Misguided Price Discovery: When Overshooting Is Mistaken for Leadership

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Highlights

1. Challenges magnitude-based price discovery measures that equate aggressive reactions with informational efficiency.

2. Proposes the *Instantaneous Pricing Error Rule*, defining leadership by accuracy relative to the long-run efficient price.

3. Introduces the *Instantaneous Pricing Efficiency Share (IPES)*, a continuous and scale-free accuracy-based measure.

4. Demonstrates that IPES penalizes both underreaction and overshooting within a structural moving-average framework.

5. Shows, through simulations and crisis episodes, that IPES reliably identifies leadership during market stress.

1 Introduction

Understanding which market leads price formation is a central question in finance, with far-reaching implications for trading strategies, market design, and regulatory oversight. Despite decades of research, however, there remains no universally accepted definition of price discovery. As emphasized by Putniņš (2013), a market dominates price discovery if it is the first to incorporate new information about fundamental value, rather than merely exhibiting greater volatility or noise. Building on this conceptual distinction, much of the recent literature operationalizes price discovery by quantifying the magnitude of a market's instantaneous response to permanent shocks, denoted $d^P_{0,i}$. Prominent examples include the Information Leadership (IL) measure of Yan and Zivot (2010) and its share-based counterpart, the Information Leadership Share (ILS) proposed by Putniņš (2013), as well as the Price Information Leadership (PIL) and Price Information Leadership Share (PILS) of Shen et al. (2025), the Covariance Information Share (CovIS) of Lautier et al. (2023), and the New Leadership Share (NLS) of Lien et al. (2025).

Under normal market conditions, these **magnitude-based measures** correctly identify the fastest-responding market as the informational leader. However, markets frequently deviate from normal conditions due to high-frequency trading, transient liquidity pressures, or other microstructure frictions. In such cases, magnitude-based measures can **misclassify volatility or overshooting as leadership**, rewarding aggressive reactions rather than accurate incorporation of information. For instance, a market that temporarily overreacts to a shock may appear dominant, even though its prices move further away from fundamentals. This limitation highlights the need for a benchmark that evaluates **accuracy rather than size**, comparing a market's initial response to the ultimate long-run impact of a permanent shock.

Informational efficiency should thus be evaluated relative to the long-run effect of a permanent shock, not merely by the instantaneous magnitude of the response. Within the Structural Moving Average (SMA) framework, price changes in cointegrated markets can be decomposed into **permanent shocks**, which determine the long-run common price, and **transitory shocks**, which capture temporary deviations such as bid–ask bounce or liquidity effects. Most existing SMA identification schemes focus on recovering the initial responses to permanent shocks, implicitly assuming that the largest initial reaction corresponds to leadership. While this assumption works under well-behaved conditions, it fails when short-run responses are exaggerated or perverse—situations associated with overshooting, illiquidity, or transient frictions.

Under the SMA normalization of **unit long-run permanent responses** (Yan and Zivot 2010), larger values of $|d^P_{0,i}|$ typically indicate closer alignment with the efficient price. The magnitude-based **instantaneous response rule**, which defines leadership as the market with the largest $|d^P_{0,i}|$, performs well in this regime. Yet, when perverse or overshooting responses occur, this rule becomes misleading: it **conflates aggressiveness with efficiency**, labeling markets with extreme reactions as leaders even if they move away from fundamentals.

To address this shortcoming, we propose the **Instantaneous Pricing Error Rule**, which defines leadership in terms of **accuracy relative to the long-run efficient price**, measured as $|d^P_{0,i} - 1|$. Unlike magnitude-based rules, this approach penalizes both under-reaction and overshooting, identifying the market whose initial response is **closest to the ultimate fundamental impact**. By construction, the instantaneous pricing error rule isolates genuine informational leadership from transient volatility or structural frictions.

We operationalize this principle through the **Instantaneous Pricing Efficiency Share (IPES)**, a continuous, scale-free metric that transforms deviations from the long-run benchmark into market leadership scores. While the magnitude-based instantaneous response rule rewards large reactions, IPES rewards **precision**: a market that reacts accurately, even if moderately, is recognized as the leader, whereas one that overreacts or moves perversely is penalized. This distinction ensures that IPES remains **robust and interpretable** in high-frequency and multi-market settings where temporary distortions are common.

Simulation evidence illustrates the practical advantages of IPES. Using the Partial Price Adjustment (PPA) and Dominant–Satellite models (Yan and Zivot 2010), we consider under-reaction, overshooting, and perverse short-run responses. Across all configurations, magnitude-based measures (NLS, PILS, CovIS) often misclassify leadership, favoring markets with larger or more volatile initial reactions—even when those responses amplify pricing errors. By contrast, IPES consistently identifies the true informational leader, providing economically meaningful rankings even under extreme overreaction or perverse responses.

These theoretical findings are reinforced by two empirical applications. During the 2010 Flash Crash, SPY and IVV—two financially identical ETFs—temporarily decoupled. Magnitude-based measures erroneously assigned leadership to IVV, which overreacted and deviated from fundamentals, whereas IPES correctly identified SPY as the leader. Similarly, in the 2024 Bitcoin liquidation cascade, magnitude-based rules favored the overreacting Futures market, while IPES consistently recognized the Spot market as the leader, reflecting its closer alignment with the efficient price. These examples highlight the **systematic advantage of accuracy-based measures** over magnitude-based rules in turbulent or distorted market conditions.

This paper makes three main contributions to the literature on price discovery. First, we clarify a **conceptual distinction between two fundamentally different rules for defining price discovery**: the prevailing **instantaneous response rule**, which equates informational leadership with the magnitude of a market's contemporaneous reaction to a permanent shock, and an **instantaneous pricing-error rule**, which defines leadership by the accuracy of that reaction relative to the long-run efficient price. By explicitly linking price discovery to deviations from the long-run benchmark, we show that magnitude-based measures implicitly impose a behavioral assumption—that larger reactions are necessarily more informative—which fails whenever short-run dynamics are distorted by overshooting, leverage, or liquidity frictions.

Second, within a Structural Moving Average (SMA) framework, we highlight the **central**

but underexplored role of long-run normalization in price discovery measurement. We show that existing structural measures—including PILS, CovIS, and NLS—recover instantaneous responses under identification schemes that normalize long-run effects of permanent shocks, yet interpret leadership solely through short-run magnitudes. By making the long-run benchmark explicit, we demonstrate how these normalization assumptions implicitly fix the efficient price path, thereby providing a natural reference point against which short-run pricing errors should be evaluated. This perspective reconciles structural identification with an economically meaningful notion of informational efficiency.

Third, building on this insight, we propose the **Instantaneous Pricing Error Rule** and operationalize it through the **Instantaneous Pricing Efficiency Share (IPES)**. IPES is a continuous, scale-free measure that assigns informational leadership to the market whose initial response lies closest to the long-run efficient benchmark. Unlike magnitude-based measures, IPES penalizes both underreaction and overshooting and remains well-defined even when instantaneous responses are negative or exceed unity. As a result, IPES provides a theoretically grounded and empirically robust measure of price discovery that is valid in both stable markets and periods of severe dislocation.

The remainder of the paper is organized as follows. Section 2 reviews the related literature and existing price discovery measures, highlighting their underlying identification rules and limitations. Section 3 introduces the Structural Moving Average (SMA) framework, discusses the identification of permanent and transitory shocks under long-run normalization, and proposes the Instantaneous Pricing Efficiency Share (IPES). Section 4 presents simulation evidence under a range of short-run dynamics, including underreaction, overshooting, and perverse responses to illustrate the performance of IPES relative to magnitude-based measures. Section 5 reports empirical results from episodes of market stress, demonstrating the practical relevance of the proposed measure. Section 6 concludes.

2 Literature Review

Early empirical measures of price discovery are largely based on reduced-form Vector Error Correction Models (VECMs). The most notable examples are Hasbrouck's Information Share (IS; Hasbrouck 1995) and Component Share (CS; Garbade and Silber 1983; Booth et al. 1999; Chu et al. 1999; Harris et al. 2002). IS decomposes the variance of permanent price innovations to infer each market's contribution, while CS relies on the permanent–transitory (PT) decomposition of cointegrated prices from Gonzalo and Granger (1995) and Gonzalo and Ng (2001). A well-known limitation of IS is the *order-dependence problem*: when idiosyncratic innovations across markets are contemporaneously correlated, IS is typically reported as a range whose upper and lower bounds depend on the ordering of the price vector, making the identification of the price leader ambiguous (Hasbrouck 1995; Baillie et al. 2002). CS, in contrast, is largely driven by contemporaneous transitory shocks and does not fully capture permanent information.

Several extensions attempt to address these limitations. Baillie et al. (2002) proposed IS-mid (or IS-mean), which averages across all possible orderings, while Lien and Shrestha (2009) and Lien and Shrestha (2014) introduced the Modified Information Share (MIS) and Generalized Information Share (GIS) using spectral decompositions. Sultan and Zivot (2015) and Shen et al. (2024) developed the Price Discovery Share (PDS) measure, which leverages portfolio volatility decomposition to assign market contributions in an order-invariant manner. De Jong and Schotman (2010) also proposed a structural unobserved components approach to identify price leadership, but their method requires restrictive assumptions for parameter identification. Despite these improvements, most IS-type measures still confound contemporaneous transitory shocks with permanent innovations and are sensitive to leverage or non-unitary cointegration.

Structural approaches offer clearer economic interpretation and greater robustness by explicitly separating permanent information from transitory noise. Yan and Zivot (2010) embed price discovery within a Structural Moving Average (SMA) framework that decomposes price innovations into permanent and transitory shocks, demonstrating that Hasbrouck's Information Share (IS) conflates informational leadership with transitory noise, while the Component Share (CS) primarily captures noise avoidance rather than information incorporation. Building on this framework, Yan and Zivot (2010) propose the Information Leadership (IL) measure, which is extended to a share variant Information Leadership Share (ILS) by Putniņš (2013). Shen et al. (2025) further develop the Price Information Leadership (PIL/PILS) measures by allowing for correlated residuals, thereby improving robustness to cross-market noise spillovers. Related SMA-based measures, including the Covariance Information Share (CovIS; Lautier et al. 2023) and the New Leadership Share (NLS; Lien et al. 2025), quantify each market's contribution to permanent price innovations directly from estimated structural parameters.

Despite their common structural foundation, these approaches differ subtly in how the SMA model is identified. The methods of Yan and Zivot (2010) and Shen et al. (2025) rely on partial identification, exploiting the fact that the contemporaneous responses to permanent shocks are identifiable functions of reduced-form price discovery measures, without fully identifying the SMA model. In contrast, Lautier et al. (2023) and Lien et al. (2025) pursue full identification of the SMA system by imposing additional assumptions—such as recursive (Cholesky) ordering or equal-variance restrictions on structural shocks—thereby enabling direct recovery of all structural parameters.

Despite these advances, existing price discovery measures continue to face important limitations. First, all structural-form approaches—including PILS, NLS, and CovIS—implicitly equate the magnitude of a market's initial response with informational leadership, without evaluating how accurately that response reflects the long-run impact of a permanent shock. This can lead to misleading conclusions when markets overreact, underreact, or differ in leverage. Second, nearly all structural measures rely on normalization assumptions that may not hold empirically—for example, the unit-variance assumption for structural shocks in Lien et al. (2025) or the equivalent-variance assumption in Lautier et al. (2023). The implications of these SMA identification assumptions for the long-run responses to permanent shocks remain underexplored and are largely ignored. These limitations underscore the need to clarify how different identification schemes affect long-run impacts and to develop measures that assess price discovery based on the alignment of initial responses with their long-run benchmarks, rather than solely on reaction magnitude.

3 Theoretical Framework

3.1 The Structural Moving Average Model

Our analysis applies to $N \geq 2$ markets; for expositional clarity, we focus on the two-market case. Let $\mathbf{p}_t = (p_{1t}, p_{2t})'$ denote the logarithmic prices of two arbitrage-linked assets. As is standard in the price discovery literature, each series is assumed to be integrated of order one, I(1), and $\mathbf{p}_t$ is cointegrated with cointegrating vector $\boldsymbol{\beta} = (1, -1)'$.

Following Yan and Zivot (2010), price changes admit a structural moving average (SMA) representation,

$$\Delta \boldsymbol{p}_t = \boldsymbol{D}(L)\boldsymbol{\eta}_t = \boldsymbol{D}_0\boldsymbol{\eta}_t + \boldsymbol{D}_1\boldsymbol{\eta}_{t-1} + \boldsymbol{D}_2\boldsymbol{\eta}_{t-2} + \dots \tag{1}$$

where $\boldsymbol{D}(L) = \sum_{k=0}^{\infty} \boldsymbol{D}_k L^k$, $\boldsymbol{D}_k$ is absolutely summable, and $\boldsymbol{D}_0$ is invertible. The number of structural shocks equals the number of prices, ensuring invertibility of $\boldsymbol{D}(L)$. The shock vector $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ consists of a permanent (efficient-price) shock η_t^P and a transitory (noise) shock η_t^T, assumed serially and mutually uncorrelated with covariance matrix $\mathbf{C} = \text{diag}(\sigma_P^2, \sigma_T^2)$. The matrix $\boldsymbol{D}_0$ encompasses the initial impacts of the structural shocks on $\Delta \mathbf{p}_t$ and defines the contemporaneous correlation structure of $\Delta \mathbf{p}_t$:

$$\boldsymbol{D}_0 = \begin{pmatrix} d_{0,1}^P & d_{0,1}^T \\ d_{0,2}^P & d_{0,2}^T \end{pmatrix}. \tag{2}$$

The identification of the SMA model in (1) starts with the empirical VECM(K-1) model:

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{p}_{t-1} + \sum_{j=1}^{K-1} \boldsymbol{\Gamma}_j \Delta \mathbf{p}_{t-j} + \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\boldsymbol{\alpha}$ is the error correction vector, $\boldsymbol{\Gamma}_j$ are the short-run coefficient matrices, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the vector of reduced-form VECM residuals assumed $\boldsymbol{\varepsilon}_t \sim iid(\mathbf{0}, \boldsymbol{\Omega})$ where the covariance matrix $\boldsymbol{\Omega}$ is a 2×2 matrix with elements σ_{ij}.

