Improving Information Leadership Share for Measuring Price Discovery

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Abstract

We propose an improvement to the information leadership (IL) measure of price discovery as introduced in Yan and Zivot (2010), and the information leadership share (ILS) measure proposed by Putniņš (2013). Our improved IL and ILS measures integrate the price discovery share (PDS) from Sultan and Zivot (2015) with the component share (CS) from Gonzalo and Granger (1995). In contrast to the IL metric by Yan and Zivot (2010), which combines the information share (IS) measure from Hasbrouck (1995) with CS, we demonstrate that our improved IL measure accurately reflects the ratio of initial responses of competing markets to a permanent shock, even when the residuals of reduced-form vector error correction models are correlated. Simulation evidence strongly supports the superiority of our measures over a wide spectrum of existing price discovery metrics (Lien and Shrestha, 2009; Putniņš, 2013; Sultan and Zivot, 2015; Patel et al., 2020). We demonstrate the effectiveness of our improved measures by examining price discovery for various Chinese stocks cross-listed in Shanghai and Hong Kong (SH-HK) both before and after the initiation of the Shanghai-Hong Kong Stock Connect.

Keywords: price discovery, information share, component share, Shanghai-Hong Kong Stock Connect

JEL Classification: C32, G10

1 Introduction

Price discovery, the process through which new pertinent information is assimilated into prices via trading activities, holds substantial importance in facilitating efficient resource allocation within a free market. Market fragmentation, characterized by the trading of the same security across multiple markets and the trading of closely related assets (such as derivatives, futures and spot contracts, ETFs tracking the same market index, etc.), is prevalent in major financial markets. Understanding which asset or market most effectively integrates new fundamental information into prices is crucial for evaluating market quality and comprehending the transmission of information into prices.

The most commonly utilized empirical price discovery metrics include the information share (IS) introduced by Hasbrouck (1995) and the component share (CS) proposed by Gonzalo and Granger (1995). As highlighted in numerous studies (e.g., Harris et al., 2002, Lien and Shrestha, 2009, Yan and Zivot, 2010, Sultan and Zivot, 2015, Patel et al., 2020), the IS measure assesses the contribution of each price series to price discovery by quantifying the share of variance in the efficient price attributable to each market. Conversely, the CS measure represents the normalized weight in the common efficient price, expressed as a linear combination of these price series. Therefore, IS can be seen as a variance-weighted version of CS. Empirically, IS is considered to provide a more robust estimate of price leadership than CS and is widely employed to quantify the price discovery process among closely related products.[1]

While the IS metric is widely utilized to gauge price discovery, it encounters a well-recognized identification challenge: its value is contingent upon the arrangement (order) of assets within the price vector used for analysis. In formulating IS, Hasbrouck (1995) employed the Cholesky decomposition of the variance-covariance matrix of residuals derived

[1]For instance, IS has been applied to various scenarios such as cross-listed stocks across different stock exchanges (Hasbrouck, 1995; Harris et al., 2002), between quotes and trade prices of stocks (Harris et al., 2002), between stock options and underlying stocks (Chakravarty et al., 2004), between futures and spot prices (Mizrach and Neely, 2008; Lien and Shrestha, 2009), and among Credit Default Swap (CDS), bonds, and stocks (Grammig and Peter, 2013), among others.

from the reduced-form vector error correction model (VECM) to derive a unique IS value. However, this method assigns greater variance to the asset positioned first in the price vector when the reduced-form residuals are correlated. Consequently, different permutations of assets in the price vector result in distinct IS values. In practice, the upper and lower bounds of IS measures from all permutations of the price vector are utilized to indicate the range of possible IS values. Yet, if reduced-form residuals exhibit high correlation, these ranges may fail to provide informative insights into which asset leads the price discovery process.

To address the order-dependence issue of IS, numerous studies have proposed various solutions and novel measures. For instance, Hasbrouck (1995) advocated for the utilization of ultra-frequency data to mitigate residual correlation, while Gonzalo and Granger (1995) recommended employing the CS measure derived from a permanent-transitory (PT) decomposition. Baillie et al. (2002) proposed the use of the mean or midpoint of all possible IS measures resulting from reordering the price vector (referred to as IS-mid), while Grammig and Peter (2013) suggested analyzing price discovery by examining the extreme tails of return distributions. Additionally, Lien and Shrestha (2009) and Lien and Shrestha (2014) introduced the Modified Information Share (MIS) measure, which is based on a spectral decomposition of the correlation matrix of reduced-form residuals.

More recently, Sultan and Zivot (2015) introduced the Price Discovery Share (PDS) measure, which relies on a straightforward additive decomposition of the volatility of the underlying efficient market innovations. Similarly, De Jong and Schotman (2010) proposed an IS measure based on a structural unobserved component model, which can be demonstrated to align with the PDS measure of Sultan and Zivot (2015) under certain restrictive assumptions. These novel measures (MIS, PDS, and the De Jong-Schotman IS measure) share the same essence of variance attribution as IS but are order-invariant, thus providing unique price discovery assessments.[2]

[2]It's worth noting that, as acknowledged in Lien and Shrestha (2009) and discussed in Sultan and Zivot (2015), the MIS measure lacks uniqueness because one can choose the negative values of the square

The price discovery measures such as IS, IS-mid, CS, MIS, and PDS all rely on residuals derived from a reduced-form VECM. Consequently, obtaining a clear structural interpretation of these measures without additional assumptions is challenging. To gain a deeper understanding of what IS and CS truly capture, Yan and Zivot (2010) utilized a structural cointegration model to elucidate how these measures are influenced by independent permanent and transitory shocks. Their analysis revealed that while both IS and CS measures account for the relative avoidance of noise trading and liquidity shocks, only IS can offer insights into the relative informativeness of individual markets.

To help sort out the confounding effects of permanent and transitory shocks on the IS and CS measures when reduced-form errors are uncorrelated, Yan and Zivot (2010) proposed an Information Leadership (IL) measure by taking the ratios of IS and CS measures. They demonstrated that IL measures solely depend on each market's initial responses to the permanent shock, providing a straightforward and intuitive gauge of price discovery. Building upon this, Putniņš (2013) transformed IL into an Information Leadership Share (ILS) measure, which ranges between 0 and 1. Putniņš (2013) conducted a series of simulations with independent transitory shocks to illustrate that only ILS, compared with IS and CS, consistently measures price discovery irrespective of the relative noise level of each market.[3]

Furthermore, Patel et al. (2020) adapted ILS into a binary Information Leadership Indicator (ILI) to mitigate bias. However, a notable limitation of the IL, ILS, and ILI measures is their derivation under the assumption of uncorrelated reduced-form idiosyncratic errors. Empirically, reduced-form errors often exhibit correlation even with high-frequency data, casting doubt on the broad applicability of these measures in practice.

In this paper, we improve upon the structural analysis initiated by Yan and Zivot (2010)

root of the eigenvalues. However, as demonstrated in Section 2, the resulting MIS measure becomes unique due to the squared sign of the associated items in the MIS calculation. This distinction might have been overlooked by Lien and Shrestha (2009) due to the complexity of their MIS expressions. In the appendix, we present a more straightforward derivation and an easily calculable formula for MIS.

[3]It's worth noting that the simulations in Putniņš (2013) relax some of the assumptions of Yan and Zivot (2010) by considering a non-invertible Structural Vector Autoregression (SVAR) with two sources of transitory noise. However, Putniņš (2013) did not provide a structural analysis of ILS and did not explicitly consider the impact of correlated reduced-form residuals on ILS.

by offering structural interpretations of price discovery measures under the general scenario of correlated reduced-form residuals. We illustrate that when reduced-form errors are correlated, the structural representations of IS and MIS derived from the Cholesky decomposition of the residual covariance matrix and the spectral decomposition of the residual correlation matrix, respectively, become intricate. In contrast, the simple additive covariance decomposition employed in the PDS measure by Sultan (2015) offers a more straightforward structural representation. However, it's crucial to note that the structural representation of PDS encompasses responses to both transitory noise and new information. Consequently, relying solely on the PDS measure can lead to misleading conclusions regarding price discovery.

Applying a similar approach to Yan and Zivot (2010) in deriving the IL measure, we combine the PDS measure with the CS measure to formulate an improved IL measure. By employing the same structural cointegration model as Yan and Zivot (2010) while avoiding the assumption of uncorrelated reduced-form residuals, we demonstrate that our refined IL measure for a specific market equates to the ratio of its initial response to a permanent shock over that of the other competing market. Consequently, this enhanced IL measure aligns with the IL measure proposed by Yan and Zivot (2010) in scenarios featuring uncorrelated reduced-form residuals. Moreover, drawing inspiration from Putniņš (2013) and Patel et al. (2020), we further define an improved ILS measure and an improved binary ILI measure.

Simulation evidence from both a partial price adjustment model and the model proposed by Putniņš (2013) with correlated transitory shocks strongly supports the superiority of our measure over a wide spectrum of existing price discovery metrics (Lien and Shrestha, 2009; Putniņš, 2013; Sultan and Zivot, 2015; Patel et al., 2020). Our simulation findings underscore the importance of integrating the CS metric with variance decomposition estimates (such as IS, MIS, or PDS) to accurately identify the price leader. Most significantly, our results demonstrate that only our improved IL measures can reliably identify the leading market when reduced-form residuals exhibit correlation or when transitory noises are correlated across markets.

We exemplify our enhanced measures by analyzing price discovery for Chinese stocks cross-listed in Shanghai and Hong Kong (SH-HK) both before and after the introduction of the Shanghai-Hong Kong Stock Connect. Empirical exploration of these SH-HK cross-listed firms using our newly proposed price discovery measure indicates a discernible improvement in the efficiency of the Chinese stock market throughout the price discovery process following the implementation of the Shanghai-Hong Kong Stock Connect.

Our findings bear relevance to certain outcomes delineated in Lautier et al. (2023). In their work, they introduce an order-invariant and correlation-robust price discovery measure termed the Covariance Information Share (CovIS). This metric is derived from the covariance of reduced-form VECM residuals with the permanent shock. While our enhanced IL measure shares similarities with CovIS, it diverges in several key aspects.

Firstly, we initiate from a reduced-form VECM and demonstrate that the structural parameter, the ratio of markets' initial responses to the permanent shock, can be extracted from the reduced-form model through the estimation of the improved IL measure. Significantly, this identification is achieved without further assumptions on the underlying structural model and remains robust even in the presence of correlated reduced-form residuals.

In contrast, the approach outlined by Lautier et al. (2023) commences with a Structural Moving Average (SMA) model and hinges upon the identification of the SMA model. Additionally, Lautier et al. (2023) place greater emphasis on dynamic analyses of the price discovery process, whereas our study predominantly focuses on contemporaneous estimates.

Secondly, our study emphasizes the importance of distinguishing between the impacts of different variance-decomposition methods on the structural interpretations of various price discovery measures, particularly in scenarios involving correlated reduced-form residuals. We demonstrate that both the Cholesky decomposition proposed by Hasbrouck (1995) and the spectral decomposition introduced in Lien and Shrestha (2009) yield intricate structural interpretations of the IS and MIS measures when correlated reduced-form residuals are present.

In comparison, the straightforward additive decomposition method outlined in Sultan and Zivot (2015) yields clear structural representations of the PDS measure, irrespective of the correlation between reduced-form residuals. While Lautier et al. (2023) also discuss these traditional price discovery measures, their emphasis lies on the structural interpretations of these measures, without specifically addressing scenarios involving correlated reduced-form residuals.

Lastly, our study endeavors to integrate variance-decomposition price discovery measures with PT decomposition measures. While Lautier et al. (2023) also provide a structural analysis to elucidate various price discovery measures, their conclusion advocates discarding all traditional measures in favor of their new CovIS measure. In contrast, our study demonstrates that traditional price discovery measures can be improved by combining variance-decomposition measures with PT decomposition measures, akin to the approach advocated by Yan and Zivot (2010).

Through our analysis, we ascertain that the most robust variance-decomposition measure is the PDS proposed by Sultan and Zivot (2015), which should be integrated with the CS measure to derive an order-invariant and correlation-robust leadership measure. Our findings contribute to resolving the ongoing debate in the literature regarding the choice between IS and CS measures. In essence, our work aligns with the objectives of Lautier et al. (2023), albeit employing a different approach.

The remainder of the paper is structured as follows: In Section 2, we outline the methodology concerning existing price discovery measures derived from a reduced cointegration model, present structural representations of these measures, and introduce our novel measures. We showcase illustrations and simulation evidence in Section 3. Subsequently, in Section 4, we apply these proposed measures to Chinese stocks cross-listed in Shanghai and Hong Kong (SH-HK stocks) both before and after the implementation of the Shanghai-Hong Kong Stock Connect. Finally, our conclusions are presented in Section 5.

2 Methodology Summary

In this section, we conduct a comprehensive review of existing price discovery share measures derived from reduced-form VECMs. To provide lucid interpretations of these measures, we subsequently adopt the structural analysis framework outlined in Yan and Zivot (2010). This allows us to derive expressions for these price discovery measures in terms of interpretable structural parameters.

2.1 Measuring price discovery in reduced-form models

For simplicity, we narrow our focus to the case of two markets.[4] Let $\mathbf{p}_t = (p_{1t}, p_{2t})'$ denote a vector of logarithmic prices for two assets closely linked by arbitrage. In the price discovery literature, it is typically assumed that these two price series are integrated of order 1, or I(1), and $\mathbf{p}_t$ is cointegrated with the cointegrating vector $\boldsymbol{\beta} = (1, -1)'$.

Price discovery measures are typically derived from a reduced-form VECM formulated as follows:

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{\Gamma}_j \Delta \mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)'$ is the error correction vector, $\boldsymbol{\Gamma}_j$ $(i = 1, \ldots, k)$ are the short-run coefficient matrices, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the vector of reduced-form VECM residuals with $E[\boldsymbol{\varepsilon}_t] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Omega}$. In what follows, we represent the residual covariance matrix $\boldsymbol{\Omega}$ as:

$$\boldsymbol{\Omega} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}, \tag{2}$$

where σ_i^2 represents the variance of each market's idiosyncratic error and ρ denotes the correlation coefficient between these two errors.

[4] Analysis for n markets is provided in Appendix A3.

Hasbrouck (1995) transforms the above VECM model into a reduced-form Vector Moving Average (VMA) model:

$$\Delta \mathbf{p_t} = \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Psi}_2\boldsymbol{\varepsilon}_{t-2} + \cdots , \tag{3}$$

and its integrated form (or Beveridge-Nelson decomposition):

$$\mathbf{p}_t = \mathbf{p}_0 + \boldsymbol{\Psi}(1)\sum_{s=1}^{t}\boldsymbol{\varepsilon}_s + \boldsymbol{\Psi}^*(L)\boldsymbol{\varepsilon}_t, \tag{4}$$

where $\boldsymbol{\Psi}(1) = \sum_{k=0}^{\infty}\boldsymbol{\Psi}_k$ with $\boldsymbol{\Psi}(L)$ and $\boldsymbol{\Psi}^*(L)$ being matrix polynomials in the lag operator, L, and $\boldsymbol{\Psi}^*(k) = -\sum_{j=k+1}^{\infty}\boldsymbol{\Psi}_j$.

The matrix $\boldsymbol{\Psi}(1)$ contains the cumulative impacts of the innovation $\boldsymbol{\varepsilon}_t$ on all future price movements, and acts as a measure of the long-run impact of $\boldsymbol{\varepsilon}_t$ on prices. As shown in Hasbrouck (1995), the rows of $\boldsymbol{\Psi}(1)$ are identical given $\boldsymbol{\beta} = (1, -1)'$. Denote $\boldsymbol{\psi} = (\psi_1, \psi_2)'$ as the common row vector of $\boldsymbol{\Psi}(1)$, and define the permanent innovation as:

$$\eta_t^P = \boldsymbol{\psi}'\boldsymbol{\varepsilon}_t = \psi_1\varepsilon_{1t} + \psi_2\varepsilon_{2t}. \tag{5}$$

The common efficient price $m_t = m_{t-1} + \eta_t^P$ evolves as a random walk driven by the permanent shock η_t^P.

2.1.1 Information Share

The information share (IS) measure proposed by Hasbrouck (1995) quantifies the contribution of each price series to price discovery by assessing the proportion of variance in the efficient price attributed to that series. There are two cases to consider.

The first case pertains to a diagonal covariance matrix $\boldsymbol{\Omega}$. Under this case, the IS measures for each market are uniquely defined as follows:

$$\text{IS}_1 = \frac{\psi_1^2 \sigma_1^2}{\boldsymbol{\psi}' \boldsymbol{\Omega} \boldsymbol{\psi}}, \quad \text{IS}_2 = \frac{\psi_2^2 \sigma_2^2}{\boldsymbol{\psi}' \boldsymbol{\Omega} \boldsymbol{\psi}}. \tag{6}$$

The IS measure takes a positive value between 0 and 1 by construction.

The second case is associated with a non-diagonal covariance matrix $\boldsymbol{\Omega}$. Under this circumstance, the IS measures are not uniquely defined, as some assumption is necessary to assign the covariance contribution of the permanent shock to each market. Hasbrouck (1995) proposed computing the Cholesky decomposition of $\boldsymbol{\Omega} = \mathbf{F}\mathbf{F}^{'}$, where $\mathbf{F}$ represents a lower triangular matrix. This decomposition attributes the covariance contribution to the asset ordered first in the price vector. Accordingly, the IS measures are defined as:

$$\text{IS}_1 = \frac{([\boldsymbol{\psi}^{'}\mathbf{F}]_1)^2}{\boldsymbol{\psi}^{'} \boldsymbol{\Omega} \boldsymbol{\psi}}, \quad \text{IS}_2 = \frac{([\boldsymbol{\psi}^{'}\mathbf{F}]_2)^2}{\boldsymbol{\psi}^{'} \boldsymbol{\Omega} \boldsymbol{\psi}}, \tag{7}$$

where $[\boldsymbol{\psi}^{'}\mathbf{F}]_i$ represents the i^{th} element of the row matrix $\boldsymbol{\psi}^{'}\mathbf{F}$. As the Cholesky factor $\mathbf{F}$ relies on the ordering of the price series, the value of IS for a given market is contingent upon the arrangement of the price series in $\mathbf{p}_t$. By examining all permutations of the price series, one can calculate the upper and lower bounds for IS measures. In empirical studies, researchers often utilize the mid-point of these upper and lower bounds for each market as the final IS measure, as suggested in Baillie et al. (2002).[5]

2.1.2 Component Share

Another method to examine the cointegration relationship between closely related market prices is through the permanent-transitory (PT) component decomposition proposed by Gonzalo and Granger (1995). Assuming $\boldsymbol{\beta} = (1, -1)'$, the common permanent component is a linear combination of observable market prices, with weights represented by the common row vector $\boldsymbol{\psi} = (\psi_1, \psi_2)'$ of matrix $\boldsymbol{\Psi}(1)$ up to a scale factor. Hence, Booth et al. (1999), Chu

[5] In empirical analysis, when we mention IS estimates, we refer to the mid-point IS estimates.

et al. (1999), and Harris et al. (2002) suggest utilizing the component share (CS) measure to quantify each market's contribution to the common component:

$$\mathrm{CS}_1 = \frac{\psi_1}{\psi_1 + \psi_2}, \quad \mathrm{CS}_2 = \frac{\psi_2}{\psi_1 + \psi_2}. \tag{8}$$

By construction, the CS values sum to one. Moreover, the value of CS remains invariant to the order of prices in $\mathbf{p}_t$ (order-invariant), where a larger (smaller) value of CS share corresponds to a higher (lower) contribution to the permanent component of prices. The aforementioned expression also highlights that IS is a variance-weighted version of CS when the reduced-form innovations are uncorrelated, as noted in Yan and Zivot (2010).

2.2 Order-Invariant Price Discovery Measures

While IS is the most widely used price discovery measure empirically, it hinges on the ordering of price series, and the resulting upper and lower bounds of these estimates can yield ambiguous conclusions regarding price discovery. In this section, we outline two order-invariant price discovery measures: the Modified Information Share (MIS) proposed by Lien and Shrestha (2009) and the Price Discovery Share (PDS) introduced by Sultan and Zivot (2015). We demonstrate that each of these order-invariant measures can be derived through a specific decomposition of the variance-covariance matrix of the reduced-form residuals.

2.2.1 Modified Information Share

To achieve a unique factorization in the presence of correlated residuals, Lien and Shrestha (2009) suggested employing the spectral decomposition of the correlation matrix $\mathbf{\Phi}$ of the reduced-form residuals $\boldsymbol{\varepsilon}_t$. As detailed in Appendix A2, this spectral decomposition results in the factorization:

$$\mathbf{\Phi} = \underbrace{\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'}_{\mathbf{M}}\underbrace{\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'}_{\mathbf{M}'}. \tag{9}$$

Therefore, the variance-covariance matrix $\mathbf{\Omega}$ can be decomposed as:

$$\mathbf{\Omega} = \mathbf{V\Phi V}' = \underbrace{\mathbf{VG\Lambda}^{1/2}\mathbf{G}'}_{\mathbf{F}^*}\underbrace{\mathbf{G\Lambda}^{1/2}\mathbf{G}'\mathbf{V}'}_{\mathbf{F}^{*\prime}}, \tag{10}$$

where $\mathbf{G}$ represents a matrix with eigenvectors of the correlation matrix $\mathbf{\Phi}$ as columns, $\mathbf{\Lambda}$ denotes the diagonal matrix with the corresponding eigenvalues as diagonal elements, and $\mathbf{V}$ denotes a diagonal matrix containing the idiosyncratic errors' standard deviations on the diagonal.

The variance decomposition matrix $\mathbf{F}^*$ presented above appears different from its original form $\mathbf{F} = [\mathbf{G\Lambda}^{-1/2}\mathbf{G}'\mathbf{V}^{-1}]^{-1}$ as proposed by Lien and Shrestha (2009). As demonstrated in Appendix A2, their factorization is equivalent to the one presented in Eq.(10) above. However, the multiple inverse calculations involved in the factorization proposed by Lien and Shrestha (2009) render the original method challenging to compute, particularly when dealing with highly correlated reduced-form residuals. Therefore, we suggest employing the factorization in Eq.(10) instead.

