Product Line Rivalry with Strategic Capacity Allocation

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Abstract

This paper analyzes the *strategic capacity allocation* of an international oligopoly. Because a line of products shares specific inputs that are fixed in the short run, a multiproduct oligopolist faces a *capacity constraint* in the production. Not being able to produce the desirable quantities to meet demand, an oligopolist has to strategically allocate its capacity among different products against its rival(s). If the market were monopolistic, a firm would mainly concern the effective profitability of a product when allocating its capacity and when responding to a capacity expansion. Identical duopolists that compete in a Cournot fashion should have identical capacity allocation. However, in a sequential game, the Stackelberg leader may allocate all its scarce capacity towards the more profitable product, while the follower will have to allocate some capacity towards the unprofitable product. This matches the observation that Boeing, the incumbent in the large commercial aircrafts (LCA) industry, specializes in smaller plane such as the 787, while Airbus makes both the superjumbo A380 and smaller planes like A350.

Keywords: Multinational Corporations; Multiproduct; Oligopoly

JEL classification: F12; F13; F23; L12; L13

1. Introduction

Some industries require firms to install expensive, specialized inputs before production starts. Naturally, there is a huge entry cost. These firms enjoy economies of scale by spreading the cost over cumulative production. With little trade barriers, the firms grow into an international oligopoly. For example, Apple, Huawei and Samsung divide most of the world's smartphone market. The plane-makers, Boeing and Airbus, form a duopoly in the world market of large civil aircrafts (LCA).[1] An international oligopolist does not pay a huge cost for inputs that can only be used to produce one product. Rather, it develops a line of products that utilize the same inputs. For examples, Apple hires computer-engineering expertise to design both iPhones and iPads. Airbus builds specially equipped factories for manufacturing both A380 and A350. Hence, an international oligopolist can spread the cost of inputs over multiple products. In this case, there are "economies of scope" because it is less costly to manufacture multiple products collectively in one firm than individually in different firms (Baumol, Panzar, & Willig, 1982).[2] Nevertheless, inputs so specific to a firm's product line must be limited, at least in the short run. Using more of a specific input to make one product means less of the input is available for making another product. Even if the world market is great enough, an international oligopolist cannot freely produce the desirable quantities to meet demand. With such *capacity constraint*, a firm has to allocate its capacity among different products wisely and it has to do so strategically when there is competition. This paper provides a framework to analyze *strategic capacity allocation* of an international oligopoly.

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[1] Large civil aircraft (LCA) is a term dubbed by the WTO and the U.S. International Trade Commission (2001). Boeing refers to them as "commercial airplanes" and Airbus calls them "passenger aircrafts."
[2] Baumol et al. (1982) assumed firms take the price of each product as given before entering the markets. Their outcome is similar to perfect competition. This paper focuses on industries in which entry is limited. Therefore, an oligopoly model is necessary, especially for analyzing strategic interactions.

It turns out that the product selection of an international oligopoly can be puzzling. The LCA industry is a novel example.[3] Boeing was a near monopoly until the entry of Airbus, and the two have become a duopoly since the 1980s.[4] Boeing and Airbus are equally resourceful and have virtually identical production technology. In particular, both are capable of manufacturing a family of planes. In early 1990s, Boeing and Airbus considered building "super-jumbo" planes together, but Boeing quit (The Economist, 2001). Airbus became the first to introduce the world's largest and the world's only double-decker jet airliner, the A380, in 2007.[5] Boeing has continued to devote all of its resources to the production of smaller planes, including the newest Boeing 787. Table 2 shows that the orders for Boeing 787 have skyrocketed and its deliveries have sped up since 2011. After witnessing the commercial success of Boeing 787, Airbus introduced the A350 as a head-to-head rival and started its deliveries in 2014. However, Airbus has not reallocated much resource from the A380 to the A350. As shown in Table 1, production of A380 has been steady since 2007.

What is puzzling here is that despite having the established advantage of an incumbent and despite having the ability to produce large planes, Boeing has given it up to Airbus. In particular, this contradicts the common argument that an incumbent often preempts the product space to deter entry into substitutes. These previous arguments emphasized on product relation on the demand side. Two products are "related" because consumers use the products as *substitutes* or *complements*. Substitutability discourages product proliferation because sales of one product hurt sales of another product. For example, Brander and Eaton (1984) modeled firms that competed in a line of four products. They assumed there were no economies of scope and products were substitutes. They

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[3] The LCA industry have always interested trade theorists because it had an oligopolistic, integrated world market where government policies were prominent (Dixit & Kyle, 1985).

[4] Baldwin and Krugman (1988) observed that Lockheed and McDonnell-Douglas were no competition to Boeing in the 1980s, and concluded that the market was not large enough to sustain more than one firm. The sizeable orders in Table 1 imply there is a much bigger demand today, so a duopoly is sustainable.

[5] A380 has 40% more floor space than the next largest airliner, Boeing 747-8. Boeing 747 is only a partial double-decker jet airliner.

showed that in spite of the discouraging circumstances, incumbents would preempt the product space to deter entry. In particular, the incumbents produced two distant substitutes, rather than two close substitutes, when facing the threat of entry. However, this does not match the observations in the LCA industry. Boeing, the incumbent, does not deter the entry of Airbus by crowding out the product space. Rather, it yields the large-plane market to Airbus. In the model in this paper, the incumbent may allocate all of its limited capacity towards the more profitable small planes, pushing the new firm to allocate some of its limited resources to the less profitable large planes. Also, this result allows for more general product relation on the demand side – products can be substitutes, complements or unrelated.

Judd (1985) assumed there was no economies of scope and firms competed in two substitutes. He examined the credibility of product proliferation as an entry deterrent. Judd (1985) included an exit stage and found that if exit cost was not prohibitive, an incumbent might exit one of the markets to avoid head-to-head competition. The result was a differentiated duopoly – each firm produced a different product. This result does not match the observation in the LCA industry either. While Boeing specializes in small planes, Airbus makes both large planes and small planes. The model in this paper allows for such a case. With a capacity constraint, Boeing cannot satisfy the entire demand for small planes, leaving some opportunity for Airbus to produce them as well.

Gilbert and Matutes (1993) also modeled competition with product line rivalry and investigated whether product preemption was credible. They considered two products. There were brand differentiation in the sense that consumers considered a product made by different firms to be different. Similar to Brander and Eaton (1984) and Judd (1985), Gilbert and Matutes (1993) assumed the products to be substitutes.[6] Different from the other two papers, Gilbert and Matutes

[6] In Gilbert and Matutes (1993), the products, "basic" and "premium" were substitutes. "Basic" ("premium") products made by different firms were also imperfect substitutes.

(1993) assumed strong economies of scope. This should provide incentives for product proliferation. They concluded an incumbent's product spatial preemption was credible if brand differentiation was sufficiently large (substitutability was sufficiently weak). However, this paper assumes no brand differentiation because airlines consider Boeing and Airbus planes of the same sizes to be virtually perfect substitutes and stock up both brands in their fleets. Boeing and Airbus themselves also consider their planes (such as Boeing 787 and A350) to be head-to-head competition. In this case, Gilbert and Matutes (1993) would say that product proliferation could not be a credible entry deterrent. Expectedly, Boeing does not preempt the product space to deter entry. Rather, Boeing seems to welcome Airbus' entry into the large-plane market. Hence, this paper focuses on a rather "entry-welcoming" product selection of the incumbent.

This paper offers a unique analysis on *strategic capacity allocation* in a model of international oligopoly, and seeks for an explanation for the puzzling, entry-welcoming product selection of an incumbent, such as what Boeing have done in the LCA industry.[7] I shall investigate capacity allocation under different market structures. The next section presents a multiproduct monopoly that faces a capacity constraint. In this simple model, I derive basic insights of capacity allocation and assess the effect of capacity expansion. Section 3 presents a duopoly model, in which firms compete in a Cournot fashion. It analyses how a firm responses to the rival's capacity allocation strategies. To investigate how an incumbent interacts with an entrant, I will then look into a model of Stackelberg competition, which is closest to the type of competition between Boeing and Airbus in Section 4. The model provides insights into how Boeing can strategically utilize the constrained

[7] This paper is not the first to model production technology that is used by different products. For example, in Röller and Tombak's (1990) and Dixon's (1994) models, a firm had to develop a costly "flexible technology" in order to produce different products. Hence, the cost increased with scope, generating diseconomies. However, the approach here is very different. Believing that firms tend to develop new products that they can manufacture using existing technology, this paper assumes that the "flexible technology" is already in place and its cost is sunk. As a result, firms spread cost by manufacturing additional products, resulting in scope economies, rather than diseconomies.

capacity to its advantage by yielding the large-plane market to Airbus. Finally, Section 5 provides concluding remarks.

2. Monopoly

Before the entry of Airbus, Boeing was a near monopoly in the LCA market. Monopoly is the simplest case in which important insights can be drawn. Hence, let's first analyze how a firm allocates its capacity in an environment that is free of competition.

2.1 Model

Consider a firm in Home, which sells planes to the rest of the world (ROW). The firm has a line of two products – "large planes" (product 1) and "small planes" (product 2). The constant marginal costs of large planes and small planes are C^1 and C^2 respectively.[8]

The production technology is simple but distinctive. Suppose production of any plane from the product line requires an input that is highly specific in its use. This can be the specially equipped factory, the high-tech components, etc. Due to this nature of the input, by the time production starts, the firm cannot alter its amount. For simplicity, think of the fixed input as "factory space" in this model. Let Z acres of factory space be the production capacity that the firm has.

[8] If there are government subsidies, S^1 and S^2, the after-subsidy marginal costs will be $C^1 - S^1$ and $C^2 - S^2$. Neven and Seabright (1995) suggested that government's subsidy to Airbus was beneficial to Europe and to the rest of the world, but hurtful to the United States. Recently, there have been many disputes filed at the WTO (The WTO, 2015). (See Pavcnik (2002) for a thorough discussion of the trade disputes in the LCA industry.) Since the US also provided subsidies to Boeing, this should neutralize the impact of the European support to Airbus (Klepper, 1994). For this reason, this paper does not give spotlight to industrial policies or their welfare effects; rather, it focuses on the multiproduct feature of present day's LCA industry.