As shown in Hasbrouck (1995), the above VECM model can be transformed into a reduced-form Vector Moving Average (VMA) model:

$$\Delta \mathbf{p_t} = \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{\varepsilon}_t + \boldsymbol{\Psi}_1 \boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Psi}_2 \boldsymbol{\varepsilon}_{t-2} + \cdots , \tag{4}$$

and its integrated form (or Beveridge-Nelson decomposition):

$$\mathbf{p}_t = \mathbf{p}_0 + \boldsymbol{\Psi}(1) \sum_{s=1}^{t} \boldsymbol{\varepsilon}_s + \boldsymbol{\Psi}^*(L)\boldsymbol{\varepsilon}_t, \tag{5}$$

where $\boldsymbol{\Psi}(1) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k$ with $\boldsymbol{\Psi}(L)$ and $\boldsymbol{\Psi}^*(L)$ being matrix polynomials in the lag operator, L, and $\boldsymbol{\Psi}^*(k) = -\sum_{j=k+1}^{\infty} \boldsymbol{\Psi}_j$. As demonstrated by Johansen (1991), the long-run impacts matrix $\boldsymbol{\Psi}(1)$ are linked to the VECM parameters as:

$$\boldsymbol{\Psi}(1) = \boldsymbol{\beta}_{\perp}(\boldsymbol{\alpha}'_{\perp}\boldsymbol{\Gamma}(1)\boldsymbol{\beta}_{\perp})^{-1}\boldsymbol{\alpha}'_{\perp} = \boldsymbol{\beta}_{\perp}\boldsymbol{\Pi}\boldsymbol{\alpha}'_{\perp}, \tag{6}$$

where $\boldsymbol{\beta}_{\perp}$ and $\boldsymbol{\alpha}_{\perp}$ are 2×1 vectors satisfying $\boldsymbol{\beta}'_{\perp}\boldsymbol{\beta} = 0$ and $\boldsymbol{\alpha}'_{\perp}\boldsymbol{\alpha} = 0$, respectively, $\boldsymbol{\Gamma}(1) = \boldsymbol{I}_2 - \sum_{j=1}^{K-1} \boldsymbol{\Gamma}_j$. Hence, $\boldsymbol{\Pi} = (\boldsymbol{\alpha}'_{\perp}\boldsymbol{\Gamma}(1)\boldsymbol{\beta}_{\perp})^{-1}$ is a scalar. The matrix $\boldsymbol{\Psi}(1)$ contains the cumulative impacts of the innovation $\boldsymbol{\varepsilon}_t$ on all future price movements, and acts as a measure of the long-run impact of $\boldsymbol{\varepsilon}_t$ on prices.

After estimating the VECM using standard methods, we can derive the parameters of the VMA model (i.e., $\boldsymbol{\Psi}_k$ coefficients) using standard algebraic manipulations. Comparing the SMA representations in (1) and the VMA representations in (4), we have:

$$\boldsymbol{\varepsilon}_t = \boldsymbol{D}_0 \boldsymbol{\eta}_t, \tag{7}$$

$$\boldsymbol{D}_k = \boldsymbol{\Psi}_k \boldsymbol{D}_0, \quad for \quad k = 1, 2, \ldots. \tag{8}$$

Then the identification of the SMA model in (1) boils down to pin down the initial impact matrix $\boldsymbol{D}_0$ and the covariance matrix ($\boldsymbol{C}$) of the structural shocks $\boldsymbol{\eta}_t$.

3.2 Identification of the Permanent Shock

As shown in Hasbrouck (1995), when $\boldsymbol{\beta} = (1, -1)'$, all rows of $\boldsymbol{\Psi}(1)$ are identical. Let $\boldsymbol{\psi}' = (\psi_1, \psi_2)$ denote this common row vector of $\boldsymbol{\Psi}(1)$, so that $\boldsymbol{\Psi}(1) = \boldsymbol{l}_2 \boldsymbol{\psi}'$, with $\boldsymbol{l}_2$ a 2×1 vector of ones. The integrated VMA in (5) can then be written as

$$\mathbf{p}_t = \mathbf{p}_0 + \boldsymbol{l}_2 \sum_{s=1}^{t} \boldsymbol{\psi}' \boldsymbol{\varepsilon}_s + \boldsymbol{\Psi}^*(L) \boldsymbol{\varepsilon}_t = \mathbf{p}_0 + \boldsymbol{l}_2 m_t + \tilde{\boldsymbol{\varepsilon}}_t, \tag{9}$$

where $m_t = \sum_{s=1}^{t} \boldsymbol{\psi}' \boldsymbol{\varepsilon}_s$ is the unobservable efficient price common to both markets, and $\tilde{\boldsymbol{\varepsilon}}_t = \boldsymbol{\Psi}^*(L) \boldsymbol{\varepsilon}_t \sim I(0)$ captures transitory pricing errors such as bid–ask bounce and inventory effects. This decomposition identifies the permanent structural shock as

$$\eta_t^P = \Delta m_t = \boldsymbol{\psi}' \boldsymbol{\varepsilon}_t = \psi_1 \varepsilon_{1t} + \psi_2 \varepsilon_{2t}, \tag{10}$$

with variance $\sigma_P^2 = \boldsymbol{\psi}' \boldsymbol{\Omega} \boldsymbol{\psi}$.

The Beveridge–Nelson decomposition (Beveridge and Nelson 1981) of the SMA model in (1) yields

$$\boldsymbol{p}_t = \boldsymbol{p}_\mathbf{0} + \boldsymbol{D}(1) \sum_{j=1}^{t} \boldsymbol{\eta}_j + \boldsymbol{s}_t, \tag{11}$$

where $\boldsymbol{D}(1) = \sum_{k=0}^{\infty} \boldsymbol{D}_k$, $\boldsymbol{s}_t = \boldsymbol{D}^*(L)\boldsymbol{\eta}_t \sim I(0)$, $\boldsymbol{D}_k^* = -\sum_{j=k+1}^{\infty} \boldsymbol{D}_j$. Consistency with (10) implies the long-run normalization:

$$\boldsymbol{D}(1) = \sum_{k=0}^{\infty} \boldsymbol{D}_k = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}. \tag{12}$$

which formalizes the distinction between permanent and transitory shocks (Yan and Zivot 2010). The permanent innovation η_t^P reflects new information about the asset's fundamental value and induces a unit long-run shift in prices across markets, while the transitory innovation η_t^T captures orthogonal, non-informational disturbances with no long-run price impact.

Noting that $\boldsymbol{\varepsilon}_t = \boldsymbol{D}_0\boldsymbol{\eta}_t$, the first column of $\boldsymbol{D}_0$ can be identified using the price discovery beta β_i defined in (Shen et al. 2024; Sultan and Zivot 2015):

$$d_{0,i}^P = \beta_i = \frac{\text{cov}(\varepsilon_{it}, \eta_t^P)}{\text{var}(\eta_t^P)} = \frac{\psi_i \sigma_i^2 + \psi_{j\neq i}\sigma_{ij\neq i}}{\psi_1^2\sigma_1^2 + 2\psi_1\psi_2\sigma_{12} + \psi_2^2\sigma_2^2}, \quad i = 1, 2. \tag{13}$$

The price discovery beta is defined as the slope coefficient from regressing ε_{it} on η_t^P, and measures the contribution of an asset's innovation to the variance of the permanent shock. As shown in Appendix A1, the second column of $\boldsymbol{D}_0$ is identified only up to a scale factor in the absence of additional normalization assumptions. For the purpose of price discovery, however, identification of the structural permanent shock is sufficient. We therefore refrain from imposing further normalization assumptions on the transitory shocks and leave full identification of the SMA model to future research.

A recent work by Lautier et al. (2023) obtains the same identification of the permanent shock as in (10), and introduce a Covariance Information Share (CovIS) based on the covariance of reduced-form residuals with the permanent shock:

$$\text{CovIS}_i = \frac{\text{cov}(\varepsilon_{it}, \eta_t^P)}{\sum_{i=j}^{n} \text{cov}(\varepsilon_{jt}, \eta_t^P)}. \tag{14}$$

To fully identify $\boldsymbol{D}_0$, Lautier et al. (2023) impose the additional restriction $\sigma_P^2 = \sigma_T^2$, which yields the following initial response matrix:

$$\breve{\boldsymbol{D}}_0 = \begin{pmatrix} \frac{\psi_1\sigma_1^2+\psi_2\sigma_{12}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & \frac{\psi_2\sigma_1\sigma_2\sqrt{1-\rho^2}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} \\ \frac{\psi_1\sigma_{12}+\psi_2\sigma_2^2}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & \frac{-\psi_1\sigma_1\sigma_2\sqrt{1-\rho^2}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} \end{pmatrix}. \tag{15}$$

Subsequent impulse responses then follow directly from $\breve{\boldsymbol{D}}_k = \boldsymbol{\Psi}_k\breve{\boldsymbol{D}}_0$. While Lautier et al. (2023) argue that the equal-variance assumption is innocuous, our simulation results show that this restriction is neither necessary nor generally satisfied in empirical applications. For the purpose of price discovery, identifying the first column of $\boldsymbol{D}_0$ is sufficient, and therefore imposing equality of structural shock variances is unwarranted.

Recently, Lien et al. (2025) achieve similar identification of permanent and transitory shocks by combining the permanent–transitory (P–T) decomposition with a Cholesky factorization of the transformed residuals following Gonzalo and Ng (2001).[1] In their framework, the VECM residuals are first transformed as:

$$\boldsymbol{\epsilon_t} = \begin{pmatrix} \epsilon_t^P \\ \epsilon_t^T \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_\perp'\boldsymbol{\varepsilon}_t \\ \boldsymbol{\beta}'\boldsymbol{\varepsilon}_t \end{pmatrix} = \boldsymbol{G}\boldsymbol{\varepsilon}_t, \tag{16}$$

where $\boldsymbol{G} = [\boldsymbol{\alpha}_\perp \vdots \boldsymbol{\beta}]'$ is a transformation matrix assumed to be nonsingular. Gonzalo and Ng (2001) showed that the permanent and transitory innovations, ϵ_t^P and $\boldsymbol{\epsilon}_t^T$, satisfy $\lim_{k\to\infty} \frac{\partial E_t[\boldsymbol{p}_{t+k}]}{\partial \epsilon_t^P} \neq 0$ and $\lim_{k\to\infty} \frac{\partial E_t[\boldsymbol{p}_{t+k}]}{\partial \boldsymbol{\epsilon}_t^T} = 0$, respectively.

To orthogonalize $\boldsymbol{\epsilon_t}$, Gonzalo and Ng (2001) apply a Cholesky factorization to its covari-

[1] Yan and Zivot (2007) were among the first to adopt the permanent–transitory decomposition of Gonzalo and Ng (2001) to identify permanent and transitory shocks within an SMA framework. Rather than applying a Cholesky factorization to the transformed residuals, Yan and Zivot (2007) employ a triangular factorization to construct orthogonal structural shocks. We leave a systematic analysis of how alternative factorizations of the transformed residuals affect SMA identification to future research.

ance matrix, $\boldsymbol{\Sigma}_\epsilon$, yielding orthogonal structural shocks:

$$\tilde{\boldsymbol{\eta}}_t = \begin{pmatrix} \tilde{\eta}_t^P \\ \tilde{\eta}_t^T \end{pmatrix} = \boldsymbol{L}^{-1}\boldsymbol{\epsilon_t} = \boldsymbol{L}^{-1}\boldsymbol{G}\boldsymbol{\varepsilon}_t, \tag{17}$$

where $\boldsymbol{L}$ is the lower-triangular Cholesky factor of $\boldsymbol{\Sigma}_\epsilon$, such that $\boldsymbol{\Sigma}_\epsilon = \boldsymbol{L}\boldsymbol{L}^{'}$. It follows that the corresponding initial response matrix can be expressed as:

$$\tilde{\boldsymbol{D}}_0 = (\boldsymbol{L}^{-1}\boldsymbol{G})^{-1} = \boldsymbol{G}^{-1}\boldsymbol{L}. \tag{18}$$

The permanent–transitory decomposition and the Cholesky factorization ensure that $\tilde{\boldsymbol{\eta}}_t = (\tilde{\eta}_t^P, \tilde{\eta}_t^T)'$ are serially uncorrelated, mutually orthogonal, and have unit variance.

As shown in Appendix A1, in the bivariate case the structural permanent shock identified by Lien et al. (2025) can be written as:[2]

$$\tilde{\boldsymbol{\eta}}_t^P = \frac{\psi_1}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}}\varepsilon_{1t} + \frac{\psi_2}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}}\varepsilon_{2t}, \tag{19}$$

and the corresponding initial impact matrix $\tilde{\boldsymbol{D}}_0$ is given as:

$$\tilde{\boldsymbol{D}}_0 = \begin{pmatrix} \tilde{d}_{0,1}^P & \tilde{d}_{0,1}^T \\ \tilde{d}_{0,2}^P & \tilde{d}_{0,2}^T \end{pmatrix} = \begin{pmatrix} \frac{\psi_1\sigma_1^2+\psi_2\sigma_{12}}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}} & \psi_2\frac{\sqrt{\sigma_1^2\sigma_2^2-\sigma_{12}^2}}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}} \\ \frac{\psi_1\sigma_{12}+\psi_2\sigma_2^2}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}} & -\psi_1\frac{\sqrt{\sigma_1^2\sigma_2^2-\sigma_{12}^2}}{\sqrt{\boldsymbol{\psi}^{'}\boldsymbol{\Omega}\boldsymbol{\psi}}} \end{pmatrix}. \tag{20}$$

[2] While Lien et al. (2025) argue that the solution for $d_{0,i}^P$, and hence for η_t^P, is unaffected by the non-uniqueness of the orthogonal complement $\boldsymbol{\alpha}_\perp$, we show in Appendix A1 that the **sign** of the scale factor ξ in $\boldsymbol{\alpha}_\perp = \xi\boldsymbol{\psi}$ is, in fact, consequential. Under the Gonzalo–Ng transformation combined with Cholesky orthogonalization, the **magnitude** of ξ is absorbed into the diagonal elements of the Cholesky factor, while its **sign** is preserved in the off-diagonal entry. Consequently, reversing the sign of ξ flips the sign of the permanent structural shock and the associated long-run impact matrix, while leaving the transitory shock unchanged. This implies that the **sign of the permanent shock is not determined** by the Cholesky decomposition alone. Imposing the economically natural normalization that a permanent shock raises the long-run efficient price uniquely resolves this ambiguity, and we adopt this normalization throughout.

