowing for procyclical leverage. However, this adjustment cost did not play a significant role in the model dynamics. The fact that the VaR constraint was able to generate the dynamics that accounted for the target correlations without relying on the ad hoc feature that is the adjustment cost is a success for this modeling technique.

Analysis of the impulse responses revealed the important characteristics of the model. The model’s response to a positive volatility shock captured all four target correlations. A positive volatility shock initiated a decline in banks’ trading book size and leverage followed by decreased lending and investment. This shock also generated a feedback effect as asset prices declined in order for households to absorb the excess supply of equity securities, resulting in a loss in banks’ equity capital. Even though the overall model did not capture the correlation of financial institution leverage with trading book size, the response to this shock showed banks’ portfolio size and leverage decreased together as is observed in the data. The model’s response to a negative equity price shock also captured all four target correlations, but only under the rational expectations case where a decline in the equity price resulted in an increase in trading book size, leverage, loans, and investment. The fact that the trading book size increased with a decline in the equity price runs counterintuitive to some empirical observations. When bank expectations about future asset prices was specified as a random walk or backward-looking, the trading book size decreased with the equity price. However, this only dampened the increase in leverage, loans, and investment.
when a decrease might have been expected. In this model, unexpected changes in volatility are most responsible for banks’ procyclical leverage behavior while unexpected changes in asset prices dampen banks’ balance sheet response to volatility shocks.

One aspect that became so important during the 2008 financial crisis that this paper did not touch on is liquidity. When repo markets froze as a result of uncertainty around the collateral value backing many mortgage-backed securities, it became difficult for banks to roll over short term debt and maintain the heightened leverage that was built in the run up to the crisis. Financial institutions became stuck with what became high-risk assets on their balance sheets and were forced to deleverage. The model I developed here should be a decent starting point to analyze the effects that illiquid asset markets have on credit supply when banks are subject to risk-weighted capital regulations. In this model, banks were able to adjust their trading book at will as households were assumed to be passive investors. If banks were unable to do this, bank balance sheet positions could outweigh the central bank response and create a positive spread between the interbank and policy rates that would amplify an initial volatility shock. This is the topic of a follow up to this paper.
References


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A Model Economy Details

A.1 Entrepreneurs and Households

The first order conditions for the entrepreneur are:

\[ c_t^E : (1 - h)^\gamma (c_{t-1}^E - hc_{t-1})^{-\gamma} = \lambda_t^E \]  
(A.1)

\[ k_t : \lambda_t^E q_t^k = E_t \left[ \psi_t^EM_t^E g_t^k \pi_{t+1}(1 - \delta) + \beta E \lambda_{t+1} \left( r_t^k + q_t^k(1 - \delta) \right) \right] \]  
(A.2)

\[ b_t : \lambda_t^E = \psi_t^E \left( (1 + r_t^b) + \beta E E_t \left( \lambda_{t+1} \frac{1 + r_t^b}{\pi_{t+1}} \right) \right) \]  
(A.3)

\[ l_t : w_t = \frac{(1 - \alpha)}{x_t^l} \frac{\gamma_t}{l_t^l} \]  
(A.4)

written without subscripts i. \( r_t^k = (\alpha / x_t) A_t^E \kappa_{t-1}^{1 - \alpha} \) is the rental rate of capital and \( x_t = P_t / P_t^w \) is the retail markup over the wholesale price. \( \lambda_t^E \) is the Lagrange multiplier on the budget constraint and \( \psi_t^E \) is the Lagrange multiplier on the collateral constraint.

The remaining first order condition for the household is:

\[ c_t^H : (1 - h)^\gamma (c_{t-1}^H - hc_{t-1})^{-\gamma} = \lambda_t^H \]  
(A.5)

A.2 Capital Producers

Perfectly competitive capital goods producers buy undepreciated capital from entrepreneurs (who also own the capital producers) and combine it with final goods purchased from retailers (defined below) to maximize profits. Old capital is converted one-to-one into new capital, but the final good is converted into new capital subject to a quadratic adjustment cost (\( \kappa_t \)). They then sell newly produced capital goods back to entrepreneurs. The capital producers problem is then to choose the level investment \( i_t \) to:

\[ \max_{(i_t)} E_0 \sum_{t=0}^{\infty} \lambda_t^E \beta E \left[ q_t^k (k_t - (1 - \delta)k_{t-1}) - i_t \right] \]

Subject to the capital accumulation equation:

\[ k_t = (1 - \delta)k_{t-1} + \left[ 1 - \frac{\kappa_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right]^2 i_t \]  
(A.6)

where \( \epsilon_t^l \) is a shock to the productivity of investment goods. The functional form of investment adjustment costs used here assumes that adjustment costs depend on the growth rate of investment rather than its level, and, up to a first-order, adjustments costs
are zero in the neighborhood of the steady state. This specification helps match the response of investment in the model to that observed in the data to monetary policy and technology shocks.