Developing planes is costly. Let K be the sunk cost, including the cost of building a factory of Z acres. By the time the firm decides on the scope, K is sunk. In other words, whether the firm makes no product, one product or two products from the line, the sunk cost is K. Joint production of the products is less costly than producing them separately. Hence, here exist *economies of scope*.

Because the factory space is fixed, once it is filled up, the firm cannot produce any more plane. Moreover, the factory space must be *rival* in its use. When the firm occupies certain area to produce a plane, it cannot produce another plane in the same area at the same time.[9] In other words, the firm's production is subject to a *capacity constraint*. More precisely, suppose producing a large plane requires θ^1 acres of factory space, and a small plane takes up θ^2 acres to build. Without a loss of generality, assume a large plane requires more factory space than a small plane such that $\theta^1 > \theta^2$. If the firm produces Q_1 large planes and Q_2 small planes, then

$$\theta^1 Q_1 + \theta^2 Q_2 \leq Z. \tag{1}$$

(1) says that the space used for producing large planes and small planes cannot sum up to more than the factory space the firm has. The emphasis of this paper is on how a firm allocates capacity among different products, so throughout the paper, there have to be two global assumptions.

(G1) *Small-capacity assumption*: Capacity, Z is sufficiently small; otherwise, there will be no constraint to the capacity.

(G2) *Large-demand assumption*: Demand for each product is sufficiently large, so that profit-maximizing Q_1 and Q_2 cannot be both equal to zero.

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[9] Without this assumption, the capacity constraint is not necessarily linear in Q_1 and Q_2. A more general capacity constraint would be some function, $\Phi(Q_1, Q_2) \leq Z$.

With these assumptions, the capacity constraint in (1) is binding. Later, I shall verify this by checking the Kuhn-Tucker conditions.

Now denote the quantities demanded of large planes and small planes as X_1 and X_2 respectively. The inverse demand of each product is a function of both quantities. That is, for $i = 1,2$, $P^i = P^i(X_1, X_2)$. Also define $P^i_j \equiv \frac{\partial P^i}{\partial X_j}$ and $P^i_{ij} \equiv \frac{\partial^2 P^i}{\partial X_i \partial X_j}$, where $i = 1, 2$ and $j = 1, 2$. Assume the demand functions satisfy standard properties that $P^i_i < 0$ and $P^i_{ii} < \xi$, where ξ is a sufficiently small positive number. Note that if consumers consider large planes and small planes to be *substitutes*, then $P^1_2 < 0$ and $P^2_1 < 0$.[10] This paper rules out the case that they are perfect substitutes; otherwise, they cannot be differentiated as two products. Hence, $P^1_2 \neq P^1_1$ and $P^2_1 \neq P^2_2$. If consumers consider them to be *complements*, then $P^1_2 > 0$ and $P^2_1 > 0$. If consumers consider them to be *unrelated* goods, then $P^1_2 = 0$ and $P^2_1 = 0$.

When the market of each product is in equilibrium, $X_i = Q_i$, and there is a single world price for each product: $P^i = P^i(Q_1, Q_2)$, for $i = 1,2$. The firm chooses the output level of each plane to maximize total profit:

$$\pi = P^1 Q_1 - C^1 Q_1 + P^2 Q_2 - C^2 Q_2 - K \quad \text{subject to} \quad \theta^1 Q_1 + \theta^2 Q_2 \leq Z, Q_1 \geq 0, Q_2 \geq 0 \qquad (2)$$

taking the demand functions, the marginal costs and the sunk cost as given.[11] Note that I assume output levels to be non-negative.

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[10] If $P^1_2 < 0$ and $P^2_1 < 0$, Brander and Eaton (1984) would call the products, "*q*-substitutes." Appendix A shows that if large planes and small planes are "*q*-substitutes," they are also "*p*-substitutes."

[11] Brander and Eaton (1984) explained how the central insights remain the same whether quantity or price is the choice variable. This paper considers quantity decisions because a firm can allocate its capacity by choosing output levels, which is the focus of this paper.

To highlight the role of the capacity constraint and for simplification, I express the variables in *effective terms* (denoted by lowercase letters).

Definitions: The capacity allocated to the production of product *i* is $q_i \equiv \theta^i Q_i$,

the effective marginal cost of product *i* is $c^i \equiv \frac{C^i}{\theta^i}$, and

the effective price of product *i* is $p^i \equiv \frac{P^i}{\theta^i}$,

for $i = 1{,}2$. In effective terms, the profit maximization problem becomes:

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0. \tag{3}$$

Notice that (3) preserves the structure of the constrained profit maximization problem in (2). I solve (3) using the Lagrangean ($\mathcal{L}$) method.[12] The Kuhn-Tucker conditions are

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda \leq 0, \quad q_1 \geq 0, \quad q_1(p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda) = 0 \tag{4a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda \leq 0, \quad q_2 \geq 0, \quad q_2(p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda) = 0 \tag{4b}$$

$$q_1 + q_2 \leq Z, \quad \lambda \geq 0, \quad \lambda(q_1 + q_2 - Z) = 0. \tag{4c}$$

Some of the conditions in (4) contradict with (G1) and (G2).[13] To be consistent with the assumptions, λ must be positive, so the capacity constraint in (1) is binding. Fig. 1 illustrates how

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[12] Alternatively, the maximization problem can be solved using the substitution method. By substituting the capacity constraint into the objective function, the multivariate constrained maximization problem becomes a univariate unconstrained maximization problem.

the monopoly allocates its capacity to large planes and small planes. I constructed the figure with linear demands and parameters that satisfy the model assumptions. The firm achieves optimal capacity allocation at the point where the isoprofit curve is tangent to the capacity line, *ZZ*. Point *M*, at which the firm allocates more capacity to small planes than to large planes, is just one possible solution. Different parameter space gives rise to different solutions along the *ZZ* line, which include the corner solutions of $(Z, 0)$ and $(0, Z)$.

To analyze the conditions for each solution, assume the demand functions are linear such that $P^1(X_1, X_2) = A^1 - B^1 X_1 - \Gamma^1 X_2$ and $P^2(X_2, X_1) = A^2 - B^2 X_2 - \Gamma^2 X_1$. A^1, A^2, B^1 and B^2 are positive constants. Γ^1 and Γ^2 determine how, if at all, the products are related. Once again, express the coefficients of the demand functions in effective terms.

Definitions: For $i = 1, 2$, $a^i \equiv \frac{A^i}{\theta^i} > 0$, $b^i \equiv \frac{B^i}{(\theta^i)^2} > 0$ and $\gamma^i \equiv \frac{\Gamma^i}{\theta^1 \theta^2}$.

The first-order conditions with respect to q_1 and q_2 are

$$a^1 - c^1 - 2b^1 q_1 - (\gamma^1 + \gamma^2) q_2 - \lambda = 0, \tag{5a}$$

$$a^2 - c^2 - 2b^2 q_2 - (\gamma^1 + \gamma^2) q_1 - \lambda = 0, \tag{5b}$$

$$q_1 + q_2 = Z. \tag{5c}$$

(5) shows that a firm's capacity allocation depends on how the products are related (or unrelated) on the demand side. The airlines (the consumers) may consider large plane and small plane as

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[13] If capacity, Z is small enough and demands are large enough (a^1 and a^2 are large enough), then it is true that $Z < \frac{a^1 - c^1}{2b^1}$, $Z < \frac{a^2 - c^2}{2b^2}$ and $Z < \frac{(2b^2 - \gamma^1 - \gamma^2)(a^1 - c^1) + (2b^1 - \gamma^1 - \gamma^2)(a^2 - c^2)}{4b^1 b^2 - (\gamma^1 + \gamma^2)^2}$.

substitutes because they function similarly in providing air transportation. In this way, $\gamma^1 > 0$ and $\gamma^2 > 0$. According to (5), marginal profit of a product will depend on the production of (and capacity devoted to) another product negatively. This is because higher sales of one product will reduce sales of the other product, which is known as "cannibalization."[14] The higher is the substitutability of the products, the stronger is the effect of cannibalization. On the other hand, airlines may want to buy both large planes and small planes in order to diversify the fleet and serve different routes. That means the products can *complement* each other. If $\gamma^1 < 0$ and $\gamma^2 < 0$ in (5), marginal profit of a product will depend on the production of (and capacity devoted to) another product positively. The more complementary the products are, the more likely the airlines will buy both products. Finally, if the airlines consider the products to be *unrelated* (or if the substitution effect and the complementary effect exactly offset each other), $\gamma^1 = 0$ and $\gamma^2 = 0$ in (5). Marginal profit of a product does not depend on the production of (and capacity devoted to) another product.

Define $\mathcal{L}_{ij} \equiv \frac{\partial^2 \mathcal{L}}{\partial q_i \partial q_j}$ where $i = 1, 2$ and $j = 1, 2$. The second-order conditions for profit maximization are such that $\mathcal{L}_{11} = -2b^1 < 0$, $\mathcal{L}_{22} = -2b^2 < 0$ and that the determinant of the Hessian matrix, $H \equiv 2(b^1 + b^2 - \gamma^1 - \gamma^2) > 0$. The first two conditions are consistent with the model assumptions that b^1 and b^2 are positive. $H > 0$ if the products are complements or unrelated goods (i.e., $\gamma^1 \leq 0$ and $\gamma^2 \leq 0$). If the products are substitutes (i.e., $\gamma^1 > 0$ and $\gamma^2 > 0$) then I have to assume $b^1 + b^2 - \gamma^1 - \gamma^2 > 0$.

