Leverage Corrections to Price Discovery Measures With an Application to Leveraged Exchange-Traded Funds*

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Abstract

This study introduces novel leverage-robust measures to examine the price discovery relationship among tradable products with leveraged returns. Structural analysis reveals that most existing price discovery measures exhibit significant leverage bias, a tendency to misidentify products with higher leverage ratios as the price leader. The proposed measures correct this bias by incorporating long-run responses to the permanent shock. A simulation study demonstrates the superiority of these new measures. Further empirical study examines price leadership among leveraged and regular ETFs tracking the S&P 500 index and shows that the regular ETF dominates the price discovery process.

Keywords: price discovery, information share, leveraged products, Exchange-traded Funds (ETFs).

JEL Classification: C32, G10, G14

1 Introduction

Price discovery is a key research question for information efficiency in financial markets with significant practical implications (Hasbrouck 1995). However, existing price discovery measures (Hasbrouck 1995; Gonzalo and Granger 1995; Yan and Zivot 2010; Putniņš 2013; Patel et al. 2020; Shen et al. 2024b) are limited to conventional one-to-one cointegrating relationships among tradable products that deliver nearly identical short-term returns (e.g., dual-listed securities, spot and forward prices, ETFs tracking the same market index), which significantly restrict the study of general price discovery among products tracking the same underling assets but with different leverage ratios or even nonlinear relationships. Using leveraged ETFs (LETFs hereafter) tracking the S&P 500 index as an example, this study makes a significant breakthrough in this literature by providing novel leveraged-robust price discovery measures. Through structural analysis, we demonstrate that existing price discovery measures suffer from a potentially significant leverage bias, a tendency to misidentify the product with a larger leverage as the price leader, and hence leading to erroneous conclusions regarding price leadership in the price discovery process. To address this issue, we propose the leverage-corrected Price Information Leadership (CPIL) measure, along with its share (CPILS) and binary form (CPILI), which offer precise identification of price discovery in the presence of leverage. These measures enable a broader exploration of information efficiency in modern financial markets.

Relaxing the one-to-one cointegration assumption generalizes the application of price discovery measures since the non-unitary cointegration relations are common in financial markets. For example, internationally cross-listed stock prices may not be unitarily cointegrated due to capital control and time-varying exchange rates (Eun and Sabherwal 2003; Grammig et al. 2005), especially for emerging markets, like the China mainland and Hong Kong dual-listed stocks. For commodity futures, Figuerola-Ferretti and Gonzalo (2010) develop a theoretical model to relate the cointegration coefficient with long-run contango or backwardation in the presence of finite elasticity of arbitrage services and convenience yields and empirically document non-unitary cointegration relations for spot and future prices of five metals. Given recent financial industry innovations, non-unitary cointegration relations commonly exist among non-traditional products, such as tradable volatility products (Fernandez-Perez et al. 2018) and crypto derivatives (Alexander and Heck 2020). More importantly, relaxing the assumption of unitary cointegration is necessary to study general price discovery questions, such as merger-acquisition pairs, which are not examined in the cointegration-based framework (Buehlmaier and Zechner 2021). In this study, we take the fast-growing LETFs as a research objective to show the necessity of leverage correction for existing price discovery measures.

Introduced in the late 2000s, LETFs, claiming to deliver amplified daily returns of the underlying assets, have gone through rapid growth and become popular tools for investors, especially retail investors. In 2008, LETFs managed a total of $21.9 billion in assets under management (AUM)[1], a figure that surged to $111.33 billion across 191 ETFs traded on U.S. markets by July 2024, according to etf.com. By providing convenient leveraged exposures to various assets, the LETFs is an important financial innovation for retail investors who cannot easily borrow in the market to create leveraged positions. However, significant long-term tracking error due to daily compounding (Avellaneda and Zhang 2010; Lu et al. 2012) and high expense ratios[2], make LETFs inappropriate for long-term investors (Cheng and Madhavan 2009). Recently, policymakers, media, and researchers have been concerned about the negative impact of LETFs, especially the rebalancing demand of LETFs (Lenkey 2024). Although statistically significant relationships between LETFs rebalancing and late-day returns and volatility have been documented by early studies (Shum et al. 2016), the relationships are not economically significant and the impact of LETFs rebalancing has declined over time (Ivanov and Lenkey 2018; Brøgger 2021; Lenkey 2024).

[1]Source: Bloomberg: https://www.bloomberg.com/news/articles/2022-02-28/is-it-smart-to-invest-in-etfs-why-retail-traders-are-flocking-to-risky-trades

[2]According to etf.com, the average expense ratio of LETFs is 1.04%, while the average expense ratio of all ETFs is approximately 0.5% (Box et al. 2020). Within our sample, the expense ratios of S&P 500 LETFs are approximately 0. 9% and the expense ratio of SPDR S&P 500 ETF Trust is only 0.09%

Given the fast-growing popularity of LETFs, a critical research question of price discovery arises: Does the LETF market provide incremental information and lead the regular ETF market? On the one hand, "smart money" may utilize LETFs to exploit their information advantages and amplify profits. If this is the case, new information is likely to emerge first in the LETF market. On the other hand, institutional investors, commonly believed to seize the information advantage, could already take leveraged long or short positions on various assets without relying on LETFs, which often come with higher costs. If this holds, the LETF market could be driven by herding retail investors and hence contains no incremental information regarding the underlying assets.

However, there is scant empirical evidence to address this question. The primary challenge hindering relevant research is that existing price discovery measures are not directly applicable to products that track an underlying asset with amplified returns, particularly when the leverage ratio is unspecified. This paper fills this gap by introducing novel leverage-robust price discovery measures. We then apply these metrics to S&P 500 leveraged ETFs to examine the price leadership among the most popular LETFs.

The empirical measures most widely used to quantify price discovery are the information share measure (IS) by Hasbrouck (1995) and the component share measure (CS) by Gonzalo and Granger (1995). As noted in Patel et al. (2020) and many other studies, IS is a variance-weighted version of CS, and IS is believed to produce a more robust leadership estimate and is the most widely used measure to quantify the process of price discovery among closely related products empirically[3]. Despite the popularity of the IS measure, it suffers from a serious identification problem, the dependence on the ordering of the price vector.

To address the order-dependence issue of IS, numerous studies have proposed different solutions and new measures. However, as noted in Sultan and Zivot (2015), no consensus has emerged so far in that many of these proposed alternatives (such as the use of high-frequency

[3] As noted in Sultan and Zivot (2015), IS has been widely used to examine price discovery between quotes and trade prices, stock options and underlying stocks, future and spot prices, credit and equity market, etc. (Hasbrouck 1995; Harris et al. 2002; Chakravarty et al. 2004; Mizrach and Neely 2008; Lien and Shrestha 2009; Grammig and Peter 2013).

data by Hasbrouck (1995), the mean or midpoint of IS measures by Baillie et al. (2002), and the discovery in the tails by Grammig and Peter (2013)) have been found to be either only effective in a particular context or have their own identification issues. More recently, Lien and Shrestha (2009) proposed the Modified Information Share (MIS) measure based on a spectral decomposition of the correlation matrix of idiosyncratic residuals, and Sultan and Zivot (2015) and Shen et al. (2024a) proposed the Price Discovery Share (PDS) based on a simple additive decomposition of the variance of the underlying efficient market innovations. These two measures share the same variance-attribution essence of the IS measure and, more importantly, are order-invariant, yielding unique price discovery measures.

However, traditional price discovery measures (IS and CS) as well as these order-invariant IS-type measures (MIS and PDS) are based on residuals from a reduced-form vector error correction model (VECM). Through a structural cointegration model that features two types of structural shocks (a permanent news innovation to the common fundamental value and a transitory liquidity/noise trading shock), Yan and Zivot (2010) and Shen et al. (2024b) show that both IS-type and CS measures account for the relative avoidance of noise trading and liquidity shocks.

To help sort out the confounding effects of responses to transitory shocks, Yan and Zivot (2010) propose the information leadership measure (IL hereafter), a combination of the IS and CS measures, which can be shown to provide clear leadership estimates in a restricted case with uncorrelated reduced-form residuals. For the more general case with possibly correlated idiosyncratic reduced errors, Shen et al. (2024b) propose the price information leadership measure (PIL), a combination of the PDS and CS measures, to unravel the confounding effects of transitory noise. The improved PIL measure is shown to reflect the ratio of initial responses of competing markets to the permanent shock and is capable of detecting the correct price leader even when reduced-form shocks are correlated.[4]

[4]Shen et al. (2024b) also define a price information leadership share measure (PILS), which takes a value between 0 and 1, and a binary price information leadership indicator measure (PILI), following the definition of the information leadership share measure (ILS) in Putniņš (2013) and the binary information leadership indicator (ILI) in Patel et al. (2020), respectively.

Despite this, applications of price discovery measures to leveraged financial products are rarely seen. For leveraged products, can traditional metrics, such as IS-type measures (IS, MIS, PDS), the CS measure, or combined IL-type measures (IL and PIL), reliably capture price leadership?

The answer is not yet. In this paper, we demonstrate that all existing price discovery measures suffer from another overlooked issue when applied to products with leveraged returns: *leverage bias*. In the reduced-form VECM from which these measures are derived, it is always assumed that the cointegrating vector is $(1, -1)'$ for the bivariate case. This one-to-one cointegration relationship implies that the long-run responses of both markets to a fundamental shock in the efficient price are identical. However, in a more general case where the cointegrating vector is $(1, -\beta)'$ with $\beta \neq 1$, the long-run responses of competing markets to the permanent shock are proportional to each market's relative leverages.[5] As a result, leverages can magnify markets' responses to shocks, causing traditional IS-type and combined IL-type measures to erroneously identify the market with greater leverage ratios as the price leader, even though it may take longer for that market to incorporate new information. We refer to this misidentification of price leadership as *leverage bias*.[6]

In light of the above analysis, we propose a correction for the IL-type measures to address leverage bias. This correction involves scaling each market's initial response by its long-run response to the permanent shock and using this scaled response to identify the price leader. We refer to this corrected measure as the Leverage-Corrected Information Leadership measure (CIL).[7] Empirically, each market's long-run response to the permanent shock can be captured by its cointegrating coefficient, which corresponds to its relative leverage ratio.

[5]The cointegrating vector $(1, -\beta)'$ implies that the second market's leverage is $1/\beta$ relative to the first market. For more details, see Section 3.

[6]Leverage bias differs from the β-neutrality concept as outlined by Lautier et al. (2024). According to Lautier et al. (2024), all IS-type measures are invariant to β-normalization, a property referred to as β-neutrality of the IS family measures. β-neutrality implies that these IS-type measures remain applicable to products with non-unitary cointegrating vectors. However, our structural analysis reveals that these β-neutral IS-type measures are also subject to leverage bias.

[7]The leverage-corrected variants for ILS, ILI, PILS, and PILI are denoted as CILS, CILI, CPILS, and CPILI, respectively.

For products with unknown leverage ratios, our simulations and empirical analysis show that leverage ratios can be consistently estimated, even when using low-frequency data.

An alternative approach to quantifying price leadership in leveraged products is to normalize the original price vector by their relative leverages, ensuring that the normalized price vector is cointegrated with the vector $(1, -1)'$ in the bivariate case. We show that IS-type measures (IS, MIS, and PDS) are invariant to β-normalization, whereas the CS measure is not. Based on this insight, we also derive the aforementioned leverage-corrected IL-type measures by integrating IS-type measures with a leverage-corrected CS measure.

A partial price adjustment model is used to illustrate the structural meanings of these measures and supports our leverage correction rationales. Simulation evidence based on this partial adjustment model with different scenarios of transitory and permanent shock responses verifies the superiority of our leverage-corrected measures, among which the Leverage-Corrected Price Information Leadership measure (CPIL), and its share (CPILS) and binary measure (CPILI) outperformed all other measures.

In the empirical investigation, we apply our leverage-corrected price discovery measures to six representative exchange traded products that track the S&P 500 index: the SPDR S&P 500 Trust ETF (SPY), the ProShares Ultra S&P500 ETF (SSO), the ProShares UltraPro S&P500 ETF (UPRO), the ProShares Short S&P500 ETF (SH), the ProShares UltraShort S&P500 ETF (SDS), and the Proshares UltraPro Short S&P500 ETF (SPXU). These ETFs are supposed to deliver one-time (+1), two-times (+2), three-times (+3), inverse one-time (-1), inverse two-times (-2) and inverse three-times (-3) daily returns of the S&P 500 index, respectively.[8] Empirical results show that SPY dominates all LETFs for over 90% of the time, with a descending order of the price discovery shares of SPY(+1) > UPRO(+3) > SSO(+2) ≈ SPXU(-3) > SDS(-2) > SH(-1).[9] These results confirm that the S&P 500 index LETF

[8]Note that our newly-proposed price discovery measures are not limited to LETFs tracking equity indexes like the S&P500 index. Based on the essential econometric tool in this paper, numerous future applications could be implemented.

[9]The notation (+1) represents the one-time daily returns of the S&P 500 index, while (-1) indicates the inverse of the one-time daily returns of the S&P 500 index, and so forth.

market is mostly driven by herding investors and contains less incremental information.

We further explore the potential determinants of LETFs' price leadership by performing a regression analysis with leverage-corrected price discovery measures of LETFs as the dependent variable and a series of fund characteristics and market condition variables as explanatory variables. We find that illiquidity and retail trading dampen LETFs' price leadership, while trading activity improves it. When investor sentiment (Baker and Wurgler 2006) is high and fear (the VIX index) is low, LETFs with long positions have higher levels of price discovery shares, and vice versa for inverse LETFs. Furthermore, LETFs exhibit higher price discovery shares when the S&P 500 index jumps (higher return kurtosis), when the economic policy uncertainty is higher (Baker and Wurgler 2006), when Fed-related monetary information is release, and during the COVID-19 and 2022 Russo-Ukrainian war outbreaks, suggesting that LETFs tend to be popular for day trading strategies when influential economic or policy information is released and the stock market is highly volatile.

The contribution of this paper is three-fold.

First, we are the first to discover and discuss leverage bias in a broad range of existing price discovery measures. Most existing price discovery measures are applicable to closely related products with a one-to-one cointegration relation. Although recent studies attempt to generalize price discovery analysis among interrelated products with non-unitary cointegrated vectors, the leverage bias is still present in these so-called generalized measures. Lien and Shrestha (2014) developed a generalized information share (GIS) which is claimed to be applicable to asset prices that are non-unitarily cointegrated. Many studies have adopted the GIS measure to quantify price discovery relationships among more generally cointegrated products, such as the VIX short-term futures and the inverse VIX short-term ETN in Fernandez-Perez et al. (2018), the perpetual swaps and futures of bitcoin in Alexander and Heck (2020), and a stock market climate risk indicator and global green bond indices in Hou et al. (2024).

However, by examining the exact definition of the GIS measure in the Appendix, we show that this GIS measure shares the same definition as their unitary-cointegrated counterpart, the MIS measure, as defined in Lien and Shrestha (2009). The equivalence of the MIS and GIS measures just reflects the fact that all IS-type measures are β-neutral, as documented in Lautier et al. (2024). Moreover, through a structural analysis, we show that the GIS (MIS) measure is not a pure price discovery measure in that it incorporates the impact of contemporaneous transitory responses as well as contemporaneous permanent responses. This GIS (MIS) measure should be combined with the CS measure to eliminate transitory noise, as proposed in Shen et al. (2024b), to form an IL-type measure. However, these improved IL-type measures proposed by Shen et al. (2024b) still suffer from leverage bias, tending to misidentify the leveraged product as the price leader.

A recent work by Lautier et al. (2024) proposes a CovIS measure based on the covariances between the reduced-form shocks and the common permanent shock. We show that this CovIS measure also suffers from the aforementioned leverage bias, and we propose a leverage-corrected version of the CovIS measure which can be directly applied to non-unitarily cointegrated products. To make this CovIS measure applicable to non-unitarily cointegrated products, Lautier et al. (2024) proposes normalizing the price vectors so that the normalized price vector is one-to-one cointegrated. The proposed leverage-corrected CovIS measure yields price discovery estimates similar to those obtained by applying the standard CovIS measure to β-normalized price series for leveraged products.

We differ from Lautier et al. (2024) in that we provide a systematic analysis of leverage bias and its corrections through a structural framework as well as a reduced-form approach, while Lautier et al. (2024) focuses more on the identification of structural responses to long-run shocks in the conventional one-to-one cointegrated situation. Our reduced-form analysis of the leverage bias and its corrections shares the same essence with Lautier et al. (2024)'s β-normalization solution for examining price discovery among non-unitarily cointegrated products. However, we also provide a more thorough structural analysis of long-run responses of leveraged products and highlight the necessity of correcting the leverage bias inherent in existing price discovery measures.

Second, we supplement the field of applied price discovery study with leverage-corrected measures, which broadly extend the scope of price discovery studies. As discussed previously, the leverage bias of existing price discovery measures prevents researchers from studying price discovery in more general scenarios where leverage is significant, such as cross-listed stock prices with capital controls, commodity futures, complex exchange-traded products like LETFs, derivatives of unconventional products like volatility or cryptocurrencies (Eun and Sabherwal 2003; Figuerola-Ferretti and Gonzalo 2010; Fernandez-Perez et al. 2018; Alexander and Heck 2020). In measuring price leadership among leveraged products, it is essential to correct for this leverage bias to obtain a pure price leadership measure. In this paper, we propose a general leverage correction method to existing measures by scaling each market's initial response by its long-run response to the permanent shock and using the scaled response as the price leader identifier. Among all these leverage-corrected measures, we prove that the Leverage-Corrected Price Information Leadership measures (CPIL, CPILS, and CPILI) provide the most accurate price leadership identification. Our contributions extend the use of recently developed price discovery measures of (Shen et al. 2024a) to contexts with non-unitary cointegrating relationships.

Third, we contribute to the literature by studying LETFs from an information perspective. With leverage-corrected price discovery measures, we examine the price discovery relationship among some of the most popular ETF and LETFs that track the S&P 500 index. Existing studies on LETFs have examined its properties, such as path-dependent performance (Avellaneda and Zhang 2010; Charupat and Miu 2011), and discuss whether LETFs amplify late-day volatility (Shum et al. 2016; Ivanov and Lenkey 2018; Lenkey 2024). We answer the research question whether LETFs contains incremental information by showing that LETFs are dominated by regular ETF (SPY) in price discovery. Consistent with DeVault et al. (2021) who find that institutional holdings of LETFs are related to poor manager skills and market timing, our empirical results suggest that LETFs are not the main venue for "smart money" to price their information advantages. More importantly, by laying the foundation for future studies on the information efficiency of various leveraged products, the contribution of this study goes beyond the examination of LETFs tracking the S&P 500 index.

The remainder of the paper is organized as follows. In Section 2, we first review existing price discovery measures defined from reduced-form cointegration models and explore their structural meanings through structural analysis. We then discuss the issue of leverage bias and propose leverage corrections to price discovery measures in Section 3. Illustration and simulation evidence based on a partial price adjustment model are shown in Section 4 and Section 5, respectively. In Section 6, we apply these proposed measures to the S&P 500 leveraged ETFs and discuss price leadership among these leveraged ETFs. Conclusions are provided in Section 7.

2 Methodology Summary

In this section, we first review existing price discovery measures based on reduced-form VECM models. In order to give clear interpretations of these price discovery measures, we then follow the structural analysis in Yan and Zivot (2010) to examine the exact meaning of these price discovery measures.

2.1 Measuring price discovery in reduced-form models

For simplicity, we focus on the case of two products.[10] Let $\mathbf{p}_t = (p_{1t}, p_{2t})'$ denote the logarithmic price series of two closely related tradable products. In the price discovery literature, it is always assumed that these two price series are integrated of order 1, or I(1), and that $\boldsymbol{p}_t$ is cointegrated with the cointegrating vector $\boldsymbol{\beta} = (1, -1)'$. Most price discovery

[10] Price discovery for multivariate products is discussed in Appendix A1.

measures are then derived from the following reduced-form VECM:

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha}\boldsymbol{\beta}' \mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{\Gamma}_j \Delta \mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)'$ is the error correction vector, $\boldsymbol{\Gamma}_j$ is the short-run coefficients, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the vector of reduced-form VECM residuals with $E[\boldsymbol{\varepsilon}_t] = 0$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \boldsymbol{\Omega}$. Denote elements of $\boldsymbol{\Omega}$ as σ_{ij} for $i, j = 1, 2$.

Hasbrouck (1995) transforms the above VECM model into a (reduced-form) vector moving average (VMA):

$$\Delta \mathbf{p}_t = \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1 \boldsymbol{\varepsilon}_{\boldsymbol{t-1}} + \boldsymbol{\Psi}_2 \boldsymbol{\varepsilon}_{\boldsymbol{t-2}} + \cdots, \tag{2}$$

and its integrated form:

$$\mathbf{p}_t = \boldsymbol{p}_0 + \boldsymbol{\Psi}(1) \sum_{s=1}^{t} \boldsymbol{\varepsilon}_s + \boldsymbol{\Psi}^*(L)\boldsymbol{\varepsilon}_t, \tag{3}$$

where $\boldsymbol{\Psi}(1) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k$ with $\boldsymbol{\Psi}(L)$ and $\boldsymbol{\Psi}^*(L)$ being matrix polynomials in the lag operator, L, and $\boldsymbol{\Psi}^*(k) = -\sum_{j=k+1}^{\infty} \boldsymbol{\Psi}_j$.

The matrix $\boldsymbol{\Psi}(1)$ contains the cumulative impacts of innovation $\boldsymbol{\varepsilon}_t$ on all future price movements. As shown in Hasbrouck (1995), the rows of $\boldsymbol{\Psi}(1)$ are identical given $\boldsymbol{\beta} = (1, -1)'$. Denote $\boldsymbol{\psi} = (\psi_1, \psi_2)'$ as the common row vector of $\boldsymbol{\Psi}(1)$, and define the permanent innovation as:

$$\eta_t^P = \boldsymbol{\psi}' \boldsymbol{\varepsilon}_t = \psi_1 \varepsilon_{1t} + \psi_2 \varepsilon_{2t}. \tag{4}$$

The common efficient price $m_t = m_{t-1} + \eta_t^P$ evolves as a random walk driven by the permanent shock η_t^P.

Booth et al. (1999), Chu et al. (1999), and Harris et al. (2002) suggest to use the component share (CS) measure to quantify each market's contribution to the common component:

$$\text{CS}_1 = \frac{\psi_1}{\psi_1 + \psi_2}, \quad \text{CS}_2 = \frac{\psi_2}{\psi_1 + \psi_2}. \tag{5}$$

Another more widely used approach to quantify price discovery is the Information Share (IS) type measures (Hasbrouck 1995). These IS-type metrics quantify the contribution of each market to the variance of the common component, which can be written as:

$$IS_i = \frac{([\boldsymbol{\psi}'\boldsymbol{F}]_i)^2}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}, \tag{6}$$

where $[\boldsymbol{\psi}'\boldsymbol{F}]_i$ represents the ith element of the row vector $\boldsymbol{\psi}'\boldsymbol{F}$ and $\boldsymbol{F}$ denotes a specific decomposition of $\boldsymbol{\Omega}$ such that $\boldsymbol{\Omega} = \boldsymbol{F}\boldsymbol{F}'$.

In the IS measure defined by Hasbrouck (1995), $\boldsymbol{F}$ is the Cholesky factorization of $\boldsymbol{\Omega}$. When the reduced-form residuals are uncorrelated (i.e., diagonal $\boldsymbol{\Omega}$), the above factorization is unique, so are the resulting IS measures. However, it is usually the case that reduced-form residuals from closely related markets are correlated. With a non-diagnal $\boldsymbol{\Omega}$, the Cholesky factor $\mathbf{F}$ depends on the ordering of the price series in $\mathbf{p}_t$, so does the value of IS. By considering all permutations of the price series, one can compute the upper and lower bounds for the IS measures. Empirically, researchers often use the midpoint of the upper and lower bounds for each market as the final IS measure, as suggested in Baillie et al. (2002).

The Modified Information Share (MIS) measure proposed by Lien and Shrestha (2009) utilizes a factorization matrix based on the spectral decomposition of the correlation matrix of the reduced-form residuals.[11] As emphasized in Lien and Shrestha (2009), this new factor structure and the resulting MIS measures are independent of ordering. As shown in Shen et al. (2024b), the MIS measure decomposes the variance contribution to each market more equally than the IS measure and coincides with the IS measure when $\boldsymbol{\Omega}$ is diagonal.

More recently, Sultan and Zivot (2015) and Shen et al. (2024a) have proposed an alter-

[11]Detailed expression for MIS see (A.17) in Section A2.

native approach to decompose the volatility $\sigma_\eta(\psi) = (\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi})^{1/2}$ of the common price shock η_t^P. Based on this alternative decomposition, they propose a new order-invariant measure of each market's price discovery share, denoted PDS, as follows:

$$PDS_i = \frac{\psi_i}{\sigma_\eta(\psi)} \frac{\partial \sigma_\eta(\psi)}{\partial \psi_i} = \frac{\psi_i}{\sigma_\eta(\psi)} \frac{[\boldsymbol{\Omega}\boldsymbol{\psi}]_i}{\sigma_\eta(\psi)} = \frac{\psi_i^2 \sigma_i^2 + \sum_{j=1}^{n} \psi_i \psi_{j\neq i} \sigma_{i,j\neq i}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}. \tag{7}$$

As discussed in Sultan and Zivot (2015) and Shen et al. (2024a), the above PDS measures are order invariant and unique, which coincide with IS measures when the reduced form residuals are uncorrelated (diagonal $\boldsymbol{\Omega}$).

2.2 A structural analysis of price discovery measures

In order to give clear interpretations of price discovery measures, Yan and Zivot (2010) start with the following structural moving average (SMA) representation of $\triangle \boldsymbol{p}_t$:

$$\triangle \boldsymbol{p}_t = \boldsymbol{D}(L)\boldsymbol{\eta}_t = \boldsymbol{D}_0\boldsymbol{\eta}_t + \boldsymbol{D}_1\boldsymbol{\eta}_{t-1} + \boldsymbol{D}_2\boldsymbol{\eta}_{t-2} + \dots \tag{8}$$

where the elements of $\{\boldsymbol{D}_k\}_{k=0}^{\infty}$ are 1-summable, $\boldsymbol{D}(L) = \sum_{k=0}^{\infty} \boldsymbol{D}_k L^k$, and $\boldsymbol{D}_0$ is invertible. Innovation in the efficient price of the asset, η_t^P, is labeled permanent and noise innovation, η_t^T, is labeled transitory so that $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$. These structural shocks are assumed to be serially and mutually uncorrelated with a diagonal covariance matrix $\boldsymbol{C} = diag(\sigma_P^2, \sigma_T^2)$. The matrix $\boldsymbol{D}_0$ contains the initial impacts of the structural shocks on $\triangle \boldsymbol{p}_t$ and defines the contemporaneous correlation structure of $\triangle \boldsymbol{p}_t$:

$$\boldsymbol{D}_0 = \begin{pmatrix} d_{0,1}^P & d_{0,1}^T \\ d_{0,2}^P & d_{0,2}^T \end{pmatrix}. \tag{9}$$

The parameters $d_{0,i}^P$ and $d_{0,i}^T$ (i=1,2) are the contemporaneous responses of p_{it} to informational and frictional innovations, respectively.