The maximization results in:

\[ i_t : 1 = q_t^k \left( 1 - \frac{\kappa_i}{2} \left( \frac{i_t \epsilon^i_t}{i_{t-1}} - 1 \right)^2 - \kappa_i \left( \frac{i_t \epsilon^i_t}{i_{t-1}} - 1 \right) \frac{i_t \epsilon^i_t}{i_{t-1}} \right) + \beta_E E_t \left[ \frac{\lambda^E_{t+1}}{\lambda^E_t} k_{t+1}^{\epsilon^{i+1}_{t+1}} k_{t+1} \kappa_i \left( \frac{i_{t+1} \epsilon^{i+1}_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right] \]  

(A.7)

which gives the condition for the price of capital is and is identical to the condition derived in Gerali et al. (2010).

I choose to separate the investment problem from the entrepreneur’s problem as a matter of convenience. Gerali et al (2010), Christiano, et al. (2005), and Smets and Wouters (2007) all use a similar formulation of adjustment costs to generate the hump-shaped response in investment to monetary policy and technology shocks observed in the data. Bernanke, Gertler, and Gilchrist (1999) note that the same condition for the capital price can be derived if this problem is folded into the capital choice of the entrepreneur’s problem.

A.3 Retailers

There is a continuum of measure one of monopolistically competitive retail goods producers indexed by \( j \) that buy intermediate goods from entrepreneurs at the wholesale price \( P^W_t \), costlessly differentiates it, and sells it to households, entrepreneurs, and capital goods producers at price \( P_t(j) \), but with a markup \( x_t \) over the wholesale price. Retailers maximize profits by setting prices subject to a Dixit-Stiglitz type CES demand curve where the price-elasticity for their product, \( \epsilon^y_t \), is assumed to be time-varying and modeled as an exogenous stochastic process. Prices are also assumed to be sticky and indexed to a combination of steady state and past inflation (\( \pi_t \) and \( \pi_{t-1} \) respectively). If retailers want to change their price to something other than what the index allows, they face a Rotemberg-type adjustment cost with parameter \( \kappa_p \). Retailers are also assumed to have issued a measure one of equity securities that promise to pay a fraction of retail profits as dividends in the following period to the holder (households and banks) of the security. The dividend rate \( \delta^e_j \) is assumed to be exogenously determined and retailers will not issue new equity shares or buyback any existing shares in this model. Retailers’ maximization problem is then:

\[
\max_{\{P_t(j)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_t^H \left[ P_t(j) y_t(j) - P^W_t y_t(j) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{1-\gamma} \right)^2 P_t y_t \right]
\]

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subject to the demand curve from households who purchase the differentiated goods:

\[ y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y^t} y_t \]

where the price index is given by:

\[ P_t = \left[ \int_0^1 P_t(j)^{1-\epsilon_y^t} \, dj \right]^{\frac{1}{1-\epsilon_y^t}} \]

The first step to derive the first order condition is to substitute the goods demand equation (A.3) in for \( y_t(j) \) to get:

\[
\max_{\{P_t(j)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_t^H \left[ P_t(j) \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y^t} y_t - P_t^W \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y^t} y_t - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{1-\epsilon_y} \right)^2 P_t y_t \right]
\]

Differentiating with respect to \( P_t(j) \), and after assuming a symmetric equilibrium which results in \( P_t(j) = P_t \), the first order condition for price setting is:

\[
[P_t] : 1 - \epsilon_y^t + \frac{\epsilon_y^t}{\pi_t} - \kappa_p \left( \pi_t - \pi_{t-1}^{1-\epsilon_y} \right) \pi_t + \beta_H E_t \left[ \frac{\lambda_t^H}{\lambda_t^H} \kappa_p \left( \pi_{t+1} - \pi_t^{1-\epsilon_y} \right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = 0
\]  
(A.8)

which produces a non-linear Phillips curve identical to the condition derived in Gerali et al. (2010).