As a result, the long-run impact matrix $\boldsymbol{D}(1)$ in Lien et al. (2025) can be expressed as:

$$\tilde{\boldsymbol{D}}(1) = \boldsymbol{\Psi}(1)\tilde{\boldsymbol{D}}_0 = \begin{pmatrix} \sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & 0 \\ \sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & 0 \end{pmatrix}. \tag{21}$$

Under the identification scheme of Lien et al. (2025), which follows Gonzalo and Ng (2001), the permanent shock induces identical long-run responses across markets. However, this common long-run effect is not normalized to unity; instead, it scales with the standard deviation of the permanent innovation η_t^P. Consistent with this normalization, our results show that the permanent shock identified by Lien et al. (2025) in (19) corresponds to a standardized version of the permanent shock in (10), and that their reported initial impact responses are proportionally scaled by the standard deviation of the permanent shock.

This distinction is central to our analysis. As shown later in the paper, price discovery measures are naturally defined as the relative importance of a market's initial response to the permanent shock, evaluated against its long-run effect. To ensure that this ratio (or discrepancy) admits a clear and invariant economic interpretation, the long-run response of prices to the permanent innovation must be explicitly normalized. By imposing the long-run normalization in (12), we fix the permanent shock to have a unit long-run impact on all prices, thereby anchoring the scale of the permanent component in economically meaningful units.

Under this unit long-run normalization, cross-market differences in price discovery arise exclusively from heterogeneity in short-run adjustment, as captured by the initial response matrix $\boldsymbol{D}_0$. In contrast, under the P–T decomposition, the scale of the permanent shock is absorbed into a variance normalization that standardizes the permanent innovation to unit variance, causing short-run responses to be scaled by the standard deviation of the permanent shock. As a result, initial impulse responses cannot be interpreted independently of the long-run normalization, and cross-market comparisons of price discovery depend on the adopted variance convention.

For these reasons, we adopt the long-run normalization in (12). This approach yields an initial permanent responses in (13), that directly measures short-run responses per unit of long-run permanent price change. Consequently, the resulting price discovery measures are scale-free, transparent, and economically interpretable as the speed and intensity with which markets incorporate permanent information.

3.3 Instantaneous Pricing Efficiency Share

With the permanent shock now formally identified within the SMA model, recent advances have largely resolved the classical ordering problem that plagued early price discovery measures, most notably Hasbrouck's Information Share. Building on the permanent–transitory decomposition of Gonzalo and Ng (2001), Yan and Zivot (2007) employ a triangular factorization to identify structural shocks without imposing the unit-variance normalization. By contrast, inheriting both the permanent–transitory decomposition and the Cholesky normalization—and hence the unit-variance assumption—of Gonzalo and Ng (2001), Lien et al. (2025) propose the New Leadership Share (NLS) to quantify each market's contribution to permanent price innovations. By clarifying the structural interpretation of price discovery, Shen et al. (2025) introduce the Price Information Leadership (PIL) measure and its share-based counterpart (PILS), showing that relative informational contributions can be consistently identified even in the presence of correlated reduced-form innovations. Along similar lines, Lautier et al. (2023) develop the Covariance Information Share (CovIS) and its quadratic variant (CovISQ) under the same permanent shock identification assumptions. For conciseness, formal definitions and structural representations of these measures are provided in Appendix A2.

Despite their methodological differences, these approaches share a common identifying principle: **price discovery is inferred exclusively from a market's contemporaneous response to the permanent shock**. As shown in Appendix A2, the PIL, NLS, and CovIS measures all classify a market as the informational leader whenever it exhibits a larger instantaneous response coefficient. We refer to this shared premise as the Instantaneous Response Rule, or equivalently, a magnitude-based criterion. Implicit in this rule is the assumption that a stronger initial response reflects superior information processing and more efficient incorporation of permanent information.

While this rule resolves important econometric issues, it embeds a conceptual limitation: it evaluates informational efficiency solely through the size of the initial adjustment, without reference to the ultimate price change dictated by the permanent shock. In a cointegrated system, the permanent information shock induces a common long-run response across markets, which converges to unity:

$$\lim_{h\to\infty} \frac{\partial E_t[\boldsymbol{p}_{t+h}]}{\partial \eta_t^P} = \lim_{h\to\infty} \sum_{k=0}^{h} \frac{\partial E_t[\Delta \boldsymbol{p}_{t+k}]}{\partial \eta_t^P} = (1,1)'. \tag{22}$$

This long-run response provides a natural benchmark for assessing how accurately markets incorporate new information. From this perspective, a market is truly efficient not when its initial permanent response is large, but when it closely aligns with the long-run effect of the shock.

Motivated by this observation, we propose a new identification criterion based on pricing accuracy rather than reaction magnitude. Specifically, we define the Instantaneous Pricing Error for market i as: [3]

$$\mathcal{E}_i = \left| d_{0,i}^P - 1 \right|. \tag{23}$$

[3]The instantaneous pricing error considered in this paper is partly motivated by the *price discovery efficiency loss* proposed by Yan and Zivot (2007). In their framework, the dynamic efficiency of market i at horizon k following a unit permanent shock is measured by the deviation of the cumulative permanent impulse response from its long-run value of unity, $f_{k,i} - 1$, where $f_{k,i} = \sum_{l=0}^{k} d_{l,i}^P$ and $d_{l,i}^P$ denotes the response of market i's price at time $t+l$ to a unit permanent shock at time t. Given a non-negative loss function L and a truncation horizon K^*, Yan and Zivot (2007) define the *price discovery efficiency loss* (PDEL) as the accumulated loss $\text{PDEL}_i = \sum_{k=0}^{K^*} L\left(f_{k,i} - 1\right)$. While Yan and Zivot (2007) emphasize pricing accuracy along the entire adjustment path, our focus is on clarifying how price discovery should be defined at the instant the permanent shock occurs. Accordingly, we abstract from dynamic efficiency considerations and focus exclusively on contemporaneous pricing accuracy, leaving the study of dynami efficiency to future research.

This instantaneous pricing error captures any failure to correctly incorporate permanent information upon impact. Importantly, it treats underreaction and overshooting symmetrically, reflecting the fact that both represent mispricing relative to the efficient equilibrium adjustment. Based on this notion of pricing accuracy, we propose an alternative identifying principle for price discovery:

Proposition 1 (Instantaneous Pricing Error Rule)**.** *The informational leader is the market that minimizes the Instantaneous Pricing Error; that is, the market whose contemporaneous response to the permanent shock is closest to its long-run equilibrium effect.*

This rule departs fundamentally from magnitude-based criteria. Rather than rewarding the market that reacts most strongly, it assigns leadership to the market that reacts *most correctly.* To operationalize this principle and obtain a continuous measure of informational leadership, we map the pricing error into a *pricing effectiveness score*:

$$\omega_i = \exp\left(-\left|d^P_{0,i} - 1\right|\right). \tag{24}$$

The exponential transformation ensures that the score is bounded between zero and one and decreases monotonically with the pricing error. This mapping captures the intuition that responses closer to the efficient benchmark are more informative, while larger deviations are increasingly penalized.

We then define the *Instantaneous Pricing Efficiency Share (IPES)* as the contribution of market i to the total pricing efficiency of the system. For a system with n markets,

$$IPES_i = \frac{\omega_i}{\sum_{j=1}^{n} \omega_j} = \frac{\exp\left(-|d^P_{0,i} - 1|\right)}{\sum_{j=1}^{n} \exp\left(-|d^P_{0,j} - 1|\right)}. \tag{25}$$

By construction, IPES is a proper share that sums to unity and admits a transparent interpretation as a relative measure of instantaneous pricing accuracy: markets with smaller pricing errors receive larger shares, while substantial underreaction or overshooting is penalized.

This definition offers several advantages over magnitude-based price discovery measures. First, any deviation from the long-run benchmark—whether underreaction or overshooting—reduces the efficiency score, so aggressive responses with $d^P_{0,i} > 1$ are not mechanically rewarded. Second, the measure extends naturally to multivariate systems without requiring pairwise comparisons or ad hoc normalization. Third, the exponential mapping implies that improvements in accuracy closer to the efficient benchmark result in proportionally higher increases in leadership, thereby distinguishing genuinely informative markets from those that merely react strongly.

These features are particularly relevant in high-frequency markets, where algorithmic trading and transient liquidity frictions often generate short-lived overshooting (Lautier et al. 2023). In such environments, large instantaneous price changes may reflect trading pressure rather than genuine information incorporation. While magnitude-based measures tend to classify these aggressive responses as price leadership, IPES assigns leadership to the market whose initial adjustment is closest to the long-run equilibrium response, effectively separating informational efficiency from reaction intensity.

It is important to emphasize that IPES is a relative measure of informational leadership rather than an absolute measure of pricing quality. When all markets respond inefficiently to the permanent shock—for example, when they overshoot by similar magnitudes—IPES will be approximately evenly split. This outcome indicates that markets are similarly inaccurate, not that pricing is efficient. In such cases, the level of the Instantaneous Pricing Error itself remains informative, capturing the absolute severity of mispricing upon impact. Accordingly, the Instantaneous Pricing Error and IPES should be viewed as complementary: the former measures how well information is incorporated, while the latter identifies which market leads in doing so.

Empirical computation of the IPES from the VECM estimates proceeds as follows. First, the optimal lag length K is determined using a VAR of $\mathbf{p}_t$, selecting the model according to the Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC). Next, a reduced-form VECM($K-1$) is estimated. The long-run impact matrix $\hat{\boldsymbol{\Psi}}(1)$ is then computed using (6), and the residual covariance matrix $\hat{\boldsymbol{\Omega}}$ is estimated. Based on these results, the initial permanent responses $\hat{d}^P_{0,i}$ are calculated according to (13), and the IPES measures are finally derived using (25).

3.4 When Does the Instantaneous Response Rule Remain Valid?

The two leadership identifying rules defined above coincide only when instantaneous price adjustments are efficient; they diverge sharply in the presence of overshooting or otherwise inefficient adjustment. Figure 1 illustrates this divergence. The horizontal and vertical axes plot the structural response coefficients $d^P_{0,1}$ and $d^P_{0,2}$, respectively.

[Insert Figure 1 about here.]

Panel A depicts the leadership regions implied by the traditional *Instantaneous Response Rule*. The green shaded area corresponds to $|d^P_{0,1}| > |d^P_{0,2}|$, under which Market 1 is identified as the price leader solely on the basis of response magnitude.

Panel B shows the leadership regions under the proposed *Instantaneous Pricing Error Rule*. The orange shaded area identifies Market 2 as the leader whenever its instantaneous response lies closer to the efficient benchmark of unity, that is, when $|d^P_{0,2} - 1| < |d^P_{0,1} - 1|$. The unshaded region therefore corresponds to Market 1 leadership.

Panel C overlays the two criteria to highlight their areas of agreement and disagreement. The shaded regions (green for Market 1 and orange for Market 2) represent *Reliable Zones*, in which both rules identify the same market as the leader. In contrast, the unshaded white regions constitute *Conflict Zones*, where the two rules yield opposite conclusions. These conflict regions arise precisely when instantaneous adjustments are inefficient—either due to overshooting, defined by ($d^P_{0,1}+d^P_{0,2} > 2$), or perverse adjustment, defined by ($d^P_{0,1}+d^P_{0,2} < 0$). In such cases, the *Instantaneous Response Rule* mechanically rewards excessive volatility, whereas the *Instantaneous Pricing Error Rule* appropriately penalizes it.

To illustrate the economic implications, consider a scenario in which Market 1 adjusts efficiently, with ($d^P_{0,1} = 1$), while Market 2 overreacts to new information, so that ($d^P_{0,2} > 1$). Under the *Instantaneous Response Rule*—employed by PIL, NLS, and CovIS—Market 2 is incorrectly identified as the leader because ($|d^P_{0,2}| > |d^P_{0,1}|$). This outcome effectively rewards volatility and noise amplification. By contrast, under the *Instantaneous Pricing Error Rule*, Market 1 is correctly identified as the leader because of its smaller pricing error ($\mathcal{E}_1 = 0 < \mathcal{E}_2$).

A similar inconsistency arises in the dominant–satellite model of Yan and Zivot (2010). When both markets exhibit large negative instantaneous responses—for example, ($d^P_{0,1} = -1, d^P_{0,2} = -2$)—the *Instantaneous Response Rule* again favors Market 2 that deviates further from equilibrium. In contrast, the *Instantaneous Pricing Error Rule* correctly assigns leadership to Market 1 whose adjustment lies closer to the fundamental update implied by the permanent shock.

Because IPES is monotonically decreasing in the Instantaneous Pricing Error, the leadership regions depicted in Figure 1 carry over directly to IPES-based leadership shares. In particular, the Reliable Zones and Conflict Zones identified in the figure correspond one-for-one to regions in which IPES and magnitude-based measures agree or disagree in their classification of informational leadership.

