# Rewarding success and failure: moral hazard and adverse selection in strategic experimentation[1]

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A principal hires an agent to learn about the cost of a project (experimentation) and then to execute it (production). The agent is privately informed about the probability that the cost is low, with the high-type agent more optimistic than the low type. Learning is modeled as a series of experiments, where the agent privately chooses his effort in each experiment. Due to the joint presence of adverse selection and moral hazard, even the low type can misrepresent his type, shirk during experimentation and earn a rent. In the optimal contract, the principal uses the length and outcome of experimentation as well as the timing of payments to induce effort and screen the agent. The principal may find it optimal to reward the low type after failure in experimentation. We provide sufficient conditions for both success and failure to be rewarded. We show that over-experimentation can be optimal as it reduces the asymmetric information after a series of failed experiments. We also consider whether it may be optimal to separate experimentation and production between two different agents. When the same agent works on both tasks, the principal can leverage the adverse selection rent to mitigate moral hazard. Therefore, integrating experimentation and production is optimal when adverse selection is severe.

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*"I haven't failed. I've just found 10,000 ways that don't work."*

Thomas Edison

## 1. Introduction

Many important tasks are organized into two stages: a preliminary stage of experimentation or learning followed by a production or implementation stage. For instance, the adoption of a new technology typically involves testing, a learning process, fraught with agency problems involving moral hazard and adverse selection[2], followed by production using the new technology. In this paper, we study such a two-stage problem where experimentation and production interact within a mixed model with both moral hazard and adverse selection.

We present a principal-agent model where a production stage follows a multi-period learning stage of strategic experimentation.[3] Each period of experimentation is subject to moral hazard by an agent who can work or shirk trying to learn the cost of the project. We define success as the agent uncovering "good news", i.e., finding out that the production cost is low. Success can only occur if the agent works. Adverse selection stems from the agent privately knowing the likelihood that the cost is low. The high-type agent is more optimistic than the low type that the cost is low. The principal offers a menu of contracts, one for each type, where the length of experimentation is endogenous. A contract specifies the number of periods of experimentation, and the payments and outputs depending on the outcomes of the experiment in each period. Both moral hazard and adverse selection raise the possibility of asymmetric beliefs between the principal and the agent in updating after each experiment, and we discuss each in turn.

[2] An example of testing a new technology is contract farming to test new seeds, which has been studied extensively in the context of developing countries (Barrett et al. (2012). Contracting firms are almost always relatively large processors, exporters, or supermarket chains that provide small farmers with new, potentially more productive seeds (Foster and Rosenzweig (1995) describe the adoption and profitability of high-yielding seed varieties associated with the Green Revolution). This environment is characterized by three key features (1) *learning* about the quality of new seeds, (2) *moral hazard* as farmers' effort and time devoted to cropping is not directly observed by the buyer, (3) and *adverse selection* because farmers may have better knowledge of local conditions (see e.g., Miyata et al. (2009), Jack et al. (2014) and Beaman et al. (2015).

[3] The exponential bandit model has been widely used as a canonical model of learning: see Bolton and Harris (1999) or Bergemann and Välimäki (2008).

If the agent works but fails to uncover good news, the expected cost of production increases. Unobserved shirking by the agent leads the principal to become more pessimistic than the agent about the true cost, creating a dynamic moral hazard rent during experimentation.[4] We define failure as the agent failing to uncover good news throughout the entire experimentation process. Once experimentation ends, either after success or failure, production takes place.

Due to the production stage, unobserved shirking during the experimentation stage pays off twice: first in the experimentation stage and then again in the production stage. We refer to this second rent due to unobserved shirking generated during the production stage as the "second moral hazard rent." In the production stage, the principal, unaware of the earlier shirking, overestimates the agent's production cost. Consequently, the production stage plays a key role since the asymmetry of beliefs between the principal and the agent creates a scope for information rent based on expected production cost.

Adverse selection generates another rent at the production stage due to asymmetric beliefs about expected production costs. Thus, adverse selection will also affect the required rent to induce effort in each period. This leads to a dynamic mixed model where both adverse selection and moral hazard affect beliefs in each period of experimentation as well as the expected production cost when experimentation fails. Depending on which adverse selection (truth-telling) constraints bind, we identify four cases. We focus on two polar cases: In Case (1), moral hazard is strong and neither adverse selection constraints bind. As can be expected, we find that the agent is rewarded, i.e., collects an information rent, only if he finds success in the experimentation stage as is true in many dynamic moral hazard models. In Case (2), adverse selection is strong and adverse selection constraints bind for both the high and low types. The first contribution of our paper is illustrated in this case. We find that rewarding failure can be optimal even in the presence of moral hazard. In our model, rewarding failure means that the agent is paid a rent even if he fails to uncover low production cost.

In our mixed model, the principal screens the types by rewarding the low type after failure but not the high type. A misreporting low-type agent can shirk during experimentation to become more optimistic than the principal about the expected cost of production and command an information rent. This leads to binding adverse selection constraints for both the high and low

[4] See, e.g., Bergemann and Hege (1998), Bergemann and Välimäki (2008), and Horner and Samuelson (2013).

types, which is not unusual for a mixed model. However, we find that rewarding failure is an effective screening tool for addressing the adverse selection problem when experimentation is highly accurate (i.e., the probability of success is high).[5] Failure is not only an indicator of shirking but also a more likely event for the low-type agent. As a result, in Case (2), when adverse selection is strong and both adverse selection constraints are binding, rewarding failure can be optimal for the low type but never for the high type. This finding contrasts with recent papers on mixed models, which have emphasized pooling in one-shot models without a production stage.[6]

To be sure, paying a screening rent to the low-type agent for failing to learn during experimentation is costly. To maintain the agent's incentives to work, the principal must appropriately increase rewards for success in each prior period. Consequently, the low-type agent is rewarded after both success and failure, but with a higher reward after success.

Besides the timing of rewards, the principal can also use the length of experimentation to provide incentives. In Case (1), the length of experimentation is used to only address moral hazard rent, and we obtain the standard result of under experimentation for both types. Case (2) is more interesting as the principal must screen *both* types. Here we find the unusual result that the low type may be asked to *over*-experiment to reduce the high type's rent. Increasing experimentation by the low type reduces the difference in the expected cost between the principal and a misreporting high type. This is because the misreporting high type does not shirk off the path, unlike the low type who may shirk off the path. For the high type, we find under-experimentation. The principal reduces the low-type's rent by shortening the high type's experimentation stage, thereby limiting the low type's ability to create asymmetric beliefs by shirking off-the-equilibrium path.

Case 2 occurs when experimentation is very accurate, and the low type is rewarded even after failure (see Claim 2). As noted above, the principal can reduce the low type's rent by having the high type under-experiment. However, if experimentation is highly accurate, this option is very costly. When experimentation is highly accurate, significantly shortening the high

[5] If experimentation is not so accurate, payments to induce effort are sufficiently high to address the adverse selection constraints for both types. We obtain the standard results in a model of experimentation without adverse selection and a production stage, where an agent is rewarded only for success and under experimentation is optimal.
[6] See, e.g., Ollier and Thomas (2013), Castro-Pires and Moreira (2021), and Gottlieb and Moreira (2022).

type's experimentation stage is suboptimal, and the principal combines under-experimentation by the high type and rewarding the low type after failure.

Another contribution of our model is to explore whether it is optimal to outsource the experimentation stage to a second agent (separation) or to retain the same agent for both stages (integration). If there were no adverse selection, moral hazard only would imply outsourcing the experimentation stage, which is typical in the literature.[7] We find that the presence of adverse selection can make integration optimal. Thus, our results suggest that isolating the experimentation stage, as is standard in the literature, is not without loss of generality if adverse selection is severe enough. A key benefit to the principal of having a single agent for both stages is that the principal can leverage the adverse selection rent to address the moral hazard problem. If adverse selection is severe, creating a large rent, the principal can effectively satisfy the moral hazard constraints "for free" by spreading the adverse selection rent over time.

Our insights have practical implications regarding the optimality of outsourcing experimentation depending on the relative strengths of moral hazard and adverse selection. Consider first the case of drug approval trials, a commonly cited example in the literature, where a pharmaceutical company (principal) typically outsources the clinical trials to a clinical research organization (a separate agent) to demonstrate the effectiveness of a new drug. In this case, moral hazard is the more significant issue, as the principal is concerned about the research organization's incentives to exert effort in conducting the trials.[8] Adverse selection is less relevant as much information about the drug's prospective efficacy is in the public domain. Therefore, separation—outsourcing the experimentation stage—is optimal as is implicitly assumed in many papers.

In contrast, consider the case of a surgeon determining the appropriate surgical procedure for a patient. The surgeon evaluates the prospects for success based on their expertise and judgment, along with the patient's medical history and a series of diagnostic tests. While the prospect of success depends largely on a surgeon's prior experience and ability, the diagnosis is a dynamic learning process. Integrating the two stages (diagnostic tests and surgery) would be

[7] More precisely, by separating experimentation, the principal saves the additional moral hazard rent at the production stage mentioned above.
[8] There are multiple examples of clinical research organizations shirking, for example, by creating fake patient profiles (see Lindblad et al. (2014), Anand et al. (2012), Pogue et al. (2013) and references therein).

optimal as seems to be the observed practice. Adverse selection is likely to be a major issue since each surgeon's expertise and experience plays a vital role. On the other hand, moral hazard is less of a concern due to strict healthcare protocols and regulations required by the health insurance company or HMO.[9] As a result, integration is optimal.

Our analysis suggests that when tasks are separated, incentive schemes are relatively simple, and the agent is rewarded only after success. In contrast, when tasks are integrated, incentives schemes become more complex, and the agent can be rewarded even after failure. This framework helps explain why CEOs are sometimes paid a hefty compensation despite failure to perform. Under pure moral hazard, CEOs should be given the lowest possible wage (typically zero) upon failure. However, some recent papers surveyed by Edmans et al. (2017) rationalize payment after failure, for example, to induce CEOs to reveal negative information or to explore risky new technologies. Our model provides an additional explanation: payment after failure can serve as a screening instrument.

***Related Literature.*** Our paper is related to the literature on contracting for experimentation following Bergemann and Hege (1998). Most of that literature considers either moral hazard or adverse selection models in isolation. See, e.g., Horner and Samuelson (2013), Sadler (2021), Escobar and Zhang (2021), Rodivilov (2022), and Moroni (2022) for experimentation models of pure moral hazard. It is a common feature in these models that unobserved behavior can lead to rent even though the agent has no prior private information. Bhaskar and Mailath (2019) show this in a dynamic moral hazard model with discrete actions where the principal can use only short-term contracts, and Bhaskar and Roketskiy (2023) allow for continuous effort choice.[10] For experimentation models with adverse selection only, see Gomes et al. (2016) and Khalil et al. (2020). Among the few exceptions that introduce both moral hazard and adverse selection are Gerardi and Maestri (2012), Guo (2016), and Halac et al. (2016).

[9] In addition, healthcare practitioners are required by law to record patient medical histories and retain detailed case histories. There is also little room for skipping tests or altering results since this behavior might be simply illegal and a surgeon might be subject to prosecution. Surgeons are of course also bound by the Hippocratic Oath.
[10] See also Bhaskar and Roketskiy (2021) for a similar insight in a dynamic consumer choice problem.

Unlike all those papers, we consider a production stage and show how the rent in one stage echoes into the other stage.[11] While the standard result in the literature is to reward success in the experimentation stage to address moral hazard, we find that the presence of adverse selection may make rewarding failure in the experimentation stage optimal.

Gerardi and Maestri (2012) also have a result akin to rewarding failure in a model with a fixed length of experimentation. The agent is rewarded if he fails to obtain a signal that the state is good, but only if his report matches the true state observed ex-post. The reason is that information is "soft" in Gerardi and Maestri (2012) (the agent's report is not verifiable), whereas information is "hard" in our model (success is observed). Manso (2011) introduces a two-period model where failure is rewarded to incentivize the agent to explore riskier projects. In our model, there is only one available project, and rewarding failure is used to screen the low type.

More broadly, the literature on mixed models has emphasized that pooling types can be optimal.[12] In our model, the principal uses the timing of payments along with the length of experimentation and outcomes to induce effort and screen the agent. With multiple screening instruments, mixed models do not necessarily imply pooling. See Foarta and Sugaya (2021) for an example.[13] Martimort et al. (2025) study a mixed model with limited liability in a static setting and find that separation can be optimal. They use a dichotomous setup where the effort only determines a separate additive stochastic benefit but does not affect the cost of production or the output. They find pooling may occur but only for inefficient agents. We study a dynamic model that is not dichotomous since the agent's effort impacts the expected cost of production through learning.

The paper also contributes to the literature on endogenous information gathering before production. The standard model is static, based on early papers by Crémer and Khalil (1992), Lewis and Sappington (1997), and Crémer, Khalil, and Rochet (1998), and typically assumes that an agent exerts effort to increase the precision of the signal of the state relevant to

[11] Khalil et al. (2020) also introduce a production stage but with adverse selection only.
[12] See, e.g., Gottlieb and Moreira (2022).
[13] Castro-Pires et al. (2024) study a mixed model when the agent is risk averse and provide sufficient conditions under which the moral hazard problem can be decoupled from the adverse selection problem. Our setting does not satisfy those sufficient conditions since our problem is multi-dimensional as the optimal contract sets the wage, the length of experimentation and the output.

production decisions).[14] By contrast, our approach introduces dynamics of learning, possibly with asymmetric speeds, by modeling effort as experimentation. In our model, the principal endogenously determines the degree of asymmetric information in the production stage by choosing the length of experimentation. Unlike the rest of the literature, we show that the principal may find it optimal to reward failure and to over-experiment to screen the types.

We also contribute to the literature on the power of incentives for innovation. As already mentioned, Manso (2011) shows that an optimal incentive scheme may exhibit a reward for early failure for a risk averse agent. Benabou and Tirole (2003) show that using high-powered incentives may be detrimental to intrinsic motivation. In a laboratory experiment, Ederer and Manso (2013) find that a combination of rewards for both failure and success can be effective in incentivizing innovation. Sadler (2021) illustrates that high-powered incentives may discourage creativity. We contribute to this literature by showing theoretically that the coexistence of low- and high-powered incentive schemes can be optimal to mitigate the effect of adverse selection when failures to innovate are informative for the subsequent production decision.

Finally, our paper is also related to the extensive literature on integration and separation of tasks between agents. See, for instance, Schmitz (2005), Khalil et al. (2006), Iossa and Martimort (2012), Hoppe and Schmitz (2013 and 2021), and Li et al. (2015). Our dynamic model of learning allows us to pinpoint the relative importance of moral hazard and adverse selection in determining the optimal organization structure.

## 2. The Model and the First Best

A principal hires an agent to deliver a quantity $q \geq 0$ of output. The constant marginal cost of the output, $c$, is initially unknown to both the principal and the agent, but it is common knowledge that the cost can be low, $c = \underline{c} > 0$, with probability $\beta_0 \in (0,1)$, or high, $c = \overline{c}$, with probability $1 - \beta_0$, where $\overline{c} - \underline{c} = \Delta c > 0$. Both the principal and the agent are risk neutral, and, for simplicity, we assume that their discount factor is one.

The agent can have two types, denoted by $\theta \in \{H, L\}$. The type determines the probability that the cost is low: $\beta_0^\theta = Pr(c = \underline{c} | \theta)$, with $0 < \beta_0^L < \beta_0^H < 1$. In other words, a

[14] For recent papers, see citations in Krähmer and Strausz (2011), Rodivilov (2021), Downs (2021), and Häfner and Taylor (2022).

high type is more optimistic that the cost is low before experimentation starts, i.e., the *high type has a lower expected cost than the low type*. The principal believes the agent is a high type ($\theta = H$) with probability $\nu \in (0,1)$ and a low type ($\theta = L$) with probability $(1 - \nu)$.

Before production occurs, the agent gathers information regarding the production cost, which we model as a standard *experimentation stage*.[15] The length of the experimentation $T \in N$ is chosen by the principal. In the *production stage*, the agent is asked to produce based on what is learned about cost during the experimentation stage.[16] We assume that the agent cannot quit after the experimentation stage.

We present the timing next and describe the key steps in more detail subsequently.

1. The agent privately learns his type $\theta \in \{H, L\}$.

2. The principal offers a menu of two contracts, from which the agent selects one. A contract specifies (i) the duration of the experimentation stage, $T^{\theta}$, (ii) wages $w_t^S(\theta)$ and $w_t^F(\theta)$ that describe what the agent is paid in period $t = 1, 2, \ldots, T^{\theta}$ if he succeeds or fails in that period, respectively, and (iii) quantities $q_S$ and $q_F$ that the agent must produce in the production stage if he was successful at some point in the experimentation stage or if he failed throughout the experimentation phase, respectively. In the benchmark model, $q_S$ and $q_F$ are exogenous.[17]

3. If the agent does not accept any contract, the game ends and both parties get payoffs normalized to zero; if the agent accepts a contract, the game proceeds to the experimentation stage, and proceeds according to the contract selected by the agent.

4. The *experimentation stage*. This stage lasts for up to $T^{\theta}$ periods but stops immediately if the agent succeeds at some period $t < T^{\theta}$. In each period, the agent privately chooses effort $e_t \in \{0,1\}$ and is paid according to the terms specified in the selected contract.

5. The *production stage*. The agent produces $q_S$ or $q_F$ depending on the outcome of the experimentation stage.

[15] See, e.g., Halac et al. (2016).
[16] In section 4, we relax the assumption that the same agent experiments and produces.
[17] In Section 5, we make the outputs endogenous.

## 2.1. The Experimentation Stage

We assume that information gathering takes the form of looking for good news. We say that the experimentation was successful in period $t$ if it reveals that the cost is low (*good news*). If the cost is actually low and the agent works at period $t$, $e_t = 1$, success occurs with probability $0 < \lambda < 1$. Success is publicly observable. Success cannot occur in a period $t$ if the cost is high, or if the agent shirks, $e_t = 0$. Experimentation at $t$ costs $\gamma e_t$ to the agent, where $\gamma > 0$. Since the agent chooses effort $e_t$ privately, the principal must induce $e_t = 1$ in each period $t = 1,2,\dots,T^\theta$ by addressing a moral hazard problem.

If success occurs in a period $t$, the experimentation stage ends, and production takes place based on $c = \underline{c}$. If the agent fails to learn that the cost is low in a period $t < T^\theta$, experimentation resumes. If the agent fails in all $T^\theta$ periods, then we say that experimentation results in failure, and production takes place based on expected cost.[18]

## 2.2. Updating Beliefs

We denote by $\beta_t^\theta$ the updated belief of a type $\theta$ agent that the cost is low at the beginning of period $t$ (after $t-1$ failures) when $e_j^\theta = 1$ for all $j \leq t-1$. We have $\beta_t^\theta = \frac{\beta_{t-1}^\theta(1-\lambda)}{\beta_{t-1}^\theta(1-\lambda)+(1-\beta_{t-1}^\theta)}$, which can be re-written in terms of $\beta_0^\theta$ as follows for $t \geq 1$:

$$\beta_t^\theta = \frac{\beta_0^\theta(1-\lambda)^{t-1}}{\beta_0^\theta(1-\lambda)^{t-1}+1-\beta_0^\theta}.$$

The expected cost for a type $\theta$ agent at the beginning of period $t \geq 0$ is then[19]

$$c_t^\theta = \beta_t^\theta \underline{c} + \left(1-\beta_t^\theta\right)\overline{c}.$$

After each failure, the agent becomes more pessimistic about the true cost being low ($\beta_t^\theta$ falls), and the expected cost rises. For the same number of failures during the experimentation stage, a low type always remains *more pessimistic* than a high type and has a higher expected cost ($c_t^L > c_t^H$). However, both $c_t^H$ and $c_t^L$ approach $\overline{c}$ in the limit.

[18] In case of failure, we assume that the agent will learn the exact cost later, but it is not contractible.

[19] If production occurs without experimentation ($T^\theta = 0$), the agent produces given the expected cost under the initial belief, $c_0^\theta = \beta_0^\theta \underline{c} + \left(1-\beta_0^\theta\right)\overline{c} = c_1^\theta$.

## 2.3. The Production Stage

Production takes place after experimentation succeeds in some $t$, or after it fails all $T^\theta$ times. Since success publicly reveals low cost, the output after success is chosen under complete information. The interesting case occurs when the agent has failed to learn during the entire experimentation stage since production then occurs under asymmetric information.[20]

A simple way to capture the impact of asymmetric beliefs in production after failure is to assume that the output after failure is fixed at $q_F > 0$. To be consistent with the extension in Section 5, where we make the outputs endogenous, we assume that the principal's value of the output is given by $V(q)$, which is strictly increasing and strictly concave.[21] We assume $V(q_S) > V(q_F) > 0$. The cost of production after success is therefore $\underline{c}q_S$ and the expected cost after $t-1$ failures is $c_t^\theta q_F$.

## 2.4. The First Best Length of Experimentation

Suppose the agent's type $\theta$ is common knowledge and the agent works in every period. The first-best length of experimentation $T^\theta$ for a type-$\theta$ agent determines the maximum expected surplus net of costs denoted by:

$$\Omega^\theta = \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1}\lambda\left[V(q_S) - \underline{c}q_S\right] + P_{T^\theta}^\theta\left[V(q_F) - c_{T^\theta+1}^\theta q_F\right] - \sum_{t=1}^{T^\theta} P_{t-1}^\theta \gamma,$$

where we denote the probability that an agent of type $\theta$ fails in all of the first $t$ periods of the experimentation stage given $e_j^\theta = 1$ for all $j \le t$ by:

$$P_t^\theta \equiv 1 - \beta_0^\theta + \beta_0^\theta(1-\lambda)^t.$$

Since the expected cost is rising until success is obtained, the first-best solution is characterized by a termination date $T_{FB}^\theta$, the maximum number of periods an agent of type $\theta$ is allowed to experiment:

$$T_{FB}^\theta \in arg\max_{T^\theta} \Omega^\theta.$$

[20] Having a productive decision after failure is a significant departure from the standard literature on strategic experimentation, where the quantity after failure is implicitly assumed to be zero, and therefore asymmetric beliefs after failure between the principal and agent do not matter for the incentives. We show that the difference in beliefs matters by assuming an explicit production stage even after failure.

[21] In Section 5, with optimally chosen outputs, we show that the key results are unaffected, except that variation in output after failure is an additional screening device. We also show that output is efficient after success for each type.

Note that $T_{FB}^{\theta}$ is bounded, and it is the highest $T^{\theta}$ such that $\Omega^{\theta}(T^{\theta}) - \Omega^{\theta}(T^{\theta} - 1) \geq 0$:

$$\beta_{T^{\theta}}^{\theta}\lambda[V(q_S) - \underline{c}q_S] + (1 - \beta_{T^{\theta}}^{\theta}\lambda)[V(q_F) - c_{T^{\theta}+1}^{\theta}q_F] \geq \gamma + [V(q_F) - c_{T^{\theta}}^{\theta}q_F].$$

The $LHS$ describes the net benefit of extending by one period at $T^{\theta}$. There is a chance to produce $q_S$ at cost $\underline{c}$ if there is success, which occurs with probability $\beta_{T^{\theta}}^{\theta}\lambda$, or to produce $q_F$ at the updated expected cost at period $T^{\theta} + 1$, denoted by $c_{T^{\theta}+1}^{\theta}$ if the agent fails. If the experimentation stage is not extended, $q_F$ is produced at the expected cost $c_{T^{\theta}}^{\theta}$, given in the $RHS$.

The first-best length of experimentation $T_{FB}^{\theta}$ is a *monotonic* function of the agent's type, which implies that the principal should allow the high type to experiment longer.[22] The reason is that the high type is more likely to learn $c = \underline{c}$. As is standard, we assume that it is always optimal to experiment at least once in the first-best case, where the agent works and the principal knows $\beta_0^{\theta}$.[23] This restriction does not apply in the optimal contract under asymmetric information, where the principal is free to choose not to experiment.

### 2.5. The Principal's Problem: contract and payoffs

We return to the problem with asymmetric information, where the principal must address both moral hazard and adverse selection. Before experimentation takes place, the principal offers the agent a menu of dynamic contracts. We restrict attention to deterministic contracts.

Without loss of generality, we use a direct truthful mechanism, where the agent is asked to announce his type, denoted by $\hat{\theta}$. A contract is defined formally by

$$\varpi^{\hat{\theta}} = \left(T^{\hat{\theta}}, \{w_t^S(\hat{\theta}), w_t^F(\hat{\theta})\}_{t=1}^{T^{\hat{\theta}}}\right),$$

where $T^{\hat{\theta}}$ is the (maximum) duration of the experimentation stage for the announced type $\hat{\theta}$, $w_t^S(\hat{\theta})$ is the agent's wage in period $t$ if he succeeded in period $t \leq T^{\hat{\theta}}$, and $w_t^F(\hat{\theta})$ is the agent's wage if he fails in period $t$. If production occurs without experimentation, the agent is paid $w_0^F(\hat{\theta})$ for delivering $q_F$.

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[22] This is different from Halac et al. (2016) and Khalil et al. (2020), where the first-best termination date is non-monotonic in type and plays a key role. The reason for the non-monotonicity in those papers is that agent's type is given by $\lambda$, and the conditional probability of success is higher for the high type early but becomes lower as the length of experimentation increases.

[23] In particular, we assume that the principal would not choose $q_S$ without experimenting.

An agent of type $\theta$, announcing his type as $\widehat{\theta}$, chooses the periods in which he works or shirks, where the number of periods the agent works is written as $\sum_{s=1}^{T^{\widehat{\theta}}} e_s^{\theta}$. Denoting the chosen effort profile by $\vec{e}^{\theta} = \{e_t^{\theta}\}_{t=1}^{t=T^{\widehat{\theta}}}$, the agent receives the expected utility $U^{\theta}(\varpi^{\widehat{\theta}}, \vec{e}^{\theta})$ at time zero from a contract $\varpi^{\widehat{\theta}}$:[24]

$$U^{\theta}(\varpi^{\widehat{\theta}}, \vec{e}^{\theta}) = (1-\beta_0^{\theta})\left[\sum_{t=1}^{T^{\widehat{\theta}}}[w_t^F(\widehat{\theta}) - \gamma e_t^{\theta}] - c^{\theta}_{\sum_{s=1}^{T^{\widehat{\theta}}} e_s^{\theta}+1} q_F\right]$$

$$+\beta_0^{\theta}\sum_{t=1}^{T^{\widehat{\theta}}}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{\theta})\right)[e_t^{\theta}\lambda(w_t^S(\widehat{\theta}) - \underline{c}q_S) + (1-\lambda e_t^{\theta})w_t^F(\widehat{\theta}) - \gamma e_t^{\theta}$$

$$-1_{\{t=T^{\widehat{\theta}}\}}(1-\lambda e_t^{\theta})c^{\theta}_{\sum_{s=1}^{T^{\widehat{\theta}}} e_s^{\theta}+1} q_F],$$

where the indicator function $1_{\{t=T^{\widehat{\theta}}\}}$ is used to denote the last period of experimentation.

