Can Daily Financial Data Help Forecast Economic Downturns?

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October 22, 2020

Abstract

Thousands of newspapers, newsletters, television shows, blogs and many more are in constant demand to report on changing economic conditions. The availability of financial data at daily frequencies allows for nowcasting of these economic conditions, potentially updating them every trading day. This paper combines daily and weekly financial data with monthly macroeconomic indicators in a mixed frequency probit (MFP) regression to forecast and nowcast US and Canadian recessions. The methodology is developed from Chen and Tsay's (2011) approach that uses polynomials to weight higher frequency data in generalized autoregressive distributed lag models. Using data from 1971/9 to 2019/1, out of sample analysis for nowcasts and forecasts of recession probabilities show improvements in the quadratic probability score (QPS) of up to 17%, compared to a benchmark model that aggregates financial data into monthly frequencies. This increases to a 30% improvement when looking at the onset of a recession. The Diebold Mariano test also indicates statistically significant improvements in out of sample prediction at the 1% level. Further analysis shows that the bond market and real activity market hold the most information about the future and current state of the economy in the US and Canada. A mixed frequency artificial neural network (MF-ANN) method is also used as comparison, showing promising results that indicate further research should be carried out in this area.

1 Introduction

Predicting when a recession will occur is of great importance and interest to policy makers, such as the government and Federal Reserve, as well as private economic agents who have vested interests in the direction of economic activity, whether it be for personal wealth reasons or general job market opportunities. Thousands of newspapers, newsletters, television shows, blogs and many more, constantly report on these changing economic conditions. However, predicting the turning point of business cycle phases in real time (constantly) is difficult as business conditions are not observable in real time. The Business Cycle Committee of the NBER, who decides when the US economy is in a recession, will make an announcement of when a recession began long after the fact. For example, the latest US recession that began in December 2007 was not officially announced until December 2008. This is a common occurrence, and over the past 30 years the NBER has made its announcements 6 to 20 months after the corresponding peak or trough in economic activity, Fossati (2015). A natural country of comparison is Canada who, similarly, use a committee called the Business Cycle Council of the C.D. Howe Institute, to determine recession periods. They also announce actual recession periods at a considerable delay. This makes it of high interest to predict if we are in or about to enter a recession, as it may not be obvious from key macroeconomic variables, and the business cycle dating committees of the US and Canada will not announce a recession to the public until long after it has begun.

The common strategy to model business conditions in real time is by generating recession probabilities using binary models, where both the recession indicator and explanatory data are recorded at the same frequency - monthly. This paper adds to the literature in multiple ways: (1) I use NBER and C.D. Howe Institute defined recession indicators as a binary dependent variable in a mixed frequency probit model (MFP), taking advantage of the readily available daily, weekly and monthly financial and macroeconomic data. I then forecast recession probabilities up to a 3 month horizon. (2) I introduce a new method to include mixed frequency data in a binary model that could be useful for topics outside of recession prediction. (3) I subset variables into different asset markets and carry out factor analysis. This allows me to see what asset markets are key leading indicators of US and Canadian recessions. (4) Part of my analysis focuses on recession 'onset' prediction, defined later as the first 5 months of a recession. Arguably this is the most important time to accurately predict a recession due to the considerable reporting lag by the respective business cycle committees. (5) The mixed frequency framework allows me to update, or 'Nowcast', current recession probabilities as frequently as daily. Hence, providing the public with useful and constant information about current business conditions. (6) As a final point of analysis and due to the ever increasing popularity of machine learning techniques, I incorporate mixed frequency data into an artificial neural network (MF-ANN) to forecast US recessions.

There has been much previous research on forecasting US recessions with matched frequency data. Popular techniques include using binary and Markov switching models. Fossati (2015) used dynamic factors estimated from panels of macroeconomic and financial indicators to predict future recessions using probit models, concluding models that include the 3-month less 10 year term spread, a stock market dynamic factor and a real economic factor achieve the best out of sample fit. Huang and Startz (2020) find that augmenting existing Markov-switching dynamic factor models with additional information on the stock return volatility improves prediction of the state of the economy. Literature on forecasting recessions with mixed frequency data is more limited, and focuses on identifying recessions using GDP turning points instead of the recession indicator directly. One example is Balcilar (2016) who found including the monthly economic policy uncertainty index improves forecasting of quarterly GDP turning points in a mixed frequency Markov switching vector autoregressive model, compared to aggregation of data into quarterly frequency.

Comparatively there has been little work focusing on Canada. Using a matched frequency probit model with dynamic factors, Fossati (2018) found that Canadian real activity factors are particularly successful at predicting recessions at short horizons, and that Canadian bond and exchange rate factors improve recession forecasts at longer horizons. I find no papers directly predicting Canadian recessions using mixed frequency. A more extensive literature review of both the US and Canada can be found in section 2.

In this paper I follow Ludvigson and Ng (2009) and Fossati (2015, 2018) in estimating factors representing the bond, stock, exchange rate and real markets from panels of macroeconomic and financial data. These daily, weekly and monthly factors are then used to predict future US and Canadian recession dates, as published by the NBER and the Business Cycle Council of the C.D. Howe Institute respectively, in a mixed frequency probit model (MFP). My main findings show that when daily financial data is included in the forecasting model for the US, out of sample predictive performance improves at the 1,2 and 3 month forecasting horizons. Reductions of up to 17% in the quadratic probability score (QPS) are found, depending on the forecast horizon, compared to aggregating data at the monthly frequency. When focusing on recession onset prediction these improvements increase up to 30%. However, for Canada, where only weekly financial data for the whole sample is available, there are mixed results. I find improved recession onset prediction performance at all forecast horizons, with up to a 22% decrease in the QPS. But evaluation statistics covering the full out of sample period show no improvement in predictive power when compared to probit models that aggregate financial data at the monthly frequency.

Additionally, I use the benefit of daily and weekly frequency data to nowcast the current months recession probabilities, updating forecasts on a daily and weekly basis in the US and Canada respectively. By including the current months financial data into the MFP I find that I can improve forecasting performance measured by the QPS in the US by up to 14%. The Diebold Mariano test also shows statistically significant improvements in forecasting performance at the 10% level, depending how many days of the current months financial data are included in the model. However, there are no significant improvements from nowcasting using financial data in Canada.

Finally, I find that daily and weekly bond market factors are key leading indicators, especially at detecting the onset of a recession, in the US and Canada respectively. The stock and real market also play important roles in improving forecasting performance. However, this is only when the stock market data is aggregated at the monthly frequency, due to daily data causing volatile results. Exchange rate data at any frequency is not useful in predicting future recessions in the US and Canada.

2 Literature Review

2.1 Matched Frequency

Estrella and Mishkin (1998) find that the 3-month less 10-year term spread and stock price indices are the most useful predictors of future US recessions. Similarly, Wright (2006) finds that using the level of the federal funds rate together with the term spread improves the performance of the predictive probit models. Katayama (2009) analyzed the performance of several binary class models for NBER recessions using combinations of 33 macroeconomic and financial indicators. He finds that the combination of the term spread, month to month changes in the SP 500 index and the growth rate of non-farm employment generate the sequence of out of sample recession probabilities that better fits NBER recession dates. Fossati (2015) uses dynamic factors estimated from panels of macroeconomic and financial indicators to predict future recessions using probit models. He concludes that at 3 month forecast horizons probit models that include the 3-month less 10 year term spread, a stock market dynamic factor and a real economic factor achieve the best out of sample fit. More recently, Huang and Startz (2020) find that augmenting existing Markov-switching dynamic factor models with additional information on the stock return volatility improves prediction of the state of the economy. They beat both the peak and trough announcements for recent recessions by the NBER by several months.

Comparatively there has been little work focusing on Canada. Atta-Mensah and Tkacz (1998) find that that the Canadian yield spread, defined as the difference between long term bond yields and the 3-month commercial paper rate, is the most useful indicator to predict recessions in Canada. Bernard and Gerlach (1998) show that when the US yield spread is included as a predictor, recession forecasts improve at medium and long term horizons. Hao and Ng (2011) include inflation, employment, monthly GDP, housing starts and a number of other financial and real activity indicators in a dynamic probit model to forecast Canadian recessions. Recently, the interest has moved to factor-augmented models, for example Fossati (2018). He uses bond and exchange rate, stock and real activity dynamic factors from Canada and the US to forecast Canadian recessions in a probit model at various horizons. Fossati finds that Canadian real activity factors are particularly successful at predicting turning points at short horizons, and that Canadian bond and exchange rate factors improve recession forecasts at 6 to 12 month horizons. Exclusion of US data results in no significant deterioration in predictive performance, but US interest rate spreads are part of the best performing model at longer forecast horizons.

2.2 Mixed Frequency

The literature on forecasting recessions with mixed frequency data is much more limited. I begin by looking at research that has focused on the US. Balcilar (2016) use a mixed frequency Markov switching vector autoregressive (MF-MS-VAR) model to predict regimes in quarterly US GDP using the monthly economic policy uncertainty index as the leading indicator. They find that their model performs better out of sample with mixed frequency data compared to that of matched frequency. Bessec (2015) uses a Markov switching factor mixed data sampling (MS-factor MIDAS) model to extract probabilities in turning points of quarterly US GDP using monthly financial variables. They also conclude that economic turn points are detected more successfully with mixed frequency models versus models that aggregate the higher frequency data.

Next I focus on the Europe. Camacho (2014) use a Markov switching dynamic factor model to forecast quarterly GDP of the euro area, using a mix of quarterly and monthly variables. Filtered probabilities are then extracted and updated whenever new data is released to forecast economic turning points in the euro area in real time. Bessec (2015) uses monthly bond market, stock price and oil price data to forecast turning points in UK quarterly GDP. They find the Markov switching MIDAS model performs better than a matched frequency model in determining US economic turning points, but has little benefit for forecasting recessions in the UK versus matched frequency. Foroni (2015) uses a MF-MS-VAR to forecast quarterly GDP growth in the euro area using four monthly indicators; the Economic Sentiment Indicator (ESI), the M1 monetary aggregate, headline industrial production and the slope of the yield curve. They find their model works well for nowcasting and short term forecasting of the euro area GDP growth. Finally, Pirschel (2016) uses a linear mixed frequency Bayesian VAR to provide monthly real time recession probabilities for the euro area using a number of monthly macroeconomic and financial indicators. The Y variable in this paper is also quarterly GDP.

Other useful, but not directly related to recession prediction, literature include Auroba (2009), Kumar (2013) and Andreou, Ghysels and Kourtellos (2013). Auroba (2009) use a dynamic factor model with daily term premium, weekly initial jobless claims, monthly employment and quarterly GDP to develop their own Business Conditions Index, where a large negative value indicates poor business conditions, and a value of 0 indicates normal business conditions. Their dynamic factors model updates the Business Conditions Index on a weekly basis. Kumar (2013) employs the same model as Auroba (2013) but for Canadian business conditions. Finally, Andreou, Ghysels and Kourtellos (2013) uses daily financial data, in combination with monthly and quarterly macroeconomic variables, in a factor MIDAS model to estimate US GDP growth. They find that the inclusion of financial data at the daily frequency improves estimation results.

2.3 Binary Mixed Frequency Models

There is very little literature that applies mixed frequency to binary variables. As to date I find three papers; 1) Freitag (2014) uses a Probit MIDAS to analyze the relationship between sovereign downgrades and sovereign CDS premiums for Euro countries. 2) Audrino (2019) uses a Logit MIDAS to obtain bank failure probabilities in the US. 3) Jiang (2020) uses a U-MIDAS Logit model to study the default of listed companies in mainland China. All papers find improvement of forecast accuracy when the model allows for inclusion of higher frequency data. Papers 1) and 2) both carry out Monte Carlo simulations.

3 Motivation

3.1 Why is Daily and Weekly Financial Data Useful?

Theory suggests that the forward looking nature of financial assets prices should contain information about the future state of the economy and therefore should be considered as extremely relevant for macroeconomic forecasting, Andreou, Ghysels and Kourtellos (2013). The fact that the Y variable of interest in this paper (recession or not) is monthly, usually restricts researchers to aggregate the financial time series to match the dependent variable frequency. For example, the log of stock returns over a whole month versus daily stock returns, and average bond yields over the month versus daily bond yields.

Not using the readily available high frequency data such as the daily financial predictors to perform the monthly forecasts of recession probabilities has two important implications: (1) you lose the possibility of having real time daily, weekly or bi-weekly updates of the recession probabilities and (2) you lose information through temporal aggregation, Andreou, Ghysels and Kourtellos (2013). In other words, important within month information that can help in predicting current and future recessions may be 'washed' out by the assumption of equal weighting of data.

Andreou, Ghysels and Kourtellos (2010) show that the estimated slope coefficient of a regression model that imposes a standard equal weighting aggregation scheme, ignoring the fact processes are generated from a mixed data environment, yield asymptotically inefficient and inconsistent estimates. Both these asymptotic inefficiencies and inconsistencies can have adverse effects on forecasting.