The permanent innovation η_t^P carries new information on the fundamental value of the asset and permanently moves the market prices. The defining characteristic of η_t^P is that it has a long-term one-to-one effect on the price levels of each market. The transitory innovation η_t^T summarizes non-information-related shocks, such as the trading by uninformed or liquidity traders. The defining characteristic of η_t^T is that it is not correlated with information innovation η_t^P and has no long-term effect on price levels. Hence, the long-run impact matrix $\boldsymbol{D}(1)$ of the structural innovations $\boldsymbol{\eta}_t$ takes the form

$$\boldsymbol{D}(1) = \begin{pmatrix} d_1^P(1) & d_1^T(1) \\ d_2^P(1) & d_2^T(1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}. \tag{10}$$

The Beveridge-Nelson (BN) decomposition (Beveridge and Nelson, 1981) applied to (8) yields the level relationship:

$$\boldsymbol{p}_t = \boldsymbol{p}_0 + \boldsymbol{D}(1) \sum_{j=1}^{t} \boldsymbol{\eta}_j + \boldsymbol{s}_t, \tag{11}$$

where $\boldsymbol{D}(1) = \sum_{k=0}^{\infty} \boldsymbol{D}_k$, $\boldsymbol{s}_t = (s_{1t}, s_{2t})' = \boldsymbol{D}^*(L)\boldsymbol{\eta}_t \sim I(0)$, and $\boldsymbol{D}^*(L) = -\sum_{j=k+1}^{\infty} \boldsymbol{D}_j$, $k = 0, \ldots, \infty$.

From the level representations (11) and (3) and by utilizing the relationship that $\boldsymbol{\varepsilon}_t = \boldsymbol{D}_0\boldsymbol{\eta}_t$, Yan and Zivot (2010) and Shen et al. (2024b) show that the structural representations of the CS measures are given as:

$$CS_1 = \frac{d_{0,2}^T}{d_{0,2}^T - d_{0,1}^T}, \quad CS_2 = -\frac{d_{0,1}^T}{d_{0,2}^T - d_{0,1}^T}. \tag{12}$$

As indicated in Yan and Zivot (2010), the CS measures only involve the structural parameters governing the price responses to the frictional innovation. Instead of measuring the relative strength of how a given market price responds to new information, the CS measures quantify each market's relative response to contemporaneous transitory frictions.

To examine the structural meaning of IS measures, Yan and Zivot (2010) consider the special case when the reduced-form innovations $\boldsymbol{\varepsilon}_t$ are uncorrelated (i.e., $\sigma_{12} = 0$). In this case, the IS measures can be uniquely defined as follows:

$$IS_1 = \frac{d_{0,1}^P d_{0,2}^T}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}, \quad IS_2 = \frac{-d_{0,1}^T d_{0,2}^P}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}. \tag{13}$$

As stated in Yan and Zivot (2010), the structural representations of IS consist of contemporaneous responses to both permanent and transitory shocks. Compared to CS measures, IS measures are more appropriate for measuring price discovery in that they contain responses to new information (i.e., $d_{0,i}^P$).

However, the presence of responses to frictional innovation (i.e., $d_{0,i}^T$) implies that the meaning of IS is not that clear. A market may have a high IS because it is more responsive to new information (a high value of $d_{0,1}^P$) or because the other market is more responsive to frictions (a high value of $d_{0,2}^T$). Noting the above structural interpretations of IS and CS measures, Yan and Zivot (2010) propose to combine IS and CS measures to form the Information Leadership (IL) metrics:

$$IL_1 = \left| \frac{IS_1/CS_1}{IS_2/CS_2} \right|, \quad IL_2 = \left| \frac{IS_2/CS_2}{IS_1/CS_1} \right|. \tag{14}$$

In the special case with uncorrelated residuals, it can be shown that:

$$IL_1 = \left| \frac{d_{0,1}^P}{d_{0,2}^P} \right|, \quad IL_2 = \left| \frac{d_{0,2}^P}{d_{0,1}^P} \right|. \tag{15}$$

As can be seen from the above expressions, the IL measures, as a combination of IS and CS, provide a more clear-cut measure of price leadership in that transitory responses to frictional innovation cancel out. This new price leadership measure, IL, quantifies the relative informational leadership of the targeting market.

It is worth noting that when idiosyncratic errors are correlated, the IS measure does not have the precise structural representations in Eq. (13), hence neither IL measure has the precise structural representation in Eq. (15). In contrast, as shown in Shen et al. (2024b), the PDS metric proposed by Sultan and Zivot (2015) and Shen et al. (2024a) has very clear structural representations even when the idiosyncratic residuals are correlated:

$$
\begin{aligned}
PDS_1 &= \frac{\psi_1^2\sigma_1^2 + \psi_1\psi_2\sigma_{12}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} = \frac{d_{0,1}^P d_{0,2}^T}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}, \\
PDS_2 &= \frac{\psi_2^2\sigma_2^2 + \psi_2\psi_1\sigma_{21}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} = \frac{-d_{0,1}^T d_{0,2}^P}{d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P}.
\end{aligned} \tag{16}
$$

The structural representations of the PDS consist of contemporaneous responses to both permanent and transitory shocks.

Because of the uniqueness (order-invariant) and the simplicity merit of PDS, Shen et al. (2024b) propose the Price Information Leadership (PIL) measure as follows:

$$
PIL_1 = \left| \frac{PDS_1/CS_1}{PDS_2/CS_2} \right|, \quad PIL_2 = \left| \frac{PDS_2/CS_2}{PDS_1/CS_1} \right|, \tag{17}
$$

which can be shown to have the following structural interpretations:

$$
PIL_1 = \left| \frac{d_{0,1}^P}{d_{0,2}^P} \right|, \quad PIL_2 = \left| \frac{d_{0,2}^P}{d_{0,1}^P} \right|. \tag{18}
$$

As can see from the above expressions, the PIL measure quantifies closely-related market's relative responsiveness to new information, clear of frictional responses. When $|d_{0,1}^P| > |d_{0,2}^P|$, PIL_1 takes a value greater than 1 and PIL_2 takes a value smaller than 1. The PIL measure coincides with the IL of Yan and Zivot (2010) when the idiosyncratic errors are uncorrelated.

Similar to the Information Leadership Share (ILS) measure defined by Putniņš (2013), Shen et al. (2024b) make the PIL into a share by defining the corresponding Price Information

Leadership Share (PILS) as:

$$PILS_1 = \frac{PIL_1}{PIL_1 + PIL_2}, \quad PILS_2 = \frac{PIL_2}{PIL_1 + PIL_2}. \tag{19}$$

Like ILS, PILS are within the unit interval with a value above (below) 0.5 indicating that the price series leads (does not lead) the price discovery process. Similar to the Information Leadership Indicator (ILI) measure defined by Patel et al. (2020), Shen et al. (2024b) further define a Price Information Leadership Indicator (PILI) as:

$$PILI_i = \begin{cases} 1, & \text{if } PILS_i > 0.5 \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

Just like ILI, the sample average of PILI can be interpreted as the proportion of days in which one market leads the other.

Shen et al. (2024b) also derive a structural representation for MIS in their Appendix, which is repeated in Appendix A2. As shown in Appendix A2, the structural representation of MIS is a complex combination of contemporaneous responses to both permanent and transitory shocks. Likewise, Shen et al. (2024b) define a Modified Information Leadership (MIL) by replacing PDS with MIS in Eq. (17), and its share (MILS) and binary (MILI) measures. Through a structural analysis and by comprehensive simulation studies, Shen et al. (2024b) highlight the necessity of combing the IS-type measures (IS, MIS, and PDS) with CS. More importantly, Shen et al. (2024b) show that the PIL (PILS and PILI) measure provides the most accurate leadership measures when reduced-form residuals are correlated.

Recently, Lautier et al. (2024) introduced a Covariance Information Share (CovIS) based on the covariance of reduced-form residuals with the permanent shock. Their measure is also order-invariant and correlation-robust with the following structural interpretations:

$$CovIS_1 = \frac{d^P_{0,1}}{d^P_{0,1} + d^P_{0,2}}, \quad CovIS_2 = \frac{d^P_{0,2}}{d^P_{0,1} + d^P_{0,2}}. \tag{21}$$

In essence, the structural representations of Shen et al. (2024b)'s PIL metric align closely with the CovIS metric, while Shen et al. (2024b)'s PILS metric mirrors the quadratic variation CovISQ defined in Lautier et al. (2024).

3 Leverage Bias

The price discovery measures discussed above are based on the assumption that the cointegration vector is $\boldsymbol{\beta} = (1, -1)'$. This unitary cointegration relationship implies that the long-run responses of the cointegrated asset prices to a unit of permanent shock are identical. However, for leveraged products, asset price pairs may exhibit cointegration with a coefficient of $\boldsymbol{\beta} = (1, -\beta)'$, where $\beta \neq 1$. For instance, consider the price discovery relationship between the SPDR S&P 500 ETF Trust (SPY) and the ProShares Ultra S&P 500 ETF (SSO). SSO aims to achieve daily investment results that correspond to two times (2×) the daily performance of the S&P 500, while SPY seeks to track the S&P 500 by holding a portfolio of common stocks included in the index. It is evident that the log price series of (SPY, SSO) are cointegrated with a vector $\boldsymbol{\beta} = (1, -0.5)'$. The presence of leverage directly impacts the long-run effects of price innovations on each price series, leading to differing responses.

To illustrate this point, note that, as demonstrated by Johansen (1991), the long-run impacts matrix in the reduced integrated VMA model (3) can be expressed as follows:

$$\boldsymbol{\Psi}(1) = \boldsymbol{\beta}_\perp \boldsymbol{\Pi} \boldsymbol{\alpha}'_\perp, \tag{22}$$

where $\boldsymbol{\beta}_\perp$ denotes the orthogonal vector to $\boldsymbol{\beta}$, and $\boldsymbol{\Pi} = \left(\boldsymbol{\alpha}'_\perp \left(\boldsymbol{I}_2 - \sum_{j=1}^{k} \boldsymbol{\Gamma}_j\right) \boldsymbol{\beta}_\perp\right)^{-1}$. Here,

$\boldsymbol{I}_2$ is the identity matrix and $\boldsymbol{\alpha}_\perp$ represents the orthogonal vector to $\boldsymbol{\alpha}$. When $\boldsymbol{\beta} = (1, -\beta)'$ and given that $\boldsymbol{\beta}'\boldsymbol{\Psi}(1) = \boldsymbol{0}$, it can be demonstrated that the rows of $\boldsymbol{\Psi}(1)$ are proportional:[12]

$$\boldsymbol{\Psi}(1) = \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1/\beta & \psi_2/\beta \end{pmatrix}. \tag{23}$$

We have demonstrated that when prices are cointegrated with the cointegrating vector $\boldsymbol{\beta} = (1, -\beta)'$, the long-run effect of the innovation $\boldsymbol{\varepsilon}_t$ on the second price series, p_{2t}, is $\frac{1}{\beta}$ times that on the first price series, p_{1t}. For products with a leverage greater than one, the condition $0 < \beta < 1$ holds. Consequently, leveraged products exhibit a more pronounced long-run response to price innovations.

We can also derive the unequal long-run responses for the structural moving average model in equation (11), resulting in the following long-run impact matrix:

$$\boldsymbol{D}(1) = \boldsymbol{\Psi}(1)\boldsymbol{D}_0 = \begin{pmatrix} 1 & 0 \\ 1/\beta & 0 \end{pmatrix}. \tag{24}$$

To measure the price discovery relationship among leveraged products, it is evident that the long-run responses of these products will differ. Leverage can amplify a product's response to shocks, leading traditional measures to identify the more leveraged product as the price leader, even if it takes longer to incorporate new information into its price. We refer to this incorrect identification of the price leader as *leverage bias*.

[12]This can also be shown by direct calculation. Let $\boldsymbol{\alpha}_\perp = (\gamma_1, \gamma_2)'$. Since $\boldsymbol{\Pi}$ in (22) is a scalar for the bivariate case and $\boldsymbol{\beta}_\perp = (1, 1/\beta)'$, we can write

$$\begin{aligned}\boldsymbol{\Psi}(1) = \pi\boldsymbol{\beta}_\perp\boldsymbol{\alpha}'_\perp &= \pi \begin{pmatrix} 1 \\ 1/\beta \end{pmatrix} \begin{pmatrix} \gamma_1 & \gamma_2 \end{pmatrix} \\ &= \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1/\beta & \psi_2/\beta \end{pmatrix},\end{aligned}$$

where $\psi_i = \pi\gamma_i$ $(i = 1, 2)$.

3.1 Leverage corrections based on structural representations

Given the presence of leverage bias, it is misleading to designate the product with a larger initial response to a permanent shock (i.e., the larger $d_{0,i}^P$) as the price leader when long-run responses differ. A more accurate definition is to identify the price leader as the market with a larger initial response relative to its long-run response to the permanent shock. In other words, rather than identifying the product with a larger $|d_{0,i}^P|$ as the price leader, it is more sensible to define the product with a larger

$$\left| \frac{d_{0,i}^P}{d_i^P(1)} \right| \tag{25}$$

as the leader, where $d_i^P(1)$ denotes the long-run response of the i-th product to a unit of permanent shock. When the price cointegration vector is given by $\boldsymbol{\beta} = (1, -\beta)'$, Eq. (24) shows that $d_1^P(1) = 1$ and $d_2^P(1) = 1/\beta$.

For products with a cointegrating vector $\boldsymbol{\beta} = (1, -\beta)'$, the second product in the price vector can be characterized as having a leverage of $1/\beta$ when $0 < \beta < 1$.[13] In this scenario, the second product exhibits a long-run response to a unit of permanent shock that is $1/\beta$ times as large as that of the first product. Thus, the ratio of the initial response to the long-run response can be interpreted as a leverage adjustment, where each price series is adjusted by its relative leverage.[14] After this leverage adjustment, the adjusted products exhibit identical long-term responses, making the adjusted initial responses a more accurate measure of price leadership.

What are the implications of the above reasoning for price discovery measures? If we accept that Eq. (25) accurately defines the price discovery process, we propose a straight-

[13] If $\beta < 0$, the second product is referred to as an inverse product.

[14] In the bivariate case, the first product always has a leverage of one, while the second product's leverage is $1/\beta$ according to the cointegrating vector $\boldsymbol{\beta} = (1, -\beta)'$.

forward correction to the Price Information Leadership (PIL) measures as follows:

$$
\begin{aligned}
CPIL_1 &= \left|\frac{(PDS_1/CS_1)/d_1^P(1)}{(PDS_2/CS_2)/d_2^P(1)}\right| = \left|\frac{PDS_1/CS_1}{PDS_2/(CS_2/\beta)}\right|, \\
CPIL_2 &= \left|\frac{(PDS_2/CS_2)/d_2^P(1)}{(PDS_1/CS_1)/d_1^P(1)}\right| = \left|\frac{PDS_2/(CS_2/\beta)}{PDS_1/CS_1}\right|,
\end{aligned} \tag{26}
$$

which can be shown to have the following structural interpretations:

$$
CPIL_1 = \left|\frac{d_{0,1}^P/d_1^P(1)}{d_{0,2}^P/d_2^P(1)}\right| = \left|\frac{d_{0,1}^P}{\beta d_{0,2}^P}\right|, \quad CPIL_2 = \left|\frac{d_{0,2}^P/d_2^P(1)}{d_{0,1}^P/d_1^P(1)}\right| = \left|\frac{\beta d_{0,2}^P}{d_{0,1}^P}\right|. \tag{27}
$$

We use the term "Corrected Price Information Leadership (CPIL)" to represent the leverage-corrected version of the PIL measure. With this new measure, a product with a higher $CPIL$ is associated with a larger initial response relative to its long-run impacts, measured by $d_{0,i}^P/d_i^P(1)$. Therefore, when identifying the price leader, the focus shifts from solely the initial response to new information to a comparison with the long-term impact, ensuring that leveraged products will not be identified as price leaders simply due to their greater leverages. This leverage correction is straightforward to implement empirically. By estimating β from the VECM model, these values can be applied to the leverage-corrected price discovery measures.

Accordingly, we can extend the CPIL measure into a share measure (CPILS) as defined in Eq. (19) and an indicator measure (CPILI) as in Eq. (20). Similarly, leverage-corrected CIL, CILS, and CILI can be derived from the IL measure, while CMIL, CMILS, and CMILI can be defined based on the MIL measure. However, we anticipate that CPIL (along with CPILS and CPILI) will provide the most accurate prediction of price leadership, where the value of CPILI can be interpreted as the proportion of days the identified leader dominates the price discovery process.

The CovIS measure, as defined in Lautier et al. (2024), is also affected by the leverage bias mentioned above. This is because the CovIS measure identifies the price leader exclusively based on the immediate impact of permanent shocks. In the case of leveraged products, CovIS is likely to designate the product with a higher leverage as the price leader. We propose the following leverage corrections to the CovIS measure:

$$\begin{aligned} \tilde{CovIS}_1 &= \frac{d_{0,1}^P/d_1^P(1)}{d_{0,1}^P/d_1^P(1) + d_{0,2}^P/d_2^P(1)} = \frac{d_{0,1}^P}{d_{0,1}^P + \beta d_{0,2}^P}, \\ \tilde{CovIS}_2 &= \frac{d_{0,2}^P/d_2^P(1)}{d_{0,1}^P/d_1^P(1) + d_{0,2}^P/d_2^P(1)} = \frac{\beta d_{0,2}^P}{d_{0,1}^P + \beta d_{0,2}^P}. \end{aligned} \tag{28}$$

Similar to the leverage corrections applied to IL-type measures, we scale each market's immediate response to a permanent shock by its long-run impact to identify the price leader. The leverage-corrected CPIL measure essentially shares the same structural interpretation as the leverage-corrected CovIS measure. Therefore, in the subsequent simulation and empirical analysis, we only report results based on the CPIL measures.

To extend the CovIS measure to non-unitarily cointegrated products, Lautier et al. (2024) propose normalizing price vectors such that the normalized price series become one-to-one cointegrated. The above leverage-corrected CovIS measure produces price discovery estimates similar to those obtained by applying the standard CovIS measure to β-normalized price series of leveraged products. The β-normalization approach for assessing price discovery among non-unitarily cointegrated products, as proposed by Lautier et al. (2024), aligns in principle with our reduced-form method for deriving leverage corrections, which is discussed in greater details in the following section. Our paper, however, differs from Lautier et al. (2024) in that we provide a systematic analysis of leverage bias and its corrections through a structural framework. In contrast, Lautier et al. (2024) emphasize the identification of structural responses to long-run shocks within the conventional one-to-one cointegrated setting. By offering a more rigorous structural analysis of the long-run responses of leveraged products, we underscore the necessity of correcting for leverage bias in existing price discovery measures and present a direct solution to this issue.

3.2 Leverage corrections based on reduced-form models

We can also derive the aforementioned leverage corrections from reduced-form models. Specifically, we utilize reduced-form VECM models to examine how β-normalization of the original price vector affects the underlying price discovery measures. Our analysis demonstrates that variance-decomposition measures (IS, MIS, and PDS) remain invariant to β-normalization, while CS measures do not. Building on these findings, we derive the same leverage-corrected measures by integrating variance-decomposition measures with leverage-corrected CS measures.

To normalize the price vector, let us define a normalization matrix as follows:

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ 0 & \beta \end{pmatrix}, \tag{29}$$

and pre-multiply the integrated moving average model in Eq.(3) by this matrix to get:

$$\boldsymbol{B}\mathbf{p}_t = \boldsymbol{B}\boldsymbol{p}_0 + \underbrace{\boldsymbol{B}\boldsymbol{\Psi}(1)\boldsymbol{B}^{-1}}_{\boldsymbol{\Psi}^*(1)} \sum_{s=1}^{t} \underbrace{\boldsymbol{B}\boldsymbol{\varepsilon}_s}_{\boldsymbol{\varepsilon}_s^*} + \boldsymbol{B}\boldsymbol{\Psi}^*(L)\boldsymbol{B}^{-1} \underbrace{\boldsymbol{B}\boldsymbol{\varepsilon}_s}_{\boldsymbol{\varepsilon}_s^*}. \tag{30}$$

It is easy to see that

$$\boldsymbol{B}\mathbf{p}_t = \begin{pmatrix} p_{1t} \\ \beta p_{2t} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\Psi}^*(1) = \boldsymbol{B}\boldsymbol{\Psi}(1)\boldsymbol{B}^{-1} = \begin{pmatrix} \psi_1 & \psi_2/\beta \\ \psi_1 & \psi_2/\beta \end{pmatrix}. \tag{31}$$

Hence, by normalizing the original price vector $\boldsymbol{p}_t$ by the matrix $\boldsymbol{B}$, we show that the long-run impacts of the new information (the rows of $\boldsymbol{\Psi}^*(1)$) are the same for this normalized price vector. Let us denote the common row of $\boldsymbol{\Psi}^*(1)$ as $\boldsymbol{\psi}^* = (\psi_1^*, \psi_2^*)'$, for which the above result indicates that $\psi_1^* = \psi_1$ and $\psi_2^* = \psi_2/\beta$.

Accordingly, we can pre-multiply the VECM model in Eq. (1) by the matrix $\boldsymbol{B}$, yielding:

$$\begin{aligned} \triangle \boldsymbol{B}\mathbf{p}_t &= \boldsymbol{B}\boldsymbol{\alpha}\boldsymbol{\beta}^{'}\mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{B}\boldsymbol{\Gamma}_j\boldsymbol{B}^{-1} \triangle \boldsymbol{B}\mathbf{p}_{t-j} + \boldsymbol{B}\boldsymbol{\varepsilon}_t, \\ &= \boldsymbol{\alpha}^*\boldsymbol{\beta}^{*'}\boldsymbol{B}\mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{\Gamma}_j^* \triangle \boldsymbol{B}\mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t^*, \end{aligned} \tag{32}$$

where $\boldsymbol{\alpha}^* = \boldsymbol{B}\boldsymbol{\alpha} = (\alpha_1, \beta\alpha_2)^{'}$ is the new error correction vector, $\boldsymbol{\beta}^* = \boldsymbol{B}^{-1}\boldsymbol{\beta} = (1, -1)^{'}$ is the new cointegrating vector, $\boldsymbol{\Gamma}_j^* = \boldsymbol{B}\boldsymbol{\Gamma}_j\boldsymbol{B}^{-1}$ is the new short-run coefficients, and $\boldsymbol{\varepsilon}_t^* = \boldsymbol{B}\boldsymbol{\varepsilon}_t$ is the new reduced-form residuals. Given the normalized integrated VMA in (31) and VECM in (32), we can rewrite the permanent shock as:

$$\eta_t^P = \boldsymbol{\psi}^{*'}\boldsymbol{\varepsilon}_t^* = \psi_1^*\varepsilon_{1t}^* + \psi_2^*\varepsilon_{2t}^*. \tag{33}$$

It is important to note that the above representation of the permanent shock is equivalent to (4), as derived from both the VMA model in (3) and the VECM model in (1) of the original price series.

Accordingly, from this new expression (33), the corresponding CS measures take values:

$$CS_1^* = \frac{\psi_1^*}{\psi_1^* + \psi_2^*} = \frac{\psi_1}{\psi_1 + \psi_2/\beta}, \quad CS_2^* = \frac{\psi_2^*}{\psi_1^* + \psi_2^*} = \frac{\psi_2/\beta}{\psi_1 + \psi_2/\beta}. \tag{34}$$

Thus, it becomes evident that the CS measures are not invariant to the β-normalization of the price series. We refer to the resulting CS^* measures as leverage-corrected CS measures.

In contrast, the IS, MIS, and PDS measures are invariant to the β-normalization of the price series (i.e., β-neutral as defined in Lautier et al. (2024)). In other words, the estimates of these IS-type measures (IS, MIS, and PDS) obtained from the VECM model (1) of the original price vector (p_{1t}, p_{2t}) are identical to those derived from the VECM model (32) of the normalized price vector $(p_{1t}, \beta p_{2t})$. To demonstrate this, we can derive the variance-covariance matrix for the normalized reduced-form residuals:

$$\boldsymbol{\Omega}^* = \boldsymbol{B}\boldsymbol{\Omega}\boldsymbol{B} = \begin{pmatrix} \sigma_1^2 & \beta\sigma_{12} \\ \beta\sigma_{12} & \beta^2\sigma_2^2 \end{pmatrix}. \tag{35}$$

It is straightforward to observe that the variance of the permanent shock remains unchanged under β-normalization:

$$\begin{aligned} \boldsymbol{\psi}^{*'}\boldsymbol{\Omega}^*\boldsymbol{\psi}^* &= \psi_1^2\sigma_1^2 + 2\psi_1(\psi_2/\beta)\beta\sigma_{12} + (\psi_2/\beta)^2\beta^2\sigma_2^2, \\ &= \psi_1^2\sigma_1^2 + 2\psi_1\psi_2\sigma_{12} + \psi_2^2\sigma_2^2. \end{aligned} \tag{36}$$

The β-neutrality property of the IS, MIS, and PDS measures can be verified by substituting $\boldsymbol{\psi}^*$ and $\boldsymbol{\Omega}^*$ into the bivariate expressions for IS, MIS, and PDS, as demonstrated in Shen et al. (2024b).

