

Population Size Effects in the Structural Development of England[†]

By OKSANA M. LEUKHINA AND STEPHEN J. TURNOVSKY*

The English structural transformation from farming to manufacturing was accompanied by rapid technological change, expansion of trade, and massive population growth. While the roles of technology and trade in this process have been investigated, the literature has largely ignored the role of population growth. We examine population size effects on various aspects of structural development, characterizing their explicit dependence on preference-side and production-side characteristics of the economy, and trade. Our quantitative analysis of the English transformation assigns a major role to population growth, with especially notable contributions to post-1750 rise in the manufacturing employment share and the relative price dynamics. (JEL J11, N13, N33, N53, N63, O33)

All industrialized countries have undergone a transition from agriculture to manufacturing, reflected in the rise of both the employment share and the value added share of the manufacturing sector. This process of structural transformation from farming to manufacturing is typically characterized by productivity gains in both sectors, an expansion of international trade, and significant population growth. While the impact of sectoral productivity gains and the growth of trade have been extensively investigated, the literature has largely ignored the role of population growth in the process of structural transformation.¹ In this paper, we focus explicitly on the impact of population growth as a factor in structural development. To this

*Leukhina: Department of Economics, University of Washington, 305 Savery Hall, Seattle, WA 98195 (e-mail: oml@uw.edu); Turnovsky: Department of Economics, University of Washington, 305 Savery Hall, Seattle, WA 98195 (e-mail: sturn@uw.edu). Acknowledgements: We thank the participants of the 2012 Midwest Macroeconomics Meetings, Barcelona GSE Summer Forum's segment "Towards sustained economic growth: Geography, Demography and Institutions," the 2013 NA Meetings of the Econometric Society, the 2013 Public Economic Theory meetings, the 2013 Growth and Development Conference held at the Washington University, and the 2015 Paris Seminar on Economic Demography, for helpful comments. We also wish to acknowledge the constructive suggestions provided by three referees. Turnovsky wishes to acknowledge support provided by the Van Voorhis endowment at the University of Washington.

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¹The role of population growth in structural development has, however, been extensively debated by economic historians, with differing views. Deane (1969) cites population growth as important, while Mokyr (1985) takes the opposite position, arguing that its significance in generating increased demand for industrial production was marginal. One paper that assigns a similar role to population in the development process to that obtained here is Goodfriend and McDermott (1995). Recently, Herrendorf, Schmitz, and Teixeira (2012) address the role of transportation in facilitating the regional distribution of population in the United States during its period of structural development, 1840–1860. Gollin and Rogerson (2014) emphasize the importance of transportation systems in the allocation of labor, particularly in developing economies, emphasizing the spatial dimension of production.

end, we construct a parsimonious two-sector general equilibrium growth model, augmented to include international trade.²

The model provides a framework that enables us to investigate qualitatively the effects of population growth on various aspects of structural development, and its interaction with sector-specific technological change and trade. Our theoretical analysis clarifies how the production and demand characteristics of the economy shape the role of population growth in this process. We find that the impact of population growth on the manufacturing employment share strengthens as (i) production flexibility in the agricultural sector declines, and as (ii) consumer demand becomes more flexible.

We employ our analytical framework to perform a quantitative investigation of the role played by population growth in the English experience of structural transformation. There are several compelling reasons for studying England. These include: (i) its central role in the world-wide industrialization process, (ii) its dramatic experience with population growth, which increased seven-fold during its period of structural change, and (iii) the ready availability of data, thus facilitating comprehensive quantitative analysis. We examine the English experience during the period 1650–1920, one characterized primarily by the movement of resources from the farming sector to the manufacturing sector, as illustrated by the empirical trends in Figure 1.³

Most of the structural transformation literature employs Cobb-Douglas sectoral production functions, together with some form of a constant elasticity of substitution (CES) utility function, often modified by the introduction of subsistence consumption.⁴ While information on economy-wide productive technology in early England is sparse, there is a consensus that the elasticity of substitution was well below unity (e.g., Mokyr 1985; von Tunzelmann 1985; Allen 2009). As will become evident, the degree of substitutability in manufacturing production is unimportant insofar as the long-run population effect is concerned. In contrast, the population effect is crucially dependent upon substitutability in the agricultural production. Thus, generalizing the production function in the agricultural sector to the CES form is critical, and especially so in light of the empirical estimates of the elasticity of substitution in agricultural production that are consistently well below 0.5.⁵

A key characteristic of our framework giving rise to the effect of population growth on structural change is the assumption that land is a fixed factor of production in the agricultural sector.⁶ If, instead, one assumes that technology in both sectors exhibits constant returns to scale in capital and labor alone, then, in the long

²A seminal paper focusing on trade is Stokey (2001). Other related papers to examine the role of trade in the process of development in a small open economy include an early theoretical analysis by Matsuyama (1992) and a more recent contribution by Teignier (2014). In both cases, the focuses are very different from that of the present paper; neither paper is concerned with identifying causes of the structural change, and in particular the role of population growth.

³All data sources used in the paper are summarized in Appendix A1.

⁴See e.g., Herrendorf, Rogerson, and Valentinyi (2014) for an extensive discussion of the benchmark model.

⁵Empirical estimates of the elasticity of substitution in agriculture are reported for a range of countries by Salhofer (2000).

⁶The role of land as a productive factor in the estimation of aggregate agricultural production functions has been long established; see e.g., Heady and Dillon (1961) and Griliches (1963). More recently, Gollin and Rogerson (2014), in emphasizing the spatial aspect of development, argue that “land enters the production function in a nontrivial way.”

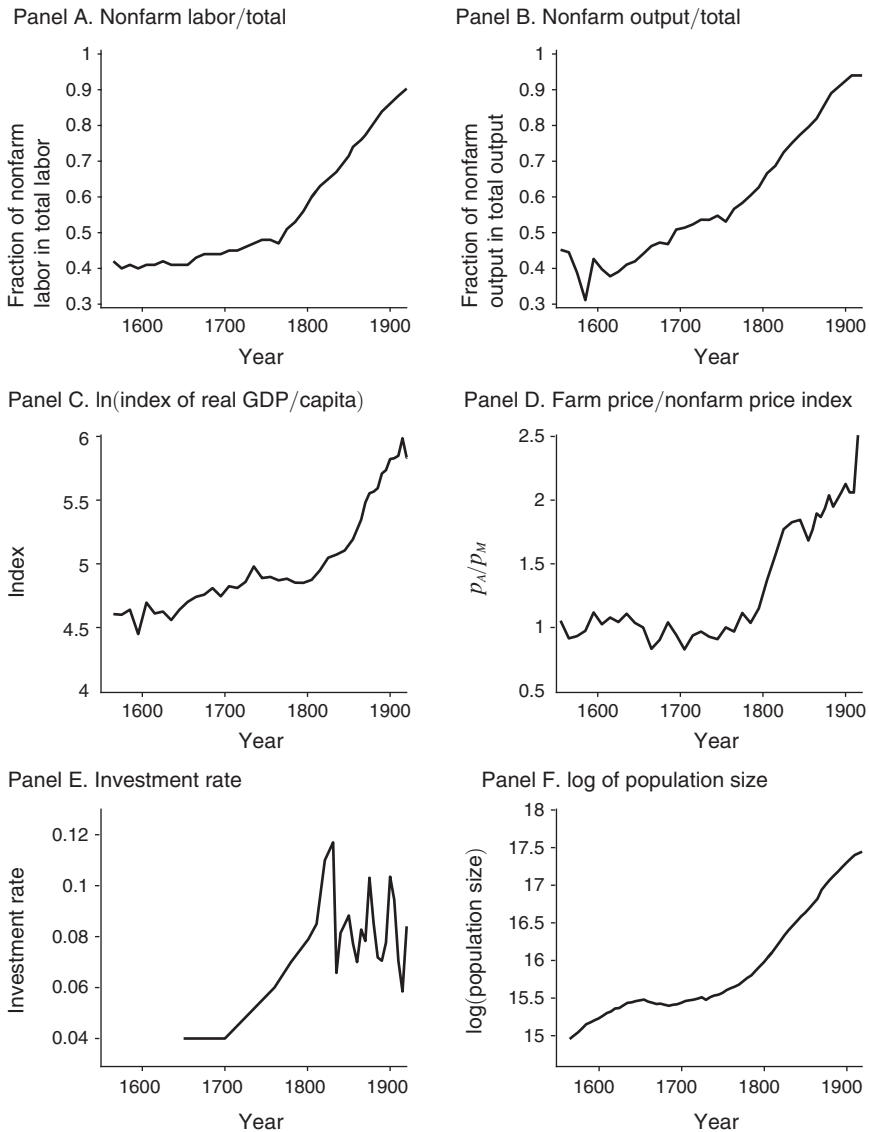


FIGURE 1. LONG-RUN EMPIRICAL TRENDS FOR THE UNITED KINGDOM

run, any population increase can be absorbed proportionately across the two sectors without imposing any structural change. The asymmetry in the technology across the agricultural and manufacturing sectors, and the inflexibility that the fixed supply of land imposes on agricultural production, is a crucial element in determining both, the impact of population growth itself and its interaction with changes in technology and trade expansion.

To conduct our quantitative analysis, we compile the historical data needed to estimate factor-specific technological change and to calibrate the model to the relatively stable period 1550–1650. We then introduce our sector-specific estimates of technological change, together with the historical records of population size and

trade volume, into the model. We find that the model, overall, generates plausible empirical trends, which, given its parsimony, we view as a success. To assess the marginal contribution of population growth, we compare the dynamics, generated by the model when all the exogenous changes are inputted (technological change, population growth, and trade expansion), with the corresponding dynamics generated under the counterfactual experiment that assumes an unchanging population. We find that the rapid population growth characterizing the eighteenth and nineteenth centuries was indeed an important contributor to the process of structural change, with its roles particularly pronounced in raising the manufacturing employment share and the relative price of the farming good.⁷ We also find that the massive trade expansion, as captured here by an exogenous growth of imports (per capita) of agricultural commodities, played a critical role in facilitating the impact of population growth on economic restructuring, alongside its direct impact.

Herrendorf, Rogerson, and Valentinyi (2014) provide a comprehensive review of the recent literature on structural transformation, so there is no need for us to discuss it further here. The main limitation of the existing structural transformation literature, from our perspective, is that population size effects have been inadequately studied, despite their prominence in the economic history literature. Our main contribution is to address this shortcoming. Also, by emphasizing the role of population size on structural change, our work brings the structural transformation literature closer to the body of literature that emphasizes the role of population size in generating a takeoff in per capita output growth (e.g., Murphy, Shleifer, and Vishny 1989; Kremer 1993; Galor and Weil 2000; Tamura 2002), thereby suggesting a more prominent role for demographics in the overall development process.

The remainder of the paper proceeds as follows. Section I introduces the model and reports the relevant analytical results. In Section II, we carry out the quantitative investigation of the English case of structural transformation and report extensive sensitivity analysis. We conclude in Section III, while the Appendix provides more details of the data sources and calibration.

I. Analytical Framework

A. *Environment*

We consider a two-sector economy, comprising an agricultural (farm) good and manufacturing (nonfarm) good production.⁸ Because the English structural transformation was accompanied by colonization, we extend the basic framework to incorporate trade.

⁷Our finding that population growth was relatively more important for the dynamics of sectoral employment shares rather than value added shares is consistent with the findings of Crafts (1980) and Mokyr (1985).

⁸Some of the structural transformation literature also considers the growth of the service sector. We abstract from this, assuming that it is absorbed in the manufacturing sector, which therefore should be viewed as representing an amalgam of the two sectors. Since most of the growth of the service sector occurred in the twentieth century, which lies outside our main period of focus, we view this assumption as adequate for our purposes.