If the actual cost is high, which happens with probability $(1-\beta_0^{\theta})$, the agent will fail for sure. After he fails in period $t < T^{\widehat{\theta}}$, experimentation continues but we allow for the agent to be paid $w_t^F(\widehat{\theta})$. If the actual cost is low, which happens with probability $\beta_0^{\theta}$, the probability of succeeding for the first time in period $t$ is given by $\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{\theta})\right)e_t^{\theta}\lambda$. If the agent succeeds in period $t$, he produces $q_S$ and is paid $w_t^S(\widehat{\theta})$. The agent can fail in each period from 1 to $t$ even if the actual cost is low, which happens with probability $\beta_0^{\theta}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{\theta})\right)(1-\lambda e_t^{\theta})$, the agent is paid $w_t^F(\widehat{\theta})$ and experimentation continues. If the agent fails $T^{\widehat{\theta}}$ times despite the cost being low, which happens with probability $\beta_0^{\theta}\prod_{s=1}^{T^{\widehat{\theta}}}(1-\lambda e_s^{\theta})$, the agent produces $q_F$ based on the expected cost at period $T+1$.

We denote by $\vec{e}^{\theta}(\varpi^{\widehat{\theta}}) \equiv argmax_{\vec{a}^{\theta}} U^{\theta}(\varpi^{\widehat{\theta}}, \vec{a}^{\theta})$ the optimal action profile for type $\theta$ in all periods $t \leq T^{\widehat{\theta}}$ facing a contract $\varpi^{\widehat{\theta}}$. Since the principal optimally induces $e_t = 1$ in every period of the experimentation stage, the optimal contract must satisfy the following global moral hazard constraint:

[24] The empty product (the result of multiplying no factors) is defined to be 1 by convention, $\prod_{s=1}^{0}(\cdot) = 1$. The updating occurs in the following period, thus the "+1" in $\sum_{s=1}^{T^{\widehat{\theta}}} e_s^{\theta} + 1$. If production occurs without experimentation ($T^{\widehat{\theta}} = 0$), we denote the agent's utility simplifies to $U^{\theta} = w_0^F(\widehat{\theta}) - c_0^{\theta} q_F$.

$(\boldsymbol{MH^{\theta}})$ $$\vec{1} \in \vec{e}^{\theta}(\varpi^{\theta}).$$

In addition, the optimal contract will also have to satisfy the following incentive compatibility constraint for all $\theta$ and $\hat{\theta}$:

$(\boldsymbol{IC^{\theta}})$ $$U^{\theta}(\varpi^{\theta}, \vec{1}) \geq U^{\theta}\left(\varpi^{\hat{\theta}}, \vec{e}^{\theta}(\varpi^{\hat{\theta}})\right).$$

We define *reward* after success and failure as the agent's wage in each event net of production cost. Thus, a reward is an information-based rent, and we denote by $y_t^{\theta}$ the reward after success in period $t$, and by $x_t^{\theta}$ the reward after failure in period $t$:

$$y_t^{\theta} \equiv w_t^S(\theta) - \underline{c}q_S,$$

$$x_t^{\theta} \equiv w_t^F(\theta) - 1_{\{t=T^{\theta}\}} c_{T^{\theta}+1}^{\theta} q_F,$$

where the indicator function $1_{\{t=T^{\theta}\}}$ is used to denote the last period of experimentation. A strictly positive reward means that the agent is receiving a rent.

We assume the agent's wage net of expected production cost must be non-negative on the equilibrium path. We impose the following limited liability ($\boldsymbol{LL}$) constraints whether experimentation succeeds or fails:

$(\boldsymbol{LLS_t^{\theta}})$ $$y_t^{\theta} \geq 0 \text{ for } t \leq T^{\theta},$$

$(\boldsymbol{LLF_t^{\theta}})$ $$x_t^{\theta} \geq 0 \text{ for } t \leq T^{\theta}.$$

The constraints are meant to capture the impact of laws governing penalties. Bankruptcy laws are well-known examples of legal restrictions that exemplify limited liability in contracts.[25] The principal's expected payoff from a contract $\varpi^{\theta}$ offered to an agent of type $\theta$, that satisfies the above constraints, is given by

[25] See, e.g., Krähmer and Strausz (2015) for more examples. This is not a constraint based on the agent's wealth, and the agent's reward net of expected production cost may well be negative off-the-equilibrium path. We assume that the contract should not be required to cover for the production cost of a misreporting agent. Additionally, without limited liability, the principal can receive first best profit since success during experimentation is a random event correlated with the agent's type (Crémer-McLean, 1985). To streamline the presentation, we assume the transfers must cover the equilibrium expected cost. This is reminiscent of the well-known cost-plus contracts in the procurement literature.

$$\pi^\theta(\varpi^\theta,\vec{1}) = \beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\left[\lambda\Big(V(q_S) - w_t^S(\theta)\Big) - (1-\lambda)w_t^F(\theta)\right]$$

$$+P_{T^\theta}^\theta V(q_F) - \left(1-\beta_0^\theta\right)\sum_{t=1}^{T^\theta} w_t^F(\theta).$$

Finally, we can state the principal's problem. The principal maximizes the objective function:

$$E_\theta\left[\pi^\theta(\varpi^\theta,\vec{1})\right] = \nu\pi^H(\varpi^H,\vec{1}) + (1-\nu)\pi^L(\varpi^L,\vec{1}).$$

s.t. $(MH^\theta)$, $(IC^\theta)$, $\left(LLS_t^\theta\right)$ and $\left(LLF_t^\theta\right)$ for $\theta \in \{H, L\}$.

# 3. Solving the Principal's Problem

## 3.1. Simplifying the $(MH^\theta)$ and $(IC^\theta)$ constraints

We first follow the standard step of replacing without loss of generality the global moral hazard constraint $(MH^\theta)$ by a sequence of local one-period moral hazard constraints $(MH_t^\theta)$.[26] The one-period moral hazard constraints $(MH_t^\theta)$ ensure that the agent will not engage in a one-shot deviation and shirk at period $t \le T^\theta$ (given that the agent has worked in all prior periods $j < t$ without success and will work in all subsequent periods $s > t$).

$$\left(MH_t^\theta\right)\ y_t^\theta - x_t^\theta \ge \frac{\gamma}{\lambda\beta_t^\theta} + \sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta + (1-\lambda)x_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta-t}}{P_{T^\theta}^\theta}\Delta c q_F.$$

The principal can motivate the agent to work by paying a higher reward for success $(y_t^\theta)$ than the reward after failure $(x_t^\theta)$. The first two terms on the *RHS* of $\left(MH_t^\theta\right)$ capture a standard rent in a dynamic model of experimentation without production (see, e.g., Bergemann and Hege (1998)).[27] A new feature relative to a standard dynamic moral hazard problem of experimentation is due to the presence of a production stage in our model. Consequently, in addition to the standard moral hazard rent, shirking in experimentation results in a *second* moral

[26] The formal proof is in Appendix A.

[27] On top of the static moral hazard rent due to limited liability, which is the first term on the $RHS$ of $\left(MH_t^\theta\right)$, there is an additional rent in the standard dynamic moral hazard problem, reflected in the second term: (i) if the agent secretly shirks at period $t$, he is *more* likely to get to any future period $s > t$ than what the principal anticipates, reflected in the different probabilities, $(1-\lambda)^{s-t-1}$, as opposed to $(1-\lambda)^{s-t}$ that the principal would anticipate.

hazard rent at the production stage $\frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta - t}}{P_{T^\theta}^\theta}\Delta c q_F$. This is because a shirking agent will have a lower expected cost compared to what the principal expects.[28]

Before presenting the $(IC^\theta)$ constraints, we discuss off-the-equilibrium effort on the $RHS$ of these constraints since the moral hazard problem is also implicitly reflected in the $(IC^\theta)$ constraints.[29] In Lemma 1 below, we show that the misreporting high type will work in all periods if he claims to be a low type as is true in a standard mixed model.[30] A payment scheme that makes a low type work will be enough to induce a misreporting high type to work. This is because the high type is less pessimistic and more likely to collect promised rewards after success.

***Lemma 1.*** *A* misreporting *high type works in all periods off the equilibrium path:* $\vec{e}^H(\varpi^L) = \vec{1}$.

*Proof*: Consider the high type's incentives to engage in a one-shot deviation and shirk at period $t \leq T^L$ after accepting a contract designed for the low type. If the misreporting high type decides to work at period $t$, his continuation value from the relationship is:

$$-\gamma + \lambda\beta_t^H y_t^L + (1-\lambda\beta_t^H)x_t^L + \beta_t^H \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t}[\lambda y_s^L + (1-\lambda)x_s^L - \gamma] +$$
$$(1-\beta_t^H)\sum_{s=t+1}^{T^L}(x_s^L - \gamma) + P_{T^L-t+1}^H\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F.$$

In contrast, upon deviating only at some period $t \leq T^L$, his continuation value from the relationship becomes:

$$x_t^L + \beta_t^H \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}[\lambda y_s^L + (1-\lambda)x_s^L - \gamma] + (1-\beta_t^H)\sum_{s=t+1}^{T^L}(x_s^L - \gamma)$$
$$+P_{T^L-t}^H\left(c_{T^L+1}^L - c_{T^L}^H\right)q_F.$$

Combining the two continuation values presented above, we can write a one-period no shirking constraint at period $t$ below for the misreporting high type, which we denote by $(NS_t^{H,L})$:

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[28] The principal's belief is based on one more period of working compared to that of a shirking agent. Thus, the shirking agent has a lower expected cost: $c_T^\theta = c_{T+1}^\theta - (\beta_T^\theta - \beta_{T+1}^\theta)\Delta c < c_{T+1}^\theta$, and he will receive an additional production stage rent as a result.

[29] A similar characterization is not easily available in Halac et al. (2016) as the agent's private information is about $\lambda$ the efficiency of learning parameter. In that case, the relative probability of success across the two types changes over time. As a result, the authors provide examples that it is possible to have multiple off-equilibrium paths for effort in the optimal contract.

[30] See, e.g., Laffont and Martimort (2002), or Chakraborty et al. (2021).

$$\left(NS_t^{H,L}\right)\ y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F.$$

Since $\beta_t^L < \beta_t^H$ for any $t$, we have $\frac{\gamma}{\lambda\beta_t^H} < \frac{\gamma}{\lambda\beta_t^L}$, and the $RHS$ of $\left(NS_t^{H,L}\right)$ is smaller than the $RHS$ of $(MH_t^L)$. Thus, $\left(NS_t^{H,L}\right)$ is implied by $(MH_t^L)$ for $t \leq T^L$. This concludes the proof of our Lemma 1. *Q.E.D.*

Denoting the difference in the expected cost across the types as $\Delta c_t = c_t^L - c_t^H$, we can now present the incentive constraint of the high type:

$$(\boldsymbol{IC^H}) \qquad \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma \geq$$

$$(1-\beta_0^H)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}[\lambda y_t^L + (1-\lambda)x_t^L - \gamma] + P_{T^L}^H \Delta c_{T^L+1} q_F.$$

The expression $P_{T^L}^H \Delta c_{T^L+1} q_F$ represents a familiar adverse selection rent at the production stage: since a low type must be paid at least his expected cost $c_{T^L+1}^L$ following $T^L$ failures, a misreporting high type will have lower expected costs if experimentation fails in all periods, which is captured by $\Delta c_{T^L+1} = c_{T^L+1}^L - c_{T^L+1}^H > 0$ in the expression.

Unlike the misreporting high type, the misreporting low type may not work in every period. Again, this is a common feature in many mixed models. The $RHS$ of the $(IC^L)$ simplifies because the low type's probability of success in any period and the expected cost after failure depend on the total number of periods worked (and failed) up to that period (not on when those failures occurred). We denote by $t^{L,H}$ the number of periods a low type works when he misreports. For expositional convenience and without loss of generality, we write the low type's off-the-equilibrium path effort as a stopping rule: he works up to period $t^{L,H} \leq T^H$ and shirks thereafter.[31] Thus, the incentive constraint of the low type is given by:

[31] The proofs in the Appendices do not rely on off the path effort of the low type being a stopping rule. But a stopping rule is without loss of generality in our model because the high type's rewards for success can be front loaded given that the relative likelihood of success $\frac{\beta_0^L(1-\lambda)^{t-1}\lambda}{\beta_0^H(1-\lambda)^{t-1}\lambda} = \frac{\beta_0^L}{\beta_0^H}$ is independent of $t$. Without the stopping rule, the second expression on the $RHS$ of $(IC^L)$ is replaced in Appendix A with the expression $\beta_0^L \sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1 - \lambda e_s^{L,H})\right)\left[e_t^{L,H}\lambda y_t^H + \left(1-\lambda e_t^{L,H}\right)x_t^H - e_t^{L,H}\gamma\right]$, where $e_t^{L,H}$ is the effort chosen by the mis-reporting low type in period $t$.

$$(\boldsymbol{IC^L}) \qquad \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\, y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma \geq$$

$$(1-\beta_0^L)\left(\sum_{t=1}^{T^H} x_t^H - \gamma t^{L,H}\right) + \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\left(\lambda y_t^H + (1-\lambda)x_t^H - \gamma\right)$$

$$+\beta_0^L(1-\lambda)^{t^{L,H}} \sum_{t^{L,H}+1}^{T^H} x_t^H - P_{t^{L,H}}^L\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right) q_F.$$

The first three expressions on the $RHS$ describe the payoffs after success and failure in each period, while the fourth term captures the impact of asymmetric beliefs if production occurs after failure. As in the case of $(IC^H)$, the difference in expected costs is due to what the principal must pay a truthful high type $c_{T^H+1}^H$, versus the expected cost for a misreporting low type who only works for $t^{L,H}$ periods, and thus has an expected cost $c_{t^{L,H}+1}^L$. If $t^{L,H}$ is much smaller than $T^H$, i.e., the misreporting low type shirks often, he could be more optimistic than the principal who mistakenly believes she is dealing with a high type who has worked for $T^H$ periods, resulting in $c_{T^H+1}^H - c_{t^{L,H}+1}^L > 0$. Thus, the $(IC^L)$ may be binding since the misreporting low type may command a rent at the production stage if he is more optimistic than the principal due to unobserved shirking.

However, if $t^{L,H}$ is close to $T^H$, i.e., the misreporting low type works often, he could be less optimistic than the principal who mistakenly believes she is dealing with a high type who has worked for $T^H$ periods, resulting in $c_{T^H+1}^H - c_{t^{L,H}+1}^L < 0$. Then, misreporting is a *gamble* for the low type with his payoff depending on the outcome of experimentation: the positive part comes from obtaining the high-type's rent if he succeeds, and the negative part comes from an expected loss in the production stage if he fails in experimentation.

In our model, there are two reasons why both $(IC)$ constraints can be binding, leaving the principal no option but to reward failure in order to screen. One reason why both the $(IC)$ constraints can be binding is standard in a mixed model: a low type may have an incentive to misreport and shirk. Another reason is that experimentation leads to a common value problem because the agent's type $\beta_0^\theta$ directly enters the principal's objective function.[32]

[32] See, e.g., Laffont and Martimort (2002), page 53. In a common value setting under pure adverse selection, that we also solve in Appendix B, both upward and downward incentive compatibility constraints can be binding because of a conflict between the principal's preference for the high type to experiment longer for pure efficiency reasons and the monotonicity condition imposed by asymmetric information.

### 3.2. Optimal contract – relative impact of moral hazard and adverse selection

The solution to the principal's problem depends on the relative importance of moral hazard and adverse selection.

We will begin discussing the optimal solution with a familiar case in the strategic experimentation literature, where experimentation is modeled primarily as a moral hazard problem – how to motivate the experimenter to work. We present this as Case 1 in Section 3.2.1 below. In contrast, if moral hazard rents are low and $(IC)$ constraints are binding, adverse selection rent may come into conflict with moral hazard incentives and induce the principal to reward failure to screen the two types. We present this as Case 2 in Section 3.2.2. The results for the remaining two intermediate cases – Case 3 with only $(IC^H)$ binding, and Case 4 with only $(IC^L)$ binding – follow from the analysis of the two main cases, and we discuss these cases in Section 3.2.3.

#### 3.2.1. Case 1. Strong moral hazard: Both IC constraints are slack

First, we consider a case where experimentation accuracy $\lambda$ is low such that moral hazard is very strong relative to adverse selection, and neither $(IC)$ constraint binds. In other words, neither type has incentives to misreport because of the high moral hazard rent they receive from their own contracts. We present the main findings along with sufficient conditions for this case to occur in Claim 1 below. They require $\lambda$ to be small, $\gamma$ sufficiently higher than $\Delta c$, and $\beta_0^L$ to be small.

**Claim 1.** *For any $\beta_0^H$ there exist $\overline{\lambda}(\beta_0^H) > 0$, $\tilde{A}(\beta_0^H) > 0$, and $\overline{\beta}_0^L(\beta_0^H) > 0$ such that the one-period moral hazard constraints are binding for each type in each period, but neither $(IC)$ constraint is binding if $\lambda < \overline{\lambda}$, $\gamma > \tilde{A}\Delta c q_F$, and $\beta_0^L < \overline{\beta}_0^L$. Furthermore, when neither IC is binding:*

(i) *Both types of agents receive two moral hazard rents: a standard rent in the experimentation stage and a second rent in the production stage.*

(ii) *Both types of agents are rewarded only after success, and the optimal reward $y_t^\theta$ is constant, given by $y_t^\theta = \frac{\gamma}{\lambda \beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$* for $t \leq T^\theta$ and $\theta \in \{H, L\}$.

(iii) *Both types of agents under-experiment relative to the first best, $T_{SB}^L < T_{FB}^L$* and *$T_{SB}^H < T_{FB}^H$; in addition the low type experiments less than the high type, $T_{SB}^L < T_{SB}^H$.*

***Proof***: See Appendix A.

Each type of agent is rewarded only after success, and the optimal reward $y_t^\theta$ is constant for $t \leq T^\theta$.[33] There is no reward after failure, i.e., $x_t^\theta = 0$ for all $t$. As explained above when describing the one-period moral hazard $(MH_t^\theta)$ constraints, the term $\frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ is what we called the second moral hazard rent. It stems from the shirking agent having a lower expected cost of production after failure than the principal. The term $\frac{\gamma}{\lambda \beta_{T^\theta}^\theta}$ represents the standard moral hazard rent in a dynamic model of experimentation without production.[34]

While *under* experimentation is optimal for both types: $T_{SB}^L < T_{FB}^L$ and $T_{SB}^H < T_{FB}^H$, the second moral hazard rent leads to a greater degree of under-experimentation than in moral hazard models of experimentation without a production stage. Each moral hazard rent increases with the length of experimentation because the divergent beliefs due to shirking increases with $T^\theta$. Furthermore, we also show that the high type experiments longer $T_{SB}^L < T_{SB}^H$ as is true under the first best.

We now discuss the intuition behind the sufficient conditions for neither $(IC)$ to be binding. From the binding $(MH_t^\theta)$, we can see that the moral hazard rents get larger if $\lambda$ is smaller and $\gamma$ is larger, which are the first two parts of the sufficient conditions for this case. If $\lambda$ is small, the outcome of an experiment is not informative about the effort making it costlier to incentivize the agent to work. We also need the cost of experimentation $\gamma$ to be sufficiently higher than $\Delta c$, which again makes moral hazard more important than adverse selection. Truth telling is obtained "for free" in this case as neither type wants to misrepresent his private information about $\beta_0^\theta$.[35] The third part of the sufficient conditions that $\beta_0^L$ is small ensures that high moral hazard payment to the low type deters him from the high type's contract.

Next, we consider the other polar case where adverse selection is very strong relative to moral hazard, and both $(IC)$ constraints are binding. The sufficient conditions on $\lambda$ and $\gamma$ are a

[33] The reward deters a one-step-deviation by the agent, and the agent's incentive to deviate does not depend on $t$ when there is no discounting (Rodivilov, 2022). Then, the optimal contract is unique up to payoff-irrelevant alterations. A similar reward structure holds in Halac et al. (2016), who argue that in the case of no discounting, the principal can be restricted to using constant bonus contracts.

[34] The standard rent has two parts, where $\frac{\gamma}{\lambda \beta_t^\theta}$ addresses the static gain, and $\gamma \sum_{s=1}^{T^\theta - t} \frac{(1-\beta_0^\theta)}{\beta_0^\theta (1-\lambda)^{t+s-1}}$ is the rent coming from a higher probability of collecting future moral hazard rents (than the principal expects in equilibrium).

[35] The high type is not attracted by the low type's contract as the high type experiments longer and receives a moral hazard rent over more periods ($T_{SB}^L < T_{SB}^H$) when he tells the truth.

mirror image of the ones above, and inducing effort on the equilibrium path is not too costly. The agent's incentives are largely driven by the impact of asymmetric beliefs at the production stage due to adverse selection concerns, while off-the-path shirking remains important and leads to both $(IC)$ constraints being binding.

### 3.2.2. Case 2. Strong adverse selection: Both $(IC^H)$ and $(IC^L)$ are binding

When $\lambda$ is high, experimentation is very effective, moral hazard payments are small, and both $(IC)$ constraints are binding. Remarkably, we find that the optimal contract requires rewarding the low type for failure, which conflicts with moral hazard constraints. While this forces the principal to increase rewards after success in each period to induce effort, a high $\lambda$ makes it optimal to do so. In Claim 2, we present the main results for this case along with sufficient conditions, which require $\lambda$ high and $\beta_0^H$ small.

**Claim 2**. *For any $\beta_0^L$ there exist $\underline{\lambda}(\beta_0^L) < 1$, $\bar{\beta}_0^H(\beta_0^L) > 0$ such that both $IC$ bind if $\lambda > \underline{\lambda}$ and $\beta_0^H < \bar{\beta}_0^H$. Furthermore, when both $IC$ are binding:*

(i) *To address moral hazard, the principal must reward each type after success in every period ($y_t^\theta > 0$ for $t \leq T^\theta$, $\theta \in \{H, L\}$).*

(ii) *The high type is not rewarded after failing in any period ($x_t^H = 0$ for $t \leq T^H$), while the low type is rewarded after failing in the very last period ($\boldsymbol{x_{T^L}^L > 0} = x_t^L$ for $t < T^L$).*

(iii) *Relative to the first best, the low type over-experiments ($T_{SB}^L > T_{FB}^L$) if $\gamma$ is small enough while the high type under-experiments ($T_{SB}^H < T_{FB}^H$).*

***Proof***: See Appendix A.

If $\lambda$ is high, the impact of experimentation on the endogenous asymmetry of information is significant. Recall that the incentive to misreport depends on $T^\theta$ through the difference in expected cost $\Delta c_{T^\theta+1}$. To understand the distortion in $T^L$, we need to examine the key positive element in the high-type's rent, $P_{T^L}^H \Delta c_{T^L+1} q_F$, which is decreasing in $T^L$. This leads to *over*-experimentation for the low type to reduce asymmetric information regarding the agent's type, which would not occur under pure moral hazard. To understand the distortion in $T^H$, recall that the low type's incentive to misreport is determined by the expected cost the principal must pay a truthful high type $c_{T^H+1}^H$ after failure, which is increasing in $T^H$. This leads to under-experimentation for the high type, reinforcing the impact of moral hazard. Lowering $T^H$ also reduces the scope for low type to create asymmetric beliefs by shirking off the path and, as a result, his incentive to misreport. However, if experimentation is very accurate, this option of reducing $T^H$ is costly. The principal cannot distort $T^H$ too much and $(IC^L)$ is binding.[36]

One important result of our model is the optimality of rewarding failure if both $(IC)$ are binding. Each type is rewarded for an event that is more likely to occur given the type, which is success for the high-type and failure for the low-type, respectively. It is without loss of generality to postpone the low-type's reward to the very last period of the relationship, $x_{T^L}^L > 0$, making it less likely for a (misreporting) high type to obtain it. In our mixed model, failure is not only an indicator of shirking but also a more likely event for the low type. If only one $(IC)$ is binding, the principal does not reward failure as we will see in the two intermediate cases below.

A high $\lambda$ also limits the moral hazard payments after success. Paying an additional screening rent after failure requires the principal to raise the reward after success (by the same amount) in each prior period. That is, the low-type agent must be given extra incentive to work in each prior period. Ultimately, the low type is still paid more after success than failure, but he gets rent even if he fails, which is not the case for the high type.[37]

Because the high type is less likely to fail, it is not optimal to reward him after failing in any period as it makes it costlier to satisfy the binding $(IC^H)$. Therefore, it is optimal to choose

$$x_t^H = 0 \textit{ for } t \leq T^H.$$

The timing of payments after success $y_t^H$ is ineffective as a screening instrument as the relative probability of success between the types $\left(\frac{\beta_0^L(1-\lambda)^{t-1}\lambda}{\beta_0^H(1-\lambda)^{t-1}\lambda} = \frac{\beta_0^L}{\beta_0^H} < 1\right)$ is constant across

[36] For instance, if the high type were asked to produce without experimentation, the low type would not be able create asymmetric beliefs by shirking off the path and would truthfully report his type.

[37] If the reward is paid in the last period, to satisfy the $(MH_t^L)$ constraints, the reward after success must increase not only in that last period, $y_{T^L}^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$, but also in the all the previous periods $t < T^L$: $y_t^L = \frac{\gamma}{\lambda\beta_t^L} + \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L - \gamma) + (1-\lambda)^{T^L-t} x_{T^L}^L$, which increases the payments $y_t^L$ strictly above the levels described in Claim 1.

periods of experimentation. Consequently, there is no restriction on when to pay the screening rent to the high type via a combination of $y_t^H$.[38]

$$y_t^H \geq \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F \text{ for } t \leq T^H \text{ (strict inequality for some } t).$$

The principal can use the screening rent to induce effort in each period and satisfy the high type's $MH$ constraints with no additional cost.