Solving (5) by Cramer's rule,

$$q_1^M = \frac{(a^1 - c^1) - (a^2 - c^2) + (2b^2 - \gamma^1 - \gamma^2)Z}{H} = Z - q_2^M, \tag{6}$$

[14] For example, the term "cannibalization" appeared in Lambertini (2003).

where the "*M*" superscript denotes optimal capacity allocation of the monopoly. Recall that the capacity in (1) has to bind in this paper, so q_1^M implies $q_2^M = Z - q_1^M$. The term, $(a^1 - c^1) - (a^2 - c^2)$ in (6) is important throughout the paper. The greater the difference between the vertical intercept of the demand curve, A, and the marginal cost, C, the greater is the marginal profit.[15] $(a - c)$ is simply $(A - C)$ adjusted for capacity requirement (θ) of the product. Hence, $(a^1 - c^1) - (a^2 - c^2)$ tells how the effective profitability of large planes compares to that of small planes. Orders for a product in Table 2 is a proxy of the demand for the product. The higher orders for small planes indicate that there is greater demand for small planes than for large planes. This implies that $a^1 < a^2$. Also, it is reasonable to believe that a small plane costs less to make than a large plane That is, $c^1 > c^2$. Hence, this paper mainly focuses on the situation when

$$(a^1 - c^1) - (a^2 - c^2) < 0\,. \tag{A1}$$

That is, small planes are *effectively more profitable* than large planes.

With (6), I can compare the monopolist allocates capacity to different products. The following assumption is useful.

$$(a^1 - c^1) - (a^2 - c^2) < (b^1 - b^2)Z\,. \tag{A2}$$

[15] Effective marginal profit of a large plane is $a^1 - 2b^1q_1 - \gamma^1 q_2 - c^1$. Effective marginal profit of a small plane is $a^2 - 2b^2q_2 - \gamma^2 q_1 - c^2$.

How b^1 and b^2 compare is unknown in general. It depends on the price elasticity of demand for each plane. If b^1 and b^2 are not too different, (A2) is basically an assumption about the relative effective profitability of small planes.

Lemma 1: If (A2) is true, then $q_2^M > q_1^M$. A monopolist allocates more capacity to small planes than to large planes because the effective profitability of small planes is sufficiently higher.

Note that capacity requirements also play a part. Recall that a large plane requires more capacity to make than a small plane (i.e., $\theta_1 > \theta_2$). Hence, even if $q_1 = q_2$, the firm will have greater output of small planes than large planes (i.e. $Q_2 > Q_1$). In other words, even if the products are effectively equally profitable and $b^1 \approx b^2$, the firm will produce more small planes simply because each large plane has a greater capacity requirement.

2.2 Capacity Expansion

(6) shows that capacity allocation depends on how big the factory is. This subsection analyzes the effect of a capacity expansion. Differentiating (6) with respect to Z:

$$\frac{\partial q_1^M}{\partial Z} = \frac{2b^2 - \gamma^1 - \gamma^2}{H}, \tag{7a}$$

$$\frac{\partial q_2^M}{\partial Z} = \frac{2b^1 - \gamma^1 - \gamma^2}{H}\,. \tag{7b}$$

If the products are complements or unrelated goods ($\gamma^1 \leq 0$ and $\gamma^2 \leq 0$), then (7a) and (7b) will be positive. If the products are substitutes ($\gamma^1 > 0$ and $\gamma^2 > 0$), then the signs of $2b^2 - \gamma^1 - \gamma^2$ and $2b^1 - \gamma^1 - \gamma^2$ are generally unknown. However, the second-order condition assumes that $b^1 + b^2 - \gamma^1 - \gamma^2 > 0$. If b^1 and b^2 are not too different, then it will as well be that $2b^2 - \gamma^1 - \gamma^2 > 0$ and $2b^1 - \gamma^1 - \gamma^2 > 0$, so (7a) and (7b) will be positive.

Lemma 2: If both (7a) and (7b) are positive, then capacity allocation to each product depends on factory space positively. Capacity expansion has a *normal* effect on capacity allocation. Otherwise, expansion has an *inferior* effect on capacity allocation.

Fig. 2 illustrates how capacity allocation changes when factory space, Z doubles. If capacity expansion is "normal," the new allocation is northeast to the original allocation at M. In the figure, M_2, M_3 and M_4 are some of the normal cases. If the capacity expansion is "inferior," the new allocation will not be at the northeast of M. Some of the inferior allocations are M_1 and M_5.

Now let's take a closer look at each of the five zones in Fig. 2. If (7a) is positive but (7b) is negative, the new capacity allocation falls into the "ultra-pro-large" zone. In other words, when the factory expands, the firm allocates more capacity to large planes, but less capacity to small planes. Small planes are "inferior." In Fig. 2, M_1 is an example of ultra-pro-large allocation. Conversely, (7a) can be negative while (7b) is positive, the new capacity allocation falls into the "ultra-pro-small" zone. That is, when the factory expands, the firm allocates more capacity to small planes, but less capacity to large planes. Large planes are "inferior." In Fig. 2, M_5 is an example of ultra-pro-small allocation.

Lemma 3: If (7a) is positive but (7b) is negative, monopolistic capacity expansion is *ultra-pro-large*. If (7b) is positive but (7a) is negative, monopolistic capacity expansion is *ultra-pro-small*.

Within the "normal" zone, there are the "pro-large," the "neutral" and the "pro-small" zones. After a factory expansion, if the proportion of capacity allocated to large planes increases, while the proportion of allocated to small planes decreases, the firm's capacity allocation is pro-large. In Fig. 2, M_2 is an example of pro-large allocation. Oppositely, if an expansion to the capacity decreases the proportion of capacity allocated to large planes, but increases that of small planes, the firm's capacity allocation is pro-small. In Fig. 2, M_4 is one pro-small allocation. Finally, if both proportions remain unchanged after a factory expansion, the firm's capacity allocation is neutral. M_3 is the neutral allocation in Fig. 2. The conditions on the proportions in the three normal zones are as follow:

Lemma 4: Assume both (7a) and (7b) are positive. If $(a^2 - c^2) > (a^1 - c^1)$, capacity expansion is *pro-large*. If $(a^1 - c^1) = (a^2 - c^2)$, capacity expansion is *neutral*. If $(a^1 - c^1) >$ $(a^2 - c^2)$, capacity expansion is *pro-small*.

Lemma 4 seems to be counter-intuitive at the first glance, but it is not. Let's take the "pro-large" case as an example. Recall from Lemma 1 that if small planes are sufficiently more profitable than large planes, the firm will allocate most of its limited capacity to small planes. That is, facing a capacity constraint, the firm has to neglect large planes to certain extent. Following a "normal" capacity expansion, the firm allocates more capacity to both products. However, since small planes already took up a lot of capacity, the firm will assign more of the additional capacity to large planes. Therefore, by proportion, the firm allocates more of the new capacity to the effectively less profitable product.

3. Duopoly – Cournot Competition

The monopoly case provides basic insights to a firm's capacity allocation. What is more interesting is how a firm strategizes its capacity allocation when facing rivalry. This section and the next section explore how competition influences capacity allocation. They can provide more understanding about the competition between Boeing and Airbus.

3.1 Model

Suppose a firm in the domestic country and a firm in the foreign country sell planes to the ROW.[16] Hereafter, I will use asterisks (*) to distinguish variables of the foreign firm. The purpose of this paper is to compare similar firms (i.e., Boeing versus Airbus), so I first simplify the model by assuming the firms to be identical. In particular, the firms have same constant marginal costs, C^1 and C^2, same sunk cost, K and same capacity, Z. Also, the factory space needed for producing a large plane is θ^1 and that for a small plane is θ^2, regardless of who produces them. In the next subsection, I shall relax this assumption and investigate how firm differentiation can affect the firms' strategic capacity allocation.

[16] As mentioned in the introduction, there is vertical (intra-firm) product differentiation, but no horizontal (inter-firm) product differentiation. For models that include both dimensions of differentiation, see Brander and Eaton (1984), Canoy and Peitz (1997) and Gilbert and Matutes (1993).

Assume the market of each product is in equilibrium. Henceforth, $X_i = Q_i + Q_i^*$ and the world price is $P^i = P^i(Q_1 + {Q_1}^*, Q_2 + {Q_2}^*)$, for $i = 1{,}2$.

In this section, I assume the firms make decisions *simultaneously*. In other words, the firms compete in *Cournot* fashion. The domestic firm chooses the acres of factory space to be allocated to each plane to maximize total profit subject to the capacity constraint, taking the foreign firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given:

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0. \tag{8}$$

Using the Lagrangean method, the Kuhn-Tucker conditions are

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda \leq 0, \quad q_1 \geq 0, \quad q_1(p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda) = 0 \tag{9a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda \leq 0, \quad q_2 \geq 0, \quad q_2(p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda) = 0 \tag{9b}$$

$$q_1 + q_2 \leq Z, \quad \lambda \geq 0, \quad \lambda(q_1 + q_2 - Z) = 0. \tag{9c}$$

Similarly, the foreign firm chooses the amount of fixed input to be allocated to each plane to maximize

$$\pi^* = p^1 q_1^* - c^1 q_1^* + p^2 q_2^* - c^2 q_2^* - K \qquad \text{subject to} \qquad q_1^* + q_2^* \leq Z, q_1^* \geq 0, q_2^* \geq 0. \tag{10}$$

taking the domestic firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given. The Kuhn-Tucker conditions are

$$p^1 + p^1_1 q^*_1 - c^1 + p^2_1 q^*_2 - \lambda^* \leq 0, \quad q^*_1 \geq 0, \quad q^*_1(p^1 + p^1_1 q^*_1 - c^1 + p^2_1 q^*_2 - \lambda^*) = 0 \tag{11a}$$

$$p^2 + p^2_2 q^*_2 - c^2 + p^1_2 q^*_1 - \lambda^* \leq 0, \quad q^*_2 \geq 0, \quad q^*_2(p^2 + p^2_2 q^*_2 - c^2 + p^1_2 q^*_1 - \lambda^*) = 0 \tag{11b}$$

$$q^*_1 + q^*_2 \leq Z, \quad \lambda^* \geq 0, \quad \lambda^*(q^*_1 + q^*_2 - Z) = 0. \tag{11c}$$

Recall that the global assumptions (G1) and (G2) must continue to hold.[17] Indeed, the conditions on the parameters have to be stricter than when there was only one firm. Now, the demands have to be big enough for both firms to profitably produce some output. Also, because the firms share the markets, each firm needs a smaller factory for production. With (G1) and (G2), the capacity constraint must be binding, so I only consider cases when $\lambda > 0$ among the Kuhn-Tucker conditions in (9) and (11).