Given the factorization in Eq.(10), Lien and Shrestha (2009) define the modified information share (MIS) measures as follows:

$$\mathrm{MIS}_i = \frac{([\boldsymbol{\psi}'\mathbf{F}^*]_i)^2}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}} = \frac{(\psi_i^*)^2}{\sum_{i=1}^{n}(\psi_i^*)^2}, \tag{11}$$

where ψ_i^* represents the i^{th} element of the row matrix $\boldsymbol{\psi}'\mathbf{F}^*$. As highlighted in Lien and Shrestha (2009), this ensuing MIS measures remain unaffected by the arrangement of prices in $\mathbf{p}_t$.[6]

In Appendix A2, we present the specific formula for the MIS measure in the bivariate

[6] As acknowledged in Lien and Shrestha (2009), due to the involvement of the square-root matrix $\mathbf{\Lambda}^{-1/2}$ in their original factorization matrix, the MIS measure is order-invariant but not uniquely determined, as one can opt for either positive or negative square roots of the diagonal elements of the matrix $\mathbf{\Lambda}^{-1}$. However, as evident from Eq.(11), the selection of the positive or negative square root of the diagonal elements of $\mathbf{\Lambda}$ does not affect the final squared values in the computation of the MIS. Therefore, we contend that the MIS measures are both order-invariant and unique, notwithstanding the possibility of multiple forms in the factorization due to the choice of positive or negative square roots for the matrix $\mathbf{\Lambda}$.

case. As shown in Eq.(A.10), the MIS measure distributes the variance contribution to each market more evenly compared to the IS measure (as depicted in Eq.(A.2)). Moreover, it aligns with the IS measure when $\rho = 0$.[7]

2.2.2 Price Discovery Share

Sultan and Zivot (2015) proposed another order-invariant measure of price discovery by employing an additive decomposition of the volatility of the permanent shock, denoted as $\sigma_\eta(\boldsymbol{\psi}) = (\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi})^{1/2}$. Their approach leverages the linear homogeneity of $\sigma_\eta(\boldsymbol{\psi})$ in $\boldsymbol{\psi}$ and applies Euler's theorem to yield:

$$\sigma_\eta(\boldsymbol{\psi}) = \boldsymbol{\psi}'\frac{\partial\sigma_\eta(\boldsymbol{\psi})}{\partial\boldsymbol{\psi}} = \sum_{i=1}^{n}\psi_i\frac{\partial\sigma_\eta(\boldsymbol{\psi})}{\partial\psi_i}. \tag{12}$$

The above decomposition implies that the volatility of the permanent shock, $\sigma_\eta(\boldsymbol{\psi})$, can be expressed as a weighted sum of marginal contributions from each market.

Based on the above decomposition, Sultan and Zivot (2015) proposed a new order-invariant measure called Price Discovery Share (PDS), defined as:

$$\text{PDS}_i = \frac{\psi_i\frac{\partial\sigma_\eta(\boldsymbol{\psi})}{\partial\psi_i}}{\sigma_\eta(\boldsymbol{\psi})}. \tag{13}$$

To see how PDS_i is related to IS_i note that:

$$\text{PDS}_i = \frac{\psi_i}{\sigma_\eta(\boldsymbol{\psi})}\frac{\partial\sigma_\eta(\boldsymbol{\psi})}{\partial\psi_i} = \frac{\psi_i}{\sigma_\eta(\boldsymbol{\psi})}\frac{[\boldsymbol{\Omega}\psi]_i}{\sigma_\eta(\boldsymbol{\psi})} = \frac{\psi_i^2\sigma_i^2 + \sum_{j=1}^{n}\psi_i\psi_{j\neq i}\sigma_{i,j\neq i}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}. \tag{14}$$

As discussed in Sultan and Zivot (2015), the PDS measures are both order-invariant and unique. Equation (14) illustrates that PDS coincides with IS when the reduced-form residuals are uncorrelated. While the PDS measure sums to one by construction, PDS_i can assume

[7] When the reduced-form residuals are uncorrelated, the correlation matrix $\boldsymbol{\Phi}$ becomes an identity matrix, as does the eigenvalue matrix $\boldsymbol{\Lambda}$ and the factorization matrix $\mathbf{M}$. Consequently, the factorization matrix $\mathbf{F}^*$ simplifies to $\mathbf{VM} = \mathbf{V}$. In this scenario, it is straightforward to demonstrate that the MIS measures defined in Eq.(11) are equivalent to the IS measures.

negative values if the weight ψ_i is negative or if the reduced-form residuals are negatively correlated (resulting in negative σ_{ij}). Sultan and Zivot (2015) argue that this scenario is highly improbable in practice.

The construction of the PDS measure corresponds to a straightforward additive decomposition of the reduced-form covariance matrix $\mathbf{\Omega}$. Consider the bivariate case, for instance:

$$\mathbf{\Omega} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & 0 \\ \sigma_{12} & 0 \end{pmatrix} + \begin{pmatrix} 0 & \sigma_{12} \\ 0 & \sigma_2^2 \end{pmatrix}, \tag{15}$$

which leads to the following decomposition of the variance of the permanent shock η_t^P:

$$\begin{aligned} \boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi} &= \boldsymbol{\psi}' \begin{pmatrix} \sigma_1^2 & 0 \\ \sigma_{12} & 0 \end{pmatrix} \boldsymbol{\psi} + \boldsymbol{\psi}' \begin{pmatrix} 0 & \sigma_{12} \\ 0 & \sigma_2^2 \end{pmatrix} \boldsymbol{\psi}, \\ &= \underbrace{(\psi_1^2\sigma_1^2 + \psi_1\psi_2\sigma_{12})}_{\text{attributed to market 1}} + \underbrace{(\psi_2^2\sigma_2^2 + \psi_1\psi_2\sigma_{12})}_{\text{attributed to market 2}}. \end{aligned} \tag{16}$$

The straightforward additive decomposition in Eq.(15) distributes the covariance between these two reduced-form residuals equally to each market, ensuring that the resulting PDS measure remains order-invariant. However, due to its additive nature, the resulting PDS measure is not necessarily guaranteed to be positive.

De Jong and Schotman (2010) also introduced an order-invariant IS measure utilizing an alternative method for attributing covariance. Derived from a structural unobserved component model, their IS measure is calculated as the product of the regression coefficient of the price innovations on the efficient price and the regression coefficient in the reverse regression of the efficient price on the price innovations. In cases where the correlation among competing markets' transitory shocks results solely from the common efficient price innovation, their IS measure expressions coincide with the PDS measure outlined in Eq.(14) under the BN normalization rule.[8]

[8]The equivalence of the De Jong-Schotman IS measure with PDS under these specific conditions is

2.3 Structural analysis of price discovery measures

As highlighted in Yan and Zivot (2010) and Lehmann (2002), since IS and CS stem from residuals of a reduced-form VECM, obtaining a straightforward structural interpretation of these measures is challenging. To elucidate this issue, Yan and Zivot (2010) employed a structural cointegration model featuring independent permanent and transitory shocks to derive structural representations for IS and CS. In this section, we revisit the structural analysis introduced by Yan and Zivot (2010) and subsequently extend this analysis to MIS and PDS.

Following Levtchenkova et al. (1999), Yan and Zivot (2010) started with the following Structural Moving Average (SMA) representation of $\Delta\mathbf{p}_t$:

$$\Delta\mathbf{p}_t = \mathbf{D}(L)\boldsymbol{\eta}_t = \mathbf{D}_0\boldsymbol{\eta}_t + \mathbf{D}_1\boldsymbol{\eta}_{t-1} + \mathbf{D}_2\boldsymbol{\eta}_{t-2} + \dots \tag{17}$$

where the elements of $\{\mathbf{D}_k\}_{k=0}^{\infty}$ are 1-summable, $\mathbf{D}(L) = \sum_{k=0}^{\infty}\mathbf{D}_k L^k$, and $\mathbf{D}_0$ is invertible. They assume that the number of structural shocks equals the number of observed prices, ensuring $\mathbf{D}(L)$ is invertible. The innovation to the common efficient price of the asset, denoted as η_t^P, represents the permanent shock, while the noise innovation, η_t^T, is termed the transitory shock, forming $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$. These structural shocks are assumed to be serially and mutually uncorrelated, with a diagonal covariance matrix $\mathbf{C} = \mathrm{diag}(\sigma_P^2, \sigma_T^2)$. The matrix $\mathbf{D}_0$ encompasses the initial impacts of the structural shocks on $\Delta\mathbf{p}_t$ and defines the contemporaneous correlation structure of $\Delta\mathbf{p}_t$:

$$\mathbf{D}_0 = \begin{bmatrix} d_{0,1}^P & d_{0,1}^T \\ d_{0,2}^P & d_{0,2}^T \end{bmatrix}. \tag{18}$$

The permanent innovation η_t^P conveys new information regarding the fundamental value of the asset, causing a permanent shift in market prices. It possesses the distinguishing

straightforward to demonstrate, as detailed in Appendix E of Lautier et al. (2023).

feature of exerting a one-to-one long-run effect on the price levels for each market. On the other hand, the transitory innovation η_t^T encapsulates non-informational shocks, such as trading by uninformed or liquidity traders. It is characterized by its lack of correlation with the informational innovation η_t^P and its absence of a long-term impact on price levels. Thus, the long-term impact matrix $\mathbf{D}(1)$ of the structural innovations $\boldsymbol{\eta}_t$ takes the form:

$$\mathbf{D}(1) = \begin{bmatrix} d_1^P(1) & d_1^T(1) \\ d_2^P(1) & d_2^T(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}. \tag{19}$$

Applying the Beveridge-Nelson decomposition to Eq. (17) results in the level relationship:

$$\mathbf{p}_t = \mathbf{p}_0 + \mathbf{D}(1) \sum_{j=1}^{t} \boldsymbol{\eta}_j + \mathbf{s}_t, \tag{20}$$

where $\mathbf{D}(1) = \sum_{k=0}^{\infty} \mathbf{D}_k$, $\mathbf{s}_t = (s_{1t}, s_{2t})' = \mathbf{D}^*(L)\boldsymbol{\eta}_t \sim I(0)$, and $\mathbf{D}_k^* = -\sum_{j=k+1}^{\infty} \mathbf{D}_j$, $k = 0, \ldots, \infty$.

From the moving average representations in Eq.(17) and Eq.(3), the reduced-form forecasting errors, $\boldsymbol{\varepsilon}_t$, are connected to the structural innovations, $\boldsymbol{\eta}_t$, through the relation $\boldsymbol{\varepsilon}_t = \mathbf{D}_0 \boldsymbol{\eta}_t$:

$$\varepsilon_{1t} = d_{0,1}^P \eta_t^P + d_{0,1}^T \eta_t^T, \quad \varepsilon_{2t} = d_{0,2}^P \eta_t^P + d_{0,2}^T \eta_t^T. \tag{21}$$

In Eq.(21), each forecasting error can be ascribed to the unobserved structural shocks η_t^P and η_t^T. The parameters $d_{0,i}^P$ and $d_{0,i}^T$ (i=1,2) represent the contemporaneous responses of p_{it} to the permanent and transitory shocks, respectively.

2.3.1 Structural interpretations of IS and CS

From the level representations in Eq.(20) and Eq.(4), and by utilizing the relationship $\boldsymbol{\varepsilon}_t = \mathbf{D}_0\boldsymbol{\eta}_t$, we can observe that

$$\mathbf{D}(1) = \boldsymbol{\Psi}(1)\mathbf{D}_0. \tag{22}$$

Hence, the elements of $\boldsymbol{\Psi}(1)$ can be represented in terms of the elements of $\mathbf{D}_0$ using the relation $\boldsymbol{\Psi}(1) = \mathbf{D}(1)\mathbf{D}_0^{-1}$ provided that $\mathbf{D}_0$ is invertible. Solving for ψ_1 and ψ_2, we obtain:

$$\psi_1 = \frac{d_{0,2}^T}{|\mathbf{D}_0|}, \quad \psi_2 = -\frac{d_{0,1}^T}{|\mathbf{D}_0|}. \tag{23}$$

Then the structural representation of the CS measures is as follows:

$$\mathrm{CS}_1 = \frac{d_{0,2}^T}{d_{0,2}^T - d_{0,1}^T}, \quad \mathrm{CS}_2 = -\frac{d_{0,1}^T}{d_{0,2}^T - d_{0,1}^T}. \tag{24}$$

As highlighted in Yan and Zivot (2010), the structural representation of CS solely comprises the parameters dictating the initial price reactions to transitory frictional innovations. Unlike gauging the relative potency of a market price in reacting to new information, CS evaluates the response of the other market to concurrent transient frictions. It's worth noting that this outcome remains unaffected by the correlation among reduced-form residuals.

To derive the structural representation for IS, Yan and Zivot (2010) examined the specific scenario where the reduced-form innovations $\boldsymbol{\varepsilon}_t$ are uncorrelated. In this instance, the allocation of the reduced-form covariance to individual markets becomes inconsequential, rendering the IS measures unique. Yan and Zivot (2010) demonstrated that $\mathrm{cov}(\varepsilon_{1t}, \varepsilon_{2t}) = 0$ when:

$$\frac{\sigma_T^2}{\sigma_P^2} = \frac{d_{0,1}^P d_{0,2}^P}{-d_{0,1}^T d_{0,2}^T}, \tag{25}$$

and all elements of $\mathbf{D}_0$ are non-zero (hence $|\mathbf{D}_0| \neq 0$).[9]

Under this specific circumstance, the IS measures can be uniquely defined in terms of the structural parameters:

$$\mathrm{IS}_1 = \frac{d^P_{0,1} d^T_{0,2}}{d^P_{0,1} d^T_{0,2} - d^T_{0,1} d^P_{0,2}}, \quad \mathrm{IS}_2 = \frac{-d^T_{0,1} d^P_{0,2}}{d^P_{0,1} d^T_{0,2} - d^T_{0,1} d^P_{0,2}}. \tag{26}$$

As delineated in Yan and Zivot (2010), the structural representation of IS encompasses contemporaneous responses to both permanent and transitory shocks. In contrast to CS, IS is better suited for assessing price discovery since it encapsulates individual market responses to permanent shocks or new information ($d^P_{0,i}$).

However, the contemporaneous impact of frictional innovation ($d^T_{0,i}$) complicates the interpretation of IS. A market may exhibit a high IS due to its heightened responsiveness to new information (a high value of $d^P_{0,1}$), or because the other market displays greater responsiveness to transitory frictions (a high value of $d^T_{0,2}$).

2.3.2 Information Leadership Measures

Under the assumption of uncorrelated reduced-form residuals, Yan and Zivot (2010) combined IS and CS measures to formulate the Information Leadership (IL) measure as follows:

$$\mathrm{IL}_1 = \left|\frac{\mathrm{IS}_1/\mathrm{CS}_1}{\mathrm{IS}_2/\mathrm{CS}_2}\right| = \left|\frac{d^P_{0,1}}{d^P_{0,2}}\right|, \quad \mathrm{IL}_2 = \left|\frac{\mathrm{IS}_2/\mathrm{CS}_2}{\mathrm{IS}_1/\mathrm{CS}_1}\right| = \left|\frac{d^P_{0,2}}{d^P_{0,1}}\right|. \tag{27}$$

The IL measure solely relies on each market's initial responses to the permanent shock, offering a straightforward and intuitive measure of price discovery.

Unlike IS and CS, IL measures do not sum to 1 as they are not shares; instead, their

[9] According to Eq.(21), the variance-covariance structure of $\boldsymbol{\varepsilon_t}$ takes a form:

$$\boldsymbol{\Omega} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} (d^P_{0,1})^2\sigma_P^2 + (d^T_{0,1})^2\sigma_T^2 & d^P_{0,1}d^P_{0,2}\sigma_P^2 + d^T_{0,1}d^T_{0,2}\sigma_T^2 \\ d^P_{0,1}d^P_{0,2}\sigma_P^2 + d^T_{0,1}d^T_{0,2}\sigma_T^2 & (d^P_{0,2})^2\sigma_P^2 + (d^T_{0,2})^2\sigma_T^2 \end{pmatrix}.$$

We omit the other two trivial cases outlined in Yan and Zivot (2010), where in the first case $d^P_{0,2} = d^T_{0,1} = 0$ and $d^P_{0,1}d^T_{0,2} \neq 0$, and in the second case $d^P_{0,1} = d^T_{0,2} = 0$ and $d^P_{0,2}d^T_{0,1} \neq 0$.

range extends from 0 to ∞. To enhance the interpretability and comparability of IL with IS and CS, Putniņš (2013) introduced the concept of Information Leadership Shares (ILS):

$$\mathrm{ILS}_1 = \frac{\mathrm{IL}_1}{\mathrm{IL}_1 + \mathrm{IL}_2}, \quad \mathrm{ILS}_2 = \frac{\mathrm{IL}_2}{\mathrm{IL}_1 + \mathrm{IL}_2}. \tag{28}$$

The ILS measures lie within the unit interval by design, with values above (below) 0.5 indicating that the price series leads (does not lead) the adjustment process to new information.

Given that the ILS measures are confined to the unit interval, their expected values tend to be biased inward from the zero/one endpoints. This bias towards zero and away from one suggests that ILS may overstate the contribution of the noisier market while understating the contribution of the leader. However, Putniņš (2013) demonstrated through simulations that when one market responds faster than the other, its expected ILS tends to exceed 0.5 regardless of the noise level. Therefore, on average, the ILS measure correctly identifies the leader but may underestimate the leading market's contribution.

To address the bias inherent in ILS, Patel et al. (2020) introduced the Information Leadership Indicator (ILI):

$$\mathrm{ILI}_i = \begin{cases} 1, & \text{if } \mathrm{ILS}_i > 0.5 \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

They conducted simulations to demonstrate that ILI is an approximately unbiased measure of information leadership even in noisy samples. Furthermore, they illustrated that the sample average of ILI across days can be interpreted as the proportion of days in which one market leads the other.[10]

Putniņš (2013) conducted simulations to demonstrate that only ILS, compared with IS and CS, reliably measures price discovery in a "who moves first" sense. In their simulation setup, each market's price series tracks the fundamental value with a time delay and contains an independent transitory noise. This leads to a specific correlation pattern between

[10] For multivariate definitions of ILS and ILI, refer to Patel et al. (2020), which are also provided in Appendix A3.

the reduced-form errors, despite not meeting the structural assumptions in Yan and Zivot (2010).[11] While the noise level of the competing markets does not affect ILS's ability to identify the market leader correctly, IS and CS metrics measure both informational leadership and relative avoidance of noise. The influence of noise on IS and CS metrics could overshadow their ability to measure informational leadership accurately, deviating from the "who moves first" view of price discovery.

The structural analysis of IS, CS, and ILS measures, along with the simulation evidence from Putniņš (2013), underscores the importance of combining IS and CS metrics for understanding price discovery. However, it remains unclear how these measures behave with correlated reduced-form residuals. It's essential to note that when the reduced-form errors are correlated, the IL measure lacks the representation in Eq.(27) due to the non-uniqueness of IS. Consequently, interpreting IL, ILS, and ILI measures requires caution. In the upcoming simulation sections, we will relax the assumption of independent noise in Putniņš (2013) and demonstrate how correlated reduced-form residuals affect the performance of ILS.

2.3.3 A structural analysis of MIS

We derive a structural representation of the MIS measure for the bivariate case in Appendix A2. As depicted in Eq.(A.11), the structural representation of MIS entails a complex combination of contemporaneous responses to both permanent and transitory shocks.

Given MIS's order invariance, one might contemplate substituting the IS measure with the MIS measure in the definition of IL in Eq.(27) to define a new Modified Information Leadership (MIL) measure as follows:

$$\mathrm{MIL}_1 = \left|\frac{\mathrm{MIS}_1/\mathrm{CS}_1}{\mathrm{MIS}_2/\mathrm{CS}_2}\right|, \quad \mathrm{MIL}_2 = \left|\frac{\mathrm{MIS}_2/\mathrm{CS}_2}{\mathrm{MIS}_1/\mathrm{CS}_1}\right|. \tag{30}$$

[11] As outlined in Putniņš (2013), the simulation model comprises three shocks: one permanent and two temporary. This setup enables each price series to possess an independent source of noise. In contrast, the structural cointegration model presented in Yan and Zivot (2010) necessitates the existence of only one permanent and one transitory shock.

Similarly, one could define the corresponding modified information leadership share measure (MILS) and the Modified Information Leadership Indicator measure (MILI) as follows:

$$\text{MILS}_1 = \frac{\text{MIL}_1}{\text{MIL}_1 + \text{MIL}_2}, \quad \text{MILS}_2 = \frac{\text{MIL}_2}{\text{MIL}_1 + \text{MIL}_2}, \tag{31}$$

and

$$\text{MILI}_i = \begin{cases} 1, & \text{if } \text{MILS}_i > 0.5 \\ 0, & \text{otherwise.} \end{cases} \tag{32}$$

The MIL measure combines the MIS and CS measures. In scenarios with correlated reduced-form residuals, MILS and MILI could serve as potential substitutes for ILS and ILI, respectively. However, due to the intricate nature of the structural representation of MIS, which involves contemporaneous impacts from both permanent and transitory shocks, the performance of MILS and MILI remains uncertain. The multivariate formulations of MILS and MILI are detailed in Appendix A3.

2.3.4 A Structural analysis of PDS

When idiosyncratic errors are correlated, the structural representations of IS, IL, ILS, and ILI become ambiguous due to IS's lack of order invariance. However, we can demonstrate that the order-invariant PDS measure proposed by Sultan and Zivot (2015) does possess a clear structural representation even when the reduced-form residuals are correlated. To illustrate this, we substitute the structural expressions of the weights in Eq.(23) and the structural representation of $\mathbf{\Omega}$ into the expressions of the PDS measures in Eq.(14), yielding:

$$\begin{aligned} \text{PDS}_1 &= \frac{\psi_1^2\sigma_1^2 + \psi_1\psi_2\sigma_{12}}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}} = \frac{d_{0,1}^P d_{0,2}^T}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}, \\ \text{PDS}_2 &= \frac{\psi_2^2\sigma_2^2 + \psi_2\psi_1\sigma_{21}}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}} = \frac{-d_{0,1}^T d_{0,2}^P}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}. \end{aligned} \tag{33}$$

The structural representation of the PDS measures comprises contemporaneous responses to both permanent and transitory shocks. Moreover, these structural representations align with those of IS in Eq.(26) under the case of uncorrelated residuals. These clear-cut structural representations advocate for the utilization of PDS over IS as a measure of price discovery.