We can summarize the results of our two claims in the following proposition:

**Proposition 1**: *If moral hazard is the dominant issue, then both types of agents receive rent only after success and each type under experiments (Claim 1). If adverse selection is the dominant issue, then the low-type agent over-experiments and receives a rent after both success and failure, while the high-type under-experiments and receives a rent after success only (Claim 2).*

Next, we consider intermediate cases when only one $(IC)$ constraint is binding.

### 3.2.3. Intermediate Cases 3 and 4: only one $(IC)$ is binding

We outline how the analysis of the two main cases above provides the intuition for the key results and sufficient conditions when only one $(IC)$ is binding, while the details are presented along with proofs in Appendix A as Claims A3 and A4.

**Claim 3**. *If either $(IC^H)$ or $(IC^L)$ is binding, but not both:*

(i) *It is without loss of generality to reward the agent only after success and not after failure.*

(ii) *There is no distortion in $T^\theta$ for the type whose $(IC^\theta)$ is binding, with $\theta \in \{H, L\}$.*

(iii) *When $(IC^L)$ is binding, under-experimentation in $T^H$ reduces the low type's rent.*

(iv) *When $(IC^H)$ is binding, there is under-experimentation in $T^L$ to reduce the high type's rent unless gamma is small enough, in which case there is over-experimentation in $T^L$.*

***Proof***: See Appendix A.

In the two intermediate cases, either $(IC^H)$ or $(IC^L)$ is binding, but not both. There are common elements in the two cases. Both types are only rewarded after success and there is no

[38] For example, it is without loss of generality to pay the extra rent to the high type after the very first success, i.e., front load the extra rent.

reward after failure. If only one $(IC)$ is binding, each type is rewarded only after success without having to worry about increasing the cost of satisfying the other $(IC)$.[39] For the type earning a screening rent, his moral hazard constraints are slack. Conversely, if a type does not earn a screening rent, all his moral hazard constraints are binding. Thus, if $(IC^H)$ is binding, the distortion in the low type's contract is only in the termination date $T^L$, while $T^H$ is first best. Similarly, the distortion is in $T^H$ when $(IC^L)$ is binding while $T^L$ is first best.

Again, moral hazard considerations tend to favor under-experimentation, while adverse selection has opposite implications on $T^L$ and $T^H$. Increasing $T^L$ reduces asymmetry of information rent for the high type, while reducing $T^H$ decreases the payoff of a misreporting low type. The optimal distortion depends on the strength of the moral hazard versus adverse selection concerns. For example, when $\gamma$ is very high, i.e., moral hazard is strong, there is no over-experimentation. Thus, over-experimentation is largely driven by experimentation having the potential to reduce the impact of asymmetric information as may be intuitive but not readily found in the literature.

## 4. Is integrating experimentation and production optimal?

In our model with experimentation and production, the interaction of adverse selection and moral hazard creates interdependent rents. In a pure moral hazard model, the principal would prefer to employ two different agents, one for experimenting and one for producing. This justifies the standard approach in the strategic experimentation literature, which studies a pure moral hazard experimentation stage model in isolation without a production stage. By separating the two stages, the principal saves what we have called the second moral hazard rent at the production stage. This begs the question of whether integrating the two tasks, as in our main model, can be optimal due to the presence of adverse selection.

A key benefit of integrating the two tasks is to use the adverse selection rent to induce effort, i.e., pay for the moral hazard rent. The rent needed to satisfy the $(IC)$ constraints can be spread across time to satisfy the dynamic moral hazard constraints. Since the relative probability of success across types is time-invariant, the exact distribution of this rent does not impact the incentive to misreport. This benefit must be balanced against the cost of the second moral

[39] Again, there is no restriction on when this rent is paid since the relative probability of success is independent of type.

hazard rent when integrating. We find that integration is optimal if the adverse selection problem is severe enough relative to the moral hazard problem in experimentation.

To establish the above result, we can use a very simple extension of our model, where the principal outsources the experimentation task to a second agent (experimenter). The first agent, the 'in-house' agent, produces output based on what is learned publicly in the experimentation stage, and on his private information about the likelihood of low cost, $\beta_0^\theta$. We discuss alternative models of separation at the end of this section. Before experimentation starts, the in-house producer is asked to publicly announce his type, which determines how experimentation proceeds. The principal pays an adverse selection rent to the producer to induce truthful reporting. The experimentation stage is a pure moral hazard problem, yielding only a standard moral hazard rent to the experimenter (based on a commonly known $\beta_0^\theta$).

When she separates the two tasks, the principal saves the moral hazard rent at the production stage but pays an adverse selection rent to the in-house producer and a moral hazard rent to the experimenter.

The decision between integration and separation hinges on the relative importance of moral hazard and adverse selection. When the moral hazard rent is relatively small compared to the adverse selection rent, the principal can effectively leverage the adverse selection rent to satisfy moral hazard constraints, and integration is optimal.[40]

We present below sufficient conditions for separation/integration to be optimal.

**Proposition 2:** ***Separation vs. integration*****:**

*(i)* *Separation is optimal if the adverse selection problem is small enough: for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\overline{\beta}_0^H(\beta_0^L)$, such that separation is optimal if $\beta_0^H < \overline{\beta}_0^H(\beta_0^L)$.*

*(ii)* *Integration is optimal if the adverse selection problem is severe enough ($\beta_0^H$ is close to one and $\beta_0^L$ sufficiently close to zero) and $\nu$ is high enough.*

***Proof***: See Appendix D.

[40] As we show in our proofs, this basic intuition holds regardless of which $(IC)$ is binding.

A possible issue regarding the model of separation above is that we assume an in-house producer publicly pre-announces the type $\beta_0^\theta$. We chose this benchmark for ease of comparison with the main model of integration. Instead, we could assume that the production agent is brought in after experimentation ends and therefore cannot announce his type before experimentation starts. Our key arguments regarding the optimality of integration would only get stronger. This would also be the case if the in-house producer privately announced his type $\beta_0^\theta$ to the principal. We briefly discuss these two sub-extensions next.

Consider first that, under separation, experimentation occurs under a common prior between the principal and the experimenter, that the production cost is low with probability $\beta_0$. There is now an additional cost of separation as the length of experimentation can no longer be based on the private information about $\beta_0^\theta$ of the (integrated) agent. Next consider the case where the in-house producer privately announces its type to the principal, who contracts with an outside experimenter. In the interim, a principal's incentive constraint would also have to be satisfied, which will again reduce the benefit of separation.

## 5. Endogenous Output

In this section, we allow the principal to optimally choose output after success and after failure, so she can now use output as an additional screening variable. While our main findings continue to hold, output after failure now serves as a screening device. Thus, the key new results occur when the experimentation stage fails: the low type is asked to under-produce relative to the first best, while the high type may over-produce. Just like over-experimentation, over-production can be used to increase the cost of misreporting.

When output is optimally chosen by the principal in the contract, the main change from the base model is that output after failure, which is denoted by $q_F^\theta$, can vary depending on the expected cost. We can replace $q_F$ by $q_F^\theta$ in the principal's problem.

We derive the formal output scheme in Appendix C but present the intuition here. When experimentation is successful, there is no asymmetric information and the marginal cost after success is type independent. We prove that there is no reason to distort the output. Both types produce the first-best output. When experimentation fails to reveal the cost, asymmetric information will induce the principal to distort the output to limit the rent.

When both $(IC^L)$ and $(IC^H)$ are slack, each type under-produces after failure to reduce the moral hazard rent. When the $(IC^H)$ constraint is slack, but $(IC^L)$ binding, the output for the low type, $q_F^\theta$, does not affect information rents and, as a result, is not distorted. The high type, however, may be asked to over-produce whenever the misreporting low type is more pessimistic than the principal after failing in experimentation. Over-production is, therefore, used to increase the cost of misreporting low type.[41]

When the $(IC^H)$ binds and $(IC^L)$ is slack, the low type is asked to under-produce in order to limit the rent of the high type. The output for the high type, $q_F^H$, does not affect information rents and, as a result, is not distorted. When both $(IC)$ constraints are binding, the low type under-produces to limit the rent of the high type, and the high type might be asked to over-produce to increase the cost of the misreporting low type.

## 6. Conclusions

We presented a dynamic model of strategic experimentation with both moral hazard and adverse selection. We offer a tractable model that explains the co-existence of both high- and low-powered incentive schemes, frequently used in practice to spur innovative activity.[42] We find that, while moral hazard always leads the principal to reward success in experimentation, the simultaneous presence of adverse selection can make it optimal for the principal to reward failure. The reason is that rewarding failure allows the principal to dynamically screen the agents, and it remains optimal even in the presence of moral hazard. We further characterize how the principal can use over and under-experimentation to provide incentives.

We derive the above insights by explicitly incorporating a production stage following a multi-period learning stage of strategic experimentation. We formally show that the principal may prefer to integrate experimentation and production by employing a single agent for both tasks. The standard model of experimentation, where experimentation is studied in isolation without a production stage, remains valid as long as adverse selection during experimentation is

[41] Since the misreporting low type may shirk off the equilibrium path, his expected cost at the production stage does not necessarily have to be greater than the expected cost for a high type on the equilibrium path. Thus overproduction is optimal when off the equilibrium path effort of the misreporting low type involves very little shirking.

[42] Technically, such a mixed model of experimentation can become intractable particularly due to the complexity of characterizing off-the-equilibrium path effort. If the adverse selection lies in the probability of success, as in Halac et al. (2016), the relative probability of success between the two types changes in ranking over time.

not a significant concern. However, when adverse selection is severe enough relative to moral hazard, integrating experimentation and production allows the principal to use the adverse selection rent to incentivize the agent to work. By optimally distributing the adverse selection rent, the principal can alleviate the moral hazard constraints.

## Appendix A: Proofs of Claims 1, 2, and 3

***Outline:*** We derive the optimal contract by solving for the payments and termination dates simultaneously. In Sections I and II, we consider four cases depending on which $IC$ constraint is binding, without determining which one applies in equilibrium. In Section I, Claims A1-A4, we characterize the structure of payments in terms of the optimal $T^\theta$ for each case. In Section II, Cases 1-4, we calculate the agent's rent using the optimal payments from $(A1)-(A4)$ in terms of the optimal $T^\theta$ for the case. Then, we determine whether the rent is increasing or decreasing in $T^\theta$ to characterize the distortion in $T^\theta$ relative to the first best. Finally, in Section III, we give sufficient conditions that determine which of the four cases applies in equilibrium.

## Section I. The optimal payment structure

First, we follow the standard steps to replace the global moral hazard $(MH^\theta)$ constraint with a sequence of one-period $(MH_t^\theta)$ constraints that we will use in the principal's problem. Second, we write the $RHS$ of $(IC^L)$ in terms of off-the path effort of the misreporting low type rather than the simplification in terms of the number of periods worked, $t^{L,H}$, we used to explain the intuition in Section 3.1. Third, we present the principal's problem and the Lagrangian, and we solve the optimal payment structure for each of the four cases, verifying the global moral hazard $(MH^\theta)$ constraint is satisfied.

***Deriving $(MH_t^\theta)$.*** Suppose a type $\theta$ agent has worked in every period until $t$ without success and will work in all periods after $t$. If the agent works in period $t$, his continuation value from the relationship is

$$-\gamma+\lambda\beta_t^\theta y_t^\theta+(1-\lambda\beta_t^\theta)x_t^\theta+\left(1-\beta_t^\theta\right)\sum_{s=t+1}^{T^\theta}\left(x_s^\theta-\gamma\right)$$

$$+\beta_t^\theta\sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t}\left(\lambda y_s^\theta+(1-\lambda)x_s^\theta-\gamma\right).$$

If the agent secretly shirks in period $t$, his continuation value from the relationship is

$$x_t^\theta+\left(1-\beta_t^\theta\right)\sum_{s=t+1}^{T^\theta}\left(x_s^\theta-\gamma\right)+\beta_t^\theta\sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta+(1-\lambda)x_s^\theta-\gamma\right)$$

$$+\left(1-\beta_t^\theta+\beta_t^\theta(1-\lambda)^{T^\theta-t}\right)\left(c_{T^\theta+1}^\theta-c_{T^\theta}^\theta\right)q_F.$$

Given that $c_{T^\theta+1}^\theta-c_{T^\theta}^\theta=\left(\beta_{T^\theta}^\theta-\beta_{T^\theta+1}^\theta\right)\left(\bar{c}-\underline{c}\right)=\frac{(1-\beta_0^\theta)\beta_0^\theta(1-\lambda)^{T^\theta-1}\lambda}{P_{T^\theta}^\theta P_{T^\theta-1}^\theta}\left(\bar{c}-\underline{c}\right)$, and

$1-\beta_t^\theta+\beta_t^\theta(1-\lambda)^{T^\theta-t}=\frac{P_{T^\theta-1}^\theta}{P_{t-1}^\theta}$, combining the two continuation values, the moral hazard constraint at period $t$ becomes

$$\left(MH_t^\theta\right)\ y_t^\theta-x_t^\theta\geq\frac{\gamma}{\lambda\beta_t^\theta}+\sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta+(1-\lambda)x_s^\theta-\gamma\right)+\frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta-t}}{P_{T^\theta}^\theta}\Delta c q_F.$$

***RHS of the $(IC^L)$.*** We express the $RHS$ of $(IC^L)$ in terms of off-the path effort of the misreporting low type and simplify the notation. Recall from Section 2.5 that off-the-path effort profile of the misreporting low type, $\vec{e}^L(\varpi^H)$, is given by

$$\vec{e}^L(\varpi^H) \equiv argmax_{\vec{a}^L}\, U^L(\varpi^H, \vec{a}^L).$$

Going forward, we simplify the notation for $\vec{e}^L(\varpi^H)$ to $\vec{e}^{L,H}$ by suppressing $(\varpi^H)$.

On the *RHS* of $(IC^L)$, we denote the expected cost by $c^L_{\sum_{s=1}^{T^H} e_s^{L,H}+1}$. Thus, the *RHS* of $(IC^L)$ is given by

$$\begin{aligned} U^L(\varpi^H, \vec{e}^{L,H}) = {} & (1-\beta_0^L) \textstyle\sum_{t=1}^{T^H}\left[x_t^H - \gamma e_t^{L,H}\right] \\ & + \beta_0^L \textstyle\sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\left[e_t^{L,H}\lambda y_t^H + (1-\lambda e_t^{L,H}) x_t^H - e_t^{L,H}\gamma\right] \\ & + \left(1-\beta_0^L + \beta_0^L\left(\textstyle\prod_{s=1}^{T^H}(1-\lambda e_s^{L,H})\right)\right)\left(c^H_{T^H+1} - c^L_{\sum_{s=1}^{T^H} e_s^{L,H}+1}\right) q_F\,. \end{aligned}$$

**The principal's optimization problem** is to choose contract $\varpi^\theta$ for $\theta \in \{H, L\}$ to maximize

$$E_\theta\left\{\beta_0^\theta \textstyle\sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V(q_S) - \underline{c}q_S - y_t^\theta\right] + P^\theta_{T^\theta}\left[V(q_F) - c^\theta_{T^\theta+1}q_F\right] - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta\right\} \text{ s.t.,}$$

$(IC^\theta)$ $$U^\theta(\varpi^\theta, \vec{1}) \geq U^\theta\left(\varpi^{\hat{\theta}}, \vec{e}^\theta(\varpi^{\hat{\theta}})\right),$$

and for $t \leq T^\theta$:

$$(MH_t^\theta)\; y_t^\theta - x_t^\theta \geq \frac{\gamma}{\lambda\beta_t^\theta} + \sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta + (1-\lambda)x_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta - t}}{P^\theta_{T^\theta}}\Delta c q_F,$$

$(LLS_t^\theta)$ $$y_t^\theta \geq 0,$$

$(LLF_t^\theta)$ $$x_t^\theta \geq 0.$$

The $(MH_t^\theta)$ constraints imply that all the $(LLS_t^H)$ and $(LLS_t^L)$ constraints are automatically satisfied and, therefore, can be ignored. Labeling $\xi^H$, $\xi^L$, $\{\mu_t^H\}_{t=1}^{T^H}$, $\{\mu_t^L\}_{t=1}^{T^L}$, $\{\eta_t^H\}_{t=1}^{T^H}$, $\{\eta_t^L\}_{t=1}^{T^L}$ as the Lagrange multipliers of the constraints associated with $(IC^H)$, $(IC^L)$, $(MH_t^H)$, $(MH_t^L)$, $(LLF_t^H)$ and $(LLF_t^L)$, respectively, and recalling $\Omega^\theta$ is the expected surplus net of cost in the first best, the optimization problem has the following Lagrangian:

$$\mathcal{L} = E\left[\beta_0^\theta \textstyle\sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V(q_S) - \underline{c}q_S - y_t^\theta\right] + P^\theta_{T^\theta}\left[V(q_F) - c^\theta_{T^\theta+1}q_F\right] - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta\right]$$

$$+\xi^H\begin{bmatrix} \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H\gamma \\ -(1-\beta_0^H)\sum_{t=1}^{T^L}[x_t^L - \gamma] - \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}[\lambda y_t^L + (1-\lambda)x_t^L - \gamma] \\ -P^H_{T^L}\Delta c_{T^L+1} q_F \end{bmatrix}$$

$$+\xi^L\begin{bmatrix} \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L\gamma \\ -(1-\beta_0^L)\sum_{t=1}^{T^H}\left[x_t^H - \gamma e_t^{L,H}\right] - \beta_0^L \sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\left[e_t^{L,H}\lambda y_t^H + (1-\lambda e_t^{L,H})x_t^H - e_t^{L,H}\gamma\right] \\ -\left(1-\beta_0^L + \beta_0^L\left(\prod_{s=1}^{T^H}(1-\lambda e_s^{L,H})\right)\right)\left(c^H_{T^H+1} - c^L_{\sum_{s=1}^{T^H} e_s^{L,H}+1}\right) q_F \end{bmatrix}$$

$$+\sum_{t=1}^{T^H}\mu_t^H\left[y_t^H - x_t^H - \frac{\gamma}{\lambda\beta_t^H} - \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) - \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c q_F\right]$$

$$+\sum_{t=1}^{T^L}\mu_t^L\left[y_t^L - x_t^L - \frac{\gamma}{\lambda\beta_t^L} - \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) - \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F\right]$$

$$+\sum_{t=1}^{T^H}\eta_t^H x_t^H + \sum_{t=1}^{T^L}\eta_t^L x_t^L.$$

The relevant first-order conditions for the optimal payments are the following: [43]

**(A1)** $$\frac{\partial\mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda - \xi^L\beta_0^L\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\lambda e_t^{L,H} + \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j-1}\lambda = 0;$$

**(A2)** $$\frac{\partial\mathcal{L}}{\partial y_t^L} = -(1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda - \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^L\beta_0^L(1-\lambda)^{t-1}\lambda + \mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda = 0;$$

**(A3)** $$\frac{\partial\mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \xi^L\left(1-\beta_0^L+\beta_0^L\prod_{s=1}^{t}(1-\lambda e_s^{L,H})\right) - \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j} + \eta_t^H = 0;$$

**(A4)** $$\frac{\partial\mathcal{L}}{\partial x_t^L} = -(1-\nu)P_t^L - \xi^H(1-\beta_0^H+\beta_0^H(1-\lambda)^t) + \xi^L P_t^L - \mu_t^L - \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j} + \eta_t^L = 0.$$

Instead of presenting the first order conditions for $T^\theta$ here, we first solve the optimal payments for each of the four cases using the above four conditions, taking as given the optimal $T^\theta$ for each case. This allows us to calculate the agent's rent in each case. Then, without explicitly computing those optimal $T^\theta$, we show whether there is over- or under-experimentation by checking if the agent's rent can be reduced by increasing or decreasing in $T^\theta$ relative to the first best.

We consider four cases depending on whether the multipliers on the $IC$ constraints, $(\xi^H, \xi^L)$, are strictly positive or not. We refer to the case with $\xi^\theta = 0$ as the corresponding $(IC^\theta)$ being slack but our proof does not require the $(IC^\theta)$ to be a strict inequality.[44]

**Case 1**: The multipliers $\xi^H = \xi^L = 0$: strong moral hazard (both IC constraints are slack).

If both the $(IC)$ constraints are slack, we find that the moral hazard constraints for both types are binding in every period. Each type is rewarded with a constant payment after success, and neither type is rewarded after failure.

[43] Recall that $e_t^{L,H}$ is endogenous and solves the agent's maximization problem $max_{\vec{a}^L}\ U^L(\varpi^H, \vec{a}^L)$. We apply an Envelope theorem for integers (see e.g., Sah and Zhao (1998), and Milgrom and Segal (2003)) with continuous parameters $x_t^H$ and $y_t^H$ and integer values for effort $e_t^{L,H} \in \{0,1\}$. Thus we have, for instance, $\frac{dU^L(\varpi^H,\vec{e}^{L,H})}{dy_t^H} = \frac{\partial U^L}{\partial y_t^H} = \beta_0^L\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\lambda e_t^{L,H}$.

[44] In a degenerate case, it is possible that a constraint is satisfied as an equality, but the multiplier is zero.

**Claim A1.** $\xi^H = \xi^L = 0 \Rightarrow \eta_t^H, \eta_t^L, \mu_t^H, \mu_t^L > 0$, and it is optimal to set

$$x_t^\theta = 0 \text{ and } y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F \text{ for } t \le T^\theta \text{ and } \theta \in \{H, L\}.$$

*Proof*:
$\boldsymbol{\mu_t^\theta} > \mathbf{0}$. We first prove that if the two $(IC)$ constraints are slack, then all the $(MH_t^\theta)$ constraints for $t \le T^\theta$ and $\theta \in \{H, L\}$ must be binding.

$\boldsymbol{\mu_t^H} > \mathbf{0}$. Given that $\xi^H = \xi^L = 0$, $(A1)$ at each period $t \le T^H$ can be rewritten as
$t = 1$: $-\nu\beta_0^H\lambda + \mu_1^H = 0 \implies \mu_1^H = \nu\beta_0^H\lambda > 0$;
$t = 2$: $-\nu\beta_0^H(1-\lambda)\lambda + \mu_2^H - \lambda\mu_1^H = 0 \implies \mu_2^H = \nu\beta_0^H\lambda > 0$;
Solving recursively for $t = 3, \dots, T^H$ we have $\mu_t^H = \nu\beta_0^H\lambda > 0$ for $t \le T^H$.
Thus, all the $(MH_t^H)$ constraints are binding.

$\boldsymbol{\mu_t^L} > \mathbf{0}$. Given that $\xi^H = \xi^L = 0$, $(A2)$ at each period $t \le T^L$ can be rewritten as
$t = 1$: $-(1-\nu)\beta_0^L\lambda + \mu_1^L = 0 \implies \mu_1^L = (1-\nu)\beta_0^L\lambda > 0$;
$t = 2$: $-(1-\nu)\beta_0^L(1-\lambda)\lambda + \mu_2^L - \lambda\mu_1^L = 0 \implies \mu_2^L = (1-\nu)\beta_0^L\lambda > 0$;
Solving recursively for $t = 3, \dots, T^L$ we have $\mu_t^L = (1-\nu)\beta_0^L\lambda > 0$ for $t \le T^L$.
Thus, all the $(MH_t^L)$ constraints are binding.

We next prove that neither type is paid after failure.

$\boldsymbol{x_t^\theta} = \mathbf{0}$. No rent after failure follows immediately from $(A3)$ and $(A4)$ as $\xi^H = \xi^L = 0$ implies $\eta_t^\theta > 0$ for all $t \le T^\theta$. Furthermore, given that $\mu_t^H = \nu\beta_0^H\lambda$, and $\mu_t^L = (1-\nu)\beta_0^L\lambda$, we find that $\eta_t^H = \nu(P_t^H + \beta_0^H\lambda) + \nu\beta_0^H\lambda\sum_{j=1}^{t-1}(1-\lambda)^{t-j} > 0$ for $t \le T^H$, and that $\eta_t^L = (1-\nu)(P_t^L + \beta_0^L\lambda) + (1-\nu)\beta_0^L\lambda\sum_{j=1}^{t-1}(1-\lambda)^{t-j} > 0$ for $t \le T^L$.
Thus, if the $(IC)$ constraints are slack, both types are rewarded only for success and all the $(MH_t^\theta)$ constraints for $t \le T^\theta$ and $\theta \in \{H, L\}$ are binding:

$$y_t^\theta = \frac{\gamma}{\lambda\beta_t^\theta} + \sum_{s=t+1}^{T^\theta}(1-\lambda)^{s-t-1}\left(\lambda y_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta - t}}{P_{T^\theta}^\theta}\Delta c q_F,$$
$$x_t^\theta = 0 \text{ for } t \le T^\theta \text{ and } \theta \in \{H, L\}.$$

Finally, we prove that the unique sequence of $y_t^\theta$ that solves the system of binding $(MH_t^\theta)$ constraints is $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$. Solving recursively binding $(MH_t^\theta)$ constraints for $y_t^\theta$ we obtain: $y_t^\theta = \frac{\gamma}{\lambda\beta_t^\theta} + \gamma\sum_{s=1}^{T^\theta - t}\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^{t+s-1}} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$.