Given q^*_1 and q^*_2, the first-order conditions of the domestic firm are:

$$p^1 + p^1_1 q_1 - c^1 + p^2_1 q_2 - \lambda = 0, \tag{12a}$$

$$p^2 + p^2_2 q_2 - c^2 + p^1_2 q_1 - \lambda = 0, \tag{12b}$$

$$q_1 + q_2 = Z. \tag{12c}$$

The second-order conditions are the same as before: $\mathcal{L}_{11} < 0$, $\mathcal{L}_{22} < 0$ and $H > 0$.

The first-order conditions of the foreign firm are:

$$p^1 + p^1_1 q^*_1 - c^1 + p^2_1 q^*_2 - \lambda^* = 0, \tag{13a}$$

$$p^2 + p^2_2 q^*_2 - c^2 + p^1_2 q^*_1 - \lambda^* = 0, \tag{13b}$$

[17] If capacity, Z is small enough and demands are large enough (a^1 and a^2 are large enough), then it is true that $Z < \frac{(b^1-\gamma^1+2b^2-2\gamma^2)(a^1-c^1)+(b^1-\gamma^1)(a^2-c^2)}{2b^1H+2b^1b^2-2\gamma^1\gamma^2-(b^1-\gamma^1)^2}$, $Z < \frac{(2b^1-2\gamma^1+b^2-\gamma^2)(a^2-c^2)+(b^2-\gamma^2)(a^1-c^1)}{2b^2H+2b^1b^2-2\gamma^1\gamma^2-(b^2-\gamma^2)^2}$ and $Z < \frac{(3b^2-\gamma^1-2\gamma^2)(a^1-c^1)+(3b^1-2\gamma^1-\gamma^2)(a^2-c^2)}{9b^1b^2-(2\gamma^1+\gamma^2)(\gamma^1+2\gamma^2)}$.

$$q_1^* + q_2^* \leq Z. \tag{13c}$$

Define $\mathcal{L}_{ij}^* \equiv \frac{\partial^2 \mathcal{L}^*}{\partial q_i \partial q_j}$ where $i = 1, 2$ and $j = 1, 2$.The second-order conditions are $\mathcal{L}_{11}^* = \mathcal{L}_{11} < 0$, $\mathcal{L}_{22}^* = \mathcal{L}_{22} < 0$ and $H > 0$.

In the next subsection, I will solve (12) and (13) for reaction functions and derive the Nash-Cournot equilibria.

3.2 Nash-Cournot Equilibrium of Identical Firms

Solving (12) gives reaction function:

$$q_1 = \varphi - \frac{q_1^*}{2}, \tag{14}$$

where $\varphi \equiv \frac{(a^1 - c^1) - (a^2 - c^2) + (3b^2 - 2\gamma^1 - \gamma^2)Z}{H}$ is a constant. Note that I have simplified the expression using the binding capacity constraint: $q_2 = Z - q_1$. Similarly, solving (13) yields reaction function:

$$q_1^* = \varphi - \frac{q_1}{2}, \tag{15}$$

Fig. 3 plots the two reaction functions. In Panel (a), the intersection point of the reaction curves, point *e*, gives the Nash-Cournot-equilibrium capacity allocation to large planes. Denote the Nash-Cournot-equilibrium capacity allocation with superscript "*D*." With a binding capacity constraint, the capacity allocation to small planes are simply, $q_2^D = Z - q_1^D$ and $q_2^{*D} = Z - q_1^{*D}$. Since the two reaction functions are symmetric, the firms have identical capacity allocation in Nash-Cournot equilibrium. Solving the two reaction functions simultaneously yield:

$$q_1^D = q_1^{*D} = \frac{2}{3}\frac{(a^1 - c^1) - (a^2 - c^2) + (3b^2 - \gamma^2 - 2\gamma^1)Z}{H}. \tag{16a}$$

which implies that

$$q_2^D = q_2^{*D} = \frac{2}{3}\frac{(a^2 - c^2) - (a^1 - c^1) + (3b^1 - \gamma^1 - 2\gamma^2)Z}{H}. \tag{16b}$$

Lemma 5: Identical duopolists that compete in a Cournot fashion have identical capacity allocation in the equilibrium.

According to (16b), if

$$(a^2 - c^2) - (a^1 - c^1) \leq -(3b^1 - 2\gamma^2 - \gamma^1)Z \quad , \tag{A3}$$

each duopolist will assign no capacity to small planes: $q_2^D = q_2^{*D} = 0$.[18] That is, each duopolist will assign all the capacity to produce large planes: $q_1^D = q_1^{*D} = Z$. Fig. 3 Panel (b) illustrates this case. When small planes are much less profitable (i.e., large planes are much more profitable), the intersection point of the reaction curves at point A is above each firm's capacity of Z. Given the foreign capacity allocation, domestic firm's best response is to allocate all the capacity to produce

[18] I assumed output levels (and so the capacity allocation) to be non-negative.

large planes at point *B*. If the domestic firm allocates all capacity to large planes, the foreign firm's best response is also to allocate all the capacity to large planes. Hence, the equilibrium point is at point *e*.

By the same token, (16a) says that if

$$(a^1 - c^1) - (a^2 - c^2) \leq -(3b^2 - 2\gamma^1 - \gamma^2)Z \quad , \tag{A4}$$

each duopolist will assign no capacity to large planes: $q_1^D = q_1^{*D} = 0$. That means each duopolist will assign all the capacity to produce small planes: $q_2^D = q_2^{*D} = Z$. Notice that $\varphi \leq 0$. As shown in Panel (c) of Fig. 3, the intersection point of the reaction functions at point *A* lies below the origin, which is not possible because capacity allocation cannot be negative. Given the foreign capacity allocation, the best the domestic firm can do is to allocate no capacity to produce large planes at point *B*. If the domestic firm allocates no capacity to large planes, the foreign firm will not allocate any capacity to large planes either. Thus, point *e*, the origin, is the equilibrium point.

Lemma 6: If (A3) is true, then $q_1^D = q_1^{*D} = Z$. In Nash-Cournot equilibrium, both duopolists allocate all the capacity to produce large planes. If (A4) is true, then $q_1^D = q_1^{*D} = 0$. In Nash-Cournot equilibrium, both duopolists allocate all the capacity to produce small planes.

Lemma 5 and Lemma 6 have crucial meaning. If Boeing and Airbus were identical and they compete in a Cournot fashion, they would have the same capacity allocation. It would not give rise to the situation that Boeing produced small planes only, while Airbus produced both small and large planes. As will be shown next, I have to adjust the assumptions in Lemma 5 and Lemma 6 in order to yield results that match real-life observations.

3.3 Nash-Cournot Equilibrium of Differentiated Firms

Many argue that Boeing and Airbus have similar production technology. That is why the previous subsections assumed the duopolists to be identical. Without the assumption, the firms can have different constant marginal costs, sunk cost, capacity, and even different capacity requirements for making each product. For example, suppose it is effectively more costly for the foreign firm to manufacture small planes than the domestic firm. In particular, assume the duopolists have the same firm characteristics except that $c^{2*} > c^2$. The reaction function of the foreign firm becomes:

$$q_1^* = \varphi^* - \frac{q_1}{2}, \tag{17}$$

where $\varphi^* \equiv \frac{(a^1 - c^{1*}) - (a^2 - c^{2*}) + (3b^2 - 2\gamma^1 - \gamma^2)Z^*}{H}$ is a constant. Notice that if $c^{2*} > c^2$, $c^{1*} = c^1$ and $Z^* = Z$, then $\varphi^* > \varphi$. As shown in Panel (a) of Fig. 4, the intersection point at point *A* gives a negative q_1^D, which is impossible for the domestic firm. Given the foreign firm's capacity allocation, the domestic firm's best response not to allocation any capacity to large planes. At the same time, given the domestic firm's action, the foreign firm's best response is to allocate some capacity to each product at point *e*. Hence, point *e* is the Nash-Cournot equilibrium point.

Similarly, suppose it costs less for the foreign firm to produce large planes, $c^{1*} < c^1$, while all other firm characteristics are the same. According to (17), $\varphi^* > \varphi$. As illustrated in Panel (b) of Fig. 4, the Nash-Cournot equilibrium point is at point *e*, just as how it was derived in Panel (a).

Lemma 7: Other things equal, if $c^{2*} > c^2$ (or $c^{1*} < c^1$), it is possible that $q_1^D = 0$ and $0 < q_1^{*D} < Z$. If it is effectively more costly for the foreign firm to produce small planes (or effectively less costly to produce large planes), the domestic firm may allocate no capacity to large planes while the foreign firm allocates capacity to both products in the Nash-Cournot equilibrium.

Therefore, if Airbus is less (more) efficient than Boeing in making small (large) planes, it is well possible that Boeing specializes in small planes, but Airbus does not.

It is also possible that the firms differ in resources. Suppose the duopolists only differ in their capacity such that the foreign firm is more resourceful, $Z^* > Z$. As indicated in (17), $\varphi^* > \varphi$. As shown in Panel (c) of Fig. 4, the capacity of the foreign firm, $Z^*$is higher than that of the domestic firm, Z. The intersection point at point *A* gives a negative q_1^D, so the domestic firm's best response is to allocate no capacity to large planes. The foreign firm's best response is to allocate some capacity to large planes (and small planes) because it has sufficient capacity to do so. Therefore, point *e* gives the Nash-Cournot equilibrium point.

Lemma 8: Other things equal, if $Z^* > Z$, it is possible that $q_1^D = 0$ and $0 < q_1^{*D} < Z$. The less resourceful domestic firm allocates no capacity to large planes, but the more resourceful foreign firm allocates capacity to both products in the Nash-Cournot equilibrium.