3.2 Why use a Mixed Frequency Probit Model?

In this section I justify my use of a reduced form binary modeling approach, the MFP, versus other potential mixed frequency empirical methods. In previous literature a common method to deal with mixed frequency data has been using state space models. However, as well as being computationally complex which increases with the number of variables involved, they also require the correct specification of the model in high frequency, which is even more complex than usual given the missing observations in the dependent variable, Foroni (2013).

Andreou, Ghysels and Kourtellos (2013) explains that a mixed data sampling (MIDAS) regression model can be viewed as a reduced form representation of the linear projection that emerges from a state space model approach - by reduced form he means that the MIDAS regression does not require the specification of a full state space system of equations. They lie out some disadvantages of state space models versus reduced form mixed frequency models as (1) they are more prone to specification errors, as a full system of equations and latent factors are required and (2) it requires a lot more parameters to achieve the same goal. The system of equations requires many parameters for the measurement equation, state dynamics and their error processes. Therefore, state space models are far more complex in terms of specification, estimation and computation of forecasts, compared to the reduced form approach I propose in this paper.

As Andreou, Ghysels and Kourtellos (2013) explain, the Kalman filter approach to estimate state space models is often feasible when dealing with a small system of mixed frequencies such as Aruoba (2009) who use 6 time series. Instead, my analysis deals with a larger number of daily and weekly variables (upwards of 20) and therefore the approach I use is regression based and reduced form - notably not requiring to model the dynamics of each and every daily predictor series and estimate a large number of parameters.

There are, however, advantages to using state space models. In particular, once a state space system is estimated it is possible to produce forecasts at any horizon. In contrast, my MFP regressions need to estimate projection equations tailored to each horizon. However, since MFP regressions are relatively easy to estimate, the fact one has potentially different MFP regressions across different horizons does not come at a substantial cost. My paper also only focuses on a handful of forecast horizons, and therefore the cost of re-estimating a model for each horizon is low. Since the Kalman filter produces multi-horizon forecasts via iteration, specification errors, which pose a serious problem with high frequency data, are compounded forward across multiple horizon forecasts. Previous literature, including Findley (1983), Findley (1985), Lin and Granger (1994), Clements and Hendry (1996), Bhanzali (1999) and Chevillon and Hendry (2005), have shown iterated forecasting can feature poor performance in the presence of specification errors.

Kuzin (2011) compares a MF-VAR, cast into state space form, with the reduced form MIDAS approach to forecast Euro area GDP. The MF-VAR does not restrict the dynamics but suffers from the curse of dimensionality. The authors argue that it is difficult to rank the different approaches, so they compare their performance empirically. They find that the MF-VAR performs better for longer horizons (up to 9 months), whereas MIDAS performs better for shorter horizons (up to 4 to 5 months). Seeming my paper is based on shorter term forecasts up to 3 months and nowcasts, Chen and Tsay's (2011) adaption of MIDAS seems applicable.

Various mixed frequency Markov switching models are also popular in the literature, see Balcilar (2016) and Bessec (2015) among others in section 2.2. However, in all previous literature this and the above methods have all required the use of GDP to forecast economic turning points. Recession probabilities are then derived from this. Using GDP as opposed to the binary recession indicator released by the business cycle dating committees has issues. Firstly, it is released on a quarterly basis versus recession indicators which are released monthly. This severely reduces the number of Y observations for when there is a recession, which are already sparse. Secondly, the amount of times and magnitude to which

GDP is revised after being first published can cause issues. Between 1990-2010, quarterly growth rates of US GDP were revised by an average of 0.26 points three years after its first publication. This revision reaches up to 0.37 points for the recession quarters, as opposed to 0.21 points in expansion, Bessec (2015). The delay in accurate GDP figures poses issues when attempting to accurately forecast recessions in real time.

Finally, my MFP is very similar to using MIDAS, but in a probit set up. Reasons for using my specific MFP vs a MIDAS probit are explained in section 4.3.

4 Methodology

4.1 Probit Model

I begin by looking at the standard probit model that will form a benchmark to compare my mixed frequency estimation to. Let y_{t+h} be a binary variable that equals 1 if month $t+h$ is subsequently declared as a recession and 0 otherwise. A forecast of the probability of a recession in month $t+h$ (p_{t+h}) from a probit regression is given by

$$p_{t+h} = Prob(y_{t+h} = 1 | z_t) = \Phi(\beta' z_t) \tag{1}$$

where $\Phi(.)$ is the standard normal cumulative distribution function, β is a vector of coefficients, and z_t is a $k \times 1$ vector of predictors including an intercept.

4.2 Mixed Frequency Probit Model

In this paper I extend the standard probit model to allow for the inclusion of daily and weekly financial and macroeconomic data. Since the NBER Business Cycle Dating Committee of the US and the Business Cycle Council of the C.D. Howe Institute of Canada announce recessions at a monthly frequency, I will need a mixed frequency probit (MFP) to forecast future recession probabilities in both countries. I extend Chen and Tsay (2011) mixed frequency approach to the probit model. In their paper they allow for a polynomial to weight daily exchange rate data to forecast quarterly changes in commodity prices. Consider first a mixed frequency model with only one predictor x_1. Following the notation in Ghysels (2007) consider the following h-month ahead predictive regression.

$$y_t^* = \beta_0 + \beta_1 W(L^{1/m}, \theta) x_{1,t-h}^m + \epsilon_t, where \tag{2}$$

$$W(L^{1/m}, \theta) = \sum_{k=1}^{K} b(k;\theta) L^{(k-1)/m}, and \tag{3}$$

$$L^{s/m} x_{1,t}^m = x_{1,t-s/m}^m \tag{4}$$

Here y_t^* is a latent variable which represents the state of the economy as measured by the Business Cycle Dating Committees in the US and Canada, t denotes the basic time unit for the lower frequency data (monthly) from 1 to T, m and x^m indicate higher sampling frequency and observations, which is indexed from 1 to K (where K is finite). $L^{1/m}$ is the lag operator in frequency-m space, $b(k;\theta)$ is the weight on each of the K lagged higher frequency predictors and ϵ_t is a white noise process. In this paper the higher frequency data is at the daily and weekly level. For example, with daily frequency if $K = 15$ we would use the last 15 trading days of the current month to forecast recession probabilities h months in the future. This is as opposed to aggregating all the current months trading days into one observation, as past literature has done when predicting future binary economic states.

I can generalize equation (2) to contain q sets of mixed-frequency predictors, as well as $r - q$ sets of matched frequency predictors as follows:

$$y_t^* = \beta_0 + \beta_1 W_1(L^{1/m}, \theta_1) x_{1,t-h}^m + ... + \beta_q W_q(L^{1/m}, \theta_q) x_{q,t-h}^m + \beta_{q+1} x_{q+1,t-h} + ... + \beta_r x_{r,t-h} + \epsilon_t, where \tag{5}$$

$$W_i(L^{1/m}, \theta_i) = \sum_{k=1}^{K} b_i(k; \theta_i) L^{(k-1)/m}, and \tag{6}$$

$$L^{s/m} x_{i,t}^m = x_{i,t-s/m}^m, \forall i = 1, ..., q \tag{7}$$

Parameters β_1,...,β_q measure the aggregate impact of predictors of the mixed frequency (daily and weekly) data $x_{1,t-h}$,...,$x_{q,t-h}$ on the lower frequency (monthly) y_t^*, provided that the sum of the weighting polynomial $W_1(L^{1/m}, \theta_1), ..., W_q(L^{1/m}, \theta_q)$, are normalized to 1. Parameters β_{q+1},...,β_{q+r} measure the aggregate impact of predictors of the matched frequency (monthly) data $x_{q+1,t-h}$,...,$x_{q+r,t-h}$ on y_t^*.

To weight the higher frequency daily financial data in my MFP a $K \times n$ Vandermonde matrix is used:

$$V = \begin{bmatrix} 1 & 1^1 & 1^2 & \dots & 1^{n-1} \\ 1 & 2^1 & 2^2 & \dots & 2^{n-1} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & K^1 & K^2 & \dots & K^{n-1} \end{bmatrix} \tag{8}$$

This follows Almon (1965) that assumes each lag coefficient can be approximated by a polynomial of degree $n - 1 < K$. Therefore, instead of having to estimate $1 + Kq$ parameters (the intercept and weights on each K higher frequency observation of the q variables) the model is reduced to $1 + nq$ parameters to estimate. This ignores matched frequency coefficients that need to be estimated in addition.

Using V from (8) I can re-write (5) as follows in matrix notation:

$$Y = \beta_0 + \beta_1 X_1 V \alpha_1 + ... + \beta_q X_q V \alpha_q + \beta_{q+1} X_{q+1} + ... + \beta_r X_r + \epsilon \tag{9}$$

Where X_i for $i = 1, 2, ..., q$ is a $(T \times K)$ matrix of higher frequency financial and macroeconomic data, sampled at a daily and weekly rate. X_i for $i = q + 1, ..., r$ is a vector of length T of matched frequency financial and macroeconomic data, sampled at a monthly rate. Y is a vector of length T of monthly binary variables indicating a recession or not. β_i measures the aggregate impact of X_i on Y. V is as described in (8) and α_i is a $n \times 1$ vector of coefficients that form the polynomial of degree $n-1$, which weights the higher frequency data. Assuming I am trying to estimate recession probabilities one month ahead, using data from every day of the month, a visual representation of the matrices for the higher frequency data are shown below.

$$Y = \begin{bmatrix} y_{feb} \\ y_2 \\ \vdots \\ y_T \end{bmatrix} X_i = \begin{bmatrix} x_{i,jan31st} & x_{i,jan30th} & \cdots & x_{i,jan1st} \\ x_{i,2-h} & x_{i,2-h-1/K} & \cdots & x_{i,2-h-(K-1)/K} \\ \vdots & \vdots & \cdots & \vdots \\ x_{i,T-h} & x_{i,T-h-1/K} & \cdots & x_{i,T-h-(K-1)/K} \end{bmatrix} \tag{10}$$

Note that the inclusion of all trading days in the month is not needed. If $K = 15$ then only the 15 most recent trading days from that month are included. Simplifying (9) further:

$$Y = \beta_0 + Z_1\gamma_1 + ... + Z_q\gamma_q + \beta_{q+1}X_{q+1} + ... + \beta_r X_r + \epsilon \tag{11}$$

Where $Z_i = X_i V$ and $\gamma_i = \beta_i \alpha_i$. The parameter γ can then be estimated via maximum likelihood, and aggregate effects of X_i, $\forall i = 1, ..., q$, on Y, as well as the higher frequency weighting function, can be solved through manipulation of the $\gamma' s$.

4.3 Identification of β

By restricting the weights on the higher frequency data to sum up to 1, we can identify $\beta_1, ..., \beta_q$. This means letting $\mathbf{1}^\top V \alpha_i = 1$ for $i = 1, ..., q$. We can the use the fact that $\gamma_i = \beta_i \alpha_i$ and therefore $V\gamma_i = \beta_i V \alpha_i$ to obtain:

$$\hat{\beta}_i = \mathbf{1}^\top (V \hat{\gamma}_i), \forall i = 1, ..., q, and \tag{12}$$

$$var(\hat{\beta}_i) = \mathbf{1}^\top V var(\hat{\gamma}_i) V^\top \mathbf{1} \tag{13}$$

Equations (12) demonstrates that the aggregate impact of X_i on Y can be obtained directly without separating out β_i's versus b_i's like in the MIDAS specification given in (5) and (6). In the MFP we combine the β_i's versus b_i's into free parameters that can be parameterized with the Vandermonde matrix as in (12). Hence, the MFP involves a one step procedure that automatically embeds the identification condition for β_i, unlike a typical MIDAS regression.

Addtionally the typical weighting restrictions used in other binary MIDAS models, for example 'Exponential Almon Lags' and 'Beta Lag', Ghysels (2007), are no longer needed. The Almon lag polynomial is well known to offer useful approximations for a variety of weighting functions, provided that n is large enough. This means (12) can deliver consistent estimates of the aggregate impact parameters under a wide range of true underlying weighting functions. Overall, the polynomial weighting function is more flexible than traditional MIDAS methods, one reason being that weights do not have to be positive.

5 Data

5.1 US

My entire data sample period for Y runs from October 1971 to January 2019, which includes the last 6 US recessions . The Y variable is a binary indicator that takes the form:

$$y_t = \begin{cases} 1, & \text{if NBER defined recession.} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

For the X variables a full table of the data can be found in Table 1 in the data appendix. I split my data into four separate markets; (1) bond market containing 15 financial indicators, (2) exchange rate market containing 4 different rates, (3) stock market containing 4 indicators, and (4) macroeconomic indicators containing 6 variables. For each of the asset classes (1)-(3) I can observe daily, weekly or monthly data, and for the macroeconomic indicator (4), data is observed at a weekly and monthly frequency depending on the variable. This variable selection has shown to have good real time predictability of US business conditions in the previous literature of Fossati (2015), Camacho (2014), Chauvet and Piger (2008) and many more.