We next prove that $y_t^\theta$ is constant:

$y_t^\theta - y_{t+1}^\theta =$

$$\left[\frac{\gamma}{\lambda\beta_t^\theta} + \gamma\sum_{s=1}^{T^\theta - t}\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^{t+s-1}} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F\right] - \left[\frac{\gamma}{\lambda\beta_{t+1}^\theta} + \gamma\sum_{s=1}^{T^\theta - t-1}\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^{t+s}} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F\right]$$

$$= \frac{\gamma}{\lambda\beta_t^\theta} - \frac{\gamma}{\lambda\beta_{t+1}^\theta} + \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^{t-1}}\sum_{s=1}^{T^\theta-t}\frac{1}{(1-\lambda)^s} - \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^t}\sum_{s=1}^{T^\theta-t-1}\frac{1}{(1-\lambda)^s}.$$

Using the formula for geometric series, we can rewrite $\sum_{s=1}^{T-t}\frac{1}{(1-\lambda)^s}$ and $\sum_{s=1}^{T-t-1}\frac{1}{(1-\lambda)^s}$ as:

$$\sum_{s=1}^{T-t}\frac{1}{(1-\lambda)^s} = \frac{1}{(1-\lambda)}\left(\frac{1-\frac{1}{(1-\lambda)^{T-t}}}{1-\frac{1}{(1-\lambda)}}\right) = \frac{\left(\frac{1}{(1-\lambda)^{T-t}}-1\right)}{\lambda} = \frac{1-(1-\lambda)^{T-t}}{\lambda(1-\lambda)^{T-t}}$$

$$\text{and } \sum_{s=1}^{T-t-1}\frac{1}{(1-\lambda)^s} = \frac{\left(\frac{1}{(1-\lambda)^{T-t-1}}-1\right)}{\lambda} = \frac{1-(1-\lambda)^{T-t-1}}{\lambda(1-\lambda)^{T-t-1}}.$$

Thus, $y_t^\theta - y_{t+1}^\theta =$

$$= \frac{\gamma}{\lambda\beta_t^\theta} - \frac{\gamma}{\lambda\beta_{t+1}^\theta} + \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^{t-1}}\left(\frac{1-(1-\lambda)^{T^\theta-t}}{\lambda(1-\lambda)^{T^\theta-t}}\right) - \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^t}\left(\frac{1-(1-\lambda)^{T^\theta-t-1}}{\lambda(1-\lambda)^{T^\theta-t-1}}\right)$$

$$= \frac{\gamma}{\lambda\beta_t^\theta} - \frac{\gamma(1-\lambda\beta_t^\theta)}{\lambda\beta_t^\theta(1-\lambda)} + \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta\lambda(1-\lambda)^{T^\theta-1}}\left(1-(1-\lambda)^{T^\theta-t} - 1 + (1-\lambda)^{T^\theta-t-1}\right)$$

$$= \frac{\gamma}{\lambda\beta_t^\theta} - \frac{\gamma(1-\lambda\beta_t^\theta)}{\lambda\beta_t^\theta(1-\lambda)} + \gamma\frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta-t-1}}{\beta_0^\theta\lambda(1-\lambda)^{T^\theta-1}}\lambda = -\gamma\frac{\lambda(1-\beta_t^\theta)}{\lambda\beta_t^\theta(1-\lambda)} + \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^t}$$

$$= -\gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^t} + \gamma\frac{(1-\beta_0^\theta)}{\beta_0^\theta(1-\lambda)^t} = 0.$$

Given that that $y_t^\theta$ is constant, we can rewrite $y_t^\theta$ by evaluating it at $t = T^\theta$:

$$y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F.$$

Finally, we prove the $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ derived from $(MH_t^\theta)$ also satisfies $(MH^\theta)$, which implies that that replacing the global $(MH^\theta)$ constraint by a sequence of local one-period constraints $(MH_t^\theta)$ is without loss of generality. Specifically, we show that $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ implies that $e_t^\theta = 1$, for all $t$, without restricting beliefs or possible effort deviations.

Consider the final period $T^\theta$, and denote by $\beta_{T^\theta}^\theta$ the belief if the agent has worked in each prior period $t < T^\theta$. The agent will work in period $T^\theta$ regardless of the effort profile in earlier periods. Note that if the agent shirked at some arbitrary period $t < T^\theta$ (i.e., worked for $\hat{T}^\theta < T^\theta$ periods only), he can only be more optimistic at period $T^\theta$. Thus, for any history of prior effort, the agent's current belief, denoted by $\hat{\beta}^\theta$, can only be greater than $\beta_{T^\theta}^\theta$. Then, the payment $y_{T^\theta}^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ satisfies $\left(MH_T^\theta\right)$ since $\hat{\beta}^\theta \geq \beta_{T^\theta}^\theta$:

$$\lambda\hat{\beta}^\theta y_{T^\theta}^\theta - (c_{T^\theta+1}^\theta - c_{T^\theta}^\theta)q_F = \lambda\hat{\beta}^\theta\left[y_{T^\theta}^\theta - \frac{1}{\lambda\hat{\beta}^\theta}(c_{T^\theta+1}^\theta - c_{T^\theta}^\theta)q_F\right]$$

$$= \lambda\hat{\beta}^{\theta}\left[\frac{\gamma}{\lambda\beta^{\theta}_{T^{\theta}}} + \left(1 - \frac{P^{\theta}_{\hat{T}^{\theta}-1}(1-\lambda)^{T^{\theta}-1}}{P^{\theta}_{T^{\theta}-1}(1-\lambda)^{\hat{T}^{\theta}-1}}\right)\left(\frac{(1-\beta^{\theta}_{0})}{P^{\theta}_{T^{\theta}}}\Delta c q_F\right)\right] \geq \gamma \text{ since } \frac{P^{\theta}_{\hat{T}^{\theta}-1}(1-\lambda)^{T^{\theta}-1}}{P^{\theta}_{T^{\theta}-1}(1-\lambda)^{\hat{T}^{\theta}-1}} \leq 1,$$

and therefore, $e^{\theta}_{T^{\theta}} = 1$.

The same argument applies to show that $e^{\theta}_{t} = 1$ for periods $t < T^{\theta} - 1$ noting that $e^{\theta}_{s} = 1$ for all $s > t$.

This concludes the characterization of the optimal payments in Claim A1.

**Case 2**: The multipliers $\xi^H > 0$, $\xi^L > 0$: strong adverse selection ($(IC^H)$ and $(IC^L)$ are binding)

We now characterize the payment structure for Case 2, when both $(IC)$ constraints are binding. We first prove that the principal can reward the high type only after success and distribute the rewards such that all the $(MH^H_t)$ constraints are satisfied at no additional cost, i.e., $\mu^H_t = 0$ for $t \leq T^H$. For the low type, we first prove that all the $(MH^L_t)$ for $t \leq T^L$ are binding. If the $(IC^L)$ binds, the low type receives a rent higher than the rent implied by $y^L_t$ from all binding $(MH^L_t)$ constraints with $x^L_t = 0$ (Case 1). Then, to satisfy $(IC^L)$, there must be an additional rent paid through a strictly positive $x^L_t$ for some $t$.[45] We then prove that this reward for failure can be paid in the final period $T^L$ without loss of generality. Finally, we verify that the global $(MH^{\theta})$ constraints are satisfied.

**Claim A2.** $\xi^H > 0, \xi^L > 0 \Rightarrow \eta^H_t > 0 = \mu^H_t$ for $t \leq T^H$, $\mu^L_t > 0$ for $t \leq T^L$, $\eta^L_t > 0 = \eta^L_{T^L}$ for $t < T^L$. Recalling that the $RHS$ of $(IC^H)$ is denoted by $U^H(\varpi^L, \vec{1})$, it is optimal to set $x^H_t = 0$ and use any combination of $y^H_t$, including front-loading to the first period, such that

$$y^H_t \geq \frac{\gamma}{\lambda\beta^H_t} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y^H_s - \gamma) + \frac{(1-\beta^H_0)(1-\lambda)^{T^H-t}}{P^H_{T^H}}\Delta c q_F \text{ for } t \leq T^H, \text{ and}$$

$$\beta^H_0 \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y^H_t - \gamma\sum_{t=1}^{T^H} P^H_t = U^H(\varpi^L, \vec{1}),$$

while, the low type's payments are given by,

$$y^L_{T^L} = x^L_{T^L} + \frac{\gamma}{\lambda\beta^L_{T^L}} + \frac{(1-\beta^L_0)}{P^L_{T^L}}\Delta c q_F,$$

$$y^L_t = \frac{\gamma}{\lambda\beta^L_t} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y^L_s - \gamma) + (1-\lambda)^{T^L-t}x^L_{T^L} + \frac{(1-\beta^L_0)(1-\lambda)^{T^L-t}}{P^L_{T^L}}\Delta c q_F \text{ for } t < T^L,$$

and

$$\beta^L_0 \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y^L_t + P^L_{T^L}x^L_{T^L} - \gamma\sum_{t=1}^{T^L} P^L_{t-1} = U^L(\varpi^H, \vec{e}^{L,H}).$$

*Proof*:

***H-type.***

$\boldsymbol{\mu^H_t = 0}$**.** There exists a solution to $(A1)$ and $(A3)$ for $t \leq T^H$ such that for all $t \leq T^H$: $\xi^H > 0, \xi^L > 0, \mu^H_t = 0 < \eta^H_t$. From $(A1)$:

[45] We ignore the knife-edge case where both the $(IC)$ constraints and all the $(MH^L_t)$ are binding simultaneously. In that case, the low type's adverse selection rent is exactly equal to the moral hazard rent and there is no additional rent to be paid.

$$\xi^H \beta_0^H (1-\lambda)^{t-1}\lambda = \nu \beta_0^H (1-\lambda)^{t-1}\lambda + \xi^L \beta_0^L \left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{L,H}\right)\right)\lambda e_t^{L,H} > 0,$$

and using it in (A3):

$$\xi^L = \eta_t^H \frac{\beta_0^H (1-\lambda)^{t-1}\lambda}{\left(\left(1-\beta_0^L+\beta_0^L \prod_{s=1}^{t}\left(1-\lambda e_s^{L,H}\right)\right)\beta_0^H(1-\lambda)^{t-1}\lambda - \beta_0^L\left(\prod_{s=1}^{t-1}\left(1-\lambda e_s^{L,H}\right)\right)\lambda e_t^{L,H} P_t^H\right)} > 0.$$

Therefore, $\eta_t^H > 0$, and the principal sets $x_t^H = 0$ and uses any combination of $y_t^H$ such that

$$y_t^H \geq \frac{\gamma}{\lambda \beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H - \gamma) + \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c q_F \text{ for } t \leq T^H, \text{ and}$$

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H - \gamma \sum_{t=1}^{T^H} P_{t-1}^H = U^H(\varpi^L, \vec{1}).$$

***L-type.***

$\boldsymbol{\mu_t^L > 0}$**.** Combining $(A2)$ and $(A4)$ we have:[46]

$$(\boldsymbol{A5}) \qquad -\xi^H \lambda (1-\lambda)^{t-1}(\beta_0^H - \beta_0^L) + \mu_t^L P_{t-1}^L$$
$$= \beta_0^L (1-\lambda)^{t-1}\lambda \eta_t^L + (1-\beta_0^L)\sum_{j=1}^{t-1}\mu_j^L (1-\lambda)^{t-j-1}\lambda \text{ for } t \leq T^L.$$

Given that the *RHS* of $(A5)$ is non-negative, we have, $\mu_t^L > 0$ for every $t \leq T^L$ and, as a result, all the $(MH_t^L)$ constraints must be binding:

$$y_t^L - x_t^L = \frac{\gamma}{\lambda \beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L.$$

$\boldsymbol{x_{T^L}^L > x_t^L = 0}$ **for** $\boldsymbol{t < T^L}$**.** Suppose that the low type is rewarded for failure in period $z < T^L$, such that $x_z^L > 0$. We next prove that the principal can decrease $x_z^L$ to zero and increase $x_{T^L}^L$ by $x_z^L$ without affecting the $(IC^L)$.

From the binding $(MH_t^L)$ constraints, lowering $x_z^L$ to zero allows lowering $y_z^L$ by $x_z^L$. Continuing, we can see $y_t^L$ is reduced by $x_z^L$ for all $t \leq z$ as follows: $y_{z-1}^L$ can also be reduced by $\lambda x_z^L + (1-\lambda)x_z^L = x_z^L$, $y_{z-2}^L$ by reduction in $\lambda y_{z-1}^L + (1-\lambda)x_{z-1}^L$ and $(1-\lambda)$ multiplied by reduction in $\lambda y_z^L + (1-\lambda)x_z^L$, so overall the reduction is $\lambda x_z^L + (1-\lambda)[\lambda x_z^L + (1-\lambda)x_z^L] = x_z^L$, and so on. Thus, $y_t^L$ is reduced by $x_z^L$ for $t \leq z$, and the total reduction in the rent paid to the low type due to lowering $x_z^L$ to zero is (we use $\sum_{t=1}^{z}(1-\lambda)^{t-1} = \frac{1-(1-\lambda)^z}{\lambda}$):

$$\beta_0^L \sum_{t=1}^{z}(1-\lambda)^{t-1}\lambda x_z^L + P_z^L x_z^L = \beta_0^L \lambda x_z^L \left[\frac{1-(1-\lambda)^z}{\lambda}\right] + [1-\beta_0^L + \beta_0^L(1-\lambda)^z]x_z^L$$
$$= x_z^L[\beta_0^L - \beta_0^L(1-\lambda)^z + 1 - \beta_0^L + \beta_0^L(1-\lambda)^z] = x_z^L.$$

Suppose we increase $x_{T^L}^L$ by $b$. From the binding $(MH_t^L)$ constraints, increasing $x_{T^L}^L$ by $b$ implies that $y_{T^L}^L$ must be increased by $b$ to satisfy $\left(MH_{T^L}^L\right)$, $y_{T^L-1}^L$ by $\lambda b + (1-\lambda)b = b$, $y_{T^L-2}^L$ by an increase in $\lambda y_{T^L-1}^L + (1-\lambda)x_{T^L-1}^L$ and $(1-\lambda)$ multiplied by the increase in $\lambda y_z^L + (1-\lambda)x_z^L$, so overall the increase is $\lambda b + (1-\lambda)[\lambda b + (1-\lambda)b] = b$, and so on. Thus, $y_t^L$ can be increased by $b$ for $t \leq T^L$ to satisfy $(MH_t^L)$, and the total increase in the rent paid to the low type due to an increase in $x_{T^L}^L$ by $b$ is

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda b + P_{T^L}^L b = \beta_0^L \lambda b \left[\frac{1-(1-\lambda)^{T^L}}{\lambda}\right] + \left[1-\beta_0^L + \beta_0^L(1-\lambda)^{T^L}\right] = b.$$

[46] We multiply (A4) by $\beta_0^L(1-\lambda)^{t-1}\lambda$ and subtract it from (A2) multiplied by $P_t^L$.

Thus, we can decrease $x_z^L$ to zero and increase $x_{T^L}^L$ by $x_z^L$, without affecting the $(IC^L)$.
We can rewrite the $RHS$ of $(IC^H)$ as

$$\sum_{t=1}^{T^L} P_t^H x_t^L + P_{T^L}^H \Delta c_{T^L+1} q_F - \gamma \sum_{t=1}^{T^L} P_{t-1}^H + \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L.$$

Denoting the altered sequence by $\{\hat{x}_t^L, \hat{y}_t^L\}$, where $x_z^L$ is decreased to zero and $x_{T^L}^L$ is increased by $x_z^L$, and the $y_t^L$ are adjusted accordingly such that all $(MH_t^L)$ for $t \le T^L$ are satisfied ($\hat{y}_t^L = y_t^L$ for $t \le z$ and $\hat{y}_t^L = y_t^L + x_z^L$ for $t > z$), we obtain the $RHS$ of $(IC_H)$: $\sum_{t=1}^{T^L} P_t^H \hat{x}_t^L + P_{T^L}^H \Delta c_{T^L+1} q_F - \gamma \sum_{t=1}^{T^L} P_{t-1}^H + \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \hat{y}_t^L$. We next show that expected value of the $RHS$ of $(IC_H)$ remains unchanged.

First, we have $\sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \hat{y}_t^L = \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L + \frac{\left(P_z^L - P_{T^L}^L\right)}{\beta_0^L} x_z^L$ by recalling that the expected value of the $LHS$ of the $(IC^L)$ is not affected by decreasing $x_z^L$ to zero and increasing $x_{T^L}^L$ by $x_z^L$. Then, the difference in the $RHS$ of $(IC_H)$ due to decreasing $x_z^L$ to zero and increasing $x_{T^L}^L$ by $x_z^L$ is:

$$\left[P_{T^L}^H x_z^L + \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \hat{y}_t^L\right] - \left[P_z^H x_z^L + \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L\right]$$

$$= \left[P_{T^L}^H x_z^L + \beta_0^H \left[\sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L + \frac{\left(P_z^L - P_{T^L}^L\right)}{\beta_0^L} x_z^L\right]\right] - \left[P_z^H x_z^L + \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L\right]$$

$$= x_z^L \left[P_{T^L}^H + \frac{\beta_0^H \left(P_z^L - P_{T^L}^L\right)}{\beta_0^L} - P_z^H\right] = x_z^L \left[\frac{P_{T^L}^H \beta_0^L + \beta_0^H \left(P_z^L - P_{T^L}^L\right) - \beta_0^L P_z^H}{\beta_0^L}\right] = 0.$$

Thus, it is without loss of generality to reward the low type for failure in only period $T^L$.

Finally, repeating the same steps as in Case 1, but using $x_{T^L}^L > 0$, we can prove that replacing the global $(MH^\theta)$ constraint by a sequence of local one-period constraints $(MH_t^\theta)$ is without loss of generality.

This concludes the proof of Claim A2. *Q.E.D.*

**Intermediate case 3**: The multipliers $\xi^H > 0$, $\xi^L = 0$ ($(IC^H)$ constraint binds)

If $(IC^H)$ binds and $(IC^L)$ is slack, the $(MH_t^L)$ are binding in each period but $(MH_t^H)$ are all slack. We first prove that the low type is rewarded only for success and all the $(MH_t^L)$ constraints are binding for $t \le T^L$. We then prove that the principal can use only rewards after success $y_t^H \ge 0 = x_t^H$ for $t \le T^H$ such that all $(MH_t^H)$ constraints are satisfied at no additional cost, i.e., $\mu_t^H = 0$ for $t \le T^H$.

**Claim A3.** $\xi^H > 0, \xi^L = 0 \Rightarrow \eta_t^L, \mu_t^L > 0$ and $\eta_t^H = \mu_t^H = 0$. It is optimal to set

$$x_t^L = 0 \text{ and } y_t^L = \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \text{ for } t \le T^L$$

and any combination of $x_t^H$ and $y_t^H$ such that, for $t \le T^H$:

$$y_t^H - x_t^H \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) + \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c q_F,$$

and $\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma = U^H(\varpi^L, \vec{1})$ is given by the binding $(IC^H)$.

*Proof*:

***L-type.***

$\boldsymbol{\mu_t^L > 0}$**.** Given that $\xi^L = 0$ and $\xi^H > 0$, condition $(A2)$ at each period $t \leq T^L$ can be rewritten as

$$\mu_t^L = (1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda + \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j-1}\lambda > 0 \text{ for } t \leq T^L.$$

Thus, all the $(MH_t^L)$ constraints are binding.

We next prove that the low type is rewarded only for success, i.e., $x_t^L = 0$ for $t \leq T^L$.

$\boldsymbol{x_t^L = 0}$**.** Given that $\xi^L = 0$ and $\xi^H > 0$ condition $(A4)$ at each period $t \leq T^L$ can be rewritten as

$$\eta_t^L = (1-\nu)P_t^L + \xi^H(1-\beta_0^H + \beta_0^H(1-\lambda)^t) + \mu_t^L + \sum_{j=1}^{t-1}\mu_j^L(1-\lambda)^{t-j} > 0 \text{ for } t \leq T^L.$$

Therefore, $\eta_t^L > 0$ for every $t \leq T^L$ and, as a result, the low type is not rewarded for failures:

$$x_t^L = 0 \text{ for } t \leq T^L.$$

Thus, the low type is rewarded only for success with the rewards given by:

$$y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L.$$

***H-type.***

In Lemma 1, we proved that the high type never shirks off-the-path, i.e., $\vec{e}^H(\varpi^L) = \vec{1}$.

$\boldsymbol{\mu_t^H = \eta_t^H = 0}$**.** Given that $\xi^L = 0$, conditions $(A1)$ and $(A3)$ can be rewritten as

$(A1')$ $\quad \frac{\partial \mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda + \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j-1}\lambda = 0;$

$(A3')$ $\quad \frac{\partial \mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \mu_t^H - \sum_{j=1}^{t-1}\mu_j^H(1-\lambda)^{t-j} + \eta_t^H = 0.$

There exists a solution to $(A1')$ and $(A3')$ for $t \leq T^H$ such that

$$\mu_t^H = \eta_t^H = 0 \text{ and } \xi^H = \nu \text{ for } t \leq T^H.$$

Therefore, the principal can use any combination of $x_t^H$ and $y_t^H$ such that, for $t \leq T^H$,

$$y_t^H - x_t^H \geq \frac{\gamma}{\lambda\beta_t^H} + \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) + \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c q_F$$

$$\text{and } \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma = U^H(\varpi^L, \vec{1})$$

Thus, we have proved that it is without loss for the principal to rely on $y_t^H \geq 0$ with $x_t^H = 0$ in the optimal contract.

Finally, repeating the same steps as in Case 1 proves that replacing the global $(MH^\theta)$ constraint by a sequence of local one-period constraints $(MH_t^\theta)$ is without loss of generality.

This concludes the proof of Claim A3. *Q.E.D.*

**Intermediate case 4**: The multipliers $\xi^H = 0$, $\xi^L > 0$: ($(IC^L)$ constraint binds)

If $(IC^L)$ binding and $(IC^H)$ is slack, the $(MH_t^H)$ are all binding but the $(MH_t^L)$ are all slack. We first prove that the high type is rewarded only for success and all the $(MH_t^H)$ constraints are binding for $t \leq T^H$. We then prove that the high type is rewarded only for success, i.e., $x_t^H = 0$ for $t \leq T^H$.

**Claim A4.** $\xi^H = 0, \xi^L > 0 \Rightarrow \eta_t^H, \mu_t^H > 0$ and $\mu_t^L = 0, \eta_t^L = 0$. It is optimal to set

$$x_t^H = 0 \text{ and } y_t^H = \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F \text{ for } t \leq T^H$$

and any combination of $x_t^L$ and $y_t^L$ such that

$$y_t^L - x_t^L \geq \frac{\gamma}{\lambda \beta_t^L} + \sum_{s=t+1}^{T^L} (1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda) x_s^L - \gamma) + \frac{(1-\beta_0^L)(1-\lambda)^{T^L - t}}{P_{T^L}^L} \Delta c q_F \text{ for } t \leq T^L$$

and $\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma = U^L(\varpi^H, \vec{e}^{L,H})$ is given by the binding $(IC^L)$.

*Proof*:

***H-type.***

$\boldsymbol{\mu_t^H > 0}$. Given that $\xi^H = 0$ and $\xi^L > 0$, condition $(A1)$ at each period $t \leq T^H$ can be rewritten as

$$\mu_t^H = \nu \beta_0^H (1-\lambda)^{t-1} \lambda + \xi^L \beta_0^L \left( \prod_{s=1}^{t-1} (1 - \lambda e_s^{L,H}) \right) \lambda e_t^{L,H}$$
$$+ \sum_{j=1}^{t-1} \mu_j^H (1-\lambda)^{t-j-1} \lambda > 0 \text{ for } t \leq T^H.$$

Thus, all the $(MH_t^H)$ constraints are binding.

$\boldsymbol{x_t^H = 0}$. Given that $\xi^H = 0$ and $\xi^L > 0$ condition $(A3)$ at each period $t \leq T^H$ can be rewritten as

$$\eta_t^H = \nu P_t^H + \xi^L \left(1 - \beta_0^L + \beta_0^L \prod_{s=1}^{t} (1 - \lambda e_s^{L,H})\right)$$
$$+ \mu_t^H + \sum_{j=1}^{t-1} \mu_j^H (1-\lambda)^{t-j} > 0 \text{ for } t \leq T^H.$$

Therefore, $\eta_t^H > 0$ for every $t \leq T^H$ and, as a result, the high type is not rewarded for failures:

$$x_t^H = 0 \text{ for } t \leq T^H.$$

Thus, the high type is rewarded only for success with the rewards given by:

$$y_t^H = \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F \text{ for } t \leq T^H.$$

***L-type.*** We next characterize the optimal contract for the low type.
We prove that the principal can use $y_t^L \geq 0 = x_t^L$ such that

$$y_t^L - x_t^L \geq \frac{\gamma}{\lambda \beta_t^L} + \sum_{s=t+1}^{T^L} (1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda) x_s^L - \gamma) + \frac{(1-\beta_0^L)(1-\lambda)^{T^L - t}}{P_{T^L}^L} \Delta c q_F \text{ for } t \leq T^L,$$

and

$$\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma = U^L\left(\varpi^H, \vec{a}^L(\varpi^H)\right).$$

$\boldsymbol{\mu_t^L = \eta_t^L = 0}$. Given that $\xi^H = 0$, conditions $(A2)$ and $(A4)$ can be rewritten as

(A2′) $\frac{\partial \mathcal{L}}{\partial y_t^L} = -(1-\nu)\beta_0^L (1-\lambda)^{t-1}\lambda + \xi^L \beta_0^L (1-\lambda)^{t-1} \lambda + \mu_t^L - \sum_{j=1}^{t-1} \mu_j^L (1-\lambda)^{t-j-1} \lambda = 0;$

(A4′) $\frac{\partial \mathcal{L}}{\partial x_t^L} = -(1-\nu) P_t^L + \xi^L P_t^L - \mu_t^L - \sum_{j=1}^{t-1} \mu_j^L (1-\lambda)^{t-j} + \eta_t^L = 0.$

There exists a solution to $(A2')$ and $(A4')$ for $t \leq T^L$ such that

$$\mu_t^L = \eta_t^L = 0 \text{ and } \xi^L = (1-\nu) \text{ for } t \leq T^L.$$

Therefore, the principal can use any combination of $x_t^L$ and $y_t^L$ such that

$$y_t^L - x_t^L \geq \frac{\gamma}{\lambda\beta_t^L} + \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) + \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F \text{ for } t \leq T^L,$$

and

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1} y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma = U^L(\varpi^H, \vec{e}^{L,H}).$$

Thus, we have proved that it is without loss for the principal to rely on $y_t^L \geq 0$ with $x_t^L = 0$ in the optimal contract.

Finally, repeating the same steps as in Case 1 proves that replacing the global $(MH^\theta)$ constraint by a sequence of local one-period constraints $(MH_t^\theta)$ is without loss of generality.