2.4 Improved Information Leadership Measures

The information leadership measures IL, ILS, and ILI derived from ratios of IS and CS lack clarity as measures of price leadership when reduced-form residuals are correlated. However, since the structural representation of PDS in Eq.(33) remains valid even for correlated reduced-form residuals, we can employ a similar approach as Yan and Zivot (2010) to define an improved IL measure that is order-invariant and robust to correlations:

$$\mathrm{PIL}_1 = \left|\frac{\mathrm{PDS}_1/\mathrm{CS}_1}{\mathrm{PDS}_2/\mathrm{CS}_2}\right| = \left|\frac{d^P_{0,1}}{d^P_{0,2}}\right|, \quad \mathrm{PIL}_2 = \left|\frac{\mathrm{PDS}_2/\mathrm{CS}_2}{\mathrm{PDS}_1/\mathrm{CS}_1}\right| = \left|\frac{d^P_{0,2}}{d^P_{0,1}}\right|. \tag{34}$$

We refer to this improved IL measure as PIL to underscore its combination of CS with PDS instead of IS. PIL quantifies the relative responsiveness of closely-related markets to new information, excluding frictional responses. When $|d^P_{0,1}| > |d^P_{0,2}|$, PIL_1 takes a value greater than 1 and PIL_2 takes a value smaller than 1. Thus, PIL effectively identifies the price leader, defined as the market with a greater contemporaneous response to the permanent shock.

Since both CS and PDS are order-invariant, PIL inherits this property. Moreover, PIL shares the same structural representation as IL, but crucially, this representation remains valid even in the presence of correlated reduced-form residuals, rendering PIL correlation-robust.

We can convert PIL into an information leadership share measure (denoted as PILS) as follows:

$$
\begin{aligned}
\text{PILS}_1 &= \frac{\text{PIL}_1}{\text{PIL}_1 + \text{PIL}_2} = \frac{(d^P_{0,1})^2}{(d^P_{0,1})^2 + (d^P_{0,2})^2}, \\
\text{PILS}_2 &= \frac{\text{PIL}_2}{\text{PIL}_1 + \text{PIL}_2} = \frac{(d^P_{0,2})^2}{(d^P_{0,1})^2 + (d^P_{0,2})^2}.
\end{aligned} \tag{35}
$$

The PILS metrics are by definition positive and fall within the range of $[0, 1]$. We can also introduce an improved binary information leadership indicator (denoted PILI) as:

$$
\text{PILI}_i = \begin{cases} 1, & \text{if } \text{PILS}_i > 0.5 \\ 0, & \text{otherwise.} \end{cases} \tag{36}
$$

Similarly to ILI, the sample average of PILI can be understood as the percentage of days in which one market takes the lead over the other. Multivariate descriptions of PILS and PILI are available in Appendix A3.

The refined information leadership gauges (PIL, PILS, and PILI) are formulated based on the PDS and CS metrics. Nevertheless, we can also establish these metrics directly regarding reduced-form VECM parameters using Eq.(8) and (14):

$$
\text{PIL}_1 = \left| \frac{\psi_1 \sigma_1^2 + \psi_2 \sigma_{12}}{\psi_2 \sigma_2^2 + \psi_1 \sigma_{12}} \right|, \quad \text{PIL}_2 = \left| \frac{\psi_2 \sigma_2^2 + \psi_1 \sigma_{12}}{\psi_1 \sigma_1^2 + \psi_2 \sigma_{12}} \right|, \tag{37}
$$

$$
\begin{aligned}
\text{PILS}_1 &= \frac{(\psi_1 \sigma_1^2 + \psi_2 \sigma_{12})^2}{(\psi_1 \sigma_1^2 + \psi_2 \sigma_{12})^2 + (\psi_2 \sigma_2^2 + \psi_1 \sigma_{12})^2}, \\
\text{PILS}_2 &= \frac{(\psi_2 \sigma_2^2 + \psi_1 \sigma_{12})^2}{(\psi_1 \sigma_1^2 + \psi_2 \sigma_{12})^2 + (\psi_2 \sigma_2^2 + \psi_1 \sigma_{12})^2}.
\end{aligned} \tag{38}
$$

Considering the structural and reduced-form representations of the PIL (and PILS) metric, one can identify the ratio of markets' initial reactions to the permanent shocks from the reduced-form VECM parameters, irrespective of the correlation among the reduced-form residuals. What's notable is that this identification is achieved without imposing additional assumptions on the structural SMA model.

Recently, Lautier et al. (2023) introduced a Covariance Information Share (CovIS) based on the covariance of reduced-form residuals with the permanent shock. Their measure is also order-invariant and correlation-robust. In essence, the structural representations of our PIL metric align closely with the CovIS metric, whereas our PILS metric mirrors the quadratic variation CovISQ. Although the CovIS (and CovISQ) metrics by Lautier et al. (2023) share the same structural representations as our PIL (and PILS) metric, they derive their results in a different manner than we do. They start from the SMA model and consider conditions under which they can identify the structural parameters. In contrast, we develop our metrics (PIL, PILS, and PILI) starting from the reduced-form VECM and considering reduced-form derived price discovery metrics.

3 Illustrations and Simulations

3.1 Illustrations: A Partial Price Adjustment Model

To elucidate the empirical performance of the discussed price discovery measures, we analyze a stylized partial adjustment microstructure model employed by Amihud and Mendelson (1987), Hasbrouck and Ho (1987), and Yan and Zivot (2010):

$$\begin{aligned}
p_{1t} &= p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b^T_{0,1}\eta^T_t, \\
p_{2t} &= p_{2,t-1} + \delta_2(m_t - p_{2,t-1}) + b^T_{0,2}\eta^T_t, \\
m_t &= m_{t-1} + \eta^P_t, \boldsymbol{\eta_t} = (\eta^P_t, \eta^T_t)' \sim i.i.d.N\left(\mathbf{0}, \begin{bmatrix} \sigma^2_P & 0 \\ & \sigma^2_T \end{bmatrix}\right).
\end{aligned} \tag{39}$$

Solving for Δp_{it} gives

$$\Delta p_{it} = d^P_i(L)\eta^P_t + d^T_i(L)\eta^T_t,$$

where

$$
\begin{aligned}
d_1^P(L) &= [1-(1-\delta_1)L]^{-1}\delta_1, \quad d_2^P(L) = [1-(1-\delta_2)L]^{-1}\delta_2, \\
d_1^T(L) &= [1-(1-\delta_1)L]^{-1}(1-L)b_{0,1}^T, \quad d_2^T(L) = [1-(1-\delta_2)L]^{-1}(1-L)b_{0,2}^T.
\end{aligned}
$$

The SMA representation of the partial adjustment model is derived from the pertinent elements of the lag polynomials $d_i^P(L)$ and $d_i^T(L)$. Specifically, the matrices for the initial impact and the long-run impact are expressed as follows:

$$
\mathbf{D}_0 = \begin{pmatrix} d_{0,1}^P & d_{0,1}^T \\ d_{0,2}^P & d_{0,2}^T \end{pmatrix} = \begin{pmatrix} \delta_1 & b_{0,1}^T \\ \delta_2 & b_{0,2}^T \end{pmatrix}, \mathbf{D}(1) = \begin{pmatrix} d_1^P(1) & d_1^T(1) \\ d_2^P(1) & d_2^T(1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.
$$

The initial responses to a one-unit permanent shock are denoted by δ_i, while the long-run responses to a one-unit permanent shock are both set to one. The initial responses to a one-unit transitory shock are denoted by $b_{0,i}^T$, while the long-run responses to a one-unit transitory shock are both set to zero. The asset exhibiting a greater initial permanent response (δ_i) is identified as the leader in the price discovery process.

The CS, PDS, and PIL measures for these two assets are devoid of any reliance on reduced form correlation and possess the following structural representations:

$$
\mathrm{CS}_1 = \frac{b_{0,2}^T}{b_{0,2}^T - b_{0,1}^T}, \quad \mathrm{CS}_2 = \frac{-b_{0,1}^T}{b_{0,2}^T - b_{0,1}^T}, \tag{40}
$$

$$
\mathrm{PDS}_1 = \frac{\delta_1 b_{0,2}^T}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2}, \quad \mathrm{PDS}_2 = \frac{-b_{0,1}^T \delta_2}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2}, \tag{41}
$$

$$
\mathrm{PIL}_1 = \left|\frac{\delta_1}{\delta_2}\right|, \quad PIL_2 = \left|\frac{\delta_2}{\delta_1}\right|. \tag{42}
$$

provided $b_{0,2}^T \neq b_{0,1}^T$.

As evident from the above expressions, only the PIL measure, which solely relies on the relative magnitude of the initial permanent responses (represented by δ_i), can accurately identify the leader in the price discovery process. The CS measure is contingent solely upon initial transitory responses (denoted by $b^T_{0,i}$), assessing the relative noise avoidance of each market rather than determining leadership in the price discovery process. While the PDS measure involves initial permanent responses (denoted by δ_i), its value also encompasses initial transitory responses (denoted by $b^T_{0,i}$). Due to the inclusion of contemporaneous avoidance to transitory shocks, PDS could also yield misleading price leadership results.

The structural representations of IS, MIS, IL, and ILS for these two products are affected by the covariance structure of the reduced-form forecasting errors. When the reduced-form innovations, $\boldsymbol{\varepsilon_t}$, are uncorrelated, the IS metric becomes unique. It can be demonstrated that $\text{cov}(\varepsilon_{1t}, \varepsilon_{2t}) = 0$ when

$$\frac{\sigma_T^2}{\sigma_P^2} = \frac{d^P_{0,1} d^P_{0,2}}{-d^T_{0,1} d^T_{0,2}} = \frac{\delta_1 \delta_2}{-b^T_{0,1} b^T_{0,2}}, \tag{43}$$

and all elements of $\mathbf{D_0}$ are non-zero (hence $|\mathbf{D_0}| \neq 0$). In such cases, the IS measures are uniquely defined as follows:

$$\text{IS}_1 = \frac{\delta_1 b^T_{0,2}}{\delta_1 b^T_{0,2} - b^T_{0,1} \delta_2}, \quad \text{IS}_2 = \frac{-b^T_{0,1} \delta_2}{\delta_1 b^T_{0,2} - b^T_{0,1} \delta_2}. \tag{44}$$

The IL measures are then:

$$\text{IL}_1 = \left| \frac{d^P_{0,1}}{d^P_{0,2}} \right| = \left| \frac{\delta_1}{\delta_2} \right|, \quad \text{IL}_2 = \left| \frac{d^P_{0,2}}{d^P_{0,1}} \right| = \left| \frac{\delta_2}{\delta_1} \right|. \tag{45}$$

As depicted by the above expressions, when the reduced-form innovations are uncorrelated, similar to PDS, the IS measure encompasses both initial permanent and transitory responses. In contrast, the IL measure emerges as a more reliable gauge of price discovery, as its value hinges solely upon the relative magnitude of the initial permanent responses. However, these structural representations of IS and IL are only applicable under the restrictive assumption of uncorrelated reduced-form innovations. In more general scenarios with correlated reduced-form innovations, their structural implications become convoluted and difficult to elucidate.

For MIL based on the MIS and CS measures, the same structural interpretations as in Eq.(45) hold, but again solely for the scenario with uncorrelated reduced-form innovations. In the more general case with correlated reduced-form innovations ($\rho \neq 0$), the structural expressions for MIS measures (as demonstrated in Eq.(A.10)) become intricate and challenging to interpret.

3.2 Simulation Evidence from Partial Price Adjustment Model

In this subsection, we undertake an empirical evaluation of the discussed price discovery measures using simulated data generated from various parameterizations of the stylized partial adjustment model. For simplicity, we set $\delta_2 = 1 - \delta_1$ and vary δ_1 from 0.9 to 0.1, with a decrement of 0.1. When $\delta_1 > \delta_2$ (i.e., $\delta_1 > 0.5$), Market 1 demonstrates a greater speed of price discovery than Market 2. We fix the variance of the permanent shock at unity ($\sigma_P^2 = 1$).

For the first parameterization, we consider the case where both markets' responses to the transitory shock are equal, set as $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$. Additionally, we set the variance of the transitory shock to $\sigma_T^2 = \frac{\delta_1 \delta_2}{-b_{0,1}^T b_{0,2}^T}$, ensuring uncorrelated reduced-form residuals for these two markets. We conduct 1000 simulations, each comprising 21600 observations of two price series, for every specified value of δ_1, mimicking 1-second sampled daily observations. Subsequently, for each simulated sample, we estimate the VECM with the restricted cointegrating vector $(1, -1)'$, and compute the corresponding price discovery measures. The average values of each price discovery measure across the 1000 samples are then summarized, and the results are presented in Panel A of Table 1.[12]

[Insert Table 1 about here.]

[12]For brevity, we report results with two decimal points. Each row of Table 1 corresponds to a specific value of δ_1.

As observed from each row in Panel A of Table 1, the IS, MIS, and PDS estimates assume values that are nearly identical to the corresponding value of δ_1.[13] Similarly, the combined measures ILS, MILS, and PILS, along with their binary counterparts ILI, MILI, and PILI, also assume identical values. This consistency aligns with the expectation that IS, MIS, and PDS would converge when reduced-form residuals are uncorrelated.

The CS estimates closely mirror the responses to transitory shocks (0.5 in this sub-case). For rows where $\delta_1 > 0.5$ ($\delta_1 < 0.5$), the IS, MIS, and PDS estimates all exceed (fall below) 0.5, and the combined measures ILS, MILS, and PILS (along with their binary counterparts) correctly identify Market 1 as the price leader.[14]

For $\delta_1 = \delta_2 = 0.5$, the sample mean (0.49, 0.51) of these binary estimates suggests that over 49% of the time, these indicator measures designate the first market as the leader. Thus, we observe that all price discovery measures equally identify each market as the leader across the 1000 samples when $\delta_1 = \delta_2 = 0.5$.

For the second parameterization, as displayed in Panel B of Table 1, we set the transitory responses as $(b^T_{0,1}, b^T_{0,2}) = (0.8, -0.2)$, with all other parameters identical to those in Panel A. Under this configuration, Market 1 exhibits larger transitory noise, and the reduced-form residuals remain uncorrelated. In this scenario, we observe that IS, MIS, and PDS can inaccurately identify the price leader.

For example, in the row where $\delta_1 = 0.7$ of Panel B, the IS (along with MIS and PDS) estimate for Market 1 is only 0.37. Despite Market 1 being anticipated to lead price discovery with $\delta_1 = 0.7$, IS, MIS, and PDS incorrectly designate Market 2 as the leader. In contrast, the combined measures ILS, MILS, and PILS (along with their binary counterparts ILI, MILI, and PILI) consistently identify Market 1 as the leader correctly when $\delta_1 > 0.5$, and Market 2 as the leader correctly when $\delta_1 < 0.5$. For the case with equal leadership ($\delta_1 = \delta_2 = 0.5$),

[13] Though not precisely identical, they are very close. Minor differences may occur if we extend the decimal points to three digits.

[14] A value greater than 0.5 for the combined price discovery measures (ILS, MILS, and PILS) pertaining to the first market indicates the identification of this market as the leader. A binary indicator value of 1 (0) for the first (second) market signifies its identification as the leader.

these combined measures identify each market as the leader approximately half the time.

For the third parameterization, as illustrated in Panel C of Table 1, we set the variance of the transitory responses to $\sigma_T^2 = 10$, with all other parameter settings identical to those in Panel B. Under this configuration, the reduced-form errors become correlated.[15]. Consequently, IS, MIS, and PDS estimates diverge due to their distinct variance decomposition methods. These measures alone may fail to correctly identify the price leader.

The combined measures ILS and MILS (along with their binary indicators ILI and MILI) offer more accurate results compared to IS and MIS alone. However, ILS (and its binary indicator ILI) tend to over-designate Market 1 as the leader even for cases with $\delta_1 = (0.4, 0.3)$. Similarly, MILS (and its binary indicator MILI) may erroneously designate Market 1 as the leader for cases with $\delta_1 = 0.4$. In contrast, PILS (and its binary indicator PILI) consistently selects the correct price leader for all scenarios where $\delta_1 \neq \delta_2$. For the case with equal leadership ($\delta_1 = \delta_2 = 0.5$), PILS (and its binary indicator PILI) identifies each market as the leader equally.

In summary, the simulation results underscore the importance of utilizing price discovery measures that combine CS with variance decomposition estimates (such as IS, MIS, or PDS) to accurately identify the price leader. Particularly, only the PILS measure (and its binary indicator PILI), which integrates CS with PDS, yields correct results when reduced-form residuals are correlated. The response of a market to the transitory shock (or noise) can potentially mislead price leadership identification when relying solely on a single metric like IS, MIS, or PDS.

In Appendix A4, we further investigate how data frequency impacts the performance of these price discovery measures. The key finding is that as price discovery measures exhibit sensitivity to data frequency, PDS and the combined measure PILS (along with its binary indicator PILI) can offer more robust price leadership estimates at lower data frequencies.

[15] The correlation coefficient of the reduced-form errors is given as $\rho = \frac{\delta_1\delta_2\sigma_P^2 + b_{0,1}^T b_{0,2}^T \sigma_T^2}{\sqrt{(\delta_1^2\sigma_P^2 + (b_{0,1}^T)^2\sigma_T^2)(\delta_2^2\sigma_P^2 + (b_{0,2}^T)^2\sigma_T^2)}}$. When we set $\delta_2 = 1 - \delta_1$, $\sigma_P^2 = 1$, $\sigma_T^2 = 10$, and $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, the correlation coefficient ρ takes an average value of -0.68 as δ_1 changes from 0.9 to 0.1 (with ρ ranging from -0.88 to -0.54).

While IS and MIS become less informative with coarser data sampling, PDS appears more resilient to changes in data frequency compared to the other measures. However, the performance of combined indicators such as ILS and MILS (along with their binary counterparts) may deteriorate due to the additional sensitivity of CS measures to data frequency variations. Further details of the simulations are provided in Appendix A4.

3.3 Simulation Evidence from Putniņš (2013)

Putniņš (2013) developed a structural model of price formation that accommodates differences between two price series in terms of noise levels and speed of adjustment to new information. In their simulation setup, each market's price series tracks the fundamental value with a time delay and includes independent transitory noise. Consequently, uncorrelated transitory noises produce a distinct correlation pattern between the reduced-form errors.

In this subsection, we adopt the simulation framework introduced by Putniņš (2013) but allow for a more general correlation pattern among the transitory shocks of the underlying markets. The simulated data are generated following the approach outlined in Putniņš (2013):

$$\begin{aligned} m_t &= m_{t-1} + u_t, \quad u_t \sim N(0,1) \\ p_{1t} &= m_{t-\delta_1} + s_{1t}, \\ p_{2t} &= m_{t-\delta_2} + s_{2t}, \end{aligned} \tag{46}$$

where m_t, p_{1t}, and p_{2t} denote the natural logarithms of the fundamental value, price series 1, and price series 2, respectively, at time t. The terms s_{1t} and s_{2t} represent the idiosyncratic noise specific to each market. Each price series i tracks the fundamental value m_t with a time delay of δ_i periods and includes transitory noise s_{it}, where $i = 1, 2$.

Instead of assuming independence between s_{1t} and s_{2t} as in Putniņš (2013), we now assume that:

$$\begin{pmatrix} s_{1t} \\ s_{2t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{s_1}^2 & \rho_s\sigma_{s_1}\sigma_{s_2} \\ \rho_s\sigma_{s_1}\sigma_{s_2} & \sigma_{s_2}^2 \end{pmatrix}\right), \tag{47}$$

where ρ_s represents the correlation coefficient of s_{1t} and s_{2t}, and σ_{s_i} measures the noise level of the price series i. The parameters δ_i and σ_{s_i} characterize price series i in terms of its speed of adjustment to innovations in the fundamental value and the magnitude of its transitory noise, respectively.

In the simulation, we fix the structural parameters of p_{1t} at $\delta_1 = 5$ and $\sigma_{s_1}^2 = 5$, while the structural parameters of p_{2t} vary with different combinations of the delay parameter δ_2, the noise level parameter $\sigma_{s_2}^2$, and the correlation parameter ρ_s. Specifically, we increase the correlation parameter ρ_s from 0 to 1 with a step size of 0.2, decrease the delay parameter δ_2 from 10 to 0 with a step size of 1, and vary the noise level parameter $\sigma_{s_2}^2$ from 0 to 10 with a step size of 1.[16]

For each parameter combination, we simulate 1000 samples, each comprising 21,600 time-series observations. Subsequently, for each simulated sample, we estimate a bivariate VECM of the two price series (p_{1t}, p_{2t}) with the restricted cointegrating vector $(1, -1)'$. From these VECM estimates, we calculate various price discovery measures discussed above. To save space, we primarily focus on the comparison of the combined price discovery measures ILS, MILS, and PILS. The sample means of these three measures for p_{2t} are reported in Tables 2 - 4, respectively.

[Insert Table 2 about here.]

[Insert Table 3 about here.]

[Insert Table 4 about here.]

[16]In the case where $\delta_1 = \delta_2 = 5$, $\sigma_{s_1}^2 = \sigma_{s_2}^2 = 5$, and $\rho_s = 1$, the simulated price series will be identical for both markets. Hence, in the simulation of this sub-case, we set the correlation coefficient as $\rho_s = 1 - 10^{-7}$.

These three tables have the same layout, with panels corresponding to different values of the correlation coefficient ρ_s, rows corresponding to different values of Market 2's delay parameter δ_2, and columns corresponding to different values of Market 2's noise level $\sigma^2_{s_2}$. Standard deviations of these measures across the 1000 samples are indicated in square brackets. To facilitate visualization, we employ a color scale where values smaller than 0.5 are marked in red, values greater than 0.5 in green, and values equal to 0.5 in yellow.