4 Simulation Evidence

4.1 Data Generation Process

To illustrate the theoretical divergence between the traditional *Instantaneous Response Rule* and the proposed *Instantaneous Pricing Error Rule* and compare the performance of competing price discovery measures, we employ the partial price adjustment (PPA) model of Yan and Zivot (2010) in which prices p_{1t} and p_{2t} track a common random-walk fundamental

m_t according to:

$$p_{it} = p_{i,t-1} + \delta_i(m_t - p_{i,t-1}) + b^T_{0,i}\eta^T_t, \quad i = 1, 2 \tag{26}$$
$$m_t = m_{t-1} + \eta^P_t,$$
$$\boldsymbol{\eta_t} = (\eta^P_t, \eta^T_t)' \sim i.i.d.N(\mathbf{0}, diag(\sigma^2_P, \sigma^2_T)).$$

In this setting, the initial impact matrix is given by:

$$\boldsymbol{D}_0 = \begin{pmatrix} \delta_1 & b^T_{0,1} \\ \delta_2 & b^T_{0,2} \end{pmatrix}. \tag{27}$$

The instantaneous structural response to a permanent information shock is therefore governed by the speed-of-adjustment parameter, $d^P_{0,i} = \delta_i$. To ensure economically meaningful and dynamically stable behavior, we restrict $\delta_i \in (0, 2)$, which guarantees cointegration with the fundamental while allowing for both under-reaction ($\delta_i < 1$) and transient overshooting ($\delta_i > 1$).

We consider four configurations of the PPA model that differ only in Market 2's speed of adjustment to permanent shocks. Specifically, we fix Market 1's adjustment speed at $\delta_1 = 0.9$ and vary Market 2's parameter over

$$\delta_2 \in \{0.5, 0.9, 1.1, 1.5\}. \tag{28}$$

These values span regimes of under-reaction, near-efficient adjustment, mild overshooting and high overshooting, allowing us to assess the robustness of alternative identifying principles across increasingly challenging environments. We calibrate the shock variances to $\sigma^2_P = 2$ and $\sigma^2_T = 5$, generating a deliberately low signal-to-noise ratio. In addition, we introduce asymmetric exposure to transitory noise by setting $b^T_{0,1} = 0.5$ and $b^T_{0,2} = -0.5$.

While the partial price adjustment model captures a wide range of empirically relevant dynamics—including under-reaction and transient overshooting—it imposes that instantaneous responses to permanent shocks remain non-negative. As a result, it cannot generate short-run *perverse* adjustments in which prices initially move in the opposite direction of the fundamental innovation. Such behavior, however, is both theoretically admissible and empirically relevant in fragmented markets, where order flow imbalances, inventory pressures, or dominant trading venues may temporarily distort the price formation process.

To examine the performance of competing price discovery measures in this more challenging environment, we therefore complement the partial adjustment framework with the dominant–satellite model of Yan and Zivot (2010), specified as:

$$\begin{aligned} p_{1t} &= m_t + s_{1t}, \quad p_{2t} = m_{t-2} + s_{2t}, \\ m_t &= m_{t-1} + \eta_t^P, \\ s_{it} &= b_{0,i}^P \eta_t^P + b_{0,i}^T \eta_t^T, \quad i = 1, 2, \\ \boldsymbol{\eta_t} &= (\eta_t^P, \eta_t^T)' \sim i.i.d.N(\mathbf{0}, diag(\sigma_P^2, \sigma_T^2)). \end{aligned} \tag{29}$$

In this model, a permanent innovation to the common efficient price m_t is incorporated contemporaneously into Market 1, up to a transient tracking error s_{1t}. By contrast, Market 2 adjusts to the efficient price with a two-period delay and is additionally subject to its own transitory tracking error s_{2t}. Consequently, Market 1 is unambiguously the informational leader in this dominant–satellite setting.

As shown in Yan and Zivot (2010), the above dominant–satellite model implies that

$$\mathbf{D}_0 = \begin{pmatrix} 1 + b_{0,1}^P & b_{0,1}^T \\ b_{0,2}^P & b_{0,2}^T \end{pmatrix}, \tag{30}$$

so that the instantaneous response to a permanent shock is $d_{0,1}^P = 1 + b_{0,1}^P$ for the dominant market and $d_{0,2}^P = b_{0,2}^P$ for the satellite market. By construction, Market 1 is the true informational leader in all scenarios, as it incorporates the permanent innovation contemporaneously, whereas Market 2 adjusts only with delay and may exhibit perverse short-run responses.

In the simulation design that follows, we consider four configurations of the dominant–satellite model that vary the degree to which transitory tracking errors respond to the permanent shock. In particular, Market 2 is allowed to exhibit a perverse (negative) transitory response to the permanent innovation, with $b^P_{0,2} \in \{-0.5, -1.5\}$, capturing moderate and severe perverse behavior. By contrast, Market 1's transitory component either does not respond to the permanent shock or responds negatively, with $b^P_{0,1} \in \{0, -0.5\}$. These parameter combinations generate environments in which the informational leader adjusts either accurately or with short-run mispricing, while the satellite market consistently displays delayed and distorted perverse reactions to permanent information.

To mimic a realistic high-frequency trading environment—characterized by substantial microstructure noise and nontrivial identification—we calibrate the shock variances to $\sigma^2_P = 1$ and $\sigma^2_T = 5$, thereby deliberately imposing a low signal-to-noise ratio. In addition, we introduce asymmetric exposure to transitory noise by setting $b^T_{0,1} = 0.1$ and $b^T_{0,2} = 0.9$.[4] This asymmetric calibration renders Market 2 substantially more exposed to transitory noise than Market 1, thereby providing a stringent and economically meaningful environment in which to assess the robustness of competing price discovery measures.

4.2 Simulation Results

For each data-generating process, we conduct 1,000 Monte Carlo replications with a sample size of $T = 23{,}400$, corresponding to second-level transaction data over a standard trading day. For each simulated sample, we estimate the VECM and recover the SMA parameters

[4] For identification of the SMA model, the initial impact matrix D_0 must be non-singular. When initial permanent responses have opposite signs, allowing transitory responses to also take opposite signs increases the risk of near-collinearity between the permanent and transitory columns of D_0, leading to singular or ill-conditioned realizations. Imposing same-signed transitory responses is therefore not a theoretical restriction, but a practical design choice that improves the likelihood of a well-conditioned D_0 in finite samples. Accordingly, instead of the specification $b^T_{0,1} = 0.5$ and $b^T_{0,2} = -0.5$ commonly used in the PPA model, we impose $b^T_{0,1} = 0.1$ and $b^T_{0,2} = 0.9$.

described in Section 3.2, from which we compute the competing price discovery measures. We organize the discussion in two parts. We first examine the performance of alternative approaches in recovering the structural parameters of the SMA model. We then provide an overview of the price discovery share results, followed by a detailed comparison across regimes.

4.2.1 Simulation Results for the Parital Price Adjustment Model

Table 1 reports sample means across 1000 simulations from the PPA model, with standard deviations in parentheses.

[Insert Table 1 about here.]

Table 1 reports structural parameter estimates obtained using three alternative approaches. Specifically, we present estimated initial permanent responses ($\hat{\delta}_i$) from our price discovery beta approach, estimated initial permanent and transitory responses ($\tilde{\delta}_i, \tilde{d}^T_{0,i}$) from Lien et al. (2025), and estimated initial transitory responses ($\breve{d}^T_{0,i}$) following Lautier et al. (2023). Across all simulation regimes, a clear and consistent pattern emerges.

Our price discovery beta approach accurately recovers the true initial permanent responses ($d^P_{0,i}$) in both relative and absolute terms. By contrast, the permanent–transitory decomposition of Lien et al. (2025) identifies permanent responses only up to a scale factor,[5] reflecting the well-known normalization indeterminacy inherent in permanent–transitory decompositions.[6] These findings indicate that while existing permanent–transitory decompositions successfully capture relative adjustment patterns, only our approach (as well as the approach in Lautier et al. (2023)) consistently delivers correctly estimates of the permanent component that governs long-run price discovery.

[5]The scale factor equals $\sqrt{2}$, corresponding to the standard deviation of the permanent shock imposed in the simulation.

[6]With respect to transitory dynamics, both Lien et al. (2025) and Lautier et al. (2023) correctly recover the relative magnitudes of transitory responses across markets, but neither approach identifies the absolute scale of transitory adjustments. However, this does not affect the identification of the permanent shocks.

We next compare three magnitude-based price discovery measures—NLS, PILS, and CovIS—with the proposed Instantaneous Pricing Efficiency Share (IPES). The comparison highlights a fundamental distinction between two identifying principles: the Instantaneous Response Rule, which assigns leadership to the market with the largest contemporaneous reaction, and the Instantaneous Pricing Error Rule, which assigns leadership to the market whose response is closest to the efficient benchmark. Under regimes featuring delayed or symmetric adjustment, the two rules coincide and all measures deliver similar conclusions. However, in the presence of overreaction, the two rules diverge sharply. In such environments, magnitude-based measures systematically favor markets that react more aggressively—even when those reactions increase pricing errors—while IPES remains aligned with informational efficiency.

Row 1 of Table 1 reports an under-reaction regime in which Market 1 adjusts rapidly to permanent innovations ($\delta_1 = 0.9$) while Market 2 under-reacts ($\delta_2 = 0.5$). In this conventional setting, both identifying rules coincide. Market 1 exhibits both the larger contemporaneous response and the smaller pricing error, and is therefore correctly identified as the informational leader. All measures assign leadership to Market 1. IPES delivers the weakest point estimate (59.86%) but exhibits the smallest standard deviation (0.50), indicating superior finite-sample stability.

Row 2 reports the equal-adjustment benchmark ($\delta_1 = \delta_2 = 0.9$). In this neutral environment, the two rules again coincide and predict symmetric leadership. All measures assign Market 1 a share close to 50%. Notably, IPES displays the tightest concentration around the theoretical benchmark, while NLS and PILS exhibit greater dispersion due to their sensitivity to noise-driven variation in response magnitudes.

The contrast between the two identifying principles becomes pronounced under overshooting. Row 3 reports a symmetric overshooting regime in which both markets deviate equally from the efficient benchmark ($|0.9 - 1| = |1.1 - 1| = 0.1$). Because both markets are equally (in)efficient, the Instantaneous Pricing Error Rule predicts symmetric leadership—a prediction exactly matched by IPES (50.00%). In contrast, the Instantaneous Response Rule mechanically favors the overshooting market with the larger contemporaneous response. As a result, NLS, PILS, and CovIS assign Market 1 only 40.08%, 40.08%, and 44.99% of leadership, respectively.

This divergence is most extreme in the high-overshooting regime reported in Row 4, where Market 2 reacts excessively ($\delta_2 = 1.5$). Despite Market 1 being substantially closer to the efficient price path, magnitude-based measures overwhelmingly favor Market 2, assigning Market 1 only 26.46% (NLS), 26.46% (PILS), and 37.49% (CovIS) of leadership. In sharp contrast, IPES continues to identify Market 1 as the informational leader, assigning it a dominant share of 59.86%.

Taken together, these simulation results demonstrate that magnitude-based price discovery measures break down in environments characterized by overreaction. By equating informational leadership with response magnitude, the Instantaneous Response Rule rewards volatility rather than pricing accuracy. In contrast, the Instantaneous Pricing Error Rule—operationalized through IPES—consistently identifies the market that most efficiently incorporates permanent information across all regimes, delivering economically meaningful and robust leadership rankings.

4.2.2 Simulation Results for the Dominant-Satellite Model

Table 2 reports simulation results from the Dominant–Satellite (DS) model under four configurations that vary the degree of perverse adjustment by the satellite market. The four configurations correspond to the four rows of Table 2, ranging from mild to severe perverse responses in Market 2 and allowing for differing degrees of short-run mispricing in Market 1.

[Insert Table 2 about here.]

Across all simulation regimes, a clear and consistent pattern emerges in the structural parameter estimates. Because the variance of the permanent shock is normalized to unity

($\sigma_P^2 = 1$), the estimated initial permanent responses $\tilde{d}_{0,i}^P$ obtained from Lien et al. (2025) coincide numerically with those ($\hat{d}_{0,i}^P$) recovered by our price discovery beta approach. Moreover, under the normalization $\sigma_P^2 = 1$, the P–T decomposition of Gonzalo and Ng (2001) becomes observationally equivalent to the identifying restrictions imposed by Lautier et al. (2023).[7]

We next compare NLS, PILS, and CovIS with the proposed Instantaneous Pricing Efficiency Share (IPES) under the Dominant–Satellite data-generating process. As in the PPA model, the results underscore a fundamental distinction between the Instantaneous Response Rule and the Instantaneous Pricing Error Rule. When the satellite market's perverse adjustment is mild, the two identifying principles coincide and all measures deliver similar leadership rankings. However, as the degree of perverse adjustment intensifies, the two rules diverge sharply. In such environments, magnitude-based measures systematically reward larger—yet economically destabilizing—responses, even when they amplify pricing errors or reverse the sign of the efficient reaction. In contrast, IPES remains tightly aligned with informational efficiency, consistently assigning leadership to the market whose response is closest to the efficient benchmark.

In Scenario 1 (mild perverse satellite response), Market 1 incorporates permanent information efficiently ($d_{0,1}^P = 1$), while Market 2 exhibits a modest negative response ($d_{0,2}^P = -0.5$). In this case, the two identifying rules coincide. Under the *Instantaneous Response Rule*, Market 1 is favored because its response is larger in absolute terms than that of Market 2. The *Instantaneous Pricing Error Rule* reaches the same conclusion, as Market 1's response lies exactly at the efficient benchmark. Correspondingly, all measures correctly assign leadership to Market 1. IPES delivers the strongest and most stable signal (81.37%), NLS and PILS assign slightly lower shares (79.97%), whereas CovIS produces an inflated

[7] In contrast, identification of transitory dynamics remains fundamentally limited. Both Lien et al. (2025) and Lautier et al. (2023) recover only the relative magnitudes of transitory responses across markets, while the absolute scale and the sign of transitory adjustments remains unidentified. These results indicate that existing P–T decomposition schemes and equal-variance normalizations can at best recover relative transitory adjustment patterns. The true scale and sign of transitory responses, however, remain unidentified under prevailing identification strategies.

value (202.96%). The inflated CovIS values occur because it is calculated as the ratio of raw $d_{0,i}^P$ to the sum of $d_{0,i}^P$. When one market exhibits a negative response, the denominator can shrink or even become negative, causing the other market's share to exceed 100% or take implausible negative values, highlighting the instability of this approach compared with using squared values.