This concludes the proof of Claim A4. *Q.E.D.*

## Section II Optimal length of experimentation

Before characterizing distortions in $T^\theta$, we express the principal's objective function in terms of rent to the agent. Recall that for a type-$\theta$ agent, first-best $T^\theta$ maximizes the surplus net of cost denoted by:

$$\Omega^\theta = \beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V(q_S) - \underline{c}q_S\right] + P_{T^\theta}^\theta\left[V(q_F) - c_{T^\theta+1}^\theta q_F\right] - \sum_{t=1}^{T^\theta} P_{t-1}^\theta \gamma.$$

Recall also that in the second-best case the principal's payoff is given by:

$$E_\theta\left\{\beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V(q_S) - \underline{c}q_S - y_t^\theta\right] + P_{T^\theta}^\theta\left[V(q_F) - c_{T^\theta+1}^\theta q_F\right] - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta\right\}$$

$$= E_\theta\left[\Omega^\theta - U^\theta(\varpi^\theta, \vec{1})\right],$$

where $U^\theta(\varpi^\theta, \vec{1}) = \beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda y_t^\theta + \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta - \sum_{t=1}^{T^\theta} P_{t-1}^\theta \gamma$ is the $\theta$ agent's rent.

For each case, we calculate the agent's rent using the optimal payments from $(A1)-(A4)$ in terms of the optimal $T^\theta$ for the case. Then, we calculate whether the rent is increasing or decreasing in $T^\theta$ to characterize distortion in $T^\theta$ relative to the first best. However, because $T^\theta$ is discrete, it may not be optimal to distort it even if the rent is increasing/decreasing in $T^\theta$. Thus, our results for each case are subject to the caveat of sufficient granularity for discrete $T^\theta$.

**Case 1**: Both the $(IC^H)$ and $(IC^L)$ constraints are slack (under-experimentation for both types).
This is the familiar case where we find under-experimentation since rent is characterized only by the moral hazard constraints, and it can be reduced by lowering $T^\theta$.
The information rent for a type $\theta$ is given by the moral hazard rent:

$$U^\theta(\varpi^\theta, \vec{1}) = \beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda y_t^\theta - \sum_{t=1}^{T^\theta} P_{t-1}^\theta \gamma,$$

$$\text{where } y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F \text{ for } t \leq T^\theta.$$

Next, we show that $U^{\theta}(\varpi^{\theta},\vec{1})$ increases in $T^{\theta}$ by proving that the difference in $U^{\theta}(\varpi^{\theta},\vec{1})$ when the length is $T^{\theta}$ versus $T^{\theta}-1$ is strictly positive as $\frac{(1-\beta_0^{\theta})}{P_{T^{\theta}}^{\theta}}\Delta c q_F > 0$:

$$\beta_0^{\theta}\sum_{t=1}^{T^{\theta}}(1-\lambda)^{t-1}\lambda y_t^{\theta} - \sum_{t=1}^{T^{\theta}} P_{t-1}^{\theta}\gamma - \left(\beta_0^{\theta}\sum_{t=1}^{T^{\theta}-1}(1-\lambda)^{t-1}\lambda y_t^{\theta} - \sum_{t=1}^{T^{\theta}-1} P_{t-1}^{\theta}\gamma\right) > 0$$

$$\Leftrightarrow \beta_0^{\theta}(1-\lambda)^{T^{\theta}-1}\lambda y_{T^{\theta}}^{\theta} - P_{T^{\theta}-1}^{\theta}\gamma > 0 \Leftrightarrow y_{T^{\theta}}^{\theta} > \frac{P_{T^{\theta}-1}^{\theta}\gamma}{\beta_0^{\theta}(1-\lambda)^{T^{\theta}-1}\lambda} = \frac{\gamma}{\beta_{T^{\theta}}^{\theta}\lambda},$$

since $y_{T^{\theta}}^{\theta} = \frac{\gamma}{\lambda\beta_{T^{\theta}}^{\theta}} + \frac{(1-\beta_0^{\theta})}{P_{T^{\theta}}^{\theta}}\Delta c q_F$ from Claim A1.

As the agent's information rent is increasing in $T^{\theta}$, there will be *under*-experimentation relative to the first best for both types, that is, $T_{SB}^{\theta} < T_{FB}^{\theta}$ for $\theta \in \{H, L\}$.

**Case 2**: Both $(IC^H)$ and $(IC^L)$ bind (over-experimentation in $T^L$, $T_{SB}^L > T_{FB}^L$, if $\gamma$ is small enough, and under-experimentation in $T^H$, $T_{SB}^H < T_{FB}^H$).

In this case, the $RHS$ of each $(IC)$ involves both moral hazard and adverse selection rent, and we determine the distortion in $T^{\theta}$ by looking at the impact on each. Just like in Case 1, the moral hazard component increases in $T^{\theta}$, but the adverse selection component has different impacts depending on the agent's type.

We denote by $t^{L,H}$ the number of periods the misreporting low type works, given the optimal contract $\varpi^H$. Since it is without loss to front load the high type's rewards after success, such that $y_1^H > \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F = y_t^H$ for $t > 1$, we can consider the mis-reporting low type to work in the first $t^{L,H}$ periods.

Recalling the definition of $P_t^{\theta}$, the information rent $U^{\theta}(\varpi^{\theta},\vec{1})$ for each type is given by the $RHS$ of $(IC^{\theta})$:

$$U^H(\varpi^L,\vec{1}) = (1-\beta_0^H)\sum_{t=1}^{T^L}[x_t^L - \gamma] + \beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}[(\lambda y_t^L + (1-\lambda)x_t^L - \gamma)]$$
$$+P_{T^L}^H\Delta c_{T^L+1}q_F;$$

which for Case 2, where $x_t^L = 0$ for $t < T^L$, is rewritten as:

$$U^H(\varpi^L,\vec{1}) = P_{T^L}^H x_{T^L}^L + \beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L - \sum_{t=1}^{T^L}P_{t-1}^H\gamma + P_{T^L}^H\Delta c_{T^L+1}q_F.$$

Similarly, $U^L(\varpi^H,\vec{e}^{L,H}) = \beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}[\lambda y_t^H - \gamma] - \gamma(1-\beta_0^L)t^{L,H}$
$$-P_{t^{L,H}}^L\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F.$$

***$T_{SB}^L > T_{FB}^L$ if $\gamma$ is small enough.*** Consider the term $P_{T^L}^H\Delta c_{T^L+1}q_F$ in $U^H(\varpi^L,\vec{1})$, which, as we prove next, is monotonically decreasing in $T^L$. Noting that $\Delta c_t = c_t^L - c_t^H = (\beta_t^H - \beta_t^L)\Delta c$, and that $\beta_t^H - \beta_t^L = \frac{\beta_0^H(1-\lambda)^{t-1}}{\beta_0^H(1-\lambda)^{t-1}+1-\beta_0^H} - \frac{\beta_0^L(1-\lambda)^{t-1}}{\beta_0^L(1-\lambda)^{t-1}+1-\beta_0^L} = \frac{(1-\lambda)^{t-1}(\beta_0^H-\beta_0^L)}{P_{t-1}^H P_{t-1}^L}$, the difference in the expected cost can be rewritten as

$$\Delta c_{T^L+1} = \frac{(1-\lambda)^{T^L}(\beta_0^H-\beta_0^L)}{P_{T^L}^H P_{T^L}^L}\Delta c.$$

Thus, $P^H_{T^L}\Delta c_{T^L+1} = \frac{(1-\lambda)^{T^L}(\beta^H_0-\beta^L_0)}{P^L_{T^L}}\Delta c = \frac{(1-\lambda)^{T^L}(\beta^H_0-\beta^L_0)}{\beta^L_0(1-\lambda)^{T^L}+1-\beta^L_0}\Delta c = \frac{(\beta^H_0-\beta^L_0)}{\beta^L_0+\frac{1-\beta^L_0}{(1-\lambda)^{T^L}}}\Delta c$, which is decreasing in $T^L$.

The first three terms of $U^H(\varpi^L,\vec{1})$ on the right-hand side of $(IC^H)$ are increasing in $T^L$. To see this, we calculate their difference when the length is $T^\theta$ versus $T^\theta-1$:

$$P^H_{T^L}x^L_{T^L} + \beta^H_0\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y^L_t - \sum_{t=1}^{T^L}P^H_{t-1}\gamma$$
$$-(P^H_{T^L-1}x^L_{T^L} + \beta^H_0\sum_{t=1}^{T^L-1}(1-\lambda)^{t-1}\lambda y^L_t - \sum_{t=1}^{T^L-1}P^H_{t-1}\gamma)$$
$$= \beta^H_0(1-\lambda)^{T^L-1}\lambda\left(y^L_{T^L}-x^L_{T^L}\right) - P^H_{T^L-1}\gamma > 0,$$

because $y^L_{T^L} = x^L_{T^L} + \frac{\gamma}{\lambda\beta^L_{T^L}} + \frac{(1-\beta^L_0)}{P^L_{T^L}}\Delta c q_F$ from Claim A2.

Furthermore, the difference in the rent due to first three terms increases with $\gamma$.[47]

Therefore, there is a trade-off when increasing $T^L$: the first three terms increase while $P^H_{T^L}\Delta c_{T^L+1}$ decreases. Thus, the high-type's rent is lowered by increasing $T^L$ if the effect of $P^H_{T^L}\Delta c_{T^L+1}$ dominates, which is the case if $\gamma$ is low. We define a low enough value of $\gamma$, called $\gamma^{2,MH}$, such that the high-type's rent is decreasing in $T^L$ if $\gamma < \gamma^{2,MH}$ so that there is over-experimentation in $T^L$.

$\boldsymbol{T^H_{SB} < T^H_{FB}}$**.** In the low-type's rent, the term $P^L_{t^{L,H}}\left(c^L_{t^{L,H}+1} - c^H_{T^H+1}\right)q_F$ is monotonically decreasing in $T^H$. Since it appears in the low-type's rent with negative sign, the low-type's rent is lowered by decreasing $T^H$. Thus, it is optimal to have under-experimentation in $T^H$ ($T^H_{SB} < T^H_{FB}$).

**Intermediate case 3**: only $(IC^H)$ constraint binds (over-experimentation in $T^L$, $T^L_{SB} > T^L_{FB}$, if $\gamma$ is small enough, and first-best $T^H$, $T^H_{SB} = T^H_{FB}$)

The information rent for each type is given by

$$U^L(\varpi^L,\vec{1}) = \beta^L_0\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y^L_t - \sum_{t=1}^{T^L}P^L_{t-1}\gamma, \text{ and}$$
$$U^H(\varpi^L,\vec{1}) = \beta^H_0\sum_{t=1}^{T^L}(1-\lambda)^{t-1}(\lambda y^L_t - \gamma) - \gamma(1-\beta^H_0)T^L + P^H_{T^L}\Delta c_{T^L+1}q_F,$$

where $y^L_t = \frac{\gamma}{\lambda\beta^L_{T^L}} + \frac{(1-\beta^L_0)}{P^L_{T^L}}\Delta c q_F$ for $t \le T^L$ from Claim A3.

$\boldsymbol{T^H_{SB} = T^H_{FB}}$**.** Since $(IC^L)$ is not binding, the stopping time for the high type, $T^H$, does not affect information rents and, as a result, is not distorted: $T^H_{SB} = T^H_{FB}$.

$\boldsymbol{T^L_{SB} > T^L_{FB}}$ ***if $\gamma$ is small enough.*** Recall from Case 2 that $P^H_{T^L}\Delta c_{T^L+1}q_F$ is decreasing in $T^L$.

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[47] The difference in the rent due to first three terms increases with $\gamma$:
$\beta^H_0(1-\lambda)^{T^L-1}\lambda(y^L_{T^L} - x^L_{T^L}) - P^H_{T^L-1}\gamma = \beta^H_0(1-\lambda)^{T^L-1}\lambda\left[\left(\frac{\gamma}{\lambda\beta^H_{T^L}\beta^L_{T^L}}(\beta^H_{T^L}-\beta^L_{T^L}) + \frac{(1-\beta^L_0)}{P^L_{T^L}}\Delta c q_F\right)\right].$

The first two terms of $U^H(\varpi^L, \vec{1})$ are increasing in $T^L$. To see this, we calculate their difference when the length is $T^L$ versus $T^L - 1$:

$$\beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} (\lambda y_t^L - \gamma) - \gamma(1-\beta_0^H)T^L$$

$$- \left( \beta_0^H \sum_{t=1}^{T^L-1} (1-\lambda)^{t-1} (\lambda y_t^L - \gamma) - \gamma(1-\beta_0^H)(T^L - 1) \right)$$

$$= \beta_0^H (1-\lambda)^{T^L-1} \left(\lambda y_{T^L}^L - \gamma\right) - \gamma(1-\beta_0^H)$$

$$= \beta_0^H (1-\lambda)^{T^L-1} \lambda y_{T^L}^L - P_{T^L-1}^H \gamma > 0$$

since $y_{T^L}^L = \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F$ from Claim A3. Furthermore, as in Case 2, the difference in the rent due to the first two terms increases with $\gamma$.

Therefore, there is a trade-off when increasing $T^L$ but the high-type's rent is lowered by increasing $T^L$ if the effect of $P_{T^L}^H \Delta c_{T^L+1}$ dominates, which is the case if $\gamma$ is low enough. Then, it is optimal to have over experimentation in $T^L$, $T_{SB}^L > T_{FB}^L$, to mitigate the high type's rent. For larger values of $\gamma$ there would be (weak) under-experimentation in $T^L$.

**Intermediate case 4**: only $(IC^L)$ constraint binds (under-experimentation in $T^H$, $T_{SB}^H < T_{FB}^H$, and first-best $T^L$, $T_{SB}^L = T_{FB}^L$)

The information rent for each type is given by

$$U^H(\varpi^H, \vec{1}) = \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma, \text{ and}$$

$$U^L(\varpi^H, \vec{e}^{L,H}) = \beta_0^L \sum_{t=1}^{t^{L,H}} (1-\lambda)^{t-1} (\lambda y_t^H - \gamma) - \gamma(1-\beta_0^L) t^{L,H}$$
$$- P_{t^{L,H}}^L \left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right) q_F,$$

where $y_t^H = \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F$ for $t \le T^H$.

$\boldsymbol{T_{SB}^L = T_{FB}^L}$**.** Since $(IC^H)$ is not binding, the stopping time for the low type, $T^L$, does not affect information rents and, as a result, is not distorted: $T_{SB}^L = T_{FB}^L$.

$\boldsymbol{T_{SB}^H < T_{FB}^H}$**.** The term $P_{t^{L,H}}^L \left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right) q_F$ is monotonically decreasing in $T^H$. Thus, under experimentation in $T^H$ lowers the rent of the low type. Similarly, the moral hazard component on the right-hand side of $(IC^L)$ is increasing in $T^H$. Thus, it is optimal to have under-experimentation in $T^H$ ($T_{SB}^H < T_{FB}^H$).

## Section III. Sufficient conditions for $IC$ constraints to be binding/slack.

We provide the formal conditions at the end of the proof, which follows two steps. In *Step 1*, we prove that (a) $(IC^H)$ is binding: if either $\beta_0^H$ is high enough or $\lambda$ is high enough, and (b) $(IC^H)$ is not binding if $\lambda$ is small and $\gamma$ is sufficiently higher than $\Delta c$. In *Step 2*, we prove that (a) $(IC^L)$ is binding if either (i) $\lambda$ is high, $\beta_0^L < 1/2$, and $\gamma$ is small or (ii) $\beta_0^H$ close to $\beta_0^L$, (b) $(IC^L)$ is slack if $\beta_0^L$ is small and $\gamma$ is high. Finally, we combine the corresponding sufficient conditions to establish 4 cases.

**Step 1**. $(IC^H)$:

***Step 1a.*** $(IC^H)$ ***is binding.*** We now prove that $(IC^H)$ is binding if *either* $\beta_0^H$ *is high enough or* $\lambda$ *is high enough*. To characterize sufficient conditions for the $(IC^H)$ to be binding, we establish the parameters under which the highest possible value of the $LHS$ of the $(IC^H)$ if $(IC^H)$ is not binding, is smaller than the lowest possible value of the $RHS$ (what he can claim by misrepresenting his type). If $(IC^H)$ is not binding, the highest payments the high type can obtain are such that $MH_t^H$ are binding without any reward for failure – Case 1 or Case 4 rewards. The lowest payoff the high type can obtain by misreporting are the low type's payments in Case 1, i.e., when $(MH_t^L)$ are binding with no rewards for failure. Thus, we evaluate both the $LHS$ and $RHS$ under the Case 1 rewards where $(MH_t^\theta)$ are binding and no rewards after failure, i.e., $x_t^\theta = 0$ and $y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F$ for $t \leq T^\theta$.

Therefore, given $e_t^{H,L} = 1$ for $t \leq T^L$ from Lemma 1, the $(IC^H)$ is satisfied if and only if:

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}(\lambda y_t^H - \gamma) - \gamma(1-\beta_0^H)T^H$$
$$\geq \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}(\lambda y_t^L - \gamma) - \gamma(1-\beta_0^H)T^L + P_{T^L}^H \Delta c_{T^L+1} q_F.$$

We next simplify $(IC^H)$ using $P_{T^L}^H \Delta c_{T^L+1} q_F = (1-\lambda)^{T^L}\left(\frac{\beta_0^H - \beta_0^L}{P_{T^L}^L}\right)\Delta c q_F$, $\sum_{t=1}^{T^\theta}(1-\lambda)^{t-1} = \frac{1-(1-\lambda)^{T^\theta}}{\lambda}$, and $\lambda y_t^\theta - \gamma = \frac{\lambda(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F + \gamma\left(\frac{1-\beta_0^\theta}{\beta_0^\theta(1-\lambda)^{T^\theta-1}}\right)$ to obtain:

$$(IC^H) \qquad \frac{\beta_0^H\gamma}{\lambda}\left(\frac{(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)}{\beta_0^H(1-\lambda)^{T^H-1}} - \frac{(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)}{\beta_0^L(1-\lambda)^{T^L-1}}\right) - \gamma(1-\beta_0^H)(T^H - T^L)$$
$$\geq \left(\frac{\beta_0^H(1-\beta_0^L) - (1-\lambda)^{T^L}\beta_0^L(1-\beta_0^H)}{P_{T^L}^L} - \frac{\beta_0^H(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)}{P_{T^H}^H}\right)\Delta c q_F.$$

Finally, given that

$$\frac{(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)}{\beta_0^H(1-\lambda)^{T^H-1}} - \frac{(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)}{\beta_0^L(1-\lambda)^{T^L-1}} = \frac{(1-\beta_0^H)\beta_0^L(1-\lambda)^{T^L} - (1-\beta_0^L)\beta_0^H(1-\lambda)^{T^H} + (\beta_0^H - \beta_0^L)(1-\lambda)^{T^L+T^H}}{\beta_0^H\beta_0^L(1-\lambda)^{T^H+T^L-1}},$$

and $\frac{\beta_0^H(1-\beta_0^L) - (1-\lambda)^{T^L}\beta_0^L(1-\beta_0^H)}{P_{T^L}^L} - \frac{\beta_0^H(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)}{P_{T^H}^H} = \frac{\beta_0^H(1-\beta_0^L)(1-\lambda)^{T^H} - \beta_0^L(1-\beta_0^H)(1-\lambda)^{T^L}}{P_{T^L}^L P_{T^H}^H}$,

the $(IC^H)$ above simplifies to

$$(IC^H) \qquad \left(\frac{(1-\beta_0^H)\beta_0^L(1-\lambda)^{T^L} - (1-\beta_0^L)\beta_0^H(1-\lambda)^{T^H} + (\beta_0^H - \beta_0^L)(1-\lambda)^{T^L+T^H}}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}} - (1-\beta_0^H)(T^H - T^L)\right)\gamma$$
$$\geq \left(\frac{\beta_0^H(1-\beta_0^L)(1-\lambda)^{T^H} - \beta_0^L(1-\beta_0^H)(1-\lambda)^{T^L}}{P_{T^L}^L P_{T^H}^H}\right)\Delta c q_F.$$

$\beta_0^H$ *is high enough.* If the $LHS$ of $(IC^H)$ is negative (or zero) and the $RHS$ is positive, the $(IC^H)$ constraint is binding. We next prove that this is the case if $\beta_0^H$ is high enough. First, consider the coefficients in front of $\gamma$ if $\beta_0^H \to 1$:

$$\lim_{\beta_0^H \to 1}\left[\frac{(1-\beta_0^H)\beta_0^L(1-\lambda)^{T^L} - (1-\beta_0^L)\beta_0^H(1-\lambda)^{T^H} + (\beta_0^H - \beta_0^L)(1-\lambda)^{T^L+T^H}}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}} - (1-\beta_0^H)(T^H - T^L)\right]$$
$$= \frac{(1-\beta_0^L)\left[(1-\lambda)^{T^L+T^H} - (1-\lambda)^{T^H}\right]}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}}$$

If $T^L \geq 1$, then $(1-\lambda)^{T^L+T^H} < (1-\lambda)^{T^H}$ and, as a result, $\frac{(1-\beta_0^L)\left[(1-\lambda)^{T^L+T^H}-(1-\lambda)^{T^H}\right]}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}} < 0$ for any $\beta_0^L > 0$. If $T^L$ becomes zero for small values of $\beta_0^L > 0$, then the numerator becomes zero while the denominator remains strictly positive. Therefore, for any $\beta_0^L > 0$, $\frac{(1-\beta_0^L)\left[(1-\lambda)^{T^L+T^H}-(1-\lambda)^{T^H}\right]}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}}$ is either negative or zero. Thus, the $LHS$ of $(IC^H)$ if $\beta_0^H \to 1$ is either negative or zero for any $\beta_0^L > 0$.

Second, note that the $RHS$ of $(IC^H)$ is strictly positive if $\beta_0^H \to 1$.

As a result, $(IC^H)$ is binding for high enough $\beta_0^H$. We define a high enough value of $\beta_0^H$, called $\underline{\beta}_0^H$, as:

$$\underline{\beta}_0^H : \quad \frac{(1-\underline{\beta}_0^H)\beta_0^L(1-\lambda)^{T^L}-(1-\beta_0^L)\underline{\beta}_0^H(1-\lambda)^{T^H}+\left(\underline{\beta}_0^H-\beta_0^L\right)(1-\lambda)^{T^L+T^H}}{\beta_0^L\lambda(1-\lambda)^{T^H+T^L-1}} = \left(1-\underline{\beta}_0^H\right)(T^H-T^L).$$

Thus, $(IC^H)$ is binding if $\beta_0^H > \underline{\beta}_0^H$.

*$\lambda$ is high enough.* We prove that $(IC^H)$ is binding if $\lambda$ is sufficiently high. First, the $LHS$ in $(IC^H)$ becomes negative if $\lambda \to 1$. As $\lambda \to 1$, there will be high enough values of $\lambda$ for which $T^H \to 1$ and $T^L \to 1$. Evaluating the coefficient of $\gamma$ on the $LHS$ at $T^L = T^H = 1$, dividing by $(1-\lambda)$ and taking the limit $\lambda \to 1$, we obtain $\frac{\beta_0^L-\beta_0^H}{\beta_0^L} < 0$.

We define a high enough value of $\lambda$, called $\lambda_1$, such that the $LHS$ is zero:

$$\lambda_1: \quad \frac{(1-\beta_0^H)\beta_0^L(1-\lambda_1)^{T^L}-(1-\beta_0^L)\beta_0^H(1-\lambda_1)^{T^H}+(\beta_0^H-\beta_0^L)(1-\lambda_1)^{T^L+T^H}}{\beta_0^L\lambda_1(1-\lambda_1)^{T^H+T^L-1}} = (1-\beta_0^H)(T^H-T^L).$$

Thus, the $LHS$ is negative if $\lambda > \lambda_1$.

Second, the coefficient in front of $\Delta cq_F$ on the $RHS$ goes to zero if $\lambda \to 1$. Thus, there exists $\lambda_2 < 1$, such that the $\lambda_2 > \lambda_1$ and the $RHS$ is close to zero for $\lambda > \lambda_2$ while the $LHS$ is strictly negative. Therefore, $(IC^H)$ is binding if $\lambda > \underline{\lambda} = \max\{\lambda_1, \lambda_2\}$.

***Step 1b.* $(IC^H)$ *is not binding.*** We now prove that $(IC^H)$ is not binding if $\lambda$ *is low enough and* $\gamma$ *is high enough*. To characterize sufficient conditions for the $(IC^H)$ not to be binding, we establish the parameters under which the lowest value of the $LHS$ of $(IC^H)$ evaluated at $x_t^H = 0$ and $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F$ for $t \leq T^H$ is greater than the highest value of the $RHS$.

Consider small values of $\lambda$. As $\lambda \to 0$, there will be small enough values for which the low type does not experiment on or off the path, i.e., $T^L = t^{L,H} = 0$. We define a small enough value of $\lambda$, called $\bar{\lambda}$, such that $T^L = t^{L,H} = 0$ if $\lambda < \bar{\lambda}$. Then, the low type has no chance to collect a reward after success. If he tells the truth, he is paid $c_1^L q_F$ for producing $q_F$ without experimentation.[48] If he misreports, he receives $c_{T^H+1}^H q_F$ for producing $q_F$ since the high type is not rewarded for failure. Thus, the low type's rent must be at least $\left(c_{T^H+1}^H - c_1^L\right)q_F$ for him to report truthfully.

[48] The term $c_t^L$ is the expected cost for the $L$ type after $t-1$ failure (so $c_1^L$ is the expected cost after no experimentation).