Initially, focusing on Panel A of these tables, where the correlation coefficient ρ_s between the two markets' idiosyncratic noise shocks is zero, we observe that ILS, MILS, and PILS all effectively measure price discovery when $\delta_2 \neq 5$. For rows where $\delta_2 < 5$, the means of ILS, MILS, and PILS estimates for Market 2 are all greater than 0.5 with very small standard deviations. Conversely, for rows where $\delta_2 > 5$, the means of ILS, MILS, and PILS estimates for Market 2 are all smaller than 0.5 with very small standard deviations. Notably, given that Market 2 moves first (rows with $\delta_2 < 5$), the estimates of ILS, MILS, and PILS decrease with Market 2's noise level (across columns with different $\sigma^2_{s_2}$). These findings align with Table 3 in Putniņš (2013), given the absence of correlation between the two markets' transitory noises.

In the scenario of equal leadership ($\delta_2 = \delta_1 = 5$) under zero correlation, we observe that ILS and MILS tend to over-assign Market 1 as the leader when the noise level of Market 2 is smaller than 5. Conversely, the PILS measure consistently designates each market as the leader evenly, regardless of the noise level of Market 2. This outcome indicates that PILS surpasses ILS and MILS in accurately identifying equal leadership when dealing with zero correlation between transitory shocks.

As the correlation coefficient ρ_s increases, both ILS and MILS demonstrate a tendency to make errors in selecting the leading market. For instance, when $\rho_s = 0.2$ as depicted in Panel B of Table 2, ILS mistakenly designates Market 2 as the leader approximately 56% of the time when $\delta_2 = 10$ and $\sigma^2_{s_2} = 2$. Additionally, ILS can erroneously choose Market 1 as the leader around 67% of the time when $\delta_2 = 0$ and $\sigma^2_{s_2} = 6$. Similarly, when $\rho_s = 0.2$ as depicted in Panel B of Table 3, MILS incorrectly designates Market 2 as the leader approximately 60% of the time when $\delta_2 = 8$ and $\sigma^2_{s_2} = 10$, and mistakenly designates Market 1 as the leader approximately 57% of the time when $\delta_2 = 3$ and $\sigma^2_{s_2} = 2$.

It's worth noting that the mis-specification of leadership for ILS and MILS seems to exacerbate with higher correlation levels ($\rho_s = 0.4$, 0.6, 0.8 as depicted in Panels C-E), although the case with the highest correlation ($\rho_s = 1$ as depicted in Panel F) appears to demonstrate a slightly less severe mis-specification issue. Additionally, it's important to note that ILS and MILS may not necessarily decrease with the noise level of Market 2 when Market 2 is the leading market.

In contrast, as Table 4 demonstrates, PILS consistently identifies the correct leading market when $\delta_2 \neq 0.5$, regardless of the values of ρ_s. In each panel of Table 4, the PILS estimates for Market 2 consistently fall below 0.5 for rows where $\delta_2 > 5$, and above 0.5 for rows where $\delta_2 < 5$, with very small standard deviatins across simulations. Even in the case of equal leadership ($\delta_2 = \delta_1 = 5$), the PILS estimates remain close to 0.5 regardless of the noise level of Market 2. This consistency is observed across all values of the correlation coefficient ρ_s. Additionally, when Market 2 is the leading market (rows with $\delta_2 < 5$), the PILS estimates decrease with the noise level of Market 2 (across columns with different $\sigma^2_{s_2}$) for all values of ρ_s.

In conclusion, the simulation confirms that ILS, MILS, and PILS effectively measure price discovery in terms of identifying "who moves first." Importantly, we demonstrate that: (1) PILS is the only measure that robustly identifies the leading market when transitory noises are correlated across markets, and (2) only PILS exhibits a monotonic negative relationship with the leading market's noise level, and (3) for independent transitory shocks, PILS is the only measure that can provide robust equal leadership identification. These findings underscore the significance of PILS as a reliable indicator of price leadership, particularly in situations with correlated transitory shocks

4 Application to Cross-listed SH-HK stocks

4.1 Data Sources

It is widely acknowledged that Chinese securities listed on the mainland market, known as A-shares, are traded at a premium (referred to as the AH premium) compared to the Hong Kong market (H-shares) (Fernald and Rogers, 2002). This departure from the principle of the law of one price has garnered significant academic and practical attention. Many researchers argue that this phenomenon stems from market segmentation within the Chinese stock market (see Wang and Jiang, 2004).

In a move towards liberalizing the Chinese capital market, the administrators of the securities market in China introduced the Shanghai-Hong Kong Stock Connect on November 17, 2014, followed by the Shenzhen-Hong Kong Stock Connect on December 5, 2016. Through these Stock Connect programs, individual investors from the mainland (Hong Kong) gained access to purchase H-shares (A-shares) without any quota or license requirements. Furthermore, retail investors from both regions were enabled to directly engage in the stock market by buying and selling shares. Additionally, the quota for institutional investors saw a notable increase of at least 50%.

However, the extent to which the Stock Connect enhances price equalization among AH shares remains a topic of ongoing debate. In this section, we aim to investigate the price leadership between AH shares both before and after the implementation of the Stock Connect policy. By doing so, we seek to provide insights into the effectiveness of the liberalization efforts in the Chinese capital market.

As of June 2023, there are 94 stocks cross-listed in Shanghai (SH) and Hong Kong (HK), and 24 stocks cross-listed in Shenzhen (SZ) and HK. Given the relatively smaller number of stocks in SZ, our focus lies on the SH-HK stocks, where we analyze the price discovery relationships between these two markets pre and post the Shanghai-Hong Kong

Stock Connect implementation.[17]

To ensure an adequate sample size for estimating the VECM, we set a minimum requirement of 5 years preceding the implementation of the Shanghai-Hong Kong Stock Connect on November 17, 2014. With this criterion, we are left with 53 SH-HK stocks. The empirical investigation utilizes daily adjusted closing prices (denominated in U.S. dollars) of these 53 SH-HK stocks spanning from January 1, 2010, to January 1, 2020. Data is sourced from Thomson Reuters Datastream.[18] The decision to utilize daily adjusted closing prices, rather than intra-day data, is deliberate. This choice results in reduced-form residuals that exhibit more significant correlations, facilitating a more discernible comparison of different price discovery measures.

The basic information of these 53 stocks are reported in Table 5.

[Insert Table 5 about here.]

For each log-price pair $(\ln P_{SH}, \ln P_{HK})$ of cross-listed SH-HK firms, we estimate a bivariate reduced-form VECM with the constrained cointegrating vector $(1, -1)'$. From this estimation, we derive a set of price discovery measures.

4.2 Estimation Results

Table 6 presents bivariate price discovery estimates of HK shares throughout the sample period spanning from 2010-01-01 to 2020-01-01. The column labeled "ρ" represents the correlation coefficient between VECM residuals. The column labeled "Rank" indicates the cointegration rank between each pair of cross-listed shares, determined through the Johansen cointegration test at the 10% significance level. Lastly, the column labeled "LR test" displays

[17] It's worth noting that Jiang and Sohn (2016) also delved into the price discovery process of Chinese stocks cross-listed in SH-HK during the periods before and after the Shanghai-Hong Kong Stock Connect. In our examination, we build upon Jiang and Sohn (2016)'s empirical investigations, with a specific focus on comparing traditional price discovery measures with our newly proposed order-invariant and correlation-robust price discovery measures.

[18] We extend our gratitude to Dr. Carlos A. Gutiérrez-Mangas for introducing us to the Compustat Global database utilized in Gutiérrez-Mangas (2023), even though we opted for an alternative data source in our study.

the p-value of the likelihood ratio test, evaluating the restriction that the cointegration vector is $(1, -1)'$.

[Insert Table 6 about here.]

From the above results, it's evident that the VECM reduced-form residuals exhibit a high degree of correlation, with an average correlation coefficient of 0.52. Across the period from 2010 to 2020, among the 53 SH-HK firms under consideration, only 29 firms demonstrate a cointegration relationship between $(\ln P_{SH}, \ln P_{HK})$, of which merely 10 firms exhibit cointegration with the coefficient $(1, -1)'$. The average PILS estimate for HK shares stands at approximately 0.60, with binary PILI estimates indicating that for 32 firms, HK shares are leading SH shares (with PILI taking a value of 1). Hence, in broad terms, over the past decade, the Hong Kong stock market has typically shown leadership over the Shanghai stock market.

Subsequently, we divide the sample period into two distinct parts: one preceding the implementation of the Shanghai-Hong Kong Stock Connect (Pre-sample), and the other succeeding it (After-sample). In the Appendix, Table A2 displays the detailed estimation outcomes for the Pre-sample, while Table A3 showcases results for the After-sample.

To enhance the readability of the estimation outcomes, we compare the mean values of these price discovery measures before and after the implementation of the Shanghai-Hong Kong Stock Connect, considering the 53 firms collectively, as well as separately for the 29 cointegrated firms, the 10 unitary cointegrated firms, and the 19 non-unitary cointegrated firms.

[Insert Table 7 about here.]

As observed in the table above, the dominance of HK shares diminishes following the implementation of the Shanghai-Hong Kong Stock Connect. The average PILS estimate decreases from 0.70 to 0.59, and the count of HK-leading-SH firms drops from 40 to 35 (with

PILI taking a value of 1). This trend persists across both the 53 firms collectively and the subset of 29 cointegrated firms.[19]

However, a focus solely on traditional price discovery metrics, such as the IS, CS, MIS, and even PDS measures, may yield contradictory conclusions, particularly evident for the 29 cointegrated firms. Among these 29 cointegrated firms, all averaged estimates of IS, CS, MIS, and PDS for the HK market witness significant increases following the implementation of the Shanghai-Hong Kong Stock Connect. In contrast, combined measures such as ILS, MILS, and PILS exhibit a declining trend for the HK market after the Shanghai-Hong Kong Stock Connect.

Furthermore, within the subset of these 29 cointegrated firms, only 10 firms successfully pass the likelihood ratio test for a $(1, -1)'$ cointegration vector. Among these 10 unitary cointegrated firms, the average PILS measure for HK shares experiences a slight increase from 0.61 to 0.64 after the implementation of the Shanghai-Hong Kong Stock Connect. Conversely, among the remaining 19 non-unitary cointegrated firms, the average PILS measure for HK shares declines from 0.67 to 0.59 post the Shanghai-Hong Kong Stock Connect. Upon comparing the average PILS estimate of these 10 unitary cointegrated firms with that of the remaining 19 non-unitary cointegrated firms, a notable enhancement in efficiency within the SH market emerges, albeit primarily observed among the non-unitary cointegrated firms.

In all, our empirical examination of SH-HK cross-listed firms using this newly proposed price discovery measure suggests that the Chinese stock market (A shares) is experiencing an enhancement in efficiency within the price discovery process following the implementation of the Shanghai-Hong Kong Stock Connect.

[19] For the 29 cointegrated firms, although the PILI estimates suggest a slight increase in the number of HK-leading-SH firms from 20 to 21 (with an average PILI changing from 0.69 to 0.72), the averaged PILS estimate for HK shares decreases from 0.65 to 0.61. Thus, we infer that for these 29 cointegrated firms, the HK market is losing some of its price discovery advantage to the SH market.

5 Conclusion

In this paper, we introduce an improved measure of price discovery termed the Price Information Leadership (PIL) measure, which is both order-invariant and correlation-robust. Our approach draws inspiration from the Information Leadership (IL) measure proposed by Yan and Zivot (2010), but we incorporate the order-invariant Price Discovery Share (PDS) introduced by Sultan and Zivot (2015) instead of the traditional Information Share (IS) measure from Hasbrouck (1995). By integrating the PDS measure with the CS measure from Gonzalo and Granger (1995), our PIL measure effectively disentangles the confounding effects arising from underlying markets' responses to the transitory shock. This leads to a straightforward structural representation of the PIL measure, expressed as the ratio of each market's initial impacts of the permanent shock, even in the presence of correlated reduced-form VECM residuals.

Additionally, we define a Price Information Leadership Share (PILS) measure and a binary Price Information Leadership Indicator (PILI), following Putniņš (2013) and Patel et al. (2020). These complementary measures provide further insights into the distribution of price leadership among the markets under consideration.

Simulation evidence based on both the partial price adjustment model of Yan and Zivot (2010) and the price delay model of Putniņš (2013) with correlated idiosyncratic errors strongly support the efficacy of our new measures. Our simulation results highlight the necessity of integrating the CS measure with a variance decomposition measure such as IS, MIS, or PDS to precisely identify the price leader. Markets' responses to transitory shocks, or noise, may misguide price leadership identification. Therefore, it is imperative to disentangle these transitory noises when assessing price discovery.

More importantly, our simulation findings unveil that only our proposed PIL measure (along with its share measure PILS and its binary indicator PILI) is capable of yielding accurate results when reduced-form residuals exhibit correlation. Additionally, our simulation evidence across different data frequencies demonstrates that while existing measures may exhibit sensitivity to data frequency, our newly proposed measures PIL, PILS, and PILI are notably less sensitive. This highlights the robustness and reliability of our measures across varying data frequencies.

We apply the aforementioned price discovery measures to analyze Shanghai and Hong Kong cross-listed firms and assess how these two markets evolve in terms of their contributions to the price discovery process following the implementation of the Shanghai-Hong Kong Stock Connect. Empirical findings, particularly based on our PILS measure, indicate that the Shanghai stock market's contribution to the price discovery process for SH-HK shares has increased subsequent to the stock market liberalization policy. However, it is noteworthy that the Hong Kong market still maintains dominance in the price discovery process. Notably, relying solely on traditional measures such as IS, CS, MIS, and even PDS may lead to contradictory conclusions.

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Table 1: Price Discovery Measures from Partial Price Adjustment Model

This table reports price discovery measure estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$

$$p_{2t} = p_{2,t-1} + \delta_2(m_t - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta_t} = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $\delta_2 = 1 - \delta_1$, $\sigma_P^2 = 1$. We simulate 1000 samples of 21600 observations.

Panel A: $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$, $\sigma_T^2 = \frac{\delta_1\delta_2}{-b_{0,1}^T b_{0,2}^T}$

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.90	0.10	0.50	0.50	0.90	0.10	0.90	0.10	0.99	0.01	0.99	0.01	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00
0.80	0.80	0.20	0.50	0.50	0.80	0.20	0.80	0.20	0.94	0.06	0.94	0.06	0.94	0.06	1.00	0.00	1.00	0.00	1.00	0.00
0.70	0.70	0.30	0.50	0.50	0.70	0.30	0.70	0.30	0.84	0.16	0.84	0.16	0.84	0.16	1.00	0.00	1.00	0.00	1.00	0.00
0.60	0.60	0.40	0.50	0.50	0.60	0.40	0.60	0.40	0.69	0.31	0.69	0.31	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.51	0.49	0.51	0.49	0.51
0.40	0.40	0.60	0.50	0.50	0.40	0.60	0.40	0.60	0.31	0.69	0.31	0.69	0.31	0.69	0.00	1.00	0.00	1.00	0.00	1.00
0.30	0.30	0.70	0.50	0.50	0.30	0.70	0.30	0.70	0.16	0.84	0.16	0.84	0.16	0.84	0.00	1.00	0.00	1.00	0.00	1.00
0.20	0.20	0.80	0.50	0.50	0.20	0.80	0.20	0.80	0.06	0.94	0.06	0.94	0.06	0.94	0.00	1.00	0.00	1.00	0.00	1.00
0.10	0.10	0.90	0.50	0.50	0.10	0.90	0.10	0.90	0.01	0.99	0.01	0.99	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00

Panel B: $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, $\sigma_T^2 = \frac{\delta_1\delta_2}{-b_{0,1}^T b_{0,2}^T}$

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.69	0.31	0.20	0.80	0.69	0.31	0.69	0.31	0.99	0.01	0.99	0.01	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00
0.80	0.50	0.50	0.20	0.80	0.50	0.50	0.50	0.50	0.94	0.06	0.94	0.06	0.94	0.06	1.00	0.00	1.00	0.00	1.00	0.00
0.70	0.37	0.63	0.20	0.80	0.37	0.63	0.37	0.63	0.84	0.16	0.84	0.16	0.84	0.16	1.00	0.00	1.00	0.00	1.00	0.00
0.60	0.27	0.73	0.20	0.80	0.27	0.73	0.27	0.73	0.69	0.31	0.69	0.31	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.20	0.80	0.20	0.80	0.20	0.80	0.20	0.80	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.51	0.49	0.51	0.49	0.51
0.40	0.14	0.86	0.20	0.80	0.14	0.86	0.14	0.86	0.31	0.69	0.31	0.69	0.31	0.69	0.00	1.00	0.00	1.00	0.00	1.00
0.30	0.10	0.90	0.20	0.80	0.10	0.90	0.10	0.90	0.16	0.84	0.16	0.84	0.16	0.84	0.00	1.00	0.00	1.00	0.00	1.00
0.20	0.06	0.94	0.20	0.80	0.06	0.94	0.06	0.94	0.06	0.94	0.06	0.94	0.06	0.94	0.00	1.00	0.00	1.00	0.00	1.00
0.10	0.03	0.97	0.20	0.80	0.03	0.97	0.03	0.97	0.01	0.99	0.01	0.99	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00

Panel C: $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, $\sigma_T^2 = 10$

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.54	0.46	0.20	0.80	0.59	0.41	0.69	0.31	0.96	0.04	0.97	0.03	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00
0.80	0.50	0.50	0.20	0.80	0.50	0.50	0.50	0.50	0.94	0.06	0.94	0.06	0.94	0.06	1.00	0.00	1.00	0.00	1.00	0.00
0.70	0.44	0.56	0.20	0.80	0.41	0.59	0.37	0.63	0.91	0.09	0.89	0.11	0.84	0.16	1.00	0.00	1.00	0.00	1.00	0.00
0.60	0.38	0.62	0.20	0.80	0.34	0.66	0.27	0.73	0.86	0.14	0.80	0.20	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.33	0.67	0.20	0.80	0.27	0.73	0.20	0.80	0.79	0.21	0.69	0.31	0.50	0.50	1.00	0.00	1.00	0.00	0.48	0.52
0.40	0.28	0.72	0.20	0.80	0.22	0.78	0.14	0.86	0.70	0.30	0.55	0.45	0.31	0.69	1.00	0.00	0.94	0.06	0.00	1.00
0.30	0.23	0.77	0.20	0.80	0.17	0.83	0.10	0.90	0.59	0.41	0.40	0.60	0.16	0.84	1.00	0.00	0.00	1.00	0.00	1.00
0.20	0.20	0.80	0.20	0.80	0.13	0.87	0.06	0.94	0.49	0.51	0.27	0.73	0.06	0.94	0.27	0.73	0.00	1.00	0.00	1.00
0.10	0.17	0.83	0.20	0.80	0.10	0.90	0.03	0.97	0.39	0.61	0.17	0.83	0.02	0.98	0.00	1.00	0.00	1.00	0.00	1.00

Notes: Numbers shown are the averages of price discovery measure estimates of 1000 samples. For each sample, the sample size is set as $N = 21600$.

Table 2: Estimates of ILS for Price Delay Model in Putniņš (2013)
This table reports ILS measure estimates from the price data simulated from the following 2-market model:

$$m_t = m_{t-1} + u_t, \quad u_t \sim N(0,1), \quad p_{1t} = m_{t-5} + s_{1t}, \quad p_{2t} = m_{t-\delta_2} + s_{2t},$$

where $\boldsymbol{s_t} = (s_{1t}, s_{2t})'$ are Guassian white noise with covariance matrix $N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}5 & \rho_s\sqrt{5}\sigma_{s_2}\\ \rho_s\sqrt{5}\sigma_{s_2} & \sigma_{s_2}^2\end{pmatrix}\right)$. We simulate 1000 samples of 21600 observations. The ILS of p_{2t} is estimated for each sample, and we calculate the mean and standard deviation (in square brackets) across the 1000 samples.