Hence, Airbus can allocate capacity to produce both products, but Boeing cannot maybe simply because Airbus has a larger factory in the first place.

Firm differentiation is one possible answer to the puzzling product selection of Boeing. However, this has not solved the whole mystery entirely yet. Boeing does not seem to act simultaneously as Airbus. Indeed, Boeing acts first as an incumbent. Rather than engaging in a Cournot competition, the two seem to engage in a Stackelberg competition, which shall be explored in the next section.

3.4 Cournot Duopoly Versus Monopoly

Before moving onto the next section, let's see how competition has affected capacity allocation by comparing the results here to the monopoly. As in Subsection 3.2, let's assume again that firms are identical in order to compare (16) to (6). When comparing (16) and (6), the comparison of γ^1 and γ^2 is necessary, which is unknown in general. They depend on how well each product acts as a substitute or a complement to the other product. If large planes are stronger substitutes (or weaker complements) to small planes than small planes are to large planes, then

$$\gamma^1 > \gamma^2 . \tag{A5}$$

If (A5) is true, (A6) is stricter than (A1):

$$(a^1 - c^1) - (a^2 - c^2) < (\gamma^2 - \gamma^1)Z. \tag{A6}$$

Lemma 9: If (A5) and (A6) are true, $q_1^D > q_1^M$. A firm allocates more capacity to large planes (less capacity to small planes) as a duopolist than as a monopolist.

If large planes more strongly substitute (or more weakly complement) small planes, large planes should be the unfavorable one along the product line. Together with (A6) that small planes are much more profitable than large planes, a monopolist should find small planes more attractive. However, when there is competition, any profit is shared away by the foreign firm. Hence, Lemma 8 concludes that a firm will allocate less capacity to small planes as a duopolist than it did as a monopolist. In Fig. 5, point *M* is the capacity allocation of the monopolist and point *D* is that of the duopolist. The figure assumes (A5) and (A6), so point *M* may lean more towards the small-plane side than point *D*.

Let's also compare the effect of capacity expansion of a duopoly to that of a monopoly. The monopoly's case is an internal expansion. As what Fig. 2 has illustrated, the domestic firm's own factory space has doubled to $2Z$ as a monopolist. When the market of large planes is in equilibrium, $x_1^M = q_1^M$. The duopoly's case is an external expansion. The capacity in the world increased because the foreign firm has joined the market, bringing along more factory space. In other words, $Z + Z^* = 2Z$. When the market of large planes is in equilibrium, $x_1^D = q_1^D + q_1^{*D}$.

Lemma 10: (A5) and (A6) imply $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $x_1^D < x_1^M$. An expanded monopoly allocates more capacity to large planes (less capacity to small planes) than a duopoly as a whole.

Lemma 9 and Lemma 10 show that under the same assumptions, q_1^D and q_1^M do not compare in the same way as x_1^D and x_1^M. If the duopolists are identical, duopolistic capacity expansion must look "neutral." However, according to Lemma 4, monopolistic capacity expansion will be "pro-large" if small planes are sufficiently effectively more profitable than large planes. Hence it is possible that the allocation of the expanded monopoly will lean more towards the large-plane side than the joint allocation of the duopoly.

I plot points *M* and *D* in Fig. 6 the same way I did in Fig. 5, assuming the same conditions. After capacity expansion, the monopoly's capacity allocation is at point *M'*. Point *D+D** refers to the combined capacity allocation of the duopoly. Fig. 6 illustrates the possible comparison that point *M'* leans more towards large planes than point *D+D**.

4. Duopoly - Stackelberg Competition

The previous section shows how competition influences a firm's capacity allocation. This section will show that the impact of rivalry is even more protruding when firms allocate capacity *sequentially*. History has it that Boeing was the incumbent and Airbus was the entrant. Boeing could make production decisions before Airbus. In other words, Boeing was the *Stackelberg leader* while Airbus was the *follower* in the competition. Previous section explains Boeing's product selection by assuming the firms have different production technology. In this section, I shall prove that even if the firms are assumed to be identical, it is possible that Boeing allocates no capacity to large planes at all simply because Boeing is taking advantage of its follower through strategic capacity allocation.

4.1 Model

Consider a situation when the domestic firm chooses its capacity allocation in the first stage and the foreign firm does so in the second stage, taking the domestic firm's actions as given. In other words, the domestic firm is a Stackelberg leader and the foreign firm is a follower. I can solve the two-stage game by backward induction. In the second stage, the foreign firm allocates capacity to each product to maximize total profit, taking the domestic firm's capacity allocation, the demand functions, the marginal costs and the sunk cost as given.

$$\pi^* = p^1 q_1^* - c^1 q_1^* + p^2 q_2^* - c^2 q_2^* - K \qquad \text{subject to} \qquad q_1^* + q_2^* \leq Z^*, q_1^* \geq 0, q_2^* \geq 0. \tag{18}$$

The first-order conditions of the foreign firm are:

$$p^1 + p_1^1 q_1^* - c^1 + p_1^2 q_2^* - \lambda^* = 0, \tag{19a}$$

$$p^2 + p_2^2 q_2^* - c^2 + p_2^1 q_1^* - \lambda^* = 0, \tag{19b}$$

$$q_1^* + q_2^* \leq Z. \tag{19c}$$

The second-order conditions are $\mathcal{L}_{11}^* < 0$, $\mathcal{L}_{22}^* < 0$ and $H > 0$. The reaction function is:

$$q_1^* = \varphi - \frac{q_1}{2}. \tag{20}$$

Again, $\varphi \equiv \frac{(a^1-c^1)-(a^2-c^2)+(3b^2-2\gamma^1-\gamma^2)Z}{H}$ is a constant, and I have simplified the expression using the binding capacity constraint: $q_2^* = Z - q_1^*$. From (20), I can derive the reaction function of capacity allocated to small planes in response to large planes, $q_2^* = q_2^*(q_1)$.

In the first stage, the domestic firm allocates capacity to each product to maximize total profit:

$$\pi = p^1 q_1 - c^1 q_1 + p^2 q_2 - c^2 q_2 - K \qquad \text{subject to} \qquad q_1 + q_2 \leq Z, q_1 \geq 0, q_2 \geq 0, \tag{21}$$

foreseeing the foreign firm's best responses in (20) and taking the demand functions, the marginal costs and the sunk cost as given. Substituting (20) into (21), the effective prices are $p^i = p^i(q_1, q_2) = p^i\big(q_1 + q_1^*(q_1), q_2 + q_2^*(q_1)\big)$ for $i = 1{,}2$. The first-order conditions of the domestic firm are:

$$p^1 + p_1^1 q_1 - c^1 + p_1^2 q_2 - \lambda = 0, \tag{22a}$$

$$p^2 + p_2^2 q_2 - c^2 + p_2^1 q_1 - \lambda = 0, \tag{22b}$$

$$q_1 + q_2 = Z. \tag{22c}$$

The second-order conditions are: $\mathcal{L}_{11} < 0$, $\mathcal{L}_{22} < 0$ and $H > 0$. Continue to assume (G1) and (G2), the solution to (22) is:

$$q_1^S = \frac{(a^1 - c^1) - (a^2 - c^2) + 2(b^2 - \gamma^1)Z}{H} = Z - q_2^S, \tag{23}$$

where "*S*" refers to Stackelberg leader's optimal capacity allocation.

To solve for the foreign firm's equilibrium capacity allocation, I substitute (23) back into (20):

$$q_1^{*F} = \frac{1}{2}\frac{(a^1 - c^1) - (a^2 - c^2) + (4b^2 - 2\gamma^2 - 2\gamma^1)Z}{H} = Z - q_2^{*F}, \tag{24}$$

where "*F*" refers to follower's optimal capacity allocation.

4.2 Stackelberg Leader Versus Follower

Let's compare (23) and (24) to see how the Stackelberg leader have a different strategy than the follower.

Lemma 11: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < (3\gamma^1 - 2\gamma^2)Z$, so $q_1^S < q_1^{*F}$.

Lemma 11 assumes small planes to be more favorable: (i) small planes are effectively much more profitable than large planes, and (ii) small planes are stronger complements (or weaker substitutes). Under these assumptions, the Stackelberg leader will take advantage of its role and allocate more capacity to small planes than the follower does. The can explain why Boeing devotes relatively more resources into manufacturing small planes, while Airbus devotes relatively more resources to large planes. However, this does not fully answer why Boeing takes on the extreme route to not produce any large planes at all. This extreme case is possible in the present model. Setting $q_1^S = 0$ in (23),

$$(a^1 - c^1) - (a^2 - c^2) = -2(b^2 - \gamma^1)Z, \tag{25}$$

and substituting (25) into (24) yields

$$q_1^{*F} = \frac{(b^2 - \gamma^2)Z}{H} \quad . \tag{26}$$

Recall that the products are not perfect substitutes, so $b^1 > \gamma^1$ and $b^2 > \gamma^2$. Hence, $0 < q_1^{*F} < Z$. Then it must be true that $0 < q_2^{*F} < Z$. While the Stackelberg leader gives up the less appealing large planes altogether, the follower still has to devote resources to produce both large planes and small planes.

Lemma 12: If $q_1^S = 0$, then $0 < q_1^{*F} < Z$. When the Stackelberg leader allocates all the capacity to produce the effectively more profitable small planes, the follower allocates capacity to produce both products.

Fig. 5 illustrates the possible relative positions of point *S*, the capacity allocation of the Stackelberg leader, and point F^*, that of the follower. Point *S* is at the end of the capacity line while point F^* is somewhere along the line.