Two issues I have with the data are: 1) Co-linearity between variables. For example, yields on 5 year Treasury Bonds are highly correlated with yields on 10 year Treasury Bonds. The same can be said for the macroeconomic indicators which vary with economic conditions and hence are correlated with one another. 2) The number of X variables compared to the Y variable observations. This is especially apparent with mixed frequency data, where, for example, 4 parameters need to be estimated per variable if using a cubic polynomial to weight the data. To overcome these problems I use Factor Analysis to capture variability among observed, correlated variables in terms of a lower number of unobserved variables.

Prior to estimation, data is first transformed to be stationary - see the data appendix for relevant transformations. Since real activity variables are usually available with some lag, I account for data availability at time t by using the last known value $x_{i,t-l}$, where l indicates the publication lag of variable i. Publication lags for US indicators are adopted from Katayama (2009) and are presented in the data appendix. Factor analysis is then run separately on each asset class (1), (2), (3) and (4) to get a set of bond, exchange rate, stock and macroeconomic market factors. The factor analysis model takes the following form

$$X = \Lambda F + \epsilon \tag{15}$$

Where X is a $(N \times T)$ matrix of observed data and Λ is an $(N \times M)$ matrix of factor loadings, where M corresponds to the number of common factors being estimated. F is a $(M \times T)$ matrix of latent factor scores and ϵ is the error term. There is no strict rule on deciding how many factors to use. Currently I analyze the cumulative variance explained by the factors and stop adding additional factors when the marginal variance explained from the extra factor is low. This leads me to use 3 factors for the bond market, 1 factor for the exchange rate market, 1 factor for the stock market and 2 factors for the real market. The MFP regression I then run is as follows:

$$Y = \beta_0 + \underbrace{F^{mkt_i}_{1,D} V_D \gamma_{D1} + ...}_{\text{daily freq}} + \underbrace{F^{mkt_i}_{1,W} V_W \gamma_{W1} + ...}_{\text{weekly freq}} + \underbrace{F^{mkt_i}_{1,M} \delta_1 + ...}_{\text{monthly freq}} + \epsilon \tag{16}$$

Where Y is a vector of binary indicators showing whether the economy is in a recession or not. $F^{mkt_i}_D$ and $F^{mkt_i}_W$ are daily and weekly frequency factors for market i, where mkt_i refers to the four asset markets; bond, stock, exchange rate and real. V_D and V_W are the associated Vandermonde matrices for the daily and weekly frequency data. The γ's are the parameters to estimate from the mixed frequency part of the model, of which can be used to find the aggregate impact of F on Y as stated in section 4.3. Finally, $F^{mkt_i}_M$ are the monthly factors for asset market i which are the same frequency as Y and hence enter the probit regression in the standard way.

Once I have dealt with missing data, the daily mixed frequency component of the MFP ($F^{mkt_i}_D$) consist of 15 trading days worth of information for each month (K=15). The weekly mixed frequency component of the MFP ($F^{mkt_i}_W$) consist of 4 weeks worth of information for each month (K=4). In this paper I do not look at the optimal choice of K, and use the highest K possible given the data restrictions. To weight the higher frequency data I use a polynomial of degree 3 and 2 for daily and weekly data respectively. I deem a polynomial of degree 3 flexible enough to capture any potential weighting function on daily data. For the weekly data, if I use a polynomial of degree higher than 3 I may as well include the weekly data points separately as it would require the same (or more) parameters to be estimated. A polynomial of degree 2 is still flexible enough to incorporate most weighting schemes.

Initially, before moving onto nowcasting, I focus on short horizon forecasts of recession probabilities - specifically 1, 2 and 3 months ahead. Recession probabilities can be estimated from equation (16) via maximum likelihood.

For direct comparison of my MFP I use a benchmark model. For each asset market, data that is used at the daily and weekly frequency in the MFP is aggregated to the monthly frequency. I then estimate monthly factors for each asset market as in equation (15). This gives me the below regression:

$$Y = \alpha_0 + F_{1,M}^{mkt_i}\theta_1 + ... + F_{p,M}^{mkt_i}\theta_p + u \tag{17}$$

Where Y is the same vector of binary indicators as in the MFP, indicating whether the economy is in a recession or not. $F_{1,M}^{mkt_i}$ through $F_{p,M}^{mkt_i}$ are monthly factors estimated from asset market i. The number of factors p included in the benchmark model depends on the MFP. For example, if the MFP contains 3 daily bond factors and 2 monthly real activity factors, the benchmark model will contain 3 monthly bond factors and 2 monthly real activity factors. This setup allows for exact comparison of using the daily and weekly frequency versus the monthly aggregated data. The benchmark matched frequency probit model is estimated via maximum likelihood.

5.2 Canada

My entire data sample period for Y runs from March 1972 to June 2018, which includes the last 5 Canadian recessions. The Y variable is a binary indicator that takes the form:

$$y_t = \begin{cases} 1, & \text{if C.D. Howe Institute defined recession.} \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

For the X variables a full table of the data can be found in Table 2 in the data appendix. In an identical way to the US I split my data into four asset markets; (1) bond market containing 11 financial indicators, (2) exchange rate market containing 4 different rates, (3) stock market containing 2 indicators, and (4) macroeconomic indicators containing 4 variables. These variables have been used in the previous literature, e.g. Fossati (2018) and Hao and Ng (2011), and have been found to be good individual predictors.

Prior to estimation, data is first transformed to be stationary - see the data appendix for relevant transformations. Again, since real activity variables are usually available with some lag, I account for data availability at time t by using the last known value $x_{i,t-l}$, where l indicates the publication lag of variable i. Publication lags for Canadian real activity indicators are obtained from Statistics Canada. I then follow the same steps as explained in section 5.1, which are summarized again below.

Step 1) Factors are estimated for each asset market, using the setup explained in equation (15), to be used in the MFP. Specifically, I extract 3 factors for the bond market, 1 factor for the exchange rate market, 1 factor for the stock market and 1 factor for the real activity market.

Step 2) Estimate equation (16) using the factors from the Canadian data, and extract the recession probabilities. The daily mixed frequency component of the MFP ($F_D^{mkt_i}$) consists of 15 trading days worth of information for each month (K=15). The weekly mixed frequency component of the MFP ($F_W^{mkt_i}$) consists of 4 weeks worth of information for each month (K=4). A polynomial of degree 3 is used to weight the daily data and of degree 2 to weight the weekly data. Before moving onto nowcasting, I focus on short horizon forecasts of recession probabilities - specifically 1, 2 and 3 months ahead.

Step 3) Compare the MFP for Canada to the benchmark model explained in equation 17.

6 Results

6.1 Forecast Evaluation

The first method used to evaluate forecast accuracy of the MFP and benchmark models is the Quadratic Probability Score (QPS). The QPS is equivalent to the mean square area when using binary models and is defined by

$$QPS = \frac{2}{T} \sum [\hat{p}_{t+h} - y_{t+h}]^2 \tag{19}$$

where T is the number of forecasts, $\hat{p}_{t+h}$ is the predicted probability of recession for month $t+h$ for a given model and y_{t+h} is the realized recession indicator in the month t+h. The QPS can take values from 0 to 2 with smaller values indicating more accurate predictions.

As discussed in the Introduction, the business cycle dating committees of the US and Canada delay their announcements of recession periods. Arguably the most important time for economic agents to know whether the economy is in a recession is the first few months, which is not always immediately obvious by observing single economic indicators. Therefore a model that is able to identify whether the economy is in a recession in the early stages is beneficial to these economic agents. I devise an additional evaluation measure which slightly adapts equation (19). I call this measure QPS_{onset}, where *onset* refers to the onset of a recession which I define as the first 5 months of a recession period. In other words, I want to see if the MFP has particularly good prediction power at the beginning of recession periods, versus that of the benchmark model.

$$QPS_{onset} = \frac{2}{T_{onset}} \sum [\hat{p}_{onset,t+h} - \mathbf{1}]^2 \tag{20}$$

T_{onset} refers to the number of forecasts during recession onsets, for example, if there are 3 recessions in the period being examined, $T_{onset} = 15$. $\hat{p}_{onset,t+h}$ is the predicted probability of a recession for month $t+h$, when period $t+h$ falls within the first 5 months of a recession. Finally, $\mathbf{1}$ is a vector indicating that the true state of the economy is a recession.

The third evaluation method I use is the log probability score (LPS), which is given by

$$LPS = -\frac{1}{T} \sum [y_{t+h} log(\hat{p}_{t+h}) - (1 - y_{t+h}) log(1 - \hat{p}_{t+h})] \tag{21}$$

The LPS can take values from 0 to $+\infty$ and smaller values indicate more accurate predictions. Compared to the QPS, the LPS score penalizes large errors more heavily.

For each of these first 3 evaluation methods I divide the statistic from the MFP model by that of the benchmark model. A value less than 1 indicates that the MFP is a better model. The amount below 1 can be interpreted as the % improvement from using the mixed frequency model versus matched frequency. For example, a value of 0.8 indicates a 20% improvement in the evaluation statistic.

The fourth evaluation method I use is the Diebold-Mariano test with a squared loss function. This test compares the predicted probabilities of the MFP and the benchmark model with the actual values of Y that occurred. For the purpose of this paper the H_A is that the MFP is more accurate than the benchmark model, and hence a low p-value is desired. I choose a squared loss function, as opposed to a linear loss function, as I deem incorrect recession predictions to be costly to the economy and therefore want to penalize these at a higher rate.

6.2 In Sample Results

6.2.1 US

The in sample period uses data for X from September 1971 to October 2018 which I use to forecast recession probabilities at alternative horizons: $h = 1$, 2 and 3 months. Given the way I have set up my data there are multiple variations of models, in terms of X variables, I can use. Firstly, I have estimated factors that represent 4 asset markets; bond, exchange rate, stock and the real market, but the best model does not necessarily contain all asset markets. Secondly, the first 3 asset markets have daily, weekly and monthly frequency available. I do not assume that I must always use the highest frequency available for the model. For example, it may be the case that monthly stock factors provide a better fit than using daily stock factors due to the fact that daily stock information is volatile.

The obvious next question is how do I classify what the best model is? For my research I deem the most important aspect to be the out of sample forecasting exercise, as this provides real time recession predictions, and hence decide the best model from the forecast evaluation of the out of sample results. These are reported in the next section. I analyze all the possible combinations of asset markets and frequencies, and report the best model based on the out of sample forecast evaluation methods explained in the previous section. For the US the best model uses 3 daily bond market factors, a monthly stock market factor and 2 monthly real market factors.

The asset markets that appear in the best model are in line with the previous literature. Estrella and Mishkin (1998), Katayama (2009), and Owyang et al. (2015), among others, find that the real economic activity indicators that form the real market factor in this paper improve recession forecasts, particularly at short horizons. Wright (2006) find that the term spread and federal funds rate have high predictive power for recessions at short and long forecast horizons. The bond market factors are functions of these variables. Fossati (2015) finds that the best performing model at a 3 month forecast horizon includes a monthly stock market factor, monthly real market factor and the monthly 3 month less 10 year spread on government bond yields. This is very similar to what I find, apart from the fact I used a daily frequency bond market factor, instead of just the monthly 3 month less 10 year spread.

Figure 1 shows the in sample recession probabilities for forecast horizons 1, 2 and 3 months. The red line shows recession probabilities calculated from the MFP, and the black line shows probabilities calculated from the benchmark model. A common occurrence is the ability of the MFP model to send a stronger signal at the onset of a recession compared to that of the benchmark matched frequency model. This is especially evident in detecting the most recent recession for all forecast horizons. The MFP also produces higher overall recession probabilities in the fourth and fifth US recessions in the sample. Recession probabilities for the first three recessions are very similar in both models. In non-recession periods the models behave very similarly, with occasional short lived false positives. These can usually be attributed to specific negative shocks, for example in Summer 1998 when the $S\&P$ 500 fell 20% in a short span of time. Fossati (2015) and other previous research detect the same false positives.

Table 3 reports the evaluation methods explained in the previous section. For the QPS and LPS I divide the statistic from the MFP model by that of the benchmark model. A value less than 1 indicates that the MFP is a better model. For all evaluation methods the MFP performs better than the matched frequency benchmark, especially when focusing on the onset of a recession, defined as the first 5 months of a recession period, with improvements up to 24%. The p value from the Diebold Mariano test is reported in the last row. For $h = 3$ months, we reject H_0 at the 1% level of significance, in favor of H_A, which states that the MFP is more accurate than the benchmark.

6.2.2 Canada

The in sample period uses data for X from February 1972 to March 2018, which I use to forecast recession probabilities at alternative horizons: $h = 1$, 2 and 3 months. Again I decide the best model based on forecast evaluation of the out of sample results, and only report these. Firstly, I must choose which of the 4 asset markets (bond, exchange rate, stock and the real market) to include in the model. Secondly, I must choose the frequencies of each of the asset market factors to include. Unlike the US, the Canadian bond market data is only available at the weekly frequency, not daily, for the full sample. The Canadian stock market factor is only available at the monthly frequency. For Canada the best model uses 3 weekly bond market factors, a monthly stock market factor and a monthly real market factor. Fossati (2018) also finds that a Canadian real market factor is a preferred variable for generating short term (1 to 3 month) recession probabilities of the Canadian economy. He finds that excluding this leads to a substantial deterioration in fit, with larger QPS and LPS values. He also finds that monthly Canadian bond yields improve prediction at the longer forecast horizons (6 to 12 months), however I find that weekly bond factors are crucial for forecasting at short horizons of 1 to 3 months. Excluding these leads to worse fit. The difference may be due to my MFP including higher frequency data.