Panel A: $\rho_s = 0$

	$\sigma_{s_2}^2$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]
9	0.03	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.05	0.06	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.08
	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.07	0.11	0.13	0.14	0.15	0.15	0.16	0.16	0.16	0.17	0.17
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
6	0.14	0.23	0.26	0.28	0.29	0.30	0.30	0.31	0.31	0.31	0.32
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.00	0.35	0.44	0.47	0.49	0.50	0.51	0.51	0.52	0.52	0.52
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	0.99	0.76	0.73	0.72	0.71	0.70	0.70	0.69	0.69	0.69	0.68
	[0.07]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	0.99	0.95	0.91	0.88	0.86	0.85	0.83	0.83	0.82	0.81	0.81
	[0.06]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	0.99	0.98	0.97	0.96	0.94	0.93	0.92	0.91	0.90	0.89	0.89
	[0.08]	[0.08]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	0.99	0.99	0.98	0.98	0.97	0.96	0.95	0.95	0.94	0.94
	[0.02]	[0.07]	[0.04]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	0.99	1.00	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.96
	[0.04]	[0.03]	[0.04]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Panel B: $\rho_s = 0.2$

	$\sigma_{s_2}^2$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.16	0.56	0.51	0.35	0.25	0.18	0.14	0.11	0.09	0.08
	[0.00]	[0.16]	[0.29]	[0.32]	[0.31]	[0.26]	[0.21]	[0.18]	[0.15]	[0.12]	[0.10]
9	0.03	0.08	0.19	0.39	0.62	0.70	0.69	0.62	0.53	0.45	0.39
	[0.01]	[0.01]	[0.09]	[0.20]	[0.24]	[0.23]	[0.27]	[0.30]	[0.31]	[0.32]	[0.30]
8	0.06	0.09	0.14	0.19	0.26	0.33	0.43	0.53	0.64	0.72	0.78
	[0.01]	[0.00]	[0.01]	[0.02]	[0.05]	[0.09]	[0.13]	[0.16]	[0.18]	[0.18]	[0.17]
7	0.07	0.15	0.19	0.23	0.26	0.29	0.32	0.36	0.39	0.43	0.47
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.03]	[0.04]	[0.05]	[0.06]
6	0.14	0.24	0.29	0.33	0.36	0.38	0.40	0.42	0.44	0.45	0.47
	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.00	0.18	0.35	0.42	0.47	0.50	0.52	0.54	0.55	0.57	0.58
	[0.00]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]
4	0.99	0.07	0.40	0.54	0.59	0.62	0.63	0.65	0.65	0.66	0.66
	[0.07]	[0.10]	[0.05]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	0.99	0.94	0.22	0.46	0.65	0.71	0.73	0.74	0.75	0.75	0.76
	[0.06]	[0.06]	[0.23]	[0.12]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	0.99	0.99	0.94	0.47	0.35	0.67	0.76	0.79	0.81	0.82	0.82
	[0.08]	[0.00]	[0.07]	[0.31]	[0.21]	[0.08]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	0.99	0.99	0.96	0.66	0.29	0.59	0.77	0.82	0.84	0.86
	[0.02]	[0.00]	[0.00]	[0.05]	[0.30]	[0.23]	[0.18]	[0.06]	[0.02]	[0.01]	[0.01]
0	0.99	0.99	0.99	0.99	0.96	0.75	0.33	0.50	0.74	0.83	0.86
	[0.04]	[0.00]	[0.00]	[0.01]	[0.05]	[0.25]	[0.28]	[0.23]	[0.10]	[0.04]	[0.02]

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Panel C: $\rho_s = 0.4$

δ_2 \ $\sigma^2_{s_2}$	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.23	0.07	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	[0.00]	[0.17]	[0.04]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.03	0.78	0.39	0.17	0.11	0.09	0.07	0.07	0.06	0.06	0.06
	[0.01]	[0.18]	[0.20]	[0.09]	[0.05]	[0.04]	[0.03]	[0.02]	[0.02]	[0.02]	[0.01]
8	0.06	0.26	0.74	0.89	0.62	0.42	0.29	0.22	0.18	0.15	0.14
	[0.01]	[0.04]	[0.14]	[0.14]	[0.20]	[0.16]	[0.12]	[0.09]	[0.07]	[0.06]	[0.05]
7	0.07	0.22	0.36	0.53	0.73	0.89	0.96	0.93	0.84	0.74	0.64
	[0.00]	[0.01]	[0.02]	[0.06]	[0.08]	[0.08]	[0.05]	[0.09]	[0.14]	[0.16]	[0.16]
6	0.14	0.24	0.34	0.42	0.49	0.57	0.64	0.71	0.78	0.84	0.90
	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.03]	[0.04]	[0.04]	[0.05]	[0.05]
5	0.00	0.01	0.19	0.33	0.43	0.50	0.55	0.60	0.64	0.67	0.70
	[0.00]	[0.01]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	0.99	0.80	0.16	0.08	0.30	0.43	0.51	0.56	0.60	0.62	0.64
	[0.07]	[0.06]	[0.11]	[0.05]	[0.04]	[0.02]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]
3	0.99	0.96	0.91	0.66	0.13	0.11	0.35	0.49	0.57	0.62	0.65
	[0.06]	[0.00]	[0.03]	[0.13]	[0.13]	[0.07]	[0.06]	[0.04]	[0.02]	[0.01]	[0.01]
2	0.99	0.97	0.97	0.95	0.86	0.58	0.13	0.11	0.34	0.50	0.60
	[0.08]	[0.00]	[0.00]	[0.01]	[0.06]	[0.16]	[0.14]	[0.09]	[0.09]	[0.05]	[0.03]
1	1.00	0.97	0.98	0.97	0.96	0.91	0.78	0.44	0.09	0.14	0.36
	[0.02]	[0.00]	[0.00]	[0.00]	[0.01]	[0.03]	[0.10]	[0.19]	[0.11]	[0.11]	[0.10]
0	0.99	0.97	0.98	0.98	0.97	0.96	0.92	0.82	0.57	0.21	0.08
	[0.04]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.03]	[0.08]	[0.18]	[0.19]	[0.09]

Panel D: $\rho_s = 0.6$

δ_2 \ $\sigma^2_{s_2}$	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.08	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08
	[0.00]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]
9	0.03	0.30	0.11	0.08	0.07	0.07	0.07	0.07	0.08	0.08	0.08
	[0.01]	[0.11]	[0.03]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]
8	0.06	0.90	0.45	0.22	0.15	0.12	0.11	0.10	0.10	0.10	0.10
	[0.01]	[0.09]	[0.12]	[0.05]	[0.03]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.07	0.36	0.83	0.94	0.67	0.46	0.34	0.27	0.23	0.20	0.18
	[0.00]	[0.03]	[0.07]	[0.07]	[0.12]	[0.10]	[0.07]	[0.05]	[0.04]	[0.03]	[0.03]
6	0.14	0.22	0.40	0.59	0.79	0.93	0.99	0.96	0.88	0.78	0.68
	[0.00]	[0.01]	[0.02]	[0.03]	[0.04]	[0.03]	[0.01]	[0.04]	[0.07]	[0.08]	[0.08]
5	0.00	0.12	0.01	0.16	0.35	0.50	0.62	0.72	0.80	0.87	0.92
	[0.00]	[0.03]	[0.01]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]
4	0.99	0.88	0.73	0.37	0.03	0.07	0.24	0.38	0.48	0.56	0.62
	[0.07]	[0.01]	[0.04]	[0.09]	[0.04]	[0.03]	[0.04]	[0.03]	[0.02]	[0.02]	[0.01]
3	0.99	0.93	0.92	0.89	0.79	0.54	0.18	0.02	0.12	0.28	0.40
	[0.06]	[0.00]	[0.00]	[0.02]	[0.04]	[0.09]	[0.10]	[0.02]	[0.05]	[0.05]	[0.04]
2	0.99	0.93	0.94	0.94	0.92	0.88	0.78	0.57	0.27	0.05	0.05
	[0.08]	[0.00]	[0.00]	[0.00]	[0.01]	[0.02]	[0.05]	[0.09]	[0.11]	[0.05]	[0.04]
1	1.00	0.93	0.94	0.95	0.95	0.93	0.90	0.83	0.71	0.51	0.25
	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.02]	[0.04]	[0.07]	[0.11]	[0.12]
0	0.99	0.93	0.94	0.95	0.95	0.94	0.93	0.90	0.84	0.75	0.60
	[0.04]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.03]	[0.06]	[0.10]

Panel E: $\rho_s = 0.8$

δ_2 \ $\sigma^2_{s_2}$	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.07	0.07	0.08	0.08	0.10	0.11	0.12	0.13	0.14	0.15
	[0.00]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
9	0.03	0.14	0.10	0.09	0.09	0.10	0.11	0.12	0.13	0.14	0.15
	[0.01]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
8	0.06	0.54	0.18	0.13	0.12	0.12	0.12	0.13	0.14	0.15	0.16
	[0.01]	[0.11]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
7	0.07	0.69	0.74	0.35	0.23	0.19	0.17	0.16	0.17	0.17	0.18
	[0.00]	[0.07]	[0.09]	[0.05]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]
6	0.14	0.18	0.53	0.95	0.92	0.67	0.50	0.39	0.34	0.31	0.29
	[0.00]	[0.02]	[0.04]	[0.03]	[0.05]	[0.06]	[0.05]	[0.04]	[0.03]	[0.02]	[0.02]
5	0.00	0.42	0.21	0.01	0.13	0.50	0.81	0.97	1.00	0.96	0.89
	[0.00]	[0.02]	[0.03]	[0.01]	[0.04]	[0.05]	[0.04]	[0.02]	[0.01]	[0.02]	[0.03]
4	0.99	0.84	0.82	0.76	0.61	0.32	0.05	0.04	0.22	0.42	0.58
	[0.07]	[0.00]	[0.01]	[0.02]	[0.04]	[0.06]	[0.03]	[0.03]	[0.05]	[0.05]	[0.04]
3	0.99	0.85	0.87	0.88	0.86	0.81	0.71	0.53	0.28	0.06	0.01
	[0.06]	[0.01]	[0.00]	[0.00]	[0.01]	[0.02]	[0.03]	[0.05]	[0.07]	[0.04]	[0.02]
2	0.99	0.85	0.87	0.89	0.90	0.88	0.85	0.80	0.70	0.56	0.37
	[0.08]	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.04]	[0.06]	[0.07]
1	1.00	0.86	0.88	0.90	0.90	0.90	0.88	0.86	0.81	0.75	0.66
	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.03]	[0.05]
0	0.99	0.86	0.88	0.90	0.91	0.91	0.90	0.88	0.86	0.82	0.76
	[0.04]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.02]	[0.03]

Panel F: $\rho_s = 1$

δ_2 \ $\sigma^2_{s_2}$	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.09	0.12	0.16	0.19	0.22	0.26	0.29	0.32	0.35	0.37
	[0.00]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
9	0.03	0.13	0.13	0.15	0.19	0.23	0.26	0.30	0.34	0.38	0.41
	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]
8	0.06	0.27	0.18	0.18	0.21	0.23	0.26	0.29	0.33	0.36	0.39
	[0.01]	[0.03]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
7	0.07	0.95	0.30	0.21	0.21	0.24	0.27	0.32	0.36	0.41	0.44
	[0.00]	[0.05]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]
6	0.14	0.09	0.94	0.48	0.27	0.24	0.28	0.34	0.40	0.45	0.48
	[0.00]	[0.02]	[0.04]	[0.03]	[0.01]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.00	0.44	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.07]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
4	0.99	0.66	0.63	0.67	0.73	0.76	0.71	0.58	0.32	0.02	0.38
	[0.07]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.01]	[0.02]	[0.03]	[0.02]	[0.11]
3	0.99	0.66	0.65	0.70	0.74	0.76	0.76	0.74	0.69	0.61	0.49
	[0.06]	[0.02]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.03]
2	0.99	0.69	0.71	0.74	0.76	0.77	0.76	0.75	0.73	0.70	0.65
	[0.08]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.02]
1	1.00	0.66	0.67	0.72	0.75	0.77	0.79	0.79	0.78	0.76	0.74
	[0.02]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]
0	0.99	0.71	0.72	0.75	0.77	0.78	0.78	0.78	0.77	0.76	0.74
	[0.04]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]

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Table 3: Estimates of MILS for Price Delay Model in Putniņš (2013)

This table reports MILS measure estimates from the price data simulated from the following 2-market model:

$$m_t = m_{t-1} + u_t, \quad u_t \sim N(0,1), \quad p_{1t} = m_{t-5} + s_{1t}, \quad p_{2t} = m_{t-\delta_2} + s_{2t},$$

where $\boldsymbol{s_t} = (s_{1t}, s_{2t})'$ are Guassian white noise with covariance matrix $N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 & \rho_s\sqrt{5}\sigma_{s_2} \\ \rho_s\sqrt{5}\sigma_{s_2} & \sigma^2_{s_2} \end{pmatrix}\right)$. We simulate 1000 samples of 21600 observations. The MILS of p_{2t} is estimated for each sample, and we calculate the mean and standard deviation (in square brackets) across the 1000 samples.

Panel A: $\rho_s = 0$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]
9	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.05	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.08
	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.07	0.11	0.12	0.13	0.14	0.14	0.15	0.15	0.15	0.16	0.16
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
6	0.16	0.23	0.26	0.27	0.28	0.29	0.29	0.29	0.30	0.30	0.30
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]
5	0.01	0.42	0.47	0.48	0.49	0.50	0.50	0.51	0.51	0.51	0.51
	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	0.99	0.81	0.77	0.74	0.72	0.71	0.70	0.70	0.69	0.69	0.69
	[0.06]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.96	0.92	0.89	0.87	0.85	0.84	0.83	0.82	0.82	0.81
	[0.05]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	0.99	0.99	0.98	0.96	0.95	0.93	0.92	0.91	0.90	0.90	0.89
	[0.07]	[0.05]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	0.99	0.99	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.94
	[0.01]	[0.05]	[0.02]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	1.00	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.96
	[0.03]	[0.02]	[0.03]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Panel B: $\rho_s = 0.2$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.08	0.37	0.31	0.18	0.10	0.06	0.04	0.02	0.02	0.01
	[0.00]	[0.13]	[0.31]	[0.32]	[0.28]	[0.21]	[0.16]	[0.12]	[0.10]	[0.07]	[0.05]
9	0.02	0.05	0.10	0.22	0.42	0.49	0.48	0.39	0.31	0.24	0.18
	[0.01]	[0.00]	[0.06]	[0.17]	[0.27]	[0.29]	[0.33]	[0.34]	[0.34]	[0.32]	[0.28]
8	0.06	0.07	0.10	0.13	0.16	0.20	0.26	0.34	0.44	0.53	0.60
	[0.01]	[0.00]	[0.00]	[0.01]	[0.03]	[0.06]	[0.10]	[0.15]	[0.19]	[0.23]	[0.24]
7	0.07	0.13	0.16	0.18	0.20	0.22	0.24	0.26	0.27	0.30	0.32
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.03]
6	0.16	0.24	0.28	0.30	0.32	0.34	0.35	0.36	0.37	0.38	0.39
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.01	0.29	0.41	0.46	0.48	0.50	0.51	0.52	0.53	0.54	0.54
	[0.02]	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	0.99	0.21	0.57	0.64	0.66	0.66	0.67	0.67	0.67	0.67	0.67
	[0.06]	[0.22]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	1.00	0.43	0.65	0.76	0.78	0.78	0.78	0.78	0.78	0.78
	[0.05]	[0.03]	[0.33]	[0.11]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	0.99	1.00	1.00	0.69	0.54	0.80	0.84	0.85	0.85	0.85	0.85
	[0.07]	[0.00]	[0.02]	[0.34]	[0.24]	[0.05]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	1.00	1.00	1.00	0.84	0.50	0.76	0.86	0.88	0.89	0.89
	[0.01]	[0.00]	[0.00]	[0.01]	[0.26]	[0.29]	[0.15]	[0.03]	[0.01]	[0.01]	[0.01]
0	1.00	1.00	1.00	1.00	1.00	0.91	0.54	0.69	0.86	0.90	0.91
	[0.03]	[0.00]	[0.00]	[0.00]	[0.03]	[0.20]	[0.33]	[0.23]	[0.06]	[0.02]	[0.01]

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Panel C: $\rho_s = 0.4$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.00]	[0.10]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
9	0.02	0.59	0.12	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	[0.01]	[0.26]	[0.15]	[0.03]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
8	0.06	0.15	0.54	0.71	0.28	0.09	0.04	0.02	0.01	0.00	0.00
	[0.01]	[0.02]	[0.19]	[0.27]	[0.24]	[0.11]	[0.05]	[0.02]	[0.01]	[0.01]	[0.00]
7	0.07	0.17	0.25	0.36	0.53	0.75	0.88	0.78	0.56	0.36	0.23
	[0.00]	[0.00]	[0.01]	[0.04]	[0.09]	[0.14]	[0.13]	[0.23]	[0.28]	[0.25]	[0.19]
6	0.16	0.25	0.31	0.36	0.40	0.45	0.50	0.56	0.62	0.70	0.77
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]	[0.03]	[0.04]	[0.06]	[0.07]
5	0.01	0.04	0.29	0.40	0.46	0.50	0.53	0.55	0.58	0.60	0.62
	[0.02]	[0.02]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
4	0.99	1.00	0.50	0.20	0.46	0.55	0.59	0.62	0.63	0.64	0.65
	[0.06]	[0.01]	[0.24]	[0.09]	[0.03]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
3	1.00	1.00	1.00	0.96	0.37	0.25	0.54	0.64	0.68	0.70	0.72
	[0.05]	[0.00]	[0.00]	[0.06]	[0.28]	[0.13]	[0.05]	[0.02]	[0.01]	[0.00]	[0.00]
2	0.99	1.00	1.00	1.00	0.99	0.90	0.35	0.26	0.55	0.67	0.73
	[0.07]	[0.00]	[0.00]	[0.00]	[0.01]	[0.11]	[0.29]	[0.16]	[0.09]	[0.03]	[0.01]
1	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.79	0.25	0.31	0.58
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.03]	[0.20]	[0.25]	[0.18]	[0.10]
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.88	0.48	0.20
	[0.03]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.02]	[0.13]	[0.30]	[0.18]

Panel D: $\rho_s = 0.6$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.01]	[0.05]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	0.06	0.76	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.01]	[0.17]	[0.07]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
	0.07	0.25	0.65	0.80	0.24	0.06	0.02	0.01	0.00	0.00	0.00
	[0.00]	[0.01]	[0.10]	[0.20]	[0.14]	[0.05]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]
	0.16	0.24	0.35	0.47	0.63	0.84	0.97	0.84	0.55	0.31	0.17
	[0.00]	[0.00]	[0.01]	[0.02]	[0.04]	[0.06]	[0.04]	[0.15]	[0.18]	[0.14]	[0.09]
	0.01	0.56	0.02	0.26	0.40	0.50	0.57	0.64	0.71	0.77	0.84
	[0.02]	[0.10]	[0.02]	[0.02]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.03]	[0.03]
	0.99	1.00	1.00	0.87	0.14	0.16	0.39	0.50	0.56	0.60	0.64
	[0.06]	[0.00]	[0.00]	[0.08]	[0.14]	[0.06]	[0.03]	[0.02]	[0.01]	[0.01]	[0.01]
	1.00	1.00	1.00	1.00	1.00	0.94	0.54	0.05	0.26	0.45	0.56
	[0.05]	[0.00]	[0.00]	[0.00]	[0.00]	[0.04]	[0.20]	[0.07]	[0.09]	[0.05]	[0.02]
	0.99	0.99	1.00	1.00	1.00	1.00	0.99	0.94	0.68	0.16	0.12
	[0.07]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.04]	[0.17]	[0.17]	[0.09]
	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.89	0.63
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.02]	[0.07]	[0.19]
	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.93
	[0.03]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.04]

Panel E: $\rho_s = 0.8$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
9	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
8	0.06	0.16	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.01]	[0.09]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
7	0.07	0.50	0.31	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	[0.00]	[0.07]	[0.13]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
6	0.16	0.21	0.44	0.87	0.69	0.16	0.04	0.01	0.00	0.00	0.00
	[0.00]	[0.01]	[0.03]	[0.06]	[0.15]	[0.07]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]
5	0.01	0.99	0.79	0.04	0.21	0.50	0.74	0.93	0.98	0.81	0.53
	[0.02]	[0.00]	[0.06]	[0.04]	[0.04]	[0.03]	[0.04]	[0.03]	[0.02]	[0.09]	[0.11]
4	0.99	0.99	1.00	1.00	0.99	0.84	0.20	0.09	0.34	0.50	0.61
	[0.06]	[0.00]	[0.00]	[0.00]	[0.01]	[0.07]	[0.13]	[0.05]	[0.05]	[0.03]	[0.02]
3	1.00	0.97	0.99	1.00	1.00	1.00	0.99	0.95	0.74	0.24	0.04
	[0.05]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.02]	[0.10]	[0.14]	[0.04]
2	0.99	0.97	0.99	1.00	1.00	1.00	1.00	1.00	0.99	0.95	0.83
	[0.07]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.02]	[0.07]
1	1.00	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
0	1.00	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	[0.03]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Panel F: $\rho_s = 1$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
9	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.04	0.06
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]
8	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
7	0.07	0.83	0.01	0.00	0.00	0.00	0.01	0.02	0.05	0.08	0.11
	[0.00]	[0.14]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.02]
6	0.16	0.13	0.89	0.04	0.00	0.00	0.01	0.05	0.11	0.16	0.20
	[0.00]	[0.02]	[0.07]	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.02]	[0.02]	[0.03]
5	0.01	0.99	0.99	0.98	0.98	0.50	0.03	0.04	0.05	0.06	0.08
	[0.02]	[0.00]	[0.00]	[0.00]	[0.00]	[0.08]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]
4	0.99	0.89	0.87	0.91	0.98	1.00	1.00	0.99	0.80	0.06	0.44
	[0.06]	[0.02]	[0.02]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.05]	[0.06]	[0.09]
3	1.00	0.89	0.90	0.95	0.99	1.00	1.00	1.00	1.00	0.98	0.93
	[0.05]	[0.02]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.02]
2	0.99	0.93	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	[0.07]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
1	1.00	0.90	0.93	0.97	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
0	1.00	0.96	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	[0.03]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

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Table 4: Estimates of PILS for Price Delay Model in Putniņš (2013)

This table reports PILS measure estimates from the price data simulated from the following 2-market model:

$$m_t = m_{t-1} + u_t, \quad u_t \sim N(0,1), \quad p_{1t} = m_{t-5} + s_{1t}, \quad p_{2t} = m_{t-\delta_2} + s_{2t},$$

where $\boldsymbol{s_t} = (s_{1t}, s_{2t})'$ are Guassian white noise with covariance matrix $N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}5 & \rho_s\sqrt{5}\sigma_{s_2}\\ \rho_s\sqrt{5}\sigma_{s_2} & \sigma_{s_2}^2\end{pmatrix}\right)$. We simulate 1000 samples of 21600 observations. The PILS of p_{2t} is estimated for each sample, and we calculate the mean and standard deviation (in square brackets) across the 1000 samples.