In Scenario 2 (high perverse satellite response), Market 1 remains efficient ($d_{0,1}^P = 1$), while Market 2 exhibits a large negative response ($d_{0,2}^P = -1.5$). In this case, the two rules diverge. The *Instantaneous Response Rule* and magnitude-based measures incorrectly favor Market 2 because its absolute response is larger ($|-1.5| > |1|$), despite being a perverse reaction. In contrast, the *Instantaneous Pricing Error Rule* correctly identifies Market 1 as the leader. The simulation results reflect this divergence: IPES assigns 92.22% to Market 1, NLS and PILS assign only 30.86%, and CovIS produces extreme negative estimates (-203.85%), highlighting its instability under strong perverse adjustments.

Scenario 3 (dominant market with underreaction and mild perverse satellite response) features Market 1 as the true dominant market, with an underreaction of $d_{0,1}^P = 0.5$, while Market 2 responds mildly perverse with $d_{0,2}^P = -0.5$. At the population level, magnitude-based rules that rely on absolute responses would assign equal leadership to both markets (50:50) because $|d_{0,1}^P| = |d_{0,2}^P|$, despite Market 1 being the true leader. In finite-sample simulations, however, the estimated responses $\hat{d}_{0,i}^P$ are slightly asymmetric, which produces the NLS and PILS measure averages of 55.34% for Market 1 in Table 2, reflecting estimation noise in a finite sample rather than a success of the identification principle. By contrast, IPES correctly identifies Market 1 as the dominant market, assigning 72.56%, while CovIS becomes numerically unstable in this configuration, producing explosive estimates with extremely large dispersion, highlighting its fragility when the satellite response is perverse and the denominator approaches zero.

In Scenario 4 (dominant market with underreaction and high perverse satellite response), Market 1 underreacts ($d_{0,1}^P = 0.5$) while Market 2 exhibits a strong negative response

($d^P_{0,2} = -1.5$). Magnitude-based rules misclassify leadership toward Market 2 due to its larger absolute response ($|0.5| < |-1.5|$). In contrast, IPES correctly identifies Market 1 as the dominant market, assigning 88.05%.

Overall, these dominant–satellite simulations reinforce and extend the lessons from the PPA model. When short-run distortions are mild (Scenario 1), the two rules agree. However, under strong perverse responses (Scenarios 2 and 4), magnitude-based measures fail, misclassifying the satellite market as the leader. The *Instantaneous Pricing Error Rule*, operationalized through IPES, consistently identifies the dominant market and provides stable, economically meaningful leadership estimates across all configurations.

5 Empirical Illustrations

To empirically illustrate the theoretical advantages of the Pricing Error Rule, we analyze two market episodes marked by extreme volatility and severe liquidity breakdowns: the 2010 Flash Crash in U.S. equity markets and the 2024 liquidation cascade in cryptocurrency markets.

In both applications, we adopt a rolling-window estimation strategy to capture the time-varying nature of price discovery and informational leadership. This dynamic framework allows us to trace the behavior of competing measures as markets transition from normal trading conditions to periods of acute dislocation. We compare the proposed Instantaneous Pricing Efficiency Share (IPES) with the magnitude-based NLS, PILS and CovIS.

5.1 Case I: The 2010 Flash Crash

We first examine the "Flash Crash" of May 6, 2010, a natural experiment in which two financially identical assets—the SPDR S&P 500 ETF (SPY) and the iShares Core S&P 500 ETF (IVV)—temporarily decoupled. Our analysis employs 1-second high-frequency quote data from NASDAQ, obtained via the TAQ database. VECM and structural parameters are estimated using a rolling window of 60 minutes with a 5-minute step size.

Figure 2 summarizes the results. Panel A illustrates the price decoupling during the crash. Between 14:40 and 15:00 EDT, a rapid evaporation of liquidity caused IVV (orange dashed line) to collapse far more severely than SPY, exhibiting extreme negative overshooting before rebounding. This episode creates a clear "conflict zone": while SPY remains relatively stable and closer to the fundamental value, IVV displays large, noise-driven price fluctuations.

[Insert Figure 2 here.]

Panel B plots the evolution of the instantaneous permanent response estimates ($d^P_{0,i}$) by our price discovery beta approach. Prior to the crash, both responses fluctuate around unity, consistent with efficient and symmetric price discovery. At the height of the crash, however, the IVV response coefficient ($d^P_{0,2}$) plunges deeply into negative territory, reaching values below -2. This sign reversal reflects a *perverse response*, whereby prices initially move violently away from the permanent information shock, indicating a qualitative breakdown of the price discovery mechanism rather than a mere overreaction in magnitude.

Panel C reports the estimated leadership shares for SPY (Market 1). The divergence across identification rules is stark. The NLS/PILS measure (gray dashed line) assigns SPY an almost negligible leadership share (approximately 5%) during the crash. Because NLS/PILS rewards the magnitude of the response through $|d^P_{0,i}|^2$, it mechanically designates IVV as the dominant leader precisely when its response is most distorted. The CovIS measure (teal dot-dash line) performs no better, falling into negative territory (around -40%), a pathological outcome driven by the perverse response of the overshooting market.

In sharp contrast, the IPES measure (blue solid line) correctly identifies SPY as the informational leader, with its leadership share jumping to around 95% during the dislocation. By defining leadership in terms of pricing-error minimization, IPES penalizes perverse and excessive responses and robustly anchors price discovery to the market that remains closest to the efficient benchmark.

In the Appendix, we further document the dynamic evolution of both the reduced-form VECM parameters and the recovered structural parameters during the Flash Crash in Figure A1. As shown in Figure A1, the variance of the IVV residuals (σ^2_{IVV}) spikes sharply during the crash, while the correlation coefficient ρ between the residuals collapses to nearly zero, signaling a temporary breakdown in cross-market information transmission.

Consistent with this disruption, the estimated common-factor weights exhibit abrupt reallocation: the weight assigned to SPY rises sharply, whereas the weight on IVV declines precipitously. Also, the error correction coefficient on SPY rapidly moves to zero indicating that SPY no longer reacts to the disequilibrium error between SPY and IVV and instead follows the permanent shock. Turning to the structural parameters, both IVV's initial permanent and transitory responses become strongly negative, indicating a pronounced short-run perverse overreaction. At the same time, the volatility of the permanent shock, σ^2_P (in logs), also spikes during the crash, reflecting an abrupt surge in fundamental uncertainty.

Taken together, these dynamic patterns clarify why magnitude-based price discovery measures perform poorly during market dislocations. By contrast, IPES remains stable precisely because it explicitly penalizes deviations from the efficient price path rather than rewarding the size of price responses.

5.2 Case II: Cryptocurrency Liquidation Cascade

Our second application examines a "liquidation cascade" in the Bitcoin market on January 3, 2024. Cryptocurrency trading is fragmented between Spot and Perpetual Futures markets, with the latter typically exhibiting amplified volatility due to high leverage. We use 1-second transaction data from Binance for BTC/USDT Spot and Perpetual Futures, and estimate vector error correction models and structural parameters using a rolling window of 60 minutes with a 5-minute step size.

Figure 3 summarizes the results. Panel A shows a sharp price decline around 12:00 UTC, followed by persistent fluctuations around a substantially lower price level. Panel B reports the evolution of the instantaneous response coefficients. Prior to the crash, the responses fluctuate around $(d^P_{0,1}, d^P_{0,2}) \in (0.5, 1)$, with the Futures market exhibiting a slightly larger contemporaneous response, suggesting that it leads price discovery under normal market conditions. Around 12:00 UTC, both markets experience a sharp drop in their instantaneous responses, with the decline being markedly stronger for the Futures market. Notably, both responses briefly enter the *perverse regime* ($d^P_{0,i} < 0$), indicating that prices initially move in the opposite direction of the permanent information shock. Such perverse responses represent a qualitative breakdown of the price discovery mechanism rather than a mere attenuation of adjustment.

[Insert Figure 3 here.]

Following this perverse phase, the two markets diverge. The Spot market response rebounds to a normal range (approximately 0.7–1), whereas the Futures market response overshoots substantially, exceeding 1.25. This pattern illustrates that perverse responses and overshooting can occur in close succession. Moreover, the Futures market reacts more aggressively—first perversely and then excessively—to the fundamental shock, consistent with automated liquidations of leveraged positions amplifying short-run price dynamics.

Panel C compares the implied leadership shares of the Spot market across different measures. The NLS/PILS measure (grey dashed line) exhibits a sharp, transient collapse around the cascade (briefly dropping toward zero), and otherwise remains materially below IPES, typically in the 30–40% range. This behavior reflects the mechanical nature of magnitude-based measures, which reward excessive reactions even when they follow a perverse response. The CovIS measure (teal dot-dash line) remains relatively stable around 35–45% but similarly fails to decisively penalize the Futures market for its inefficiency during the cascade. In contrast, the IPES measure (blue solid line) diverges sharply around 12:00 UTC. By recognizing that the Spot market's response remains closer to the efficient benchmark of unity, IPES assigns a dominant leadership share to the Spot market during the cascade, peaking above 50% and remaining above the magnitude-based alternatives.

In the Appendix, we further document the dynamic evolution of both the reduced-form VECM parameters and the recovered structural parameters during the Bitcoin crash in Figure A2. As shown in Figure A2, both reduced-form and structural parameters exhibit substantially greater volatility in the Bitcoin spot and futures markets than in the Flash Crash episode, reflecting the more fragmented and less regulated nature of cryptocurrency trading. Specifically, the variances of both spot and futures residuals spike sharply during the crash, while the correlation coefficient ρ declines markedly but remains at a relatively high level, indicating partial—but not complete—disruption of common information transmission. The estimated common-factor loadings also display pronounced turbulence, fluctuating sharply during the crash and remaining highly volatile in its aftermath.

At the structural level, both markets exhibit a sharp initial contraction in their instantaneous permanent responses, followed by a rapid rebound, with both the decline and the rebound being substantially more pronounced in the futures market. By contrast, the initial transitory responses display a persistent structural polarity: the spot market maintains consistently positive adjustments, whereas the futures market remains in negative territory. This asymmetry intensifies markedly during the crash and around 19:00 UTC, when the futures market experiences explosive negative excursions while the spot market remains comparatively resilient, highlighting pronounced heterogeneity in short-run adjustment dynamics across trading venues. At the same time, the volatility of the permanent shock spikes sharply, reflecting an abrupt surge in fundamental uncertainty. Taken together, these dynamics clarify the mechanism underlying the divergence in price discovery measures: periods characterized by elevated residual variance, collapsing cross-market correlation, and explosive permanent responses are precisely those in which magnitude-based rules systematically reward instability rather than informational efficiency.

Collectively, these two case studies show that magnitude-based measures (NLS, PILS, and CovIS) break down in conflict zones, where overshooting and perverse responses dominate short-run price dynamics. By mechanically rewarding response magnitude, these measures misidentify leadership precisely when markets are most inefficient. In contrast, IPES remains stable and economically interpretable, providing a robust measure of price discovery efficiency even in extreme market conditions.

6 Conclusion

This paper revisits the foundations of price discovery measurement in fragmented markets and identifies a fundamental limitation shared by a broad class of modern metrics. We show that existing measures—including PILS, NLS, and CovIS—are unified by an implicit *instantaneous response rule* that equates informational leadership with the magnitude of a market's contemporaneous reaction to permanent shocks. While this rule performs adequately when short-run price adjustments are well behaved, it becomes systematically misleading in environments characterized by overshooting or perverse responses, where large reactions reflect transient frictions rather than efficient information incorporation.

To address this limitation, we propose an alternative identifying principle—the *instantaneous pricing error rule*—which defines leadership by the accuracy, rather than the aggressiveness, of the initial price adjustment. Building on this principle, we introduce the Instantaneous Pricing Efficiency Share (IPES), a structurally grounded, scale-free measure that evaluates how closely each market's contemporaneous response aligns with the long-run equilibrium effect of permanent information.

Simulation evidence from both partial price adjustment and dominant–satellite models demonstrates that the instantaneous response rule and the pricing error rule coincide only under efficient adjustment. When overshooting or perverse responses occur, magnitude-based measures systematically misclassify leadership, often rewarding the least efficient market. In contrast, IPES consistently identifies the true informational leader across all regimes, remains stable under strong noise, and avoids pathological outcomes such as negative or explosive shares.

Empirical applications to the 2010 Flash Crash and the 2024 cryptocurrency liquidation cascade further highlight the practical relevance of these findings. In both cases, magnitude-based measures mislabel markets exhibiting extreme volatility and perverse adjustments as leaders, whereas IPES robustly identifies the market closest to the efficient price path, yielding economically interpretable leadership dynamics even under extreme stress.

More broadly, our results suggest that price discovery should be understood as a problem of *pricing accuracy* rather than reaction intensity. In modern high-frequency, algorithmically driven markets, large instantaneous price movements may reflect liquidity constraints, forced trading, or feedback effects rather than superior information processing. Measures that fail to distinguish between these forces risk conflating noise amplification with informational leadership.

By disentangling reaction magnitude from adjustment accuracy, the Instantaneous Pricing Efficiency framework establishes a coherent, economically grounded foundation for price discovery analysis. Rather than inferring leadership from the size of the initial response, IPES evaluates whether markets incorporate permanent information correctly upon impact, using the long-run equilibrium response as a meaningful benchmark. In this sense, IPES is not merely an incremental refinement but a conceptual substitute for magnitude-based measures, particularly in environments characterized by transient frictions, overshooting, or noisy trading. By penalizing mispricing rather than rewarding aggressiveness, IPES provides a more reliable and interpretable measure of informational leadership precisely when conventional approaches are most prone to failure.