Figure 2 shows the in sample recession probabilities for forecast horizons 1, 2 and 3 months. The red line shows recession probabilities calculated from the MFP, and the black line shows probabilities calculated from the benchmark model. For the first 4 recessions there is slight improvement in recession onset prediction and the MFP probability usually peaks at a higher value than the benchmark model. Both models perform similarly in non-recession periods with less false positives than the US. The most noticeable false positive occurs in the mid 1980's and is found in other literature, see Fossati (2018). Overall, in sample improvements for Canada are visually not so obvious compared to the US, which may emphasize the importance of daily frequency bond market factors over weekly frequency.

Table 4 reports the evaluation methods explained in the previous section. All QPS and LPS values are below 1, indicating that the MFP model performs better than the benchmark matched frequency model. I observe improvements up to 12%, which is less than those reported for the US. Of the first 3 evaluation methods, the only one that is better than the US is the QPS on the full sample at the 1 month forecast horizon. Again this may emphasize the importance of daily versus weekly bond data frequency in forecast accuracy in mixed frequency regressions. Finally, the Diebold Mariano test suggests that the MFP is more accurate than the benchmark model at the 5% and 10% significance level for the 1 and 2 month forecast horizons respectively.

6.3 Out of Sample Results

The out of sample forecasting of recessions comes with some complications. The Y variable indicating whether the US and Canadian economies are in a recession or not is observed with a lag. Nyberg (2010) assumes this 'publication lag' in the recession indicator is 9 months when forecasting US recessions. However, because the Great Recession was not officially declared a recession in the US until 12 months after the fact I will assume a 12 month lag for both the US and Canada. This assumption can be relaxed to be less or more than 12 months with ease, however it does not significantly impact results.

It is important that when carrying out the out of sample forecast I only use information known at time t to forecast recession probabilities in $t + h$. For example, if I am standing at the end of July 2018 and I want to forecast the probability that a recession will begin in August 2018 then only X and Y data known at the end of July 2018 can be used. Since financial markets data is released in real time I can use the exchange rate, bond and stock market factors up to and including the end of July 2018. This is with the exception of the $S\&P$ 500 price to earnings ratio which is observed with a 2 month lag due to earnings data being released on a quarterly basis. The real market factor contains data that is released with a publication lag. I follow Fossati (2015) and assume this is a 1 period lag, so in the example above I would only use values up to the end of June 2018. The rest of the variables with publication lags are stated in the Data Appendix.

The publication lag in the Y binary indicator is where the main complication comes from. Since I assume there is a 12 month lag, using the example above, I would only be able to estimate the probit model up to August 2017. The estimated parameters from this probit model along with the X data known at the end July 2018 can be used to extract the real time 1 month ahead recession probability for August 2018. This method can easily be generalized to h month ahead predictions.

With this in mind, I use recursive estimation for the out of sample period, expanding the time window one month at a time. Factors for the MFP and benchmark model are re-estimated every time an additional month of X data becomes available. Finally, I re-estimate the probit model as I add each additional month of data. Rolling windows is another potential technique to use, however because of the lack of recessions that have occurred in my sample in the US (6) and Canada (5) I do not. The above methodology allows me to estimate real time h month ahead recession probabilities for the US and Canada.

6.3.1 US

The out sample period forecasts real time recession probabilities from November 1988 to January 2019, which includes the last 3 US recessions. As explained in the in sample results section there are multiple variations of the model that can be run. After running all variations the best out of sample model at the 1, 2 and 3 month ahead forecast horizons includes 3 daily bond market factors, a monthly stock market factor and 2 monthly real market factors. This setup is used for both the MFP and the benchmark model for direct comparison. I explain why these particular asset markets are chosen in more detail later in this section. Figure 3 shows the out of sample recession probabilities for forecast horizons 1, 2 and 3 months. The red line shows recession probabilities calculated from the MFP, and the black line shows probabilities calculated from the benchmark model.

Visually we can see that the MFP gives a higher probability of a recession for all recessionary periods at all forecast horizons, $h = 1, 2, 3$ months. More importantly, arguably, there is much stronger detection of a recession onset in every recessionary period for all the forecast horizons when comparing the MFP against the benchmark. This is especially evident in the most recent US recession. In non recession periods both models perform similarly with the only significant false positive occurring in 1998. As explained in previous sections this can be attributed to the $S\&P$ 500 falling 20% in a short span of time. The false positive is more prevalent in the MFP model, indicating that the daily bond market factors are driving this additional sensitivity compared to the benchmark - this is because the only difference between the 2 models is the daily bond market factors versus the monthly bond market factors.

Table 5 reports the evaluation methods used to assess the performance of the MFP against the benchmark. For the QPS and LPS I divide the statistic from the MFP model by that of the benchmark model, as I did for the in sample results. A value less than 1 indicates that the MFP is a better model. For all evaluation methods the MFP performs better than the matched frequency benchmark, especially when focusing on the onset of a recession, defined as the first 5 months of a recession period. Improvements can reach up to 30%. This confirms the visual assessment given above. The p value from the Diebold Mariano test is reported in the last row. For all forecast horizons, we reject H_0 at the 1% level of significance, in favor of H_A, which states that the MFP is more accurate than the benchmark over the whole out of sample period.

By including and excluding certain asset market factors in the models I can gain insights into how these markets interact with the US economy, and which are key leading indicators for recessions. Monthly real market factors are key in predicting recessions, and model performance drops significantly without these. However, in the US I also had the ability to use Weekly Initial Jobless Claims as a higher frequency weekly macroeconomic variable, but it did not improve model performance and hence was omitted from the best model. Daily bond market factors are also key for the superior performance of the MFP over the benchmark model. Specifically, these are what drive the improved recession onset predictability compared to a model that uses monthly bond market factors. Weekly corporate bond data do not improve model prediction and are therefore excluded. Daily stock market factors introduce much volatility and false positives into the recession prediction and are hence omitted, however using just monthly stock factors improve model prediction. Finally, daily and monthly exchange rate factors do not improve recession prediction and are not included in the best models.

6.3.2 Canada

The out sample period forecasts real time recession probabilities from August 1988 to June 2018, which includes the last 2 Canadian recessions. As explained in the in sample results section there are multiple variations of the model that can be run. After running all variations the best out of sample model at the 1, 2 and 3 month ahead forecast horizons includes 3 daily bond market factors, a monthly stock market factor and a monthly real market factor - similar to the best model found for the US. This setup is used for both the MFP and the benchmark model for direct comparison. I explain why these particular asset markets are chosen in more detail later in this section. Figure 4 shows the out of sample recession probabilities for forecast horizons 1, 2 and 3 months. The red line shows recession probabilities calculated from the MFP, and the black line shows probabilities calculated from the benchmark model.

Visually, unlike the US we do not observe out of sample improvements from the higher frequency data in the most recent recession, which may emphasize the importance of daily frequency bond market factors over weekly frequency. For the recession in the early 1990's results between the MFP and benchmark look different, with improvements in recession onset prediction for all forecast horizons. Both models perform similarly in non-recession periods with no notable false positives, besides one in 1995/96 at the 1 month forecast horizon.

Table 6 reports the evaluation methods used to assess the performance of the MFP against the benchmark. For the QPS and LPS I divide the statistic from the MFP model by that of the benchmark model, as I did for the US. For the QPS and LPS which cover the full out of sample period, the MFP performs worse than the matched frequency benchmark. The QPS onset, which measures the models effectiveness at detecting the beginning of a recession, shows significant improvement in results for the MFP versus the benchmark at all forecast horizons. This improvement is up to 22% depending on the forecast horizon. The Diebold Mariano statistic does not conclude that the MFP improves prediction performance over the whole out of sample period. These results outline the potential importance that daily financial data can have in predicting future recessions over weekly and monthly frequency. It may also indicate that news and economic shocks are incorporated in Canadian financial market prices at a slower rate than in the US. Unfortunately, daily bond market data is not available for the whole of the sample period for Canada, and is only recorded, with missing observations, beginning in 1991. However, like the US I have still improved in recession onset prediction by using higher frequency financial market data in the model.

In terms of asset markets that are key leading indicators for Canadian recessions, results are similar to those of the US. Monthly real market factors are key in predicting recessions, and model performance drops significantly without these. Bond market factors, whether they be monthly or weekly frequency improve prediction especially at the recession onset. Stock market factors were not available at the daily frequency for Canada, but the monthly frequency improves forecasting performance. Finally, as with the US, daily and monthly exchange rate factors do not improve recession prediction and are not included in the best models.

7 Nowcasting

As I discussed at the beginning of the paper, one of the main advantages to using mixed frequency data is the ability to update estimates at a more frequent rate than with matched frequency data. In this paper I have explained the importance of the constant interest and need from economic agents to know current economic conditions. With the MFP framework, that uses daily and weekly financial data, I can potentially update the current and future months recession probabilities every trading day once data is released, and hence serve as constant source for forecasting the current un-observable state of the US and Canadian economies. I plan to follow Andreou, Elena and Ghysels (2013) and Gomez-Zamudio and Ibarra (2017) method of nowcasting with leads. Both these papers use current quarter daily financial data to update current and future quarter GDP estimates for the US and Mexico. As explained in Andreou, Elena and Ghysels (2013) there are quite a few variations in the specification of a nowcasting with leads regression model. One particular way which I will focus on is shown below:

$$y_t^* = \beta_0 + \beta_1 W_i(L^{1/m}, \theta) x_{1,t-h+1}^m + \beta_2 W_j(L^{1/m}, \theta) x_{1,t-h}^m + \epsilon_t, where \tag{22}$$

$$W(L^{1/m}, \theta) = \sum_{k=1}^{K} b(k; \theta) L^{(k-1)/m}, and \tag{23}$$

$$L^{s/m} x^m_{1,t} = x^m_{1,t-s/m} \tag{24}$$

Where y^*_t is a latent variable which represents the state of the economy, t denotes the basic time unit for the lower frequency data (monthly) from 1 to T, m and x^m indicate higher sampling frequency and observations, which is indexed from 1 to K (where K is finite). $L^{1/m}$ is the lag operator in frequency-m space, $b(k;\theta)$ is the weight on each of the K lagged higher frequency predictors and ϵ_t is a white noise process.The difference between nowcasting with leads and equation 2 is the second term on the RHS of equation 22. This $x^m_{1,t-h+1}$ term refers to the 'lead' and will be weighted differently compared to the previous month. For example, if the forecast horizon is 1 month and we are forecasting a recession probability for February, we usually only use the information known up to the end of January. However, with nowcasting, once we enter the month of February we do not need to wait until the end of the month to update our X data to forecast a March recession probability. Instead we can take account of the fact that financial data is released without delay or measurement error, and update the current February recession probability, potentially, as every trading day of February passes. This allows for constant updating of economic conditions. Visually, and intuitively, the matrices may look like the following:

$$Y = \begin{bmatrix} y_{feb} \\ y_{mar} \\ \vdots \\ y_T \end{bmatrix} X_i = \begin{bmatrix} x_{i,feb15th} & \cdots & x_{i,feb1st} \\ x_{i,mar15th} & \cdots & x_{i,mar1st} \\ \vdots & \cdots & \vdots \\ x_{i,T-h+j/K} & \cdots & x_{i,T-h+1/K} \end{bmatrix} X_j = \begin{bmatrix} x_{i,jan31st} & \cdots & x_{i,jan1st} \\ x_{i,feb28th} & \cdots & x_{i,feb1st} \\ \vdots & \cdots & \vdots \\ x_{i,T-h-1/K} & \cdots & x_{i,T-h-(K-1)/K} \end{bmatrix} \tag{25}$$

Where X_i is the matrix referring to the 'lead' if we were 15 trading days into February, which can be updated as new information is released, and X_j is information from 1 month previous. As Andreou, Elena and Ghysels (2013) explains, conventional nowcasting typically refers to within period updates of forecasts. For example, updating the current months recession probability as shown above. Nowcasting with leads can be viewed as current month updates of current months recession probabilities, but also of any future horizon recession probability forecast (i.e. for h= $2, 3$ months).

A common method for nowcasting is to use state space models. While both state space and my MFP model can produce multiple horizon forecasts, a subtle difference is that the MFP can produce direct, as opposed to iterated h step ahead forecasts. Arguably iteration-based forecasts can suffer from misspecification, which can be compounded across multiple horizons that may produce inferior forecasts; see Marcellino, Stock and Watson (2006). However, for the purpose of my paper I will focus on the conventional definition of nowcasting, updating just the current months recession probability to see if this improves out of sample prediction.