Building on the leverage-corrected CS measures in Eq. (34), we can define the corresponding leverage-corrected PIL measures as follows:

$$\begin{aligned} CPIL_1 &= \left|\frac{PDS_1/CS_1^*}{PDS_2/CS_2^*}\right| = \left|\frac{PDS_1/CS_1}{PDS_2/(CS_2/\beta)}\right|, \\ CPIL_2 &= \left|\frac{PDS_2/CS_2^*}{PDS_1/PDSS_1^*}\right| = \left|\frac{PDS_2/(CS_2/\beta)}{PDS_1/CS_1}\right|. \end{aligned} \tag{37}$$

As seen, the above expressions are equivalent to those in Eq. (26), which were derived from the structural approach. Thus, we have also derived leverage-corrected price discovery measures from the reduced-form models. The leverage-corrected CIL and CMIL measures can be defined similarly by replacing PDS with IS and MIS in the above expressions, respectively. Our structural analysis suggests that leverage-corrected CPIL measures provide more accurate assessments of price leadership compared to CIL and CMIL. Additionally, these leverage-corrected information leadership measures can be converted into shares (CILS, CMILS, CPILS) and their respective binary indicators (CILI, CMILI, CPILI).

3.3 Discussion on leverage bias versus β-neutrality

Building on the MIS measure proposed by Lien and Shrestha (2009), Lien and Shrestha (2014) introduced the Generalized Information Share (GIS), which is claimed to be applicable to asset prices that are non-unitarily cointegrated. The GIS has been widely cited in empirical studies such as Fernandez-Perez et al. (2018), Alexander and Heck (2020), and Hou et al. (2024). However, as demonstrated in Appendix A2, these two measures are essentially identical, highlighting the fact that all IS-type measures are β-neutral, as documented by Lautier et al. (2024) and discussed in the previous section.

Although IS-type measures can be directly applied to products with leveraged returns (i.e., β-neutral), structural analysis suggests that these measures are not pure indicators of price leadership. Instead, they capture both permanent and transitory responses, and therefore should be combined with the CS measure to filter out transitory noise. Yan and Zivot (2010) and Shen et al. (2024b) propose combining IS-type measures with the CS measure to form IL-type measures, which aim to isolate immediate responses of competing markets to permanent shocks. However, as demonstrated in this paper, these improved IL-type measures still suffer from leverage bias, as leverage can amplify both the initial and long-term responses of products to permanent shocks. We propose further correcting for this leverage bias by accounting for long-term responses to permanent shocks.

Although Lien and Shrestha (2014) recognized that long-run responses to permanent shocks differ due to non-unitary cointegration, they did not explore how leverage impacts underlying price discovery measures. Through structural analysis in Appendix A2, we demonstrate that the GIS (MIS) measure is not an ideal price discovery tool, as it incorporates immediate transitory responses. To eliminate this noise, the GIS (MIS) measure should be combined with the CS measure, resulting in the MIL measure, as proposed by Shen et al. (2024b). However, this combined MIL measure tends to misidentify products with higher leverage as the price leader. Applying a similar leverage correction to the MIL measure, we derive a leverage-bias-free CMIL measure, as outlined in Appendix A2.

4 Illustration: A Partial Price Adjustment Model

To illustrate the empirical relevance of the leverage-corrected price discovery measures, we consider the stylized partial adjustment micro-structure model as in Amihud and Mendelson (1987), Hasbrouck and Ho (1987), and Yan and Zivot (2010):

$$\begin{aligned}
p_{1t} &= p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T, \\
p_{2t} &= p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T, \\
m_t &= m_{t-1} + \eta_t^P, \boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)' \sim i.i.d.N\left(\mathbf{0}, \begin{pmatrix} \sigma_P^2 & 0 \\ 0 & \sigma_T^2 \end{pmatrix}\right).
\end{aligned} \tag{38}$$

This above data generating process differs from the one in Yan and Zivot (2010) in that the second price series is a leveraged product with a leverage of $1/\beta$. It is easy to show that the above price series $\boldsymbol{p_t} = (p_{1t}, p_{2t})'$ is cointegrated with $\boldsymbol{\beta} = (1, -\beta)'$.

Solving for $\triangle p_{it}$ gives

$$\triangle p_{it} = d_i^P(L)\eta_t^P + d_i^T(L)\eta_t^T,$$

where

$$\begin{aligned}
d_1^P(L) &= [1-(1-\delta_1)L]^{-1}\delta_1, \quad d_2^P(L) = [1-(1-\delta_2)L]^{-1}\delta_2/\beta, \\
d_1^T(L) &= [1-(1-\delta_1)L]^{-1}(1-L)b_{0,1}^T, \quad d_2^T(L) = [1-(1-\delta_2)L]^{-1}(1-L)b_{0,2}^T.
\end{aligned}$$

The SMA representation of the above DGP is determined from the appropriate elements of the lag polynomials $d_i^P(L)$ and $d_i^T(L)$. In particular, the intial impact and the long-run impact matrices are given as

$$\boldsymbol{D}_0 = \begin{pmatrix} d^P_{0,1} & d^T_{0,1} \\ d^P_{0,2} & d^T_{0,2} \end{pmatrix} = \begin{pmatrix} \delta_1 & b^T_{0,1} \\ \delta_2/\beta & b^T_{0,2} \end{pmatrix}, \quad \boldsymbol{D}(1) = \begin{pmatrix} d^P_1(1) & d^T_1(1) \\ d^P_2(1) & d^T_2(1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1/\beta & 0 \end{pmatrix}.$$

In the original one-to-one cointegration case of Yan and Zivot (2010), the initial responses of these two products to one-unit permanent shock are denoted by δ_i, while their long-run responses are of the same value one. In this context, the product exhibiting the largest initial response is identified as the leader in the price discovery process.

However, when the second product is associated with a leverage ratio of $1/\beta > 1$ as in Eq. (38), the leverage amplifies both initial and long-term responses to a one-unit permanent shock by a factor of $1/\beta$. As a result, if we disregard the leverage, we may incorrectly identify the second product as the leader when $\delta_2/\beta > \delta_1$, even though $\delta_2 < \delta_1$. This partial adjustment model clearly illustrates how the cointegration coefficient, or more fundamentally, the leverages of products, affect the responses of products to permanent shocks. It underscores the importance of accounting for leverage in order to accurately identify the true leader in the price discovery process.

To further explore how product leverage impacts IS and CS measures, as well as IL-type measures, we extend the structural analysis of Yan and Zivot (2010) to derive the structural representation of the CS measure within this partial price adjustment model:

$$CS_1 = \frac{b^T_{0,2}}{b^T_{0,2} - b^T_{0,1}}, \quad CS_2 = \frac{-b^T_{0,1}}{b^T_{0,2} - b^T_{0,1}}, \tag{39}$$

provided $b^T_{0,2} \neq b^T_{0,1}$. As is evident from the above expressions, leverage does not directly influence the CS measures.

The structural representation of the IS metric is complicated by the covariance structure of the reduced-form forecasting errors. When the reduced-form innovations, $\boldsymbol{\varepsilon}_t$, are uncorrelated, the IS metric remains unique. Let us consider the case where

$$\frac{\sigma_T^2}{\sigma_P^2} = \frac{d_{0,1}^P d_{0,2}^P}{-d_{0,1}^T d_{0,2}^T} = \frac{\delta_1 \delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}, \tag{40}$$

and all elements of $\boldsymbol{D}_0$ are non-zero (hence $|\boldsymbol{D}_0| \neq 0$). In this case, we can show that $cov(\varepsilon_{1t}, \varepsilon_{2t}) = 0$, and the IS measures can be uniquely defined as:

$$IS_1 = \frac{\delta_1 b_{0,2}^T}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2/\beta}, \quad IS_2 = \frac{-b_{0,1}^T \delta_2/\beta}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2/\beta}. \tag{41}$$

As in Yan and Zivot (2010), the structural representation of IS includes contemporaneous responses to both permanent and transitory shocks. Furthermore, when the underlying products have different leverage ratios ($\beta \neq 1$), the IS measures implicitly account for this leverage, leading to a potential leverage bias.

The information leadership (IL) measure, as defined in Yan and Zivot (2010), combines the IS and CS measures to create a more refined metric for assessing price discovery:

$$IL_1 = \left| \frac{IS_1/CS_1}{IS_2/CS_2} \right| = |\frac{d_{0,1}^P}{d_{0,2}^P}| = \left| \frac{\delta_1}{\delta_2/\beta} \right|. \tag{42}$$

From the above expression, it is evident that the leverage distorts the IL metric, causing it to fail as an indicator of the price leader, even when $cov(\varepsilon_{1t}, \varepsilon_{2t}) = 0$. Consider a scenario where the first product leads the market (i.e., $\delta_1 > \delta_2$), but the second product has a leverage ratio such that $\delta_1 < \delta_2/\beta$. Notably, the leverage of the second product does not have to be large for this condition to hold. In practice, initial responses to permanent shocks may be very similar (i.e., δ_i are close in value), making it highly likely that a leveraged product with $1/\beta > 1$ will be incorrectly identified as the market leader, even when $\delta_1 > \delta_2$. Therefore, we conclude that the IL measure fails to accurately identify the true price leader for leveraged products, as it tends to incorrectly label the product with a greater leverage as the leader, despite its potential lag in the price discovery process.

In the case with uncorrelated idiosyncratic residuals, we propose to correct the IL measure as follows:

$$CIL_1 = \left| \frac{IS_1/CS_1}{IS_2/(CS_2/\beta)} \right| = \left| \frac{\delta_1}{\delta_2} \right|. \tag{43}$$

With this leverage-corrected IL measure, the product with a larger initial response to the permanent shock, $|\delta_i|$, will be identified as the leader. However, the above structural interpretation of the leverage-corrected IL measure holds only in cases where the idiosyncratic residuals are uncorrelated.

For the more general case with possibly correlated residuals, we examine the structural interpretations of the PDS and PIL measures. Substituting relevant values into Eq. (16), we can get:

$$PDS_1 = \frac{\delta_1 b_{0,2}^T}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2/\beta}, \quad PDS_2 = \frac{-b_{0,1}^T \delta_2/\beta}{\delta_1 b_{0,2}^T - b_{0,1}^T \delta_2/\beta}. \tag{44}$$

As noted in Sultan and Zivot (2015), the PDS measures coincide with the IS measures when the reduced-form residuals are uncorrelated. Furthermore, the above structural representations remain valid even when $\mathbf{\Omega}$ is not diagonal.

The PIL, which is derived from a combination of PDS and CS measures, is expressed as follows:

$$PIL_1 = \left| \frac{PDS_1/CS_1}{PDS_2/CS_2} \right| = |\frac{\delta_1}{\delta_2/\beta}|, \quad PIL_2 = \left| \frac{PDS_2/CS_2}{PDS_1/CS_1} \right| = \left| \frac{\delta_2/\beta}{\delta_1} \right|. \tag{45}$$

As can be seen from the above expression, the PIL measure, like the IL measure, is influenced by leverage bias through the inclusion of leverage information β. To address this bias, we propose the following leverage-corrected CPIL measure:

$$CPIL_1 = \left| \frac{PDS_1/CS_1}{PDS_2/(CS_2/\beta)} \right| = \left| \frac{\delta_1}{\delta_2} \right|, \quad CPIL_2 = \left| \frac{PDS_2/(CS_2/\beta)}{PDS_1/CS_1} \right| = \left| \frac{\delta_2}{\delta_1} \right|. \tag{46}$$

As shown in the above expressions, the leverage-corrected CPIL measures coincide with the leverage-corrected CIL measures when $\mathbf{\Omega}$ is diagonal. Moreover, due to the simple additive decomposition of the variance-covariance matrix, the leverage-corrected CPIL (and thus CPILS and CPILI) retains a clear structural interpretation even when the idiosyncratic residuals are correlated. More importantly, this leverage-corrected CPIL measure can accurately identify the market with the largest initial permanent response, $|\delta_i|$, as the price leader, regardless of product leverage.

In Appendix A2, we demonstrate that the structural interpretations in Eq. (42) also apply to the MIL measure, which is derived from the MIS and CS measures, when $\mathbf{\Omega}$ is diagonal. In the more general case, the presence of a non-zero correlation coefficient ρ complicates the structural expressions for the MIS measures. Therefore, we hypothesize that the leverage-corrected CPIL measure, along with its corresponding share measure (CPILS) and binary indicator (CPILI), will provide the most accurate estimates of price leadership.

5 Simulation Evidence

In this section, we compare the estimates of traditional measures (IS and CS), order-invariant measures (MIS and PDS), combined binary measures (ILI, MILI, and PILI), and leverage-corrected binary measures (CILI, CMILI, and CPILI) using simulated data derived from various parameterizations of the stylized partial adjustment model in Eq. (38).[15]

We set $\delta_2 = 1 - \delta_1$, allowing δ_1 to vary from 0.9 to 0.1 in decrements of 0.1. When $\delta_1 > \delta_2$ (i.e., $\delta_1 > 0.5$), Market 1 exhibits a higher speed of price discovery than Market 2. The variance of the permanent shock is fixed at one ($\sigma_P^2 = 1$). To demonstrate the impact of leverage on the effectiveness of these price discovery measures, we vary the leverage ($1/\beta$) of Market 2 from 1 to 3.

In this setting, we first examine the case where both markets exhibit equal responses to

[15]Results based on combined IL-type measures (IL, MIL, PIL), their share measures (ILS, MILS, PILS), and their binary counterparts (ILI, MILI, PILI) are qualitatively similar. Therefore, to save space, we report only the results based on the binary measures.

transitory shocks, with $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$. The variance of the transitory shock is set as $\sigma_T^2 = \frac{\delta_1\delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$, ensuring that the reduced-form residuals of the two markets are uncorrelated. We simulate 1,000 samples of 216,000 observations for each price series, for each specified value of δ_1. For each simulated sample, we estimate the VECM model and compute the corresponding price discovery measures.[16] The averages of each price discovery measure over the 1,000 samples are then summarized and reported in Table 1.[17]

[Insert Table 1 about here.]

In the simplest case, where transitory shock responses are equal and reduced-form residuals are uncorrelated, we observe that as the leverage of Market 2 increases from 1 to 3, the IS, MIS, and PDS measures become less effective at correctly identifying the price leader. For example, when $\delta_1 = 0.9$, the IS measure (along with MIS and PDS) identifies Market 1 as the leader 90% of the time when $1/\beta = 1$. However, as Market 2's leverage rises to 3, this proportion declines to 75%.

Similarly, the combined indicators (ILI, MILI, and PILI) also become less accurate in identifying the correct leader as Market 2's leverage increases. These measures tend to incorrectly designate Market 2 as the leader as its leverage grows. For instance, when $\delta_1 = 0.7$, ILI (as well as MILI and PILI) correctly identifies Market 1 as the leader at $1/\beta = 2$, but fails to do so at $1/\beta = 3$. In contrast, the leverage-corrected indicators (CILI, CMILI, and CPILI) consistently provide accurate estimates of the price leader for cases where $\delta_1 \neq 0.5$, regardless of Market 2's leverage. In the scenario of equal leadership ($\delta_1 = \delta_2 = 0.5$), these leverage-corrected indicators assign leadership to each market approximately half the time.

In Table 2, we modify the parameter settings to account for unequal transitory shock responses, with $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, and maintain uncorrelated reduced-form residuals

[16] To save space, we only report estimates for binary combined indicators (ILI, MILI, PILI) and their leverage-corrected versions. Estimates on combined measures (IL, MIL, PIL) and their share measures (ILS, MILS, PILS) are qualitatively similar and are available upon request.

[17] To conserve space, results are rounded to two decimal places. Each panel in Table 1 corresponds to a specific value of $1/\beta$.

by setting $\sigma_T^2 = \frac{\delta_1\delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$. In this scenario, Market 1 experiences greater noise due to its larger response to transitory shocks. The results are largely consistent with those in Table 1, confirming that the leverage of the products further diminishes the accuracy of the information measures (IS, MIS, and PDS) in identifying the price leader.[18] Similarly, the combined indicator measures (ILI, MILI, and PILI) also show reduced effectiveness. Once again, the leverage-corrected indicators (CILI, CMILI, and CPILI) consistently identify the price leader correctly, regardless of Market 2's leverage.

[Insert Table 2 about here.]

For the case with unequal transitory shock responses $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$ and correlated reduced-form residuals $\sigma_T^2 = 10$, as presented in Table 3, the estimates for IS, MIS, and PDS differ due to the distinct variance decomposition method. As a result, the combined measures (ILI, MILI, and PILI) and their leverage-corrected counterparts also produce varying results. In this setting, the combined indicators (ILI and MILI) can yield highly misleading results, even when there is no leverage (for $\delta_1 < 0.5$ in Panel A of Table 3). The leverage of Market 2 further exacerbates this issue, leading these indicators to incorrectly identify Market 2 as the leader. While applying leverage corrections to these indicators mitigates this bias to some extent, no measure rivals the performance of the leverage-corrected indicator, CPILI.

When comparing the results of all measures in Table 3, CPILI consistently provides the most accurate identification of the price leader. It correctly identifies the leader 100% of the time in cases of clear-cut leadership ($\delta_1 \neq 0.5$), regardless of the leverage level. For scenarios with equal leadership ($\delta_1 = \delta_2 = 0.5$), CPILI assigns leadership to each market approximately half the time.

[Insert Table 3 about here.]

[18] With unequal transitory shock responses, these information measures perform even worse compared to the case with equal transitory shock responses.

In summary, for products with leveraged returns, we demonstrate that applying leverage corrections is essential, with the leverage-corrected indicator, CPILI, providing the most accurate estimate of price leadership. Leverage amplifies both the initial and long-run responses of products to permanent shocks. Without accounting for the effects of leverage, even combined indicators (ILI, MILI, and PILI) can yield misleading results on price leadership. Furthermore, the simplest additive form of the variance decomposition in PDS offers a more straightforward structural representation, which in turn allows CPILI to deliver the most accurate leadership estimates. These findings align with the implications of our model.

In Appendix A3, we also present a series of simulations to examine how data frequency affects the performance of these price discovery measures. The results indicate that price discovery measures are sensitive to data frequency, with leverage compounding this complexity. As IS, MIS, and PDS become less informative with rougher data, PDS appears to be more resilient to changes in data frequency compared to the other two measures. The performance of the combined indicators (ILI, MILI, and PILI) degrades significantly, as the CS measures are particularly sensitive to data frequency, and the same holds for the leverage-corrected CILI and CMILI measures. However, our simulation results across different data frequencies again confirm the superiority of the leverage-corrected CPILI measure. Further details can be found in Appendix A3.

6 Application to S&P 500 Leveraged ETFs

6.1 Data Sources

In this study, we select the SPY ETF and five corresponding leveraged ETFs (SSO, UPRO, SH, SDS, and SPXU) to empirically examine the price discovery of LETFs and further investigate the determinants of their price discovery. We retrieve tick-level trade data from the TAQ database. Following Holden and Jacobsen (2014), we keep trades that are uncorrected, with a price greater than zero, without symbol suffixes and within trading hours according to the New York Stock Exchange calendar. The sample period spans from January 1, 2014, to December 23, 2023. Lower-frequency trade prices (e.g., 1-second, 5-second, 10-second intervals) are derived from tick-level data by selecting the last available trade price at each interval. Panel A of Table 4 presents basic information, including inception times, assets under management (AUM), and expense ratios for the selected ETFs. Notably, SPY, being one of the most traded ETFs globally, boasts significantly higher assets under management and a lower expense ratio compared to LETFs.

[Insert Table 4 about here.]

The simulation study indicates that data frequency significantly impacts the measure of price discovery. Panel B of Table 4 summarizes the daily average time (in seconds) between trades for SPY and LETFs. As expected, SPY is the most frequently traded product, while LETFs require, on average, more than 5 to 10 seconds between trades. This suggests that for LETFs, prices with a frequency higher than 10 seconds may lack informativeness due to substantial price stagnation caused by illiquidity.

Panel C of Table 4 summarizes the daily averages of the liquidity and pricing efficiency measures that may influence price discovery shares. Consistent with the existing literature (Lesmond et al. 2004; Chen et al. 2007), we use the bid-ask spread percentage (B/A Spread) and zero trading seconds (Zeros)—defined as the percentage of non-trading seconds relative to the total number of seconds within trading hours each day—to assess illiquidity. In addition, we report the turnover ratio (Turnover) and retail trading ratio (Retail), which represent the percentage of retail investor trading volume relative to the total trading volume for the day. Detailed definitions of these variables can be found in Table 5. The variables B/A Spread, Turnover, and Retail are retrieved from the TAQ database, while Zeros and RKT are calculated from high-frequency trading data. All other variables are publicly available.

[Insert Table 5 about here.]

Overall, LETFs exhibit higher illiquidity, larger turnover ratios, and a greater proportion of retail investor trading compared to SPY. Consequently, we anticipate that SPY will dominate price discovery on average.

6.2 Price Discovery of LETFs

For each day in our sample, we estimate a reduced-form VECM and calculate price discovery measures across various data frequencies. In the VECM estimation, the cointegrating vector and the other model coefficients are estimated using Johansen's Maximum Likelihood Estimation (MLE) method, with the optimal lag length determined by the Akaike Information Criterion (AIC).

For the six ETF products, we first estimate bivariate price discovery measures for each pair of products, followed by price discovery measures derived from the multivariate model encompassing all six products. Table 6 presents the sample averages of daily bivariate price discovery measures between SPY(+1) and each of the LETFs, while Table A4 in the Appendix summarizes bivariate estimates among the remaining LETFs. The numbers reported represent sample means of these measures, with sample standard deviations provided in square brackets below. To conserve space, we display only the estimates for the first product in each bivariate model, which corresponds to SPY for the results in Table 6.[19] Multivariate price discovery estimates are presented in Table A5 in the Appendix.

[Insert Table 6 about here.]

The first observation from Table 6 is that even with high-frequency data (such as at the 1-second level), the correlation coefficient ρ between the bivariate idiosyncratic errors can reach as high as 0.25 for SPY(+1) and SSO(+2). This highlights the importance of the variance decomposition method and the selection of measures such as IS, MIS, or PDS, even in the context of high-frequency data. Additionally, we find that the cointegration coefficient

[19] For each measure shown in the table, the price discovery measure for the other market can be calculated as one minus the estimate for the first market.

$(1, -\beta)$ (and thus the relative leverage $|1/\beta|$) can be estimated quite accurately, even with relatively rough data. For instance, using the 5-minute data, the cointegration vector for SPY(+1) and SSO(+2) is estimated at $(1, -0.5)$, with a sample standard deviation of 0.01 for the daily estimated cointegrating vector. Similarly, the cointegration vector for SPY(+1) and UPRO(+3) is estimated at $(1, -0.33)$, with a sample standard deviation of 0.03.

As shown in Table 6, for any combination of two products, the IS measures converge to 0.5 as the data frequency decreases from 1-second to 5-minute. This pattern of convergence to 0.5 is less pronounced for MIS. In contrast, PDS does not converge to 0.5 when the data frequency decreases from 1-second to 5-minute. For SPY(+1) and any leveraged product (including the inverse product), estimates of IS, CS, MIS, and PDS consistently indicate that SPY (+1) leads the other product approximately 60%- 90% of the time at the 1-second frequency.

However, the combined share measures (ILS, MILS and PILS) and the corresponding indicator measures (ILI, MILI and PILI) produce opposite leadership results across most data frequencies. For instance, for SPY(+1) and UPRO(+3), the estimates from ILS, MILS, and PILS (as well as ILI, MILI, and PILI) suggest that UPRO(+3) leads SPY(+1) most of the time, regardless of the frequency. Our structural analysis indicates that these combined measures can produce misleading price leadership estimates due to leverage bias. In contrast, our leverage-corrected measures—CILS, CMILS, and CPILS (along with CILI, CMILI, and CPILI)—enhance the accuracy of these combined estimates. Specifically, our CPILI measure indicates that SPY(+1) leads the other products approximately 90% of the time across nearly all data frequencies higher than the 5-minute level. At the 5-minute level, SPY(+1) still demonstrates leadership over the other products, but only about 60% of the time.

The bivariate price discovery relationships among the other products, as shown in Table A4 in the Appendix, indicate that the inverse product, SH(-1), is dominated by all other products approximately 90% of the time across nearly all data frequencies, and 60% of the time at the 5-minute level, based on our CPILI estimates. For products with positive leverages, we observe that UPRO(+3) dominates SSO(+2) about 60% to 71% of the time at higher data frequencies. Additionally, at frequencies exceeding 5-minute, UPRO(+3) also displays dominance over negatively leveraged products: approximately 56% to 67% of the time for SPXU(-3), about 64% to 77% for SDS(-2), and roughly 90% for SH(-1). Our CPILI estimates further indicate that SSO(+2) is slightly dominated by UPRO(+3). The price discovery relationship between SSO(+2) and SPXU(-3) is less clear, with nearly equal leadership observed between these two products. However, SSO(+2) demonstrates clear leadership over SDS(-2) approximately 60% of the time and over SH(-1) about 90% of the time at higher data frequencies.

[Insert Figure 1 here.]

In Figure 1, we plot the time series of the bivariate CPILS measures of five LETFs relative to SPY, using the 1-second trade prices. Price discovery leadership of the reverse and LETFs exhibit some upward trends after 2020, especially after the 2019 COVID-19 market crash and the outbreak of the 2022 Ukraine war, as indicated by the shaded areas of Figure 1. Since the CPILS measures of all LETFs are generally below 0.2 for most of the sample period, the figure confirms the dominance of SPY in price discovery relative to LETFs. However, under certain circumstances, LETFs, especially UPRO, SSO and SPXU may have relatively higher price discovery share. In the following Section 6.3, we further investigate the determinants of LETFs' price discovery shares.

In summary, our bivariate estimation results reveal that SPY(+1) dominates all other products across nearly all frequencies for approximately 90% of the time (60% of the time at the 5-minute frequency). Conversely, the inverse product, SH(-1), is dominated by all other products for over 90% of the time. Among all six ETFs, we observe a moderate descending order of the price discovery shares as SPY(+1) $>$ UPRO(+3) $>$ SSO(+2) $\approx$ SPXU(-3) $>$ SDS(-2) $>$ SH(-1).

For the multivariate results presented in Table A5, the fundamental findings remain consistent. Our multivariate CPILS estimates indicate that SPY(+1) continues to dominate all other products, while SH(-1) is dominated by all other products across all frequencies, with the exception of the 5-minute data. The ranking of price leadership among the other four leveraged products is consistent with the pairwise estimates. Overall, the multivariate CPILS estimates reinforce the same descending order of price discovery shares: SPY(+1) > UPRO(+3) > SSO(+2) ≈ SPXU(-3) > SDS(-2) > SH(-1) for data frequencies exceeding 1-minute. The multivariate estimates for minute-level data may be unreliable due to the small sample size at this low frequency and the corresponding loss of degrees of freedom in the multivariate VECM model. To compare price leadership among any two products, we recommend referring to the results from the bivariate model, as these estimates provide a direct comparison. The multivariate model's increased loss of degrees of freedom may lead to misleading conclusions regarding price leadership identification.