An important direction for future research is to extend this framework to the full identification of transitory shocks and their dynamic responses, enabling a unified assessment of permanent information incorporation and short-run pricing distortions. Further extensions may also explore how pricing errors interact with market design, trading frictions, and institutional features that shape price formation.

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Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT in order to improve readability and language. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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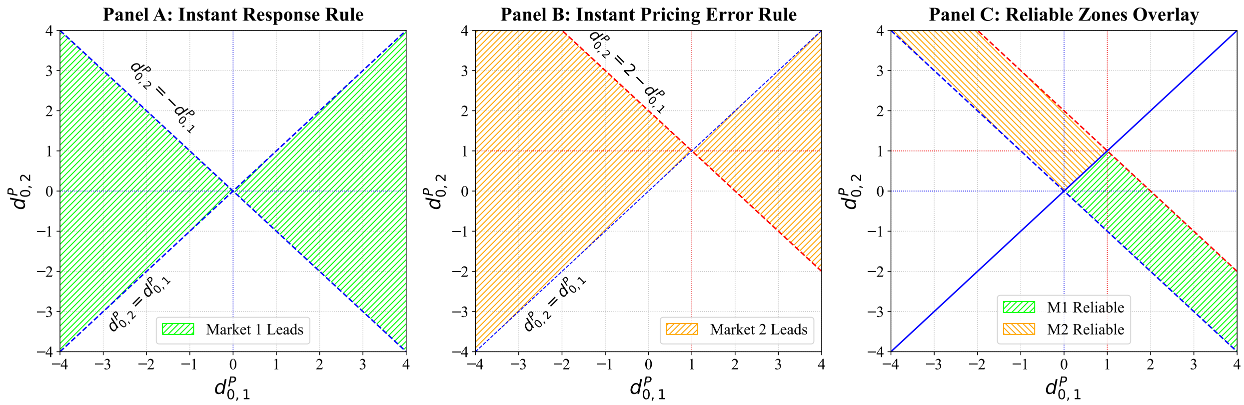

Figure 1: Comparison of Leadership Identification Rules. Panel A: Market 1 leadership under the Instantaneous Response Rule (magnitude-based). **Panel B:** Market 2 leadership under the Instantaneous Pricing Error Rule (pricing-error based). **Panel C:** Overlay of the two criteria: colored areas indicate agreement (*Reliable Zones*), while unshaded white areas indicate disagreement (*Conflict Zones*) arising from overshooting or perverse adjustments.

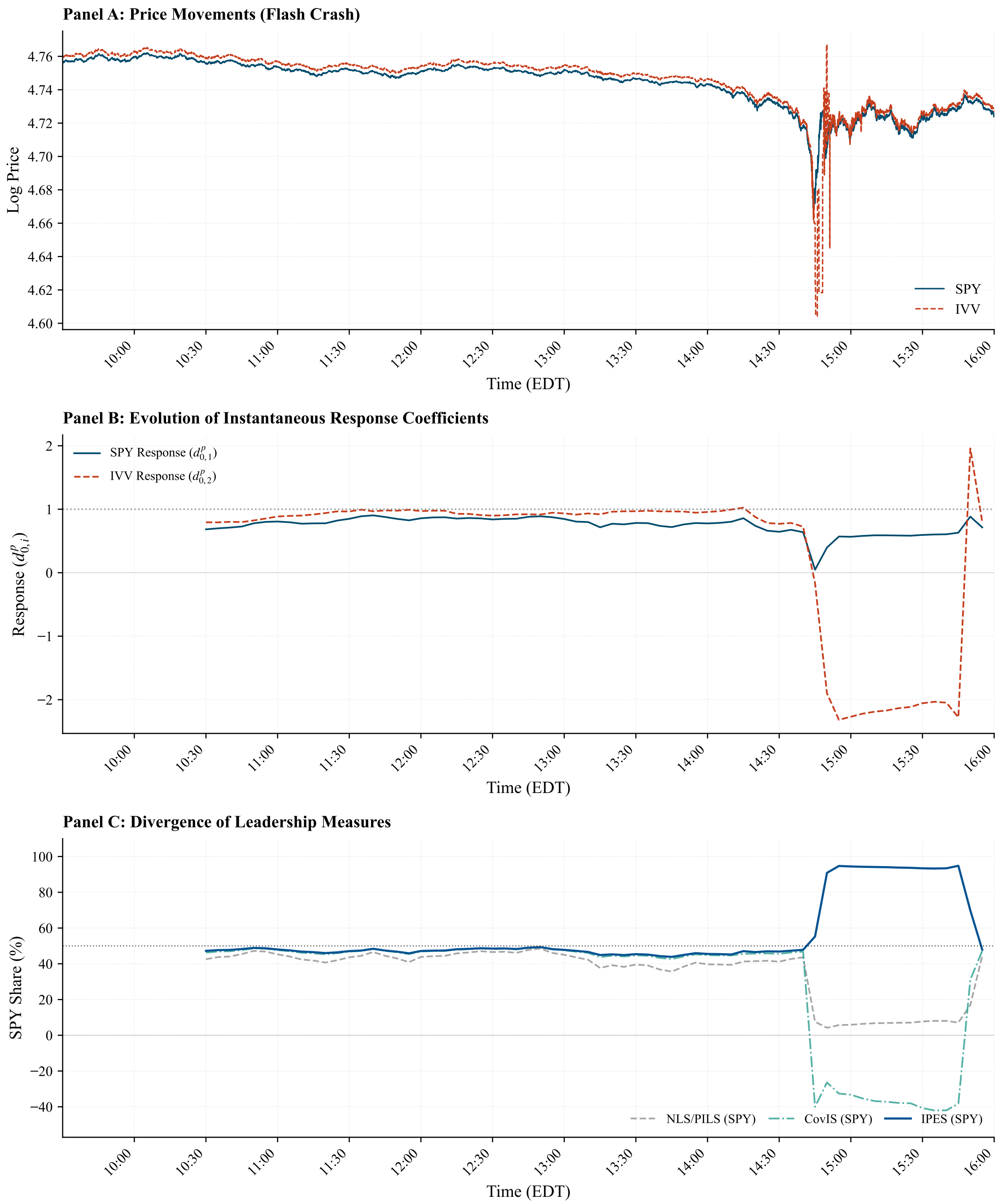

Figure 2: Dynamic Price Discovery during the Flash Crash (May 6, 2010). **Panel A:** Log prices of SPY and IVV, highlighting the severe perverse response of IVV (orange dashed) relative to SPY (blue solid). **Panel B:** Instantaneous response coefficients; note the pronounced negative spike in the IVV response. **Panel C:** Leadership shares for SPY. During the crash (around 14:45), NLS/PILS (gray dashed) and CovIS (teal dot-dash) erroneously assign leadership to the crashing IVV market, whereas IPES (blue solid) correctly identifies SPY as the leader.

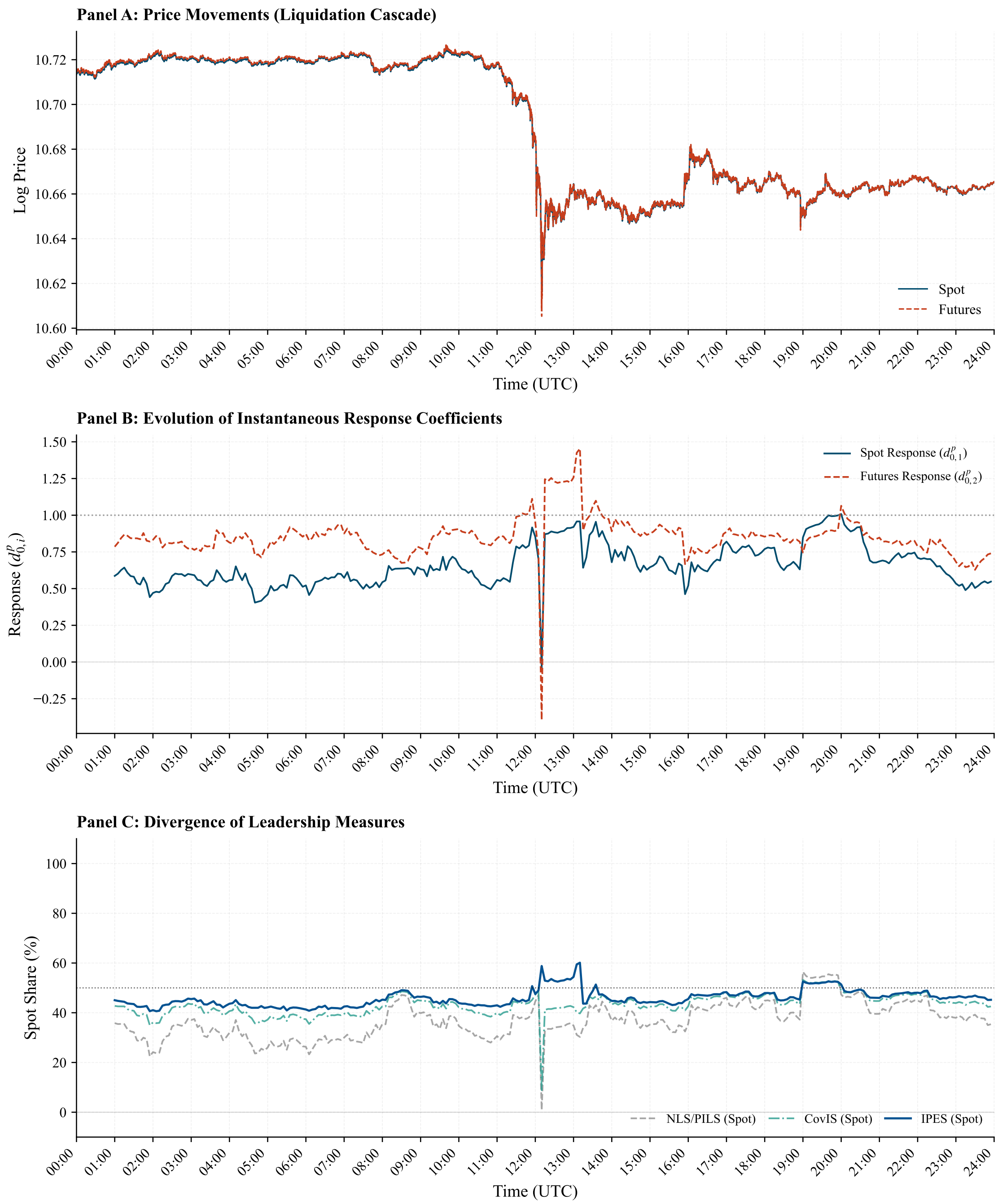

Figure 3: Dynamic Price Discovery during the Bitcoin Crash (Jan 3, 2024). Panel A: Price dislocation between BTC/USDT Spot and Perpetual Futures. **Panel B:** Instantaneous permanent response coefficients, highlighting extreme volatility in the Futures market (red/orange). **Panel C:** Leadership measures for the Spot market. NLS/PILS (gray dashed) erroneously assign leadership to the Futures market due to its large response magnitude, whereas IPES (blue solid) correctly identifies the Spot market as the leader by penalizing the Futures market's inefficient adjustment during the crash.

Table 1: Partial Price Adjustment Model with Under-reaction and Overshooting

	Setup		Estimates of $d_{0,i}^P$		Estimates from NLS				Estimates of $d_{0,i}^T$		Market 1 Shares (%)			
Scenario	δ_1	δ_2	$\hat{\delta}_1$	$\hat{\delta}_2$	$\tilde{\delta}_1$	$\tilde{\delta}_2$	$\tilde{d}_{0,1}^T$	$\tilde{d}_{0,2}^T$	$\breve{d}_{0,1}^T$	$\breve{d}_{0,2}^T$	NLS_1	$PILS_1$	$CovIS_1$	$IPES_1$
1. Under-reaction	0.9	0.5	0.90	0.50	1.27	0.71	1.12	-1.12	0.79	-0.79	**76.38**	**76.38**	**64.27**	**59.86**
			(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.01)	(0.02)	(0.01)	(1.20)	(1.20)	(0.76)	(0.50)
2. Equal-adjustment	0.9	0.9	0.90	0.90	1.27	1.27	1.12	-1.12	0.79	-0.79	**49.98**	**49.98**	**49.99**	**49.99**
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(1.04)	(1.04)	(0.52)	(0.47)
3. Symmetric Overshoot	0.9	1.1	0.90	1.10	1.27	1.56	1.12	-1.12	0.79	-0.79	40.08	40.08	44.99	**50.00**
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.89)	(0.89)	(0.46)	(0.39)
4. High Overshoot	0.9	1.5	0.90	1.50	1.27	2.12	1.12	-1.12	0.79	-0.79	26.46	26.46	37.49	**59.86**
			(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.02)	(0.01)	(0.02)	(0.61)	(0.61)	(0.36)	(0.42)

Note: This table reports average leadership shares for Market 1 across 1,000 Monte Carlo simulations (standard deviations in brackets) based on the partial price adjustment model in (26). Each simulation contains 23,400 observations. $\hat{\delta}_i$ denotes the initial permanent response estimated using the price discovery beta in (13). $\tilde{\delta}_i$ and $\tilde{d}_{0,i}^T$ denote the initial impact matrix identified by Lien et al. (2025). $\breve{d}_{0,i}^T$ denotes the initial transitory responses identified by Lautier et al. (2023). Simulation parameters are set as $\sigma_P^2 = 2$, $\sigma_T^2 = 5$, $b_{0,1}^T = 0.5$, and $b_{0,2}^T = -0.5$. **Bold values** indicate that the corresponding measure correctly identifies the price leader.