7.1 Results

I follow the same out of sample forecasting procedure as described before, using the model explained in equation 16, adding additional leads for the financial data as laid out in equation 22. However, one small complication is the number of variables that are needed to be estimated. For example, in the case of the US the best model found for out of sample estimation included 3 daily bond market factors, 1 monthly stock market factor and 2 monthly real market factors. Assuming I use a polynomial of degree 3, I need to estimate 16 parameters (including a constant). When I add the lead terms for the daily bond market factors this could potentially increase to 28 parameters to estimate, which is high relative to my number of observations. To overcome this problem I have two potential solutions; 1) adjust down the degree of the polynomial on the 'lead' data as less data points do not need such a flexible function to weight them. 2) Remove some of the lagged mixed frequency factors which represent older information, but keep all the 'lead' factors which represent the most up to date information. Financial markets absorb shocks quickly into prices so the most up to date information is the most relevant. Both these strategies decrease the number of parameters to estimate, and allow the maximum likelihood estimation to converge.

Specifically, the degree of polynomial I use to weight the 'lead' data points increases as the number of leads increases. For one lead the data is included directly into the MFP, for 2 leads I use a polynomial of degree 1, for 3 to 9 leads I use a polynomial of degree 2 and for 10 to 14 leads I use a polynomial of degree 3. For each model I only include 2 of the lagged bond factors, but all 3 of the lead bond factors. This decision was made after trying the different variations, and was based on the best evaluation results.

7.2 US

Figure 5 compares the out of sample evaluation statistics of the MFP model containing various numbers of leads, with the MFP model that contains no leads for the 1 month forecast horizon. The top panel of Figure 5 divides the QPS from the MFP containing leads by the QPS of the MFP with no leads, for the full out of sample period. The x-axis refers to the number of trading days I use as leads. For example, $lead = 8$ compares the MFP with 8 extra days of bond market data from the current month, to the MFP that contains no leads. A ratio of less than 1 indicates that the extra information from the leads reduces the error rate of the current months recession prediction. The red dot concludes whether the the extra information improves recession probability prediction at the 10% level of significance, according to the Diebold Mariano Test. For all leads examined (1-14) the additional bond market information used to nowcast the current months recession probability reduces the QPS by up to 14%. For 6 of the leads (2,3,4,8,13 and 14 days) I find that the recession probability predictions are more accurate than the MFP containing no leads, as shown by the red dots. Further analysis also shows a reduction in the false positive that was found in the top panel of Figure 3 in 1998. Surprisingly there is no clear relationship between number of leads and model performance, shown by a correlation of 0.11 between the QPS ratio and the number of leads included.

The bottom panel of Figure 5 divides the QPS onset from the MFP containing leads, by the QPS onset of the MFP with no leads. For all leads examined, the additional daily bond market data improves the QPS onset error rate by up to 15%. Unlike the top panel, there is a stronger relationship between the QPS onset ratio and the number of leads. The correlation is -0.29, which is still weak but is in the direction I would expect.

7.3 Canada

The same process of nowcasting was carried out for Canada using weekly bond market data as leads. When leads of 1,2 and 3 weeks of bond data were included in the model there were no improvements in forecasting performance at the 10% significance level according to the Diebold Mariano test, compared to the forecasts of the MFP used for Canada with no leads. As with the out of sample results this may explain the importance of using daily financial data in the model, and may also indicate that financial markets in Canada incorporate news and economic shocks into financial prices at a slower rate than the US.

8 Machine Learning - Neural Networks

With the rise in popularity of machine learning methods, and as a final part of my analysis, I compare my MFP results with forecasts from a mixed frequency artificial neural network (MF-ANN). In order to include mixed frequency data in the network, I will follow the methodology proposed by Xu (2019). They develop a simple ANN model for mixed frequency data through introducing an unrestricted mixed data sampling (U-MIDAS) approach into the ANNs framework. In their paper the model is applied to an empirical task of forecasting inflation in China, and improvements are found in forecasting performance compared to a model that aggregates data into the same frequencies.

Figure 6 shows the mixed frequency feed forward ANN proposed by Xu (2019). X in the input layer represents the chosen variables used for the analysis. The frequency alignment layer changes the dimension of the higher frequency data so that it is the correct dimensions to run through the network. For example, if I use daily financial data to predict the monthly recession indicator then I will have K days of explanatory data per month. Originally X is a vector of length $(T \times K)$ where K represents the number of higher frequency data points in the lower frequency time frame, and T represents the number of lower frequency data points in the sample. The frequency alignment step transforms this vector to a $(T \times K)$ matrix, where the first column represents the most recent trading days data of each month, the second column represents the second most recent trading days data of each month and so on.

Previously in the (MFP) I was restricted by the number of variables I could include in the regression, hence I used a polynomial to weight the higher frequency data. This greatly reduced the number of parameters to estimate. With the ANN I do not necessarily have the degrees of freedom concern. I can therefore add the higher frequency daily data directly into the network as its own variable. For example, using the unrestricted MIDAS approach proposed by Xu (2019) with 3 daily financial variables as my X, and $K = 15$, would mean 45 inputs into the mixed frequency ANN. With the MFP this would cause extreme in sample over-fitting, which is why a polynomial would be used to weight the higher frequency data. Xu (2019) calls this the U-MIDAS ANN approach as we are not restricting the weighting functions for the higher frequency data to follow a particular function. Theoretically the network will decide which days of the month are the most important in predicting recessions and weight them accordingly.

The activation function used is a sigmoid function transforming the output to a value between 0 and 1, representing the probability of a recession. After data is passed through the network, the cross entropy loss function, which is the same as the log probability score shown in equation 21, is calculated and back propagation is used to train the network by adjusting the weights and bias.

For this comparison I focus only on the US, using the same variables found to be the best predictors in the MFP; 3 daily bond factors, 1 monthly stock market factor and 2 monthly real activity factors. The MF-ANN is recursively estimated month by month with the publication lag for Y assumed to be 3 months. Unlike the MFP this publication lag can impact results significantly. As this is just a comparison, and not a thorough exploration of mixed frequency data in neural networks, I report results only for a MF-ANN with 1 hidden layer containing 5 nodes.

8.1 US Results

Figure 7 shows the out of sample recession probabilities for forecast horizons 1, 2 and 3 months between November 1988 and January 2019. The red line shows recession probabilities calculated from the MFP, and the black line shows probabilities calculated from the MF-ANN. The MFP visually appears to pick up the first recession period more accurately than the MF-ANN, at all forecast horizons. Results for the second recession period are similar between the 2 models, with the MFP outperforming the MF-ANN at the 3 month forecast horizon. In the final recession period the MF-ANN peaks at a higher probability at the beginning of the recession at all forecast horizons. For false positives, the MF-ANN detects an additional one compared to the MFP at the 2 month horizon.

Table 7 reports the evaluation methods used to assess the performance of the MFP against the MF-ANN. For the QPS and LPS I divide the statistic from the MFP model by that of the MF-ANN, as I did in the previous sections. A value less than 1 indicates that the MFP is a better model. Results are mixed, with the MF-ANN generally outperforming the MFP over the whole out of sample period. This is shown by values greater than 1 for the QPS $Full$ and LPS $Full$, and Diebold Mariano test failing to reject the null hypothesis. The null states that the MFP is less accurate then the MF-ANN over the whole out of sample period. However, when focusing on the recession onset prediction performance, the MFP far outperforms the MF-ANN, sometimes by up to 21%.

Overall the results show promise for using mixed frequency data in a neural network to forecast recessions, and should be explored in more depth. This brief study did not explore all possible input variables and hyper-parameters settings in the network, and therefore forecast performance could be optimized more by doing so.

9 Conclusion

This paper uses factors representing the bond, stock, exchange rate and real markets estimated from panels of macroeconomic and financial data. These daily, weekly and monthly factors are then used to predict future US and Canadian recession dates, as published by the NBER and the Business Cycle Council of the C.D. Howe Institute respectively, in a mixed frequency probit model. My main findings show that when daily financial data is included in the forecasting model for the US, out of sample predictive performance improves at the 1,2 and 3 month forecasting horizons with reductions of up to 17% in the quadratic probability score (QPS) depending on the forecast horizon. This is compared to aggregating data at the monthly frequency. When focusing on recession onset prediction these improvements increase up to 30%. However, for Canada, where only weekly financial data for the whole sample is available, there are mixed results. I find improved recession onset prediction performance at all forecast horizons, with up to a 22% decrease in the QPS. But evaluation statistics covering the full out of sample period show no improvement in predictive power when compared to probit models that aggregate financial data at the monthly frequency.

Additionally, I use the benefit of daily and weekly frequency data to nowcast the current months recession probabilities, updating forecasts on a daily and weekly basis in the US and Canada respectively. By including the current months financial data into the MFP I find that I can improve forecasting performance measured by the QPS in the US by up to 14%. The Diebold Mariano test also shows statistically significant improvements in forecasting performance at the 10% level, depending how many days of the current months financial data is included in the model. However, there are no significant improvements from nowcasting using financial data in Canada.

By dividing data into the 4 asset markets and extracting factors from each of these, I am able to gain valuable insights into which markets are key leading indicators for US and Canadian recessions. Results are similar for both the US and Canada. I find that daily and weekly bond market factors are key leading indicators, especially at detecting the onset of a recession, in the US and Canada respectively, compared to aggregated monthly bond market data. The stock and real market also play important roles in improving forecasting performance. However, this is only when the stock market data is aggregated at the monthly frequency, due to daily data causing volatile results. Exchange rate data at any frequency is not useful in predicting future recessions in the US and Canada.

Finally, I compare my MFP with the forecasting performance of a neural network that incorporates mixed frequency data. Results show promise that inclusion of mixed frequency data into machine learning techniques can improve forecasting accuracy further, and this should be explored in greater detail in the future.

10 Data Appendix

For US data in Table 1, the Bond and Exchange rate market data are from FRED (St. Louis Fed) unless AC (authors calculation) is stated. All macroeconomic indicators are from FRED. The SP500 Industrial Index and SP500 PE ratio are from GFD (Global Financial Data). Finally, the S&P 500 and Dow Jones index closing price is from Yahoo Finance.

For Canadian data in Table 2, the Bond market data are from Statistics Canada, Exchange rate data is from FRED, Stock market data is from GFD and all Macroeconomic indicators are from FRED, apart from Housing Starts which is from Statistics Canada. AC indicates authors calculation.

Table 1: US Variables

Variable	Daily	Weekly	Monthly	Description
Bond Market				
Fed Funds 2 (0)	Y	N	Y	Interest Rate: Federal Funds (Effective) (% per annum)
3m Tbill 2 (0)	Y	N	Y	Interest Rate: US Treasury Bills, 3-Mo. (% per annum)
6m Tbill 2 (0)	Y	N	Y	Interest Rate: US Treasury Bills, 6-Mo. (% per annum)
1y Tbond 2 (0)	Y	N	Y	Interest Rate: US Treasury, 1-Yr. (% per annum)
5y Tbond 2 (0)	Y	N	Y	Interest Rate: US Treasury, 5-Yr. (% per annum)
10y Tbond 2 (0)	Y	N	Y	Interest Rate: US Treasury, 10-Yr. (% per annum)
AAA bond 2 (0)	N	Y	Y	Bond Yield: Moody's AAA Corporate (% per annum)
BAA bond 2 (0)	N	Y	Y	Bond Yield: Moody's BAA Corporate (% per annum)
3m spread 1 (0)	Y	N	Y	3m Tbill - Fed Funds (AC)
6m spread 1 (0)	Y	N	Y	6m Tbill - Fed Funds (AC)
1y spread 1 (0)	Y	N	Y	1y Tbill - Fed Funds (AC)
5y spread 1 (0)	Y	N	Y	5y Tbill - Fed Funds (AC)
10y spread 1 (0)	Y	N	Y	10y Tbill - Fed Funds (AC)
AAA spread 1 (0)	N	Y	Y	AAA bond - Fed Funds (AC)
BAA spread 1 (0)	N	Y	Y	BAA bond - Fed Funds (AC)
Exchange Rate Market				
Ex. Rate Switzerland 3 (0)	Y	N	Y	Foreign Exchange Rate: Swiss Franc per US$
Ex. Rate Japan 3 (0)	Y	N	Y	Foreign Exchange Rate: Yen per US$
Ex. Rate UK 3 (0)	Y	N	Y	Foreign Exchange Rate: Cents per Pound
Ex. Rate Canada 3 (0)	Y	N	Y	Foreign Exchange Rate: Canadian$ per US$
Stock Market				
SP 500 3 (0)	Y	N	Y	S&P 500 Index Closing Price
DJ Index 3 (0)	Y	N	Y	Dow Jones Index, Closing Price
SP Industrials 3 (0)	N	N	Y	S&P 500 Industrials Index Closing Price
SP PE ratio 3 (2)	N	N	Y	S&P 500 Index: Price Earnings Ratio (%)
Macroeconomic Indicators				
IPI 3 (1)	N	N	Y	Industrial Production Index, Total Index
PILT 3 (1)	N	N	Y	Personal Income Less Transfer Payments
MTS 3 (1)	N	N	Y	Manufacturing and Trade Sales
Emp: Total 3 (1)	N	N	Y	Employees On Nonfarm Payrolls: Total Private
Housing Starts 3 (1)	N	N	Y	Total New Privately Owned Housing Units Started
Unemp claims: Weekly 3 (1)	N	Y	N	Unemployment Insurance Weekly Claims