In fact,

$$\pi^S - \pi^{*F} = [(p^2 - c^2) - (p^1 - c^1)]q_1^{*F} > 0\,. \tag{27}$$

In other words, the Stackelberg leader earns higher profit than the follower. The analysis shows how the Stackelberg leader can take advantage of its position to strategize its capacity allocation against the follower. The orders for Boeing 787 and Airbus A350 in Table 2 and the fact that small planes should be less costly to make imply that the small planes are the more profitable product. Hence, Boeing has the established advantage to allocate its limited acres of factory space into manufacturing small planes; and Airbus can only respond by satisfying the remaining demand for small planes and using its limited resources to produce large planes. Therefore, what seems to be an *entry-welcoming* move by the incumbent is indeed a profitable one.

The results can also have implications on the firms' position in the competition. Recall in Section 3 that identical duopolists that compete in a Cournot fashion must have identical capacity allocation. If they are not identical, it is possible that the domestic firm specializes in small planes, while the foreign firm does not. In this section, the firms are assumed to be identical, but since the domestic firm is a Stackelberg leader, it can specialize in small planes, making the foreign firm produces both products.

Lemma 13: Given $q_1 = 0$ and $0 < q_1^* < Z$. If the firms are identical, the domestic firm must be a Stackelberg leader and the foreign firm must be a follower.

If Boeing and Airbus had different costs of production and/or different capacity, it is possible that the two competed in a Cournot fashion. If Boeing and Airbus were identical, it must be that Boeing was a Stackelberg leader while Airbus was a follower.

4.3 Stackelberg Duopoly, Cournot Duopoly and Monopoly

To see how different types of competition affects a firm's capacity allocation, this section compares results in (23) and (24) to those in previous sections. Comparing (23) to (6) shows that

Lemma 14: If (A5) is true, then $q_1^S < q_1^M$. A Stackelberg leader allocates less capacity to large planes (more capacity to small planes) than a monopolist.

When there was no rivalry, the domestic firm might not care about how its own large planes substitute (or complement) its small planes as long as it occupies both markets. However, as a Stackelberg leader, the domestic firm cannot ignore how the rival large planes strongly substitute (or weakly complement) its small planes. Therefore, the Stackelberg leader's capacity allocation inclines more towards small planes than it did as a monopolist. Since Fig. 5 assumes (A5), point *S* is farther on the small-plane side compared to point *M*.

The lemma below is based on (24) and (16).

Lemma 15: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $q_1^{*F} > q_1^D$. A follower allocates more capacity to large planes (less capacity to small planes) than a duopolist in a Cournot competition.

The follower allocates its capacity in a disadvantageous way. When small planes are more favorable, a firm can allocate less capacity toward small planes as a follower in the competition than when it could act simultaneously as the rival. Fig. 5 assumes (A5) and (A6), point F^* must lean more towards the large-plane side than point *D*.

With assumptions (A5) and (A6), Lemma 9, Lemma 14 and Lemma 15 together yield the result that $q_1^{*F} > q_1^D > q_1^M > q_1^S$, which is illustrated in Fig. 5.

As in Subsection 3.4, let's also compare the effect of an internal capacity expansion in the monopoly and an external capacity expansion in the duopoly. Under Stackelberg competition, $x_1^{SF} = q_1^S + q_1^{*F}$ when the market of large planes is in equilibrium.

Lemma 16: (A5) and (A6) imply that $(a^1 - c^1) - (a^2 - c^2) < 2(\gamma^1 - \gamma^2)Z$, so $x_1^{SF} < x_1^D$.

Together with Lemma 10, which has the same assumptions, $x_1^{SF} < x_1^D < x_1^M$. As explained in Subsection 3.4, with competition, the industry as a whole allocates more capacity to the product that is more demanded. Duopolistic capacity allocation also differs under different types of competition. Stackelberg competition results in capacity allocation that inclines towards the more demanded small planes than Cournot competition. This is because the Stackelberg leader has the advantage to allocate much more capacity towards small planes, and may even give up large planes altogether. In Fig. 6, point *S+F** refers to the combined capacity allocation of the Stackelberg leader and the follower. With the assumptions that (A5) and (A6), Fig. 6 shows how point *S+F** has the greatest capacity allocated to small planes, followed by point *D+D** and point *M'*.

5. Concluding Remarks

In some industries, the huge set-up cost is a natural entry barrier. The resulting economies of scale lead to an international oligopolistic market structure. The characteristics of and the competition in these international oligopolies motivate this paper. It is well known that the rivalry between the gigantic oligopolists is fierce. They do not compete in one product but a line of products. These products are not only related on the demand side – they can be substitutes or complements. On the cost side, products can share an input. Because the sunk cost for the input can be spread over scope, there are economies of scope. This paper maintains that the input specific to a firm's product line must be scarce. When such capacity constraint is binding, an oligopolist has to strategize its capacity allocation among different products because it cannot produce the desirable quantities to satisfy the large world demand. The simple model of monopoly illustrates how capacity allocation is done. It turns out that if small planes are sufficiently (effectively) more profitable, a monopolist already allocates more capacity towards small planes. Hence when there is a capacity expansion, the extra capacity will be left for producing large planes. That is, a "pro-large" allocation results. Competition has a significant effect on the capacity expansion. The duopolists' combined capacity allocation would lean more towards the more demanded small planes than the expanded monopoly. Whether the duopolists make production decisions simultaneously or sequentially is also crucial. The model shows that if the duopolists are identical, their capacity allocation must also be identical under Cournot competition. If duopolists are different, such as having different marginal cost in producing a product, it is possible that one duopolist allocates all capacity to one product, but the duopolist does not. Since many believe Boeing is more efficient, this is a possible reason why Boeing produces small planes only. However, many argue that Boeing and Airbus have similar production technology. Also, since Boeing was the incumbent, it should have acted like a Stackelberg leader in the game. The model of Stackelberg competition shows that the leader has the advantage to allocate all its capacity to the profitable product, leaving the follower to spend some precious capacity on the unprofitable product. This provides insight into why Boeing, the incumbent "yielded" the large-plane market altogether to Airbus. When indeed, Boeing was at the advantageous position to earn greater profit than Airbus by doing so. The results also have implications on the firms' position in the competition. If Boeing and Airbus had different costs of production and/or different capacity, it is possible that the two were competing in a Cournot fashion. If Boeing and Airbus were identical, to have such capacity allocation, it must be that Boeing was a Stackelberg leader while Airbus was a follower.

The present model is a simple framework that highlights strategic capacity allocation of multiproduct oligopolies, but it is open to possible extensions. For example, there could be a more complex product set, such as the one in Gilbert and Matutes (1993). To introduce brand differentiation, I can assume the Boeing and Airbus produce differentiated small planes and differentiated large planes. Nonetheless, I believe the central insights of this paper would remain, especially when small planes like Boeing 787 and Airbus's A350 were seen as very close substitutes.

While this model sheds some light on how strategic capacity allocation works, there is a lack of dynamics. It is of my interest to extend the model to incorporate cost adjustment over time. One natural approach is to allow a firm to invest (whose cost is included in K) to obtain greater amount of the fixed specific inputs, Z in the future. We can also consider the learning effect on the marginal costs, c^1 and c^2 if "the fixed input" in the model includes not only physical capital, but also human capital. Even more intriguing will be to allow the capacity requirements, θ^1 and θ^2, to vary over time. *Learning* may cause both θ^1 and θ^2 to drop over time. θ^1 and θ^2 may drop at different rates, so the learning curves of large planes and small planes may not be equally steep. The capacity constraint itself can also take a more general functional form, rather than linear.

It is also possible to allow one market to clear before the other. There can be a monopoly or a duopoly in each market. The duopolists can act simultaneously or sequentially. One interesting variation is a four-stage game, in which one market clears before the other market and the Stackelberg leader acts first in each market. In this case, products are produced in different periods. To analyze strategic capacity allocation, an intertemporal capacity constraint is necessary.

Finally, it will be interesting to test this theoretical model empirically, using detailed data of the four aircraft models. Certainly, a more complicated model will be necessary for the purpose. For example, determinants of demand should include the prices of pre-owned LCAs and fuel prices.[19]

[19] Benkard (2004) treated aircraft purchases as rentals because the market for used LCAs is efficient. There are low transaction costs.

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Appendix

The conventional definition of product relationship uses the demand functions instead of the inverse demand functions. This yields the "*p*-definitions." Differentiate the demand functions:

$$\frac{\partial X^1}{\partial P^1} = -\frac{B^2}{B^1B^2 - \Gamma^1\Gamma^2}, \qquad \text{(i)}$$

$$\frac{\partial X^2}{\partial P^2} = -\frac{B^1}{B^1B^2 - \Gamma^1\Gamma^2}, \qquad \text{(ii)}$$

which are negative only if $B^1B^2 - \Gamma^1\Gamma^2 \geq 0$. Hence the condition, $B^1B^2 - \Gamma^1\Gamma^2 \geq 0$, must be satisfied in order to be consistent with the law of demand. Also,

$$\frac{\partial X^1}{\partial P^2} = \frac{\Gamma^1}{B^1B^2 - \Gamma^1\Gamma^2}, \qquad \text{(iii)}$$

$$\frac{\partial X^2}{\partial P^1} = \frac{\Gamma^2}{B^1B^2 - \Gamma^1\Gamma^2}. \qquad \text{(iv)}$$

If the products are *p*-substitutes, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ will be positive, requiring $\Gamma^1 > 0$ and $\Gamma^2 > 0$. If the products are *p*-complements, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ will be negative, so $\Gamma^1 < 0$ and $\Gamma^2 < 0$. If the products are unrelated, $\frac{\partial X^1}{\partial P^2} = \frac{\partial X^2}{\partial P^1} = 0$, so it requires $\Gamma^1 = \Gamma^2 = 0$.

Also, if the products were perfect substitutes, $\frac{\partial X^1}{\partial P^2}$ and $\frac{\partial X^2}{\partial P^1}$ would approach infinity. This would happen when $B^1B^2 - \Gamma^1\Gamma^2 = 0$, but this paper rules out perfect substitutability. Together with the requirement to be consistent with the law of demand, this paper assumes $B^1B^2 - \Gamma^1\Gamma^2 > 0$. It is useful to note that, in effective terms, $b^1b^2 - \gamma^1\gamma^2 = \frac{B^1}{\theta^{1^2}}\frac{B^2}{\theta^{2^2}} - \frac{\Gamma^1}{\theta^1\theta^2}\frac{\Gamma^2}{\theta^1\theta^2} = \frac{B^1B^2-\Gamma^1\Gamma^2}{(\theta^1\theta^2)^2} > 0$.