Panel A: $\rho_s = 0$

	$\sigma_{s_2}^2$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
9	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.04	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07
	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.10	0.12	0.13	0.13	0.14	0.14	0.14	0.14	0.15	0.15
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
6	0.17	0.23	0.25	0.26	0.27	0.27	0.28	0.28	0.28	0.28	0.29
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]	[0.02]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	1.00	0.86	0.80	0.76	0.74	0.72	0.71	0.71	0.70	0.69	0.69
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.97	0.93	0.90	0.88	0.86	0.85	0.84	0.83	0.82	0.81
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	1.00	0.99	0.98	0.96	0.95	0.94	0.93	0.92	0.91	0.90	0.89
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	1.00	0.99	0.99	0.98	0.97	0.96	0.96	0.95	0.94	0.94
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	1.00	1.00	1.00	0.99	0.99	0.98	0.98	0.97	0.97	0.97
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Panel B: $\rho_s = 0.2$

	$\sigma_{s_2}^2$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.05	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.08
	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.11	0.13	0.14	0.14	0.15	0.15	0.15	0.15	0.16	0.16
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
6	0.17	0.25	0.27	0.28	0.28	0.29	0.29	0.29	0.29	0.29	0.30
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	1.00	0.86	0.79	0.75	0.73	0.71	0.70	0.69	0.69	0.68	0.68
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.97	0.93	0.90	0.87	0.85	0.84	0.82	0.81	0.81	0.80
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	1.00	0.99	0.98	0.96	0.94	0.93	0.91	0.90	0.89	0.88	0.88
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	1.00	1.00	0.99	0.99	0.98	0.97	0.97	0.96	0.96	0.95
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

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Panel C: $\rho_s = 0.4$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.02	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.02	0.03	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.06	0.07	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09
	[0.01]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.12	0.14	0.15	0.15	0.16	0.16	0.16	0.16	0.16	0.16
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
6	0.17	0.26	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.30	0.30
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	1.00	0.87	0.79	0.75	0.72	0.70	0.69	0.68	0.68	0.67	0.67
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.97	0.93	0.89	0.86	0.84	0.82	0.81	0.80	0.79	0.79
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	1.00	0.99	0.98	0.95	0.93	0.92	0.90	0.89	0.88	0.87	0.86
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	1.00	0.99	0.98	0.96	0.95	0.94	0.93	0.92	0.91	0.91
	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	1.00	0.99	0.98	0.97	0.97	0.96	0.95	0.94	0.94	0.93
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]

Panel D: $\rho_s = 0.6$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.03	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.06
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.02	0.04	0.05	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.07	0.08	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10
	[0.01]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.13	0.15	0.16	0.16	0.16	0.16	0.17	0.16	0.16	0.16
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
6	0.17	0.26	0.29	0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	1.00	0.88	0.80	0.75	0.72	0.70	0.68	0.67	0.67	0.66	0.66
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.98	0.94	0.90	0.86	0.84	0.82	0.80	0.79	0.78	0.77
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	1.00	0.99	0.98	0.95	0.93	0.90	0.89	0.87	0.86	0.85	0.84
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	0.99	0.98	0.97	0.95	0.93	0.92	0.91	0.90	0.89	0.88
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	0.99	0.98	0.97	0.96	0.95	0.93	0.93	0.92	0.91	0.91
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]

Panel E: $\rho_s = 0.8$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.04	0.06	0.08	0.08	0.09	0.09	0.10	0.10	0.10	0.10
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.02	0.06	0.08	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
8	0.06	0.08	0.10	0.11	0.11	0.12	0.12	0.12	0.11	0.11	0.11
	[0.01]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.14	0.16	0.17	0.17	0.17	0.17	0.16	0.16	0.15	0.15
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
6	0.17	0.27	0.29	0.30	0.30	0.30	0.29	0.29	0.29	0.28	0.28
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
4	1.00	0.92	0.84	0.77	0.73	0.70	0.68	0.67	0.66	0.66	0.65
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.00	0.99	0.96	0.91	0.87	0.83	0.81	0.79	0.77	0.77	0.76
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
2	1.00	0.99	0.97	0.94	0.91	0.88	0.86	0.85	0.84	0.83	0.82
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
1	1.00	0.99	0.97	0.94	0.92	0.90	0.89	0.87	0.86	0.86	0.85
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
0	1.00	0.99	0.97	0.94	0.92	0.91	0.90	0.89	0.88	0.87	0.87
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]

Panel F: $\rho_s = 1$

	$\sigma^2_{s_2}$										
δ_2	0	1	2	3	4	5	6	7	8	9	10
10	0.01	0.07	0.12	0.16	0.19	0.21	0.23	0.25	0.26	0.27	0.28
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
9	0.02	0.08	0.12	0.15	0.17	0.18	0.18	0.18	0.17	0.17	0.16
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
8	0.06	0.11	0.15	0.18	0.20	0.22	0.23	0.24	0.24	0.24	0.23
	[0.01]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
7	0.06	0.16	0.19	0.20	0.21	0.20	0.18	0.16	0.13	0.11	0.09
	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]	[0.02]
6	0.17	0.28	0.30	0.29	0.26	0.22	0.16	0.10	0.05	0.03	0.02
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
4	1.00	1.00	1.00	0.98	0.89	0.78	0.72	0.68	0.66	0.65	0.65
	[0.00]	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
3	1.00	1.00	0.98	0.93	0.86	0.80	0.77	0.74	0.73	0.73	0.72
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
2	1.00	0.98	0.91	0.85	0.80	0.78	0.77	0.76	0.75	0.75	0.75
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]
1	1.00	0.99	0.96	0.90	0.85	0.82	0.80	0.79	0.78	0.78	0.78
	[0.00]	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]
0	1.00	0.95	0.88	0.83	0.80	0.79	0.78	0.78	0.78	0.78	0.78
	[0.00]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.00]	[0.00]	[0.00]	[0.00]

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Table 5: Firms cross-listed in Shanghai (SHSE) and Hong Kong (SEHK)

Stock No.	Company	SHSE code	SEHK code	SHSE listing date	SEHK listing date
1	Air China Limited	601111.SH	0753.HK	2006-08-18	2004-12-15
2	Aluminum Corporation of China Limited	601600.SH	2600.HK	2007-04-30	2001-12-12
3	Anhui Conch Cement Co.,Ltd.	600585.SH	0914.HK	2002-02-07	1997-10-21
4	Anhui Expressway Co., Ltd.	600012.SH	0995.HK	2003-01-07	1996-11-13
5	Bank of China	601988.SH	3988.HK	2006-07-05	2006-06-01
6	Bank of Communications Co.,Ltd.	601328.SH	3328.HK	2007-05-15	2005-06-23
7	Beijing Jingcheng Machinery Electric Co.,Ltd.	600860.SH	0187.HK	1994-05-06	1993-08-06
8	Beijing North Star Co.,Ltd.	601588.SH	0588.HK	2006-10-16	1997-05-14
9	China CITIC Bank Corp., Ltd.	601998.SH	0998.HK	2007-04-27	2007-04-27
10	China Coal Energy Co.,Ltd.	601898.SH	1898.HK	2008-02-01	2006-12-19
11	China Construction Bank Corporation	601939.SH	0939.HK	2007-09-25	2005-10-27
12	China Eastern Airlines Corp., Ltd.	600115.SH	0670.HK	1997-11-05	1997-02-05
13	China Life Insurance Co.,Ltd.	601628.SH	2628.HK	2007-01-09	2003-12-18
14	China Merchants Bank Co., Ltd.	600036.SH	3968.HK	2002-04-09	2006-09-22
15	China Minsheng Banking Corp., Ltd.	600016.SH	1988.HK	2000-12-19	2009-11-26
16	China Oilfield Services Limited	601808.SH	2883.HK	2007-09-28	2002-11-20
17	China Pacific Insurance (Group) Co., Ltd.	601601.SH	2601.HK	2007-12-25	2009-12-23
18	China Petroleum & Chemical Corporation	600028.SH	0386.HK	2001-08-08	2000-10-19
19	China Railway Construction Corp., Ltd.	601186.SH	1186.HK	2008-03-10	2008-03-13
20	China Railway Group Limited	601390.SH	0390.HK	2007-12-03	2007-12-07
21	China Shenhua Energy Co.,Ltd.	601088.SH	1088.HK	2007-10-09	2005-06-15
22	China Southern Airlines Co., Ltd.	600029.SH	1055.HK	2003-07-25	1997-07-31
23	Chongqing Iron & Steel Co.,Ltd.	601005.SH	1053.HK	2007-02-28	1997-10-17
24	COSCO Shipping Development Co., Ltd	601866.SH	2866.HK	2007-12-12	2004-06-16
25	COSCO Shipping Energy Transportation Co., Ltd	600026.SH	1138.HK	2002-05-23	1994-11-11
26	COSCO Shipping Holdings Co., Ltd	601919.SH	1919.HK	2007-06-26	2005-06-30
27	CRRC Corporation Limited	601766.SH	1766.HK	2008-08-18	2008-08-21
28	CSSC Offshore & Marine Engineering (Group) Co.,Ltd.	600685.SH	0317.HK	1993-10-28	1993-08-06
29	Datang International Power Generation Co.,Ltd.	601991.SH	0991.HK	2006-12-20	1997-03-21
30	Dongfang Electric Corp., Ltd.	600875.SH	1072.HK	1995-10-10	1994-06-06
31	Guangshen Railway Co.,Ltd.	601333.SH	0525.HK	2006-12-22	1996-05-14
32	Guangzhou Baiyunshan Pharmaceutical Holdings Co.,Ltd.	600332.SH	0874.HK	2001-02-06	1997-10-30
33	Huadian Power International Co., Ltd.	600027.SH	1071.HK	2005-02-03	1999-06-30
34	Huaneng Power International,Inc.	600011.SH	0902.HK	2001-12-06	1998-01-21
35	Industrial and Commercial Bank of China Limited	601398.SH	1398.HK	2006-10-27	2006-10-27
36	Jiangsu Expressway Co.,Ltd.	600377.SH	0177.HK	2001-01-16	1997-06-27
37	Jiangxi Copper Co.,Ltd.	600362.SH	0358.HK	2002-01-11	1997-06-12
38	Maanshan Iron & Steel Co.,Ltd.	600808.SH	0323.HK	1994-01-06	1993-11-03
39	Metallurgical Corporation of China Ltd.	601618.SH	1618.HK	2009-09-21	2009-09-24
40	Nanjing Panda Electronics Co.,Ltd.	600775.SH	0553.HK	1996-11-18	1996-05-02
41	Petrochina Co.,Ltd.	601857.SH	0857.HK	2007-11-05	2000-04-07
42	Ping An Insurance (Group) Company of China,Ltd.	601318.SH	2318.HK	2007-03-01	2004-06-24
43	Shanghai Electric Group Co.,Ltd.	601727.SH	2727.HK	2008-12-05	2005-04-28
44	Shenji Group Kunming Machine Tool Co.,Ltd.	600806.SH	0300.HK	1994-01-03	1993-12-07
45	Shenzhen Expressway Co.,Ltd.	600548.SH	0548.HK	2001-12-25	1997-03-12
46	Sichuan Expressway Co.,Ltd.	601107.SH	0107.HK	2009-07-27	1997-10-07
47	Sinopec Oilfield Service Corporation	600871.SH	1033.HK	1995-04-11	1994-03-29
48	Sinopec Shanghai Petrochemical Co.,Ltd.	600688.SH	0338.HK	1993-11-08	1993-07-26
49	Tianjin Capital Environmental Protection Group	600874.SH	1065.HK	1995-06-30	1994-05-17
50	Triumph New Energy Company Limited	600876.SH	1108.HK	1995-10-31	1994-07-08
51	Tsingtao Brewery Co.,Ltd.	600600.SH	0168.HK	1993-08-27	1993-07-15
52	Yanzhou Coal Mining Co.,Ltd.	600188.SH	1171.HK	1998-07-01	1998-04-01
53	Zijin Mining Group Co.,Ltd.	601899.SH	2899.HK	2008-04-25	2003-12-23

Note: Data is sourced from Thomson Reuters Datastream. Listing dates are adapted from Table 1 of Jiang and Sohn (2016) on page 34.

Table 6: Price Discovery Estimates for HK shares from 2010-01-01 to 2020-01-01

Stock No.	IS	CS	MIS	PDS	ILS	MILS	PILS	ILI	MILI	PILI	ρ	Rank	LR test
1	0.16	0.06	0.11	0.04	0.89	0.75	0.24	1	1	0	0.52	1	0.00
2	0.79	0.89	0.86	0.93	0.18	0.33	0.71	0	0	1	0.56	1	0.36
3	0.30	0.13	0.22	0.11	0.88	0.77	0.38	1	1	0	0.70	1	0.03
4	0.14	0.19	0.11	0.08	0.31	0.22	0.13	0	0	0	0.35	1	0.00
5	0.47	0.46	0.46	0.46	0.51	0.50	0.49	1	1	0	0.41	0	0.19
6	0.17	0.37	0.12	0.07	0.11	0.05	0.02	0	0	0	0.48	0	0.02
7	0.77	0.61	0.81	0.87	0.82	0.89	0.95	1	1	1	0.52	0	0.10
8	0.93	0.75	1.00	1.08	0.95	1.00	0.96	1	1	1	0.50	1	0.04
9	0.32	0.35	0.30	0.26	0.44	0.38	0.31	0	0	0	0.51	1	0.00
10	0.71	0.57	0.74	0.78	0.77	0.82	0.87	1	1	1	0.49	1	0.00
11	0.11	0.04	0.07	0.02	0.91	0.77	0.18	1	1	0	0.45	1	0.01
12	0.18	0.08	0.12	0.05	0.87	0.73	0.28	1	1	0	0.54	0	0.04
13	0.38	0.50	0.35	0.31	0.27	0.22	0.17	0	0	0	0.60	0	0.01
14	0.26	0.20	0.21	0.16	0.65	0.53	0.35	1	1	0	0.54	1	0.00
15	0.27	0.18	0.22	0.15	0.75	0.63	0.40	1	1	0	0.59	0	0.06
16	0.93	0.73	0.98	1.04	0.96	1.00	0.99	1	1	1	0.46	1	0.08
17	0.12	0.34	0.02	-0.11	0.06	0.00	0.03	0	0	0	0.61	1	0.00
18	0.87	1.00	0.93	1.00	0.00	0.00	0.80	0	0	1	0.52	1	0.70
19	0.42	0.41	0.41	0.40	0.52	0.50	0.47	1	0	0	0.48	0	0.01
20	0.20	0.11	0.14	0.07	0.80	0.64	0.29	1	1	0	0.54	1	0.00
21	0.87	0.66	0.93	1.01	0.92	0.98	1.00	1	1	1	0.53	1	0.00
22	0.09	0.21	0.00	-0.11	0.13	0.00	0.12	0	0	0	0.57	1	0.01
23	0.48	0.44	0.47	0.47	0.57	0.57	0.56	1	1	1	0.51	1	0.08
24	0.73	0.57	0.77	0.82	0.82	0.87	0.92	1	1	1	0.51	0	0.08
25	0.73	0.54	0.76	0.81	0.83	0.88	0.93	1	1	1	0.52	0	0.24
26	0.38	0.43	0.36	0.32	0.40	0.34	0.28	0	0	0	0.58	0	0.09
27	0.34	0.29	0.31	0.28	0.61	0.55	0.46	1	1	0	0.53	1	0.00
28	0.91	0.70	0.98	1.07	0.95	1.00	0.98	1	1	1	0.54	1	0.67
29	0.84	0.85	0.88	0.91	0.46	0.60	0.77	0	1	1	0.42	0	0.28
30	0.91	0.71	0.99	1.08	0.95	1.00	0.97	1	1	1	0.54	1	0.00
31	0.80	0.87	0.85	0.91	0.28	0.43	0.69	0	0	1	0.51	0	0.73
32	0.84	0.91	0.93	1.06	0.21	0.67	0.76	0	1	1	0.63	1	0.00
33	0.65	0.54	0.67	0.69	0.72	0.75	0.78	1	1	1	0.44	0	0.05
34	0.61	0.55	0.62	0.64	0.61	0.63	0.66	1	1	1	0.44	0	0.02
35	0.09	0.01	0.04	0.00	0.99	0.96	0.18	1	1	0	0.43	1	0.02
36	0.45	0.44	0.45	0.45	0.52	0.51	0.50	1	1	1	0.40	1	0.01
37	0.81	0.97	0.89	0.98	0.02	0.06	0.67	0	0	1	0.58	1	0.85
38	0.09	0.25	0.01	-0.08	0.08	0.00	0.05	0	0	0	0.54	0	0.19
39	0.30	0.26	0.26	0.22	0.59	0.50	0.38	1	1	0	0.54	0	0.04
40	0.90	0.81	0.99	1.11	0.83	1.00	0.85	1	1	1	0.58	0	0.02
41	0.94	0.74	0.99	1.06	0.97	1.00	0.98	1	1	1	0.46	0	0.05
42	0.13	0.26	0.00	-0.16	0.15	0.00	0.14	0	0	0	0.67	1	0.01
43	0.61	0.60	0.63	0.64	0.52	0.56	0.59	1	1	1	0.49	1	0.00
44	0.78	0.59	0.82	0.88	0.86	0.91	0.96	1	1	1	0.50	1	0.16
45	0.92	1.00	0.96	1.00	0.00	0.01	0.84	0	0	1	0.41	1	0.58
46	0.80	0.85	0.84	0.89	0.34	0.47	0.66	0	0	1	0.47	1	0.71
47	0.75	0.71	0.77	0.80	0.59	0.66	0.74	1	1	1	0.44	0	0.19
48	0.85	0.65	0.87	0.90	0.90	0.93	0.96	1	1	1	0.38	0	0.13
49	0.91	0.81	0.99	1.10	0.85	1.00	0.87	1	1	1	0.56	0	0.14
50	0.80	0.96	0.88	0.98	0.03	0.09	0.75	0	0	1	0.60	1	0.07
51	0.80	0.85	0.84	0.90	0.31	0.47	0.71	0	0	1	0.51	0	0.03
52	0.91	0.72	0.99	1.08	0.94	1.00	0.96	1	1	1	0.55	0	0.10
53	0.91	0.72	1.00	1.11	0.94	1.00	0.94	1	1	1	0.58	0	0.27
Mean	0.57	0.54	0.58	0.60	0.58	0.59	0.60	0.64	0.66	0.60	0.52	0.55	0.14
S.D.	0.30	0.28	0.35	0.42	0.32	0.33	0.31	0.48	0.47	0.49	0.07	0.50	0.22

Note: This table shows bivariate price discovery estimates of HK shares in each pair (lnP_{SH}, lnP_{HK}) for the sample period 2010-01-01 to 2020-01-01. Column "ρ" stands for the correlation coefficient between VECM residuals. Column "Rank" stands for the cointegration rank between each pair of cross-listed shares based on the Johansen cointegration test at the 10% significance level. Column "LR test" stands for the p-value of the likelihood ratio test of the restriction that the cointegration vector is $(1, -1)'$. All price discovery measures are calculated based on OLS estimates of VECM with the restricted cointegrating vector $(1, -1)$.

Table 7: Price Discovery Estimates for HK shares: Before and After SH Stock Connect

	53 Firms		**29 coint. Firms**		**10 unitary coint. Firms**		**19 non-unitary coint. Firms**	
	Pre	After	Pre	After	Pre	After	Pre	After
IS_{HK}	0.65	0.65	0.60	0.68	0.58	0.70	0.60	0.67
	[0.29]	[0.25]	[0.28]	[0.24]	[0.29]	[0.24]	[0.29]	[0.25]
CS_{HK}	0.58	0.67	0.54	0.70	0.55	0.72	0.54	0.69
	[0.27]	[0.26]	[0.28]	[0.27]	[0.30]	[0.29]	[0.28]	[0.27]
MIS_{HK}	0.67	0.68	0.61	0.72	0.59	0.74	0.62	0.70
	[0.33]	[0.31]	[0.32]	[0.29]	[0.33]	[0.29]	[0.33]	[0.30]
PDS_{HK}	0.69	0.72	0.63	0.76	0.60	0.80	0.64	0.74
	[0.38]	[0.38]	[0.38]	[0.36]	[0.37]	[0.36]	[0.39]	[0.37]
ILS_{HK}	0.63	0.46	0.61	0.44	0.57	0.46	0.63	0.43
	[0.29]	[0.30]	[0.28]	[0.29]	[0.28]	[0.33]	[0.29]	[0.27]
$MILS_{HK}$	0.67	0.52	0.64	0.52	0.61	0.54	0.66	0.51
	[0.29]	[0.33]	[0.27]	[0.33]	[0.27]	[0.37]	[0.28]	[0.31]
$PILS_{HK}$	0.70	0.59	0.65	0.61	0.61	0.64	0.67	0.59
	[0.28]	[0.25]	[0.28]	[0.22]	[0.30]	[0.20]	[0.28]	[0.23]
ILI_{HK}	0.70	0.42	0.69	0.41	0.60	0.50	0.74	0.37
	[0.46]	[0.50]	[0.47]	[0.50]	[0.52]	[0.53]	[0.45]	[0.50]
$MILI_{HK}$	0.79	0.49	0.79	0.52	0.70	0.60	0.84	0.47
	[0.41]	[0.50]	[0.41]	[0.51]	[0.48]	[0.52]	[0.37]	[0.51]
$PILI_{HK}$	0.75	0.66	0.69	0.72	0.60	0.70	0.74	0.74
	[0.43]	[0.48]	[0.47]	[0.45]	[0.52]	[0.48]	[0.45]	[0.45]
Coint. Rank	0.46	0.57	0.47	0.58	0.45	0.58	0.49	0.57
	[0.09]	[0.07]	[0.10]	[0.08]	[0.09]	[0.06]	[0.11]	[0.09]
ρ	0.47	0.55	0.59	0.62	0.60	0.50	0.58	0.68
	[0.50]	[0.50]	[0.50]	[0.49]	[0.52]	[0.53]	[0.51]	[0.48]
LR test	0.23	0.28	0.24	0.32	0.27	0.35	0.22	0.30
	[0.29]	[0.31]	[0.29]	[0.32]	[0.30]	[0.32]	[0.30]	[0.32]

Note: This table shows sample averages of bivariate price discovery estimates of HK shares in each pair (lnP_{SH}, lnP_{HK}) for the sample period of 2010-01-01 to 2014-11-16 (Pre) and in the sample period of 2014-11-17 to 2020-01-01 (After). Numbers in square brackets are standard deviations across firms. 53 firms include all the firms as listed in Table 5. 29 cointegrated firms include firms that exhibit a cointegration relation over the full sample period 2010-01-01 to 2020-01-01, and 10 unitary cointegrated firms include cointegrated firms with a cointegration vector $(1, -1)$. Estimates for 19 cointegrated firms with a non-unitary cointegration vector are displayed in the last two columns. Row "Coint. Rank" stands for the sample averaged number of the cointegration relation between each pair of cross-listed shares based on the Johansen cointegration test. Row "ρ" stands for the sample average of the correlation coefficient between VECM residuals. Row "LR test" stands for the sample average of the p-value of the likelihood ratio test of the restriction that the cointegration vector is $(1, -1)$. All price discovery measures are calculated based on OLS estimates of VECM with the restricted cointegrating vector $(1, -1)$.