Technology and Firms.—The manufacturing sector comprises a large number of identical firms, endowed with a technology F_t at time t that requires capital and labor as inputs. We assume that F_t has the usual neoclassical properties, including homogeneity of degree 1 in capital and labor, which allows us to restrict attention to a single aggregate firm, exhibiting competitive behavior. Thus aggregate output of the manufacturing sector is described by $Y_{M,t} = F_t(K_{M,t}, L_{M,t})$, where $K_{M,t}$ and $L_{M,t}$ denote aggregate employment of capital and labor in that sector. Taking factor rental rates w_t and r_t as given, the aggregate firm hires inputs to maximize profit:

$$(1) \quad \max_{K_{M,t}, L_{M,t}} F_t(K_{M,t}, L_{M,t}) - w_t L_{M,t} - r_t K_{M,t}.$$

Analogously, there is a large number of identical firms in the agricultural sector, endowed with a neoclassical technology G_t at time t , homogeneous of degree 1 in capital, labor, and land. Aggregate output of the agricultural sector is specified by $Y_{A,t} = G_t(K_{A,t}, L_{A,t}, N_t)$, where $K_{A,t}$, $L_{A,t}$, and N_t denote aggregate employment of capital, labor and land in the agricultural sector. Taking factor rental rates w_t , r_t , s_t and the price of the agricultural good p_t , expressed in terms of the (numeraire) manufacturing good, as given, the aggregate firm hires inputs to maximize profit:

$$(2) \quad \max_{K_{A,t}, L_{A,t}, N_t} p_t G_t(K_{A,t}, L_{A,t}, N_t) - w_t L_{A,t} - r_t K_{A,t} - s_t N_t.$$

Preferences and Families.—There is a large number of identical infinitely-lived families, each composed of L_t identical individuals at time t . We normalize the mass of families to one, so L_t also denotes population size. We assume the evolution of population size over time is given exogenously by $\{L_t\}_{t=0}^{\infty}$.

Families own land N_t , given in fixed supply, and initial capital stock K_0 (in the form of the manufacturing good). Each individual is also endowed with one unit of productive time per period. Families seek to maximize $\sum_{t=0}^{\infty} \beta^t L_t u(c_{A,t} - \bar{c}_A, c_{M,t})$, where $u(c_{A,t} - \bar{c}_A, c_{M,t})$ is individual period utility satisfying standard assumptions, defined over individual consumption of agricultural good $c_{A,t}$ and manufacturing good $c_{M,t}$, \bar{c}_A is a nonnegative constant denoting subsistence food consumption, and $\beta \in (0, 1)$ is the discount factor.⁹ Incorporating a subsistence level requirement of food consumption, as widely adopted in the structural transformation literature, renders the utility function nonhomothetic, with the income elasticity of the demand for food being less than unity.

Each period, families inelastically supply labor at a wage rate w_t and rent out their capital and land holdings at rates r_t and s_t , respectively. Capital stock depreciates at the rate of δ but may be augmented via investment purchased from the

⁹Family utility can be rationalized as follows. Families maximize a weighted sum of individual utilities, assigning weight β^t to individuals born in period t . Utility of an individual born in t is given by $\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_{A,\tau} - \bar{c}_A, c_{M,\tau})$.

manufacturing sector. Given $\{w_t, r_t, s_t, p_t\}_{t=0}^{\infty}$, families make consumption and capital accumulation choices to solve

$$(3) \quad \max_{\{c_{A,t}, c_{M,t}, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t u(c_{A,t} - \bar{c}_A, c_{M,t})$$

subject to $p_t c_{A,t} + c_{M,t} + k_{t+1} \frac{L_{t+1}}{L_t} - (1 - \delta) k_t = r_t k_t + w_t + s_t n_t$, for all t ,

$$c_{A,t}, c_{M,t}, k_{t+1} \geq 0, k_0, n_0 \text{ given,}$$

where k_t and n_t denote per capita capital and land holdings in period t .¹⁰

Trade and Market Clearing.—We use upper case letters to denote aggregate quantities of the manufacturing good consumption, agricultural good consumption, and aggregate supply of capital: $C_{M,t} = c_{M,t} L_t$, $C_{A,t} = c_{A,t} L_t$, $K_t = k_t L_t$.

Labor, capital, and land markets are assumed to clear every period, with labor and capital being perfectly mobile across the two sectors:

$$(4) \quad L_{M,t} + L_{A,t} = L_t,$$

$$(5) \quad K_{M,t} + K_{A,t} = K_t,$$

$$(6) \quad N_t = N.$$

The expansion of the British Empire during the eighteenth and nineteenth centuries was accompanied by an expansion of trade, set in an environment highly protective of the domestic manufacturing sector. High tariffs were imposed on imports of manufactured commodities. In effect, British trade during this time period can be largely characterized by imports of agricultural commodities, such as sugar, cotton and tobacco, from its colonies in exchange for exports of the manufacturing good. To incorporate the expansion of trade into the model, we assume that the amount of imports per capita is given exogenously by $\{M_{A,t}\}$, as a consequence of colonization. The amount of exports $\{X_{M,t}\}$ adjusts endogenously to ensure a trade balance:¹¹

$$(7) \quad X_{M,t} = p_t M_{A,t}.$$

¹⁰The solution exists because the constraint set is nonempty and compact, and the objective function is continuous. Since the constraint set is also convex and we assume a strictly concave utility function, the solution is also unique.

¹¹We model trade similar to Stokey's (2001) approach, where imports are also taken to be exogenous, although specified as a fraction of domestic consumption. An alternative way would be to assume an exogenous sequence of prices $\{p_t\}$, as if it were determined in world-wide markets, and allow for both imports and exports to be determined endogenously. Our choice of modeling trade more accurately captures the trade environment faced by eighteenth–nineteenth century England. Especially during the Golden era following Napoleon's defeat, England was the main trade partner of the colonized world. The exogenous relative price assumption—which would work better for a small open economy—appears less appropriate in our case. We also abstract from the presence of international capital markets the development of which occurred beyond our time frame of study.

Market clearing in the manufacturing good requires that the aggregate output of the manufacturing good is allocated between the aggregate consumption $C_{M,t}$, investment $K_{t+1} - (1 - \delta)K_t$, and exports $X_{M,t}$,

$$(8) \quad C_{M,t} + K_{t+1} - (1 - \delta)K_t + X_{M,t} = F_t(K_{M,t}, L_{M,t}).$$

Market clearing in the agricultural good is given by

$$(9) \quad C_{A,t} = G_t(K_{A,t}, L_{A,t}, N_t) + M_{A,t}.$$

For simplicity, we assume that domestically supplied and imported farm products are perfect substitutes.

B. Equilibrium

DEFINITION 1: A competitive equilibrium consists of allocations $\{C_{A,t}, C_{M,t}, K_{t+1}, K_{A,t}, K_{M,t}, L_{A,t}, L_{M,t}, N_t, X_{M,t}\}_{t=0}^{\infty}$ and prices $\{p_t, w_t, r_t, s_t\}_{t=0}^{\infty}$ such that firms' maximization problems, given in (1) and (2), and families' maximization problem, given in (3), are solved, and market clearing and trade balance conditions (4)–(9) are satisfied.

Because the population effect is insensitive to production conditions in the manufacturing sector, as will be noted in Section IC, we assume that output in that sector is determined by the Cobb-Douglas technology, $F_t(K_{M,t}, L_{M,t}) = B_{M,t}K_{M,t}^{\nu}L_{M,t}^{1-\nu}$. However, we allow for a general constant elasticity of substitution (CES) production function in the agricultural sector:

$$G_t(K_{A,t}, L_{A,t}, N_t) = B_{A,t} \left[\omega_{k,t} K_{A,t}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_{l,t} L_{A,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \omega_{k,t} - \omega_{l,t}) N_t^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where ε denotes the elasticity of substitution in agricultural production.¹² Firms' maximization requires that factor marginal products equal their respective rental prices and can be summarized by

$$(10) \quad r_t = \nu B_{M,t} \frac{L_{M,t}^{1-\nu}}{K_{M,t}^{1-\nu}} = p_t B_{A,t} \omega_{k,t} K_{A,t}^{-\frac{1}{\varepsilon}} \left[\cdot \right]^{\frac{1}{\varepsilon-1}},$$

$$(11) \quad w_t = (1 - \nu) B_{M,t} \frac{K_{M,t}^{\nu}}{L_{M,t}^{\nu}} = p_t B_{A,t} \omega_{l,t} L_{A,t}^{-\frac{1}{\varepsilon}} \left[\cdot \right]^{\frac{1}{\varepsilon-1}},$$

$$(12) \quad s_t = p_t B_{A,t} (1 - \omega_{k,t} - \omega_{l,t}) N_t^{-\frac{1}{\varepsilon}} \left[\cdot \right]^{\frac{1}{\varepsilon-1}},$$

¹² As Uzawa (1962) first showed, to have a CES production function with more than two factors, the pairwise elasticities of substitution across all factor pairs must be equal. More general production functions can accommodate different pairwise partial elasticities of substitution across factor pairs, but they are no longer constant.

where $[\cdot]$ refers to the bracketed term in the production function G .

We specify the individual utility function by

$$u(c_{A,t} - \bar{c}_A, c_{M,t}) = \frac{1}{\gamma} \left\{ \left[\alpha (c_{A,t} - \bar{c}_A)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) c_{M,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\gamma},$$

where σ is the *intra*temporal elasticity of substitution between $c_{A,t} - \bar{c}_A$ and $c_{M,t}$, and $1/(1 - \gamma)$ is the *inter*temporal elasticity of substitution of the composite good appearing inside the curly brackets.¹³ Performing the optimization yields the following first-order conditions describing families' intratemporal and intertemporal tradeoffs, expressed in terms of aggregate variables:

$$(13) \quad \left(\frac{\alpha}{1 - \alpha} \right) \left(\frac{C_{A,t} - \bar{c}_A L_t}{C_{M,t}} \right)^{-\frac{1}{\sigma}} = p_t,$$

$$(14) \quad \left[\frac{\tilde{C}_t/L_t}{\tilde{C}_{t+1}/L_{t+1}} \right]^{\frac{\gamma\sigma+2-\sigma}{\sigma}} \left[\frac{C_{M,t+1}/L_{t+1}}{C_{M,t}/L_t} \right]^{\frac{1}{\sigma}} = \beta(r_{t+1} + 1 - \delta),$$

where $\tilde{C}_t \equiv \left[\alpha (C_{A,t} - \bar{c}_A L_t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) C_{M,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$. Together with the transversality condition, the above two first-order conditions from family maximization characterize its solution.

The equilibrium quantities and prices are then characterized by the market clearing and trade balance conditions (4)–(9), the first-order conditions (10)–(14), and the transversality condition. In order to solve for the model dynamics in Section II, we assume that the exogenously given series $\{L_t, M_{A,t}, B_{M,t}, B_{A,t}, \omega_{k,t}, \omega_{l,t}\}_{t=0}^{\infty}$ converge to constant values in the distant future. The equilibrium solution is then found by replacing the transversality condition by the assumption of a steady state in the distant future.

C. Analytical Results on Comparative Steady States

Assuming that population size L_t , technological parameters $B_{M,t}$, $B_{A,t}$, $\omega_{k,t}$, $\omega_{l,t}$, and trade all remain constant over time, the equilibrium dynamics described by the model, summarized in Section IC, converge to a steady state. In our quantitative analysis carried out in Section II, we will trace out the evolution of the model economy between two steady state equilibria, after we introduce the historical changes in population, technology, and trade. Performing comparative steady state analysis with respect to each of these factors, while holding other factors fixed, will provide insight into their separate effects on structural change, as well as the dependence of those on the key parameters of the model. In assessing the role of international trade we quantify its significance by the level of imports per capita, $m_{A,t} \equiv M_{A,t}/L_t$, with

¹³ Because of the nonhomotheticity, the elasticity of substitution between a and c is not constant.

its rise parameterizing an increase in trade intensity. The intuition gained from our analytical results will considerably enhance our understanding of the quantitative study of England carried out in Section II.