Table 2: Dominant–Satellite Model with Perverse Adjustments

	DGP				Estimates of $d_{0,i}^P$		Estimates from NLS				Estimates of $d_{0,i}^T$		Market 1 Shares (%)			
Scenario	$b_{0,1}^P$	$b_{0,2}^P$	$d_{0,1}^P$	$d_{0,2}^P$	$\hat{d}_{0,1}^P$	$\hat{d}_{0,2}^P$	$\tilde{d}_{0,1}^P$	$\tilde{d}_{0,2}^P$	$\tilde{d}_{0,1}^T$	$\tilde{d}_{0,2}^T$	$\breve{d}_{0,1}^T$	$\breve{d}_{0,2}^T$	NLS_1	$PILS_1$	$CovIS_1$	$IPES_1$
1. Mild perverse satellite	0	-0.5	1	-0.5	1.00	-0.50	1.00	-0.50	-0.22	-2.01	-0.22	-2.01	**79.97**	**79.97**	**202.96**	**81.37**
					(0.03)	(0.06)	(0.01)	(0.06)	(0.03)	(0.02)	(0.03)	(0.06)	(3.82)	(3.82)	(26.01)	(0.92)
2. High perverse satellite	0	-1.5	1	-1.5	1.00	-1.50	1.00	-1.50	-0.22	-2.01	-0.22	-2.01	30.86	30.86	-203.85	**92.22**
					(0.03)	(0.07)	(0.01)	(0.06)	(0.03)	(0.04)	(0.03)	(0.08)	(1.88)	(1.88)	(28.46)	(0.51)
3. Underreaction + mild perverse	-0.5	-0.5	0.5	-0.5	0.51	-0.46	0.52	-0.47	-0.21	-2.02	-0.21	-2.00	**55.34**	**55.34**	**122749.61**	**72.60**
					(0.02)	(0.06)	(0.01)	(0.06)	(0.01)	(0.02)	(0.02)	(0.06)	(6.78)	(6.78)	(3871936.28)	(1.14)
4. Underreaction + high perverse	-0.5	-1.5	0.5	-1.5	0.50	-1.50	0.50	-1.50	-0.22	-2.01	-0.22	-2.01	10.07	10.07	-50.34	**88.05**
					(0.02)	(0.07)	(0.01)	(0.06)	(0.01)	(0.04)	(0.02)	(0.08)	(0.94)	(0.94)	(3.96)	(0.78)

Note: This table reports average leadership shares for Market 1 across 1,000 Monte Carlo simulations (standard deviations in brackets) based on the Dominant–Satellite model in (29). The true informational leader is Market 1 in all designs. $\hat{d}_{0,i}^P$ denotes the initial permanent response estimated using the price discovery beta in (13). $\tilde{d}_{0,i}^P$ and $\tilde{d}_{0,i}^T$ denote the initial impact matrix identified by Lien et al. (2025). $\breve{d}_{0,i}^T$ denotes the initial transitory responses identified by Lautier et al. (2023). Simulation parameters are set as $\sigma_P^2 = 1$, $\sigma_T^2 = 5$, $b_{0,1}^T = 0.1$, and $b_{0,2}^T = 0.9$. **Bold values** indicate that the corresponding measure correctly identifies the price leader.

Internet Appendix to

Misguided Price Discovery: When Overshooting Is Mistaken for Leadership

A1 Identification of the SMA Model

As shown in Yan and Zivot (2010), when the initial impact matrix $\boldsymbol{D}_0$ is invertible, the structural shocks can be expressed as linear transformations of the reduced-form innovations, $\boldsymbol{\eta}_t = \boldsymbol{D}_0^{-1}\boldsymbol{\varepsilon}_t$. Elementary algebra yields

$$\eta_t^P = \frac{d_{0,2}^T}{\Delta}\varepsilon_{1t} - \frac{d_{0,1}^T}{\Delta}\varepsilon_{2t}, \quad \eta_t^T = -\frac{d_{0,2}^P}{\Delta}\varepsilon_{1t} + \frac{d_{0,1}^P}{\Delta}\varepsilon_{2t}, \tag{A.1}$$

where $\Delta = |\boldsymbol{D}_0| = d_{0,1}^P d_{0,2}^T - d_{0,1}^T d_{0,2}^P$ denotes the determinant of $\boldsymbol{D}_0$.

Given the identification of the permanent shock in (10), the parameters ψ_1 and ψ_2 are linked to the initial impact impact structural parameters in $\boldsymbol{D}_0$ through:

$$\psi_1 = \frac{d_{0,2}^T}{\Delta}, \quad \psi_2 = -\frac{d_{0,1}^T}{\Delta}, \tag{A.2}$$

and hence the full matrix $\boldsymbol{D}_0$ can be solved as

$$\boldsymbol{D}_0 = \begin{pmatrix} \frac{\psi_1\sigma_1^2 + \psi_2\sigma_{12}}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & -\psi_2\Delta \\ \frac{\psi_1\sigma_{12} + \psi_2\sigma_2^2}{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} & \psi_1\Delta \end{pmatrix}. \tag{A.3}$$

This representation makes clear that the transitory impact vector is identified only up to a scalar factor: while the relative loading $-\psi_2/\psi_1$ is uniquely determined, the overall scale Δ remains unrestricted without an additional normalization. However, for the purpose of price discovery, identification of the permanent shock and its associated impact vector is sufficient. We therefore refrain from imposing further normalization assumptions on the transitory shock and leave full identification of the SMA model to future research.

To illustrate identification of the bivariate SMA model in Lien et al. (2025), note that the long-run impact matrix $\boldsymbol{\Psi}(1)$ can be expressed in terms of VECM estimates as in (6). When $\boldsymbol{\beta} = (1, -1)'$, the orthogonal complement $\boldsymbol{\beta}_\perp$ is a 2×1 vector with equal elements, implying that

$$\boldsymbol{\alpha}_\perp = \xi \boldsymbol{\psi}, \tag{A.4}$$

where $\boldsymbol{\psi} = (\psi_1, \psi_2)'$ is the common row vector of $\boldsymbol{\Psi}(1)$ and $\xi \neq 0$ is a scalar. Although $\boldsymbol{\alpha}_\perp$ is defined only up to scale, we show below that the sign of ξ is not innocuous once orthogonalization is imposed.

Under the transformation of Gonzalo and Ng (2001), the rotation matrix is

$$\boldsymbol{G} = \begin{pmatrix} \boldsymbol{\alpha}_\perp' \\ \boldsymbol{\beta}' \end{pmatrix} = \begin{pmatrix} \xi\psi_1 & \xi\psi_2 \\ 1 & -1 \end{pmatrix}, \tag{A.5}$$

with an inverse:

$$\boldsymbol{G}^{-1} = \frac{1}{\xi(\psi_1 + \psi_2)} \begin{pmatrix} 1 & \xi\psi_2 \\ 1 & -\xi\psi_1 \end{pmatrix}. \tag{A.6}$$

The variance-covariance matrix of the transformed residual $\boldsymbol{\epsilon_t} = \boldsymbol{G}\boldsymbol{\varepsilon}_t$ is given as:

$$\boldsymbol{\Sigma}_\epsilon = \boldsymbol{G}\boldsymbol{\Omega}\boldsymbol{G}' = \begin{pmatrix} \xi^2(\psi_1^2\sigma_1^2 + 2\psi_1\psi_2\sigma_{12} + \psi_2^2\sigma_2^2) & \xi(\psi_1\sigma_1^2 - (\psi_1 - \psi_2)\sigma_{12} - \psi_2\sigma_2^2) \\ \xi(\psi_1\sigma_1^2 - (\psi_1 - \psi_2)\sigma_{12} - \psi_2\sigma_2^2) & \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \end{pmatrix}. \tag{A.7}$$

For notational convenience, define

$$s \equiv \sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}} = \sqrt{\psi_1^2\sigma_1^2 + 2\psi_1\psi_2\sigma_{12} + \psi_2^2\sigma_2^2}, \qquad m \equiv \psi_1\sigma_1^2 - (\psi_1 - \psi_2)\sigma_{12} - \psi_2\sigma_2^2.$$

Then the covariance matrix $\boldsymbol{\Sigma}\epsilon$ has elements:

$$\Sigma_{\epsilon,11} = \xi^2 s^2, \quad \Sigma_{\epsilon,12} = \xi m.$$

.

A Cholesky factorization $\boldsymbol{\Sigma}_\epsilon = \boldsymbol{L}\boldsymbol{L}'$ with positive diagonal elements yields

$$\boldsymbol{L} = \begin{pmatrix} \sqrt{\Sigma_{\epsilon,11}} & 0 \\ \Sigma_{\epsilon,12}/\sqrt{\Sigma_{\epsilon,11}} & \sqrt{\Sigma_{\epsilon,22} - \Sigma^2_{\epsilon,12}/\Sigma_{\epsilon,11}} \end{pmatrix} = \begin{pmatrix} |\xi|\, s & 0 \\ \frac{\xi}{|\xi|}\, a & b \end{pmatrix}, \qquad (\xi \neq 0), \tag{A.8}$$

where

$$a \equiv m/s, \quad b \equiv \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} - a^2} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} - \frac{m^2}{s^2}} > 0.$$

Crucially, the Cholesky convention absorbs the magnitude of ξ into L_{11} while preserving its sign in the off-diagonal element L_{21}.

The inverse of $\boldsymbol{L}$ can be shown as:

$$\boldsymbol{L}^{-1} = \begin{pmatrix} \frac{1}{|\xi|\, s} & 0 \\ -\frac{(\xi/|\xi|)\, a}{|\xi|\, s\, b} & \frac{1}{b} \end{pmatrix}.$$

Then the structural shocks $\tilde{\boldsymbol{\eta}}_t = \boldsymbol{L}^{-1}\boldsymbol{G}\boldsymbol{\varepsilon}_t$ can be shown as

$$\tilde{\boldsymbol{\eta}}_t = \boldsymbol{L}^{-1}\boldsymbol{G}\boldsymbol{\varepsilon}_t = \begin{pmatrix} \frac{\xi}{|\xi|}\frac{\psi_1}{s} & \frac{\xi}{|\xi|}\frac{\psi_2}{s} \\ \frac{1}{b}\left(1 - \frac{a\psi_1}{s}\right) & \frac{1}{b}\left(-1 - \frac{a\psi_2}{s}\right) \end{pmatrix} \boldsymbol{\varepsilon}_t. \tag{A.9}$$

It follows that the permanent shock satisfies

$$\tilde{\eta}_t^P = \frac{\xi}{|\xi|}\frac{\boldsymbol{\psi}'\boldsymbol{\varepsilon}_t}{\sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}},$$

so reversing the sign of ξ flips the sign of the permanent shock.

The initial impact matrix of the structural shocks can be shown as:

$$\tilde{\boldsymbol{D}}_0 = \boldsymbol{G}^{-1}\boldsymbol{L} = \frac{1}{\xi(\psi_1 + \psi_2)} \begin{pmatrix} 1 & \xi\psi_2 \\ 1 & -\xi\psi_1 \end{pmatrix} \begin{pmatrix} |\xi| s & 0 \\ (\xi/|\xi|)a & b \end{pmatrix}.$$

Carrying out the multiplication and simplifying using $|\xi|/\xi = \xi/|\xi|$ yields

$$\tilde{\boldsymbol{D}}_0 = \frac{1}{s} \begin{pmatrix} \frac{\xi}{|\xi|}\left(\psi_1\sigma_1^2 + \psi_2\sigma_{12}\right) & \psi_2\sqrt{\sigma_1^2\sigma_2^2 - \sigma_{12}^2} \\ \frac{\xi}{|\xi|}\left(\psi_1\sigma_{12} + \psi_2\sigma_2^2\right) & -\psi_1\sqrt{\sigma_1^2\sigma_2^2 - \sigma_{12}^2} \end{pmatrix}. \tag{A.10}$$

This expression makes clear that the permanent-impact vector, which is the first column, inherits the sign factor $\xi/|\xi|$, while the transitory-impact vector, which is the second column, does not.

Since the long-run impact matrix is $\tilde{\boldsymbol{D}}(1) = \boldsymbol{\Psi}(1)\tilde{\boldsymbol{D}}_0$, we obtain

$$\tilde{\boldsymbol{D}}(1) = \begin{pmatrix} \boldsymbol{\psi}' \\ \boldsymbol{\psi}' \end{pmatrix} \tilde{\boldsymbol{D}}_0 = \begin{pmatrix} \frac{\xi}{|\xi|} s & 0 \\ \frac{\xi}{|\xi|} s & 0 \end{pmatrix},$$

and the long-run response to the permanent shock is

$$\boldsymbol{\psi}' \, \tilde{\boldsymbol{D}}_{0,1} = \boldsymbol{\psi}' \left(\frac{\xi}{|\xi|} \frac{\boldsymbol{\Omega\psi}}{s} \right) = \frac{\xi}{|\xi|} \frac{\boldsymbol{\psi}'\boldsymbol{\Omega\psi}}{s} = \frac{\xi}{|\xi|} \, s,$$

where $\tilde{\boldsymbol{D}}_{0,1}$ denotes the first column of $\tilde{\boldsymbol{D}}_0$.

Since $s = \sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega\psi}} > 0$, the economically natural normalization that the permanent shock has a positive long-run effect implies

$$\tilde{D}_{11}(1) > 0 \quad \Longleftrightarrow \quad \xi > 0.$$

This result clarifies an important identification point. While $\boldsymbol{\alpha}_\perp = \xi\boldsymbol{\psi}$ is indeed defined only up to a nonzero scalar and the absolute value of ξ does not impact $\tilde{\boldsymbol{D}}_0$ and $\tilde{\boldsymbol{D}}_1$, the sign of ξ determines the sign of the permanent shock and of the long-run impact matrix. Consequently, the sign of ξ is not arbitrary. Imposing the natural convention that the permanent shock raises the long-run efficient price uniquely selects $\xi > 0$, which we adopt throughout the paper.

A2 Structural and Reduced-form Representations of Price Discovery Measures

To better illustrate the difference among existing price discovery measures, we repeat the definitions of PIL, PILS, NLS, and CovIS in this section. All of these measures are defined based on the VECM estimates from Equation (3).