1 = no transformation, 2 = first difference, 3 = log of first difference, number in parentheses = publication lag in weeks or months

Table 2: Canada Variables

Variable	Daily	Weekly	Monthly	Description
Bond Market				
10y+ Gov Bond 2 (0)	N	Y	Y	Government marketable bonds average yield, 10 years
5-10y Gov Bond 2 (0)	N	Y	Y	Government marketable bonds average yield, 5-10 years
3-5y Gov Bond 2 (0)	N	Y	Y	Government marketable bonds average yield, 3-5 years
1-3y Gov Bond 2 (0)	N	Y	Y	Government marketable bonds average yield, 1-3 years
3m Prime Corp Paper 2 (0)	N	Y	Y	3 months prime corporate paper
2m Prime Corp Paper 2 (0)	N	Y	Y	2 months prime corporate paper
1m Prime Corp Paper 2 (0)	N	Y	Y	1 month prime corporate paper
10y+ spread 1 (0)	N	Y	Y	Yield Spread b/t 10-yr bond and 3-m prime (AC)
5-10y spread 1 (0)	N	Y	Y	Yield Spread b/t 5-10-yr bond and 3-m prime (AC)
3-5y spread 1 (0)	N	Y	Y	Yield Spread b/t 3-5-yr bond and 3-m prime (AC)
1-3y spread 1 (0)	N	Y	Y	Yield Spread b/t 1-3-yr bond and 3-m prime (AC)
Exchange Rate Market				
Ex. Rate Switzerland 3 (0)	Y	N	Y	Foreign Exchange Rate: Swiss Franc per Can$
Ex. Rate Japan 3 (0)	Y	N	Y	Foreign Exchange Rate: Yen per Can$
Ex. Rate US 3 (0)	Y	N	Y	Foreign Exchange Rate: US$ per Can$
Ex. Rate UK 3 (0)	Y	N	Y	Foreign Exchange Rate: Pound Sterling per Can$
Stock Market				
TSX Value 3 (0)	N	N	Y	Toronto Stock Exchange, value of shares traded
TSX Vol 3 (0)	N	N	Y	Toronto Stock Exchange, volume of shares traded
Macroeconomic Indicators				
Consumer Credit 3 (1)	N	N	Y	Consumer Credit, month-end, sa, Total outstanding balances
Manufacturing Prod 3 (1)	N	N	Y	Production in total manufacturing, sa
Emp: Total 2 (1)	N	N	Y	Employed population, total
Housing Starts 1 (1)	N	N	Y	Housing starts 12 month growth

1 = no transformation, 2 = first difference, 3 = log of first difference, number in parentheses = publication lag in weeks or months

References

[1] Shirley Almon. The distributed lag between capital appropriations and expenditures. *Econometrica: Journal of the Econometric Society*, pages 178–196, 1965.

[2] Elena Andreou, Eric Ghysels, and Andros Kourtellos. Regression models with mixed sampling frequencies. *Journal of Econometrics*, 158(2):246–261, 2010.

[3] Elena Andreou, Eric Ghysels, and Andros Kourtellos. Should macroeconomic forecasters use daily financial data and how? *Journal of Business & Economic Statistics*, 31(2):240–251, 2013.

[4] S Boragan Aruoba, Francis X Diebold, and Chiara Scotti. Real-time measurement of business conditions. *Journal of Business & Economic Statistics*, 27(4):417–427, 2009.

[5] Joseph Atta-Mensah, Greg Tkacz, et al. *Predicting Canadian recessions using financial variables: A probit approach*, volume 98. Bank of Canada, 1998.

[6] Francesco Audrino, Alexander Kostrov, and Juan-Pablo Ortega. Predicting us bank failures with midas logit models. *Journal of Financial and Quantitative Analysis*, 54(6):2575–2603, 2019.

[7] Mehmet Balcilar, Rangan Gupta, and Mawuli Segnon. The role of economic policy uncertainty in predicting us recessions: A mixed-frequency markov-switching vector autoregressive approach. *Economics: The Open-Access, Open-Assessment E-Journal*, 10(2016-27):1–20, 2016.

[8] Henri Bernard and Stefan Gerlach. Does the term structure predict recessions? the international evidence. *International Journal of Finance & Economics*, 3(3):195–215, 1998.

[9] Marie Bessec and Othman Bouabdallah. Forecasting gdp over the business cycle in a multi-frequency and data-rich environment. *Oxford Bulletin of Economics and Statistics*, 77(3):360–384, 2015.

[10] R Bhansali. for multistep prediction of a time series: A review. *Asymptotics, nonparametrics, and time series*, 201, 1999.

[11] Maximo Camacho, Gabriel Perez Quiros, and Pilar Poncela. Green shoots and double dips in the euro area: A real time measure. *International Journal of Forecasting*, 30(3):520–535, 2014.

[12] Marcelle Chauvet and Jeremy Piger. A comparison of the real-time performance of business cycle dating methods. *Journal of Business & Economic Statistics*, 26(1):42–49, 2008.

[13] Yu-chin Chen and Wen-Jen Tsay. Forecasting commodity prices with mixed-frequency data: An ols-based generalized adl approach. *Available at SSRN 1782214*, 2011.

[14] Guillaume Chevillon and David F Hendry. Non-parametric direct multi-step estimation for forecasting economic processes. *International Journal of Forecasting*, 21(2):201–218, 2005.

[15] Michael P Clements and David F Hendry. Multi-step estimation for forecasting. *Oxford Bulletin of Economics and Statistics*, 58(4):657–684, 1996.

[16] Arturo Estrella and Frederic S Mishkin. Predicting us recessions: Financial variables as leading indicators. *Review of Economics and Statistics*, 80(1):45–61, 1998.

[17] David F Findley. On the use of multiple models for multi-period forecasting. In *Proceedings of Business and Economic Statistics, American Statistical Association*, pages 528–531, 1983.

[18] David F Findley. Model selection for multi-step-ahead forecasting. *IFAC Proceedings Volumes*, 18(5):1039–1044, 1985.

[19] Claudia Foroni, Pierre Guérin, and Massimiliano Marcellino. Markov-switching mixed-frequency var models. *International Journal of Forecasting*, 31(3):692–711, 2015.

[20] Claudia Foroni and Massimiliano Giuseppe Marcellino. A survey of econometric methods for mixed-frequency data. *Available at SSRN 2268912*, 2013.

[21] Sebastian Fossati. Forecasting us recessions with macro factors. *Applied Economics*, 47(53):5726–5738, 2015.

[22] Sebastian Fossati, Rodrigo Sekkel, and Max Sties. Forecasting recessions in canada. 2018.

[23] Lennart Freitag et al. Default probabilities, cds premiums and downgrades: A probit-midas analysis. Technical report, 2014.

[24] Eric Ghysels, Arthur Sinko, and Rossen Valkanov. Midas regressions: Further results and new directions. *Econometric Reviews*, 26(1):53–90, 2007.

[25] Luis M Gomez-Zamudio and Raul Ibarra. Are daily financial data useful for forecasting gdp? evidence from mexico. *Journal of the Latin American and Caribbean Economic Association*, 17(2):173–203, 2017.

[26] Lili Hao and Eric CY Ng. Predicting canadian recessions using dynamic probit modelling approaches. *Canadian Journal of Economics/Revue canadienne d'économique*, 44(4):1297–1330, 2011.

[27] Yu-Fan Huang and Richard Startz. Improved recession dating using stock market volatility. *International Journal of Forecasting*, 36(2):507–514, 2020.

[28] Cuixia Jiang, Wei Xiong, Qifa Xu, and Yezheng Liu. Predicting default of listed companies in mainland china via u-midas logit model with group lasso penalty. *Finance Research Letters*, page 101487, 2020.

[29] Munechika Katayama. Improving recession probability forecasts in the us economy, 2009.

[30] Gitanjali Kumar. High-frequency real economic activity indicator for canada. Technical report, Bank of Canada Working Paper, 2013.

[31] Vladimir Kuzin, Massimiliano Marcellino, and Christian Schumacher. Midas vs. mixed-frequency var: Nowcasting gdp in the euro area. *International Journal of Forecasting*, 27(2):529–542, 2011.

[32] Jin-Lung Lin and Clive WJ Granger. Forecasting from non-linear models in practice. *Journal of Forecasting*, 13(1):1–9, 1994.

[33] Sydney C Ludvigson and Serena Ng. A factor analysis of bond risk premia. Technical report, National Bureau of Economic Research, 2009.

[34] Massimiliano Marcellino, James H Stock, and Mark W Watson. A comparison of direct and iterated multistep ar methods for forecasting macroeconomic time series. *Journal of econometrics*, 135(1-2):499–526, 2006.

[35] Henri Nyberg. Dynamic probit models and financial variables in recession forecasting. *Journal of Forecasting*, 29(1-2):215–230, 2010.

[36] Michael T Owyang, Jeremy Piger, and Howard J Wall. Forecasting national recessions using state-level data. *Journal of Money, Credit and Banking*, 47(5):847–866, 2015.

[37] Inske Pirschel. Forecasting euro area recessions in real-time. Technical report, Kiel Working Paper, 2016.

[38] Jonathan H Wright. The yield curve and predicting recessions. *Finance and Economics Discussion Series, Federal Reserve Board*, 2006.

[39] Qifa Xu, Xingxuan Zhuo, Cuixia Jiang, and Yezheng Liu. An artificial neural network for mixed frequency data. *Expert Systems with Applications*, 118:127–139, 2019.

Tables

Table 3: In Sample, US

Forecast Horizon		**1 month**	**2 month**	**3 month**
QPS Full	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.93	0.94	0.88
QPS Onset	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.88	0.83	0.76
LPS Full	$\frac{LPS_{MFP}}{LPS_{benchmark}}$	0.88	0.89	0.87
Diebold Mariano	p value	0.15	0.13	0.01

Table 4: In Sample, Canada

Forecast Horizon		**1 month**	**2 month**	**3 month**
QPS Full	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.88	0.94	0.95
QPS Onset	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.95	0.96	0.95
LPS Full	$\frac{LPS_{MFP}}{LPS_{benchmark}}$	0.90	0.94	0.95
Diebold Mariano	p value	0.03	0.08	0.11

In sample results of the best model for the US and Canada at various forecast horizons. Best model of the US consists of 3 bond market factors, 1 stock market factor and 2 real market factors. Best model for Canada consists of 3 bond market factors, 1 stock market factor and 1 real market factor.

Table 5: Out of Sample, US

Forecast Horizon		**1 month**	**2 month**	**3 month**
QPS Full	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.86	0.83	0.87
QPS Onset	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.89	0.70	0.71
LPS Full	$\frac{LPS_{MFP}}{LPS_{benchmark}}$	0.92	0.88	0.90
Diebold Mariano	p value	0.006	0.002	0.019

Table 6: Out of Sample, Canada

Forecast Horizon		**1 month**	**2 month**	**3 month**
QPS Full	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	1.13	1.13	1.11
QPS Onset	$\frac{QPS_{MFP}}{QPS_{benchmark}}$	0.78	0.83	0.85
LPS Full	$\frac{LPS_{MFP}}{LPS_{benchmark}}$	1.26	1.20	1.25
Diebold Mariano	p value	0.95	0.93	0.88

Out of sample results of the best model for the US and Canada at various forecast horizons. Best model of the US consists of 3 bond market factors, 1 stock market factor and 2 real market factors. Best model for Canada consists of 3 bond market factors, 1 stock market factor and 1 real market factor.

Table 7: MFP vs MF-ANN, Out of Sample, US

Forecast Horizon		**1 month**	**2 month**	**3 month**
QPS Full	$\frac{QPS_{MFP}}{QPS_{MF-ANN}}$	1.13	1.05	1.06
QPS Onset	$\frac{QPS_{MFP}}{QPS_{MF-ANN}}$	0.91	0.82	0.79
LPS Full	$\frac{LPS_{MFP}}{LPS_{MF-ANN}}$	1.00	0.91	1.04
Diebold Mariano	p value	0.89	0.61	0.70

Out of sample results of the best model for the US at various forecast horizons, compared to a mixed frequency ANN (MF-ANN). Best model of the US consists of 3 bond market factors, 1 stock market factor and 2 real market factors.