Because labor migration from agriculture into manufacturing is the main characteristic of structural change, we focus our analytical results on the manufacturing employment share L_M/L . log-linearizing the system of steady-state conditions and abstracting from capital depreciation to sharpen the analysis,¹⁴ we derive the following partial effects of an increase in population L , an increase in trade m_A , and the sectoral levels of technology, B_A and B_M :

$$(15) \quad \frac{d\hat{L}_M}{d\hat{L}} - 1 = \frac{L_A e_N}{Q} \left[\frac{C_M}{Y_M} \left(\frac{C_A}{Y_A} \frac{1}{\varepsilon} - \frac{1}{\sigma(1-\chi)} \right) + \frac{X_M}{Y_M} \left(\frac{C_A}{Y_A} \frac{1}{\sigma\varepsilon} \right) \right],$$

$$(16) \quad \frac{d\hat{L}_M}{d\hat{m}_A} = \frac{L_A}{Q} \left[\frac{C_M M_A}{Y_M Y_A} \frac{1}{\sigma(1-\chi)} + \frac{X_M C_A}{Y_M Y_A} \frac{1}{\sigma} \right] > 0,$$

$$(17) \quad \frac{d\hat{L}_M}{d\hat{B}_A} = \frac{L_A}{Q} \left[\frac{C_M}{Y_M} \left(\frac{1}{\sigma(1-\chi)} - \frac{C_A}{Y_A} \right) - \frac{X_M C_A}{Y_M Y_A} \frac{1}{\sigma} \right],$$

$$(18) \quad \frac{d\hat{L}_M}{d\hat{B}_M} = \frac{L_A}{Q(1-\nu)} \left[\frac{C_A C_M}{Y_A Y_M} \left(1 - \frac{1}{\sigma} \right) + e_K \left(\frac{\varepsilon}{\sigma(1-\chi)} \frac{C_M}{Y_M} - \frac{C_A}{Y_A} \left[\frac{C_M}{Y_M} + \frac{1}{\sigma} \frac{X_M}{Y_M} \right] \right) \right],$$

which we express in terms of the underlying parameters and endogenous ratios, the latter evaluated at steady state. We denote by $e_K \equiv rK/(pY_A)$ and $e_N \equiv sN/(pY_A)$ the income shares earned by capital and land in the agricultural sector, while $\chi \equiv \bar{c}_A L/C_A$ refers to the steady-state ratio of subsistence to total farm consumption, and $\hat{\cdot}$ denotes percentage changes.¹⁵ In addition, we see that the level of per capita imports m_A exerts direct positive influence on the consumption-output ratio in agriculture, $C_A/Y_A = 1 + m_A L/Y_A$, import-output ratio in agriculture, $M_A/Y_A = m_A L/Y_A$, and the exports-output ratio in manufacturing, $X_M/Y_M = m_A p L/Y_M$, but negatively affects the consumption-output ratio in manufacturing, $C_M/Y_M = 1 - X_M/Y_M$.

One important observation, evident from (15), is that by characterizing a population increase as having a structural effect we mean that it has a non-proportionate impact on sectoral labor allocations. We should also note that if one were to assume a CES technology in the nonfarm sector, as opposed to the Cobb-Douglas function

¹⁴Capital depreciation is incorporated in quantitative analysis (Section II).

¹⁵ $Q \equiv L_M \left[\frac{e_N}{\varepsilon} \left(1 + \frac{X_M}{\sigma Y_M} \right) \frac{C_A}{Y_A} + \frac{(1-e_N) C_M}{\sigma(1-\chi) Y_M} \right] + L_A \frac{1}{\sigma} \frac{C_A}{Y_A} > 0$.

considered here, the qualitative signs of the first two effects, $\frac{d\hat{L}_M}{d\hat{L}} - 1$ and $\frac{d\hat{L}_M}{d\hat{B}_A}$, would remain unchanged.¹⁶

From (15)–(18), it is clear that the above effects depend crucially on four sets of factors: (i) the flexibility of production, as reflected in the elasticity of factor substitution in agricultural production ε ; (ii) the complementarity/substitutability of the two goods ($c_A - \bar{c}_A$) and c_M in utility, as reflected in σ ; (iii) the degree of nonhomotheticity in demand, as reflected in the subsistence term χ ; and (iv) the importance of international trade, as parameterized by m_A , and reflected in the various equilibrium consumption-output ratios specified above.

Population Effects.—It is evident from (15) that the population effect depends crucially upon the presence of land in farm production.¹⁷ Indeed, if $e_N = 0$, changes in population size produce no movement in the manufacturing employment share. In this case, all quantities adjust in proportion to the population change, with the relative price remaining unchanged. Intuitively, the economy-wide production structure is sufficiently flexible so that the additional labor can be absorbed proportionately in both sectors, without any structural adjustment. Henceforth, we maintain the assumption that land is a necessary input in farm production, $e_N > 0$.

We begin by considering a closed economy ($m_A = 0$), in which case the population effect reported in (15) simplifies to

$$(19) \quad \frac{d\hat{L}_M}{d\hat{L}} - 1 = \frac{L_A e_N}{Q} \left[\frac{1}{\varepsilon} - \frac{1}{\sigma(1 - \chi)} \right].$$

An increase in population will raise the employment share in manufacturing if the bracketed expression is positive. The first term inside the brackets captures the degree of flexibility of farm production. If factors are not easily substitutable (low ε), the population effect is strong. This is because additional labor is not easily absorbed in farm production, and is therefore pushed into the manufacturing sector, where production possibilities are more flexible. In the extreme case of a Leontief production function ($\varepsilon = 0$), all of the additional labor will be employed in manufacturing as long as $\chi < 1$.¹⁸ The second term inside the brackets captures demand flexibility. If the two goods are easily substitutable (high σ) and nonhomotheticity is not too strong (low χ), the labor pushed out of agriculture is easily absorbed in manufacturing, with the consumption of the manufacturing product substituting for

¹⁶In the case of the last effect $d\hat{L}_M/d\hat{B}_M$, there is an additional positive influence of B_M arising due to a lower elasticity of factor substitution in manufacturing. The formal expressions in this more general case are available in an expanded version of this paper, available on request. There we report formal expressions for the long-run responses of all key variables, including relative sectoral outputs. Here we report only the responses most directly relevant to understanding the role of population.

¹⁷This is analogous to the role played by population growth in the model developed by Goodfriend and McDermott (1995). In their analysis, the importance of population is due to the assumption that per capita output is an average of output produced under a diminishing returns technology and an increasing returns technology, so its allocation across the two sectors matters.

¹⁸If $\chi = 1$ and population increases, the equilibrium will cease to exist in this case, because subsistence consumption needs increase but food production cannot be raised.

the consumption of the agricultural good. In our quantitative analysis, population growth will prove important in reallocating labor to the manufacturing sector.

Note further from equation (19) that under the common assumptions of Cobb-Douglas technology for agriculture ($\varepsilon = 1$), log utility ($\sigma = 1$), and homotheticity ($\chi = 0$), population size has no effect on sectoral employment shares. It is, therefore, unsurprising that it has been largely overlooked in the literature. Moreover, with added nonhomotheticity ($\chi > 0$), the population effect is negative. In this case, population growth increases the subsistence consumption requirement and, hence, the demand for farm good consumption, shifting labor toward the farming sector.¹⁹ This analysis makes it clear that generalizing the farm production function to allow for a lower elasticity of factor substitution bears important consequences for the quantitative analysis of population growth.

To the extent that population growth contributes to labor reallocation away from the farm, it will also influence the relative price. Consider the intratemporal equation (13), log-linearized around the steady state:

$$(20) \quad d\hat{p} = \frac{1}{\sigma}(d\hat{C}_M - d\hat{C}_A) + \frac{\chi}{\sigma(1 - \chi)}(d\hat{L} - d\hat{C}_A).$$

In light of the steady-state intertemporal condition $\beta = (\nu B_M(L_M/K_M)^{1-\nu} + 1 - \delta)^{-1}$, an increase in the labor input L_M , induced by population growth, will be accompanied by a proportional change in capital K_M , output Y_M , and consumption C_M , if in autarky. Meanwhile, the same (or smaller) increase in the labor input L_A will induce a smaller increase in the farm output Y_A and consumption C_A , if in autarky, because land is in fixed supply. To support the shift in consumption away from farming, $d\hat{C}_M - d\hat{C}_A$, the relative price p must rise. There is an additional pressure for p to rise if $\chi > 0$, as it must also balance the increased demand for subsistence food consumption associated with a larger population. The relationship above shows the overall price increase is mitigated by more flexible demand conditions (high σ and low χ).

Arguably the most notable impact of the trade expansion that we uncover in Section II is that it significantly moderated the positive impact of population growth on the relative price p , thereby allowing the model to generate an increase in the manufacturing value-added share alongside the increase in the manufacturing employment share. The intuition can be seen by invoking the argument above. In the presence of trade, the immediate link between output and consumption of a particular good is broken, which relaxes the constraints imposed by inflexible demand conditions. The price effect in (20) is significantly moderated, as changes in the relative output levels induced by labor reallocation no longer imply proportional changes in the relative consumption levels.

The presence of trade also strengthens the partial effect of population growth on structural change, given in (15). To see this, we compare the partial effect in

¹⁹ A similar mechanism is at work in Gollin and Rogerson (2014). Using a static framework of a multiregional model of a poor economy with costly transportation, they show how population growth can lead to a higher share of the labor force being employed in agriculture and living in subsistence.

the stationary equilibrium of a closed economy ($m_A = 0$) to that of an open economy ($m_A > 0$).²⁰ Recall that the inflexible demand conditions implied by our preferred case of complementarity ($\sigma < 1$), i.e., inability to easily substitute between consumption of the two goods, is what made it difficult for the relatively flexible manufacturing technology to absorb a larger fraction of additional labor. As trade breaks the immediate link between output and consumption of a particular good, inflexible demand conditions matter less, and additional labor is more readily employed by the manufacturing sector as some of its output is now exchanged for the farm good through trade.²¹

In our quantitative analysis, we indeed find that population growth was an important driver of the manufacturing employment and value-added shares and the relative price dynamics. However, in the absence of the trade expansion, population growth would imply an overly sharp increase in the price of agricultural commodities, which would effectively stall the rise in the manufacturing value added share.

Increase in Trade Intensity.—Equation (16) indicates that an increase in trade intensity m_A leads to an unambiguous migration of population to the manufacturing sector. Intuitively, an inflow of farm goods prompts a restructuring of the economy to ensure a sufficient rise in the output of the manufacturing good in order to balance trade with exports and to increase domestic consumption of the manufacturing good. Importantly, the effect strengthens with more complementarity (lower σ), as more restructuring is needed when the farm good cannot be easily substituted for the manufacturing good in consumption. The magnitude also depends on the current level of trade as captured by the endogenous quantities appearing in (16). In addition to its direct impact on structural change, Section II also emphasizes the role of trade in facilitating population growth effects.

Technological Change in the Agricultural Sector.—The effect of the factor-neutral technological change in agricultural production, given in (17), depends on demand flexibility and import volume. If the two goods are easily substitutable in consumption, then the sector experiencing productivity gains (farming) will be used more intensively, with its product substituted for the consumption of the good produced in the less productive manufacturing sector. Labor would reallocate toward the farm sector. In contrast, if the demand conditions are inflexible ($\sigma < 1$), resources will flow toward the relatively less productive manufacturing sector, to ensure it generates sufficient output to satisfy the inflexible demand. The positive effect on the manufacturing employment share is reinforced by nonhomotheticity ($\chi > 0$): As income increases due to productivity gains, families disproportionately demand more of the manufacturing good. We find this effect to be quantitatively important in the period prior to 1800.

²⁰Formally, when m_A increases from zero to a positive value, more weight (X_M/Y_M versus C_M/Y_M) is placed on the second larger term in (15), making the overall expression more likely to be positive.

²¹In fact, the population effect is viable even in the extreme case of the Leontief utility function, provided that the trade volume is sufficiently large.

The presence of trade ($m_A > 0$) weakens the magnitude of this described effect. In the presence of trade, some of the manufacturing good is exported to pay for agricultural imports. Technological progress on the farm implies a drop in agricultural prices, which means that fewer exports are needed to pay for the imported food. This reduces the pressure to reallocate resources away from agriculture.

The generality of farm technology allows for technological progress to augment the various factors of production to differential degrees. The type of technological progress that occurs has consequences for the sectoral labor reallocation. In Section II, we examine the nature of technological change in agriculture quantitatively, finding that during the period 1750–1850, technological progress disproportionately augmented capital.