As shown in Shen et al. (2025), the improved information leadership measures (PIL) and its share variant PILS can be derived from the reduced-form VECM parameters as:

$$\begin{aligned}
\text{PIL}_1 &= \left|\frac{\psi_1\sigma_1^2 + \psi_2\sigma_{12}}{\psi_2\sigma_2^2 + \psi_1\sigma_{12}}\right|, \quad \text{PIL}_2 = \left|\frac{\psi_2\sigma_2^2 + \psi_1\sigma_{12}}{\psi_1\sigma_1^2 + \psi_2\sigma_{12}}\right|, \\
\text{PILS}_1 &= \frac{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2}{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2 + (\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2}, \\
\text{PILS}_2 &= \frac{(\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2}{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2 + (\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2},
\end{aligned} \tag{A.11}$$

with structural representations:

$$\begin{aligned}
\text{PIL}_1 &= \left|\frac{d_{0,1}^P}{d_{0,2}^P}\right|, \quad \text{PIL}_2 = \left|\frac{d_{0,2}^P}{d_{0,1}^P}\right|. \\
\text{PILS}_1 &= \frac{(d_{0,1}^P)^2}{(d_{0,1}^P)^2 + (d_{0,2}^P)^2}, \quad \text{PILS}_2 = \frac{(d_{0,2}^P)^2}{(d_{0,1}^P)^2 + (d_{0,2}^P)^2}.
\end{aligned} \tag{A.12}$$

The Normalized Leadership Share (NLS) of Lien et al. (2025) is defined as

$$\text{NLS}_1 = \frac{(\tilde{d}_{0,1}^P)^2}{(\tilde{d}_{0,1}^P)^2 + (\tilde{d}_{0,2}^P)^2}, \quad \text{NLS}_2 = \frac{(\tilde{d}_{0,2}^P)^2}{(\tilde{d}_{0,1}^P)^2 + (\tilde{d}_{0,2}^P)^2}. \tag{A.13}$$

where the initial permanent responses $\tilde{d}_{0,i}^P$ have the following reduced-form solution:

$$\tilde{d}_{0,i}^P = \sqrt{\boldsymbol{\psi}'\boldsymbol{\Omega}\boldsymbol{\psi}}\, d_{0,i}^P = \frac{\psi_i\sigma_i^2 + \psi_{j\neq i}\sigma_{ij\neq i}}{\sqrt{\psi_1^2\sigma_1^2 + 2\psi_1\psi_2\sigma_{12} + \psi_2^2\sigma_2^2}}. \tag{A.14}$$

We can see that the scaling of the initial permanent responses cancels out in the calculation of NLS. As a result, NLS admits the same reduced-form representation as PILS, as shown in (A.11), and empirical estimates of NLS therefore coincide exactly with those of PILS.

Meanwhile, the CovIS metric of Lautier et al. (2023) is defined as the covariance of reduced-form residuals with the permanent shock:

$$\text{CovIS}_i = \frac{\text{cov}(\varepsilon_{it}, \eta_t^P)}{\sum_{i=j}^n \text{cov}(\varepsilon_{jt}, \eta_t^P)} = \frac{d_{0,i}^P}{\sum_{i=1}^n d_{0,i}^P}, \tag{A.15}$$

with reduced-form representations as follows:

$$\text{CovIS}_1 = \frac{\psi_1\sigma_1^2 + \psi_2\sigma_{12}}{\psi_1\sigma_1^2 + (\psi_1 + \psi_2)\sigma_{12} + \psi_2\sigma_2^2}, \quad \text{CovIS}_2 = \frac{\psi_2\sigma_2^2 + \psi_1\sigma_{12}}{\psi_1\sigma_1^2 + (\psi_1 + \psi_2)\sigma_{12} + \psi_2\sigma_2^2}. \tag{A.16}$$

Lautier et al. (2023) also define a quadratic variation, denoted as CovISQ:

$$\begin{aligned}\text{CovISQ}_1 &= \frac{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2}{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2 + (\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2}, \\ \text{CovISQ}_2 &= \frac{(\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2}{(\psi_1\sigma_1^2 + \psi_2\sigma_{12})^2 + (\psi_2\sigma_2^2 + \psi_1\sigma_{12})^2},\end{aligned} \tag{A.17}$$

with structural representations:

$$\text{CovISQ}_1 = \frac{(d_{0,1}^P)^2}{(d_{0,1}^P)^2 + (d_{0,2}^P)^2}, \quad \text{CovISQ}_2 = \frac{(d_{0,2}^P)^2}{(d_{0,1}^P)^2 + (d_{0,2}^P)^2}. \tag{A.18}$$

As shown in the representations above, PILS, NLS, and CovISQ share the same structural as well as reduced-form representations, despite being derived from different approaches. More importantly, all of these measures rank markets according to the absolute magnitude of their contemporaneous response, $|d_{0,i}^P|$, assigning leadership to the market that reacts most strongly upon impact. As we argue in this paper, this magnitude-based criterion for price leadership can be misleading when markets overreact or exhibit perverse responses to information shocks.

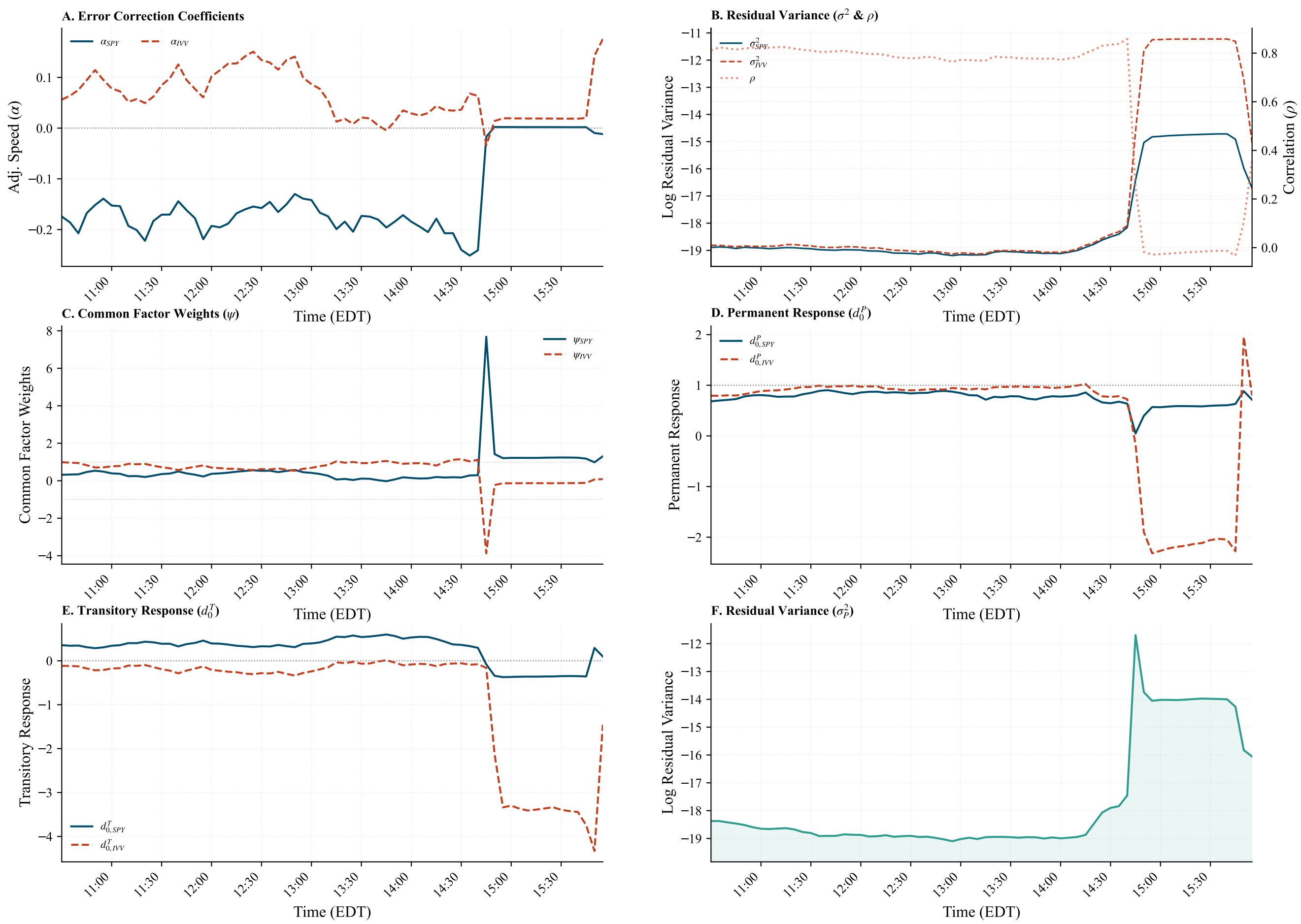

Figure A1: Dynamic Evolution of Structural and Reduced-Form Parameters during the Flash Crash (May 6, 2010). The figure illustrates the dynamic evolution of VECM and SMA estimates for the log price pair of SPY and IVV during the Flash Crash. **Panel A:** Error correction coefficients of the VECM. **Panel B:** Variances and correlation coefficient of the VECM residuals. **Panel C:** Common row vector of the long-run impact matrix in the VMA. **Panel D:** Recovered initial permanent responses estimated using the price discovery beta approach. **Panel E:** Recovered initial transitory responses identified by Lautier et al. (2023). **Panel F:** Log volatility of the recovered permanent shock.

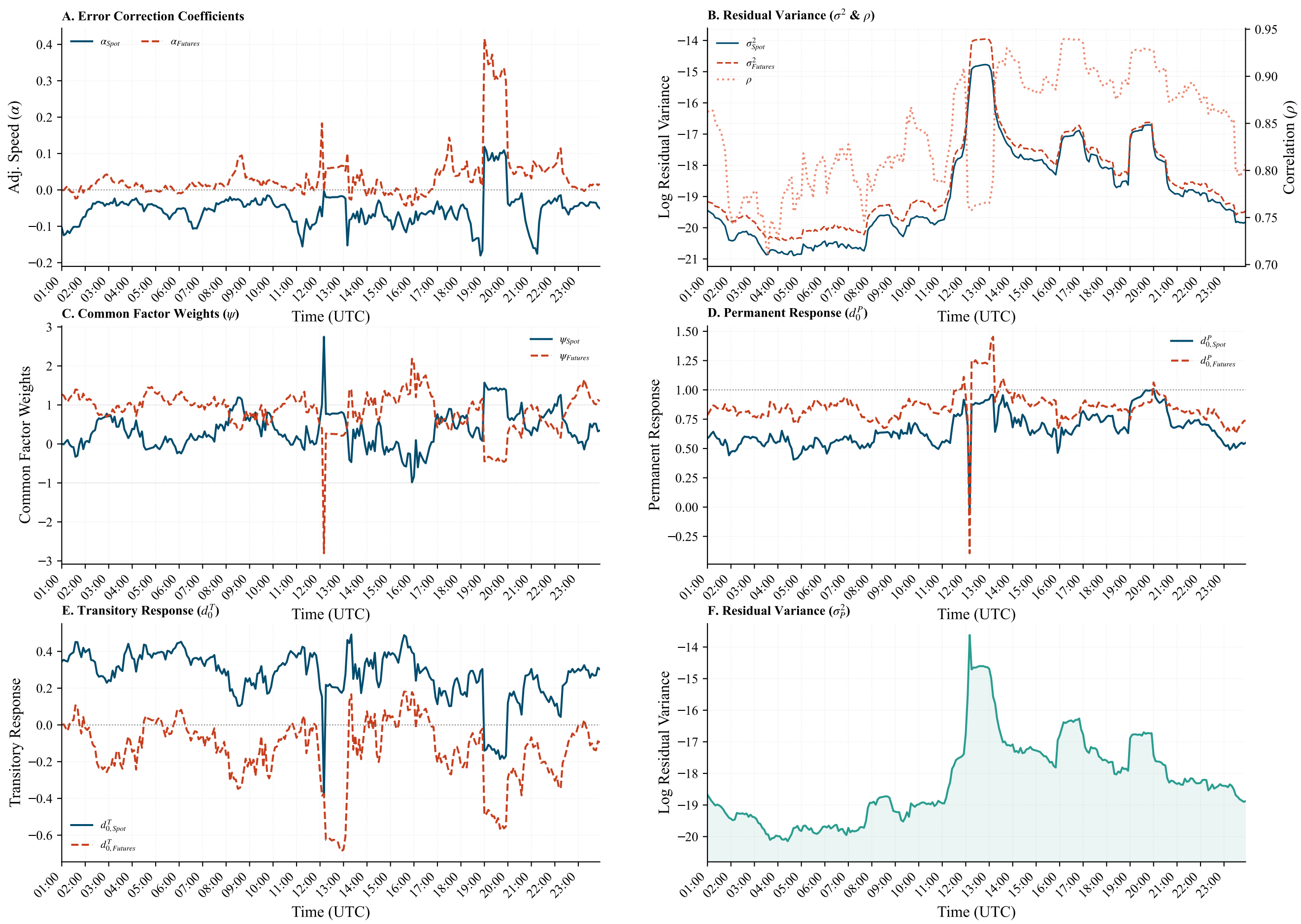

Figure A2: Dynamic Evolution of Structural and Reduced-Form Parameters during the Bitcoin Crash (Jan 3, 2024). The figure illustrates the dynamic evolution of VECM and the SMA estimates for the log price pair of BTC/USDT Spot and Perpetual Futures on Jan 3, 2024. **Panel A:** Error correction coefficients of the VECM. **Panel B:** Variances and correlation coefficient of the VECM residuals. **Panel C:** Common row vector of the long-run impact matrix in the VMA. **Panel D:** Recovered initial permanent responses estimated using the price discovery beta approach. **Panel E:** Recovered initial transitory responses identified by Lautier et al. (2023). **Panel F:** Log volatility of the recovered permanent shock.