If the high type misreports, he must produce without experimenting while collecting the low type's rent $\left(c^H_{T^H+1} - c^L_1\right)q_F$. Since the low type's rent is positive if $c^H_{T^H+1} > c^L_1$ and negative if $c^H_{T^H+1} \leq c^L_1$, it is sufficient to characterize sufficient conditions for $(IC^H)$ to be satisfied in the former ($c^H_{T^H+1} > c^L_1$) case only. Consider the former case ($c^H_{T^H+1} > c^L_1$). The $(IC^H)$ constraint is now:[49]

$$\beta^H_0 \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\left(\lambda\left[\frac{\gamma}{\lambda\beta^H_{T^H}} + \frac{(1-\beta^H_0)}{P^H_{T^H}}\Delta c q_F\right] - \gamma\right) - \gamma(1-\beta^H_0)T^H \geq \left(c^H_{T^H+1} - c^H_1\right)q_F,$$

$$\gamma\left(\beta^H_0\left(\frac{1}{\beta^H_{T^H}} - 1\right)\left(\frac{1-(1-\lambda)^{T^H}}{\lambda}\right) - (1-\beta^H_0)T^H\right) + \beta^H_0\frac{(1-\beta^H_0)}{P^H_{T^H}}\Delta c q_F\left(1-(1-\lambda)^{T^H}\right) \geq \left(c^H_{T^H+1} - c^H_1\right)q_F.$$

Since $\Delta c > c^H_{T^H+1} - c^H_1$, it is sufficient to prove the following condition (we replace $c^H_{T^H+1} - c^H_1$ with $\Delta c$ on the $RHS$ of the $(IC^H)$ above) and rearrange:

$$\gamma\left(\beta^H_0\left(\frac{1}{\beta^H_{T^H}} - 1\right)\left(\frac{1-(1-\lambda)^{T^H}}{\lambda}\right) - (1-\beta^H_0)T^H\right) \geq \left(1 - \beta^H_0\frac{(1-\beta^H_0)}{P^H_{T^H}}\left(1-(1-\lambda)^{T^H}\right)\right)\Delta c q_F,$$

It can be verified that the coefficient of $\gamma$ is strictly positive (it is the moral hazard rent in a model of experimentation without production), the $(IC^H)$ is satisfied if

$$\gamma \geq \frac{\left(1-\beta^H_0\frac{(1-\beta^H_0)}{P^H_{T^H}}\left(1-(1-\lambda)^{T^H}\right)\right)}{\left(\beta^H_0\left(\frac{1}{\beta^H_{T^H}}-1\right)\left(\frac{1-(1-\lambda)^{T^H}}{\lambda}\right)-(1-\beta^H_0)T^H\right)}\Delta c q_F,$$

Therefore, $(IC^H)$ is not binding if $\lambda < \bar{\lambda}$ and $\gamma > \tilde{A}\Delta c q_F$, where

$$\tilde{A} \equiv \max_{T^H}\frac{\left(1-\beta^H_0\frac{(1-\beta^H_0)}{P^H_{T^H}}\left(1-(1-\lambda)^{T^H}\right)\right)}{\left(\beta^H_0\left(\frac{1}{\beta^H_{T^H}}-1\right)\left(\frac{1-(1-\lambda)^{T^H}}{\lambda}\right)-(1-\beta^H_0)T^H\right)}.$$

**Step 2**. $(IC^L)$:

***Step 2a.*** $(\boldsymbol{IC^L})$ ***is binding.*** We now prove that $(IC^L)$ is binding if $\beta^H_0$ *is not too high*. To characterize sufficient conditions for the $(IC^L)$ to be binding, we establish the parameters under which the highest possible value of the $LHS$ of the $(IC^L)$ if $(IC^L)$ is not binding, is smaller than the lowest possible value of the $RHS$ (what he can claim by misrepresenting his type). Similar to the reasoning for step 1a, we evaluate both the $LHS$ and $RHS$ under the Case 1 rewards where $(MH^\theta_t)$ are binding and no rewards after failure, i.e., $x^\theta_t = 0$ and $y^\theta_t = \frac{\gamma}{\lambda\beta^\theta_{T^\theta}} + \frac{(1-\beta^\theta_0)}{P^\theta_{T^\theta}}\Delta c q_F$ for $t \leq T^\theta$.

The $(IC^L)$ can be rewritten:

$$\beta^L_0\sum_{t=1}^{T^L}(1-\lambda)^{t-1}(\lambda y^L_t - \gamma) - \gamma(1-\beta^L_0)T^L$$
$$\geq \beta^L_0\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}(\lambda y^H_t - \gamma) - \gamma(1-\beta^L_0)t^{L,H} + P^L_{t^{L,H}}\left(c^H_{T^H+1} - c^L_{t^{L,H}+1}\right)q_F.$$

[49] Given that $P^H_0 = 1$, the $RHS$ of $(IC^H)$ simplifies: $\left(c^H_{T^H+1} - c^L_1\right)q_F + P^H_0\Delta c_1 q_F = \left(c^H_{T^H+1} - c^L_1\right)q_F + (c^L_1 - c^H_1)q_F = \left(c^H_{T^H+1} - c^H_1\right)q_F$.

Since our goal is to identify parameter values under which $(IC^L)$ binds, we evaluate the constraint at $t^{L,H} = 1$. If choosing $t^{L,H} = 1$ is suboptimal for a low type who mimics the high type, then optimizing over $t^{L,H}$ can only increase the right-hand side of $(IC^L)$. Thus, establishing that $(IC^L)$ is binding for $t^{L,H} = 1$ is sufficient for our purposes.
For $t^{L,H} = 1$, the $(IC^L)$ can be simplified to:

$$(IC^L)\ \left(\frac{(1-\beta_0^L)(1-(1-\lambda)^{T^L})}{\lambda(1-\lambda)^{T^L-1}} - \frac{(1-\beta_0^H)\beta_0^L}{\beta_0^H(1-\lambda)^{T^H-1}} - (1-\beta_0^L)(T^L-1)\right)\gamma$$
$$\geq \left(\frac{\beta_0^L(1-\beta_0^H)(1-\lambda)^{T^L} - \beta_0^H(1-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^L}^L P_{T^H}^H}\right)\Delta c q_F.$$

We now prove that, if $\beta_0^H$ is not too high, then the coefficient of $\gamma$ is negative. Consider the first two terms of the LHS, $\frac{(1-\beta_0^L)(1-(1-\lambda)^{T^L})}{\lambda(1-\lambda)^{T^L-1}} - \frac{(1-\beta_0^H)\beta_0^L}{\beta_0^H(1-\lambda)^{T^H-1}}$:

$$\frac{(1-\beta_0^L)(1-(1-\lambda)^{T^L})}{\lambda(1-\lambda)^{T^L-1}} - \frac{(1-\beta_0^H)\beta_0^L}{\beta_0^H(1-\lambda)^{T^H-1}} < 0,$$
$$\beta_0^H < \frac{\beta_0^L\lambda(1-\lambda)^{T^L-1}}{\lambda\beta_0^L(1-\lambda)^{T^L-1} + (1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)(1-\lambda)^{T^H-1}}.$$

Therefore, there exists a $\beta_0^{H,LHS} \in (0,1)$ such that the coefficient of $\gamma$ is negative if $\beta_0^H < \beta_0^{H,LHS}$, where

$$\beta_0^{H,LHS} = \min_{T^H,T^L} \frac{\beta_0^L\lambda(1-\lambda)^{T^L-1}}{\beta_0^L\lambda(1-\lambda)^{T^L-1} + (1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)(1-\lambda)^{T^H-1}}.$$

Moreover, the RHS of $(IC^L)$ is non-negative if $\beta_0^H$ is not too high:

$$\beta_0^L(1-\beta_0^H)(1-\lambda)^{T^L} - \beta_0^H(1-\beta_0^L)(1-\lambda)^{T^H} > 0 \quad \Leftrightarrow \quad \beta_0^H < \frac{\beta_0^L(1-\lambda)^{T^L}}{\beta_0^L(1-\lambda)^{T^L} + (1-\beta_0^L)(1-\lambda)^{T^H}}.$$

Therefore, there exists a $\beta_0^{H,RHS} \in (0,1)$ such that the RHS of $(IC^L)$ is non-negative if $\beta_0^H < \beta_0^{H,RHS}$, where

$$\beta_0^{H,RHS} = \min_{T^H,T^L} \frac{\beta_0^L(1-\lambda)^{T^L}}{\beta_0^L(1-\lambda)^{T^L} + (1-\beta_0^L)(1-\lambda)^{T^H}}.$$

Thus, $(IC^L)$ is binding if $\beta_0^H < \bar{\beta}_0^H$, where $\bar{\beta}_0^H = \min\{\beta_0^{H,LHS}, \beta_0^{H,RHS}\}$.
***Step 2b.*** $\boldsymbol{(IC^L)}$ ***is not binding.*** We now prove that $(IC^L)$ is not binding if $\beta_0^L$ *is small*. To characterize sufficient conditions for the $(IC^L)$ not to be binding, we establish the parameters under which the lowest value of the $LHS$ of $(IC^L)$, evaluated at $x_t^L = 0$ and $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for $t \leq T^L$, is greater than the highest value of the $RHS$. To characterize this highest value, consider small values of $\beta_0^L$ and take the limit $\beta_0^L \to 0$. As $\beta_0^L \to 0$, there will be small enough values of $\beta_0^L$ for which $T^L = t^{L,H} = 0$. We define a small enough value of $\beta_0^L$, called $\overline{\beta}_0^L$, such that $T^L = t^{L,H} = 0$ if $\beta_0^L < \overline{\beta}_0^L$. Using similar steps as in Step 1b and recalling that $x_t^H = 0$,

for $\beta_0^L < \overline{\beta}_0^L$, the $RHS$ of the $(IC^L)$ is $U^L(\varpi^H, \vec{e}^{L,H}) = \left(c^H_{T^H+1} - c^L_1\right)q_F$. Again, it is sufficient to consider the case $(c^H_{T^H} > c^L_1)$. The $(IC^L)$ constraint is satisfied if the following condition holds:

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\left(\lambda\left[\frac{\gamma}{\lambda\beta^L_{T^L}} + \frac{(1-\beta_0^L)}{P^L_{T^L}}\Delta c q_F\right] - \gamma\right) - \gamma(1-\beta_0^L)T^L \geq \left(c^H_{T^H+1} - c^L_1\right)q_F.$$

The $LHS$ is zero while the $RHS$ is negative for small enough $\beta_0^L$ as $T^H$ is finite.

We finally state the sufficient conditions for each of the 4 cases formally.

### Sufficient conditions:

***Case 1***: We established that there exist $\overline{\lambda}$ and $\tilde{A}$ such that $(IC^H)$ is automatically satisfied if $\lambda < \overline{\lambda}$ and $\gamma > \tilde{A}\Delta c q_F$. In addition, we established that there exist $\overline{\beta}_0^L$ such that $(IC^L)$ is automatically satisfied if $\beta_0^L < \overline{\beta}_0^L$. Combining the two results, we obtain:

Neither $(IC^H)$ nor $(IC^L)$ are binding if $\lambda < \overline{\lambda}$, $\beta_0^L < \overline{\beta}_0^L$, and $\gamma > \tilde{A}\Delta c q_F$.

**Case 2**: We established that there exist $\underline{\lambda}$ such that $(IC^H)$ is binding if $\lambda > \underline{\lambda}$. In addition, we established that $(IC^L)$ is binding if $\beta_0^H < \bar{\beta}_0^H$. Combining the two results, we obtain:

Both $IC$ constraints are binding if $\lambda > \underline{\lambda}$ and $\beta_0^H < \bar{\beta}_0^H$.

**Intermediate Case 3**: We established that there exist $\underline{\beta}_0^H$ such that $(IC^H)$ is binding if $\beta_0^H > \underline{\beta}_0^H$. In addition, we established that there exist $\overline{\beta}_0^L$ and $\tilde{A}$ such that $(IC^L)$ is automatically satisfied if $\beta_0^L < \overline{\beta}_0^L$. Combining the two results, we obtain:

$(IC^H)$ is binding and $(IC^L)$ is not binding if $\beta_0^H > \underline{\beta}_0^H$, $\beta_0^L < \overline{\beta}_0^L$.

**Intermediate Case 4**: We established that there exist $\overline{\lambda}$ and $\tilde{A}$ such that $(IC^H)$ is not binding if $\lambda < \overline{\lambda}$ and $\gamma > \tilde{A}\Delta c q_F$. In addition, we established that $(IC^L)$ is binding if $\beta_0^H < \bar{\beta}_0^H$. Combining the two results, we obtain:

$(IC^H)$ is not binding and $(IC^L)$ is binding if $\beta_0^H < \bar{\beta}_0^H$, $\gamma > \tilde{A}\Delta c q_F$, and $\lambda < \overline{\lambda}$.

*Q.E.D.*

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## Supplementary Material

***Outline:*** In Appendix B, we derive the optimal contract under adverse selection only (the agent's effort is publicly observable). In Appendix C, we consider an extension of the main model with endogenous output. In Appendix D, we provide sufficient conditions for separation/integration to be optimal.

### Appendix B: Adverse Selection (No Moral Hazard)

***Outline:*** We now derive the optimal contract under adverse selection only (agent's effort is publicly observable). Thus, we ignore the moral hazard constraints. In Proposition B, we characterize the optimal contract depending on whether $(IC^L)$ is binding or not. The main takeaway is that, if $(IC^L)$ is binding, the high type is rewarded only after success ($x_t^H = 0$ for $t \leq T^H$ and $y_t^H > 0$ for some $t \leq T^H$), while the low type is rewarded only after failure in the last period ($x_{T^L}^L > 0 = x_t^L$ for $t < T^L$ and $y_t^L = 0$ for $t \leq T^L$). If only $(IC^H)$ is binding, the low type receives no rent and there is no restriction on when to pay the high type. In Claims B2 and B3 we provide sufficient conditions for $(IC^L)$ to be binding (high $\lambda$) and slack (low $\lambda$), respectively.

**Proposition B.** ***The optimal contract under adverse selection (no moral hazard)***

*(i) If both $(IC^H)$ and $(IC^L)$ are binding, the high type is rewarded only after success ($x_t^H = 0$ for $t \leq T^H$ and $y_t^H > 0$ for some $t \leq T^H$), while the low type is rewarded only after failure in the last period ($\boldsymbol{x_{T^L}^L} > \mathbf{0} = x_t^L$ for $t < T^L$ and $y_t^L = 0$ for $t \leq T^L$). If only $(IC^H)$ is binding, the low type receives no rent and there is no restriction on when to reward the high type to pay his rent.*

*(ii) Relative to the first best, the low type strictly over-experiments, while the high type weakly under-experiments (strictly, when both the incentive compatibility constraints are binding).*

*Proof*: We first characterize the optimal payment structure $\{x_t^H\}_{t=1}^{T^H}$, $\{y_t^H\}_{t=1}^{T^H}$, $\{x_t^L\}_{t=1}^{T^L}$ and $\{y_t^L\}_{t=1}^{T^L}$ (part (i) of Proposition B) given the lengths of experimentation, $T^L$ and $T^H$. Then, we characterize the optimal lengths of experimentation, $T^L$ and $T^H$ (part (ii) of Proposition B).

The principal's optimization problem is to choose contracts $\varpi^H$ and $\varpi^L$ to maximize the expected net surplus minus the rent of the agent, subject to the respective (*IC*) and (*LL*) constraints given below:

$$E_\theta \left\{ \Omega^\theta - \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1} \lambda y_t^\theta - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta \right\} \text{ s.t.}$$

$$(IC^H)\ \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$$
$$\geq \beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda y_t^L + \sum_{t=1}^{T^L} P_t^H x_t^L + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

$(IC^L)$ $\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L$

$$\geq \beta_0^L \sum_{t=1}^{T^H} (1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^L x_t^H - P_{T^H}^L \Delta c_{T^H+1} q_F,$$

$(LLS_t^H)$ $y_t^H \geq 0$ for $t \leq T^H$,

$(LLS_t^L)$ $y_t^L \geq 0$ for $t \leq T^L$,

$(LLF_t^H)$ $x_t^H \geq 0$ for $t \leq T^H$,

$(LLF_t^L)$ $x_t^L \geq 0$ for $t \leq T^L$.

We first prove the following claim.

Claim B1: The constraint $(\boldsymbol{IC^H})$ is binding ($\xi^H > 0$).

*Proof*: If the $(IC^H)$ constraint was not binding, it would be possible to decrease the payment to the high type until $(LLS_t^H)$ and $(LLF_t^H)$ are binding, but that would violate the $(IC^H)$ constraint since $P_{T^L}^H \Delta c_{T^L+1} q_F > 0$ given $\beta_0^H > \beta_0^L$.

*Q.E.D.*

### I. Optimal payment structure (Proof of part (i) of Proposition B)

Labeling $\xi^H$, $\xi^L$, $\{\alpha_t^H\}_{t=1}^{T^H}$, $\{\alpha_t^L\}_{t=1}^{T^L}$, $\{\eta_t^H\}_{t=1}^{T^H}$, $\{\eta_t^L\}_{t=1}^{T^L}$ as the Lagrange multipliers of the constraints associated with $(IC^H)$, $(IC^L)$, $(LLS_t^H)$, $(LLS_t^L)$, $(LLF_t^H)$ and $(LLF_t^L)$ respectively, the optimization problem has the following Lagrangian:

$$\mathcal{L} = \nu\left[\Omega^H - \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1}\lambda y_t^H - \sum_{t=1}^{T^H} P_t^H x_t^H\right]$$
$$+(1-\nu)\left[\Omega^L - \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1}\lambda y_t^L - \sum_{t=1}^{T^L} P_t^L x_t^L\right]$$
$$+\xi^H \begin{bmatrix} \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H \\ -\beta_0^H \sum_{t=1}^{T^L} (1-\lambda)^{t-1}\lambda y_t^L - \sum_{t=1}^{T^L} P_t^H x_t^L - P_{T^L}^H \Delta c_{T^L+1} q_F \end{bmatrix}$$
$$+\xi^L \begin{bmatrix} \beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L \\ -\beta_0^L \sum_{t=1}^{T^H} (1-\lambda)^{t-1}\lambda y_t^H - \sum_{t=1}^{T^H} P_t^L x_t^H + P_{T^H}^L \Delta c_{T^H+1} q_F \end{bmatrix}$$
$$+\sum_{t=1}^{T^H} \alpha_t^H y_t^H + \sum_{t=1}^{T^L} \alpha_t^L y_t^L + \sum_{t=1}^{T^H} \eta_t^H x_t^H + \sum_{t=1}^{T^L} \eta_t^L x_t^L.$$

The relevant Kuhn-Tucker conditions for the optimization problem are:

**(B1)** $\frac{\partial \mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda - \xi^L\beta_0^L(1-\lambda)^{t-1}\lambda + \alpha_t^H = 0;$

**(B2)** $\frac{\partial \mathcal{L}}{\partial y_t^L} = -(1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda - \xi^H\beta_0^H(1-\lambda)^{t-1}\lambda + \xi^L\beta_0^L(1-\lambda)^{t-1}\lambda + \alpha_t^L = 0;$

**(B3)** $\frac{\partial \mathcal{L}}{\partial x_t^H} = -\nu P_t^H + \xi^H P_t^H - \xi^L P_t^L + \eta_t^H = 0;$

**(B4)** $\frac{\partial \mathcal{L}}{\partial x_t^L} = -(1-\nu)P_t^L - \xi^H P_t^H + \xi^L P_t^L + \eta_t^L = 0.$

From Claim B1, we know that $\xi^H > 0$. We will show that the solution to the principal's optimization problem depends on whether the $(IC^L)$ constraint is binding or not. We refer to the case where only $(IC^H)$ constraint is binding as Case A, and to the case where both the (*IC*) constraints are binding as Case B. We explore each case separately in what follows.

*Case A: Only $(IC^H)$ constraint is binding* ($\xi^L = 0, \xi^H > 0$).

In this case the low type does not receive any rent ($U_A^L = 0$) and it immediately follows that $x_t^L = 0$ and $y_t^L = 0$ for $1 \leq t \leq T^L$. Thus, the rent of the high type can be derived from the *RHS* of $(IC^H)$ as $P_{T^L}^H \Delta c_{T^L+1} q_F$. Therefore, the expected rent in case A is

$$\nu U_A^H + (1-\nu) U_A^L = \nu P_{T^L}^H \Delta c_{T^L+1} q_F.$$

There are no restrictions in choosing $\{y_t^H\}_{t=1}^{T^H}$ and $\{x_t^H\}_{t=1}^{T^H}$ except those imposed by the $(IC^H)$ constraint. To see this, pick any period $s$ where $y_s^H > 0$ (alternatively, $x_s^H > 0$), which implies $\alpha_s^H = 0$ (alt. $\eta_s^H = 0$). Then, using $(B1)$ (alt. $(B3)$), we have $\xi^H = \nu$, and $\alpha_t^H = \eta_t^H = 0$ for all $t \leq T^H$ by $(B1)$ and $(B3)$. In other words, the principal can choose any combinations of nonnegative payments to the high type $\left(\{x_t^H\}_{t=1}^{T^H}, \{y_t^H\}_{t=1}^{T^H}\right)$ such that

$$\beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H = P_{T^L}^H \Delta c_{T^L+1} q_F.$$

*Case B: Both $IC$ constraints are binding* ($\xi^L > 0, \xi^H > 0$).

We will now show that when the $(IC^L)$ is binding, there are restrictions on the payment structure to both types. We show first that the high type is only paid after success while the low type is only paid after failing all $T^L$ times.

$\boldsymbol{x_t^H = 0}$**.** We first prove that the high type is not rewarded for failures, i.e., $x_t^H = 0$ for all $t \leq T^H$. Combining (B1) and (B3) we have:

$$\xi^L \left(\frac{P_t^L}{P_t^H} - \frac{\beta_0^L}{\beta_0^H}\right) + \frac{\alpha_t^H}{\beta_0^H (1-\lambda)^{t-1} \lambda} = \frac{\eta_t^H}{P_t^H} \text{ for } t \leq T^H.$$

Since $\xi^L > 0$ and $\frac{P_t^L}{P_t^H} - \frac{\beta_0^L}{\beta_0^H} > 0$, we have $\eta_t^H > 0$ for $t \leq T^H$. This implies that the high type is not rewarded for failures, i.e.,

$$x_t^H = 0 \text{ for all } t \leq T^H.$$

Since the high type gets a rent, we must have $y_t^H \geq 0$ with a strict inequality for some $t$.

$\boldsymbol{y_t^L = 0}$ **for all $\boldsymbol{t \leq T^L}$.** We next prove that the low type is not rewarded for success, i.e., $y_t^L = 0$ for all $t \leq T^L$. Combining (B2) and (B4) we have:

$$\xi^H \left(\frac{P_t^H}{P_t^L} - \frac{\beta_0^H}{\beta_0^L}\right) + \frac{\alpha_t^L}{\beta_0^L (1-\lambda)^{t-1} \lambda} = \frac{\eta_t^L}{P_t^L} \text{ for } t \leq T^L.$$

Since $\xi^H > 0$ and $\frac{P_t^H}{P_t^L} - \frac{\beta_0^H}{\beta_0^L} < 0$, we have $\alpha_t^L > 0$ for $t \leq T^L$. This implies the low type is not rewarded for success, i.e.,

$$y_t^L = 0 \text{ for all } t \leq T^L.$$

Since the low type gets a rent, we must have $x_t^L \geq 0$ with a strict inequality for some $t$.

We show next that, as the relative probability of failure $\frac{P_t^L}{P_t^H}$ is increasing in $t$, the low type will be paid in the last period after failure to deter the high type from pretending to be the low type. Similarly, we also show below that, as the relative likelihood of success $\frac{\beta_0^L(1-\lambda)^{t-1}\lambda}{\beta_0^H(1-\lambda)^{t-1}\lambda} = \frac{\beta_0^L}{\beta_0^H}$ is independent of $t$, the principal can use any combination of $y_t^H$ to pay the rent to the high type.

$\boldsymbol{x_{T^L}^L > x_t^L = 0}$ **for** $\boldsymbol{t < T^L}$**.** We next prove in two steps that the low type is rewarded for failure in the very last period only. First, we prove by contradiction that the low type is rewarded for failure in only one period $s \leq T^L$. Second, we prove that it is optimal to reward the low type for the very last failure, i.e., $s = T^L$.

*Step1:* Suppose that the low type is rewarded for failures in two distinct periods $s \leq T^L$ and $k \leq T^L$. Therefore, $x_s^L, x_k^L > 0$ ($\eta_s^L = 0 = \eta_k^L$). Evaluating (B4) at $t = s$ and $t = k$, we derive

$$-(1-\nu)P_s^L - \xi^H P_s^H + \xi^L P_s^L = 0 = -(1-\nu)P_k^L - \xi^H P_k^H + \xi^L P_k^L = 0,$$

$$\frac{P_s^L}{P_s^H}[\xi^L - (1-\nu)] = \xi^H = \frac{P_k^L}{P_k^H}[\xi^L - (1-\nu)],$$

which leads to a contradiction since $\frac{P_t^L}{P_t^H}$ is strictly increasing in $t$:

$$\frac{d\left(\frac{P_t^L}{P_t^H}\right)}{dt} = \frac{d\left(\frac{1-\beta_0^L+\beta_0^L(1-\lambda)^t}{1-\beta_0^H+\beta_0^H(1-\lambda)^t}\right)}{dt} = -\frac{(\beta_0^H-\beta_0^L)}{\left(P_t^H\right)^2}(1-\lambda)^t \ln(1-\lambda) > 0.$$

*Step2:* We now prove that the low type is rewarded for failure in the last period only.

Expressing $\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H$ from the $(IC^H)$:

$$\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H = \frac{P_s^H x_s^L + P_{T^L}^H \Delta c_{T^L+1} q_F}{\beta_0^H},$$

and plugging it to the $(IC^L)$ we obtain

$$P_s^L x_s^L = \beta_0^L \left[\frac{P_s^H x_s^L + P_{T^L}^H \Delta c_{T^L+1} q_F}{\beta_0^H}\right] - P_{T^H}^L \Delta c_{T^H+1} q_F,$$

$$x_s^L = \frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{(\beta_0^H - \beta_0^L)}.$$

as we know $P_s^L x_s^L - \beta_0^L P_s^H = (\beta_0^H - \beta_0^L)$.

The agent's rent is then

$$U_B^L = P_s^L x_s^L = \frac{P_s^L\left(\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F\right)}{(\beta_0^H - \beta_0^L)},$$

$$U_B^H = \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H = \beta_0^H \frac{P_s^H x_s^L + P_{T^L}^H \Delta c_{T^L+1} q_F}{\beta_0^H} = P_s^H x_s^L + P_{T^L}^H \Delta c_{T^L+1} q_F$$

$$= P_s^H \left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{(\beta_0^H - \beta_0^L)}\right] + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

where the subscript B refers to Case B.