6.3 Determinants of LETFs' Price Discovery

For each LETF, we conduct the following regression to explore the determinants of its price discovery shares relative to SPY:

$$logit\ CPILS_t = \alpha + \boldsymbol{\beta_1} \cdot \boldsymbol{Fund_t} + \boldsymbol{\beta_2} \cdot \boldsymbol{Market_t} + \boldsymbol{\beta_3} \cdot \boldsymbol{Event_t} + \boldsymbol{y_t} + \epsilon_t, \tag{47}$$

where $logit\ CPILS_t = \log \frac{CPILS_t}{1-CPILS_t}$ represents the logit transformation of the CPILS shares of a specific LETF relative to SPY on day t, estimated from prices sampled at 1-second intervals. $\boldsymbol{Fund_t}$ denotes a vector of daily LETF-specific variables, including the bid-ask spread percentage (B/A Spread), turnover ratio (Turnover), retail investor trading (Retail), and zero-trading seconds (Zeros) as previously defined. $\boldsymbol{Market_t}$ refers to a vector of daily market-level variables, comprising the scaled realized kurtosis of the S&P 500 index (RKT), the logarithm of the CBOE VIX index (VIX), the logarithm of the daily Economic Policy Uncertainty index from Baker et al. (2016) (EPU), and the Investor Sentiment index orthogonal to macro from Baker and Wurgler (2006) (Sentiment). $\boldsymbol{Event_t}$ represents a vector of indicator variables of events, including days when there are events related to the Federal Reserve and interest rate decisions (Fed)[20], the 2020 stock market crash due to COVID-19 (Covid), and the outbreak of the 2022 Russo-Ukrainian war (Ukraine). We also include a vector of year dummies $\boldsymbol{y_t}$ to control time effects and use White standard errors to address heteroskedasticity. Detailed variable definitions are presented in Table 5, and the summary statistics are reported in Table 7.

[Insert Table 7 about here.]

We expect that illiquidity, reflected by a higher bid-ask spread (B/A Spread) and a greater number of zero-trading seconds (Zeros), as well as increased retail trading (Retail), are likely to reduce price discovery shares of LETFs. Conversely, higher trading activity, as indicated by a higher turnover ratio (Turnover), is expected to enhance price discovery. Additionally, we anticipate that LETFs will exhibit greater price discovery shares when the market index experiences significant movements (higher RKT) and when the uncertainty of economic policy is greater (higher EPU). Differentiating long and inverse LETFs, we expect greater price discovery shares for long LETFs when the investor sentiment is high (higher Sentiment) and fear is low (lower VIX), and vice versa for inverse LETFs. Finally, on days when there are significant economic or policy events and the S&P 500 reacts accordingly, LETFs could be used as convenient instruments, even by institutional investors, as part of short-term strategies aimed at maximizing profits. This could result in higher price discovery shares. Hence, we expect the coefficients on the event indicator variables (Fed, Covid, and Unkraine) to be positive.

[Insert Table 8 about here.]

Table 8 presents the estimation results of Eq. (47). As expected, higher illiquidity is associated with a reduction in price leadership, as evidenced by the negative and significant

[20] We retrieve the economic calendar from FXStreet and label trading days with events tied to the Federal Reserve, including FOMC meetings, Fed Chair speeches, monetary policy statements, and other events related to the Fed and monetary policies.

coefficients for both B/A Spread and Zeros. In contrast, Turnover is positively associated with price discovery shares, suggesting that LETFs contribute more to price discovery when trading activity intensifies. Furthermore, higher levels of retail trading are associated with a significant decrease in LETF price discovery shares.

Regarding market conditions, coefficients on realized kurtosis of S&P 500 index (RKT) are significantly positive for all LETFs, indicating that LETFs exhibit higher price discovery shares when the S&P 500 index jumps. In addition, the coefficients on economic policy uncertainty (EPU) are significantly positive for SSO(+2), SH(-1), and SDS(-2), suggesting that the price discovery shares of LETFs are generally higher in the presence of higher policy uncertainty.

From a behavioral perspective, a higher level of VIX index is associated with significantly lower price discovery shares of SSO(+2) and UPRO(+3) but higher shares of SPXU(-3). Comparatively, a higher level of investor sentiment is associated with significantly higher price discovery shares of SSO(+2) and UPRO(+3). In general, when sentiment is high and fear (the VIX index) is low, LETFs with long positions tend to be popular among investors. However, when the VIX index is high, the inverse LETF could be used to short the S&P 500 index.

In terms of economic or other events, price discovery shares of all LETFs are significantly higher on days when Fed-related information is released. During period of Covid market crash and outbreak of the 2022 Russo-Ukrainian war, some LETFs, including both long and short LETFs, have significantly higher price discovery shares. The results confirm that LETFs exhibit higher price discovery shares when substantial information, especially related to interest rate decisions, is incorporated into the S&P 500 index.

In general, for ETFs and LETFs tracking the S&P 500 index, the regular ETF (SPY) serves as the primary venue for pricing new information. LETFs are typically favored by retail speculators who may lack the capacity to efficiently take (inversely) leveraged positions. Illiquidity significantly hampers the price discovery of LETFs. While LETFs may contribute more to price discovery during periods of major macroeconomic shocks and market jumps, SPY remains the dominant vehicle through which price discovery occurs.

7 Conclusion

While existing measures have enabled researchers to examine price discovery relationships among financial products with nearly identical returns, such as classic future-spot relations, further improvements are required to accommodate modern financial innovations. In this paper we expand the scope of price discovery research by introducing leverage-robust measures. Through a structural analysis, we highlight the significant leverage bias inherent in existing price discovery measures. Without appropriate corrections, these measures tend to misidentify products with higher leverage ratios as price leaders, leading to misleading conclusions about the information efficiency of tradable products that deliver leveraged returns.

We propose incorporating long-run responses to permanent shocks to correct for leverage bias. This approach results in leverage-corrected price discovery measures, and we demonstrate that the Leverage-Corrected Price Information Leadership (CPIL) measure, along with its share (CPILS) and binary counterpart (CPILI), provides the most accurate estimates of price leadership. Simulation evidence based on a partial price adjustment model supports the superiority of our proposed measures. For leveraged products, we show that leverage corrections are crucial, with CPIL, CPILS, and CPILI outperforming all other leverage-corrected measures. Furthermore, in the absence of leverage, these measures yield results consistent with those of their uncorrected counterparts, underscoring their robustness.

We apply these price discovery measures to six ETF and LETFs tracking the S&P 500 index. Empirical results show that SPY(+1) dominates all other LETFs in terms of price discovery, with a descending order of price discovery shares: SPY(+1) $>$ UPRO(+3) $>$ SSO(+2) $\approx$ SPXU(-3) $>$ SDS(-2) $>$ SH(-1). Regression analysis further reveals that illiquidity, trading inactivity, and retail trading dampen the price discovery of LETFs. Although

LETFs may exhibit higher price discovery shares during periods of influential information releases or significant market jumps, the regular ETF (SPY) remains the primary venue for pricing new information.

The contributions of this paper extend beyond leveraged ETFs. We offer the first comprehensive exploration of price discovery among financial products with leveraged returns and provide robust solutions for correcting the leverage bias embedded in existing measures. Our findings represent a notable advancement in this area and establish a foundation for future research on price discovery in the presence of more general cointegration relationships.

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Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT in order to improve readability and language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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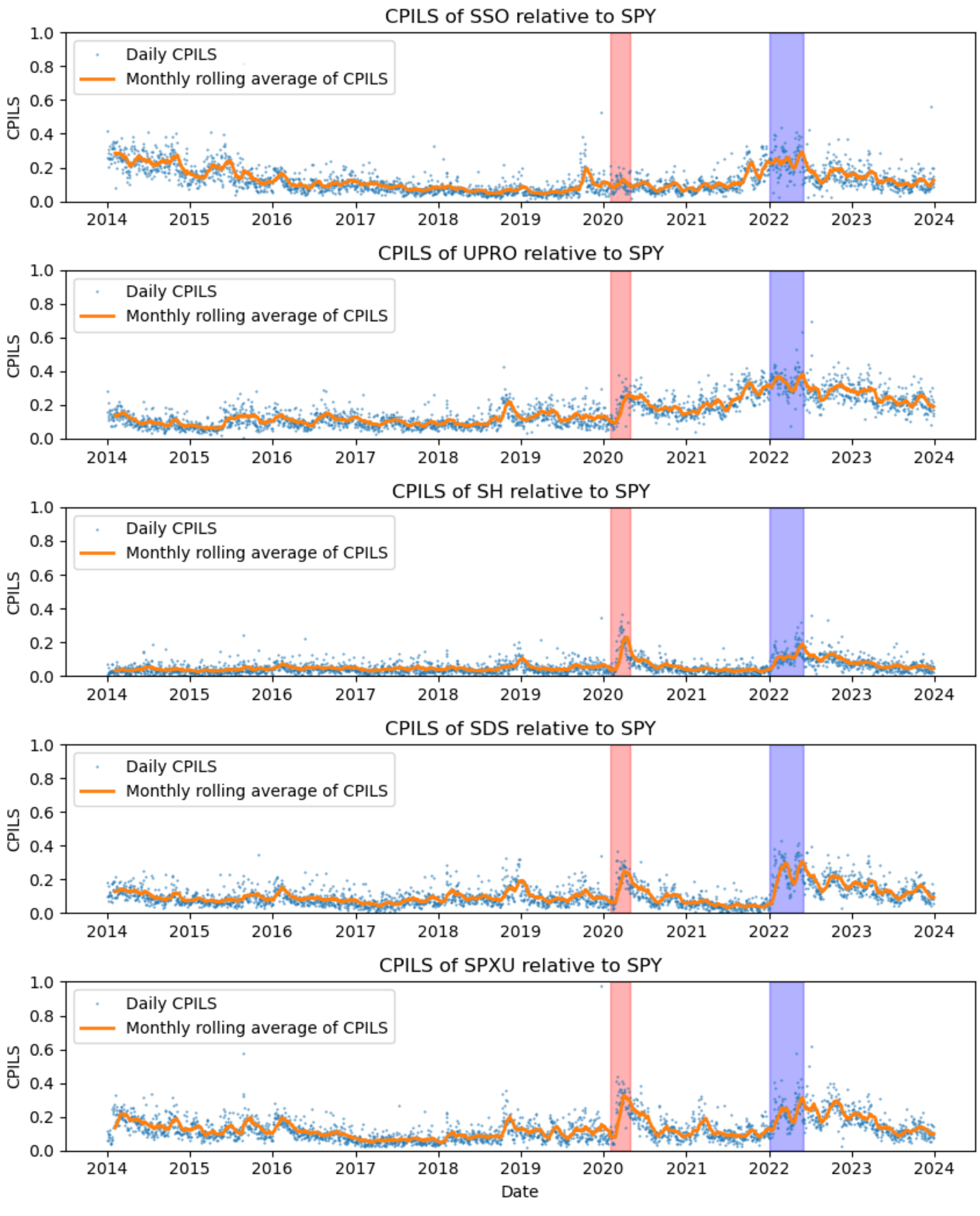

Figure 1: The Price Discovery Shares (CPILS) of LETFs relative to SPY.

From top to bottom, Figure 1 shows the time series of daily bivariate price discovery shares (CPILS) of SSO(+2), UPRO(+3), SH(-1), SDS(-2), and SPXU(-3) relative to SPY, respectively. The daily bivariate CPILS estimates are estimated with the 1-second trade prices. Shaded areas represent the 2020 stock market crash due to COVID-19 (red) and the outbreak of the 2022 Russo-Ukrainian war (blue).

Table 1: Leverage with Equal Transitory Responses and Uncorrelated Residuals
This table reports price discovery estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $\delta_2 = 1 - \delta_1$, $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$, $\sigma_P^2 = 1$, $\sigma_T^2 = \frac{\delta_1\delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$. We simulate 1000 samples of 21600 observations.

Panel A: $1/\beta = 1$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.90	0.10	0.50	0.50	0.90	0.10	0.90	0.10	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.80	0.20	0.50	0.50	0.80	0.20	0.80	0.20	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.70	0.30	0.50	0.50	0.70	0.30	0.70	0.30	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.60	0.40	0.50	0.50	0.60	0.40	0.60	0.40	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.48	0.52	0.48	0.52	0.48	0.52	0.48	0.52	0.48	0.52	0.48	0.52
0.4	0.40	0.60	0.50	0.50	0.40	0.60	0.40	0.60	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.30	0.70	0.50	0.50	0.30	0.70	0.30	0.70	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.20	0.80	0.50	0.50	0.20	0.80	0.20	0.80	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.10	0.90	0.50	0.50	0.10	0.90	0.10	0.90	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Panel B: $1/\beta = 2$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.82	0.18	0.50	0.50	0.82	0.18	0.82	0.18	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.67	0.33	0.50	0.50	0.67	0.33	0.67	0.33	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.54	0.46	0.50	0.50	0.54	0.46	0.54	0.46	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.43	0.57	0.50	0.50	0.43	0.57	0.43	0.57	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.33	0.67	0.50	0.50	0.33	0.67	0.33	0.67	0.00	1.00	0.00	1.00	0.00	1.00	0.48	0.52	0.48	0.52	0.48	0.52
0.4	0.25	0.75	0.50	0.50	0.25	0.75	0.25	0.75	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.18	0.82	0.50	0.50	0.18	0.82	0.18	0.82	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.11	0.89	0.50	0.50	0.11	0.89	0.11	0.89	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.05	0.95	0.50	0.50	0.05	0.95	0.05	0.95	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Panel C: $1/\beta = 3$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.75	0.25	0.50	0.50	0.75	0.25	0.75	0.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.57	0.43	0.50	0.50	0.57	0.43	0.57	0.43	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.44	0.56	0.50	0.50	0.44	0.56	0.44	0.56	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.33	0.67	0.50	0.50	0.33	0.67	0.33	0.67	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.25	0.75	0.50	0.50	0.25	0.75	0.25	0.75	0.00	1.00	0.00	1.00	0.00	1.00	0.48	0.52	0.48	0.52	0.48	0.52
0.4	0.18	0.82	0.50	0.50	0.18	0.82	0.18	0.82	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.12	0.88	0.50	0.50	0.12	0.88	0.12	0.88	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.08	0.92	0.50	0.50	0.08	0.92	0.08	0.92	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.04	0.96	0.50	0.50	0.04	0.96	0.04	0.96	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Notes: Numbers shown are the averages of price discovery measure estimates of 1000 samples. For each sample, the sample size is set as $N = 21600$.

Table 2: Leverage with Unequal Transitory Responses and Uncorrelated Residuals
This table reports price discovery estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $\delta_2 = 1 - \delta_1$, $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, $\sigma_P^2 = 1$, $\sigma_T^2 = \frac{\delta_1 \delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$. We simulate 1000 samples of 21600 observations.

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
Panel A: $1/\beta = 1$																				
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.69	0.31	0.20	0.80	0.69	0.31	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.50	0.50	0.20	0.80	0.50	0.50	0.50	0.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.37	0.63	0.20	0.80	0.37	0.63	0.37	0.63	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.27	0.73	0.20	0.80	0.27	0.73	0.27	0.73	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.20	0.80	0.20	0.80	0.20	0.80	0.20	0.80	0.49	0.52	0.48	0.52	0.48	0.52	0.49	0.52	0.48	0.52	0.48	0.52
0.4	0.14	0.86	0.20	0.80	0.14	0.86	0.14	0.86	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.10	0.90	0.20	0.80	0.10	0.90	0.10	0.90	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.06	0.94	0.20	0.80	0.06	0.94	0.06	0.94	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.03	0.97	0.20	0.80	0.03	0.97	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
Panel B: $1/\beta = 2$																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.53	0.47	0.20	0.80	0.53	0.47	0.53	0.47	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.33	0.67	0.20	0.80	0.33	0.67	0.33	0.67	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.23	0.77	0.20	0.80	0.23	0.77	0.23	0.77	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.16	0.84	0.20	0.80	0.16	0.84	0.16	0.84	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.11	0.89	0.20	0.80	0.11	0.89	0.11	0.89	0.00	1.00	0.00	1.00	0.00	1.00	0.49	0.52	0.49	0.52	0.48	0.52
0.4	0.08	0.92	0.20	0.80	0.08	0.92	0.08	0.92	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.05	0.95	0.20	0.80	0.05	0.95	0.05	0.95	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.03	0.97	0.20	0.80	0.03	0.97	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.01	0.99	0.20	0.80	0.01	0.99	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
Panel C: $1/\beta = 3$																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.43	0.57	0.20	0.80	0.43	0.57	0.43	0.57	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.25	0.75	0.20	0.80	0.25	0.75	0.25	0.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.16	0.84	0.20	0.80	0.16	0.84	0.16	0.84	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.11	0.89	0.20	0.80	0.11	0.89	0.11	0.89	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.08	0.92	0.20	0.80	0.08	0.92	0.08	0.92	0.00	1.00	0.00	1.00	0.00	1.00	0.49	0.52	0.49	0.52	0.48	0.52
0.4	0.05	0.95	0.20	0.80	0.05	0.95	0.05	0.95	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.3	0.03	0.97	0.20	0.80	0.03	0.97	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.02	0.98	0.20	0.80	0.02	0.98	0.02	0.98	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.01	0.99	0.20	0.80	0.01	0.99	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Notes: Numbers shown are the averages of price discovery measure estimates of 1000 samples. For each sample, the sample size is set as $N = 21600$.

Table 3: Leverage with Unequal Transitory Responses and Correlated Residuals
This table reports price discovery estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b^T_{0,1}\eta^T_t,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b^T_{0,2}\eta^T_t,$$

where $m_t = m_{t-1} + \eta^P_t$, $\boldsymbol{\eta}_t = (\eta^P_t, \eta^T_t)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma^2_P, \sigma^2_T)$. The simulation parameterization is set as $\delta_2 = 1-\delta_1$, $(b^T_{0,1}, b^T_{0,2}) = (0.8, -0.2)$, $\sigma^2_P = 1$, $\sigma^2_T = 10$. We simulate 1000 samples of 21600 observations.

Panel A: $1/\beta = 1$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.54	0.46	0.20	0.80	0.59	0.41	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.50	0.50	0.20	0.80	0.50	0.50	0.50	0.50	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.44	0.56	0.20	0.80	0.41	0.59	0.37	0.63	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.38	0.62	0.20	0.80	0.34	0.66	0.27	0.73	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.33	0.67	0.20	0.80	0.27	0.73	0.20	0.80	1.00	0.00	1.00	0.00	0.48	0.52	1.00	0.00	1.00	0.00	0.48	0.52
0.4	0.28	0.72	0.20	0.80	0.22	0.78	0.14	0.86	1.00	0.00	0.94	0.06	0.00	1.00	1.00	0.00	0.94	0.06	0.00	1.00
0.3	0.23	0.77	0.20	0.80	0.17	0.83	0.10	0.90	1.00	0.00	0.00	1.00	0.00	1.00	1.00	0.00	0.00	1.00	0.00	1.00
0.2	0.20	0.80	0.20	0.80	0.13	0.87	0.06	0.94	0.26	0.74	0.00	1.00	0.00	1.00	0.26	0.74	0.00	1.00	0.00	1.00
0.1	0.17	0.83	0.20	0.80	0.10	0.90	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Panel B: $1/\beta = 2$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.51	0.49	0.20	0.80	0.52	0.48	0.53	0.47	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.40	0.60	0.20	0.80	0.37	0.63	0.33	0.67	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.30	0.70	0.20	0.80	0.26	0.74	0.23	0.77	1.00	0.00	1.00	0.00	0.99	0.02	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.22	0.78	0.20	0.80	0.19	0.81	0.16	0.84	1.00	0.00	0.11	0.89	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.16	0.84	0.20	0.80	0.14	0.86	0.11	0.89	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	0.48	0.52
0.4	0.12	0.88	0.20	0.80	0.10	0.90	0.08	0.92	0.00	1.00	0.00	1.00	0.00	1.00	0.96	0.04	0.03	0.97	0.00	1.00
0.3	0.09	0.91	0.20	0.80	0.07	0.93	0.05	0.95	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.07	0.93	0.20	0.80	0.05	0.95	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.06	0.94	0.20	0.80	0.04	0.96	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Panel C: $1/\beta = 3$

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
δ_1	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
0.9	0.46	0.54	0.20	0.80	0.45	0.55	0.43	0.57	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.8	0.31	0.69	0.20	0.80	0.28	0.72	0.25	0.75	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
0.7	0.20	0.80	0.20	0.80	0.18	0.82	0.16	0.84	0.54	0.46	0.01	0.99	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.6	0.14	0.86	0.20	0.80	0.12	0.88	0.11	0.89	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
0.5	0.09	0.91	0.20	0.80	0.09	0.91	0.08	0.92	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00	0.94	0.06	0.48	0.52
0.4	0.07	0.93	0.20	0.80	0.06	0.94	0.05	0.95	0.00	1.00	0.00	1.00	0.00	1.00	0.02	0.98	0.00	1.00	0.00	1.00
0.3	0.05	0.95	0.20	0.80	0.04	0.96	0.03	0.97	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.2	0.04	0.96	0.20	0.80	0.03	0.97	0.02	0.98	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
0.1	0.03	0.97	0.20	0.80	0.02	0.98	0.01	0.99	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00

Notes: Numbers shown are the averages of price discovery measure estimates of 1000 samples. For each sample, the sample size is set as $N = 21600$.

Table 4: Comparison of Exchange Traded Products Tracking the S&P500 Index.

	SPY	SSO	UPRO	SH	SDS	SPXU
Panel A. Basic Information						
Leverage	100%	200%	300%	-100%	-200%	-300%
Issuer	State Street	ProShares	ProShares	ProShares	ProShares	ProShares
Inception	Jan 22, 1993	Jun 19, 2006	Jun 25, 2009	Jun 19, 2006	Jul 11, 2006	Jun 25, 2009
AUM (Million $)	563,690.0	5,584.1	4,042.9	900.8	531.0	555.7
Shares (Million)	1,012.0	64.8	49.5	80.5	24.0	20.5
Expense Ratio	0.09%	0.91%	0.92%	0.88%	0.90%	0.90%
Panel B. Daily average time between two trades (in seconds)						
Min	1.00	1.25	1.14	1.55	1.34	1.16
Mean	1.18	6.72	5.98	13.03	6.94	6.34
Max	2.77	25.16	28.50	59.85	28.92	44.57
Panel C. Trading characteristics						
B/A Spread	0.00	0.02	0.03	0.05	0.05	0.06
Zeros	0.13	0.80	0.73	0.87	0.79	0.76
Turnover	0.10	0.11	0.21	0.10	0.24	0.34
Retail	0.11	0.15	0.17	0.16	0.17	0.18

Note: Basic information is retrieved from VettaFi, as of July 21, 2024.

Table 5: Variable definitions.

Variable	Definition
logit CPILS	The logit transformation of LETFs' price discovery share (CPILS) relative to the SPY.
B/A Spread	The percentage of bid-ask spread over ask price, scaled by 100.
Turnover	The turnover ratio defined as trading volume over the outstanding shares.
Retail	The percentage of retail trading volume over total volume.
Zeros	The percentage of non-trading seconds over total trading seconds within trading hours in a day.
RKT	The scaled realized kurtosis of S&P 500 index, calculated as $\frac{1}{100} N \sum_{i=1}^{N} r_i^4 / \left(\sum_{i=1}^{N} r_i^2 \right)^2$, where r_i is the 1-minute SPY return within trading hours.
VIX	The logarithm of CBOE VIX index.
EPU	The logarithm of daily economic policy uncertainty index from Baker et al. (2016).
Sentiment	The investor sentiment index orthogonal to macro from Baker and Wurgler (2006).
Fed	A dummy variable equals 1 on days when there are events related to the Federal Reserve and interest rate decisions.
Covid	A dummy variable equals 1 between 2020-02-01 to 2020-04-30, indicating the 2020 stock market crash due to COVID-19.
Ukraine	A dummy variable equals 1 between 2022-01-01 to 2022-05-31, indicating the outbreak of the 2022 Russo-Ukrainian war.

Table 6: Empirical Bivariate Price Discovery Estimates for SPY and other LETFs
This table presents the sample means of the price discovery measures for SPY in the bivariate model involving SPY and the other LETF product. The symbol ρ stands for the correlation coefficient between the VECM residuals (sample averages), while $-\beta$ denotes the cointegrating coefficient in the cointegration vector $(1, -\beta)$ (sample averages). Values in square brackets correspond to sample standard deviations.