This paper uses the less conventional "*q*-definitions" instead of "*p*-definitions," but the two have same requirements. In effective terms, $\frac{\partial p^1}{\partial x^1} = b^1$, $\frac{\partial p^2}{\partial x^2} = b^2$, $\frac{\partial p^1}{\partial x^2} = \gamma^1$ and $\frac{\partial p^2}{\partial x^1} = \gamma^2$. If the products are *q*-substitutes, γ^1 and γ^2 will be positive. If the products are *q*-complements, γ^1 and γ^2 will be negative. If the products are unrelated, γ^1 and γ^2 will equal zero. Also, if the products were perfect *q*-substitutes, $\frac{\partial p^1}{\partial x^1} = \frac{\partial p^1}{\partial x^2}$ and $\frac{\partial p^2}{\partial x^2} = \frac{\partial p^2}{\partial x^1}$. This would happen when $b^1 - \gamma^1 = 0$ and $b^2 - \gamma^2 = 0$. However, since this paper rules out perfect substitutability, it assumes $b^1 - \gamma^1 > 0$ and $b^2 - \gamma^2 > 0$. These are slightly stricter than the requirement of the "*p*-definitions" that $b^1b^2 - \gamma^1\gamma^2 > 0$.

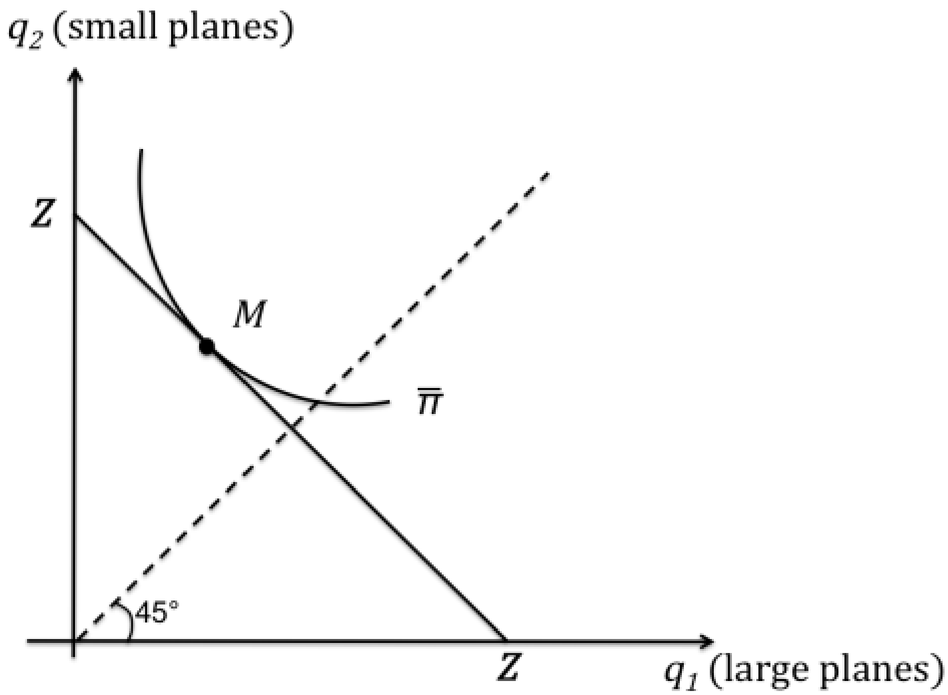

Fig. 1. Optimal Capacity Allocation of a Multiproduct Monopolist

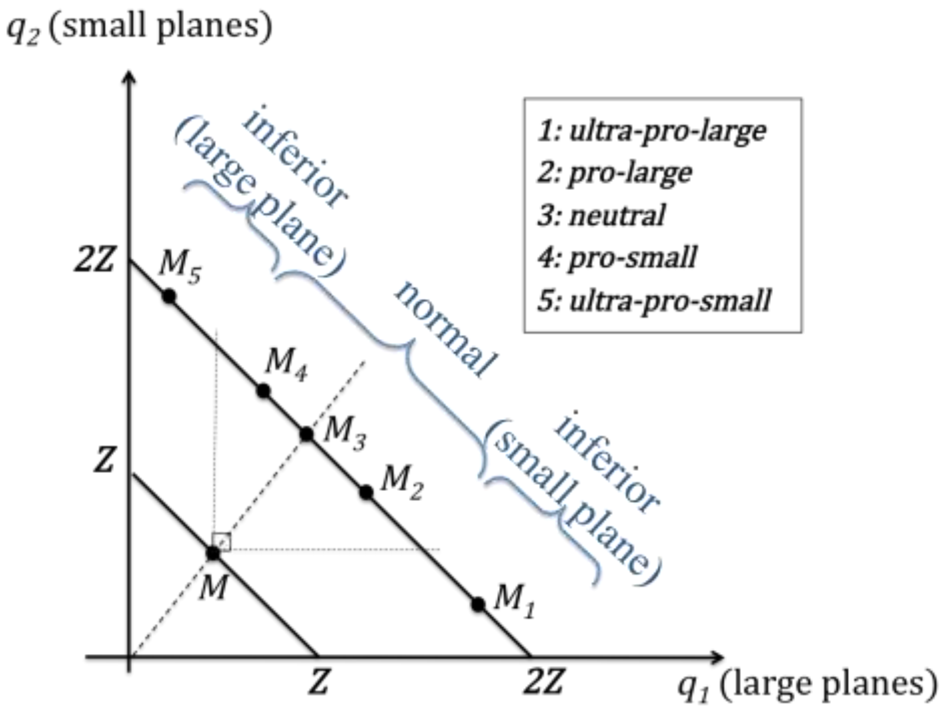