### A.4 Wage Setting

Each household \( i \) is assumed to supply differentiated labor and is able to set its nominal wage rate \( W_t(i) \) through a wage setting process that maximizes utility subject to a Dixit-Stiglitz type CES demand curve from entrepreneurs who hire household labor. The wage-elasticity, \( \epsilon_l^t \), is assumed to time-varying and modeled as an exogenous stochastic process. Wages are thus assumed to be sticky and indexed to a combination of steady state and past inflation (\( \pi \) and \( \pi_{t-1} \) respectively). If households want to change their wage to something other than what the index allows, they face a Rotemberg-type adjustment cost with parameter \( \kappa_w \). This is achieved by:

\[
\max_{\{W_t(i)\}} E_0 \sum_{t=0}^{\infty} \lambda_t^H \beta_t^H \left[ \frac{W_t(i)}{P_t} l_t(i) - \frac{\kappa_w}{2} \left( \frac{W_t(i)}{W_{t-1}(i)} - \frac{p_{t-1}^{1-\epsilon_y}}{p_t} \right)^2 \frac{W_t(i)}{P_t} \right] - \frac{l_t(i)^{1+\phi}}{1+\phi}
\]
subject to the labor demand curve:

\[ l_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\epsilon_t^i} l_t \]

where the wage index is given by:

\[ W_t = \left[ \int_0^1 W_t(i)^{1-\epsilon_t^i} di \right]^{\frac{1}{1-\epsilon_t^i}} \]

Nominal wage inflation can then be defined as:

\[ \pi_t^w = \frac{w_t}{w_{t-1}} \pi_t \]

Following the same procedure as in the retailer problem, substitute the labor demand equation (A.4) in for \( l_t(i) \) to get:

\[
\max_{\{W_t(i)\}} \sum_{t=0}^{\infty} \lambda^H_t \beta^H_t \left[ \frac{W_t(i)}{P_t} \left( \frac{W_t(i)}{W_t} \right)^{-\epsilon_t^i} l_t - \frac{\kappa_w}{2} \left( \frac{W_t(i)}{W_{t-1}(i)} - \pi_{t-1} \pi^{1-\epsilon} \right)^2 \frac{W_t(i)}{P_t} \right] - \left[ \frac{W_t(i)}{W_t} \right]^{1-\epsilon_t^i} l_t \right]^{1+\phi} \]

Differentiating with respect to \( W_t(i) \), and after assuming a symmetric equilibrium which results in \( W_t(i) = W_t \), the first order condition for wage setting is:

\[
[W_t] : (1 - \epsilon_t^i)l_t + \frac{\epsilon_t^i l_t^{1+\phi}}{w_t \lambda^H_t} - \kappa_w \left( \pi_t^w - \pi_{t-1}^w \pi^{1-\epsilon} \right) \pi_t^w + \beta_H E_t \left[ \frac{\lambda^H_{t+1}}{\lambda^H_t} \kappa_w \left( \pi_{t+1}^w - \pi_t^w \pi^{1-\epsilon} \right) \pi_{t+1}^w \pi_{t+1} \right] = 0
\]

which produces a non-linear wage-Phillips curve and is identical to the condition derived in Gerali et al. (2010).

A.5 Wholesale Branch

The problem for the wholesale branch is to maximize discounted cash flows and can be written as:

\[
\max_{\{B_t, D_t, S^b_{j,t}\}} \sum_{t=0}^{\infty} \lambda^R_t \beta^R_t \left[ (1 + r_t^b)B_t - B_{t+1} + D_{t+1} - (1 + r_t^b)D_t + \int_j (q^{e}_{j,t+1} \pi_{t+1} + \delta_j \Pi^R_{j,t}) S^b_{j,t} dj - \int_j q^{e}_{j,t+1} S^b_{j,t+1} \pi_{t+1} dj + K^b_{t+1} \pi_{t+1} - K^b_t - \frac{\kappa_K}{2} \left( \frac{K^b_t}{R W A_t - v_b} \right)^2 \right]
\]
where the index \( j \) denotes the equity price and shares from retailer \( j \). \( RWA_t \) stands for risk-weighted assets and will be defined in more detail below. Combine this with the balance sheet identity:

\[
B_t + \int_j q_j^e s_j^b \, dj = D_t + K_t^b
\]

which shows that the wholesale branch problem reduces to maximizing period profits:

\[
\max_{\{B_t, D_t, S_j^b, t\}} r_t^b B_t - r_t^b D_t + \int_j E_t \left[ q_j^e (s_j^b + \delta_j^R) \right] s_j^b \, dj - \frac{K_t^b}{2} \left( \frac{K_t^b}{RWA_t} - v_b \right)^2 K_t^b
\]