Therefore, the expected rent $\nu U_B^H + (1-\nu) U_B^L =$

$$\nu\left[P_s^H\left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}\right] + P_{T^L}^H \Delta c_{T^L+1} q_F\right]$$

$$+(1-\nu)P_s^L \frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}$$

$$= \nu P_{T^L}^H \Delta c_{T^L+1} q_F + \left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}\right](\nu P_s^H + (1-\nu)P_s^L),$$

which is strictly decreasing in $s$. Thus, the agent's rent is minimized at $s = T^L$. Thus, the expected rent in case B is

$$\nu U_B^H + (1-\nu)U_B^L = \nu P_{T^L}^H \Delta c_{T^L+1} q_F + \left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}\right] EP_{T^L}^{\theta},$$

where $EP_{T^L}^{\theta} = \left(\nu P_{T^L}^H + (1-\nu)P_{T^L}^L\right)$.

$\boldsymbol{y_t^H \geq 0}$ **for all** $\boldsymbol{t \leq T^H}$**.** We finally prove that the principal can use any combination of $y_t^H$ to pay rent to the high type. Given that the high type's rent is strictly positive and $x_t^H = 0$ for all $t$, there must be some period $s$ such that $y_s^H > 0$ (and $\alpha_s^H = 0$). Then, from (B1), we have $\xi^H \beta_0^H - \xi^L \beta_0^L = \nu \beta_0^H$ because the relative likelihood of success is independent of $t$. Thus, for all $t$, $(B1)$ can be written as

$$\frac{\partial \mathcal{L}}{\partial y_t^H} = -\nu\beta_0^H + \xi^H \beta_0^H - \xi^L \beta_0^L + \frac{\alpha_t^H}{(1-\lambda)^{t-1}\lambda} = \frac{\alpha_t^H}{(1-\lambda)^{t-1}\lambda} = 0,$$

which implies that $\alpha_t^H = 0$ for all $t \leq T^H$.

To summarize, if both (*IC*) are binding then $x_t^H = 0$, $y_t^H \geq 0$ for $t \leq T^H$ and $y_t^L = 0$, $x_{T^L}^L > x_t^L = 0$ for $t < T^L$. Thus, the principal can use any combination of $y_t^H$ such that both $(IC^H)$ and $(IC^L)$ hold as equalities simultaneously:

$$x_{T^L}^L = \frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}, \text{ and}$$

$$\beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1}\lambda y_t^H = P_{T^L}^H \left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}\right] + P_{T^L}^H \Delta c_{T^L+1} q_F.$$

This completes the proof of part (i) of Proposition B.

*Q.E.D.*

## II. Optimal length of experimentation (Proof of part (ii) of Proposition B)

In Case A above, only $(IC^H)$ is binding, and the rents to the high and low types are

$$U_A^H = P_{T^L}^H \Delta c_{T^L+1} q_F \text{ and } U_A^L = 0,$$

where the subscript A refers to Case A.

In Case A, the high type's rent $P^H_{T^L}\Delta c_{T^L+1}q_F$ is not affected by $T^H$ and therefore, $T^H_{SB} = T^H_{FB}$ when $(IC^L)$ is not binding. We next prove that the informational rent of the high-type agent, $P^H_{T^L}\Delta c_{T^L+1}q_F$, is monotonically decreasing in $T^L$ and therefore over experimentation is optimal. Noting that $\Delta c_t = c^L_t - c^H_t = (\beta^H_t - \beta^L_t)(\overline{c} - \underline{c})$, and that $\beta^H_t - \beta^L_t = \frac{\beta^H_0(1-\lambda)^{t-1}}{\beta^H_0(1-\lambda)^{t-1}+1-\beta^H_0} - \frac{\beta^L_0(1-\lambda)^{t-1}}{\beta^L_0(1-\lambda)^{t-1}+1-\beta^L_0} = \frac{(1-\lambda)^{t-1}(\beta^H_0-\beta^L_0)}{P^H_{t-1}P^L_{t-1}}$, the difference in the expected cost can be rewritten as

$$\Delta c_{T^L+1} = \frac{(1-\lambda)^{T^L}(\beta^H_0-\beta^L_0)}{P^H_{T^L}P^L_{T^L}}(\overline{c} - \underline{c}).$$

Thus, $P^H_{T^L}\Delta c_{T^L+1} = \frac{(1-\lambda)^{T^L}(\beta^H_0-\beta^L_0)}{P^L_{T^L}}(\overline{c} - \underline{c}) = \frac{(1-\lambda)^{T^L}(\beta^H_0-\beta^L_0)}{\beta^L_0(1-\lambda)^{T^L}+1-\beta^L_0}(\overline{c} - \underline{c}) = \frac{(\beta^H_0-\beta^L_0)}{\beta^L_0+\frac{1-\beta^L_0}{(1-\lambda)^{T^L}}}(\overline{c} - \underline{c})$, which is decreasing in $T^L$. Thus, we have *over* experimentation, i.e.,

$$T^L_{SB} > T^L_{FB}.$$

In Case B above, both $(IC^H)$ and $(IC^L)$ are binding, and the agent's rents are

$$U^L_B = \frac{P^L_{T^L}\left(\beta^L_0 P^H_{T^L}\Delta c_{T^L+1}q_F - \beta^H_0 P^L_{T^H}\Delta c_{T^H+1}q_F\right)}{(\beta^H_0-\beta^L_0)},$$

$$U^H_B = P^H_{T^L}\left[\frac{\beta^L_0 P^H_{T^L}\Delta c_{T^L+1}q_F - \beta^H_0 P^L_{T^H}\Delta c_{T^H+1}q_F}{(\beta^H_0-\beta^L_0)}\right] + P^H_{T^L}\Delta c_{T^L+1}q_F,$$

where the subscript B refers to Case B.

We have already shown that $P^H_{T^L}\Delta c_{T^L+1}$ is decreasing in $T^L$. Following similar steps, we can show that $P^L_{T^H}\Delta c_{T^H+1} = \frac{(\beta^H_0-\beta^L_0)}{\beta^H_0+\frac{1-\beta^H_0}{(1-\lambda)^{T^H}}}(\overline{c} - \underline{c})$ is decreasing in $T^H$. Therefore, we will have *under* experimentation in $T^H$.

This completes the proof of part (ii) of Proposition B.

*Q.E.D.*

*Claim B2 (Sufficient conditions for* $(\boldsymbol{IC^L})$ *to be binding): There exist* $\mathbf{0} < \overline{\boldsymbol{\lambda}} < \mathbf{1}$ *and* $\mathbf{0} < \underline{\boldsymbol{\nu}} < \mathbf{1}$*, such that* $(\boldsymbol{IC^L})$ *is binding if* $\boldsymbol{\lambda} > \overline{\boldsymbol{\lambda}}$ *and* $\boldsymbol{\nu} < \underline{\boldsymbol{\nu}}$*.*

*Proof:* Suppose that $(IC^L)$ is not binding. We will prove by contradiction that if $\lambda$ is large and $\nu$ is small, both $(IC)$ must be binding. In Step 1, we show when $\lambda$ is high, the two $(IC)$ constraints require $T^H \leq T^L$. In Step 2, we show that if $(IC^L)$ is ignored, then the equilibrium value of $T^H_{FB}$ is strictly larger than $T^L$ if $\nu$ is small enough. This implies a contradiction, and that both $(IC)$ constraints must be binding.

***Step 1: There exists $\bar{\lambda} \in (0, 1)$, such that, if $\lambda > \bar{\lambda}$, the $(IC^H)$ and $(IC^L)$ imply $T^L \geq T^H$.***

Consider the two $(IC)$ constraints:

$$(IC^H)\ \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$$

$$\geq \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^H x_t^L + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

$$(IC^L)\ \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L$$

$$\geq \beta_0^L \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^L x_t^H - P_{T^H}^L \Delta c_{T^H+1} q_F.$$

Since we proved that in the optimal contract (i) there is no restriction on when to reward the high type when only the $(IC^H)$ is binding, it is without loss of generality to consider $x_t^H = 0$ for all $t \geq 1$ and $y_t^H$ given by

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H = P_{T^L}^H \Delta c_{T^L+1} q_F,$$

derived from the binding $(IC^H)$ evaluated at $y_t^L = x^L = 0$ for all $t \geq 1$.

Replacing $\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H$ with the expression above in $(IC^L)$ and plugging $y_t^L = x^L = 0$ for all $t \geq 1$, we have the 'gamble' for the low type on the RHS of $(IC^L)$ below. We will show that this gamble becomes positive when $\lambda$ is high enough.

$$(IC^L)\ \ 0 \geq \beta_0^L \left[\frac{P_{T^L}^H \Delta c_{T^L+1} q_F}{\beta_0^H}\right] - P_{T^H}^L \Delta c_{T^H+1} q_F,$$

which can be rewritten as

$$\beta_0^H P_{T^H}^L \Delta c_{T^H+1} \geq \beta_0^L P_{T^L}^H \Delta c_{T^L+1}, \text{ or}$$

$$(1-\lambda)^{T^H-T^L} \geq \frac{\frac{\beta_0^L}{(1-\beta_0^L)}}{\frac{\beta_0^H}{(1-\beta_0^H)}} \in (0,1)$$

since $\Delta c_t = \frac{(1-\lambda)^{t-1}(\beta_0^H-\beta_0^L)}{P_{t-1}^H P_{t-1}^L}(\bar{c}-\underline{c})$.

If $\lambda$ is high enough, the inequality can only be satisfied if $T^H < T^L$. Thus, we introduce a value of $\lambda$, called $\bar{\lambda}$, as the highest value of $\lambda$ such that the inequality is satisfied with $T^H > T^L$ (we evaluate $T^H - T^L = 1$ to achieve the highest value for the *LHS* of the inequality above), where $\bar{\lambda} \in (0,1)$ is given by

$$\bar{\lambda} \equiv 1 - \frac{\frac{\beta_0^L}{(1-\beta_0^L)}}{\frac{\beta_0^H}{(1-\beta_0^H)}}.$$

Thus, for any $\lambda > \bar{\lambda}$, the two $IC$s can only be satisfied if $T^H < T^L$.

***Step 2: There exists $\underline{\nu} \in (0,1)$, such that, if $\nu < \underline{\nu}$, we have $T_{SB}^{L} < T_{SB}^{H} = T_{FB}^{H}$.***

We will show that if $\nu$ is small, then the first best order of termination dates is preserved in the second best when $(IC^L)$ is ignored. Recall that if only the $(IC^H)$ is binding, then the stopping time for the high type is not distorted ($T^H = T_{FB}^H$), and the distortions in $T^L$ are determined by the following F.O.C.:

$$\frac{\partial\left(E_\theta\, \Omega^\theta(\varpi^\theta) - \nu P_{T^L}^H \Delta c_{T^L+1} q_F\right)}{\partial T^L},$$

where $P_{T^L}^H \Delta c_{T^L+1} q_F$ is the rent of the high type. It follows that when $\nu$ is small, the benefit of distorting $T^L$ is also small, and the equilibrium value of $T^L$ will remain close to its first-best value $T_{FB}^L$. Define $\underline{\nu}$ such that the equilibrium value of $T^L$ is strictly less than $T^H$ if $\nu < \underline{\nu}$.

Thus, we have a contradiction, and both $(IC)$ constraints are binding when $\lambda > \overline{\lambda}$ and $\nu < \underline{\nu}$.

This conclues the proof of the sufficient conditions for $(IC^L)$ to be binding.

*Q.E.D.*

Claim B3 (Sufficient conditions for $(\boldsymbol{IC^L})$ to be slack): *There exist $\mathbf{0} < \underline{\boldsymbol{\lambda}} < \mathbf{1}$, such that $(\boldsymbol{IC^L})$ is not binding if $\boldsymbol{\lambda} < \underline{\boldsymbol{\lambda}}$.*

*Proof:* Consider the two $(IC)$ constraints:

$$(IC^H)\ \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$$
$$\geq \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^H x_t^L + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

$$(IC^L)\ \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L$$
$$\geq \beta_0^L \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^L x_t^H - P_{T^H}^L \Delta c_{T^H+1} q_F.$$

Since we proved that in the optimal contract (i) there is no restriction on when to reward the high type when only the $(IC^H)$ is binding, it is without loss of generality to consider $x_t^H = 0$ for all $t \geq 1$ and $y_t^H$ given by

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H = P_{T^L}^H \Delta c_{T^L+1} q_F,$$

derived from the binding $(IC^H)$ evaluated at $y_t^L = x^L = 0$ for all $t \geq 1$.

Replacing $\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H$ with the expression above in $(IC^L)$ and plugging $y_t^L = x^L = 0$ for all $t \geq 1$, we have

$$(IC^L)\ \ 0 \geq \beta_0^L \left[\frac{P_{T^L}^H \Delta c_{T^L+1} q_F}{\beta_0^H}\right] - P_{T^H}^L \Delta c_{T^H+1} q_F,$$

which can be rewritten as

$$\beta_0^H P_{T^H}^L \Delta c_{T^H+1} \geq \beta_0^L P_{T^L}^H \Delta c_{T^L+1}, \text{ or}$$

$$(1-\lambda)^{T^H-T^L} \geq \frac{\frac{\beta_0^L}{(1-\beta_0^L)}}{\frac{\beta_0^H}{(1-\beta_0^H)}} \in (0,1),$$

since $\Delta c_t = \frac{(1-\lambda)^{t-1}(\beta_0^H-\beta_0^L)}{P_{t-1}^H P_{t-1}^L}(\overline{c}-\underline{c})$.

Therefore, the gamble is negative if $(1-\lambda)^{T^H-T^L} \geq \frac{\frac{\beta_0^L}{(1-\beta_0^L)}}{\frac{\beta_0^H}{(1-\beta_0^H)}} \in (0,1)$.

First, if $T^H < T^L$, then the gamble is positive for all parameter values. Second, if $T^H > T^L$, then the $LHS$ goes to 1 as $\lambda \to 0$. Thus, there exists a value of $\lambda$, called $\underline{\lambda}$, as the highest value of $\lambda$ such that the inequality is satisfied. Therefore, for any $\lambda < \underline{\lambda}$, the gamble is negative for the low type and, as a result, the $(IC^L)$ is not binding.

This concludes the proof of the sufficient conditions for $(IC^H)$ to be not binding.

*Q.E.D.*

## Appendix C. Endogenous Output

We now characterize the results for the model with endogenous output. We denote the output after success at period $t$ by type $\theta$ by $q_t^S(\theta)$, and the output after failure is denoted by $q_F^\theta$. We then replace $y_t^\theta \equiv w_t^S(\theta) - \underline{c}q_t^S(\theta)$ and $x_t^\theta \equiv w_t^F(\theta) - 1_{\{t=T^\theta\}} c_{T^\theta+1}^\theta q_F^\theta$ for $t \leq T^\theta$ and $\theta \in \{L, H\}$ in the constraints and the principal's objective function.

The principal's optimization problem is to choose contracts $\varpi^\theta$ for $\theta \in \{H, L\}$ to maximize

$$E_\theta \left\{ \beta_0^\theta \sum_{t=1}^{T^\theta} (1-\lambda)^{t-1} \lambda \left[ V\left(q_t^S(\theta)\right) - \underline{c}q_t^S(\theta) - y_t^\theta \right] + P_{T^\theta}^\theta \left[ V\left(q_F^\theta\right) - c_{T^\theta+1}^\theta q_F^\theta \right] - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta \right\} \text{ s.t.}$$

$(IC^\theta)$ $\quad U^\theta(\varpi^\theta, \vec{1}) \geq U^\theta\left(\varpi^{\hat{\theta}}, \vec{e}^\theta\left(\varpi^{\hat{\theta}}\right)\right),$

$(MH_t^\theta)$ $\quad y_t^\theta - x_t^\theta$

$$\geq \frac{\gamma}{\lambda \beta_t^\theta} + \sum_{s=t+1}^{T^\theta} (1-\lambda)^{s-t-1} \left(\lambda y_s^\theta + (1-\lambda) x_s^\theta - \gamma\right) + \frac{(1-\beta_0^\theta)(1-\lambda)^{T^\theta - t}}{P_{T^\theta}^\theta} \Delta c q_F^\theta \text{ for } t \leq T^\theta,$$

$(LLS_t^\theta)$ $\quad y_t^\theta \geq 0$ for $t \leq T^\theta$,

$(LLF_t^\theta)$ $\quad x_t^\theta \geq 0$ for $t \leq T^\theta$.

Labeling $\xi^H$, $\xi^L$, $\{\mu_t^H\}_{t=1}^{T^H}$, $\{\mu_t^L\}_{t=1}^{T^L}$, $\{\eta_t^H\}_{t=1}^{T^H}$, $\{\eta_t^L\}_{t=1}^{T^L}$ as the Lagrange multipliers of the constraints associated with $(IC^H)$, $(IC^L)$, $(MH_t^H)$, $(MH_t^L)$, $(LLF_t^H)$ and $(LLF_t^L)$ respectively, the optimization problem has the following Lagrangian:

$$\mathcal{L} = E\left[\beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda\left[V\left(q_t^S(\theta)\right) - \underline{c}q_t^S(\theta) - y_t^\theta\right] + P_{T^\theta}^\theta\left[V\left(q_F^\theta\right) - c_{T^\theta+1}^\theta q_F^\theta\right] - \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta\right]$$

$$+\xi^H\begin{bmatrix}\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H \gamma \\ -(1-\beta_0^H)\sum_{t=1}^{T^L}[x_t^L - \gamma] - \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}[\lambda y_t^L + (1-\lambda)x_t^L - \gamma] \\ -P_{T^L}^H \Delta c_{T^L+1} q_F^L\end{bmatrix}$$

$$+\xi^L\begin{bmatrix}\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L - \sum_{t=1}^{T^L} P_{t-1}^L \gamma \\ -(1-\beta_0^L)\sum_{t=1}^{T^H}\left[x_t^H - \gamma e_t^{L,H}\right] - \beta_0^L \sum_{t=1}^{T^H}\left(\prod_{s=1}^{t-1}(1-\lambda e_s^{L,H})\right)\left[e_t^{L,H}\lambda y_t^H + \left(1-\lambda e_t^{L,H}\right)x_t^H - e_t^{L,H}\gamma\right] \\ -\left(1-\beta_0^L + \beta_0^L\left(\prod_{s=1}^{T^H}(1-\lambda e_s^{L,H})\right)\right)\left(c_{T^H+1}^H - c_{\sum_{s=1}^{T^H} e_s^{L,H}+1}^L\right) q_F^H\end{bmatrix}$$

$$+\sum_{t=1}^{T^H}\mu_t^H\left[y_t^H - x_t^H - \frac{\gamma}{\lambda\beta_t^H} - \sum_{s=t+1}^{T^H}(1-\lambda)^{s-t-1}(\lambda y_s^H + (1-\lambda)x_s^H - \gamma) - \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c q_F^H\right]$$

$$+\sum_{t=1}^{T^L}\mu_t^L\left[y_t^L - x_t^L - \frac{\gamma}{\lambda\beta_t^L} - \sum_{s=t+1}^{T^L}(1-\lambda)^{s-t-1}(\lambda y_s^L + (1-\lambda)x_s^L - \gamma) - \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c q_F^L\right]$$

$$+\sum_{t=1}^{T^H}\eta_t^H x_t^H + \sum_{t=1}^{T^L}\eta_t^L x_t^L.$$

### Optimal output

The relevant Kuhn-Tucker conditions for the optimization problem are:

$$\frac{\partial\mathcal{L}}{\partial q_F^H} = V'(q_F^H) - c_{T^H+1}^H - \xi^L\left(1-\beta_0^L + \beta_0^L\left(\prod_{s=1}^{T^H}(1-\lambda e_s^{L,H})\right)\right)\left(c_{T^H+1}^H - c_{\sum_{s=1}^{T^H+1} e_s^{L,H}}^L\right) - \sum_{t=1}^{T^H}\mu_t^H \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H}\Delta c = 0;$$

$$\frac{\partial\mathcal{L}}{\partial q_F^L} = V'(q_F^L) - c_{T^L+1}^L - \xi^H P_{T^L}^H \Delta c_{T^L+1} - \sum_{t=1}^{T^L}\mu_t^L \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L}\Delta c = 0;$$

$$\frac{\partial\mathcal{L}}{\partial q_t^S(H)} = \nu\beta_0^H(1-\lambda)^{t-1}\lambda\left[V'\left(q_t^S(H)\right) - \underline{c}\right] = 0;$$

$$\frac{\partial\mathcal{L}}{\partial q_t^S(L)} = (1-\nu)\beta_0^L(1-\lambda)^{t-1}\lambda\left[V'\left(q_t^S(L)\right) - \underline{c}\right] = 0;$$

There is no distortion in the output after success, i.e., it is at the first-best level:

$$V'\left(q_t^S(H)\right) = \underline{c} \text{ for } t \le T^H;$$

$$V'\left(q_t^S(L)\right) = \underline{c} \text{ for } t \le T^L.$$

Distortions in the output after failure depend on Cases 1-4.

Case 1: *Both the* $(IC^H)$ *and* $(IC^L)$ *constraints are slack.* ($\xi^H = \xi^L = 0$ *and* $\mu_t^H, \mu_t^L > 0$)

Both types *under* produce after failure:

$$V'(q_F^H) = c_{T^H+1}^H + \sum_{t=1}^{T^H} \mu_t^H \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H} \Delta c > c_{T^H+1}^H;$$

$$V'(q_F^L) = c_{T^L+1}^L + \sum_{t=1}^{T^L} \mu_t^L \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L} \Delta c > c_{T^L+1}^L.$$

Case 2*: Both* $(IC^H)$ *and* $(IC^L)$ *bind.* ($\xi^H > 0$, $\xi^L > 0$, $0 = \mu_t^H, \mu_t^L > 0$)

Both *under* and *over* production possible for the high type after failure:

$$V'(q_F^H) = c_{T^H+1}^H + \xi^L \left(1 - \beta_0^L + \beta_0^L \left(\prod_{s=1}^{T^H} (1 - \lambda e_s^{L,H})\right)\right) \left(c_{T^H+1}^H - c_{\sum_{s=1}^{T^H+1} e_s^{L,H}}^L\right).$$

The low type *under* produces:

$$V'(q_F^L) = c_{T^L+1}^L + \xi^H P_{T^L}^H \Delta c_{T^L+1} + \sum_{t=1}^{T^L} \mu_t^L \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L} \Delta c > c_{T^L+1}^L.$$

Case 3*: The* $(IC^H)$ *constraint binds and* $(IC^L)$ *is slack.* ($\xi^H > 0$, $\xi^L = 0$, $\mu_t^L > 0$ $\mu_t^H = 0$)

The high type produces at the first best level:

$$V'(q_F^H) = c_{T^H+1}^H.$$

The low type *under* produces:

$$V'(q_F^L) = c_{T^L+1}^L + \xi^H P_{T^L}^H \Delta c_{T^L+1} + \sum_{t=1}^{T^L} \mu_t^L \frac{(1-\beta_0^L)(1-\lambda)^{T^L-t}}{P_{T^L}^L} \Delta c > c_{T^L+1}^L.$$

Case 4: *The* $(IC^L)$ *constraint binds and* $(IC^H)$ *is slack.* ($\xi^H = 0$, $\xi^L > 0$, $\mu_t^H > 0$ *and* $\mu_t^L = 0$)

Both *under* and *over* production possible for the high type after failure:

$$V'(q_F^H) = c_{T^H+1}^H$$
$$+\xi^L \left(1 - \beta_0^L + \beta_0^L \left(\prod_{s=1}^{T^H} (1 - \lambda e_s^{L,H})\right)\right) \left(c_{T^H+1}^H - c_{\sum_{s=1}^{T^H+1} e_s^{L,H}}^L\right) +$$
$$\sum_{t=1}^{T^H} \mu_t^H \frac{(1-\beta_0^H)(1-\lambda)^{T^H-t}}{P_{T^H}^H} \Delta c.$$

The low type produces at the first best level:

$$V'(q_F^L) = c_{T^L+1}^L.$$

## Appendix D: Sufficient Conditions for Separation/Integration

**Claim D1 (Sufficient Conditions for Separation to be optimal):** ***Separation is optimal if the adverse selection problem is small enough ($\beta_0^H$ is close to $\beta_0^L$).***

*Proof*: We prove that separation is optimal in all 4 cases of the main model (depending on which $IC$ are binding) if the adverse selection problem is small enough. That is, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\bar{\beta}}_0^H(\beta_0^L)$, such that separation is optimal if $\beta_0^H < \bar{\bar{\beta}}_0^H(\beta_0^L)$. We consider each of the four cases in turn and prove that in each of them the principal is better off with separating contracts for experimentation and production than under integration if the adverse selection problem is not severe.

From the principal's problem in Appendix A, the expected payment by the principal to both types under integration is given by:

$$E_\theta\left[\beta_0^\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda y_t^\theta + \sum_{t=1}^{T^\theta} P_t^\theta x_t^\theta\right]$$

$$= \nu\left[\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H\right] + (1-\nu)\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + P_{T^L}^L x_{T^L}^L\right],$$

where the two $(IC)$ constraints are

$$(IC^H)\ \ \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H - \gamma\sum_{t=1}^{T^H} P_{t-1}^H \geq \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + P_{T^L}^H x_{T^L}^L$$
$$-\gamma\sum_{t=1}^{T^L} P_{t-1}^H + P_{T^L}^H\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F,$$

$$(IC^L)\ \ \beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + P_{T^L}^L x_{T^L}^L - \gamma\sum_{t=1}^{T^L} P_{t-1}^L \geq \beta_0^L \sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H$$
$$-\gamma\sum_{t=1}^{t^{L,H}} P_{t-1}^L - P_{t^{L,H}}^L\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F.$$

The expected payment by the principal to each type under separation is the sum of the standard moral hazard rent paid to the experimentation experimenter ($y_t^\theta = \frac{\gamma}{\lambda\beta_{T^\theta}^\theta}$ and $x_t^\theta = 0$ for $t \leq T^\theta$) and the adverse selection rent paid to the producer. Recalling from the proof of part (i) of Proposition B in Appendix B that this adverse selection rent depends on whether one or both $IC$ are binding (Case A or B under separation):

Case A: $\nu U_A^H + (1-\nu)U_A^L = \nu P_{T^L}^H \Delta c_{T^L+1} q_F$ since $U_A^L = 0$.