To help understand the model's quantitative dynamics, it is therefore useful to consider the effect of a capital-augmenting technological increase in farm production. Abstracting from trade for convenience ($m_A = 0$), this can be shown to be

$$(21) \quad \frac{\partial \hat{L}_M}{\partial \hat{\omega}_k} = \frac{L_A}{Q} \varepsilon e_K \left[\frac{1}{\sigma(1-\chi)} - \frac{1}{\varepsilon} \right].$$

It is immediately seen that the condition for a positive effect of capital-augmenting progress on the manufacturing employment share is precisely opposite to the population effect condition (15) in the absence of trade. The above effect is negative whenever $\sigma(1-\chi) > \varepsilon$. An increase in the efficiency of capital results in the relative scarcity of labor in farming. If capital cannot be easily substituted for labor in agricultural production and if demand is sufficiently flexible to allow for a shift of consumption in favor of the agricultural good, then labor will flow to the farming sector. This negative impact explains our quantitative finding that the overall technological change in farming worked against the process of structural change in the post-1800 period.

Factor-Neutral Technological Change in the Manufacturing Sector.—The factor-neutral technological change in the manufacturing sector, reported in (18), depends on the structural characteristics of the entire economy. The reason why this effect is more complicated is because the manufacturing sector produces capital which is employed in both sectors. If capital production did not bear any influence on farm output ($e_K = 0$) and the economy were closed ($m_A = 0$), the bracketed term would simplify to $\left[1 - \frac{1}{\sigma}\right]$. It is negative in the preferred case of complementarity ($\sigma < 1$), so gains in B_M result in labor reallocation toward the less productive farm sector. But with $e_K > 0$, there is an additional term, $e_K \left[\frac{\varepsilon}{\sigma(1-\chi)} - 1 \right]$, which may be positive and dominate if the farm production is sufficiently flexible and the degree of nonhomotheticity is very strong. Intuitively, productivity improvements in the capital-producing sector encourage capital accumulation. If capital can be easily substituted for labor in agriculture, labor will shift toward the manufacturing sector. This effect is reinforced by nonhomotheticity of demand ($\chi > 0$), which implies that families disproportionately demand more of the manufacturing good as a result of income gains associated with higher productivity B_M .

In our quantitative analysis, we find the overall effect $\frac{d\hat{L}_M}{d\hat{B}_M}$ to be negative because the farm production function is inflexible (low ε). Because of the offsetting effects of trade on the consumption-output ratios C_M/Y_M and C_A/Y_A , the effect of introducing trade ($m_A > 0$) on the response of the sectoral labor allocation to technological change in the manufacturing sector is ambiguous.

II. Quantitative Investigation

Our objective is to investigate quantitatively the extent to which population growth influenced structural development of England. We focus on the time period between 1650 and 1920, during which England experienced both dramatic population growth and massive reallocation of labor from the farm sector into the manufacturing sector. We also separately consider the pre-Industrial Revolution period of 1650–1750, which precedes the dramatic expansion of trade. We exclude the twentieth century from our analysis, in part because it saw the rise of the service industry, which we do not model explicitly.

Our general strategy is to calibrate the model parameters by assuming that our model in steady state adequately represents the experience of England during the relatively stable period of 1550–1650, and by targeting several empirical moments from this period. Figure 1 reveals that this time period is indeed relatively stable, with approximately zero growth in output per capita, which makes it a good period for implementing our calibration strategy.²²

To assess the overall success of the model in capturing the English experience, and in particular the contribution of population growth, we introduce the estimated series for the technological change, $B_{M,t}$, $B_{A,t}$, $\omega_{k,t}$, $\omega_{l,t}$, the actual historical time series for population L_t and per capita imports $m_{A,t}$ into the calibrated economy, and compare the resulting dynamics to the historical record of manufacturing share of employment and output, output per capita, investment rate, and the relative price of the farm good. We can then assess the separate contribution of any of the exogenous factors by shutting it down and that channel to the overall model success.

A. Calibration on a Grid of Values for ε , σ , and χ

The earlier theoretical discussion pinpointed ε , σ , and χ as the most critical determinants of the population size effects. In light of these results, our strategy is to conduct the proposed quantitative analysis on a grid of empirically plausible values for ε , σ , and the calibration target for χ . We calibrate the remaining parameters according to the strategy outlined above.

We assume each period in the model corresponds to five years in the data. This implies that targets such as the annual interest rate and the annual depreciation rate

²²We do not force the model to match anything post 1650 because there are factors excluded from the model that have likely contributed to the process of structural change. For example, the enclosure movement, i.e., the process that ended traditional rights enjoyed by the commoners, such as access to common land for hay and grazing of livestock, was also likely an important factor as it effectively raised the cost of living on the farm. The process of colonization may also have contributed through channels other than the expansion of trade modeled here.

must be adjusted to five year rates. Maintaining values for ε , σ , and the target value for χ as unknown, we set the rest of the parameters by targeting the following empirical moments that we document for the period 1550–1650: [i]: $r + 1 - \delta = 1.04^5$, [ii]: $pY_A/(pY_A + Y_M) = 0.65$, [iii]: $L_A/L = 0.59$, [iv]: $wL_M/Y_M = 0.75$, [v]: $\frac{wL_A/(pY_A)}{rK_A/(pY_A)} = 3.64$, [vi]: $\delta K/K = 1 - 0.975^5$, [vii]: $m_A = 0$. These moments, in the order listed, refer to the interest rate, the farm output and employment shares, labor income share in manufacturing, the ratio between labor and capital income shares in agriculture, the rate of capital depreciation, and per capita imports.²³ The data sources underlying these targets are detailed in the Appendix. Broadly speaking, the steady-state equations are rewritten in terms of the empirical moments [i]–[vii] and χ (i.e., eight targets). The equations are then used to solve for the following eight model parameters ($\delta, \beta, \nu, B_M, \alpha, \bar{c}_A, L, m_A$) after normalizations are made and values of ε and σ are chosen. The details of the calibration procedure are reported in the Appendix.

Because the intertemporal elasticity of substitution $(1 - \gamma)^{-1}$ bears no influence on steady-state quantities, it cannot be identified via the proposed calibration strategy. We set $\gamma = -1$ yielding an intertemporal elasticity of substitution of 0.5, which is well within the consensus range reported by Guvenen (2006). The main impact of this parameter is on the speed of the transitional dynamics, and thus it has limited consequences for our quantitative results pertaining to the long run.

Hence, for any given choice of ε , σ , and target χ , the calibration procedure pinpoints the remaining model parameters in a way that ensures the empirical moments [i]–[vii] are matched in the initial steady state. Thus, we calibrate the model on a grid defined by various values of ε , σ , and χ and discuss implications for the main results in each case. This comprises the main bulk of our sensitivity analysis, discussed in Section IID.

Benchmark Case.—The benchmark case corresponds to our preferred choice of ε , σ , and target for χ , and we report the results for this case in more detail.

Mokyr (1993, 1977) and von Tunzelmann (1985), although informally, argue that the extreme case of Leontief technology ($\varepsilon = 0$) may be an appropriate approximation for the case of eighteenth century England. Allen (2009) estimates the elasticity of substitution of 0.2 for the overall economy of England.²⁴ We choose $\varepsilon = 0.25$ for the benchmark case, although setting it lower would strengthen the population effect. This is evident from equation (18) and confirmed quantitatively in Section IID.

As discussed earlier, the elasticity of substitution in consumption, σ , is a critical quantity for the population effect. Its choice varies extensively in the structural change literature. Recalling our analytical discussion in Section IC, we know that population growth will strongly contribute to the process of labor reallocation away from the farm if one assumes perfect substitutability between the two consumption

²³Note that matching the moments [ii], [iii], and [iv] will imply matching the labor income share in agriculture $wL_A/(pY_A) = 0.57$. Therefore, we do not list this moment separately.

²⁴As previously noted, the empirical estimates of the elasticity of substitution in agriculture, reported by Salhofer (2000), generally fall well below 0.5.

TABLE 1—BENCHMARK MODEL: CALIBRATED PARAMETER VALUES

Technology	Preferences	Other
$B_{A,1650} = 1$	$\bar{c}_A = 0.18496$	$N = 100$
$\omega_{k,1650} = 0.20$	$\alpha = 0.53$	$\delta = 0.12$
$\omega_{l,1650} = 0.50$	$\beta = 0.82$	$M_A = 0$
$\varepsilon = 0.25$	$\sigma = 0.5$	$L = 156$
$B_{M,1650} = 1.71$	$\gamma = -1$	
$\nu = 0.25$		

Note: This table summarizes the benchmark calibration.

goods ($\sigma \rightarrow \infty$), as do Hansen and Prescott (2002), Doepke (2004), and Bar and Leukhina (2010). But it will not apply if the utility function is of the Leontief form ($\sigma = 0$) discussed by Buera and Kaboski (2009) and Herrendorf, Rogerson, and Valentinyi (2013) in the case of the value added definition of production sectors, unless trade is given a more prominent role.

As explained in Herrendorf, Rogerson, and Valentinyi (2013), because we define sector-specific production as its value added, it is important to also interpret sector-specific consumption as value added components of final consumption. Unfortunately, detailed consumption expenditure data, needed for calibrating σ via a procedure developed in Herrendorf, Rogerson, and Valentinyi (2013), are unavailable for this time period. Hence, we simply follow Buera and Kaboski (2009) and set $\sigma = 0.5$ in the benchmark case. This choice is slightly below the range of estimates obtained in Comin, Lashkari, and Mestieri (2015), where the demand system is estimated under the assumption of more general nonhomothetic CES preferences.²⁵

The target value for χ is difficult to obtain. Note that, independent of this target, the calibration implies that the fraction of the consumption expenditure spent on the farm good $pC_A/(pC_A + C_M)$ is already fixed at 70 percent. We set $\chi = 0.3$ to maintain a relatively strong degree of nonhomotheticity, given the prevalence of this assumption in the structural transformation literature.²⁶ By assuming a positive subsistence level of food consumption, we weaken the relative influence of the population effect. This is seen from equation (15) and confirmed quantitatively in Section IID.

The calibrated parameters for the benchmark case are reported in Table 1.

B. Estimating Technological Progress

To estimate technological change in the two sectors of production, needed for inputting in the model, we draw upon the assumption of profit maximization, together with the historical time series compiled by Gregory Clark and detailed in

²⁵Comin, Lashkari, and Mestieri (2015) obtain $\sigma = 0.67$ between agricultural and manufacturing goods in the demand estimation based on country panel data, and $\sigma = 0.57$ based on the US consumption time series with value-added definition of production sectors.

²⁶Some information is provided by Steger (2000) for developing economies. Our choice of $\chi = 0.3$ is fairly close to a measure of the subsistence ratio of GNP for lower-middle-income countries based on World Bank (1990) data.

the Appendix. Additional details on how we map the available prices into the model prices needed for this estimation procedure are reported in the Appendix.

From profit-maximizing relationships in the manufacturing sector (10) and (11), we obtain

$$(22) \quad B_{M,t} = \left(\frac{r_t}{v}\right)^v \left(\frac{w_t}{1-v}\right)^{1-v},$$

and thus movements in productivity $B_{M,t}$ can be inferred from movements in factor prices, given the calibrated value of v .²⁷ Because prices are available only as indices, this procedure is useful only for identifying the growth rate of $B_{M,t}$. To infer the actual input series $\{B_{M,t}\}$ we apply these estimated growth rates to the initial (calibrated) value $B_{M,1650}$.

The generality of production technology in the agricultural sector requires that we estimate both factor-neutral and factor-specific progress $\{B_{A,t}, \omega_{k,t}, \omega_{l,t}\}$. To do so, we apply the following procedure.

Drawing on factor price and factor ratios data in the agricultural sector, the sources that are detailed in the Appendix, we derive indices for $\omega_{k,t}/\omega_{l,t}$ and $(1 - \omega_{k,t} - \omega_{l,t})/\omega_{l,t}$ by manipulating profit maximizing relationships in the agricultural sector (10)–(12) and substituting from the land market clearing condition (6):

$$(23) \quad \frac{\omega_{l,t}}{\omega_{k,t}} = \frac{w_t}{r_t} \left(\frac{K_{A,t}}{L_{A,t}}\right)^{-\frac{1}{\varepsilon}},$$

$$(24) \quad \frac{\omega_{l,t}}{1 - \omega_{k,t} - \omega_{l,t}} = \frac{w_t}{s_t} \left(\frac{N}{L_{A,t}}\right)^{-\frac{1}{\varepsilon}}.$$

Note these estimates depend on the calibrated value for ε . We adjust the obtained indices so that the initial values correspond to the calibrated values $\omega_{k,1650}$ and $\omega_{l,1650}$. The time series for $\omega_{l,t}$ and $\omega_{k,t}$ are then implied.²⁸ What we observe in the data is that wages w_t grow faster than capital rental rates r_t , but capital intensity in the agricultural sector also intensifies. We find that the second effect dominates for the benchmark case value of $\varepsilon = 0.25$ (as well as for the case $\varepsilon = 0.5$, considered in Section IID, although to a lesser extent), and results in our estimate of capital-biased technological change in agricultural production.