Figures

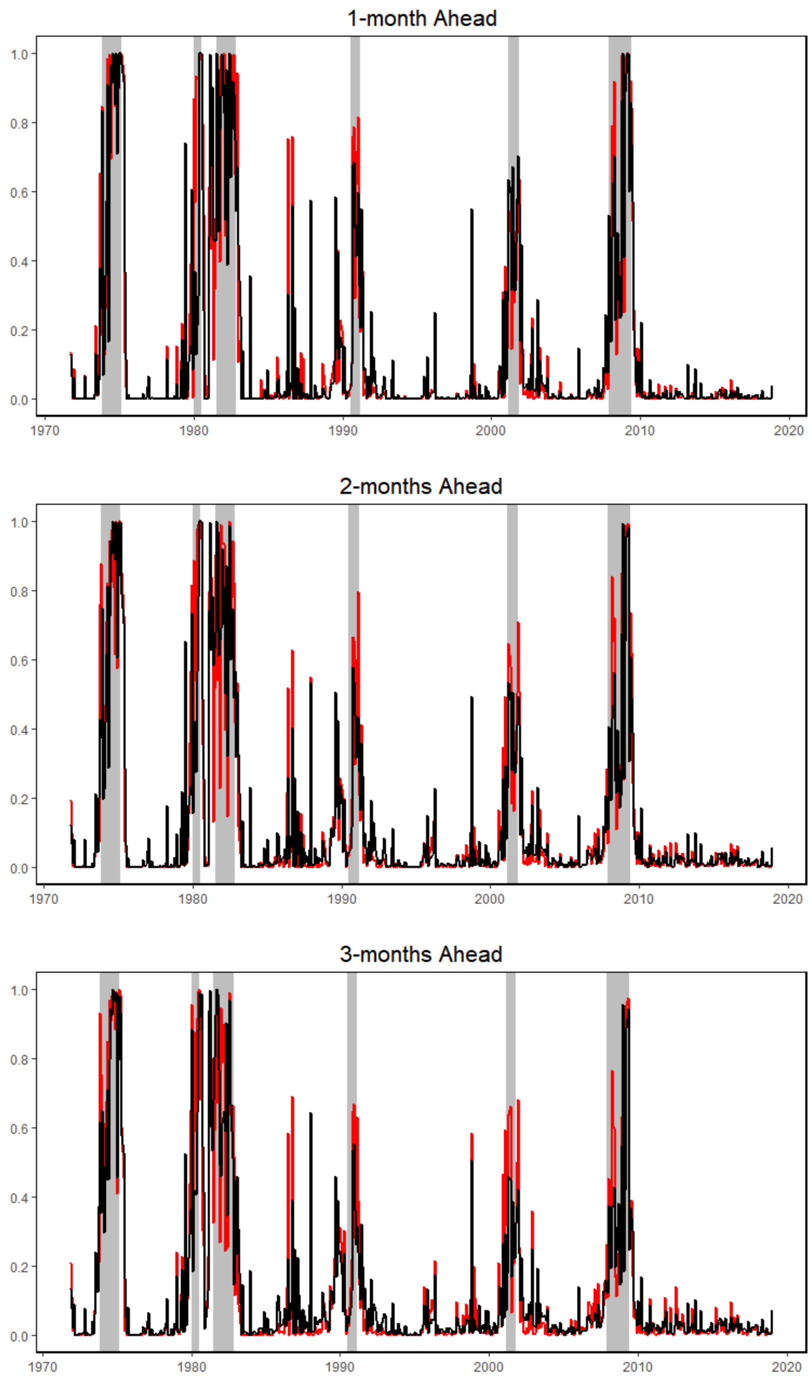

Figure 1: In sample US predicted probabilities of a recession for the best performing probit models at different forecast horizons: mixed frequency probit (red); benchmark model (black). Shaded areas show recessions as defined by the NBER.

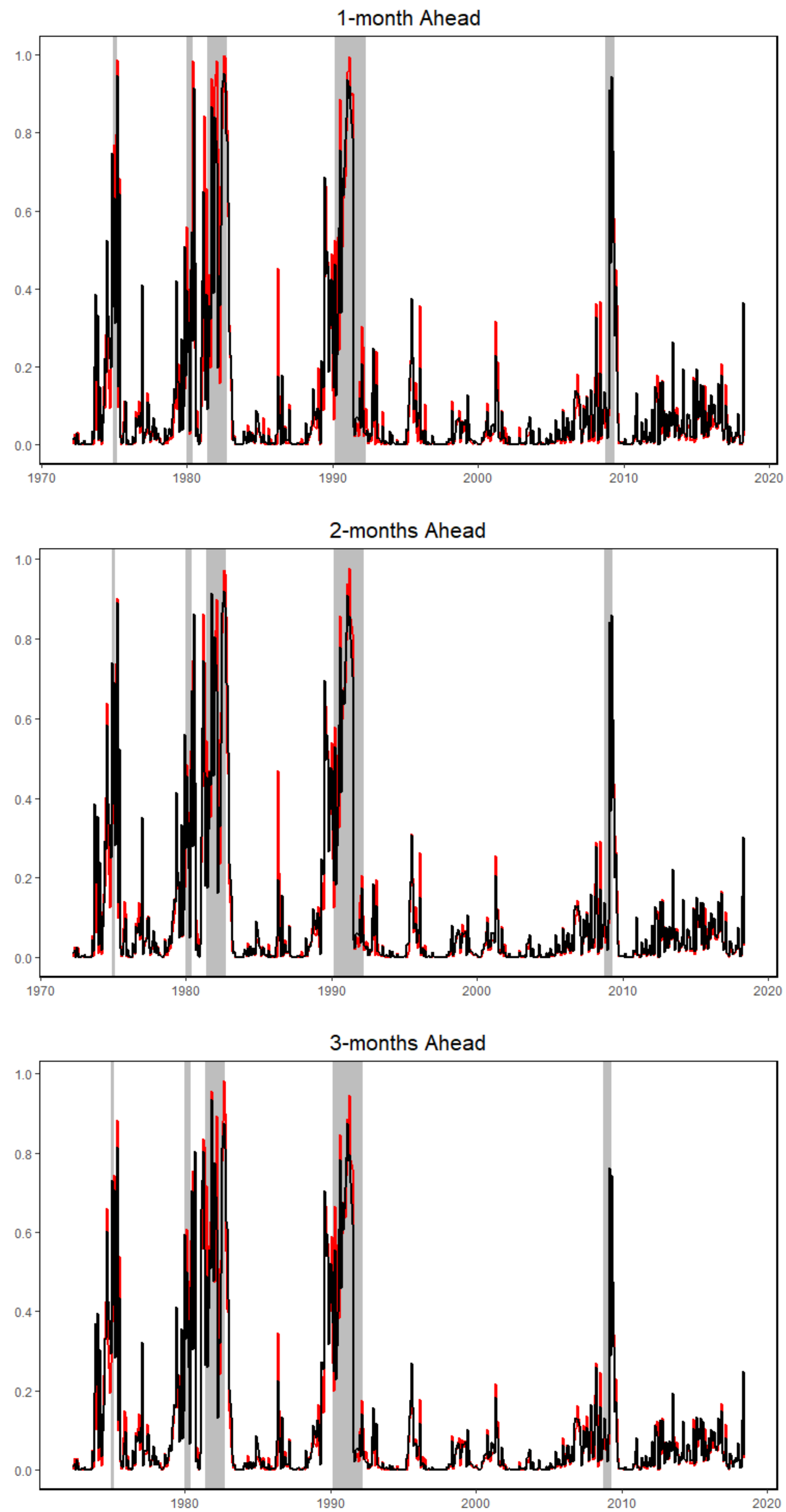

Figure 2: In sample Canadian predicted probabilities of a recession for the best performing probit models at different forecast horizons: mixed frequency probit (red); benchmark model (black). Shaded areas show recessions as defined by the Business Cycle Council of the C.D. Howe Institute.

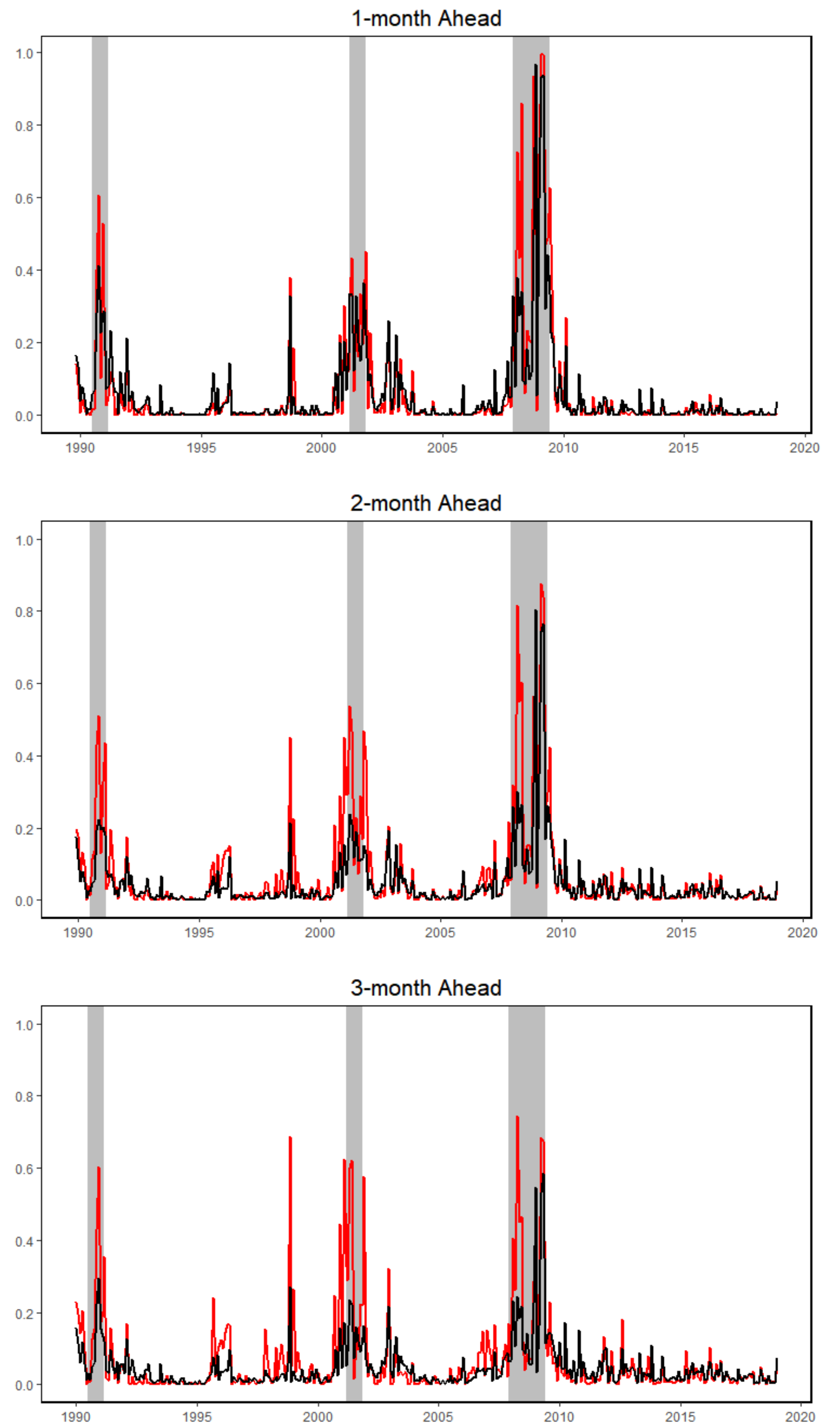

Figure 3: Out of sample US predicted probabilities of a recession for the best performing probit models at different forecast horizons: mixed frequency probit (red); benchmark model (black). Shaded areas show recessions as defined by the NBER.

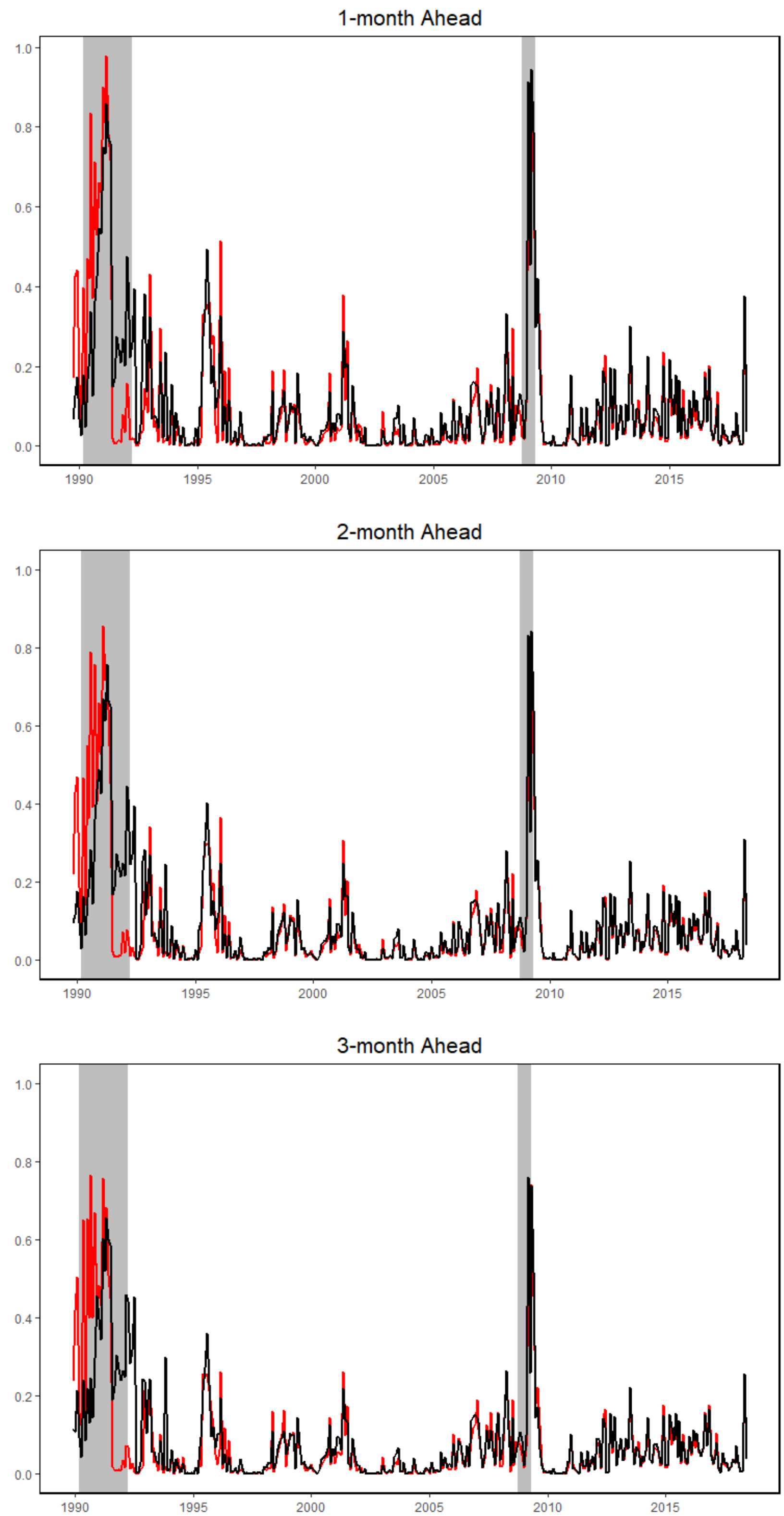

Figure 4: Out of sample Canadian predicted probabilities of a recession for the best performing probit models at different forecast horizons: mixed frequency probit (red); benchmark model (black). Shaded areas show recessions as defined by the Business Cycle Council of the C.D. Howe Institute.