Case B: $\nu U_B^H + (1-\nu)U_B^L = \nu P_{T^L}^H \Delta c_{T^L+1} q_F + \left[\frac{\beta_0^L P_{T^L}^H \Delta c_{T^L+1} q_F - \beta_0^H P_{T^H}^L \Delta c_{T^H+1} q_F}{[\beta_0^H - \beta_0^L]}\right] EP_{T^L}^\theta,$

where $EP_{T^L}^\theta = \left(\nu P_{T^L}^H + (1-\nu)P_{T^L}^L\right)$.

Thus, we have the expected payment by the principal under separation:

Case A: $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H} + \nu P_{T^L}^H \Delta c_{T^L+1} q_F$.

Case B: $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\frac{\beta_0^L E P_{T^L}^{\theta}\, P_{T^L}^H \Delta c_{T^L+1}}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^{\theta}\, \beta_0^H P_{T^H}^L \Delta c_{T^H+1}}{(\beta_0^H-\beta_0^L)} q_F + \nu P_{T^L}^H \Delta c_{T^L+1} q_F.$$

Given that

$$(\boldsymbol{D1}) \qquad \Delta c_{t+1} = (\beta_{t+1}^H - \beta_{t+1}^L)\Delta c = \frac{\beta_0^H(1-\lambda)^t}{P_t^H} - \frac{\beta_0^L(1-\lambda)^t}{P_t^L}\Delta c = \left(\frac{\beta_0^H(1-\lambda)^t}{P_t^H} - \frac{\beta_0^L(1-\lambda)^t}{P_t^L}\right)\Delta c$$

$$= \frac{\beta_0^H(1-\lambda)^t P_t^L - \beta_0^L(1-\lambda)^t P_t^H}{P_t^H P_t^L}\Delta c = \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^t}{P_t^H P_t^L}\Delta c \text{ for any } t,$$

we rewrite the expected payment by the principal under separation:

Case A: $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F.$$

Case B: $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$

$$+\frac{\beta_0^L E P_{T^L}^{\theta}\, P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^{\theta}\, \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F.$$

Note that the standard MH payment during experimentation, denoted as $MH_e$:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$$

is paid under both integration and separation.

We compare the adverse selection rent to the producer against the second MH rent in production under integration, denoted by $MH_p$:

$$E_\theta \sum_{t=1}^{T^\theta}(1-\lambda)^{t-1}\lambda \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta}\Delta c q_F =$$

$$\left[\nu\beta_0^H \frac{(1-\beta_0^H)}{P_{T^H}^H}\left(1-(1-\lambda)^{T^H}\right) + (1-\nu)\beta_0^L \frac{(1-\beta_0^L)}{P_{T^L}^L}\left(1-(1-\lambda)^{T^L}\right)\right]\Delta c q_F.$$

We will check this for each of the Cases 1-4 depending on which $IC$ is binding under integration.

### Case 1 under integration: both IC are slack

In Case 1, when both ICs are slack, from Claim A1 of Appendix A, we have $x_t^\theta = 0$ for $t \le T^\theta$ and $\theta \in \{H, L\}$ and $y_t^\theta = \frac{\gamma}{\lambda \beta_{T^\theta}^\theta} + \frac{(1-\beta_0^\theta)}{P_{T^\theta}^\theta} \Delta c q_F$. Thus, the expected rent under integration is given by:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L} (1-\lambda)^{t-1} \lambda \left( \frac{\gamma}{\lambda \beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L} \Delta c q_F \right)$$

$$+\nu \beta_0^H \sum_{t=1}^{T^H} (1-\lambda)^{t-1} \lambda \left( \frac{\gamma}{\lambda \beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H} \Delta c q_F \right).$$

***Case A under separation (only*** $(IC^H)$ ***is binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$. Separation is optimal if

$$\nu P_{T^L}^H \frac{(\beta_0^H - \beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L} \Delta c q_F < \left[ \nu \frac{(1-\beta_0^H)\beta_0^H \left(1-(1-\lambda)^{T^H}\right)}{P_{T^H}^H} + (1-\nu) \frac{(1-\beta_0^L)\beta_0^L \left(1-(1-\lambda)^{T^L}\right)}{P_{T^L}^L} \right] \Delta c q_F,$$

$$(\boldsymbol{D1A}) \qquad \nu(1-\lambda)^{T^L}(\beta_0^H - \beta_0^L)$$

$$< \nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right) \frac{P_{T^L}^L}{P_{T^H}^H} + (1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right).$$

Since the *RHS* stays strictly positive and the *LHS* goes to zero as $\beta_0^H \to \beta_0^L$, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H1a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H1a}(\beta_0^L)$.

***Case B under separation (both*** $(IC)$ ***are binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$. Separation is optimal if

$$\frac{\beta_0^L E P_{T^L}^\theta \, P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L} \Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^\theta \, \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L} \Delta c}{(\beta_0^H-\beta_0^L)} q_F + \nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L} \Delta c q_F$$

$$< \left[ \frac{\nu(1-\beta_0^H)\beta_0^H\left(1-(1-\lambda)^{T^H}\right)P_{T^L}^L + (1-\nu)(1-\beta_0^L)\beta_0^L\left(1-(1-\lambda)^{T^L}\right)P_{T^H}^H}{P_{T^H}^H \, P_{T^L}^L} \right] \Delta c q_F.$$

Or, equivalently

$$(\boldsymbol{D1B}) \qquad \frac{\beta_0^L E P_{T^L}^\theta (1-\lambda)^{T^L} P_{T^H}^H - E P_{T^L}^\theta \, \beta_0^H (1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H} + \nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}$$

$$< \frac{\nu(1-\beta_0^H)\beta_0^H\left(1-(1-\lambda)^{T^H}\right)P_{T^L}^L + (1-\nu)(1-\beta_0^L)\beta_0^L\left(1-(1-\lambda)^{T^L}\right)P_{T^H}^H}{P_{T^H}^H \, P_{T^L}^L},$$

Since the *RHS* stays strictly positive and the *LHS* goes to zero as $\beta_0^H \to \beta_0^L$, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H1b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H1b}(\beta_0^L)$.

### Case 3 under integration: only ($\boldsymbol{IC^H}$) binding

We first determine the expected payment to the low type. From Claim A3 in Case 3 of Appendix A, we have $x_t^L = 0$, and $y_t^L = \frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for all $t \leq T^L$. Therefore, the expected payment to the low type in the principal's objective function is

$$(1-\nu)\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^L} P_t^L x_t^L\right]$$

$$= (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right).$$

We next determine the expected payment to the high type. Given $x_t^L = 0$, we have the binding ($IC^H$) constraint:

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H - \sum_{t=1}^{T^H} P_{t-1}^H\gamma$$

$$= \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F,$$

and by moving $\sum_{t=1}^{T^H} P_{t-1}^H\gamma$ to the *RHS*, the expected payment to the high type in the principal's objective function can be written as:

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H + \sum_{t=1}^{T^H} P_t^H x_t^H$$

$$= \beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F.$$

Thus, the expected payment by the principal to both types under integration in Case 3 is given by:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right).$$

***Case A under separation (only*** $(IC^H)$ ***is binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$. Separation is optimal if the expected payment under Case A separation is smaller than that under Case 2 integration:

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H} + \nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right),$$

$$(\boldsymbol{D3A}) \qquad \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H} + \nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right).$$

The *LHS* goes to $\nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}$ as $\beta_0^H \to \beta_0^L$ and the *RHS* goes to:

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right) + \nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} \text{ as } \beta_0^H \to \beta_0^L.$$

Since $\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right) > 0$, the condition is satisfied as $\beta_0^H \to \beta_0^L$. Therefore, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H3a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H3a}(\beta_0^L)$.

***Case B under separation (both $(IC)$ are binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$. Separation is optimal if

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\frac{\beta_0^L E P_{T^L}^\theta P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{E P_{T^L}^\theta \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right)$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] + \sum_{t=1}^{T^H} P_{t-1}^H\gamma - \sum_{t=1}^{T^L} P_{t-1}^H\gamma + P_{T^L}^H \Delta c_{T^L+1} q_F\right),$$

$$(\boldsymbol{D3B}) \qquad \frac{\beta_0^L E P_{T^L}^\theta (1-\lambda)^{T^L} P_{T^H}^H - E P_{T^L}^\theta \beta_0^H (1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H}\Delta c q_F + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< (1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$$

$$+\nu\left(\beta_0^H \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]+\sum_{t=1}^{T^H} P_{t-1}^H\gamma-\sum_{t=1}^{T^L} P_{t-1}^H\gamma+P_{T^L}^H\Delta c_{T^L+1}q_F\right),$$

The *LHS* goes to $\nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}$ as $\beta_0^H \to \beta_0^L$ and the *RHS* goes to:

$$\nu\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F+\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F \text{ as } \beta_0^H \to \beta_0^L.$$

Since $\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F > 0$, the condition is satisfied as $\beta_0^H \to \beta_0^L$.

Therefore, for any for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H3b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H3b}(\beta_0^L)$.

### Case 4 under integration: only ($\boldsymbol{IC^L}$) binding

We first determine the rent paid to the high type. In Case 4, from Claim A4 of Appendix A, we have $x_t^H = 0$ and $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$ for all $t \le T^H$. Therefore, the rent paid to the high type is

$$\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H+\sum_{t=1}^{T^H} P_t^H x_t^H = \beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right).$$

We next determine the rent paid to the low type. From Claim A4 of Appendix A, we have the binding ($IC^L$) constraint:

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}y_t^L+\sum_{t=1}^{T^L} P_t^L x_t^L-\sum_{t=1}^{T^L} P_{t-1}^L\gamma =$$
$$\beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H-\sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F,$$

and by moving $\sum_{t=1}^{T^L} P_{t-1}^L\gamma$ to the *RHS*, the rent paid to the low type can be written as:

$$\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L+\sum_{t=1}^{T^H} P_t^H x_t^L$$
$$= \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda y_t^H+\sum_{t=1}^{T^L} P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F.$$

Thus, the expected rent under integration is

$$\nu\left[\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H+\sum_{t=1}^{T^H} P_t^H x_t^H\right]+(1-\nu)\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L+\sum_{t=1}^{T^L} P_t^L x_t^L\right]$$
$$= \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F \end{pmatrix}.$$

***Case A under separation (only*** $(IC^H)$ ***is binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$.

Separation is optimal if

$$(1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}+\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}+$$

$$\nu P_{T^L}^H\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$<\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F \end{pmatrix},$$

$(\boldsymbol{D2A})$

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta c q_F$$

$$<\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)+$$

$$(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} \end{pmatrix}.$$

The *LHS* goes to zero and as $\beta_0^H\to\beta_0^L$. Since $t^{L,H}\to T^H\to T^L$ as $\beta_0^H\to\beta_0^L$, the *RHS* goes to:[50]

$$\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F>0.$$

Since the *RHS* stays strictly positive and the *LHS* goes to zero as $\beta_0^H\to\beta_0^L$, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H2a}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H<\bar{\beta}_0^{H2a}(\beta_0^L)$.

[50] The value of $t^{L,H}\to T^H$ as $\beta_0^H\to\beta_0^L$ because the low type's disadvantage with the probability of success goes down as $\beta_0^H\to\beta_0^L$. To see this, recall that the payment $y_t^H$ induces a lying low type to only work for $t^{L,H}\le T^H$ periods, and this difference in relative probabilities of success disappears as $\beta_0^H\to\beta_0^L$. Therefore, the value of $t^{L,H}\to T^H$ as $\beta_0^H\to\beta_0^L$.

***Case B under separation (both $(IC)$ are binding).*** We prove that separation is optimal if $\beta_0^H$ is close to $\beta_0^L$. Separation is optimal if

$$(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^L}^L} + \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda \frac{\gamma}{\lambda\beta_{T^H}^H}$$

$$+\frac{\beta_0^L EP_{T^L}^\theta P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F - \frac{EP_{T^L}^\theta \beta_0^H P_{T^H}^L \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^H}}{P_{T^H}^H P_{T^H}^L}\Delta c}{(\beta_0^H-\beta_0^L)} q_F +$$

$$\nu P_{T^L}^H \frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right)$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma - \sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F \end{pmatrix},$$

**(*D*2*B*)**
$$\frac{\beta_0^L EP_{T^L}^\theta (1-\lambda)^{T^L} P_{T^H}^H - EP_{T^L}^\theta \beta_0^H (1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H}\Delta c q_F + \nu\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^L}\Delta c q_F$$

$$< \nu\beta_0^H \sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$$

$$+(1-\nu)\begin{pmatrix} \beta_0^L \sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ +\sum_{t=1}^{T^L} P_{t-1}^L\gamma - \sum_{t=1}^{t^{L,H}} P_{t-1}^L\gamma + P_{t^{L,H}}^L\left(c_{T^H+1}^H - c_{t^{L,H}+1}^L\right)q_F \\ -\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L} \end{pmatrix},$$

Since the *LHS* goes to zero and the *RHS* goes to $(1-\nu)\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F > 0$ as $\beta_0^H \to \beta_0^L$, for any $\beta_0^L$ there exists a value of $\beta_0^H$, called $\bar{\beta}_0^{H2b}(\beta_0^L)$, such that the inequality is satisfied if $\beta_0^H < \bar{\beta}_0^{H2b}(\beta_0^L)$.

### Case 2 under integration: both IC are binding

Since the rent under integration in Case 2 is greater than in Cases 3 and 4, separation is optimal in Case 2 under the same parameters as in those two cases.

To complete the proof, define

$$\bar{\bar{\beta}}_0^H(\beta_0^L) = \min\{\bar{\beta}_0^{H1a}(\beta_0^L), \bar{\beta}_0^{H1b}(\beta_0^L), \bar{\beta}_0^{H2a}(\beta_0^L), \bar{\beta}_0^{H2b}(\beta_0^L), \bar{\beta}_0^{H3a}(\beta_0^L), \bar{\beta}_0^{H3b}(\beta_0^L)\}$$

*Q.E.D.*

Claim D2 (Sufficient Conditions for Integration to be optimal): *Integration is optimal if the adverse selection problem is severe enough ($\boldsymbol{\beta_0^H}$ is sufficiently close to one and $\boldsymbol{\beta_0^L}$ sufficiently close to zero) and $\boldsymbol{\nu}$ is high enough.*

*Proof*: We fix the experimentation lengths to the optimal values under separation. We will find conditions such that integration dominates separation given these experimentation lengths. Then, integration will also dominate (for the same parameter conditions) for the optimal $T^\theta$ under integration by revealed preference.

We argue first that we will only need to consider Case A under separation. As $\beta_0^L \to 0$ and $\beta_0^H \to 1$ then $T^L \to 0$, $T^H \to 1$, $t^{L,H} \to 0$, and $t^{H,L} \to 0$. Thus, the additional adverse selection rent in Case B under integration becomes negative as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\frac{\beta_0^L EP_{T^L}^\theta (1-\lambda)^{T^L} P_{T^H}^H - EP_{T^L}^\theta \, \beta_0^H (1-\lambda)^{T^H} P_{T^L}^L}{P_{T^L}^L P_{T^H}^H} \Delta c q_F \to \frac{0-1\,(1-\lambda)^1 1}{1(1-\lambda)} \Delta c q_F = -\Delta c q_F < 0.$$

Therefore, Case B is not relevant as $\beta_0^L \to 0$ and $\beta_0^H \to 1$.

### Case 1 under integration: both IC are slack

***Case A under separation (only $(IC^H)$ is binding).*** From condition $(F1A)$, integration is optimal if

$$\nu(1-\lambda)^{T^L}(\beta_0^H-\beta_0^L)$$

$$> \nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H}+(1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right).$$

Since the $LHS$ stays strictly positive as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\nu(1-\lambda)^{T^L}(\beta_0^H-\beta_0^L) \to \nu(1-\lambda)^0(1-0)=\nu>0,$$

and the $RHS$ goes to zero as $\beta_0^L \to 0$ and $\beta_0^H \to 1$:

$$\nu(1-\beta_0^H)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H}+(1-\nu)(1-\beta_0^L)\left(1-(1-\lambda)^{T^L}\right)$$

$$\to \nu(1-1)\left(1-(1-\lambda)^{T^H}\right)\frac{P_{T^L}^L}{P_{T^H}^H}+(1-\nu)(1-0)\left(1-(1-\lambda)^0\right)=0,$$

there exist $\bar{\beta}_0^{L1a}>0$ and $\underline{\beta}^{H1a}<1$, such that the inequality is satisfied if $\beta_0^L<\bar{\beta}_0^{L1a}$ and $\beta_0^H>\underline{\beta}^{H1a}$.

### Case 3 under integration: only $(IC^H)$ binding

***Case A under separation (only $(IC^H)$ is binding).*** From condition $(E3A)$, integration is optimal if

$$\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^H}^H}+\nu P_{T^L}^H\frac{(\beta_0^H-\beta_0^L)(1-\lambda)^{T^L}}{P_{T^L}^H P_{T^L}^L}\Delta cq_F$$

$$> (1-\nu)\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F\right)$$

$$+\nu\left(\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F\right]+\sum_{t=1}^{T^H}P_{t-1}^H\gamma-\sum_{t=1}^{T^L}P_{t-1}^H\gamma+\right.$$

$$\left.P_{T^L}^H\Delta c_{T^L+1}q_F\right).$$

Since the $LHS$ goes to $\nu\frac{\gamma}{\beta_1^H}+\nu\Delta cq_F$ and the $RHS$ goes to $\nu\gamma+\nu\Delta cq_F$ as $\beta_0^L \to 0$ and $\beta_0^H \to 1$, there exist $\bar{\beta}_0^{L3a}>0$ and $\underline{\beta}^{H3a}<1$, such that the inequality is satisfied if $\beta_0^L<\bar{\beta}_0^{L3a}$ and $\beta_0^H>\underline{\beta}^{H3a}$.

### Case 2 under integration: both IC are binding

***Case A under separation (only $(IC^H)$ is binding).***

Recalling from Claim A3 of Appendix A, we have $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F$ for $t > 1$, and $y_t^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L - t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F$ for all $t \le T^L$, we can rewrite the binding $(IC)$ constraints:

$$(IC^L)\quad x_{T^L}^L\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right] = \beta_0^L\lambda y_1^H$$
$$+\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}$$
$$-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L - t^{L,H})\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]$$
$$+\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]$$
$$-\left(1-\beta_0^L + \beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F,$$

$$(IC^H)\quad \beta_0^H\lambda y_1^H = x_{T^L}^L\left[\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^H\right]$$
$$+\gamma\sum_{t=1}^{T^H}P_t^H + \gamma\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1} - \gamma T^L(1-\beta_0^H) - \gamma\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}$$
$$-\beta_0^H\sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right]$$
$$+\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L - t^{L,H}))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right]$$
$$+\left(1-\beta_0^H + \beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L - c_{T^L+1}^H\right)q_F.$$

Solving $y_1^H$ and $x_{T^L}^L$ from the two binding $(IC)$ constraints we obtain:

$$x_{T^L}^L = \frac{\beta_0^L\lambda}{\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right]}\, y_1^H$$
$$+\frac{1}{\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^L\right]}\Bigg(\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) -$$
$$\gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1} - \beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L - t^{L,H})\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] +$$
$$\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] - \left(1-\beta_0^L + \beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F\Bigg)$$

$$y_1^H =$$
$$\frac{\left[\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda + P_{T^L}^H\right]}{\lambda\left(\beta_0^H P_{T^L}^L - \beta_0^L P_{T^L}^H\right)}\left[\begin{array}{c}\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma + \gamma\sum_{t=1}^{T^L}P_t^L - \gamma t^{L,H}(1-\beta_0^L) - \gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1} \\ -\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L} + \left(1+\lambda(T^L - t^{L,H})\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta c q_F\right] \\ +\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta c q_F\right] \\ -\left(1-\beta_0^L + \beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L - c_{T^H+1}^H\right)q_F\end{array}\right]$$

$$+\frac{\left[\beta_0^L \sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda+P_{T^L}^L\right]}{\lambda\left(\beta_0^H P_{T^L}^L-\beta_0^L P_{T^L}^H\right)}\begin{bmatrix}\gamma\sum_{t=1}^{T^H}P_t^H+\gamma\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}-\gamma T^L(1-\beta_0^H)-\gamma\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\\ +\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+(1+\lambda(T^L-t^{L,H}))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F\right]\\ -\beta_0^H\sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right]\\ +\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L-c_{T^L+1}^H\right)q_F\end{bmatrix}$$

$x_{T^L}^L =$

$$\frac{\beta_0^H}{\left(\beta_0^H P_{T^L}^L-\beta_0^L P_{T^L}^H\right)}\begin{bmatrix}\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\gamma+\gamma\sum_{t=1}^{T^L}P_t^L-\gamma t^{L,H}(1-\beta_0^L)-\gamma\beta_0^L\sum_{t=1}^{t=t^{L,H}}(1-\lambda)^{t-1}\\ -\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+\left(1+\lambda(T^L-t^{L,H})\right)\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F\right]\\ +\beta_0^L\sum_{t=2}^{t=t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right]\\ -\left(1-\beta_0^L+\beta_0^L(1-\lambda)^{t^{L,H}}\right)\left(c_{t^{L,H}+1}^L-c_{T^H+1}^H\right)q_F\end{bmatrix}$$

$$+$$

$$\frac{\beta_0^L}{\left(\beta_0^H P_{T^L}^L-\beta_0^L P_{T^L}^H\right)}\begin{bmatrix}\gamma\sum_{t=1}^{T^H}P_t^H+\gamma\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}-\gamma T^L(1-\beta_0^H)-\gamma\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\\ +\beta_0^H\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^L}^L}+(1+\lambda(T^L-t^{L,H}))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F\right]\\ -\beta_0^H\sum_{t=2}^{T^H}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right]\\ +\left(1-\beta_0^H+\beta_0^H(1-\lambda)^{T^L}\right)\left(c_{T^L+1}^L-c_{T^L+1}^H\right)q_F\end{bmatrix}.$$

Note that $y_1^H \to \frac{(c_2^H-c_1^L)q_F}{\lambda}$ and $x_{T^L}^L \to (c_2^H-c_1^L)q_F$ as $\beta_0^L \to 0$ and $\beta_0^H \to 1$.

Since $y_t^L = x_{T^L}^L + \frac{\gamma}{\lambda\beta_{T^L}^L} + (1+\lambda(T^L-t))\frac{(1-\beta_0^L)}{P_{T^L}^L}\Delta cq_F$ for $t \geq 1$ and $y_t^H = \frac{\gamma}{\lambda\beta_{T^H}^H} + \frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F \to$ for $t > 1$, the expected rent paid by the principal under integration converges to

$$(1-\nu)\left[\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda y_t^L+P_{T^L}^L x_{T^L}^L\right]+\nu\left[\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda y_t^H\right]$$

$$\to \nu\lambda y_1^H = \nu\lambda\frac{(c_2^H-c_1^L)q_F}{\lambda} = \nu(c_2^H-c_1^L)q_F.$$

***Case A under separation (only*** $(IC^H)$ ***is binding).*** The rent under separation converges to $\nu\gamma+\nu\Delta cq_F$ as $\beta_0^L \to 0$ and $\beta_0^H \to 1$. Therefore, integration is optimal as $\beta_0^L \to 0$ and $\beta_0^H \to 1$ if

**(*D4A*)** $$\nu(c_2^H-c_1^L)q_F < \nu\gamma+\nu\Delta cq_F,\ (c_2^H-c_1^L)q_F < \gamma+\Delta cq_F,$$

which holds for any parameters. Thus, there exist $\bar{\beta}_0^{L4a} > 0$ and $\underline{\beta}^{H4a} < 1$, such that integration is optimal if $\beta_0^L < \bar{\beta}_0^{L4a}$, $\beta_0^H > \underline{\beta}^{H4a}$.

Case 4 under integration: only $(IC^L)$ binding

***Case A under separation (only*** $(IC^H)$ ***is binding).*** From condition $(E2A)$, integration is optimal if

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta cq_F > \nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right)+$$

$$(1-\nu)\begin{pmatrix}\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right]\\+\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}\end{pmatrix},$$

Since the *LHS* goes to $\nu\Delta cq_F$ as $\beta_0^L\to 0$ and $\beta_0^H\to 1$:

$$\nu(1-\lambda)^{T^L}\left(\frac{\beta_0^H-\beta_0^L}{P_{T^L}^L}\right)\Delta cq_F\to\nu(1-\lambda)^0\left(\frac{1-0}{P_0^L}\right)\Delta cq_F=\nu\Delta cq_F,$$

and the *RHS* goes to $(1-\nu)(c_2^H-c_1^L)q_F$ as $\beta_0^L\to 0$ and $\beta_0^H\to 1$:

$$\nu\beta_0^H\sum_{t=1}^{T^H}(1-\lambda)^{t-1}\lambda\left(\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right)+$$

$$(1-\nu)\begin{pmatrix}\beta_0^L\sum_{t=1}^{t^{L,H}}(1-\lambda)^{t-1}\lambda\left[\frac{\gamma}{\lambda\beta_{T^H}^H}+\frac{(1-\beta_0^H)}{P_{T^H}^H}\Delta cq_F\right]\\+\sum_{t=1}^{T^L}P_{t-1}^L\gamma-\sum_{t=1}^{t^{L,H}}P_{t-1}^L\gamma+P_{t^{L,H}}^L\left(c_{T^H+1}^H-c_{t^{L,H}+1}^L\right)q_F-\beta_0^L\sum_{t=1}^{T^L}(1-\lambda)^{t-1}\lambda\frac{\gamma}{\lambda\beta_{T^L}^L}\end{pmatrix},$$

$$\to\nu\sum_{t=1}^{1}(1-\lambda)^{t-1}\lambda\frac{(1-1)}{P_1^H}\Delta cq_F+(1-\nu)P_0^L(c_{1+1}^H-c_{0+1}^L)q_F=(1-\nu)(c_2^H-c_1^L)q_F,$$

there exist $\bar{\beta}_0^{L2a}>0$ and $\underline{\beta}^{H2a}<1$, such that the inequality is satisfied if $\beta_0^L<\bar{\beta}_0^{L2a}$, $\beta_0^H>\underline{\beta}^{H2a}$ and $\nu>\frac{(c_2^H-c_1^L)}{\Delta c+c_2^H-c_1^L}$.

*Q.E.D.*