Combining the optimality condition (11), with the share of labor income in agricultural production, $e_{L,t}$, we obtain the following estimate for $\{B_{A,t}\}$:

$$(25) \quad B_{A,t} = \frac{w_t}{p_t} \left[\frac{e_{L,t}^{1/\varepsilon}}{\omega_{l,t}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

²⁷The same procedure is applied to infer productivity movements in both sectors of production in Bar and Leukhina (2010), where a Cobb-Douglas technology is employed in both sectors.

²⁸Precisely, we have $\omega_{l,t} = \left(1 + \frac{\omega_{k,t}}{\omega_{l,t}} + \frac{1 - \omega_{k,t} - \omega_{l,t}}{\omega_{l,t}}\right)^{-1}$ and $\omega_{k,t} = \left(1 + \frac{\omega_{k,t}}{\omega_{l,t}} + \frac{1 - \omega_{k,t} - \omega_{l,t}}{\omega_{l,t}}\right)^{-1} \frac{\omega_{k,t}}{\omega_{l,t}}$.

Again, we adjust the estimated index so the initial value corresponds to the calibrated value $B_{A,1650}$.²⁹

The resulting time series are reported in Figure 2. Panel A reports the estimates of $B_{A,t}$ and $B_{M,t}$; panel B reports our estimates of $\omega_{k,t}$ and $\omega_{l,t}$. The turn of the nineteenth century, which marks the beginning of the Industrial Revolution, is particularly significant for the takeoff in the growth rate of manufacturing productivity. Whereas prior to 1800, the manufacturing sector experienced a rather slow progress, after 1800, it exhibits rapid and steady growth. Factor-neutral technological change in the agricultural sector is evident prior to 1750, when in fact it grew faster than did manufacturing productivity, and after 1850, when a steady progress reemerges. During the period 1750–1850, technological progress in agricultural production is best described as capital-augmenting.

C. Benchmark Results

During the overall period we examine, 1650 and 1920, the population of England increased by a factor of 7 (Figure 1, panel F), total factor productivity in manufacturing grew by a factor of 3, technological change in agriculture exhibited strong capital-augmenting progress, and the per capita import volume $m_{A,t} = M_{A,t}/L_t$ expanded from being almost negligible to 80 percent of the 1650 level of per capita output (Figure 2, panel C).³⁰ In order to assess quantitatively the capability of our model to generate the main features of structural change in England, we solve for the dynamics of the calibrated model, while exogenously varying the time series for population, per capita volume of imports, and sector-specific technological progress in accordance with Figure 1, panel F and Figure 2. This constitutes our main “*all effects*” experiment.

To solve for the model dynamics in practice, we assume that the exogenously given series $\{L_t, m_{A,t}, B_{M,t}, B_{A,t}, \omega_{k,t}, \omega_{l,t}\}_{t=0}^{\infty}$ stop changing beginning with the year 2000.³¹ We then search for sequences of endogenous variables satisfying the market clearing and trade balance conditions (4)–(9) and the first-order conditions (10)–(14), and converging to a steady state in the distant future.

²⁹ Although wages are equalized across sectors in the model, this is not the case in the data. To improve the accuracy of our estimates of sector-specific technology, we therefore employ sector-specific wages (rather than an average wage), and choose the estimation procedure based on wage indices, rather than the actual levels. Appendix C provides the details. In our data, the manufacturing wage index increased by a factor of 4 during our estimation period, while the farming wage index increased by a factor of 3.5, implying an increase in the nonfarm-farm wage differential. Understanding what factors contributed to the presence of the nonfarm-farm wage premium (e.g., differential mortality, cost of living, or selection on human capital) and its increase should certainly prove useful in further analysis of structural transformation.

³⁰ See Appendix A for sources on import data.

³¹ The data employed in the estimation of technological progress, discussed in Section IIB, are available until 1920. We impute future technological progress, which is a necessary input for obtaining the model solution, by assuming that the factor-neutral technological progress observed since the latter half of the nineteenth century continues into the future, while factor-specific technological progress on the farm stalls in 1920. Assuming that growth continues later into the future makes little difference for the model dynamics up to 1920. Likewise, working with the assumption of a balanced growth path in the distant future, instead of the steady state assumption made here, makes little difference for the model dynamics up to 1920.

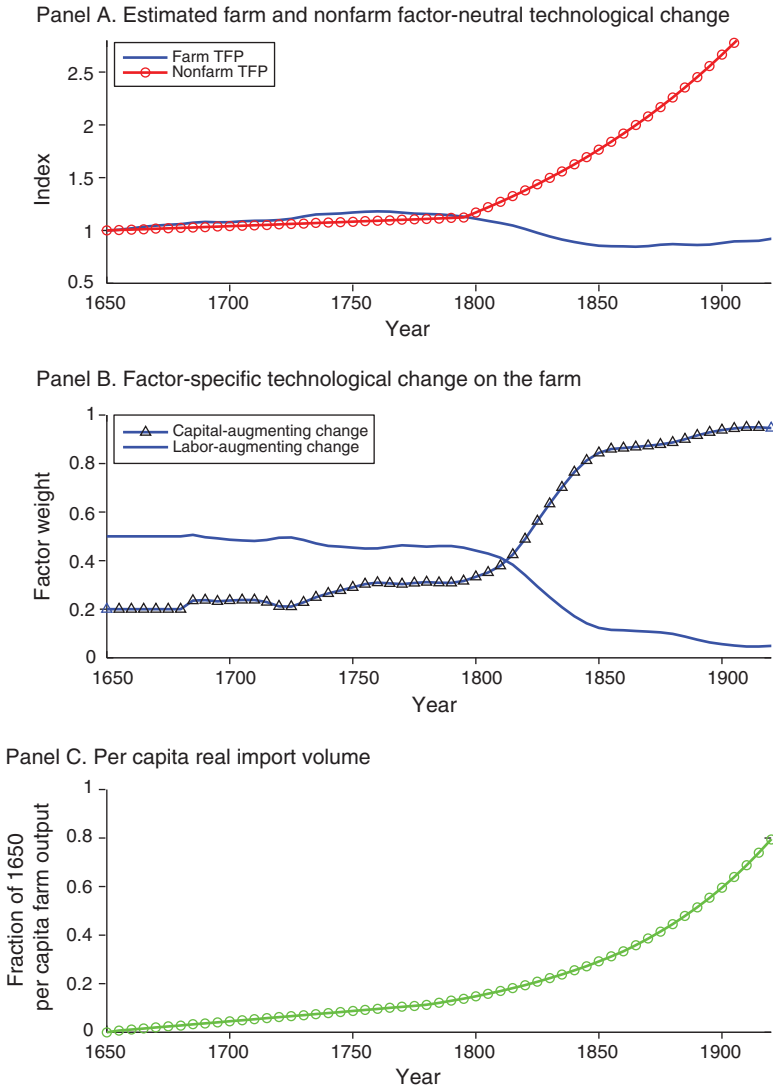


FIGURE 2. TECHNOLOGICAL PROGRESS ESTIMATES AND IMPORTS

In light of our analytical results, the benchmark calibration ($\sigma = 0.5 < 1$) implies that total factor productivity growth in the manufacturing sector is expected to work against the observed process of labor reallocation. However, population growth and technological progress in agriculture are likely to contribute to this process. In order to assess the separate contribution of population growth to the model’s overall success, we perform an additional experiment, which differs from the main experiment only in that we keep population constant at its 1650 level (“*all but population*” experiment). We then assess the marginal contribution of population growth by comparing the resulting dynamics from the “*all but population*” experiment to the performance of the model under the “*all effects*” experiment. For comparison, we also perform a similar counterfactual to highlight the marginal contribution of

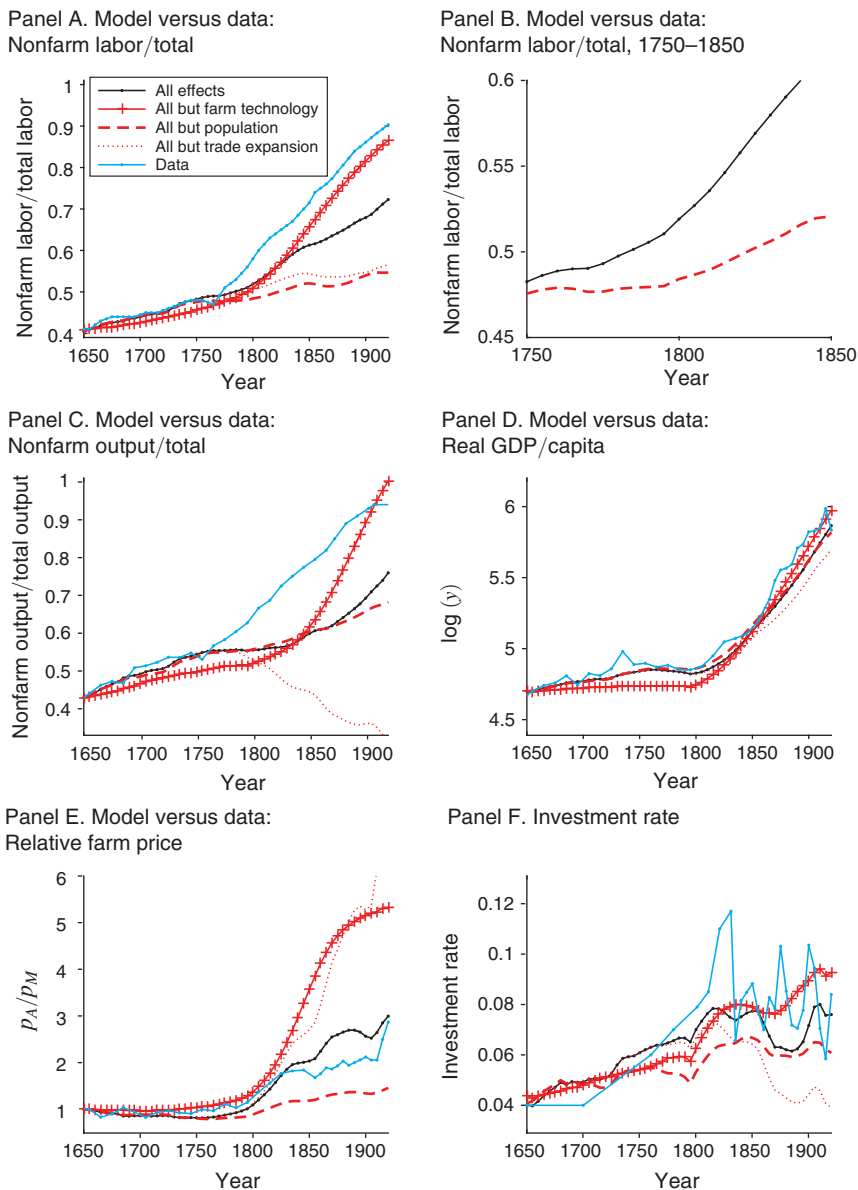


FIGURE 3. BENCHMARK RESULTS: MODEL VERSUS DATA

technological progress on the farm (“*all but farm technology*” experiment) and expansion of trade (“*all but trade expansion*” experiment).³²

Figure 3 reports the model dynamics for the share of employment in the manufacturing sector, manufacturing share of output, per capita output, relative price, and investment rate, resulting from each experiment, along with the empirical counterpart for comparison. Table 2 summarizes the results.

³²To facilitate the comparison with the results in Stokey (2001), the last counterfactual simply shuts down all import growth beginning with 1780.