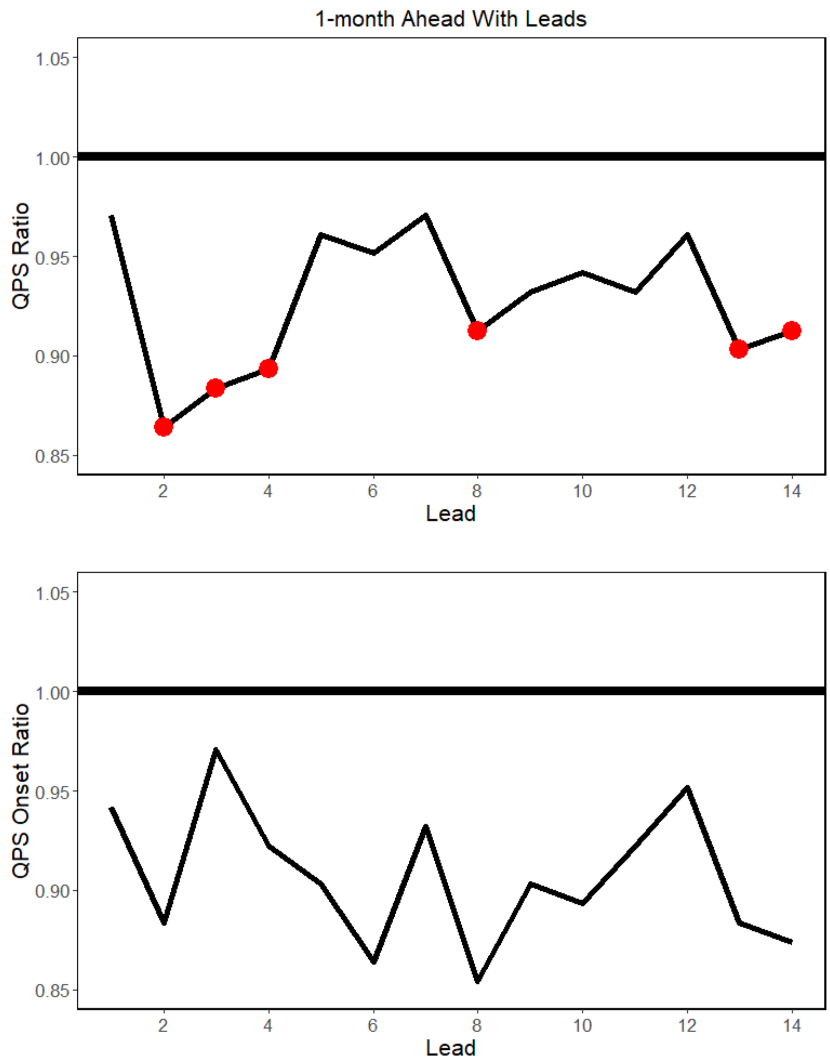

Figure 5: Top: US out of sample QPS of the MFP nowcast with leads model divided by QPS of the MFP model with no leads at the 1-month forecast horizon. Red dots show when the nowcast with leads model is more accurate than the MFP 1-month ahead model at the 10% significance level, as per the Diebold Mariano Test. Bottom: Out of sample QPS onset of the MFP nowcast with leads model divided by QPS onset of the MFP model with no leads at the 1-month forecast horizon. Lead refers to the number of days of financial data into the current month used to nowcast the current months recession probability. Horizontal line at 1 used to show when the QPS of the nowcast with leads model is lower than QPS of MFP with no leads model.

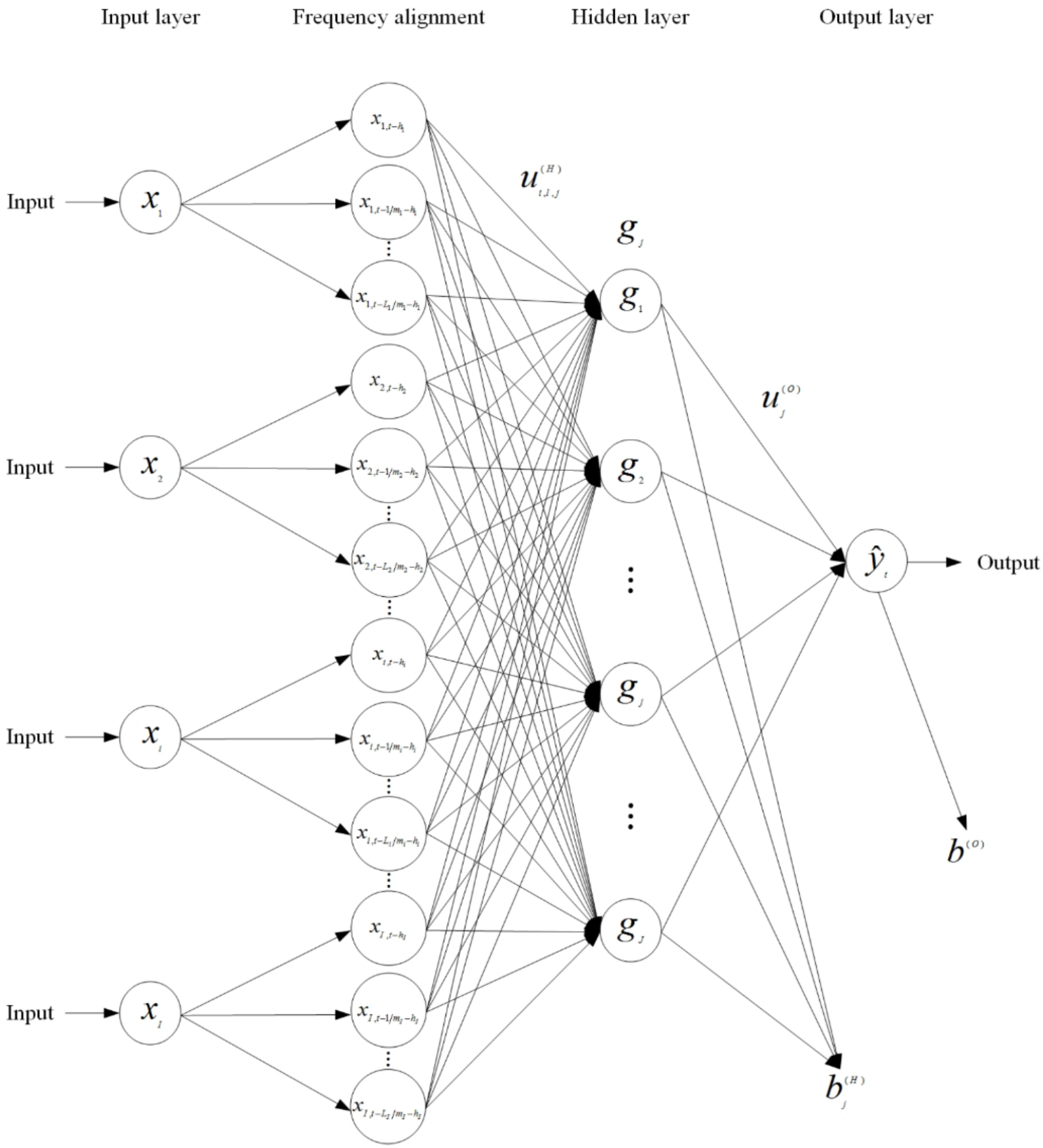

Figure 6: Unrestricted Mixed Frequency (U-MIDAS) ANN

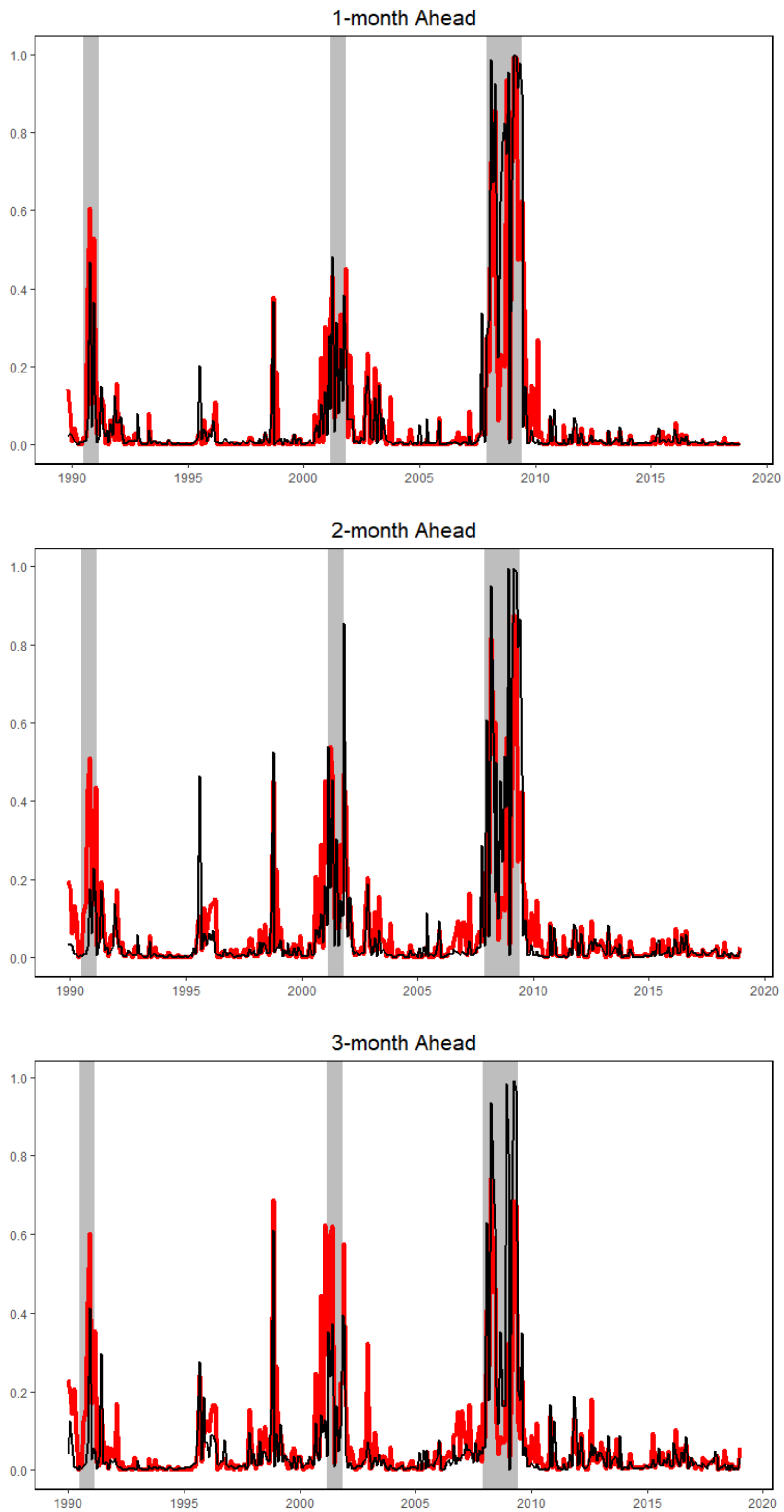

Figure 7: Out of sample US predicted probabilities of a recession at different forecast horizons: mixed frequency probit (red); mixed frequency ANN (black). Shaded areas show recessions as defined by the NBER.