The conditions used to reduce the problem to a representative equity in section 5.3.3 arise from the first order conditions for the trading desk of wholesale branch, which are:

\[
[s_j^b]: E_t \left[ q_j^e (s_j^b + \delta_j^R) \right] - r_t^b q_j^e - \frac{dAdj_t^b}{ds_j^b} = 0
\]

\[
\frac{dAdj_t^b}{ds_j^b} = \kappa Kb \left( \frac{K_t^b}{RWA_t} - v_b \right) \left( \frac{K_t^b}{RWA_t} \right)^2 dRWA_t
\]

\[
\frac{dRWA_t}{ds_j^b} = M \left( E_t \left[ q_j^e (s_j^b + \delta_j^R) \right] + \frac{Z}{\sigma^2} \left( \sigma_j^2 s_j^b + \sum_{i \neq j} \rho_{ij} \sigma_j s_i^b \right) \right) = 0
\]

and the first order condition for the household’s equity choice (4.6).

### A.6 Deposit Branch

The first order condition is found by maximizing deposit branch profits (5.1.1) subject to the deposit demand (5.1.1). The first step is to substitute the deposit demand in for \( d_t(i) \) in deposit profits to get:

\[
\max_{\{r_t^d(i)\}} E_0 \sum_{t=0}^{\infty} \lambda^H_t \beta^H \left[ r_t^d \left( \frac{r_t^d(i)}{r_t^d} \right)^{-\epsilon_t^d} d_t - r_t^d(i) \left( \frac{r_t^d(i)}{r_t^d} \right)^{-\epsilon_t^d} d_t - \frac{\kappa d}{2} \left( \frac{r_t^d(i)}{r_{t-1}^d(i)} - 1 \right)^2 r_t^d(i) d_t \right]
\]

Differentiating with respect to \( r_t^d(i) \), and after assuming a symmetric equilibrium which results in \( r_t^d(i) = r_t^d \), the first order condition is:
\[ [r^d_t] : -(1 - \epsilon^d_t) - \epsilon^d_t \frac{r^b_t}{r_t} - \kappa_d \left( \frac{r^d_t}{r^d_{t-1}} - 1 \right) \frac{r^d_t}{r^d_{t-1}} \]

\[ + \beta_H E_t \left[ \frac{\lambda^H_{t+1}}{\lambda^H_t} \kappa_d \left( \frac{r^d_{t+1}}{r^d_t} - 1 \right) \left( \frac{r^d_t}{r^d_{t-1}} \right)^2 \frac{d_{t+1}}{d_t} \right] = 0 \] (A.10)

which is identical to the condition derived in Gerali et al. (2010).

Given the deposit demand (5.1.1), the corresponding Dixit-Stiglitz deposit rate index is:

\[ r^d_t = \left[ \frac{1}{r^d_t(i)^{1-\epsilon^d_t}} \right]^{1-\epsilon^d_t} \]

A.7 Loan Branch

The first order condition is found by maximizing loan branch profits (5.1.2) subject to the loan demand (5.1.2). Again, the first step is to substitute the loan demand in for \( b_t(i) \) into loan profits to get:

\[ \max_{\{r^b_t(i)\}} \sum_{t=0}^{\infty} \lambda_t^H \beta_H \left[ r^b_t(i) \left( \frac{r^b_t}{r_t} \right)^{1-\epsilon^b_t} b_t - r^b_t \left( \frac{r^b_t}{r_t} \right)^{1-\epsilon^b_t} b_t - \kappa_b \left( \frac{r^b_t}{r^b_{t-1}(i)} - 1 \right)^2 \frac{r^b_t}{r^b_t(i)b_t} \right] \]

Differentiating with respect to \( r^b_t(i) \), and after assuming a symmetric equilibrium which results in \( r^b_t(i) = r^b_t \), the first order condition is:

\[ [r^b_t] : (1 - \epsilon^b_t) + \epsilon^b_t \frac{r^b_t}{r^b_t} - \kappa_b \left( \frac{r^b_t}{r^b_{t-1}} - 1 \right) \frac{r^b_t}{r^b_{t-1}} \]

\[ + \beta_H E_t \left[ \frac{\lambda^H_{t+1}}{\lambda^H_t} \kappa_b \left( \frac{r^b_{t+1}}{r^b_t r^b_t} - 1 \right) \left( \frac{r^b_t}{r^b_{t-1}(i)} - 1 \right)^2 \frac{b_{t+1}}{b_t} \right] = 0 \] (A.11)

which is again identical to the condition derived in Gerali et al. (2010).