Internet Appendix to

Improving Price Leadership Share for Measuring Price Discovery

A1 Bivariate formula for IS

For the bivariate case, it is easy to show that the Cholesky decomposition of the variance-covariance matrix $\mathbf{\Omega} = \mathbf{F}\mathbf{F}'$ yields:

$$\mathbf{F} = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sigma_2(1-\rho^2)^{1/2} \end{pmatrix}, \tag{A.1}$$

and hence according to Eq. (7), the IS measures take the following values:

$$\text{IS}_1 = \frac{\psi_1^2\sigma_1^2 + \rho^2\psi_2^2\sigma_2^2 + 2\psi_1\psi_2\sigma_{12}}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}}, \quad \text{IS}_2 = \frac{\psi_2^2\sigma_2^2(1-\rho^2)}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}}. \tag{A.2}$$

Empirically, mid-points of IS estimates from all possible permutations of the price vector are used for the final leadership estimates, which can be shown to be:

$$\bar{\text{IS}}_1 = \frac{\psi_1^2\sigma_1^2 + \psi_1\psi_2\sigma_{12} + \rho^2(\psi_2^2\sigma_2^2 - \psi_1^2\sigma_1^2)/2}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}}, \quad \bar{\text{IS}}_2 = \frac{\psi_2^2\sigma_2^2 + \psi_1\psi_2\sigma_{12} + \rho^2(\psi_1^2\sigma_1^2 - \psi_2^2\sigma_2^2)/2}{\boldsymbol{\psi}'\mathbf{\Omega}\boldsymbol{\psi}}. \tag{A.3}$$

A2 Alternative Derivation of MIS

To illustrate the alternative derivation of the MIS measure of Lien and Shrestha (2009), we repeat the symbol definitions in Lien and Shrestha (2009) as the following:

- let $\mathbf{\Phi}$ denote the innovation correlation matrix,
- let $\mathbf{\Lambda}$ represent the diagonal matrix with diagonal elements being the eigenvalues of the correlation matrix $\mathbf{\Phi}$,

- let $\mathbf{G}$ denote a matrix with columns being the corresponding eigenvectors,

- let $\mathbf{V}$ denote a diagonal matrix containing the innovation standard deviations on the diagonal; i.e., $\mathbf{V} = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$.

By definition, the correlation matrix $\mathbf{\Phi}$ is a real symmetric matrix (and can prove that it is a positive definite matrix). By the spectral theorem, we know that the eigenvalues (diagonal elements of $\mathbf{\Lambda}$) of $\mathbf{\Phi}$ are real and the eigenvectors (columns of $\mathbf{G}$) can be chosen real and orthonormal. Hence, the spectral theorem leads to the following decomposition:

$$\mathbf{\Phi G} = \mathbf{G\Lambda}, \tag{A.4}$$

$$\mathbf{\Phi} = \mathbf{G\Lambda G}^{-1} = \mathbf{G\Lambda G}', \tag{A.5}$$

where $\mathbf{G}^{-1}$ denote the inverse and $\mathbf{G}'$ denote the transpose of $\mathbf{G}$, respectively. By the spectral decomposition of real symmetric matrices, we have $\mathbf{G}'\mathbf{G} = \mathbf{I}_n$ and $\mathbf{G}^{-1} = \mathbf{G}'$. Hence, the second equal sign in the above equation holds. Note that Eq.(A.5) is the spectral decomposition of the correlation matrix $\mathbf{\Phi}$, even though Lien and Shrestha (2009) do not recognize this fact.

In order to get order-invariant price discovery measures, we further rewrite the above spectral decomposition of $\mathbf{\Phi}$ as the following:

$$\begin{aligned}
\mathbf{\Phi} &= \mathbf{G\Lambda G}', \\
&= \mathbf{G\Lambda}^{1/2}\mathbf{\Lambda}^{1/2}\mathbf{G}', \\
&= \mathbf{G\Lambda}^{1/2}\mathbf{G}'\mathbf{G\Lambda}^{1/2}\mathbf{G}', \\
&= \mathbf{MM}', \qquad (A.6)
\end{aligned}$$

where $\mathbf{M} = \mathbf{G\Lambda}^{1/2}\mathbf{G}'$. Note that in this new decomposition, the matrix $\mathbf{M}$ is a symmetric matrix (hence $\mathbf{M} = \mathbf{M}'$). The reason why one needs to decompose the correlation matrix $\mathbf{\Phi}$ into the product of a symmetric matrix ($\mathbf{M}$) and its transpose is for the order-invariant requirement of the final information share measures. For example, in the Cholesky decomposition $\mathbf{\Phi} = \mathbf{LL}'$ with $\mathbf{L}$ being a lower triangular matrix. Since the matrix $\mathbf{L}$ is not symmetric, the resulting IS measures depend on the ordering of the price series entering the price vector.

As to the decomposition of the variance matrix, using the decomposition of $\mathbf{\Phi}$ in Eq.(A.6), we have:

$$
\begin{aligned}
\mathbf{\Omega} &= \mathbf{V}\mathbf{\Phi}\mathbf{V}', \\
&= \mathbf{V}\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'\mathbf{V}', \\
&= \mathbf{V}\mathbf{M}\mathbf{M}'\mathbf{V}', \qquad \text{(A.7)}
\end{aligned}
$$

which leads to the following factorization:

$$\mathbf{\Omega} = \mathbf{F}^*\mathbf{F}^{*\prime}, \quad \text{with} \quad \mathbf{F}^* = \mathbf{V}\mathbf{M} = \mathbf{V}\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}'. \qquad \text{(A.8)}$$

The original factorization in Lien and Shrestha (2009) looks slightly different from the above factorization. In Lien and Shrestha (2009), the $\mathbf{F}$ matrix takes the form $\mathbf{F}^* = [\mathbf{G}\mathbf{\Lambda}^{-1/2}\mathbf{G}'\mathbf{V}^{-1}]^{-1}$. We can show that their factorization is equivalent to the above factorization in Eq. (A.8). To illustrate, the F matrix define in Lien and Shrestha (2009) can be re-written as:

$$
\begin{aligned}
\mathbf{F}^* &= [\mathbf{G}\mathbf{\Lambda}^{-1/2}\mathbf{G}'\mathbf{V}^{-1}]^{-1}, \\
&= \mathbf{V}(\mathbf{G}')^{-1}\mathbf{\Lambda}^{1/2}\mathbf{G}^{-1}, \\
&= \mathbf{V}\mathbf{G}\mathbf{\Lambda}^{1/2}\mathbf{G}', \qquad \text{(A.9)}
\end{aligned}
$$

by noting the relation that $\mathbf{G}'\mathbf{G} = \mathbf{I}_n$. Hence, the original factorization in Lien and Shrestha (2009) is equivalent to the factorization in Eq.(A.8). However, these multiple inverse calculations in the original factorization of Lien and Shrestha (2009) are difficult to calculate, especially so when these reduced-form residuals are highly correlated. Hence, we propose to use the factorization in Eq.(A.8) instead.

Moreover, for the bivariate case, we can show that

$$
\mathbf{\Phi} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{\text{G}} \underbrace{\begin{pmatrix} 1+\rho & 0 \\ 0 & 1+\rho \end{pmatrix}}_{\Lambda} \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{G'},
$$

$$
= \underbrace{\begin{pmatrix} 0.5(\sqrt{1+\rho}+\sqrt{1-\rho}) & 0.5(\sqrt{1+\rho}-\sqrt{1-\rho}) \\ 0.5(\sqrt{1+\rho}-\sqrt{1-\rho}) & 0.5(\sqrt{1+\rho}+\sqrt{1-\rho}) \end{pmatrix}}_{\text{M}} \underbrace{\begin{pmatrix} 0.5(\sqrt{1+\rho}+\sqrt{1-\rho}) & 0.5(\sqrt{1+\rho}-\sqrt{1-\rho}) \\ 0.5(\sqrt{1+\rho}-\sqrt{1-\rho}) & 0.5(\sqrt{1+\rho}+\sqrt{1-\rho}) \end{pmatrix}}_{M'}.
$$

and

$$\mathbf{F}^* = \mathbf{VM} = \begin{pmatrix} 0.5(\sqrt{1+\rho}+\sqrt{1-\rho})\sigma_1 & 0.5(\sqrt{1+\rho}-\sqrt{1-\rho})\sigma_1 \\ 0.5(\sqrt{1+\rho}-\sqrt{1-\rho})\sigma_2 & 0.5(\sqrt{1+\rho}+\sqrt{1-\rho})\sigma_2 \end{pmatrix}.$$

Hence, the bivariate formula for MIS becomes:

$$MIS_i = \frac{\psi_i^2\sigma_i^2(1+\sqrt{1-\rho^2})/2 + \psi_j^2\sigma_j^2(1-\sqrt{1-\rho^2})/2 + \psi_i\psi_j\sigma_{i,j}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}. \tag{A.10}$$

As the above expression shows, the MIS measure decomposes the variance contribution to each market more equally than the IS measure does (as in Eq.(A.2)) and coincides with the IS measure when $\rho = 0$.

We can also provide a structural representation for the MIS measure for the bivariate case. By substituting relevant terms in the variance-covariance matrix into the bivariate expression of MIS in Eq.(A.10), we get:

$$\begin{aligned} \text{MIS}_1 &= \frac{1}{2} + \frac{1}{2}\frac{d_{0,1}^P d_{0,2}^T + d_{0,1}^T d_{0,2}^P}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}\sqrt{1-\rho^2}, \\ \text{MIS}_2 &= \frac{1}{2} - \frac{1}{2}\frac{d_{0,1}^P d_{0,2}^T + d_{0,1}^T d_{0,2}^P}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}\sqrt{1-\rho^2}. \end{aligned} \tag{A.11}$$

As we can see, because of the complex (spectral) variance decomposition of the MIS, the structural representation of MIS is a complex combination of contemporaneous responses to both permanent and transitory shocks.

A3 Multivariate Price Discovery Measures

Let $\mathbf{p_t} = (p_{1t}, p_{2t}, ..., p_{nt})'$ denote a vector of log prices for n assets that are closely related by arbitrage, of which each price series is intergated of order 1, or I(1). Assume that these price series are cointegrated with the following $(n-1)\times n$ cointegrating matrix:

$$\boldsymbol{\beta}' = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{pmatrix}. \tag{A.12}$$

Then, the multivariate reduced-form VECM is given as:

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{\Gamma}_j \Delta \mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{A.13}$$

where $\boldsymbol{\alpha}$ is the matrix of error correction coeffiicents, $\boldsymbol{\Gamma}_j$ $(i = 1, \dots, k)$ are the short-run coefficient matrices, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{nt})'$ is the vector of reduced-form VECM residuals with $E[\boldsymbol{\varepsilon}_t] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Omega}$.

The VMA and the integrated form for the multivariate VECM model take the same forms of Eq.(3) and Eq.(4), respectively. Denote the common row of $\boldsymbol{\Psi}(1)$ as $\boldsymbol{\psi} = (\psi_1, \psi_2, ..., \psi_n)'$ and define the permanent innovation as:

$$\eta_t^P = \boldsymbol{\psi}'\boldsymbol{\varepsilon}_t = \psi_1\varepsilon_{1t} + \psi_2\varepsilon_{2t} + \cdots + \psi_n\varepsilon_{nt}. \tag{A.14}$$

Given the Cholesky decomposition of $\boldsymbol{\Omega} = \mathbf{F}\mathbf{F}'$, with $\mathbf{F}$ being a lower triangular $n \times n$ matrix, the IS measure for the i^{th} market is defined as:

$$\text{IS}_i = \frac{([\boldsymbol{\psi}'\mathbf{F}]_i)^2}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}. \tag{A.15}$$

where $[\boldsymbol{\psi}'\mathbf{F}]_i$ is the i^{th} element of the row matrix $\boldsymbol{\psi}'\mathbf{F}$. The CS measure for the i^{th} market is given as:

$$\text{CS}_i = \frac{\psi_i}{\psi_1 + \psi_2 + \cdots + \psi_n}. \tag{A.16}$$

The multivariate representations for MIS and PDS are the same as in Eq.(11) and Eq.(14).

It is hard to define a multivariate IL measure in that the ratio of one market's measures over those of the other market depends on the choice of the competing market. However, one can still define the share version of the IL measure, i.e., the ILS measure. Patel et al. (2020) defined the following β measure for each market:

$$\beta_i^{IS} = \frac{\mathrm{IS}_i}{\mathrm{CS}_i}, \tag{A.17}$$

and then define the Information Leadership Shares (ILS) as:

$$\mathrm{ILS}_i = \frac{\beta_i^{IS}}{\beta_1^{IS} + \beta_2^{IS} + \cdots + \beta_n^{IS}}. \tag{A.18}$$

The multivariate binary indicator ILI is then defined as:

$$\mathrm{ILI}_i = \begin{cases} 1, & \text{if } \mathrm{ILS}_i > \mathrm{ILS}_k \quad \forall \quad k \neq i \\ 0, & \text{otherwise.} \end{cases} \tag{A.19}$$

We follow Patel et al. (2020) to define the multivariate Modified Information Leadership (MILS) and its binary indicator MILI by replacing the IS measure in Eq.(A.17) with the MIS measure of Lien and Shrestha (2009) defined in Eq.(11). Multivariate definitions for PILS and PILI are defined by replacing the IS measure with the PDS measure of Sultan and Zivot (2015) in Eq.(14).

A4 Simulation Evidence With Different Data frequency

In this section, we further examine how data frequency affect the accuracy of our price discovery measures. We generate data from the partial adjustment model with the sample size of 21600, mimicking the 1 second-level data frequency. Then, we re-sample the 1-second data at the 5-second (5s), 10-second (10s), 15-second (15s), 30-second (30s), 1-minute (1min), and 5-minute (5min) intervals, respectively. With each re-sampled data, we re-estimate the VECM model with the restricted cointegrating vector $(1, -1)$ and calculate price discovery estimates. Results are summarized in Table A1.

[Insert Table A1 about here.]

The parameter settings in Table A1 are the same as those in Panel A of Table 1, with equal transitory shock responses and uncorrelated reduced-form errors. Panel A of Table A1 repeats

Panel A of Table 1 with 1s level data, while Panel B to Panel G exhibit results for 5s to 5min data, respectively.

As the results show, IS, MIS and PDS now yield different estimates for data frequency lower than 1s, even though the original data generating process involves uncorrelated reduced-form errors. Data sampled at more coarse frequency render the variance-variance matrix of the reduced-forms errors no longer diagonal even for the case with originally uncorrelated 1s level errors. As the data frequency decreases from 1s to 5min (from Panel A to Panel G), we see that IS estimates become less informative as a leader identifier. For the case with $\delta_1 = 0.9$, IS estimate of Market 1 decreases from 0.90 to 0.51 when data frequency decreases from 1s to 5min. We find the same deterioration of the MIS's performance as data become coarse. The PDS measure seems to perform much better than these two measures. For 5min-level data, PDS can correctly identify Market 1 as the leader for 70% of the time when $\delta_1 = 0.7$, compared with 50% (52%) estimates of the IS (MIS) measure.

As these information share measures (IS, MIS, and PDS) exhibit sensitiveness to data frequency, we find that CS measure is also sensitive to data frequency, but in a non-monotonic pattern. As the 1s level, CS estimates take values very close to their theoretical values as shown from our structural analysis. However, as data frequency decrease, for the case with $\delta_1 = 0.9$, we see that CS estimates of Market 1 firstly increase, peak at 15s level data, and then decrease. As a result, the combined measures ILS and MILS (and their binary indicator ILI and MILI) behave very poorly in identifying the correct leader for data frequency lower than 1s. For the 5s level data, the performance of MILI seems a little better than ILI. However, for even lower frequency data, these two measures' performances are unacceptable as they always pick the follower as the leader for the majority of the time.

Interestingly, PILS (and its binary indicator PILI), the measure based on PDS and CS, is able to provide accurate identification of the leadership up to 30s level data. For the 30s level data, PILI can correctly identify Market 1 as the leader for 96% of the time when $\delta_1 = 0.6$, and evenly pick either market as the leader when $\delta_1 = 0.5$. The identification accuracy pf PDS and PILI decreases a little bit when it comes to the 1min level data. Still, PILI can correctly identify Market 1 as the leader for 83% of the time when $\delta_1 = 0.6$, and evenly pick either market as the leader when $\delta_1 = 0.5$. For 5min level data, PILI can correctly pick the leader for above half the time (56%) when $\delta_1 = 0.6$. When Market 1 is leading the other market with $\delta_1 > 0.6$, we can see that PILI provides much better leadership estimates in comparison to ILI and MILI.

To sum up, as price discovery measures are sensitive to data frequency, PDS and the combined measure PILS (and its binary indicator PILI) can provide more robust leadership estimates with the data sampled at lower frequencies. As IS and MIS become less informative as data become rougher, PDS seems to be more resilient to data frequency than the other two measures. Performances of combined indicators ILS and MILS (and their binary indicator ILI and MILI) can be very poor because of the added sensitiveness of CS measures to data frequency.

Even though we can not provide a structural explanation for this superiority of PILS (and PILI) under different data frequency, we conjecture that the simple additive decomposition of the variance-covariance matrix of PDS and its straightforward structural explanation may contribute to its frequency resiliency. Therefore, we propose to use PILS (and its binary indicator PILI) as the price leadership measure for empirical investigations. Also, theoretical investigations on data frequency merit further studies which are beyond this paper's scope.

Table A1: Data Frequency: Equal Transitory Responses and Uncorrelated Residuals

This table reports price discovery measure estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta_t} = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $\delta_2 = 1 - \delta_1$, $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$, $\sigma_P^2 = 1$, $\sigma_T^2 = \frac{\delta_1 \delta_2}{-b_{0,1}^T b_{0,2}^T}$. We simulate 1000 samples of 21600 observations.

Panel A: Frequency=1s

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.90	0.10	0.50	0.50	0.90	0.10	0.90	0.10	0.99	0.01	0.99	0.01	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00
0.80	0.80	0.20	0.50	0.50	0.80	0.20	0.80	0.20	0.94	0.06	0.94	0.06	0.94	0.06	1.00	0.00	1.00	0.00	1.00	0.00
0.70	0.70	0.30	0.50	0.50	0.70	0.30	0.70	0.30	0.84	0.16	0.84	0.16	0.84	0.16	1.00	0.00	1.00	0.00	1.00	0.00
0.60	0.60	0.40	0.50	0.50	0.60	0.40	0.60	0.40	0.69	0.31	0.69	0.31	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.51	0.49	0.51	0.49	0.51
0.40	0.40	0.60	0.50	0.50	0.40	0.60	0.40	0.60	0.31	0.69	0.31	0.69	0.31	0.69	0.00	1.00	0.00	1.00	0.00	1.00
0.30	0.30	0.70	0.50	0.50	0.30	0.70	0.30	0.70	0.16	0.84	0.16	0.84	0.16	0.84	0.00	1.00	0.00	1.00	0.00	1.00
0.20	0.20	0.80	0.50	0.50	0.20	0.80	0.20	0.80	0.06	0.94	0.06	0.94	0.06	0.94	0.00	1.00	0.00	1.00	0.00	1.00
0.10	0.10	0.90	0.50	0.50	0.10	0.90	0.10	0.90	0.01	0.99	0.01	0.99	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00

Panel B: Frequency=5s

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.75	0.25	0.78	0.22	0.82	0.18	0.93	0.07	0.41	0.59	0.63	0.37	0.93	0.07	0.22	0.78	0.93	0.07	1.00	0.00
0.80	0.64	0.36	0.71	0.29	0.72	0.28	0.83	0.17	0.34	0.66	0.51	0.49	0.80	0.20	0.00	1.00	0.60	0.40	1.00	0.00
0.70	0.58	0.42	0.65	0.35	0.64	0.36	0.73	0.27	0.37	0.63	0.48	0.52	0.68	0.32	0.00	1.00	0.28	0.72	1.00	0.00
0.60	0.54	0.46	0.57	0.43	0.57	0.43	0.62	0.38	0.43	0.57	0.48	0.52	0.59	0.41	0.06	0.94	0.28	0.71	1.00	0.00
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.52	0.48	0.52	0.48	0.49	0.51
0.40	0.46	0.54	0.42	0.58	0.43	0.57	0.38	0.62	0.58	0.42	0.52	0.48	0.41	0.59	0.95	0.05	0.74	0.26	0.00	1.00
0.30	0.42	0.58	0.35	0.65	0.36	0.64	0.27	0.73	0.64	0.36	0.53	0.47	0.31	0.69	1.00	0.00	0.74	0.26	0.00	1.00
0.20	0.36	0.64	0.28	0.72	0.28	0.72	0.16	0.84	0.67	0.33	0.50	0.50	0.20	0.80	1.00	0.00	0.43	0.57	0.00	1.00
0.10	0.25	0.75	0.22	0.78	0.17	0.83	0.07	0.93	0.60	0.40	0.37	0.63	0.07	0.93	0.80	0.20	0.10	0.90	0.00	1.00

Panel C: Frequency=10s

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.64	0.36	0.86	0.14	0.75	0.25	0.94	0.06	0.09	0.91	0.19	0.81	0.85	0.15	0.00	1.00	0.01	0.99	1.00	0.00
0.80	0.58	0.42	0.81	0.19	0.67	0.33	0.86	0.14	0.11	0.89	0.19	0.81	0.69	0.31	0.00	1.00	0.00	1.00	1.00	0.00
0.70	0.55	0.45	0.72	0.28	0.61	0.39	0.76	0.24	0.18	0.82	0.26	0.74	0.60	0.40	0.00	1.00	0.01	0.99	1.00	0.00
0.60	0.52	0.48	0.62	0.38	0.55	0.45	0.64	0.36	0.32	0.68	0.37	0.63	0.54	0.46	0.07	0.93	0.08	0.92	1.00	0.00
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.52	0.48	0.47	0.52
0.40	0.48	0.52	0.38	0.62	0.45	0.55	0.36	0.64	0.68	0.32	0.63	0.37	0.46	0.54	0.94	0.06	0.92	0.08	0.00	1.00
0.30	0.45	0.55	0.27	0.73	0.39	0.61	0.24	0.76	0.82	0.18	0.74	0.26	0.40	0.60	1.00	0.00	0.99	0.00	0.00	1.00
0.20	0.42	0.58	0.19	0.81	0.33	0.67	0.13	0.87	0.90	0.10	0.81	0.19	0.31	0.69	1.00	0.00	1.00	0.00	0.00	1.00
0.10	0.36	0.64	0.13	0.87	0.25	0.75	0.06	0.94	0.92	0.08	0.83	0.17	0.15	0.85	1.00	0.00	1.00	0.00	0.00	1.00