Panel A: SPY(+1) and SSO(+2)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.63	0.59	0.63	0.64	0.59	0.62	0.66	0.83	0.85	0.87	0.73	0.76	0.83	0.97	0.99	1.00	0.25	-0.50
	[0.26]	[0.27]	[0.26]	[0.27]	[0.18]	[0.16]	[0.15]	[0.13]	[0.10]	[0.08]	[0.44]	[0.43]	[0.37]	[0.17]	[0.10]	[0.03]	[0.12]	[0.01]
5s	0.59	0.70	0.62	0.67	0.25	0.27	0.40	0.47	0.52	0.71	0.18	0.10	0.20	0.49	0.60	1.00	0.55	-0.50
	[0.23]	[0.29]	[0.26]	[0.31]	[0.22]	[0.18]	[0.12]	[0.30]	[0.26]	[0.09]	[0.38]	[0.30]	[0.40]	[0.50]	[0.49]	[0.06]	[0.18]	[0.01]
10s	0.57	0.74	0.62	0.71	0.18	0.19	0.32	0.34	0.38	0.64	0.14	0.06	0.03	0.32	0.37	0.99	0.69	-0.50
	[0.19]	[0.29]	[0.24]	[0.32]	[0.23]	[0.19]	[0.08]	[0.32]	[0.29]	[0.08]	[0.35]	[0.24]	[0.18]	[0.47]	[0.48]	[0.10]	[0.18]	[0.01]
15s	0.56	0.77	0.62	0.75	0.16	0.16	0.28	0.28	0.32	0.60	0.14	0.08	0.01	0.25	0.28	0.98	0.77	-0.50
	[0.17]	[0.28]	[0.22]	[0.33]	[0.24]	[0.20]	[0.06]	[0.32]	[0.29]	[0.07]	[0.34]	[0.27]	[0.08]	[0.43]	[0.45]	[0.14]	[0.17]	[0.01]
30s	0.53	0.80	0.60	0.80	0.14	0.15	0.24	0.23	0.28	0.56	0.12	0.10	0.00	0.19	0.22	0.96	0.87	-0.50
	[0.13]	[0.26]	[0.18]	[0.37]	[0.26]	[0.23]	[0.04]	[0.31]	[0.29]	[0.05]	[0.33]	[0.30]	[0.02]	[0.39]	[0.41]	[0.20]	[0.14]	[0.01]
1min	0.52	0.78	0.59	0.85	0.15	0.18	0.22	0.26	0.31	0.53	0.12	0.12	0.00	0.21	0.26	0.89	0.93	-0.50
	[0.09]	[0.25]	[0.16]	[0.49]	[0.27]	[0.26]	[0.03]	[0.31]	[0.31]	[0.04]	[0.33]	[0.33]	[0.04]	[0.40]	[0.44]	[0.31]	[0.10]	[0.03]
5min	0.50	0.66	0.55	0.93	0.27	0.34	0.20	0.45	0.52	0.51	0.21	0.29	0.00	0.41	0.53	0.63	0.99	-0.50
	[0.04]	[0.23]	[0.20]	[1.85]	[0.29]	[0.32]	[0.02]	[0.32]	[0.34]	[0.03]	[0.41]	[0.45]	[0.03]	[0.49]	[0.50]	[0.48]	[0.04]	[0.01]
Panel B: SPY(+1) and UPRO(+3)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.57	0.61	0.57	0.58	0.37	0.39	0.41	0.81	0.82	0.84	0.22	0.25	0.32	0.97	0.99	1.00	0.28	-0.33
	[0.27]	[0.28]	[0.28]	[0.29]	[0.15]	[0.15]	[0.15]	[0.12]	[0.10]	[0.09]	[0.41]	[0.43]	[0.47]	[0.18]	[0.10]	[0.03]	[0.14]	[0.01]
5s	0.55	0.71	0.58	0.63	0.17	0.16	0.21	0.48	0.53	0.68	0.05	0.02	0.00	0.51	0.60	0.99	0.58	-0.33
	[0.23]	[0.29]	[0.27]	[0.33]	[0.17]	[0.13]	[0.08]	[0.31]	[0.26]	[0.10]	[0.21]	[0.13]	[0.05]	[0.50]	[0.49]	[0.08]	[0.19]	[0.01]
10s	0.54	0.76	0.59	0.68	0.14	0.13	0.16	0.37	0.41	0.62	0.08	0.04	0.00	0.34	0.40	0.98	0.72	-0.33
	[0.19]	[0.29]	[0.24]	[0.35]	[0.22]	[0.16]	[0.05]	[0.34]	[0.29]	[0.08]	[0.28]	[0.19]	[0.00]	[0.47]	[0.49]	[0.15]	[0.18]	[0.01]
15s	0.54	0.79	0.59	0.73	0.13	0.12	0.14	0.31	0.35	0.59	0.09	0.05	0.00	0.27	0.31	0.96	0.79	-0.33
	[0.17]	[0.28]	[0.22]	[0.38]	[0.23]	[0.19]	[0.04]	[0.34]	[0.31]	[0.07]	[0.29]	[0.22]	[0.02]	[0.45]	[0.46]	[0.19]	[0.17]	[0.01]
30s	0.52	0.81	0.59	0.80	0.12	0.13	0.12	0.27	0.32	0.55	0.11	0.08	0.00	0.22	0.26	0.93	0.88	-0.33
	[0.12]	[0.26]	[0.19]	[0.48]	[0.25]	[0.22]	[0.03]	[0.33]	[0.32]	[0.05]	[0.31]	[0.28]	[0.02]	[0.41]	[0.44]	[0.26]	[0.14]	[0.01]
1min	0.51	0.81	0.58	0.86	0.13	0.14	0.11	0.28	0.34	0.53	0.11	0.09	0.00	0.23	0.29	0.87	0.94	-0.33
	[0.09]	[0.24]	[0.16]	[0.68]	[0.25]	[0.23]	[0.02]	[0.33]	[0.32]	[0.03]	[0.31]	[0.29]	[0.00]	[0.42]	[0.46]	[0.34]	[0.10]	[0.01]
5min	0.50	0.73	0.55	0.94	0.19	0.25	0.10	0.46	0.53	0.51	0.12	0.19	0.00	0.42	0.54	0.62	0.99	-0.33
	[0.04]	[0.21]	[0.20]	[2.41]	[0.25]	[0.29]	[0.03]	[0.31]	[0.33]	[0.03]	[0.32]	[0.39]	[0.03]	[0.49]	[0.50]	[0.49]	[0.04]	[0.03]

Panel C: SPY(+1) and SH(-1)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.87	0.73	0.87	0.87	0.91	0.92	0.94	0.91	0.92	0.94	0.98	0.99	1.00	0.98	0.99	1.00	-0.10	1.00
	[0.18]	[0.25]	[0.18]	[0.18]	[0.12]	[0.09]	[0.04]	[0.12]	[0.09]	[0.04]	[0.14]	[0.10]	[0.00]	[0.14]	[0.10]	[0.00]	[0.06]	[0.02]
5s	0.85	0.81	0.87	0.88	0.57	0.66	0.85	0.58	0.66	0.85	0.65	0.76	1.00	0.65	0.77	1.00	-0.26	1.00
	[0.18]	[0.23]	[0.19]	[0.20]	[0.31]	[0.28]	[0.09]	[0.31]	[0.28]	[0.09]	[0.48]	[0.42]	[0.03]	[0.48]	[0.42]	[0.03]	[0.14]	[0.02]
10s	0.82	0.84	0.85	0.88	0.39	0.50	0.78	0.39	0.50	0.78	0.42	0.56	1.00	0.42	0.56	1.00	-0.37	1.00
	[0.19]	[0.23]	[0.20]	[0.21]	[0.31]	[0.30]	[0.11]	[0.31]	[0.31]	[0.11]	[0.49]	[0.50]	[0.07]	[0.49]	[0.50]	[0.07]	[0.17]	[0.02]
15s	0.80	0.85	0.84	0.89	0.30	0.41	0.74	0.30	0.41	0.74	0.29	0.43	0.99	0.29	0.43	0.99	-0.45	1.00
	[0.19]	[0.23]	[0.20]	[0.22]	[0.29]	[0.31]	[0.11]	[0.29]	[0.31]	[0.11]	[0.45]	[0.49]	[0.09]	[0.45]	[0.49]	[0.08]	[0.19]	[0.02]
30s	0.74	0.86	0.81	0.91	0.19	0.29	0.67	0.19	0.30	0.67	0.14	0.25	0.98	0.14	0.25	0.99	-0.60	1.00
	[0.17]	[0.21]	[0.18]	[0.23]	[0.24]	[0.28]	[0.10]	[0.25]	[0.28]	[0.10]	[0.35]	[0.43]	[0.13]	[0.35]	[0.43]	[0.12]	[0.19]	[0.02]
1min	0.67	0.85	0.76	0.94	0.15	0.26	0.60	0.15	0.26	0.60	0.09	0.18	0.95	0.08	0.19	0.95	-0.74	1.00
	[0.14]	[0.19]	[0.17]	[0.26]	[0.21]	[0.27]	[0.08]	[0.21]	[0.27]	[0.08]	[0.28]	[0.39]	[0.22]	[0.28]	[0.39]	[0.21]	[0.16]	[0.02]
5min	0.55	0.65	0.63	0.91	0.34	0.46	0.52	0.34	0.47	0.52	0.26	0.44	0.66	0.26	0.44	0.69	-0.93	1.00
	[0.09]	[0.25]	[0.20]	[0.80]	[0.31]	[0.34]	[0.07]	[0.31]	[0.34]	[0.06]	[0.44]	[0.50]	[0.47]	[0.44]	[0.50]	[0.46]	[0.07]	[0.29]

Panel D: SPY(+1) and SDS(-2)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.72	0.68	0.73	0.73	0.63	0.66	0.70	0.85	0.87	0.89	0.80	0.84	0.91	0.97	0.99	1.00	-0.19	0.50
	[0.26]	[0.27]	[0.26]	[0.27]	[0.17]	[0.15]	[0.13]	[0.13]	[0.10]	[0.07]	[0.40]	[0.36]	[0.29]	[0.16]	[0.10]	[0.00]	[0.12]	[0.01]
5s	0.68	0.76	0.71	0.75	0.26	0.30	0.44	0.48	0.55	0.74	0.16	0.14	0.31	0.53	0.64	1.00	-0.44	0.50
	[0.25]	[0.28]	[0.27]	[0.30]	[0.22]	[0.20]	[0.14]	[0.30]	[0.26]	[0.10]	[0.36]	[0.35]	[0.46]	[0.50]	[0.48]	[0.06]	[0.21]	[0.01]
10s	0.65	0.79	0.70	0.77	0.18	0.20	0.35	0.34	0.41	0.67	0.10	0.07	0.09	0.33	0.41	0.99	-0.58	0.50
	[0.22]	[0.27]	[0.25]	[0.31]	[0.22]	[0.20]	[0.10]	[0.31]	[0.28]	[0.09]	[0.29]	[0.26]	[0.28]	[0.47]	[0.49]	[0.12]	[0.22]	[0.01]
15s	0.64	0.81	0.69	0.79	0.15	0.17	0.31	0.27	0.34	0.63	0.09	0.07	0.04	0.25	0.31	0.98	-0.66	0.50
	[0.21]	[0.27]	[0.24]	[0.33]	[0.22]	[0.21]	[0.09]	[0.30]	[0.29]	[0.09]	[0.29]	[0.26]	[0.19]	[0.43]	[0.46]	[0.15]	[0.22]	[0.01]
30s	0.60	0.83	0.67	0.83	0.12	0.14	0.26	0.21	0.27	0.58	0.09	0.09	0.01	0.17	0.22	0.95	-0.78	0.50
	[0.17]	[0.25]	[0.21]	[0.40]	[0.23]	[0.22]	[0.06]	[0.29]	[0.29]	[0.07]	[0.28]	[0.28]	[0.09]	[0.38]	[0.41]	[0.22]	[0.19]	[0.01]
1min	0.56	0.82	0.64	0.85	0.12	0.14	0.23	0.21	0.27	0.54	0.09	0.09	0.00	0.16	0.21	0.89	-0.87	0.50
	[0.13]	[0.23]	[0.18]	[0.53]	[0.23]	[0.22]	[0.04]	[0.29]	[0.29]	[0.05]	[0.29]	[0.29]	[0.04]	[0.37]	[0.40]	[0.31]	[0.14]	[0.01]
5min	0.52	0.69	0.57	0.82	0.24	0.31	0.21	0.41	0.48	0.51	0.18	0.26	0.00	0.35	0.47	0.62	-0.97	0.50
	[0.06]	[0.24]	[0.20]	[1.68]	[0.29]	[0.31]	[0.03]	[0.32]	[0.34]	[0.04]	[0.38]	[0.44]	[0.04]	[0.48]	[0.50]	[0.49]	[0.06]	[0.03]

Panel E: SPY(+1) and SPXU(-3)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.67	0.70	0.68	0.68	0.40	0.43	0.46	0.82	0.84	0.87	0.29	0.33	0.42	0.97	0.99	1.00	-0.22	0.33
	[0.26]	[0.26]	[0.27]	[0.28]	[0.16]	[0.15]	[0.14]	[0.13]	[0.10]	[0.08]	[0.45]	[0.47]	[0.49]	[0.18]	[0.11]	[0.04]	[0.12]	[0.01]
5s	0.63	0.77	0.66	0.70	0.15	0.16	0.23	0.46	0.53	0.71	0.02	0.01	0.02	0.50	0.59	0.99	-0.50	0.33
	[0.24]	[0.27]	[0.27]	[0.31]	[0.15]	[0.13]	[0.10]	[0.30]	[0.26]	[0.10]	[0.15]	[0.12]	[0.13]	[0.50]	[0.49]	[0.10]	[0.19]	[0.01]
10s	0.61	0.80	0.65	0.73	0.12	0.12	0.18	0.34	0.40	0.64	0.05	0.03	0.00	0.32	0.39	0.98	-0.64	0.33
	[0.21]	[0.27]	[0.25]	[0.33]	[0.18]	[0.15]	[0.06]	[0.32]	[0.28]	[0.09]	[0.21]	[0.16]	[0.04]	[0.47]	[0.49]	[0.16]	[0.19]	[0.01]
15s	0.59	0.82	0.65	0.76	0.11	0.11	0.15	0.29	0.35	0.61	0.06	0.04	0.00	0.26	0.31	0.96	-0.72	0.33
	[0.19]	[0.26]	[0.23]	[0.35]	[0.20]	[0.17]	[0.05]	[0.32]	[0.29]	[0.08]	[0.24]	[0.19]	[0.03]	[0.44]	[0.46]	[0.20]	[0.18]	[0.01]
30s	0.56	0.84	0.63	0.80	0.10	0.11	0.13	0.24	0.29	0.56	0.07	0.06	0.00	0.21	0.25	0.92	-0.83	0.33
	[0.14]	[0.24]	[0.20]	[0.43]	[0.21]	[0.20]	[0.03]	[0.31]	[0.30]	[0.06]	[0.26]	[0.24]	[0.02]	[0.41]	[0.43]	[0.27]	[0.15]	[0.01]
1min	0.54	0.83	0.61	0.83	0.10	0.11	0.11	0.24	0.30	0.53	0.08	0.07	0.00	0.20	0.26	0.86	-0.91	0.33
	[0.10]	[0.23]	[0.17]	[0.57]	[0.22]	[0.21]	[0.02]	[0.31]	[0.31]	[0.04]	[0.27]	[0.26]	[0.00]	[0.40]	[0.44]	[0.35]	[0.11]	[0.01]
5min	0.51	0.73	0.55	0.80	0.19	0.24	0.10	0.45	0.51	0.51	0.13	0.18	0.00	0.41	0.52	0.60	-0.98	0.33
	[0.05]	[0.22]	[0.21]	[1.80]	[0.26]	[0.29]	[0.03]	[0.32]	[0.34]	[0.04]	[0.34]	[0.38]	[0.03]	[0.49]	[0.50]	[0.49]	[0.05]	[0.02]

Table 7: Summary statistics for CPILS measures, fund characteristics and Market variables.

This table shows the summary statistics for logit transformed CPILS measures of LETFs reltive to SPY estimated from 1-second prices, fund characteristics, and market variables used in Eq. (47). The daily sample ranges from January 1, 2014, to December 31, 2023.

Variable	N	Mean	SD	Min	p25	p50	p75	Max
Panel A: SSO								
logit CPILS	2516	-2.09	0.69	-7.20	-2.59	-2.15	-1.60	1.48
B/A Spread	2516	0.02	0.02	0.01	0.01	0.02	0.02	0.38
Turnover	2516	0.11	0.09	0.01	0.06	0.09	0.14	1.37
Retail	2516	0.15	0.05	0.01	0.11	0.14	0.17	0.64
Zeros	2516	0.80	0.11	0.15	0.74	0.82	0.88	0.96
Panel B: UPRO								
logit CPILS	2516	-1.79	0.67	-4.68	-2.33	-1.85	-1.27	0.82
B/A Spread	2516	0.03	0.02	0.01	0.02	0.03	0.03	0.34
Turnover	2516	0.21	0.13	0.02	0.13	0.18	0.25	1.48
Retail	2516	0.17	0.05	0.02	0.14	0.17	0.20	0.38
Zeros	2516	0.73	0.18	0.06	0.59	0.80	0.88	0.96
Panel C: SH								
logit CPILS	2516	-3.11	0.85	-11.11	-3.54	-3.09	-2.60	-0.55
B/A Spread	2516	0.05	0.02	0.02	0.04	0.05	0.06	0.41
Turnover	2516	0.10	0.09	0.01	0.04	0.07	0.14	0.66
Retail	2516	0.16	0.06	0.02	0.12	0.15	0.20	0.75
Zeros	2516	0.87	0.10	0.30	0.84	0.91	0.94	0.98
Panel D: SDS								
logit CPILS	2516	-2.31	0.75	-11.45	-2.73	-2.32	-1.86	-0.28
B/A Spread	2516	0.05	0.03	0.02	0.03	0.04	0.06	0.18
Turnover	2516	0.24	0.21	0.02	0.11	0.18	0.29	2.15
Retail	2516	0.17	0.05	0.05	0.14	0.16	0.20	0.43
Zeros	2516	0.79	0.13	0.20	0.74	0.84	0.89	0.97
Panel E: SPXU								
logit CPILS	2516	-2.04	0.65	-4.04	-2.47	-2.07	-1.60	3.84
B/A Spread	2516	0.06	0.04	0.02	0.03	0.05	0.07	0.93
Turnover	2516	0.34	0.23	0.03	0.18	0.28	0.43	2.27
Retail	2516	0.18	0.05	0.03	0.14	0.17	0.21	0.37
Zeros	2516	0.76	0.16	0.14	0.68	0.80	0.88	0.97
Panel F: Market and Event								
RKT	2516	0.07	0.05	0.03	0.04	0.05	0.07	0.80
VIX	2516	2.84	0.34	2.21	2.58	2.78	3.06	4.42
EPU	2516	4.55	0.63	1.20	4.13	4.52	4.93	6.69
Sentiment	2516	0.18	0.56	-0.35	-0.19	0.03	0.26	2.01
Fed	2516	0.13	0.34	0.00	0.00	0.00	0.00	1.00
Covid	2516	0.02	0.16	0.00	0.00	0.00	0.00	1.00
Ukraine	2516	0.04	0.20	0.00	0.00	0.00	0.00	1.00

Table 8: Determinants of LETFs' price discovery shares.
Table 8 summarizes the estimation results of the following regression:

$$logit\ CPILS_t = \alpha + \boldsymbol{\beta_1} \cdot \boldsymbol{Fund_t} + \boldsymbol{\beta_2} \cdot \boldsymbol{Market_t} + \boldsymbol{\beta_3} \cdot \boldsymbol{Event_t} + \boldsymbol{y_t} + \epsilon_t.$$

The dependent variable are logit-transformed CPLIS measures of SSO in column (1), UPRO in column (2), SH column (3), SDS in column (4), and SPXU in column (1). The t-statistics calculated with White standard errors are reported in brackets. ***, ** and * indicate the coefficients are statistically significant at the 1%, 5% and 10% levels, respectively.

Dependent Variable	logit CPILS				
	(1) SSO	(2) UPRO	(3) SH	(4) SDS	(5) SPXU
B/A Spread	-0.58***	-0.72***	-0.35***	-0.46***	-0.30***
	(-3.36)	(-6.78)	(-2.65)	(-7.50)	(-4.73)
Turnover	0.07*	0.06***	0.08***	0.06***	0.04***
	(1.91)	(4.12)	(3.80)	(5.92)	(3.80)
Retail	-0.14***	-0.12***	-0.02	-0.06***	-0.12***
	(-6.04)	(-4.71)	(-1.29)	(-3.31)	(-4.81)
Zeros	-0.40***	-0.39***	-0.24***	-0.29***	-0.35***
	(-14.03)	(-19.82)	(-8.83)	(-12.09)	(-15.78)
RKT	0.18***	0.14***	0.09***	0.15***	0.15***
	(6.76)	(6.62)	(6.70)	(8.06)	(6.88)
VIX	-0.02**	-0.01**	-0.00	0.00	0.01**
	(-2.31)	(-2.25)	(-0.47)	(0.00)	(2.18)
EPU	0.01***	0.00	0.00**	0.00***	0.00
	(3.09)	(1.43)	(2.35)	(3.52)	(1.12)
Sentiment	0.02***	0.01***	0.00	0.00	-0.00
	(4.61)	(3.72)	(0.36)	(1.32)	(-0.22)
Fed	0.01***	0.01***	0.00*	0.01***	0.01***
	(5.68)	(4.41)	(1.95)	(4.79)	(5.23)
Covid	0.01	-0.00	0.02***	0.01	-0.01
	(1.42)	(-0.21)	(3.07)	(1.09)	(-1.02)
Ukraine	0.02***	0.00	0.02***	0.01	-0.00
	(2.87)	(0.51)	(3.05)	(0.93)	(-0.02)
Year Dummy	Yes	Yes	Yes	Yes	Yes
Obs.	2516	2516	2516	2516	2516
Adj. R^2	0.70	0.81	0.57	0.75	0.72
Robust SE	Yes	Yes	Yes	Yes	Yes

Internet Appendix to

Leverage Corrections to Price Discovery Measures With an Application to Leveraged Exchange-Traded Funds

A1 Leverage Corrections for Multivariate Price Discovery

Let $\mathbf{p}_t = (p_{1t}, p_{2t}, ..., p_{nt})'$ denote a vector of log prices for n assets that are closely related by arbitrage, of which each price series is intergated of order 1, or I(1). Assume that these price series are cointegrated with the following $(n-1) \times n$ cointegrating matrix:

$$\boldsymbol{\beta}' = \begin{pmatrix} 1 & -\beta_2 & 0 & \cdots & 0 \\ 1 & 0 & -\beta_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & -\beta_n \end{pmatrix}. \tag{A.1}$$

Let us denote $\beta_1 = 1$.

The multivariate reduced-form VECM is given as:

$$\Delta\mathbf{p}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{p}_{t-1} + \sum_{j=1}^{k} \boldsymbol{\Gamma}_j \Delta\mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{A.2}$$

where $\boldsymbol{\alpha}$ is the matrix of error correction coefficients, $\boldsymbol{\Gamma}_j$ $(i = 1, \ldots, k)$ are the short-run coefficient matrices, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{nt})'$ is the vector of reduced-form VECM residuals with $E[\boldsymbol{\varepsilon}_t] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Omega}$. In the reduced multivariate integrated VMA model, the long-run impact matrix is expressed as $\boldsymbol{\Psi}(1) = \boldsymbol{\beta}_\perp \boldsymbol{\Pi} \boldsymbol{\alpha}_\perp'$, where $\boldsymbol{\beta}_\perp$ is the orthogonal matrix for $\boldsymbol{\beta}$ and $\mathbf{\Pi} = \left(\boldsymbol{\alpha}_{\perp}^{'}(\boldsymbol{I}_n - \sum_{j=1}^{k}\mathbf{\Gamma}_j)\boldsymbol{\beta}_{\perp}\right)^{-1}$, with $\boldsymbol{I}_n$ denoting the identity matrix and $\boldsymbol{\alpha}_{\perp}$ being the orthogonal matrix for $\boldsymbol{\alpha}$. When $\boldsymbol{\beta}$ takes the value in (A.1), the rows of $\mathbf{\Psi}(1)$ are proportional, as $\boldsymbol{\beta}^{'}\mathbf{\Psi}(1) = \mathbf{0}$. Let us denote the first row of $\mathbf{\Psi}(1)$ as $\boldsymbol{\psi} = (\psi_1, \psi_2, ..., \psi_n)'$, then we have:

$$\mathbf{\Psi}(1) = \begin{pmatrix} \psi_1 & \psi_2 & \cdots & \psi_n \\ \psi_1/\beta_2 & \psi_2/\beta_2 & \cdots & \psi_n/\beta_2 \\ \psi_1/\beta_n & \psi_2/\beta_n & \cdots & \psi_n/\beta_n \end{pmatrix}. \tag{A.3}$$

Thus, the long-run impact of innovation $\boldsymbol{\varepsilon}_t$ on the i-th price series (p_{it}) is $1/\beta_i$ times the impact on the first price series (p_{1t}). For products with leverage greater than one, where $0 < |\beta_i| < 1$, the long-run response to a permanent shock will be larger.

In this multivariate case, we define the permanent innovation as:

$$\eta_t^P = \boldsymbol{\psi}'\boldsymbol{\varepsilon}_t = \psi_1\varepsilon_{1t} + \psi_2\varepsilon_{2t} + \cdots + \psi_n\varepsilon_{nt}. \tag{A.4}$$

Given the Cholesky decomposition of $\mathbf{\Omega} = \mathbf{F}\mathbf{F}^{'}$, where $\mathbf{F}$ is a lower triangular matrix of size $n \times n$, the information share (IS) measure for the i^{th} market is defined as:

$$\text{IS}_i = \frac{([\boldsymbol{\psi}^{'}\mathbf{F}]_i)^2}{\boldsymbol{\psi}^{'}\mathbf{\Omega}\boldsymbol{\psi}}. \tag{A.5}$$

where $[\boldsymbol{\psi}^{'}\mathbf{F}]_i$ is the i^{th} element of the row vector $\boldsymbol{\psi}^{'}\mathbf{F}$. The CS measure for the i^{th} market is given as:

$$\text{CS}_i = \frac{\psi_i}{\psi_1 + \psi_2 + \cdots + \psi_n}. \tag{A.6}$$

The multivariate representations for MIS and PDS are the same as in Eq.(A.17) and Eq.(7).

Defining a multivariate IL measure is challenging, as the ratio of one market's measures to those of another depends on the selection of the competing market. However, a share-based version of the IL measure, known as the ILS measure, can still be defined. Patel et al.

(2020) introduce the following β measure for each market:

$$\beta_i^{IS} = \frac{\text{IS}_i}{\text{CS}_i}, \tag{A.7}$$

and define the Information Leadership Shares (ILS) as:

$$\text{ILS}_i = \frac{\beta_i^{IS}}{\beta_1^{IS} + \beta_2^{IS} + \cdots + \beta_n^{IS}}. \tag{A.8}$$

The multivariate binary indicator ILI is then defined as:

$$\text{ILI}_i = \begin{cases} 1, & \text{if } \text{ILS}_i > \text{ILS}_k \quad \forall \quad k \neq i \\ 0, & \text{otherwise.} \end{cases} \tag{A.9}$$

We follow Patel et al. (2020) to define the multivariate Modified Information Leadership (MILS) and its binary indicator MILI by replacing the IS measure with the MIS measure of Lien and Shrestha (2009). The multivariate definitions for PILS and PILI can be defined by replacing the IS measure with the PDS measure of Sultan and Zivot (2015) and Shen et al. (2024a) in Eq.(7).