TABLE 2—BENCHMARK MODEL: MAIN RESULTS

	L_M/L	Y_M/Y	$\ln(y)$	$Inv./Y$	p
% change data, 1650–1920	120	124	26	110	195
Percent of the empirical change accounted for by					
“all effects” experiment	65	62	96	82	102
Farm technology effect	–42	–74	–9	–24	–117
Population effect	56	23	4	45	77
% change data, 1650–1750	17	30	3	38	–7
Percent of the empirical change accounted for by					
“all effects” experiment	112	87	95	139	270
Farm technology effect	40	40	74	55	121
Population effect	10	5	–3	37	–8
% change data, 1750–1920	88	72	22	53	217
Percent of the empirical change accounted for by					
“all effects” experiment	57	55	97	47	123
Farm technology effect	–80	–155	–24	–186	–55
Population effect	70	32	5	51	69

Notes: This table summarizes the results obtained in the benchmark model. Figure 3 helps visualize these results. The “all effects” experiment refers to all effects introduced in the model. It captures the overall success of the model in generating the observed data trends in the manufacturing employment share, L_M/L , the manufacturing value added share, Y_M/Y , per capita output, $\ln(y)$, investment-output ratio, $Inv./Y$, and relative price of the agricultural good, p . We assess the contribution of population growth by comparing the model dynamics obtained under the “all effects” to that generated by “all but population effect” experiment. The difference is attributed to the population effect. The contribution of technological change in agriculture is assessed by comparing the “all effects” experiment to the “all but farm technology” experiment.

When all changes are implemented simultaneously (“all effects”), the model generates massive labor reallocation away from agriculture, quite comparable to the actual labor reallocation recorded in the data. This is evident from a visual inspection of Figure 3, panel A. The darkest solid line corresponds to the manufacturing employment share implied by the “all effects” experiment. It rises nearly as much as its empirical counterpart. Table 2 reports the formal accounting for the entire time period considered and for its two subperiods, the pre-Industrial Revolution period (1650–1750) and the Industrialization period (1750–1920). The main experiment accounts for 65 percent of the (120 percent) empirical rise in the manufacturing employment share. In fact, during the first subperiod (1650–1750), the model successfully generates the entire increase in the manufacturing employment share observed in the data. This result is quite remarkable, given that our calibration procedure did not target any changes over time.

Figure 3, panel C reveals that the “all effects” experiment also implies a substantial increase in the manufacturing output share, accounting for 62 percent of its empirical counterpart. Once again, the model captures nearly all of the increase in the manufacturing output share during the first subperiod considered (1650–1750), but struggles somewhat to generate a sufficient rise during the period of 1750–1800.

The rise of the manufacturing sector implied by the main experiment occurs simultaneously with a dramatic increase in per capita output growth (Figure 3 panel D) and substantial increases in both the relative price of the farm good (Figure 3 panel E) and the economy-wide investment rate (Figure 3 panel F). It is clearly seen that the implications of the model for the relative price, investment, and output dynamics are very closely aligned with their empirical counterparts. Table 2 reports that the model successfully accounts for 96 percent of the observed rise in the log of per capita output and 102 percent of the rise in the relative farm price. Even though we did not target investment rate as a part of our calibration procedure, the implied investment rate in 1650 is comparable to the 4 percent reported in Crafts (1985). The investment rate rises to approximately 8–9 percent by 1920, and the model accounts for 82 percent of this change (Table 2).

To summarize: when we input the estimates of technological change, population size, and trade volume into the model, it generates plausible empirical trends. Given its parsimonious nature, we deem this quantitative result to be a considerable success of the model, and proceed to inquire further into the importance of the contribution of population growth.

To assess the marginal contribution of dramatic population growth to the overall model success, we compare the dynamics resulting from the “*all effects*” experiment with the dynamics resulting from “*all but population*” counterfactual, which differs from the main (“*all effects*”) experiment only in that it keeps the population size fixed at its 1650 level. We attribute the additional success of the “*all effects*” experiment—which can be visualized via Figure 3 as the difference between the darkest solid line and the dashed line corresponding to the “*all but population*” counterfactual—to population growth.

Similarly, to assess the marginal contribution of technological change in agriculture and the expansion of trade, we compare the dynamics resulting from the “*all effects*” experiment with the dynamics resulting from the “*all but farm technology*” and “*all but trade expansion*” counterfactuals.

Figure 3, panels A and B reveal that population growth was unambiguously a major factor behind labor movement away from the farming sector, especially during the period of 1750–1850 illustrated separately in Figure 3, panel B. Table 2 reports the formal accounting. The population effect accounts for 56 percent of the rise in the manufacturing employment share generated by the “*all effects*” experiment during the total period. However, it plays very little role in the early subperiod, accounting for only 10 percent of the overall model success during that period. The main reason is that population itself did not change much during this initial period. Figure 3, panel A also reveals that the model’s overall success in generating labor reallocation is significantly weakened, especially in the last half of the nineteenth century. This effect is partly due to the weakening of the population effect in the absence of trade.³³

³³The importance of trade expansion in the process of the development of England has been already shown in Stokey (2001). In addition to its direct role, we emphasize that the expansion of trade significantly strengthened the role of population effects in structural development.

In contrast, technological change in agricultural production plays a much more prominent role early on, accounting for 40 percent of the model's overall success during the first subperiod, but begins working against the observed process of labor reallocation as of around 1800 when technological change becomes predominately capital-augmenting. These findings are unsurprising, given our analytical result that capital-augmenting progress on the farm and population growth cannot both contribute to labor reallocation. As expected, in light of our calibration and earlier analytical results, technological progress in manufacturing does not contribute to labor reallocation.³⁴ The remaining model success is due to nontrivial interactions of technology, population, and trade effects.

Population growth is essential for generating plausible relative price dynamics, accounting for roughly 80 percent of the near doubling of the relative price of agricultural goods. Figure 3, panel E reveals that the dashed line corresponding to the "all but population" counterfactual is nearly flat, which means that, without the population change, there would be very little increase in the relative price of agricultural goods. As explained in Section IC, population growth makes the farm good relatively more expensive in the face of inflexible demand conditions, which is the case in the benchmark model ($\sigma < 1$ and $\chi > 0$). As an upshot, population growth also keeps the value of farm output relatively high. It is therefore less successful at explaining the rise in the manufacturing value added share. It accounts for only a quarter of the overall model success along this dimension, although it assumes a more prominent role in the later subperiod (Figure 3, panel C).

Figure 3, panels B and C also reveal that without the trade expansion the model significantly overstates an increase in the relative price of the agricultural good, and as a consequence, implies a drop in the manufacturing value added share despite the growth of its relative production. As explained in Section IC, population growth tends to push labor out of the inflexible agricultural sector. The relative price must increase to support the implied shift in the relative consumption in favor of the manufacturing good, and even more so because of the increased demand for subsistence food consumption associated with population growth. Rising agricultural imports alleviate the upward pressure on the price. Intuitively, rising imports allow the economy to shift resources toward the more flexible manufacturing sector without sacrificing agricultural consumption because manufacturing output can be used to pay for food imports.

We conclude that, in addition to its direct effect on labor reallocation, the dramatic expansion of trade played an incredibly important role in facilitating the impact of population growth on the process of structural transformation. It successfully moderated the strong positive effect of population growth on the relative farm price and thus played a critical role in generating the rise in the value added share of manufacturing.

In contrast to the population effect, technological change on the farm reduces the relative price of the agricultural good (Figure 3 panel E) and, as in the case with the employment share, accounts for 40 percent of the rise in the manufacturing output

³⁴ Because of space limitations we omit reporting these results.

TABLE 3—SENSITIVITY: GREATER SUBSTITUTABILITY IN UTILITY, RECALIBRATED PARAMETER VALUES

Technology	Preferences	Other
$B_{A, 1,650} = 1$	$\bar{c}_A = 0.1852$	$N = 100$
$\omega_{k, 1,650} = 0.20$	$\alpha = 0.61$	$\delta = 0.12$
$\omega_{l, 1,650} = 0.50$	$\beta = 0.82$	$M_A = 0$
$\varepsilon = 0.25$	$\sigma = 0.9$	$L = 156$
$B_{M, 1,650} = 1.71$	$\gamma = -1$	
$\nu = 0.25$		

Notes: This table summarizes the calibrated parameters in the case of $\sigma = 0.9$. See the main text for the description of the general calibration strategy.

share during the early subperiod while working against it in the second subperiod. Technological change in agriculture also helps explain the presence of positive, although slow-paced, economic growth in the first subperiod. The nineteenth century economic growth, however, is driven by productivity growth in the manufacturing sector, aided by the expansion of trade. We know from Section IC that growing import volume helps the economy shift resources away from the farm. Because the manufacturing sector experiences higher productivity growth, this shift helps with the overall economic growth.

The same cannot be said of population growth. Population growth is unimportant for economic growth—the major characteristic of the second subperiod (Figure 3, panel D). Although it pushes labor toward the faster growing manufacturing sector, just as in the case of the trade expansion, there is an offsetting (denominator) effect on per capita output. Population growth, however, is an important factor behind the rising economy-wide investment rate, accounting for approximately half of the total model success (Figure 3, panel F), although the trade expansion becomes more important in the nineteenth century.

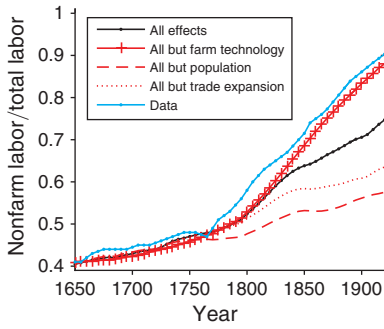
To summarize the role of population effects: population size plays an important role in the process of economic development as a whole. It matters for the dynamics of the manufacturing employment and value added shares, especially in the later period of 1750 to 1920. Population growth is also important in accounting for the rise in agricultural prices and investment rate. The role of trade expansion is critical in facilitating these effects.

D. Sensitivity Analysis

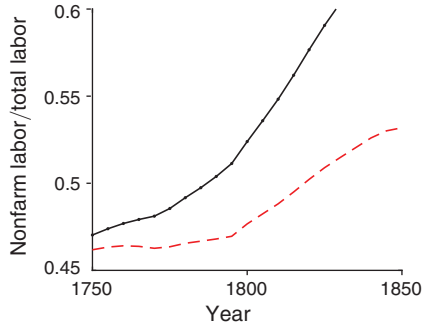
Although we view our benchmark calibration as a plausible representation of pre-Industrial England, it is important to undertake extensive sensitivity analysis.

First, to get a sense of the robustness of the main results to the demand conditions, we repeat the entire quantitative exercise under the assumption of greater demand flexibility ($\sigma = 0.9$), although we still maintain the assumption of complementary in utility ($\sigma < 1$). Recalibrated parameters are reported in Table 3. Utility parameters \bar{c}_A and α adjust to the change in the value for σ in order to ensure all the calibration targets are met.

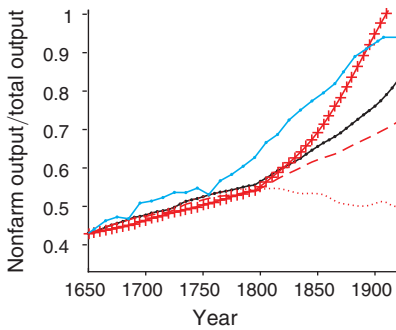
Panel A. Model versus data:
Nonfarm labor/total



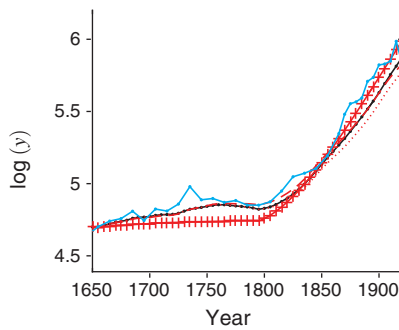
Panel B. Model versus data:
Nonfarm labor/total, 1750–1850



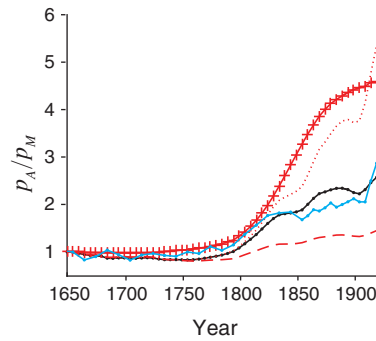
Panel C. Model versus data:
Nonfarm output/total