Panel D: Frequency=15s

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.60	0.40	0.89	0.11	0.71	0.29	0.95	0.05	0.05	0.95	0.11	0.89	0.78	0.22	0.00	1.00	0.00	1.00	1.00	0.00
0.80	0.56	0.44	0.84	0.16	0.65	0.35	0.88	0.12	0.07	0.93	0.12	0.88	0.63	0.37	0.00	1.00	0.00	1.00	1.00	0.00
0.70	0.53	0.47	0.75	0.25	0.59	0.41	0.77	0.23	0.15	0.85	0.20	0.80	0.57	0.43	0.01	0.99	0.01	0.99	1.00	0.00
0.60	0.51	0.49	0.63	0.37	0.54	0.46	0.64	0.36	0.30	0.70	0.34	0.66	0.53	0.47	0.13	0.87	0.14	0.86	1.00	0.00
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.50	0.50	0.50	0.50	0.51	0.49	0.51	0.49	0.49	0.51
0.40	0.49	0.51	0.36	0.64	0.45	0.55	0.35	0.65	0.71	0.29	0.67	0.33	0.47	0.53	0.88	0.12	0.87	0.13	0.00	1.00
0.30	0.47	0.53	0.24	0.76	0.41	0.59	0.22	0.78	0.86	0.14	0.81	0.19	0.43	0.57	0.99	0.01	0.99	0.01	0.00	1.00
0.20	0.44	0.56	0.15	0.85	0.35	0.65	0.12	0.88	0.93	0.07	0.88	0.12	0.36	0.64	1.00	0.00	1.00	0.00	0.00	1.00
0.10	0.40	0.60	0.11	0.89	0.28	0.72	0.05	0.95	0.95	0.05	0.90	0.10	0.22	0.78	1.00	0.00	1.00	0.00	0.00	1.00

Panel E: Frequency=30s

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.56	0.44	0.87	0.13	0.67	0.33	0.96	0.04	0.06	0.94	0.11	0.89	0.66	0.34	0.00	1.00	0.01	0.99	1.00	0.00
0.80	0.53	0.47	0.82	0.18	0.61	0.39	0.89	0.11	0.09	0.91	0.13	0.87	0.57	0.43	0.02	0.98	0.03	0.97	1.00	0.00
0.70	0.52	0.48	0.74	0.26	0.57	0.43	0.79	0.21	0.18	0.82	0.21	0.79	0.53	0.47	0.12	0.88	0.12	0.88	1.00	0.00
0.60	0.51	0.49	0.63	0.37	0.53	0.47	0.65	0.35	0.32	0.68	0.34	0.66	0.51	0.49	0.27	0.73	0.27	0.73	0.96	0.04
0.50	0.50	0.50	0.49	0.51	0.50	0.50	0.49	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.49
0.40	0.49	0.51	0.36	0.64	0.47	0.53	0.34	0.66	0.68	0.32	0.67	0.33	0.49	0.51	0.73	0.27	0.73	0.27	0.02	0.98
0.30	0.48	0.52	0.25	0.75	0.43	0.57	0.21	0.79	0.82	0.18	0.80	0.20	0.47	0.53	0.90	0.10	0.90	0.10	0.00	1.00
0.20	0.47	0.53	0.17	0.83	0.39	0.61	0.10	0.90	0.91	0.09	0.87	0.13	0.43	0.57	0.97	0.03	0.97	0.03	0.00	1.00
0.10	0.44	0.56	0.13	0.87	0.33	0.67	0.04	0.96	0.95	0.05	0.89	0.11	0.34	0.66	1.00	0.00	0.99	0.01	0.00	1.00

Panel F: Frequency=1min

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.53	0.47	0.81	0.19	0.63	0.37	0.96	0.04	0.11	0.89	0.18	0.82	0.58	0.42	0.05	0.95	0.09	0.91	1.00	0.00
0.80	0.52	0.48	0.74	0.26	0.58	0.42	0.90	0.10	0.20	0.80	0.24	0.76	0.53	0.47	0.15	0.85	0.18	0.82	1.00	0.00
0.70	0.51	0.49	0.67	0.33	0.55	0.45	0.81	0.19	0.28	0.72	0.32	0.68	0.52	0.48	0.25	0.75	0.27	0.73	0.97	0.03
0.60	0.50	0.50	0.59	0.41	0.53	0.47	0.67	0.33	0.38	0.62	0.40	0.60	0.51	0.49	0.34	0.66	0.35	0.65	0.83	0.17
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.49	0.51	0.49	0.51	0.50	0.50	0.49	0.51	0.49	0.51	0.53	0.47
0.40	0.50	0.50	0.42	0.58	0.48	0.52	0.36	0.64	0.61	0.39	0.60	0.40	0.49	0.51	0.61	0.39	0.60	0.40	0.16	0.84
0.30	0.49	0.51	0.34	0.66	0.45	0.55	0.22	0.78	0.71	0.29	0.68	0.32	0.48	0.52	0.74	0.26	0.73	0.27	0.02	0.98
0.20	0.48	0.52	0.27	0.73	0.42	0.58	0.11	0.89	0.80	0.20	0.75	0.25	0.47	0.53	0.85	0.15	0.83	0.17	0.00	1.00
0.10	0.47	0.53	0.19	0.81	0.37	0.63	0.04	0.96	0.88	0.12	0.82	0.18	0.42	0.58	0.95	0.05	0.92	0.08	0.00	1.00

Panel G: Frequency=5min

	IS		CS		MIS		PDS		ILS		MILS		PILS		ILI		MILI		PILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.90	0.51	0.49	0.54	0.46	0.55	0.45	0.91	0.09	0.44	0.56	0.52	0.48	0.51	0.49	0.38	0.62	0.53	0.47	0.86	0.14
0.80	0.50	0.50	0.51	0.49	0.53	0.47	0.82	0.18	0.49	0.51	0.54	0.46	0.51	0.49	0.44	0.56	0.55	0.45	0.72	0.28
0.70	0.50	0.50	0.51	0.49	0.52	0.48	0.70	0.30	0.49	0.51	0.51	0.49	0.50	0.50	0.46	0.54	0.51	0.49	0.63	0.37
0.60	0.50	0.50	0.49	0.51	0.50	0.50	0.54	0.46	0.51	0.49	0.52	0.48	0.50	0.50	0.51	0.49	0.53	0.47	0.56	0.44
0.50	0.50	0.50	0.49	0.51	0.49	0.51	0.38	0.62	0.51	0.49	0.51	0.49	0.50	0.50	0.52	0.48	0.51	0.49	0.49	0.51
0.40	0.50	0.50	0.49	0.51	0.48	0.52	0.27	0.73	0.51	0.49	0.49	0.51	0.50	0.50	0.53	0.47	0.48	0.52	0.42	0.57
0.30	0.50	0.50	0.49	0.51	0.47	0.53	0.16	0.84	0.51	0.49	0.47	0.53	0.50	0.50	0.55	0.45	0.46	0.54	0.33	0.67
0.20	0.50	0.50	0.48	0.52	0.46	0.54	0.07	0.93	0.53	0.47	0.46	0.54	0.49	0.51	0.58	0.42	0.45	0.55	0.25	0.75
0.10	0.49	0.51	0.45	0.55	0.44	0.56	0.06	0.94	0.57	0.43	0.49	0.51	0.49	0.51	0.63	0.37	0.47	0.52	0.14	0.86

Notes: Numbers shown are the averages of price discovery measure estimates of 1000 samples. For each sample, the sample size is set as $N = 21600$.

Table A2: Price Discovery Estimates for HK shares from 2010-01-01 to 2014-11-16

Stock No.	IS	CS	MIS	PDS	ILS	MILS	PILS	ILI	MILI	PILI	ρ	Rank	LR test
1	0.12	0.08	0.08	0.04	0.71	0.53	0.22	1	1	0	0.40	1	0.00
2	0.84	0.93	0.90	0.96	0.15	0.32	0.80	0	0	1	0.51	0	0.45
3	0.27	0.14	0.20	0.11	0.84	0.72	0.39	1	1	0	0.63	1	0.66
4	0.53	0.49	0.53	0.53	0.57	0.57	0.57	1	1	1	0.20	0	0.85
5	0.93	0.66	0.97	1.02	0.98	1.00	1.00	1	1	1	0.40	0	0.06
6	0.90	0.94	0.95	1.02	0.28	0.72	0.89	0	1	1	0.48	1	0.01
7	0.82	0.60	0.85	0.90	0.90	0.94	0.97	1	1	1	0.44	1	0.01
8	0.84	0.58	0.89	0.96	0.93	0.97	1.00	1	1	1	0.52	0	0.09
9	0.87	0.98	0.93	1.01	0.01	0.06	0.80	0	0	1	0.52	1	0.45
10	0.93	0.83	0.98	1.06	0.87	1.00	0.93	1	1	1	0.49	1	0.02
11	0.96	0.79	0.99	1.05	0.97	1.00	0.97	1	1	1	0.40	1	0.08
12	0.07	0.11	0.01	-0.05	0.24	0.01	0.13	0	0	0	0.47	1	0.00
13	0.85	0.91	0.94	1.05	0.24	0.71	0.79	0	1	1	0.60	0	0.18
14	0.30	0.24	0.26	0.22	0.66	0.56	0.43	1	1	0	0.56	1	0.00
15	0.86	0.98	0.91	0.99	0.02	0.06	0.85	0	0	1	0.51	1	0.03
16	0.90	0.92	0.92	0.96	0.39	0.58	0.84	0	1	1	0.37	0	0.14
17	0.75	0.59	0.80	0.86	0.81	0.89	0.95	1	1	1	0.56	0	0.07
18	0.13	0.00	0.07	0.00	1.00	1.00	0.25	1	1	0	0.50	1	0.42
19	0.94	0.75	0.99	1.07	0.96	1.00	0.96	1	1	1	0.49	0	0.48
20	0.91	0.71	0.99	1.10	0.94	1.00	0.95	1	1	1	0.57	0	0.41
21	0.57	0.54	0.58	0.60	0.55	0.58	0.61	1	1	1	0.55	1	0.00
22	0.15	0.31	0.10	0.05	0.13	0.06	0.01	0	0	0	0.46	1	0.00
23	0.63	0.51	0.64	0.66	0.73	0.75	0.78	1	1	1	0.44	1	0.00
24	0.94	0.69	0.98	1.04	0.98	1.00	0.99	1	1	1	0.42	1	0.01
25	0.74	0.65	0.76	0.79	0.71	0.76	0.80	1	1	1	0.39	0	0.73
26	0.79	0.78	0.84	0.88	0.55	0.68	0.82	1	1	1	0.47	0	0.89
27	0.25	0.15	0.20	0.15	0.76	0.66	0.47	1	1	0	0.53	1	0.07
28	0.19	0.33	0.15	0.12	0.19	0.12	0.07	0	0	0	0.45	0	0.05
29	0.48	0.43	0.48	0.48	0.59	0.58	0.59	1	1	1	0.38	0	0.05
30	0.84	0.88	0.89	0.94	0.36	0.56	0.84	0	1	1	0.47	1	0.09
31	0.22	0.36	0.19	0.15	0.20	0.14	0.09	0	0	0	0.44	0	0.04
32	0.86	0.92	0.94	1.05	0.23	0.70	0.79	0	1	1	0.59	0	0.01
33	0.87	0.61	0.90	0.93	0.94	0.97	0.99	1	1	1	0.40	0	0.02
34	0.94	0.73	0.98	1.02	0.97	1.00	1.00	1	1	1	0.39	0	0.01
35	0.83	0.54	0.85	0.88	0.94	0.96	0.98	1	1	1	0.38	0	0.23
36	0.47	0.40	0.47	0.46	0.63	0.62	0.62	1	1	1	0.36	0	0.93
37	0.35	0.31	0.32	0.27	0.61	0.53	0.42	1	1	0	0.59	0	0.02
38	0.09	0.04	0.03	-0.03	0.82	0.34	0.34	1	0	0	0.48	0	0.84
39	0.93	0.83	0.98	1.05	0.88	1.00	0.94	1	1	1	0.46	0	0.09
40	0.88	0.65	0.94	1.00	0.93	0.98	1.00	1	1	1	0.49	1	0.00
41	0.25	0.18	0.22	0.18	0.71	0.63	0.51	1	1	1	0.47	0	0.12
42	0.68	0.73	0.72	0.79	0.36	0.48	0.65	0	0	1	0.62	0	0.25
43	0.37	0.33	0.36	0.34	0.60	0.57	0.54	1	1	1	0.41	1	0.01
44	0.43	0.45	0.41	0.41	0.45	0.43	0.42	0	0	0	0.42	1	0.01
45	0.88	0.84	0.90	0.93	0.65	0.75	0.84	1	1	1	0.33	1	0.95
46	0.71	0.63	0.72	0.73	0.66	0.69	0.71	1	1	1	0.31	1	0.44
47	0.40	0.41	0.39	0.39	0.48	0.46	0.45	0	0	0	0.35	0	0.09
48	0.74	0.61	0.75	0.76	0.77	0.78	0.80	1	1	1	0.23	0	0.62
49	0.94	0.76	1.00	1.07	0.95	1.00	0.96	1	1	1	0.49	0	0.02
50	0.79	0.56	0.84	0.91	0.90	0.95	0.98	1	1	1	0.54	1	0.26
51	0.79	0.77	0.82	0.86	0.56	0.65	0.77	1	1	1	0.43	1	0.00
52	0.93	0.80	1.00	1.08	0.92	1.00	0.92	1	1	1	0.50	0	0.53
53	0.78	0.82	0.83	0.90	0.37	0.55	0.80	0	1	1	0.55	1	0.17
Mean	0.65	0.58	0.67	0.69	0.63	0.67	0.70	0.70	0.79	0.75	0.46	0.47	0.23
S.D.	0.29	0.27	0.33	0.38	0.29	0.29	0.28	0.46	0.41	0.43	0.09	0.50	0.29

Note: This table shows bivariate price discovery estimates of HK shares in each pair (lnP_{SH}, lnP_{HK}) for the sample period 2010-01-01 to 2014-11-16. Column "ρ" stands for the correlation coefficient between VECM residuals. Column "Rank" stands for the cointegration rank between each pair of cross-listed shares based on the Johansen cointegration test at the 10% significance level. Column "LR test" stands for the p-value of the likelihood ratio test of the restriction that the cointegration vector is $(1, -1)'$. All price discovery measures are calculated based on OLS estimates of VECM with the restricted cointegrating vector $(1, -1)$.

Table A3: Price Discovery Estimates for HK shares from 2014-11-17 to 2020-01-01

Stock No.	IS	CS	MIS	PDS	ILS	MILS	PILS	ILI	MILI	PILI	ρ	Rank	LR test
1	0.88	0.87	0.96	1.09	0.53	0.96	0.78	1	1	1	0.60	1	0.18
2	0.25	0.16	0.19	0.11	0.75	0.59	0.29	1	1	0	0.60	0	0.19
3	0.62	0.79	0.69	0.80	0.15	0.25	0.53	0	0	1	0.78	1	0.53
4	0.30	0.41	0.28	0.25	0.28	0.23	0.18	0	0	0	0.46	0	0.00
5	0.94	0.88	0.99	1.05	0.83	1.00	0.89	1	1	1	0.44	1	0.43
6	0.93	0.83	0.99	1.08	0.89	1.00	0.89	1	1	1	0.50	1	0.22
7	0.09	0.28	0.00	-0.11	0.06	0.00	0.06	0	0	0	0.58	1	0.00
8	0.83	0.91	0.88	0.95	0.19	0.35	0.73	0	0	1	0.50	0	0.04
9	0.84	0.96	0.91	0.97	0.05	0.13	0.61	0	0	1	0.51	1	0.23
10	0.75	0.81	0.80	0.84	0.32	0.44	0.60	0	0	1	0.51	1	0.02
11	0.80	0.87	0.85	0.90	0.25	0.39	0.62	0	0	1	0.50	1	0.70
12	0.21	0.41	0.14	0.05	0.13	0.05	0.01	0	0	0	0.59	0	0.02
13	0.77	0.92	0.84	0.93	0.08	0.17	0.56	0	0	1	0.60	0	0.26
14	0.66	0.68	0.69	0.72	0.45	0.51	0.59	0	1	1	0.52	0	0.94
15	0.78	0.99	0.87	1.01	0.00	0.00	0.72	0	0	1	0.67	1	0.03
16	0.67	0.70	0.70	0.74	0.44	0.51	0.60	0	1	1	0.53	0	0.09
17	0.27	0.15	0.19	0.10	0.80	0.64	0.27	1	1	0	0.65	0	0.38
18	0.75	0.81	0.80	0.85	0.33	0.46	0.65	0	0	1	0.54	0	0.87
19	0.54	0.60	0.54	0.55	0.38	0.39	0.40	0	0	0	0.53	0	0.03
20	0.12	0.13	0.04	-0.06	0.45	0.07	0.13	0	0	0	0.57	1	0.00
21	0.84	0.65	0.89	0.96	0.88	0.95	0.99	1	1	1	0.52	1	0.00
22	0.70	0.80	0.75	0.84	0.25	0.38	0.62	0	0	1	0.64	1	0.33
23	0.30	0.23	0.26	0.20	0.69	0.59	0.43	1	1	0	0.58	0	0.04
24	0.27	0.21	0.22	0.14	0.67	0.53	0.29	1	1	0	0.61	1	0.00
25	0.84	0.64	0.95	1.11	0.90	0.99	0.97	1	1	1	0.66	0	0.69
26	0.20	0.04	0.10	-0.03	0.97	0.86	0.30	1	1	0	0.67	0	0.02
27	0.43	0.42	0.41	0.39	0.53	0.49	0.45	1	0	0	0.59	0	0.05
28	0.80	0.97	0.88	0.98	0.02	0.05	0.68	0	0	1	0.61	1	0.54
29	0.67	0.70	0.69	0.72	0.44	0.49	0.55	0	0	1	0.46	1	0.00
30	0.88	0.82	0.98	1.12	0.72	1.00	0.80	1	1	1	0.62	0	0.31
31	0.87	0.94	0.95	1.04	0.19	0.64	0.72	0	1	1	0.56	0	0.93
32	0.77	0.97	0.87	1.02	0.01	0.04	0.70	0	0	1	0.69	1	0.01
33	0.86	0.95	0.91	0.97	0.11	0.25	0.76	0	0	1	0.48	1	0.15
34	0.93	0.80	0.99	1.07	0.92	1.00	0.93	1	1	1	0.49	1	0.01
35	0.78	0.81	0.81	0.86	0.39	0.51	0.66	0	1	1	0.48	1	0.35
36	0.94	0.84	1.00	1.06	0.91	1.00	0.91	1	1	1	0.45	1	0.88
37	0.89	0.87	0.98	1.10	0.57	0.98	0.73	1	1	1	0.60	1	0.82
38	0.40	0.47	0.37	0.34	0.36	0.31	0.25	0	0	0	0.61	0	0.04
39	0.33	0.30	0.29	0.23	0.56	0.47	0.34	1	0	0	0.59	0	0.00
40	0.64	0.74	0.68	0.73	0.28	0.37	0.50	0	0	0	0.64	0	0.64
41	0.77	0.78	0.81	0.85	0.47	0.58	0.71	0	1	1	0.47	1	0.64
42	0.38	0.33	0.33	0.26	0.62	0.51	0.34	1	1	0	0.72	1	0.76
43	0.87	0.91	0.96	1.06	0.33	0.84	0.76	0	1	1	0.58	1	0.00
44	0.77	0.96	0.86	0.97	0.02	0.07	0.68	0	0	1	0.66	1	0.27
45	0.92	0.75	0.97	1.04	0.94	1.00	0.99	1	1	1	0.46	1	0.00
46	0.76	0.91	0.83	0.91	0.08	0.17	0.49	0	0	0	0.61	1	0.16
47	0.58	0.56	0.60	0.61	0.54	0.57	0.60	1	1	1	0.52	1	0.01
48	0.91	0.70	0.98	1.06	0.95	1.00	0.98	1	1	1	0.51	0	0.46
49	0.70	0.81	0.75	0.82	0.24	0.34	0.53	0	0	1	0.61	0	0.77
50	0.87	0.81	0.98	1.12	0.72	0.99	0.82	1	1	1	0.63	0	0.49
51	0.46	0.48	0.45	0.44	0.46	0.44	0.42	0	0	0	0.56	1	0.34
52	0.83	0.65	0.90	0.99	0.88	0.96	1.00	1	1	1	0.57	0	0.01
53	0.41	0.47	0.38	0.35	0.37	0.32	0.26	0	0	0	0.61	0	0.01
Mean	0.65	0.67	0.68	0.72	0.46	0.52	0.59	0.42	0.49	0.66	0.57	0.55	0.28
S.D.	0.25	0.26	0.31	0.38	0.30	0.33	0.25	0.50	0.50	0.48	0.07	0.50	0.31

Note: This table shows bivariate price discovery estimates of HK shares in each pair (lnP_{SH}, lnP_{HK}) for the sample period 2014-11-16 to 2020-01-01. Column "ρ" stands for the correlation coefficient between VECM residuals. Column "Rank" stands for the cointegration rank between each pair of cross-listed shares based on the Johansen cointegration test at the 10% significance level. Column "LR test" stands for the p-value of the likelihood ratio test of the restriction that the cointegration vector is $(1, -1)'$. All price discovery measures are calculated based on OLS estimates of VECM with the restricted cointegrating vector $(1, -1)$.