To correct the leverage bias of the multivariate price discovery, let us define the following normalization matrix:

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 & & 0 \cdots & 0 \\ 0 & \beta_2 & 0 & \cdots & 0 \\ 0 & 0 & \beta_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \beta_n \end{pmatrix}, \tag{A.10}$$

and pre-multiply the integrated moving average model in Eq.(3) by this matrix:

$$\boldsymbol{B}\mathbf{p}_t = \boldsymbol{B}\boldsymbol{p}_0 + \underbrace{\boldsymbol{B}\boldsymbol{\Psi}(1)\boldsymbol{B}^{-1}}_{\boldsymbol{\Psi}^*(1)} \sum_{s=1}^{t} \underbrace{\boldsymbol{B}\boldsymbol{\varepsilon}_s}_{\boldsymbol{\varepsilon}_s^*} + \boldsymbol{B}\boldsymbol{\Psi}^*(L)\boldsymbol{B}^{-1} \underbrace{\boldsymbol{B}\boldsymbol{\varepsilon}_s}_{\boldsymbol{\varepsilon}_s^*}. \tag{A.11}$$

It is evident that:

$$\boldsymbol{B}\mathbf{p}_t = \begin{pmatrix} p_{1t} \\ \beta_2 p_{2t} \\ \vdots \\ \beta_n p_{nt} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\Psi}^*(1) = \boldsymbol{B}\boldsymbol{\Psi}(1)\boldsymbol{B}^{-1} = \begin{pmatrix} \psi_1 & \psi_2/\beta_2 & \cdots & \psi_n/\beta_n \\ \psi_1 & \psi_2/\beta_2 & \cdots & \psi_n/\beta_n \\ \vdots & \vdots & \cdots & \vdots \\ \psi_1 & \psi_2/\beta_2 & \cdots & \psi_n/\beta_n \end{pmatrix}. \tag{A.12}$$

By normalizing the original price vector $\boldsymbol{p}_t$ by the matrix $\boldsymbol{B}$, we demonstrate that the long-run impacts of new information (the rows of $\boldsymbol{\Psi}^*(1)$) are identical for the normalized price vector. Let the common row of $\boldsymbol{\Psi}^*(1)$ be denoted as $\boldsymbol{\psi}^* = (\psi_1^*, \psi_2^*, \cdots, \psi_n^*)'$. The above result implies that $\psi_1^* = \psi_1$, $\psi_2^* = \psi_2/\beta_2$, $\cdots$, and $\psi_n^* = \psi_n/\beta_n$.

The multivariate leverage-corrected CS measures take the following values:

$$CS_i^* = \frac{\psi_i^*}{\psi_1^* + \psi_2^* + \cdots + \psi_n^*} = \frac{\psi_i/\beta_i}{\psi_1 + \psi_2/\beta_2 + \cdots + \psi_n/\beta_n}, \tag{A.13}$$

with $\beta_1 = 1$. Thus, it is evident that the CS measures are not invariant to β-normalization of the price series. We call the above CS^* measures the leverage-corrected CS measures.

The β-neutrality of IS, MIS, and PDS measures can be verified in the multivariate case by substituting $\boldsymbol{\psi}^*$ and $\boldsymbol{\Omega}^*$ into the multivariate expressions of IS, MIS, and PDS. Based on the leverage-corrected CS measures in Eq. (A.13), we can define the corresponding leverage-corrected ILS, MILS, and PILS measures as:

$$\begin{aligned} CILS_i &= \frac{IS_i/CS_i^*}{IS_1/CS_1^* + IS_2/CS_2^* + \cdots + IS_n/CS_n^*} \\ &= \frac{IS_i/(CS_i/\beta_i)}{IS_1/(CS_1/\beta_1) + IS_2/(CS_2/\beta_2) + \cdots + IS_n/(CS_n/\beta_n)}, \end{aligned} \tag{A.14}$$

$$\begin{aligned} CMILS_i &= \frac{MIS_i/CS_i^*}{MIS_1/CS_1^* + MIS_2/CS_2^* + \cdots + MIS_n/CS_n^*} \\ &= \frac{MIS_i/(CS_i/\beta_i)}{MIS_1/(CS_1/\beta_1) + MIS_2/(CS_2/\beta_2) + \cdots + MIS_n/(CS_n/\beta_n)}, \end{aligned} \tag{A.15}$$

$$\begin{aligned} CPILS_i &= \frac{PDS_i/CS_i^*}{PDS_1/CS_1^* + PDS_2/CS_2^* + \cdots + PDS_n/CS_n^*} \\ &= \frac{PDS_i/(CS_i/\beta_i)}{PDS_1/(CS_1/\beta_1) + PDS_2/(CS_2/\beta_2) + \cdots + PDS_n/(CS_n/\beta_n)}. \end{aligned} \tag{A.16}$$

We can convert the leverage-corrected information leadership share measures into their corresponding binary leadership indicators (CILI, CMILI, CPILI), as in (A.9).

A2 Leverage bias and β-neutrality of MIS and GIS

The Modified Information Share (MIS) proposed by Lien and Shrestha (2009) utilizes a factorization matrix that is based on the spectral decomposition of the correlation matrix (denoted by $\mathbf{\Phi}$) of the reduced-form residuals $\boldsymbol{\varepsilon}_t$. Based on the MIS measure, Lien and Shrestha (2014) further propose the GIS measure, which was claimed to be applicable to cases where the prices are cointegrated but the cointegration vector does not have to be one-to-one. In this section, we show that theoretically these two measures are the same, which just reflects the β-neutrality property of all IS-type measures, as indicated in Lautier et al. (2024). More importantly, we show that β-neutral measures also suffer from the leverage bias as defined in the main text.

As defined in Lien and Shrestha (2009), MIS takes the following expression:

$$MIS_i = \frac{([\boldsymbol{\psi}^{'} \boldsymbol{F}^*]_i)^2}{\boldsymbol{\psi}^{'} \mathbf{\Omega} \boldsymbol{\psi}} = \frac{\boldsymbol{\psi}_i^{*2}}{\sum_{i=1}^n \boldsymbol{\psi}_i^{*2}}, \tag{A.17}$$

where $\boldsymbol{\psi}^{*'} = \boldsymbol{\psi}^{'} \boldsymbol{F}^*$, $\boldsymbol{F}^* = [\boldsymbol{G} \mathbf{\Lambda}^{-1/2} \boldsymbol{G}^{'} \boldsymbol{V}^{-1}]^{-1}$, $\boldsymbol{G}$ denotes a matrix with eigenvectors of the correlation matrix $\mathbf{\Phi}$ as columns, $\mathbf{\Lambda}$ represents the diagonal matrix with the corresponding eigenvalues as diagonal elements, and $\boldsymbol{V}$ denotes a diagonal matrix containing the idiosyncratic errors' standard deviations on the diagonal.

For the generalized cointegating coefficient given in (A.1), we can show that the rows of the long-run impact matrix $\mathbf{\Psi}(1)$ are proportional as shown in (A.3). Let us denote the j-th row of $\mathbf{\Psi}(1)$ as vector $\boldsymbol{\psi}_j$. Then, Lien and Shrestha (2014) define the GIS measure as:

$$GIS_i = \frac{([\boldsymbol{\psi}_1^{'} \boldsymbol{F}^*]_i)^2}{\boldsymbol{\psi}_1^{'} \mathbf{\Omega} \boldsymbol{\psi}_1}, \tag{A.18}$$

with $\boldsymbol{F}^*$ as defined above.

So, we can see that the MIS and GIS take the same formula except that GIS is defined under a generalized non-unitary cointegating case. To define the price discovery measure under the generalized case, Lien and Shrestha (2014) choose to pick the first row $\boldsymbol{\psi}_1$ of $\boldsymbol{\Psi}(1)$ (instead of the common row $\boldsymbol{\psi}$ in the unitary cointegrated case) and decompose the variance of the permanent shock based on this choice. In fact, any other row of $\boldsymbol{\Psi}(1)$ will yield the same value of GIS since the rows $\boldsymbol{\psi}_j$ are proportional. Thus, we have proved that MIS and GIS are basically the same measure, which just reflects the β-neutrality of MIS.[1]

While the β-neutrality property implies that these β-neutral price discovery measures can be directly applicable to markets with leveraged returns without further modifications, it does not guarantee that they can yield the correct identification of the leader in the price discovery process. Taking the MIS (i.e., GIS) for example, Shen et al. (2024b) show that the MIS measure has the following structural representations for the bivariate case:

$$\begin{aligned} \mathrm{MIS}_1 &= \frac{1}{2} + \frac{1}{2}\frac{d^P_{0,1}d^T_{0,2} + d^T_{0,1}d^P_{0,2}}{d^P_{0,1}d^T_{0,2} - d^T_{0,1}d^P_{0,2}}\sqrt{1-\rho^2}, \\ \mathrm{MIS}_2 &= \frac{1}{2} - \frac{1}{2}\frac{d^P_{0,1}d^T_{0,2} + d^T_{0,1}d^P_{0,2}}{d^P_{0,1}d^T_{0,2} - d^T_{0,1}d^P_{0,2}}\sqrt{1-\rho^2}, \end{aligned} \tag{A.19}$$

where ρ denotes the correlation coefficient between the reduced-form residuals. For the case with uncorrelated residuals ($\rho = 0$), the MIS measure shares the same structural representations with IS:

$$\begin{aligned} \mathrm{MIS}_1 &= \frac{d^P_{0,1}d^T_{0,2}}{d^P_{0,1}d^T_{0,2} - d^T_{0,1}d^P_{0,2}}, \\ \mathrm{MIS}_2 &= \frac{-d^T_{0,1}d^P_{0,2}}{d^P_{0,1}d^T_{0,2} - d^T_{0,1}d^P_{0,2}}. \end{aligned} \tag{A.20}$$

As we can see from the above expressions, the MIS (as well as GIS) measure encompasses contemporaneous responses to both permanent and transitory shocks. Hence, it cannot work as an efficient price leader identifier. Shen et al. (2024b) propose to combine the IS-type with the CS measure to cancel out transitory responses to frictional innovations. For the

[1]For the biavariate case, Section 3.2 also derives the β-neutrality of the IS, MIS, and PDS measures by normalizing the original price vector. Proposition 10 of Lautier et al. (2024) also proves that all IS-type measures are β-neutral.

MIS measure, Shen et al. (2024b) propose the following combinations:

$$MIL_1 = \left| \frac{MIS_1/CS_1}{MIS_2/CS_2} \right|, \quad MIL_2 = \left| \frac{MIS_2/CS_2}{MIS_1/CS_1} \right|, \tag{A.21}$$

which can be shown to have the following structural interpretations when $\rho = 0$:

$$MIL_1 = \left| \frac{d_{0,1}^P}{d_{0,2}^P} \right|, \quad MIL_2 = \left| \frac{d_{0,2}^P}{d_{0,1}^P} \right|. \tag{A.22}$$

The improved MIL measure now depends soly on the contemporaneous responses to the permanent shock (but only when $\rho = 0$), which make it a more appropriate price discovery measure. However, as discussed in the main text, this improved MIL still suffers from the so-called leverage bias, in that it does not take the long-run response to the permanent shock into consideration and thus tends to pick the market with a large leverage as the leader. In other words, the improved MIL measure still suffers from leverage bias.

When we agree that a price leader should be the one with a larger initial responses as a fractional of its long-run response to the permanent shock, we propose the following leverage-correction to MIL:

$$\begin{aligned} CMIL_1 &= \left| \frac{(MIS_1/CS_1)/d_1^P(1)}{(MIS_2/CS_2)/d_2^P(1)} \right| = \left| \frac{MIS_1/CS_1}{MIS_2/(CS_2/\beta)} \right|, \\ CMIL_2 &= \left| \frac{(MIS_2/CS_2)/d_2^P(1)}{(MIS_1/CS_1)/d_1^P(1)} \right| = \left| \frac{MIS_2/(CS_2/\beta)}{MIS_1/CS_1} \right|, \end{aligned} \tag{A.23}$$

which can be shown to have the following structural interpretations when $\rho = 0$:

$$CMIL_1 = \left| \frac{d_{0,1}^P/d_1^P(1)}{d_{0,2}^P/d_2^P(1)} \right| = \left| \frac{d_{0,1}^P}{\beta d_{0,2}^P} \right|, \quad CMIL_2 = \left| \frac{d_{0,2}^P/d_2^P(1)}{d_{0,1}^P/d_1^P(1)} \right| = \left| \frac{\beta d_{0,2}^P}{d_{0,1}^P} \right|, \tag{A.24}$$

The correlation among reduced-form residuals make the structural representations of MIS and hence MIL complex. Hence, Shen et al. (2024b) propose to use the improve PIL measure instead.

In summary, even though IS-type measures can be directly applicable to markets with leveraged returns (β-neutral), structural analysis of these IS-type measures implies that they are not pure measures of price leader. Instead, they incorporate instant transitory responses and should be combined with the CS measure to cancel out the transitory noises. However, these improved IL-type measures proposed by Shen et al. (2024b) still suffer from leverage bias. We need to further correct for this leverage bias by taking into account the long-run permanent responses.

A3 Data frequency

In this Appendix, we further examine how data frequency affect the accuracy of our price discovery measures. We generate data from the partial adjustment model with the sample size of 21600, mimicking the 1-second data frequency. Then, we resample the 1-second data at an interval of 5 seconds (5s), 10 seconds (10s), 15 seconds (15s), 30 seconds (30s), 1 minute (1min), and 5 minutes (5min). With each re-sampled data, we re-estimate the VECM model and calculate price discovery estimates.

To see how leverages affect these price discovery measures under different data frequency, we next fix the leadership parameter as $(\delta_1, \delta_2) = (0.9, 0.1)$ and let Market 2 have a leverage (i.e., $1/\beta$) of 1, 2, 3, respectively. Table A1 summarizes the results for the case with equal transitory shock responses ($(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$) and uncorrelated reduced-form residuals ($\sigma_T^2 = \frac{\delta_1 \delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$). Table A2 summarizes the results for the case with unequal transitory shock responses ($(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$) and uncorrelated reduced-form residuals ($\sigma_T^2 = \frac{\delta_1 \delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$). Table A3 summarizes the results for the case with unequal transitory shock responses ($(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$) and correlated reduced-form residuals ($\sigma_T^2 = 10$).

[Insert Table A1 about here.]

As Table A1 indicates, leverages make traditional measures (IS and CS), as well as these order-invariant measures (MIS and PDS), more sensitive to data frequency. However, as IS and MIS become less informative in identifying the leader (Market 1) as the data becomes coarser, we find that PDS seems to be able to provide a satisfactory measure of leadership even under the 5-minute frequency. For example, for the 5-minute level data and when Market 2 is associated with a leverage of 3, IS and MIS seem to take a random guess on the choice of the leader, while PDS can still choose the right leader for 86% of the time.

As for these combined indicators (ILI, MILI, and PILI), we can see that leverages also amplify their inefficacy in leadership identification. For 1-second level data, their performances for different leverages are almost the same. However, as data frequency become lower than 5-second, ILI seems to always pick the followers as the leader, while MILI behaves a little bit better than ILI but still picks the wrong leader for data frequency lower than 10-second. And the chances of these measures picking the wrong leader become higher for products with a larger leverage. PILI also exhibits this pattern in identification of the leader, although it performs slightly better than ILI and MILI.

Leverage-corrected CILI and CMILI measures seem to alleviate the bias of their uncorrected correspondents only for 5-second level data. For other frequencies lower than 5-second, these two leverage-corrected measures behave as poorly as their uncorrected correspondents. Interestingly, the leverage-corrected CPILI measure provides the most satisfactory leadership estimates for all leverage levels and for all data frequencies. For the 5-minute level data, CPILI identifies Market 1 as the leader for 86% of the time when Market 2 is associated with a leverage of 3.

The results in Table A2 with unequal transitory responses basically reveal the same findings as in Table A1. Unequal transitory responses make IS, MIS, and PDS measures worse at identifying the leader. Again, the combined indicators (ILI and MILI) and their leverage corrections (CILI and CMILI) do not perform as well as PILI and CPILI, and CPILI exhibits its superiority in the identification of the price leader among all these measures.

[Insert Table A2 about here.]

The results in Table A3 with unequal transitory responses and correlated residuals also provide evidence of the superiority of CPILI. With correlated residuals, IS and MIS are poor at identifying the leader even under the original data-generating frequency 1-second. However, as the data frequency becomes coarser, PDS appears not to be able to outperform IS and MIS. And as a result, the combined indicators (ILI and MILI) slightly outperform PILI for all frequencies lower than 5-second. All of these leverage-corrected measures (CILI, CMILI, and CPILI) provide satisfactory leadership measures for frequencies higher than 30-second. For lower frequency data at the 1-minute and 5-minute level, CPILI can correctly identify the price leader more often than the other two counterparts (CILI and CMILI).

[Insert Table A3 about here.]

In summary, price discovery measures are sensitive to data frequency, and leverages can make things worse in a complex way. As IS, MIS, and PDS become less informative as data become rougher, PDS seems to be more resilient to data frequency than the other two measures. The performance of the combined indicators (ILI, MILI, and PILI) can be very poor due to the added sensitiveness of the CS measures to the frequency of the data. So do the leverage-corrected CILI and CMILI measures. However, our simulation results with data frequency again prove the superiority of the leverage-corrected CPILI measure.

Table A1: Data Frequency and Leverages: Equal Transitory Responses and Uncorrelated Residuals

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
Panel A: Frequency=1s																				
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.90	0.10	0.50	0.50	0.90	0.10	0.90	0.10	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.82	0.18	0.50	0.50	0.82	0.18	0.82	0.18	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.75	0.25	0.50	0.50	0.75	0.25	0.75	0.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Panel B: Frequency=5s																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.75	0.25	0.78	0.22	0.82	0.18	0.93	0.07	0.21	0.79	0.92	0.08	1.00	0.00	0.21	0.79	0.92	0.08	1.00	0.00
2	0.67	0.33	0.77	0.23	0.75	0.25	0.86	0.14	0.00	1.00	0.05	0.95	1.00	0.00	0.90	0.11	1.00	0.00	1.00	0.00
3	0.63	0.37	0.77	0.23	0.70	0.30	0.80	0.20	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Panel C: Frequency=10s																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.64	0.36	0.87	0.13	0.75	0.25	0.94	0.06	0.00	1.00	0.01	0.99	1.00	0.00	0.00	1.00	0.01	0.99	1.00	0.00
2	0.60	0.40	0.88	0.12	0.70	0.30	0.89	0.11	0.00	1.00	0.00	1.00	1.00	0.00	0.00	1.00	0.07	0.93	1.00	0.00
3	0.59	0.41	0.88	0.12	0.68	0.32	0.85	0.15	0.00	1.00	0.00	1.00	0.00	1.00	0.02	0.98	0.25	0.75	1.00	0.00
Panel D: Frequency=15s																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.60	0.40	0.89	0.11	0.71	0.29	0.95	0.05	0.00	1.00	0.00	1.00	1.00	0.00	0.00	1.00	0.00	1.00	1.00	0.00
2	0.58	0.42	0.91	0.09	0.68	0.32	0.91	0.09	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	1.00	0.00
3	0.57	0.43	0.92	0.08	0.67	0.33	0.88	0.12	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.02	0.98	1.00	0.00
Panel E: Frequency=30s																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.56	0.44	0.87	0.13	0.67	0.33	0.96	0.04	0.00	1.00	0.01	0.99	1.00	0.00	0.00	1.00	0.01	0.99	1.00	0.00
2	0.55	0.45	0.92	0.08	0.65	0.35	0.94	0.06	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.01	0.99	1.00	0.00
3	0.55	0.45	0.94	0.06	0.64	0.36	0.91	0.09	0.00	1.00	0.00	1.00	0.00	1.00	0.01	0.99	0.02	0.98	1.00	0.00
Panel F: Frequency=1min																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.53	0.47	0.81	0.19	0.63	0.37	0.97	0.03	0.05	0.95	0.09	0.92	1.00	0.00	0.05	0.95	0.08	0.92	1.00	0.00
2	0.53	0.47	0.88	0.12	0.62	0.38	0.95	0.05	0.01	0.99	0.01	0.99	0.00	1.00	0.06	0.94	0.09	0.91	1.00	0.00
3	0.53	0.47	0.91	0.09	0.61	0.39	0.93	0.07	0.01	0.99	0.01	0.99	0.00	1.00	0.07	0.93	0.10	0.90	1.00	0.00
Panel G: Frequency=5min																				
	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.51	0.49	0.54	0.46	0.55	0.45	0.90	0.10	0.40	0.60	0.54	0.46	0.83	0.17	0.40	0.60	0.54	0.46	0.85	0.15
2	0.51	0.49	0.66	0.34	0.55	0.45	0.88	0.12	0.24	0.76	0.29	0.71	0.00	1.00	0.41	0.59	0.55	0.46	0.86	0.14
3	0.51	0.49	0.72	0.28	0.54	0.46	0.86	0.14	0.15	0.85	0.17	0.83	0.00	1.00	0.42	0.59	0.54	0.46	0.86	0.15

Notes: This table reports price discovery measure estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$

$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. We set $(\delta_1, \delta_2) = (0.9, 0.1)$, $(b_{0,1}^T, b_{0,2}^T) = (0.5, -0.5)$, $\sigma_P^2 = 1$, $\sigma_T^2 = \frac{\delta_1 \delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$. Numbers shown are the averages of price discovery measure estimates of 1000 samples with a sample size of $N = 21600$.

Table A2: Data Frequency and Leverages: Unequal Transitory Responses and Uncorrelated Residuals

1/β	IS P_1	IS P_2	CS P_1	CS P_2	MIS P_1	MIS P_2	PDS P_1	PDS P_2	ILI P_1	ILI P_2	MILI P_1	MILI P_2	PILI P_1	PILI P_2	CILI P_1	CILI P_2	CMILI P_1	CMILI P_2	CPILI P_1	CPILI P_2
Panel A: Frequency=1s																				
1	0.69	0.31	0.20	0.80	0.69	0.31	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.53	0.47	0.20	0.80	0.53	0.47	0.53	0.47	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.43	0.57	0.20	0.80	0.43	0.57	0.43	0.57	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Panel B: Frequency=5s																				
1	0.60	0.40	0.45	0.55	0.66	0.34	0.75	0.25	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.53	0.47	0.44	0.56	0.55	0.45	0.58	0.42	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.49	0.51	0.44	0.56	0.49	0.51	0.48	0.52	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
Panel C: Frequency=10s																				
1	0.58	0.42	0.66	0.34	0.66	0.34	0.82	0.18	0.09	0.92	0.53	0.48	1.00	0.00	0.09	0.92	0.53	0.48	1.00	0.00
2	0.55	0.45	0.66	0.34	0.60	0.40	0.70	0.30	0.00	1.00	0.01	0.99	1.00	0.00	0.85	0.15	0.99	0.01	1.00	0.00
3	0.53	0.47	0.67	0.33	0.55	0.45	0.61	0.39	0.00	1.00	0.00	1.00	0.00	1.00	0.99	0.01	1.00	0.00	1.00	0.00
Panel D: Frequency=15s																				
1	0.57	0.43	0.76	0.24	0.65	0.35	0.85	0.15	0.01	0.99	0.05	0.95	1.00	0.00	0.01	0.99	0.05	0.95	1.00	0.00
2	0.55	0.45	0.77	0.23	0.61	0.39	0.75	0.25	0.00	1.00	0.00	1.00	0.00	1.00	0.14	0.86	0.42	0.58	1.00	0.00
3	0.54	0.46	0.77	0.23	0.58	0.42	0.68	0.32	0.00	1.00	0.00	1.00	0.00	1.00	0.54	0.46	0.85	0.15	1.00	0.00
Panel E: Frequency=30s																				
1	0.55	0.45	0.83	0.17	0.64	0.36	0.89	0.11	0.01	0.99	0.03	0.97	1.00	0.00	0.01	0.99	0.03	0.97	1.00	0.00
2	0.54	0.46	0.85	0.15	0.61	0.39	0.81	0.19	0.00	1.00	0.00	1.00	0.00	1.00	0.05	0.95	0.07	0.93	1.00	0.00
3	0.53	0.47	0.86	0.14	0.59	0.41	0.75	0.25	0.00	1.00	0.00	1.00	0.00	1.00	0.10	0.90	0.16	0.84	1.00	0.00
Panel F: Frequency=1min																				
1	0.53	0.47	0.78	0.22	0.61	0.39	0.91	0.09	0.08	0.92	0.11	0.89	1.00	0.00	0.08	0.92	0.11	0.89	1.00	0.00
2	0.53	0.47	0.84	0.16	0.59	0.41	0.84	0.16	0.03	0.97	0.03	0.97	0.00	1.00	0.12	0.88	0.14	0.86	1.00	0.00
3	0.52	0.48	0.86	0.14	0.58	0.42	0.78	0.22	0.02	0.98	0.02	0.98	0.00	1.00	0.16	0.84	0.19	0.81	1.00	0.00
Panel G: Frequency=5min																				
1	0.51	0.49	0.52	0.48	0.54	0.46	0.83	0.17	0.42	0.58	0.55	0.46	0.83	0.17	0.42	0.58	0.55	0.45	0.85	0.15
2	0.50	0.50	0.64	0.36	0.53	0.47	0.76	0.24	0.26	0.74	0.29	0.71	0.00	1.00	0.44	0.56	0.54	0.46	0.84	0.16
3	0.50	0.50	0.70	0.30	0.53	0.47	0.71	0.29	0.17	0.83	0.18	0.83	0.00	1.00	0.45	0.55	0.53	0.47	0.82	0.18

Notes: This table reports price discovery measure estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $(\delta_1, \delta_2) = (0.9, 0.1)$, $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, $\sigma_P^2 = 1$, $\sigma_T^2 = \frac{\delta_1\delta_2/\beta}{-b_{0,1}^T b_{0,2}^T}$. Numbers shown are the averages of price discovery measure estimates of 1000 samples with a sample size of $N = 21600$.