Panel D. Model versus data:
Real GDP/capita



Panel E. Model versus data:
Relative farm price



Panel F. Investment rate

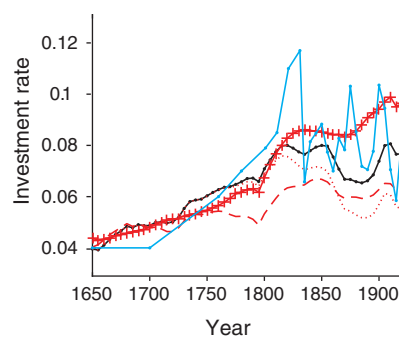


FIGURE 4. SENSITIVITY: RESULTS UNDER FLEXIBLE DEMAND, $\sigma = 0.9$

Figure 4 is the counterpart of Figure 3 and helps visualize how the model dynamics change with more flexibility in demand conditions. As expected, in light of results from Section IC, more flexibility in demand allows for the population effect to assume an even more prominent role in driving the dynamics. Population growth accounts for much more of the rise in the manufacturing value added share, without sacrificing its contribution to the relative price dynamics. The model’s overall fit actually improves as the population effect is strengthened. Table 4 reports the results. Note that the separate role of trade weakens, primarily because, in the face of more flexible demand,

TABLE 4—SENSITIVITY: RESULTS UNDER FLEXIBLE DEMAND, $\sigma = 0.9$

	L_M/L	Y_M/Y	$\ln(y)$	$Inv./Y$	p
% change data, 1650–1920	120	124	26	110	195
Percent of the empirical change accounted for by					
“all effects” experiment	70	75	97	86	80
Farm technology effect	–36	–55	–10	–28	–128
Population effect	50	27	4	44	72
% change data, 1650–1950	17	30	3	38	–7
Percent of the empirical change accounted for by					
“all effects” experiment	95	73	96	140	247
Farm technology effect	25	23	74	52	121
Population effect	14	9	–3	37	–8
% change data, 1750–1920	88	72	22	53	217
Percent of the empirical change accounted for by					
“all effects” experiment	67	81	97	52	96
Farm technology effect	–56	–88	–24	–179	–64
Population effect	58	32	5	46	63

Notes: This table summarizes the results obtained by the calibration that sets $\sigma = 0.9$. All other parameters are recalibrated to meet the calibration targets. Figure 4 reports the model dynamics for this calibration. The table illustrates that raising flexibility in demand conditions strengthens the role of population growth in the overall process of structural development. See notes to Table 2.

population growth relies less on import growth to propagate its effects. Therefore, the model without the trade expansion performs better than it did in the benchmark case.

The opposite occurs when we reduce σ : the population effect weakens and the overall performance of the model deteriorates.³⁵ This happens because movements in the manufacturing output share reflect both price and quantity adjustments. Inflexible demand conditions cause population growth to overstate the rise in the relative price of farm goods, thereby increasing the value of farm production and dampening the positive effect on the manufacturing output share.

To quantify more generally how the contribution of population growth depends on the two key parameters (ε and σ) and the target value for χ , we repeat the same quantitative exercise for each possible calibration obtained by varying ε , σ , and χ across the plausible ranges $\varepsilon \in \{0.25, 0.5\}$, $\sigma \in \{0.33, 0.5, 2\}$, and $\chi = \{0, 0.3, 0.5\}$. This exercise should be viewed as the main part of our quantitative investigation as it focuses around the three quantities identified in Section IC as the main determinants of the quantitative role of population in the process of structural transformation. Notice that an alternative approach to sensitivity analysis would be to vary various parameters one by one, while recalibrating the remaining parameters to meet the specified empirical targets. However, we know from our analytical results that the influence of various parameters for the relative importance of population growth transpires primarily through their influence on the main determinants (ε , σ , and χ).

³⁵This result is reported formally in Table 5 and discussed below.

TABLE 5—GENERAL SENSITIVITY ANALYSIS: MAIN RESULTS

ε	χ	σ	Effect	L_M/L	Y_M/Y	$\ln(y)$	$Imv./Y$	p
0.25	0	0.33	Total	61	52	96	67	122
			Population	71	34	5	46	79
0.25	0.3	0.33	Total	63	56	96	77	112
			Population	58	21	4	43	79
0.25	0.5	0.33	Total	65	62	96	91	102
			Population	45	11	3	44	78
0.25	0	0.5	Total	63	57	97	71	111
			Population	68	35	5	47	78
0.25	0.3	0.5	Total	65	62	96	82	102
			Population	56	23	4	45	77
0.25	0.5	0.5	Total	68	67	96	97	92
			Population	44	14	3	46	76
0.25	0	2	Total	81	99	101	113	52
			Population	48	35	6	58	56
0.25	0.3	2	Total	87	106	106	153	54
			Population	42	31	10	65	60
0.25	0.5	2	Total	89	109	107	150	54
			Population	26	18	4	34	46
0.5	0	0.33	Total	43	45	92	77	77
			Population	55	27	-2	53	69
0.5	0.3	0.33	Total	48	52	92	91	72
			Population	41	16	-3	51	69
0.5	0.5	0.33	Total	54	60	93	113	68
			Population	30	9	-2	54	69
0.5	0	0.5	Total	47	50	93	79	74
			Population	54	29	-2	53	68
0.5	0.3	0.5	Total	52	57	93	94	69
			Population	41	19	-2	52	68
0.5	0.5	0.5	Total	58	65	94	111	64
			Population	31	12	-2	52	68
0.5	0	2	Total	77	94	100	113	47
			Population	45	36	3	59	56
0.5	0.3	2	Total	81	99	100	131	44
			Population	38	29	3	59	57
0.5	0.5	2	Total	85	104	101	161	43
			Population	32	24	4	63	58

Notes: This table summarizes the results obtained by calibrating the model on a grid of the target for proximity to subsistence χ , farm elasticity of substitution ε , and elasticity of substitution in utility σ . For each calibration, total and population effects are reported for the period 1650–1920. The total effect refers to the percentage of empirical change accounted for by the main (“all effects”) experiment. The population effect reports the percentage of the main experiment success that is accounted for by population growth.

It is, therefore, more informative to directly vary ε , σ , and χ . This reasoning justifies our general calibration strategy.

Hence, the model is recalibrated for each combination of ε , σ , and χ , according to the procedure outlined above. Because our agricultural technology estimates depend on the calibrated value of ε , we repeat the estimation procedure for the case of $\varepsilon = 0.5$.

Table 5 reports the results. For each calibration, we repeat the same quantitative exercise as in the benchmark model. We report the overall success of the model, and

the contribution of population growth to the overall success. To save on space, we report only the total effect and the contribution of population growth.

For comparison, the benchmark calibration case is reported in rows 9 and 10 of the table ($\varepsilon = 0.25$, $\chi = 0.3$, and $\sigma = 0.5$). As already discussed, the benchmark model explains 65 percent and 62 percent of the rise in the manufacturing employment and output shares during the period between 1650 and 1920. It also accounts for 96 percent, 82 percent, and 102 percent of the observed change in $\log y$, investment-output ratio and relative price during this time period. Population growth plays a major role, accounting for 56 percent of the model success in generating labor reallocation toward the manufacturing sector.

The calibrations presented in bold in the first half of the table differ from the benchmark only in terms of σ . Our sensitivity analysis clearly reveals that with less flexibility in demand ($\sigma = 0.3$), the model's overall success weakens, especially along the dimension of the manufacturing output share and investment rate. The reason for this is that population growth, as explained in Section IC, implies a response in the relative farm price that is inversely related to demand flexibility, σ . In this case, the response is implausibly high, overstating the empirical rise in p by 23 percent. This raises the value of agricultural output. If we reduced σ further, this effect would be exacerbated.³⁶ Even though the role of population growth in driving the employment share diminishes only slightly, because the low flexibility in farm production remains highly effective at pushing the additional labor into the manufacturing sector, the overall model success in generating the rise in the manufacturing output share is substantially hampered.

In contrast, increasing σ to 2 substantially strengthens the success of the model and the role of population growth in driving both the employment and output share dynamics. In fact, the overall model success in accounting for the rise in the employment share in the second period, during which population changes are most rapid, is maximized in the case of $\varepsilon = 0.25$, $\chi = 0.3$, and $\sigma = 2$. The model accounts for 87 percent of its empirical rise (96 percent in the second subperiod), with population accounting for a little over 40 percent of the model success. However, the relative price response is insufficient in this case of flexible demand conditions. Indeed, the model accounts for only 54 percent of the observed increase in p . Consequently, the rise in the manufacturing value added share and investment rate are both overstated.

An increase in χ to 0.5, i.e., the case immediately below the benchmark, has a similar impact to that of a decrease in σ , as it too makes the demand conditions less flexible and therefore slightly weakens the population effect. However, the overall model fit remains good, as the influence of technological progress in agriculture becomes more prominent in the first subperiod.

The lower half of Table 5 reports the results for $\varepsilon = 0.5$. Because our estimates of agricultural technology depend on ε , new estimates had to be obtained for the case of $\varepsilon = 0.5$, and they are illustrated in Figure 5. It reveals that greater elasticity of substitution weakens the degree of capital-augmenting change and strengthens the degree of factor-neutral change in agriculture.

³⁶These results are not reported in the table.

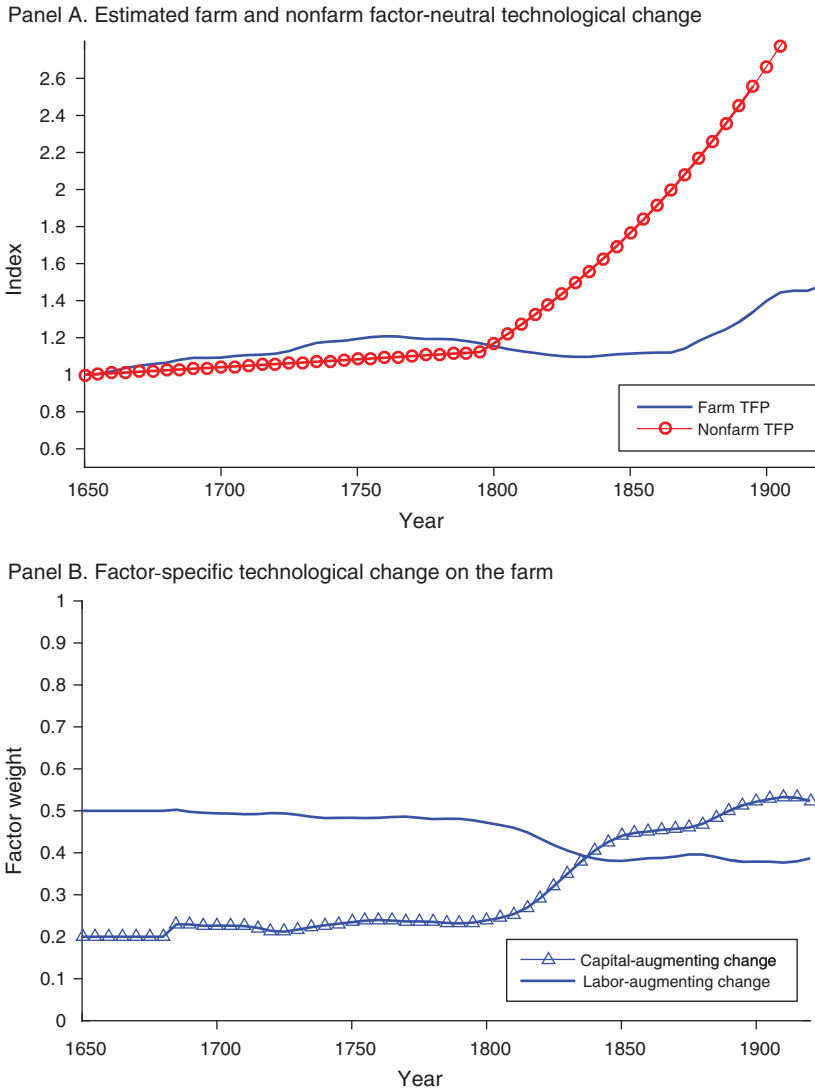


FIGURE 5. TECHNOLOGICAL PROGRESS ESTIMATES UNDER $\epsilon = 0.5$

To compare the case where only ϵ is changed (from 0.25 to 0.5) to the benchmark case, recall from Section IC that with greater substitutability between factors of production, the extra labor should be more easily absorbed in agriculture and the role of population growth in channeling labor toward the manufacturing sector should diminish. However, even in this case, with substantially more flexibility in farm production, the model explains over 50 percent of movement in the manufacturing employment and output shares, with population growth still playing a major role, especially in the second subperiod.