Fig. 2. Effect of an Increase of Capacity, *Z*

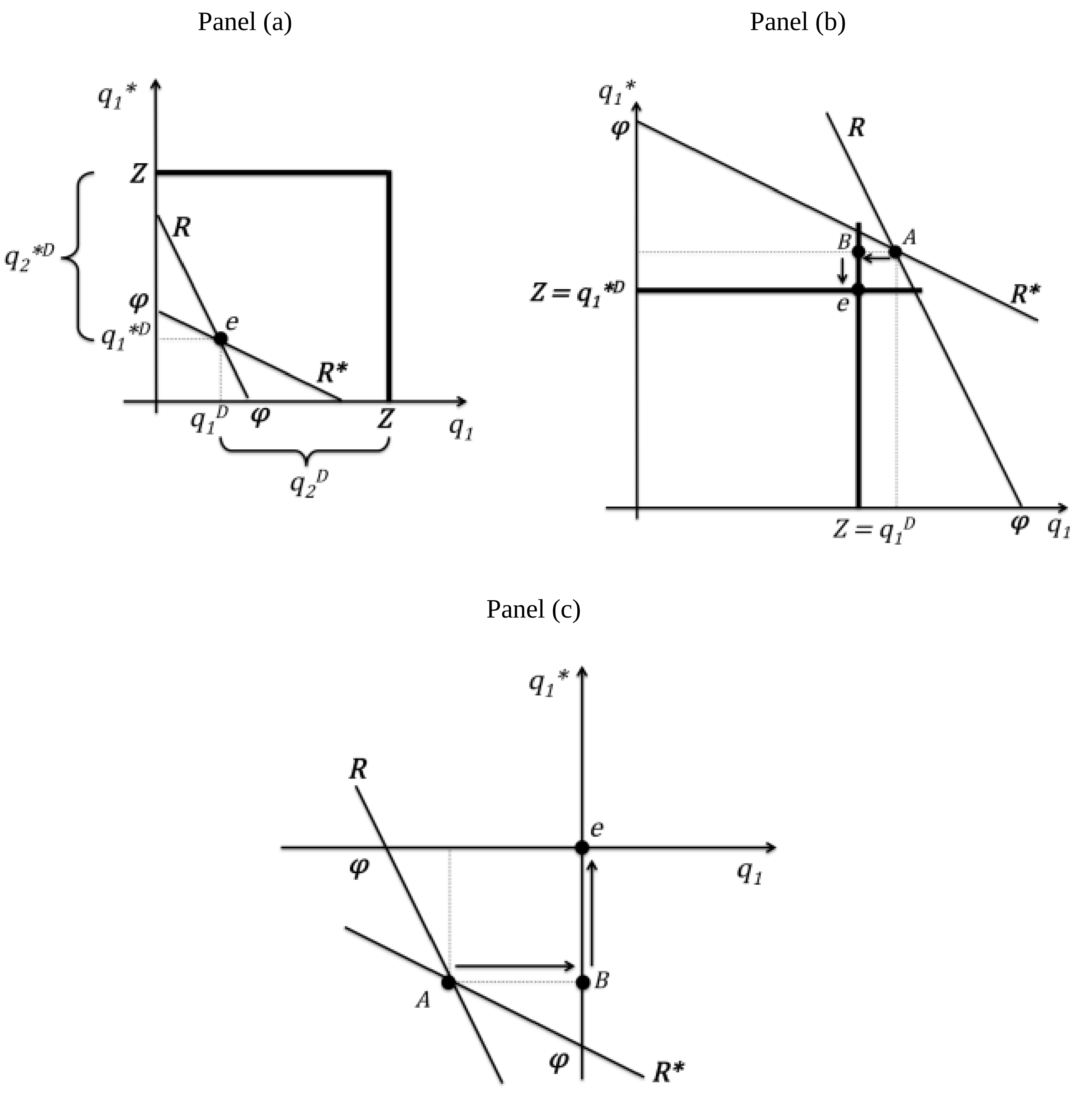

Fig. 3. Nash-Cournot-Equilibrium Capacity Allocation of Identical Firms

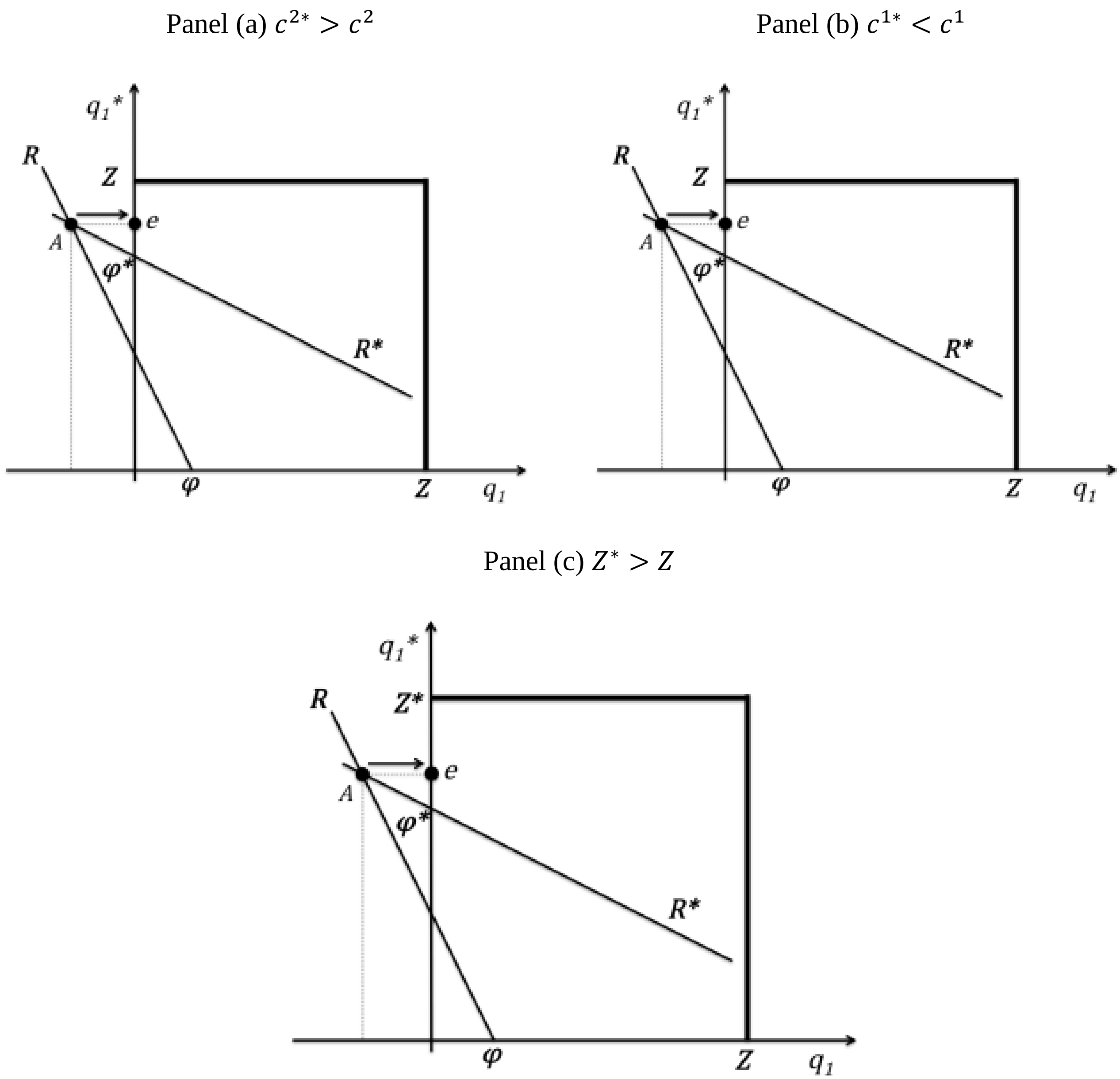

Fig. 4. Nash-Cournot-Equilibrium Capacity Allocation of Differentiated Firms

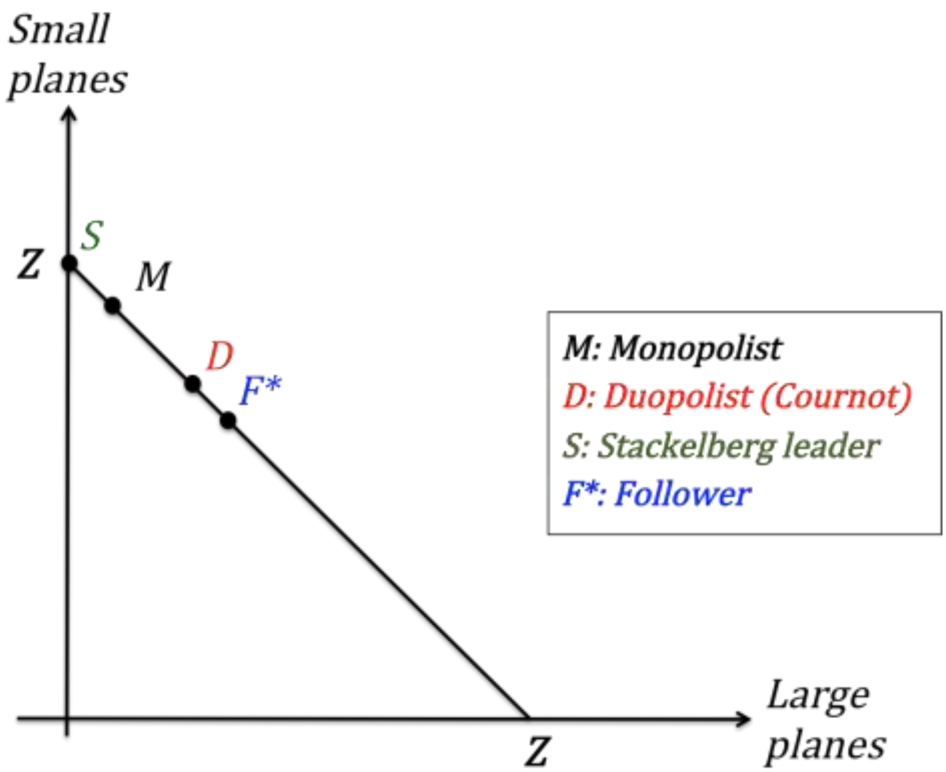

Fig. 5. Strategic Capacity Allocation Under Different Market Structures

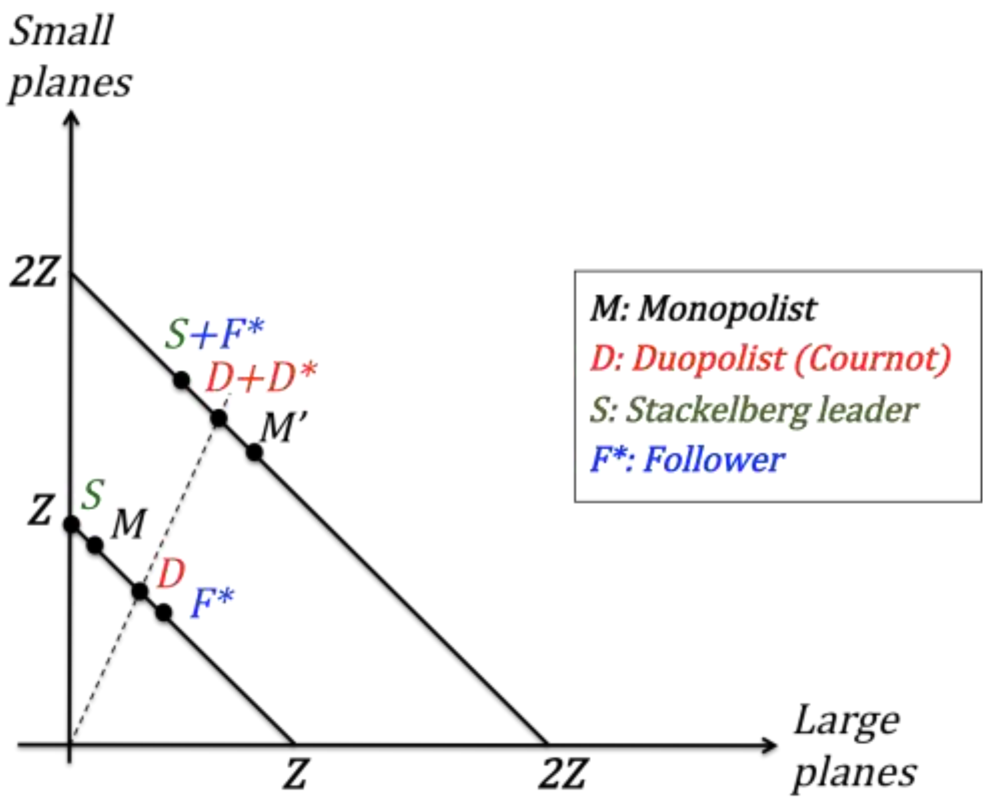

Fig. 6. Effect of Capacity Expansion

Table 1.
Deliveries of Boeing 787 and Airbus A380 and A350

Product Year	Boeing 787 “small plane”	A380 “large plane”	A350 “small plane”
2007	0	1	0
2008	0	12	0
2009	0	10	0
2010	0	18	0
2011	3	26	0
2012	46	30	0
2013	65	25	0
2014	114	30	1
2015	135	27	14
Total	363	179	15

Source: Airbus, Boeing.

Table 2.
Orders of Boeing 787 and Airbus A380 and A350

Product Year	Boeing 787 “small plane”	A380 “large plane”	A350 “small plane”
2001	0	85	0
2002	0	10	0
2003	0	34	0
2004	56	10	0
2005	235	20	0
2006	157	7	2
2007	369	23	292
2008	93	9	163
2009	-59	4	51
2010	-4	32	78
2011	13	19	-31
2012	-12	9	27
2013	182	42	230
2014	41	13	-32
2015	71	2	-3
Total	1142	319	777

Source: Airbus, Boeing.