Table A3: Data Frequency and Leverages: Unequal Transitory Responses and Correlated Residuals

Panel A: Frequency=1s

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.54	0.46	0.20	0.80	0.59	0.41	0.69	0.31	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.51	0.49	0.20	0.80	0.52	0.48	0.53	0.47	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.46	0.54	0.20	0.80	0.45	0.55	0.43	0.57	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

Panel B: Frequency=5s

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.62	0.38	0.34	0.66	0.62	0.38	0.62	0.38	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.45	0.55	0.33	0.67	0.45	0.55	0.45	0.55	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.37	0.63	0.32	0.68	0.36	0.64	0.35	0.65	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00

Panel C: Frequency=10s

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.57	0.43	0.39	0.61	0.58	0.42	0.58	0.42	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.44	0.56	0.39	0.61	0.43	0.57	0.41	0.59	1.00	0.00	1.00	0.00	0.99	0.01	1.00	0.00	1.00	0.00	1.00	0.00
3	0.40	0.60	0.40	0.60	0.37	0.63	0.33	0.67	0.36	0.64	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00

Panel D: Frequency=15s

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.54	0.46	0.41	0.59	0.55	0.45	0.56	0.44	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00
2	0.45	0.55	0.43	0.57	0.43	0.57	0.40	0.60	0.86	0.14	0.47	0.53	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00
3	0.43	0.57	0.47	0.53	0.39	0.61	0.34	0.66	0.08	0.92	0.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00

Panel E: Frequency=30s

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.51	0.49	0.44	0.56	0.51	0.49	0.52	0.48	0.95	0.05	1.00	0.00	1.00	0.00	0.95	0.05	1.00	0.00	1.00	0.00
2	0.47	0.53	0.51	0.49	0.45	0.55	0.42	0.58	0.27	0.73	0.08	0.92	0.00	1.00	0.99	0.01	1.00	0.00	1.00	0.00
3	0.47	0.53	0.57	0.43	0.44	0.56	0.38	0.62	0.07	0.93	0.01	0.99	0.00	1.00	1.00	0.00	1.00	0.00	1.00	0.00

Panel F: Frequency=1min

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.50	0.50	0.48	0.52	0.51	0.49	0.51	0.49	0.60	0.40	0.69	0.32	1.00	0.00	0.60	0.40	0.69	0.32	1.00	0.00
2	0.49	0.51	0.55	0.45	0.47	0.53	0.43	0.57	0.29	0.72	0.19	0.81	0.00	1.00	0.77	0.23	0.81	0.19	1.00	0.00
3	0.48	0.52	0.61	0.39	0.46	0.54	0.41	0.59	0.17	0.83	0.10	0.90	0.00	1.00	0.80	0.20	0.83	0.17	1.00	0.00

Panel G: Frequency=5min

	IS		CS		MIS		PDS		ILI		MILI		PILI		CILI		CMILI		CPILI	
$1/\beta$	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
1	0.50	0.50	0.48	0.52	0.49	0.51	0.46	0.54	0.54	0.46	0.55	0.45	0.67	0.33	0.54	0.46	0.55	0.45	0.69	0.31
2	0.50	0.50	0.58	0.42	0.48	0.52	0.40	0.60	0.37	0.63	0.31	0.70	0.00	1.00	0.57	0.44	0.53	0.47	0.73	0.27
3	0.50	0.50	0.64	0.36	0.48	0.52	0.40	0.60	0.25	0.75	0.21	0.79	0.00	1.00	0.57	0.43	0.54	0.46	0.73	0.27

Notes: This table reports price discovery measure estimates from the price data simulated from the following 2-market model:

$$p_{1t} = p_{1,t-1} + \delta_1(m_t - p_{1,t-1}) + b_{0,1}^T \eta_t^T,$$
$$p_{2t} = p_{2,t-1} + \delta_2(m_t/\beta - p_{2,t-1}) + b_{0,2}^T \eta_t^T,$$

where $m_t = m_{t-1} + \eta_t^P$, $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are Guassian white noise with diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$. The simulation parameterization is set as $(\delta_1, \delta_2) = (0.9, 0.1)$, $(b_{0,1}^T, b_{0,2}^T) = (0.8, -0.2)$, $\sigma_P^2 = 1$, $\sigma_T^2 = 10$. Numbers shown are the averages of price discovery measure estimates of 1000 samples with a sample size of $N = 21600$.

Table A4: Empirical Bivariate Price Discovery Estimates for other LETFs

This table shows the sample means of price discovery measures for the first product in the bivariate model of any other two LETFs pair (except SPY). ρ stands for correlation coefficient between bivariate VECM residuals (sample averages). $-\beta$ stands for the cointegrating coefficient in the cointegration vector $(1, -\beta)$ (sample averages). Numbers in square brackets are sample standard deviations.

Panel A: UPRO(+3) and SSO(+2)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.58	0.49	0.59	0.60	0.71	0.72	0.73	0.53	0.55	0.58	0.95	0.93	0.89	0.62	0.67	0.71	0.34	-1.50
	[0.22]	[0.20]	[0.23]	[0.24]	[0.11]	[0.12]	[0.15]	[0.13]	[0.14]	[0.17]	[0.22]	[0.25]	[0.32]	[0.49]	[0.47]	[0.45]	[0.12]	[0.02]
5s	0.55	0.51	0.57	0.60	0.59	0.63	0.72	0.43	0.46	0.54	0.71	0.84	0.94	0.39	0.44	0.70	0.61	-1.50
	[0.17]	[0.23]	[0.20]	[0.26]	[0.21]	[0.16]	[0.11]	[0.20]	[0.15]	[0.12]	[0.46]	[0.37]	[0.24]	[0.49]	[0.50]	[0.46]	[0.15]	[0.02]
10s	0.53	0.51	0.55	0.60	0.54	0.59	0.71	0.40	0.42	0.53	0.59	0.71	0.97	0.34	0.37	0.69	0.74	-1.50
	[0.14]	[0.25]	[0.18]	[0.27]	[0.28]	[0.22]	[0.08]	[0.26]	[0.21]	[0.09]	[0.49]	[0.46]	[0.16]	[0.47]	[0.48]	[0.46]	[0.14]	[0.02]
15s	0.52	0.52	0.54	0.60	0.52	0.56	0.70	0.39	0.41	0.52	0.55	0.64	0.99	0.34	0.35	0.68	0.81	-1.50
	[0.11]	[0.26]	[0.17]	[0.29]	[0.31]	[0.26]	[0.07]	[0.29]	[0.25]	[0.07]	[0.50]	[0.48]	[0.09]	[0.47]	[0.48]	[0.47]	[0.12]	[0.02]
30s	0.51	0.51	0.53	0.60	0.50	0.53	0.70	0.40	0.41	0.51	0.51	0.58	1.00	0.37	0.37	0.65	0.89	-1.50
	[0.08]	[0.28]	[0.14]	[0.35]	[0.34]	[0.31]	[0.04]	[0.33]	[0.30]	[0.05]	[0.50]	[0.49]	[0.05]	[0.48]	[0.48]	[0.48]	[0.08]	[0.02]
1min	0.50	0.49	0.52	0.59	0.53	0.56	0.69	0.43	0.45	0.50	0.53	0.58	1.00	0.42	0.42	0.60	0.95	-1.50
	[0.05]	[0.29]	[0.13]	[0.48]	[0.37]	[0.33]	[0.03]	[0.37]	[0.33]	[0.03]	[0.50]	[0.49]	[0.03]	[0.49]	[0.49]	[0.49]	[0.06]	[0.02]
5min	0.50	0.42	0.50	0.57	0.61	0.62	0.69	0.50	0.51	0.50	0.65	0.66	1.00	0.49	0.51	0.52	0.99	-1.50
	[0.03]	[0.24]	[0.19]	[1.77]	[0.32]	[0.33]	[0.03]	[0.33]	[0.33]	[0.03]	[0.48]	[0.47]	[0.05]	[0.50]	[0.50]	[0.50]	[0.02]	[0.07]

Panel B: UPRO(+3) and SPXU(-3)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.63	0.63	0.64	0.64	0.52	0.54	0.56	0.52	0.54	0.56	0.55	0.61	0.67	0.55	0.61	0.67	-0.29	1.00
	[0.23]	[0.22]	[0.24]	[0.25]	[0.11]	[0.12]	[0.14]	[0.11]	[0.12]	[0.14]	[0.50]	[0.49]	[0.47]	[0.50]	[0.49]	[0.47]	[0.12]	[0.05]
5s	0.60	0.62	0.61	0.63	0.43	0.46	0.54	0.43	0.46	0.54	0.37	0.41	0.66	0.38	0.42	0.66	-0.56	1.00
	[0.20]	[0.25]	[0.23]	[0.28]	[0.20]	[0.16]	[0.11]	[0.20]	[0.16]	[0.11]	[0.48]	[0.49]	[0.48]	[0.49]	[0.49]	[0.47]	[0.18]	[0.05]
10s	0.58	0.61	0.60	0.62	0.41	0.43	0.53	0.41	0.43	0.53	0.35	0.38	0.64	0.35	0.39	0.63	-0.69	1.00
	[0.16]	[0.26]	[0.21]	[0.32]	[0.27]	[0.22]	[0.09]	[0.27]	[0.22]	[0.08]	[0.48]	[0.49]	[0.48]	[0.48]	[0.49]	[0.48]	[0.16]	[0.06]
15s	0.56	0.61	0.58	0.61	0.40	0.43	0.52	0.40	0.43	0.52	0.35	0.36	0.62	0.35	0.37	0.62	-0.76	1.00
	[0.14]	[0.27]	[0.20]	[0.36]	[0.29]	[0.25]	[0.07]	[0.29]	[0.25]	[0.07]	[0.48]	[0.48]	[0.48]	[0.48]	[0.48]	[0.48]	[0.15]	[0.05]
30s	0.54	0.60	0.57	0.61	0.39	0.41	0.51	0.39	0.41	0.51	0.36	0.36	0.60	0.36	0.36	0.60	-0.86	1.00
	[0.10]	[0.28]	[0.18]	[0.47]	[0.33]	[0.29]	[0.06]	[0.33]	[0.29]	[0.05]	[0.48]	[0.48]	[0.49]	[0.48]	[0.48]	[0.49]	[0.11]	[0.06]
1min	0.52	0.59	0.55	0.59	0.40	0.41	0.51	0.40	0.41	0.51	0.38	0.38	0.57	0.37	0.37	0.56	-0.92	1.00
	[0.07]	[0.29]	[0.17]	[0.62]	[0.36]	[0.32]	[0.04]	[0.36]	[0.32]	[0.04]	[0.48]	[0.48]	[0.50]	[0.48]	[0.48]	[0.50]	[0.07]	[0.06]
5min	0.50	0.51	0.51	0.49	0.49	0.49	0.50	0.49	0.49	0.50	0.49	0.50	0.49	0.49	0.50	0.50	-0.98	1.00
	[0.04]	[0.26]	[0.21]	[1.67]	[0.33]	[0.35]	[0.05]	[0.33]	[0.34]	[0.04]	[0.50]	[0.50]	[0.50]	[0.50]	[0.50]	[0.50]	[0.02]	[0.07]

Panel C: UPRO(+3) and SDS(-2)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.71	0.62	0.71	0.72	0.73	0.75	0.77	0.56	0.59	0.62	0.98	0.99	0.99	0.67	0.74	0.77	-0.26	1.50
	[0.19]	[0.20]	[0.19]	[0.20]	[0.11]	[0.11]	[0.11]	[0.13]	[0.13]	[0.15]	[0.13]	[0.10]	[0.09]	[0.47]	[0.44]	[0.42]	[0.15]	[0.06]
5s	0.67	0.63	0.69	0.71	0.59	0.65	0.75	0.41	0.47	0.58	0.76	0.88	1.00	0.26	0.39	0.76	-0.50	1.50
	[0.18]	[0.22]	[0.19]	[0.22]	[0.19]	[0.16]	[0.09]	[0.18]	[0.16]	[0.12]	[0.43]	[0.33]	[0.04]	[0.44]	[0.49]	[0.43]	[0.21]	[0.06]
10s	0.64	0.62	0.66	0.71	0.52	0.59	0.74	0.36	0.42	0.56	0.61	0.76	1.00	0.20	0.25	0.74	-0.64	1.50
	[0.16]	[0.23]	[0.19]	[0.27]	[0.23]	[0.20]	[0.07]	[0.21]	[0.19]	[0.10]	[0.49]	[0.43]	[0.04]	[0.40]	[0.43]	[0.44]	[0.21]	[0.07]
15s	0.62	0.62	0.65	0.70	0.48	0.54	0.73	0.33	0.39	0.55	0.52	0.68	1.00	0.20	0.22	0.73	-0.71	1.50
	[0.15]	[0.25]	[0.19]	[0.32]	[0.27]	[0.24]	[0.06]	[0.24]	[0.22]	[0.08]	[0.50]	[0.46]	[0.03]	[0.40]	[0.41]	[0.45]	[0.20]	[0.07]
30s	0.58	0.61	0.62	0.68	0.43	0.48	0.71	0.31	0.35	0.53	0.41	0.52	1.00	0.23	0.23	0.69	-0.82	1.50
	[0.13]	[0.26]	[0.18]	[0.44]	[0.30]	[0.28]	[0.05]	[0.28]	[0.26]	[0.06]	[0.49]	[0.50]	[0.04]	[0.42]	[0.42]	[0.46]	[0.16]	[0.08]
1min	0.55	0.58	0.59	0.67	0.43	0.48	0.70	0.33	0.36	0.52	0.41	0.48	1.00	0.27	0.28	0.64	-0.89	1.50
	[0.10]	[0.28]	[0.17]	[0.59]	[0.34]	[0.31]	[0.04]	[0.32]	[0.30]	[0.05]	[0.49]	[0.50]	[0.04]	[0.45]	[0.45]	[0.48]	[0.11]	[0.07]
5min	0.51	0.47	0.53	0.57	0.56	0.59	0.69	0.45	0.48	0.50	0.59	0.63	1.00	0.43	0.47	0.53	-0.97	1.50
	[0.06]	[0.26]	[0.20]	[1.61]	[0.33]	[0.33]	[0.04]	[0.34]	[0.34]	[0.04]	[0.49]	[0.48]	[0.05]	[0.50]	[0.50]	[0.50]	[0.04]	[0.08]
Panel D: UPRO(+3) and SH(-1)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.91	0.72	0.91	0.92	0.95	0.96	0.97	0.72	0.76	0.80	0.99	1.00	1.00	0.93	0.97	0.99	-0.13	2.99
	[0.11]	[0.18]	[0.11]	[0.11]	[0.08]	[0.06]	[0.03]	[0.14]	[0.12]	[0.10]	[0.08]	[0.06]	[0.02]	[0.25]	[0.17]	[0.11]	[0.07]	[0.11]
5s	0.88	0.75	0.90	0.92	0.80	0.87	0.96	0.46	0.56	0.75	0.88	0.94	1.00	0.53	0.70	0.99	-0.30	2.99
	[0.11]	[0.18]	[0.10]	[0.11]	[0.24]	[0.18]	[0.02]	[0.24]	[0.22]	[0.10]	[0.33]	[0.24]	[0.02]	[0.50]	[0.46]	[0.11]	[0.15]	[0.11]
10s	0.84	0.76	0.88	0.92	0.68	0.78	0.95	0.32	0.44	0.71	0.76	0.87	1.00	0.25	0.46	0.98	-0.41	2.99
	[0.12]	[0.18]	[0.11]	[0.12]	[0.29]	[0.24]	[0.03]	[0.23]	[0.24]	[0.10]	[0.43]	[0.34]	[0.02]	[0.43]	[0.50]	[0.12]	[0.18]	[0.11]
15s	0.81	0.76	0.86	0.92	0.59	0.72	0.95	0.25	0.36	0.68	0.66	0.81	1.00	0.12	0.29	0.98	-0.49	2.99
	[0.12]	[0.18]	[0.12]	[0.13]	[0.30]	[0.27]	[0.03]	[0.21]	[0.24]	[0.09]	[0.47]	[0.39]	[0.02]	[0.33]	[0.45]	[0.14]	[0.19]	[0.11]
30s	0.74	0.75	0.81	0.91	0.45	0.60	0.94	0.15	0.25	0.63	0.48	0.68	1.00	0.05	0.11	0.96	-0.64	2.99
	[0.12]	[0.19]	[0.13]	[0.18]	[0.30]	[0.30]	[0.03]	[0.18]	[0.22]	[0.08]	[0.50]	[0.47]	[0.02]	[0.21]	[0.32]	[0.18]	[0.18]	[0.11]
1min	0.67	0.71	0.75	0.91	0.41	0.56	0.92	0.14	0.23	0.58	0.40	0.62	1.00	0.05	0.11	0.93	-0.77	2.99
	[0.11]	[0.21]	[0.13]	[0.24]	[0.30]	[0.30]	[0.02]	[0.19]	[0.23]	[0.07]	[0.49]	[0.48]	[0.00]	[0.23]	[0.31]	[0.26]	[0.15]	[0.11]
5min	0.55	0.44	0.62	0.84	0.64	0.73	0.90	0.35	0.46	0.52	0.69	0.78	1.00	0.30	0.44	0.64	-0.94	2.98
	[0.08]	[0.26]	[0.20]	[0.85]	[0.32]	[0.31]	[0.03]	[0.32]	[0.34]	[0.06]	[0.46]	[0.41]	[0.03]	[0.46]	[0.50]	[0.48]	[0.06]	[0.76]
Panel E: SSO(+2) and SPXU(-3)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.58	0.67	0.58	0.58	0.30	0.30	0.31	0.48	0.48	0.49	0.03	0.04	0.08	0.41	0.45	0.48	-0.28	0.67
	[0.22]	[0.19]	[0.23]	[0.24]	[0.10]	[0.11]	[0.13]	[0.11]	[0.12]	[0.15]	[0.18]	[0.20]	[0.28]	[0.49]	[0.50]	[0.50]	[0.12]	[0.03]
5s	0.56	0.65	0.57	0.57	0.30	0.30	0.31	0.46	0.48	0.50	0.08	0.05	0.03	0.35	0.35	0.48	-0.54	0.67
	[0.19]	[0.22]	[0.21]	[0.26]	[0.16]	[0.12]	[0.09]	[0.17]	[0.13]	[0.11]	[0.28]	[0.22]	[0.18]	[0.48]	[0.48]	[0.50]	[0.17]	[0.03]
10s	0.55	0.64	0.56	0.57	0.30	0.31	0.32	0.45	0.47	0.50	0.16	0.10	0.02	0.36	0.37	0.51	-0.68	0.67
	[0.16]	[0.24]	[0.20]	[0.29]	[0.23]	[0.18]	[0.07]	[0.24]	[0.19]	[0.08]	[0.37]	[0.30]	[0.13]	[0.48]	[0.48]	[0.50]	[0.17]	[0.03]
15s	0.54	0.64	0.55	0.57	0.31	0.30	0.32	0.43	0.45	0.51	0.21	0.14	0.01	0.37	0.37	0.51	-0.75	0.67
	[0.14]	[0.25]	[0.19]	[0.33]	[0.26]	[0.21]	[0.06]	[0.28]	[0.23]	[0.07]	[0.41]	[0.35]	[0.10]	[0.48]	[0.48]	[0.50]	[0.15]	[0.03]
30s	0.53	0.64	0.54	0.56	0.31	0.31	0.31	0.42	0.44	0.51	0.25	0.21	0.00	0.38	0.37	0.52	-0.86	0.67
	[0.10]	[0.26]	[0.17]	[0.42]	[0.30]	[0.27]	[0.05]	[0.32]	[0.28]	[0.05]	[0.43]	[0.41]	[0.07]	[0.49]	[0.48]	[0.50]	[0.11]	[0.03]
1min	0.52	0.63	0.53	0.56	0.33	0.32	0.31	0.42	0.43	0.50	0.29	0.26	0.00	0.40	0.40	0.50	-0.92	0.67
	[0.07]	[0.28]	[0.16]	[0.54]	[0.34]	[0.31]	[0.04]	[0.36]	[0.32]	[0.04]	[0.45]	[0.44]	[0.04]	[0.49]	[0.49]	[0.50]	[0.08]	[0.12]
5min	0.50	0.59	0.51	0.48	0.37	0.38	0.31	0.48	0.48	0.50	0.32	0.34	0.01	0.47	0.48	0.49	-0.98	0.67
	[0.04]	[0.25]	[0.21]	[1.61]	[0.32]	[0.33]	[0.05]	[0.33]	[0.34]	[0.04]	[0.47]	[0.47]	[0.07]	[0.50]	[0.50]	[0.50]	[0.02]	[0.05]

Panel F: SSO(+2) and SDS(-2)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.64	0.64	0.64	0.64	0.53	0.54	0.55	0.53	0.54	0.55	0.55	0.57	0.58	0.54	0.56	0.58	-0.24	1.00
	[0.25]	[0.23]	[0.26]	[0.26]	[0.13]	[0.15]	[0.17]	[0.13]	[0.15]	[0.17]	[0.50]	[0.50]	[0.49]	[0.50]	[0.50]	[0.49]	[0.13]	[0.03]
5s	0.62	0.63	0.63	0.63	0.47	0.50	0.55	0.47	0.50	0.55	0.34	0.43	0.58	0.35	0.42	0.58	-0.48	1.00
	[0.22]	[0.25]	[0.24]	[0.28]	[0.17]	[0.13]	[0.13]	[0.17]	[0.13]	[0.13]	[0.47]	[0.49]	[0.49]	[0.48]	[0.49]	[0.49]	[0.20]	[0.03]
10s	0.61	0.63	0.62	0.63	0.42	0.46	0.54	0.42	0.46	0.54	0.32	0.33	0.60	0.32	0.33	0.59	-0.62	1.00
	[0.20]	[0.27]	[0.23]	[0.31]	[0.24]	[0.19]	[0.10]	[0.24]	[0.19]	[0.10]	[0.46]	[0.47]	[0.49]	[0.47]	[0.47]	[0.49]	[0.20]	[0.03]
15s	0.60	0.63	0.61	0.62	0.40	0.43	0.53	0.40	0.43	0.53	0.32	0.32	0.60	0.32	0.32	0.59	-0.70	1.00
	[0.18]	[0.27]	[0.22]	[0.33]	[0.27]	[0.23]	[0.09]	[0.27]	[0.23]	[0.09]	[0.47]	[0.47]	[0.49]	[0.47]	[0.47]	[0.49]	[0.19]	[0.03]
30s	0.57	0.62	0.59	0.62	0.37	0.40	0.52	0.38	0.40	0.52	0.33	0.33	0.61	0.33	0.33	0.59	-0.81	1.00
	[0.14]	[0.28]	[0.20]	[0.42]	[0.32]	[0.28]	[0.07]	[0.32]	[0.28]	[0.06]	[0.47]	[0.47]	[0.49]	[0.47]	[0.47]	[0.49]	[0.16]	[0.04]
1min	0.55	0.61	0.57	0.61	0.38	0.40	0.51	0.38	0.40	0.51	0.34	0.34	0.59	0.34	0.35	0.58	-0.89	1.00
	[0.10]	[0.29]	[0.17]	[0.54]	[0.35]	[0.32]	[0.05]	[0.35]	[0.32]	[0.05]	[0.47]	[0.48]	[0.49]	[0.48]	[0.48]	[0.49]	[0.11]	[0.04]
5min	0.51	0.54	0.53	0.55	0.45	0.48	0.50	0.45	0.48	0.50	0.44	0.47	0.51	0.44	0.47	0.50	-0.97	1.00
	[0.05]	[0.27]	[0.20]	[1.46]	[0.34]	[0.34]	[0.05]	[0.34]	[0.34]	[0.04]	[0.50]	[0.50]	[0.50]	[0.50]	[0.50]	[0.50]	[0.04]	[0.05]

Panel G: SSO(+2) and SH(-1)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.87	0.75	0.88	0.88	0.89	0.90	0.91	0.69	0.71	0.74	0.99	0.99	1.00	0.90	0.93	0.95	-0.13	2.00
	[0.15]	[0.18]	[0.15]	[0.15]	[0.09]	[0.08]	[0.07]	[0.15]	[0.14]	[0.14]	[0.09]	[0.07]	[0.04]	[0.30]	[0.26]	[0.23]	[0.07]	[0.06]
5s	0.85	0.77	0.87	0.88	0.74	0.81	0.90	0.48	0.57	0.71	0.88	0.95	1.00	0.53	0.70	0.96	-0.29	2.00
	[0.14]	[0.18]	[0.14]	[0.15]	[0.21]	[0.16]	[0.06]	[0.21]	[0.19]	[0.13]	[0.32]	[0.22]	[0.00]	[0.50]	[0.46]	[0.20]	[0.15]	[0.06]
10s	0.82	0.77	0.85	0.88	0.60	0.71	0.89	0.35	0.45	0.68	0.71	0.85	1.00	0.26	0.46	0.96	-0.41	2.00
	[0.14]	[0.19]	[0.15]	[0.16]	[0.26]	[0.22]	[0.05]	[0.22]	[0.22]	[0.11]	[0.45]	[0.36]	[0.00]	[0.44]	[0.50]	[0.21]	[0.18]	[0.06]
15s	0.79	0.78	0.83	0.88	0.51	0.63	0.88	0.27	0.38	0.66	0.57	0.77	1.00	0.14	0.30	0.96	-0.49	2.00
	[0.14]	[0.19]	[0.15]	[0.17]	[0.27]	[0.25]	[0.05]	[0.21]	[0.22]	[0.11]	[0.50]	[0.42]	[0.00]	[0.35]	[0.46]	[0.20]	[0.19]	[0.06]
30s	0.73	0.78	0.79	0.89	0.37	0.51	0.86	0.18	0.27	0.62	0.35	0.57	1.00	0.05	0.13	0.95	-0.64	2.00
	[0.14]	[0.19]	[0.14]	[0.19]	[0.27]	[0.27]	[0.04]	[0.19]	[0.22]	[0.09]	[0.48]	[0.50]	[0.00]	[0.22]	[0.33]	[0.22]	[0.18]	[0.06]
1min	0.66	0.75	0.74	0.89	0.31	0.44	0.84	0.15	0.24	0.58	0.24	0.46	1.00	0.06	0.11	0.91	-0.77	2.00
	[0.12]	[0.20]	[0.14]	[0.24]	[0.27]	[0.29]	[0.04]	[0.19]	[0.23]	[0.07]	[0.43]	[0.50]	[0.00]	[0.25]	[0.32]	[0.29]	[0.15]	[0.06]
5min	0.55	0.51	0.62	0.84	0.54	0.65	0.81	0.36	0.47	0.52	0.56	0.69	1.00	0.30	0.45	0.63	-0.94	1.99
	[0.08]	[0.26]	[0.19]	[0.76]	[0.34]	[0.33]	[0.04]	[0.33]	[0.34]	[0.06]	[0.50]	[0.46]	[0.04]	[0.46]	[0.50]	[0.48]	[0.06]	[0.29]

Panel H: SPXU(-3) and SDS(-2)