Given the loan demand (5.1.2), the corresponding Dixit-Stiglitz loan rate index is:

\[ r^b_t = \left[ \frac{1}{r^b_t(i)^{1-\epsilon^b_t}} \right]^{1-\epsilon^b_t} \]
A.8 Monetary Policy

The central bank is assumed to set its policy rate $r_{cb}^t$ according to the Taylor rule given by:

\[
(1 + r_{cb}^t) = (1 + r^b) \left( 1 + r_{cb}^{t-1} \right)^{\phi_R} \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y(1-\phi_R)} \epsilon_t^r \quad (A.12)
\]

$\phi_R$, $\phi_\pi$, and $\phi_y$ are the weights assigned to interest rate persistence, inflation, and output responses of monetary policy respectively. $\epsilon_t^r$ is assumed to be an exogenous stochastic shock meant to capture monetary policy shocks. The steady state policy rate ($r_{cb}^*)$ is pinned down through the rate of time preference and the steady state markup over the deposit rate.

A.9 Market Clearing

Market clearing for the model is given by:

\[
y_t + \delta^e \frac{\Pi_{t-1}^R}{\Pi_t} = c_t + q_t^k \left[ k_t - (1 - \delta) k_{t-1} \right] + \delta \frac{K_{t-1}^b}{\Pi_t} + \delta^s \Pi_t^R + F s_{t-1}^H + \text{Adj}_t
\]

\[
1 = s_t^H + s_t^b
\]

Market clearing states that income is given in period $t$ from retailer distributed dividends out of period $t - 1$ profits but a fraction of retail profits today must be stored to be distributed out as dividends in the next period. The fraction of bank capital that depreciates, the portfolio management fee, as well as any adjustment costs that occur take away from consumption and investment in capital in period $t$. 
B Data

**Output**: Gross domestic product, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Consumption**: Personal consumption expenditures, quarterly, nominal, billions of dollars, seasonally adjusted annual rate (St. Louis Federal Reserve Economic Data)

**Investment**: Gross fixed capital formation, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Loans to Entrepreneurs**: Commercial and industrial loans, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Deposits**: Total savings deposits at all depository institutions, quarterly, nominal, billions of dollars, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Equity Price**: S&P 500 price, quarterly, nominal, not seasonally adjusted, (Robert Shiller’s *Irrational Exhuberance* Online Data)

**Consumer Price Index**: Consumer price index for all urban consumers: all items, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Gross Inflation Rate**: Ratio of CPI$_t$ to CPI$_{t-1}$, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Wage**: Nonfarm business sector compensation per hour, nominal, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Gross Wage Inflation Rate**: Ratio of W$_t$ to W$_{t-1}$, quarterly, seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Central Bank Rate**: Effective federal funds rate, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Interest Rate on Loans to Entrepreneurs**: Weighted-average effective loan rate of all C&I loans, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Interest Rate on Deposits**: M2 OWN rate, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Dividend Rate**: Dividends-per-share over earnings-per-share, quarterly, nominal, not seasonally adjusted (Robert Shiller’s *Irrational Exhuberance* Online Data)
**Loan-to-Value Ratio**: Total debt-to-equity for the United States converted to total debt-to-assets, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

**Volatility**: VIX, quarterly, not seasonally adjusted (St. Louis Federal Reserve Economic Data)

All nominal data are converted to real data by dividing by the consumer price index in decimal form found by dividing the consumer price index by 100.

Figure B.1: Raw Macro Data

Note: Data are logged.
Figure B.2: Transformed Macro Data

Note: Data are logged and detrended using the HP filter with smoothing parameter set at 1,600 as suggested by Ravn-Uhlig (2002) except for the VIX which is logged and demeaned.

Figure B.3: Interest Rates

Note: Rates are expressed on a quarterly basis and demeaned.
Note: Rates are expressed on a quarterly basis and demeaned.
C Bayesian Estimation: Shock Posterior Distributions, Convergence, and Identification

Figure C.1: Posterior Distribution of AR(1) Coefficients

Figure C.2: Posterior Distribution of Shock Standard Deviations

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm. Identification for each parameter is assessed by the magnitude of the bar.

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Figure C.5: Multivariate Convergence Diagnostics

![Graph](image)

Figure C.6: Univariate Convergence Diagnostics: Structural Parameters

![Graph](image)

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Figure C.7: Univariate Convergence Diagnostics: Structural Parameters

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Figure C.8: Univariate Convergence Diagnostics: Shock Standard Deviations

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Figure C.9: Univariate Convergence Diagnostics: Shock Standard Deviations

Figure C.10: Univariate Convergence Diagnostics: AR(1) Coefficients

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.
Figure C.11: Univariate Convergence Diagnostics: AR(1) Coefficients

Note: Results based on 16 chains of 100,000 draws each from the MH-MCMC algorithm.