	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.59	0.50	0.59	0.60	0.73	0.74	0.74	0.55	0.56	0.57	0.99	0.99	0.99	0.78	0.78	0.75	0.23	-1.50
	[0.23]	[0.23]	[0.24]	[0.24]	[0.07]	[0.08]	[0.09]	[0.09]	[0.09]	[0.11]	[0.09]	[0.09]	[0.10]	[0.42]	[0.42]	[0.43]	[0.14]	[0.02]
5s	0.59	0.51	0.59	0.60	0.68	0.70	0.73	0.50	0.52	0.55	0.91	0.98	1.00	0.48	0.60	0.73	0.47	-1.50
	[0.21]	[0.24]	[0.22]	[0.25]	[0.13]	[0.09]	[0.07]	[0.14]	[0.10]	[0.09]	[0.29]	[0.16]	[0.07]	[0.50]	[0.49]	[0.44]	[0.21]	[0.02]
10s	0.57	0.51	0.58	0.60	0.63	0.66	0.72	0.46	0.49	0.53	0.77	0.89	1.00	0.41	0.46	0.72	0.60	-1.50
	[0.18]	[0.25]	[0.21]	[0.26]	[0.20]	[0.15]	[0.06]	[0.20]	[0.15]	[0.07]	[0.42]	[0.31]	[0.04]	[0.49]	[0.50]	[0.45]	[0.21]	[0.02]
15s	0.56	0.51	0.58	0.59	0.59	0.63	0.71	0.44	0.46	0.53	0.67	0.82	1.00	0.39	0.42	0.71	0.68	-1.50
	[0.16]	[0.25]	[0.20]	[0.27]	[0.24]	[0.19]	[0.05]	[0.24]	[0.19]	[0.06]	[0.47]	[0.39]	[0.05]	[0.49]	[0.49]	[0.45]	[0.20]	[0.02]
30s	0.55	0.52	0.56	0.59	0.54	0.58	0.70	0.42	0.44	0.52	0.55	0.66	1.00	0.38	0.38	0.68	0.79	-1.50
	[0.13]	[0.27]	[0.18]	[0.31]	[0.30]	[0.25]	[0.04]	[0.30]	[0.25]	[0.05]	[0.50]	[0.47]	[0.04]	[0.48]	[0.49]	[0.47]	[0.16]	[0.02]
1min	0.53	0.51	0.55	0.58	0.51	0.55	0.70	0.41	0.43	0.51	0.51	0.57	1.00	0.39	0.39	0.60	0.88	-1.50
	[0.10]	[0.29]	[0.16]	[0.41]	[0.35]	[0.31]	[0.04]	[0.35]	[0.31]	[0.04]	[0.50]	[0.49]	[0.03]	[0.49]	[0.49]	[0.49]	[0.12]	[0.03]
5min	0.51	0.46	0.53	0.59	0.57	0.60	0.69	0.47	0.49	0.50	0.58	0.63	1.00	0.45	0.50	0.52	0.97	-1.50
	[0.06]	[0.27]	[0.19]	[1.40]	[0.34]	[0.33]	[0.04]	[0.35]	[0.34]	[0.04]	[0.49]	[0.48]	[0.04]	[0.50]	[0.50]	[0.50]	[0.04]	[0.06]

Panel I: SPXU(-3) and SH(-1)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.85	0.59	0.85	0.85	0.95	0.96	0.96	0.71	0.73	0.75	1.00	1.00	1.00	0.97	0.98	0.99	0.12	-2.99
	[0.13]	[0.18]	[0.13]	[0.13]	[0.03]	[0.02]	[0.02]	[0.10]	[0.10]	[0.10]	[0.03]	[0.02]	[0.00]	[0.18]	[0.13]	[0.11]	[0.08]	[0.06]
5s	0.83	0.64	0.85	0.87	0.86	0.91	0.96	0.52	0.60	0.72	0.95	0.98	1.00	0.65	0.79	0.99	0.29	-2.99
	[0.12]	[0.19]	[0.12]	[0.12]	[0.17]	[0.12]	[0.02]	[0.20]	[0.17]	[0.10]	[0.22]	[0.15]	[0.00]	[0.48]	[0.41]	[0.10]	[0.16]	[0.06]
10s	0.80	0.66	0.83	0.87	0.76	0.83	0.95	0.38	0.47	0.68	0.85	0.93	1.00	0.39	0.56	0.99	0.41	-2.99
	[0.12]	[0.20]	[0.12]	[0.14]	[0.24]	[0.19]	[0.02]	[0.22]	[0.20]	[0.09]	[0.36]	[0.26]	[0.00]	[0.49]	[0.50]	[0.11]	[0.19]	[0.07]
15s	0.77	0.67	0.81	0.87	0.67	0.77	0.94	0.31	0.40	0.66	0.76	0.87	1.00	0.23	0.41	0.98	0.49	-2.99
	[0.12]	[0.20]	[0.12]	[0.14]	[0.28]	[0.23]	[0.02]	[0.21]	[0.21]	[0.09]	[0.43]	[0.33]	[0.00]	[0.42]	[0.49]	[0.14]	[0.19]	[0.07]
30s	0.71	0.67	0.77	0.87	0.55	0.67	0.93	0.21	0.30	0.61	0.61	0.77	1.00	0.09	0.18	0.97	0.63	-2.99
	[0.12]	[0.21]	[0.13]	[0.18]	[0.31]	[0.28]	[0.02]	[0.19]	[0.21]	[0.08]	[0.49]	[0.42]	[0.00]	[0.28]	[0.38]	[0.18]	[0.18]	[0.07]
1min	0.64	0.65	0.72	0.86	0.48	0.60	0.92	0.17	0.25	0.57	0.50	0.68	1.00	0.06	0.11	0.92	0.76	-2.99
	[0.11]	[0.22]	[0.13]	[0.24]	[0.31]	[0.30]	[0.02]	[0.18]	[0.21]	[0.06]	[0.50]	[0.46]	[0.00]	[0.24]	[0.32]	[0.27]	[0.15]	[0.07]
5min	0.54	0.44	0.61	0.84	0.64	0.73	0.90	0.35	0.45	0.52	0.69	0.80	1.00	0.29	0.43	0.64	0.93	-3.00
	[0.08]	[0.26]	[0.19]	[0.80]	[0.32]	[0.30]	[0.02]	[0.32]	[0.33]	[0.06]	[0.46]	[0.40]	[0.02]	[0.45]	[0.50]	[0.48]	[0.07]	[0.20]
Panel J: SDS(-2) and SH(-1)																		
	IS	CS	MIS	PDS	ILS	MILS	PILS	CILS	CMILS	CPILS	ILI	MILI	PILI	CILI	CMILI	CPILI	ρ	$-\beta$
1s	0.78	0.59	0.78	0.78	0.88	0.89	0.90	0.67	0.68	0.70	1.00	1.00	1.00	0.95	0.97	0.98	0.12	-1.99
	[0.17]	[0.20]	[0.17]	[0.17]	[0.06]	[0.05]	[0.04]	[0.11]	[0.10]	[0.10]	[0.03]	[0.00]	[0.00]	[0.23]	[0.18]	[0.15]	[0.08]	[0.04]
5s	0.77	0.63	0.78	0.80	0.78	0.82	0.89	0.53	0.58	0.68	0.90	0.96	1.00	0.69	0.77	0.98	0.28	-1.99
	[0.16]	[0.20]	[0.16]	[0.17]	[0.19]	[0.14]	[0.04]	[0.20]	[0.16]	[0.09]	[0.30]	[0.19]	[0.02]	[0.46]	[0.42]	[0.15]	[0.16]	[0.03]
10s	0.74	0.65	0.77	0.81	0.67	0.74	0.88	0.42	0.49	0.65	0.78	0.87	1.00	0.49	0.61	0.98	0.40	-1.99
	[0.15]	[0.21]	[0.15]	[0.17]	[0.26]	[0.21]	[0.04]	[0.22]	[0.20]	[0.09]	[0.41]	[0.34]	[0.02]	[0.50]	[0.49]	[0.15]	[0.19]	[0.03]
15s	0.72	0.66	0.76	0.81	0.60	0.69	0.87	0.35	0.43	0.63	0.70	0.81	1.00	0.35	0.47	0.97	0.48	-1.99
	[0.14]	[0.21]	[0.15]	[0.18]	[0.28]	[0.24]	[0.04]	[0.22]	[0.21]	[0.08]	[0.46]	[0.39]	[0.02]	[0.48]	[0.50]	[0.18]	[0.20]	[0.04]
30s	0.67	0.67	0.72	0.81	0.49	0.59	0.85	0.26	0.33	0.60	0.53	0.70	1.00	0.18	0.27	0.94	0.63	-2.00
	[0.13]	[0.21]	[0.14]	[0.20]	[0.29]	[0.27]	[0.04]	[0.21]	[0.21]	[0.07]	[0.50]	[0.46]	[0.00]	[0.38]	[0.45]	[0.23]	[0.19]	[0.05]
1min	0.62	0.65	0.68	0.81	0.43	0.53	0.83	0.22	0.29	0.56	0.43	0.61	1.00	0.13	0.18	0.88	0.76	-1.99
	[0.11]	[0.22]	[0.14]	[0.25]	[0.30]	[0.28]	[0.03]	[0.21]	[0.21]	[0.06]	[0.50]	[0.49]	[0.00]	[0.34]	[0.38]	[0.32]	[0.16]	[0.07]
5min	0.53	0.48	0.59	0.78	0.57	0.65	0.81	0.38	0.46	0.52	0.60	0.72	1.00	0.34	0.46	0.63	0.93	-2.00
	[0.08]	[0.27]	[0.19]	[0.80]	[0.33]	[0.32]	[0.04]	[0.33]	[0.33]	[0.06]	[0.49]	[0.45]	[0.04]	[0.47]	[0.50]	[0.48]	[0.07]	[0.12]

Table A5: Empirical Multivariate Price Discovery Estimates for All LETFs

This table shows the sample means of price discovery measures for each product in the multivariate model of all these six LETFs together. Numbers in square brackets are sample standard deviations.

Panel A: IS, MIS, PDS, and CS

IS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.39	0.23	0.16	0.12	0.09	0.02
	[0.23]	[0.15]	[0.11]	[0.10]	[0.08]	[0.02]
5s	0.34	0.22	0.17	0.14	0.11	0.03
	[0.18]	[0.11]	[0.09]	[0.08]	[0.07]	[0.03]
10s	0.30	0.21	0.18	0.15	0.12	0.04
	[0.14]	[0.08]	[0.07]	[0.06]	[0.07]	[0.04]
15s	0.28	0.20	0.18	0.15	0.13	0.05
	[0.12]	[0.06]	[0.06]	[0.05]	[0.06]	[0.04]
30s	0.24	0.19	0.18	0.16	0.14	0.08
	[0.08]	[0.04]	[0.04]	[0.04]	[0.05]	[0.04]
1min	0.21	0.18	0.18	0.17	0.15	0.11
	[0.06]	[0.03]	[0.03]	[0.03]	[0.04]	[0.04]
5min	0.17	0.17	0.17	0.17	0.17	0.16
	[0.09]	[0.09]	[0.09]	[0.10]	[0.10]	[0.12]

MIS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.40	0.23	0.15	0.12	0.08	0.01
	[0.24]	[0.17]	[0.12]	[0.10]	[0.08]	[0.02]
5s	0.39	0.21	0.16	0.13	0.10	0.02
	[0.22]	[0.14]	[0.11]	[0.09]	[0.08]	[0.03]
10s	0.37	0.20	0.16	0.13	0.11	0.03
	[0.19]	[0.11]	[0.09]	[0.08]	[0.08]	[0.03]
15s	0.36	0.19	0.16	0.14	0.11	0.04
	[0.16]	[0.09]	[0.08]	[0.07]	[0.07]	[0.04]
30s	0.32	0.18	0.17	0.15	0.12	0.06
	[0.13]	[0.08]	[0.07]	[0.07]	[0.07]	[0.05]
1min	0.28	0.17	0.17	0.15	0.14	0.08
	[0.11]	[0.08]	[0.08]	[0.07]	[0.07]	[0.05]
5min	0.17	0.16	0.16	0.17	0.17	0.17
	[0.18]	[0.18]	[0.18]	[0.19]	[0.18]	[0.19]

PDS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.43	0.23	0.14	0.11	0.08	0.01
	[0.27]	[0.19]	[0.13]	[0.11]	[0.09]	[0.02]
5s	0.50	0.20	0.12	0.10	0.07	0.01
	[0.31]	[0.22]	[0.15]	[0.14]	[0.11]	[0.03]
10s	0.56	0.17	0.10	0.09	0.06	0.01
	[0.33]	[0.25]	[0.17]	[0.17]	[0.14]	[0.06]
15s	0.62	0.14	0.09	0.09	0.06	0.01
	[0.34]	[0.27]	[0.19]	[0.19]	[0.16]	[0.07]
30s	0.68	0.11	0.07	0.07	0.06	0.01
	[0.39]	[0.35]	[0.25]	[0.25]	[0.24]	[0.11]
1min	0.74	0.05	0.06	0.07	0.07	0.01
	[0.58]	[0.59]	[0.44]	[0.46]	[0.40]	[0.17]
5min	0.33	0.00	0.18	0.20	0.16	0.13
	[2.74]	[2.43]	[2.08]	[1.85]	[1.81]	[0.88]

CS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.41	0.17	0.17	0.09	0.10	0.05
	[0.25]	[0.13]	[0.13]	[0.08]	[0.09]	[0.05]
5s	0.55	0.13	0.13	0.07	0.08	0.05
	[0.29]	[0.14]	[0.13]	[0.08]	[0.09]	[0.05]
10s	0.60	0.11	0.11	0.06	0.07	0.05
	[0.29]	[0.14]	[0.13]	[0.08]	[0.09]	[0.05]
15s	0.62	0.10	0.10	0.06	0.07	0.05
	[0.28]	[0.12]	[0.12]	[0.08]	[0.08]	[0.06]
30s	0.60	0.09	0.10	0.06	0.07	0.07
	[0.25]	[0.11]	[0.11]	[0.07]	[0.08]	[0.06]
1min	0.52	0.10	0.13	0.07	0.09	0.09
	[0.23]	[0.10]	[0.12]	[0.08]	[0.09]	[0.08]
5min	0.35	0.12	0.17	0.10	0.12	0.13
	[0.19]	[0.09]	[0.12]	[0.08]	[0.10]	[0.11]

Panel B: ILS, MILS, PILS

ILS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.15	0.29	0.14	0.27	0.11	0.05
	[0.10]	[0.13]	[0.12]	[0.13]	[0.11]	[0.14]
5s	0.03	0.24	0.18	0.31	0.17	0.07
	[0.07]	[0.24]	[0.24]	[0.27]	[0.22]	[0.18]
10s	0.02	0.24	0.18	0.30	0.18	0.08
	[0.08]	[0.28]	[0.26]	[0.30]	[0.26]	[0.19]
15s	0.02	0.24	0.18	0.28	0.19	0.08
	[0.09]	[0.29]	[0.26]	[0.31]	[0.27]	[0.20]
30s	0.02	0.23	0.17	0.28	0.20	0.10
	[0.09]	[0.29]	[0.26]	[0.31]	[0.28]	[0.21]
1min	0.03	0.23	0.16	0.28	0.19	0.12
	[0.12]	[0.30]	[0.26]	[0.32]	[0.27]	[0.22]
5min	0.05	0.20	0.12	0.26	0.20	0.16
	[0.15]	[0.26]	[0.21]	[0.27]	[0.25]	[0.23]

MILS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.17	0.30	0.13	0.26	0.10	0.04
	[0.10]	[0.12]	[0.10]	[0.11]	[0.10]	[0.12]
5s	0.04	0.26	0.18	0.31	0.16	0.06
	[0.06]	[0.22]	[0.22]	[0.25]	[0.20]	[0.16]
10s	0.03	0.25	0.18	0.30	0.18	0.07
	[0.07]	[0.28]	[0.25]	[0.29]	[0.25]	[0.17]
15s	0.02	0.25	0.18	0.29	0.19	0.07
	[0.08]	[0.29]	[0.25]	[0.30]	[0.26]	[0.18]
30s	0.02	0.24	0.17	0.28	0.19	0.08
	[0.09]	[0.30]	[0.26]	[0.30]	[0.27]	[0.19]
1min	0.03	0.24	0.16	0.28	0.19	0.10
	[0.12]	[0.31]	[0.26]	[0.32]	[0.27]	[0.20]
5min	0.05	0.17	0.12	0.26	0.21	0.20
	[0.14]	[0.26]	[0.21]	[0.31]	[0.28]	[0.28]

PILS

	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.22	0.33	0.11	0.24	0.08	0.01
	[0.09]	[0.11]	[0.06]	[0.08]	[0.04]	[0.01]
5s	0.09	0.35	0.14	0.30	0.11	0.01
	[0.03]	[0.08]	[0.05]	[0.07]	[0.04]	[0.01]
10s	0.07	0.35	0.14	0.31	0.12	0.02
	[0.02]	[0.06]	[0.04]	[0.05]	[0.03]	[0.01]
15s	0.06	0.34	0.14	0.31	0.13	0.02
	[0.02]	[0.05]	[0.03]	[0.05]	[0.03]	[0.01]
30s	0.05	0.34	0.14	0.32	0.13	0.02
	[0.01]	[0.04]	[0.02]	[0.04]	[0.03]	[0.01]
1min	0.04	0.33	0.14	0.32	0.14	0.03
	[0.01]	[0.03]	[0.01]	[0.03]	[0.02]	[0.01]
5min	0.04	0.30	0.14	0.31	0.15	0.06
	[0.04]	[0.11]	[0.07]	[0.13]	[0.11]	[0.08]

Panel C: CILS, CMILS, CPILS

	CILS						CMILS						CPILS					
	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.44	0.12	0.12	0.11	0.10	0.10	0.50	0.12	0.11	0.10	0.09	0.08	0.62	0.12	0.09	0.09	0.07	0.03
	[0.21]	[0.11]	[0.13]	[0.10]	[0.12]	[0.19]	[0.20]	[0.09]	[0.11]	[0.09]	[0.10]	[0.16]	[0.12]	[0.06]	[0.05]	[0.04]	[0.03]	[0.02]
5s	0.11	0.15	0.20	0.20	0.19	0.15	0.15	0.16	0.20	0.19	0.18	0.13	0.37	0.17	0.14	0.14	0.12	0.06
	[0.15]	[0.21]	[0.25]	[0.24]	[0.23]	[0.25]	[0.15]	[0.19]	[0.23]	[0.23]	[0.22]	[0.22]	[0.09]	[0.05]	[0.05]	[0.04]	[0.04]	[0.03]
10s	0.07	0.17	0.20	0.21	0.20	0.17	0.09	0.18	0.20	0.20	0.19	0.14	0.29	0.18	0.16	0.16	0.14	0.07
	[0.14]	[0.25]	[0.26]	[0.27]	[0.27]	[0.26]	[0.14]	[0.24]	[0.26]	[0.26]	[0.26]	[0.24]	[0.07]	[0.04]	[0.04]	[0.03]	[0.04]	[0.03]
15s	0.05	0.17	0.19	0.20	0.21	0.17	0.06	0.18	0.20	0.20	0.21	0.15	0.26	0.18	0.17	0.16	0.15	0.09
	[0.13]	[0.26]	[0.27]	[0.27]	[0.28]	[0.27]	[0.13]	[0.26]	[0.26]	[0.27]	[0.27]	[0.25]	[0.06]	[0.03]	[0.03]	[0.03]	[0.03]	[0.04]
30s	0.05	0.16	0.18	0.19	0.21	0.20	0.06	0.17	0.19	0.19	0.21	0.17	0.22	0.18	0.17	0.17	0.16	0.11
	[0.14]	[0.26]	[0.27]	[0.27]	[0.29]	[0.29]	[0.14]	[0.26]	[0.27]	[0.27]	[0.28]	[0.27]	[0.04]	[0.02]	[0.02]	[0.02]	[0.03]	[0.03]
1min	0.06	0.16	0.16	0.19	0.19	0.24	0.08	0.17	0.17	0.19	0.19	0.21	0.19	0.17	0.17	0.17	0.16	0.13
	[0.17]	[0.26]	[0.26]	[0.27]	[0.27]	[0.30]	[0.17]	[0.26]	[0.26]	[0.27]	[0.27]	[0.28]	[0.02]	[0.02]	[0.02]	[0.02]	[0.03]	[0.03]
5min	0.11	0.12	0.11	0.15	0.18	0.34	0.10	0.10	0.11	0.16	0.19	0.34	0.16	0.16	0.16	0.17	0.17	0.19
	[0.21]	[0.21]	[0.20]	[0.22]	[0.25]	[0.30]	[0.21]	[0.21]	[0.21]	[0.25]	[0.27]	[0.36]	[0.08]	[0.08]	[0.08]	[0.10]	[0.11]	[0.16]

Panel D: ILI, MILI, PILI

	ILI						MILI						PILI					
	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.09	0.45	0.06	0.32	0.04	0.05	0.11	0.50	0.04	0.29	0.03	0.03	0.19	0.54	0.02	0.25	0.00	0.00
	[0.28]	[0.50]	[0.23]	[0.47]	[0.19]	[0.21]	[0.32]	[0.50]	[0.19]	[0.45]	[0.16]	[0.17]	[0.39]	[0.50]	[0.15]	[0.43]	[0.02]	[0.00]
5s	0.01	0.25	0.15	0.38	0.14	0.07	0.01	0.29	0.14	0.39	0.12	0.06	0.00	0.67	0.00	0.33	0.00	0.00
	[0.09]	[0.43]	[0.36]	[0.48]	[0.34]	[0.26]	[0.08]	[0.45]	[0.34]	[0.49]	[0.33]	[0.23]	[0.00]	[0.47]	[0.06]	[0.47]	[0.00]	[0.00]
10s	0.01	0.24	0.17	0.33	0.17	0.08	0.01	0.26	0.16	0.35	0.16	0.06	0.00	0.66	0.00	0.34	0.00	0.00
	[0.11]	[0.43]	[0.38]	[0.47]	[0.37]	[0.27]	[0.10]	[0.44]	[0.37]	[0.48]	[0.36]	[0.24]	[0.00]	[0.47]	[0.03]	[0.47]	[0.03]	[0.00]
15s	0.01	0.24	0.18	0.30	0.19	0.08	0.01	0.26	0.17	0.32	0.18	0.06	0.00	0.64	0.00	0.36	0.00	0.00
	[0.12]	[0.43]	[0.38]	[0.46]	[0.39]	[0.27]	[0.10]	[0.44]	[0.38]	[0.46]	[0.39]	[0.24]	[0.00]	[0.48]	[0.02]	[0.48]	[0.00]	[0.00]
30s	0.02	0.23	0.16	0.29	0.20	0.09	0.02	0.26	0.16	0.30	0.19	0.07	0.00	0.62	0.00	0.38	0.00	0.00
	[0.13]	[0.42]	[0.37]	[0.45]	[0.40]	[0.29]	[0.13]	[0.44]	[0.37]	[0.46]	[0.39]	[0.26]	[0.00]	[0.49]	[0.00]	[0.49]	[0.02]	[0.00]
1min	0.03	0.24	0.15	0.29	0.19	0.11	0.03	0.26	0.15	0.30	0.18	0.09	0.00	0.58	0.00	0.42	0.00	0.00
	[0.16]	[0.43]	[0.36]	[0.45]	[0.39]	[0.31]	[0.16]	[0.44]	[0.36]	[0.46]	[0.39]	[0.28]	[0.00]	[0.49]	[0.00]	[0.49]	[0.02]	[0.00]
5min	0.05	0.21	0.11	0.29	0.20	0.15	0.04	0.17	0.10	0.27	0.21	0.21	0.00	0.43	0.02	0.47	0.05	0.02
	[0.21]	[0.41]	[0.31]	[0.45]	[0.40]	[0.35]	[0.20]	[0.38]	[0.30]	[0.45]	[0.41]	[0.40]	[0.04]	[0.50]	[0.14]	[0.50]	[0.22]	[0.14]

Panel E: CILI, CMILI, CPILI

	CILI						CMILI						CPILI					
	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)	SPY(+1)	UPRO(+3)	SSO(+2)	SPXU(-3)	SDS(-2)	SH(-1)
1s	0.75	0.03	0.05	0.04	0.04	0.08	0.83	0.02	0.04	0.03	0.03	0.06	1.00	0.00	0.00	0.00	0.00	0.00
	[0.43]	[0.17]	[0.22]	[0.19]	[0.20]	[0.28]	[0.37]	[0.15]	[0.19]	[0.16]	[0.16]	[0.23]	[0.02]	[0.00]	[0.00]	[0.00]	[0.02]	[0.00]
5s	0.15	0.13	0.19	0.20	0.18	0.16	0.23	0.12	0.17	0.19	0.17	0.13	0.98	0.01	0.00	0.00	0.00	0.00
	[0.36]	[0.33]	[0.39]	[0.40]	[0.38]	[0.36]	[0.42]	[0.32]	[0.38]	[0.39]	[0.37]	[0.34]	[0.13]	[0.08]	[0.04]	[0.07]	[0.05]	[0.02]
10s	0.06	0.16	0.19	0.21	0.20	0.17	0.08	0.17	0.19	0.21	0.19	0.15	0.96	0.02	0.00	0.01	0.00	0.00
	[0.24]	[0.37]	[0.39]	[0.41]	[0.40]	[0.38]	[0.27]	[0.38]	[0.39]	[0.41]	[0.40]	[0.36]	[0.19]	[0.13]	[0.07]	[0.10]	[0.06]	[0.04]
15s	0.04	0.17	0.20	0.20	0.21	0.18	0.06	0.17	0.20	0.20	0.21	0.16	0.94	0.02	0.01	0.02	0.01	0.00
	[0.20]	[0.38]	[0.40]	[0.40]	[0.41]	[0.39]	[0.23]	[0.38]	[0.40]	[0.40]	[0.41]	[0.37]	[0.23]	[0.14]	[0.10]	[0.13]	[0.08]	[0.06]
30s	0.04	0.16	0.18	0.19	0.23	0.21	0.04	0.18	0.19	0.19	0.22	0.18	0.88	0.04	0.02	0.04	0.02	0.01
	[0.20]	[0.36]	[0.38]	[0.39]	[0.42]	[0.41]	[0.20]	[0.38]	[0.39]	[0.40]	[0.41]	[0.38]	[0.32]	[0.20]	[0.12]	[0.19]	[0.13]	[0.07]
1min	0.06	0.16	0.15	0.19	0.19	0.26	0.06	0.17	0.17	0.19	0.18	0.22	0.71	0.07	0.05	0.10	0.05	0.02
	[0.24]	[0.36]	[0.36]	[0.39]	[0.39]	[0.44]	[0.24]	[0.38]	[0.38]	[0.39]	[0.39]	[0.41]	[0.45]	[0.26]	[0.22]	[0.29]	[0.21]	[0.15]
5min	0.09	0.10	0.09	0.13	0.16	0.41	0.09	0.09	0.10	0.15	0.19	0.38	0.12	0.11	0.12	0.16	0.18	0.30
	[0.29]	[0.30]	[0.29]	[0.34]	[0.37]	[0.49]	[0.28]	[0.29]	[0.30]	[0.35]	[0.39]	[0.49]	[0.33]	[0.31]	[0.32]	[0.37]	[0.39]	[0.46]