As a part of sensitivity analysis omitted here for brevity, we also reduced the elasticity of substitution in agricultural production, moving the technology closer to the Leontief case. This increases the role of population growth in driving labor

reallocation toward the manufacturing sector. More generally, we considered a broader range of values for ε , σ , and χ .

In addition, we investigated the alternative strategy of introducing trade into the model described in footnote 11. This made little difference to our main results. This should be clear from the fact that the benchmark model generates the rise in the relative price commensurate with its empirical rise.³⁷

The main conclusion of our sensitivity analysis is that population effects are important regardless of the choice of ε , σ , and χ . The role of population growth is particularly important in accounting for the dynamics of the sectoral employment share, the relative price of the agricultural good, and investment rate. However, its relevance in explaining the sectoral value added share dynamics is somewhat sensitive to the choice of parameters.

III. Conclusions

The role of population size in structural transformation has not been adequately examined, despite its prodigious growth that has accompanied structural developments. In this paper, we assessed the role of population growth in the structural development of England. To this end, we have built a two-sector general equilibrium growth model, with general CES forms of utility and farming technology, which features population size effects on structural development alongside the more standard technology and trade-based channels. We then employed this framework to examine the effects of population growth on structural development of England, both analytically and quantitatively.

Our analytical results reveal that the population effect on the manufacturing employment share strengthens with less production flexibility in agriculture and greater flexibility of demand. The overall level of development, and therefore proximity to subsistence consumption, also plays an important role—the effect that was pointed out in Gollin and Rogerson (2014).

In order to examine carefully the case of England, we compiled the historical data needed to estimate factor-specific technological change and to calibrate the model to the relatively stable period around prior to 1650. To assess the marginal contribution of population growth, we compared the model dynamics with all the exogenous changes inputted in the model (technological change, population growth and trade intensity) to the model dynamics under the counterfactual experiment that shuts down population growth. We argued that the rapid population growth characterizing the eighteenth and nineteenth centuries was likely an important contributor to the process of structural change, with its roles particularly pronounced in raising the manufacturing employment share, the relative price of the farming good and the aggregate investment rate. We also argued that the expansion of trade facilitated the quantitative effects of population growth, primarily by moderating its strong positive effect on the relative price of the agricultural good.

³⁷Since the first-order conditions are the same under this alternative specification of trade, except that imports become endogenous while the relative price becomes exogenous, it is not surprising that the results are robust.

Finally, we have previously referred to two related papers on structural change in the United States, namely Buera and Kaboski (2009) and Herrendorf, Rogerson, and Valentinyi (2013) who suggest that Leontief utility may be a good approximation. These papers were also concerned with the rise of the service sector, and the success of the Leontief specification is driven mainly by the observation that the share of value added of services (or consumption expenditure on services in the case of Herrendorf, Rogerson, and Valentinyi 2013) expanded together with the rise in the relative price of services.³⁸ We would like to suggest that, even if we were to pursue the strategy of calibrating the model by targeting the empirical trends during 1650–1920, which would be more in line with the methodologies implemented in these papers, we would still likely rule out the Leontief form of utility for our case study. The reason for this is the data: the share of value added of farm based production expanded together with the *fall* in its relative price. In fact, the sensitivity analysis described above clearly indicates that the overall fit of the model deteriorates as the elasticity of substitution σ declines. Even in the case of $\sigma = 0.3$, the model overstates the rise in the farm price by 36 percent in the post-1750 period and consequently struggles to generate a sufficient rise in the manufacturing value added share and investment rate. Whether introducing a service sector would substantially improve the performance of the Leontief utility function, as in these studies, is an interesting question that may merit further study.

APPENDIX

A. Data Sources³⁹

Index of Real GDP per capita, [1565–1865]: Clark (2001b, 30, table 7) (England and Wales); [1820–1990]: Maddison (1995, 194).

Investment Rate: [1700–1830]: Crafts (1985), [1830–1921] calculated from nominal GDP and gross capital formation reported in Mitchell (1988, 831).

Fraction of non-farm labor in total labor: [1565–1865]: Clark (2001b, 8, table 1) (England); [1820–1992]: Maddison (1995, 253) (UK).

Fraction of non-farm output in total output: [1555–1865]: Imputed by dividing the nominal net farm output obtained from Clark (2002, 14, table 4) (England), by the nominal GDP obtained from Clark (2001b, 19, table 3) (England and Wales), but adjusted for population differential between England and Wales; [1788–1991]: Mitchell (1978) (UK).

Population Size Index: [1541–1836]: Wrigley et al. (1997) (England); [1841–1999]: Human Mortality Database (England and Wales). Ideally, we would like to use a working age population time series, but population by age group is available

³⁸ In both papers, three consumption goods are considered with a fixed elasticity of substitution between them.

³⁹ Due to data limitations for England alone, we were forced to draw on the data sources available for England and Wales and the United Kingdom. This inconsistency should not introduce a significant error for the following reasons: (1) We do not consider level variables, such as GDP or population size, but instead growth rates, indices, and fractions of level variables. (2) For the period under consideration, the population of Wales is less than 6 percent of that of England. (3) Scotland's population size relative to that of England and Wales falls from 17 percent in 1820 (the earliest date for which we are forced to use UK data sources) to less than 10 percent today. (4) Appropriate rescaling was made in all cases.

only beginning with the first population census of 1841, reported in Mitchell (1978). For the overlapping period 1841–1921, the increase in the total population size is comparable to the increase in the population size of 15–70 year olds (2.38 versus 2.46).

Price Indices: [1555–1865]: We take $P_{A,t}$ and the aggregate index P_t from Clark (2001a, 30). We use the Schumpeter-Gilboy Price Index of producer goods for $P_{M,t}$ for [1661–1915] in Mitchell and Deane (1962). For earlier years, we infer $P_{M,t}$ using P_t , $P_{A,t}$ and fraction of output produced on the farm.

Trade Volume: [1690–1910]: Data on imports by category is taken from Mitchell (1962, 285). The imports basically consist of food items and raw materials such as raw silk and timber. In the benchmark model, we include grain, coffee, sugar, tea, wine, tobacco, flax, hemp, oil, and seeds in our calculation of $M_{A,t}$.⁴⁰ We use the appropriate price index, population time series and farm output to calculate the index $\{m_{A,t}/y_{A,1650}\}$. The series that is inputted into the model $\{m_{A,t}\}$ is then inferred from this index using the calibrated value for $y_{A,1650}$.⁴¹

Land Rental Rate (\tilde{s}_t , in percent): table 2 in Clark (2002).

Capital Rental Rate (r_t , in percent): Following Clark (2002, 6), we infer $r_t = \tilde{s}_t + 0.04$, allowing 1.5 percent for risk premium and 2.5 percent for depreciation.

Total Land Rents ($\tilde{s}_t P_{N,t}$, where $P_{N,t}$ is the price of land measured in \$/acre): Table 4 in Clark (2002) gives $\tilde{s}_t P_{N,t} N$, divided by $N = 26.5$ million acres, taken from Clark (2002, 10).

Total Wage Bill: Table 3 in Clark (2001b).

Nominal Wages: Table 1 in Clark (2002) reports nominal wages in the farm sector $W_{A,t}$. We take $W_{M,t}$ from Table 6 in Clark (2005). The rise in $W_{M,t}$ is comparable to the rise in industrial wages reported in Mitchell (1978, 193–98) for the overlapping period of 1809–1920.

Factor Income Shares on the Farm: [1500–1912]: Derived from tables 3 and 4 in Clark (2002).

Labor-Land Ratio in Agriculture: We impute this index by multiplying the fraction of farm labor in total labor by the total population.

Capital-Labor Ratio in Agriculture: The nominal capital per acre is reported in table 4 of Clark (2002). Dividing it by $P_{M,t}$ and by the labor-land ratio obtains the real capital-labor ratio in agriculture.

Labor Share in Total Income: [1585–1865]: Clark (2001b, 46, table 9) (England); [1924–1973]: Matthews, Feinstein, and Odling-Smee (1982, 164) (UK).

Land Share in Total Income: [1585–1865]: Clark (2001b, 46, table 9) (England); [1873–1913]: Matthews, Feinstein, and Odling-Smee (1982, 643) (UK).

Capital Share in Total Income: Imputed as a residual income share.

⁴⁰Our results do not change significantly if we use the series for all imports.

⁴¹The reason why we do not simply use the index for the growth rate of m_t is because the initial value m_{1650} is a zero.

B. Calibration Details

In order to apply our calibration procedure, it is convenient to rewrite the steady-state equations in terms of our moments of interest and under the assumption of no trade (moment [viii]), characteristic of the initial period 1550–1650. We use the lower case variables to refer to the corresponding aggregates divided by the population size (e.g., $n = N/L$, $l_A = L_A/L$, etc):

$$(B1) \quad \frac{\alpha}{1 - \alpha} \left(\frac{y_M - \delta k}{y_A - \bar{c}_A} \right)^{\frac{1}{\sigma}} = p,$$

$$(B2) \quad \beta(r + 1 - \delta) = 1,$$

$$(B3) \quad \frac{pB_A \omega_l}{l_A^{\frac{1}{\varepsilon}}} \left(\omega_k k_A^{\frac{\varepsilon-1}{\varepsilon}} + \omega_l l_A^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \omega_k - \omega_l) n^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} = (1 - \nu) B_M \left(\frac{k - k_A}{l - l_A} \right)^{\nu},$$

$$(B4) \quad \frac{\omega_l}{\omega_k} \left(\frac{k_A}{l_A} \right)^{\frac{1}{\varepsilon}} = \frac{(1 - \nu)(k - k_A)}{\nu(1 - l_A)},$$

$$(B5) \quad r = \nu B_M \left(\frac{1 - l_A}{k - k_A} \right)^{1-\nu},$$

$$(B6) \quad w = (1 - \nu) B_M \left(\frac{1 - l_A}{k - k_A} \right)^{-\nu}.$$

Broadly speaking, substituting for the empirical targets, we can use the above equations to solve for the model parameter values in terms of the flexible quantities ε , σ , and χ .

We set $\delta = 1 - 0.975^5 = 0.12$ to realize the 2.5 percent annual depreciation rate (moment [vii]) reported in Clark (2002). In order to match the annual interest rate of 4 percent (moment [i]) reported in Clark (2001b), the steady-state level of r must equal $1.04^5 - (1 - 0.12)$, which pinpoints $\beta = 1.04^{-5} = 0.822$ through equation (B2).

To ensure a match with the labor income share in manufacturing (moment [iv]), we set $\nu = 0.25$. The steady-state expressions for r and w , given in (B5) and (B6), together with an accounting identity $\frac{k_A}{l_A} = \frac{rK_A/pY_A}{wL_A/pY_A} \frac{w}{r}$ and empirical moments [iii] and [v], then imply the steady-state relationships: $k_A = 0.335B_M^{4/3}$ and $k = 0.641B_M^{4/3}$.

We set $\omega_k = 0.2$ and $\omega_l = 0.5$ and normalize $B_A = 1$. Equations (B3) and (B4) and moment [ii] then pin down the calibrated value for B_M and n and the steady-state level of p – all as functions of ε .

Equation (B1) gives the utility parameter α as a function of ε , σ , and the target value for χ . Finally, \bar{c}_A is pinned down as the product of y_A and the target value for χ .

C. Estimating Technological Progress: Additional Details

This Appendix clarifies the mapping between prices taken from the data and model prices, needed for estimating technological progress (Section IIB). In the model, p_t is the relative price of the farm good (units of manufacturing good per unit of farm good), r_t is the rental rate of capital (percent), w_t is the real wage (manufacturing goods per unit of labor), and s_t is the rental price of land (manufacturing goods per acre).

We infer the relative price, real wages and the real land rental price as $p_t = P_{A,t}/P_{M,t}$, $w_{i,t} = W_{i,t}/P_{M,t}$ and $s_t = \tilde{s}_t P_{N,t}/P_{M,t}$, where $P_{A,t}$ and $P_{M,t}$ are the historical price indices of the farm and nonfarm goods respectively, $W_{A,t}$ and $W_{M,t}$ are the nominal wages reported for the two sectors (\$), \tilde{s}_t is the return on land rent (in percent), and $P_{N,t}$ is the price of land (in \$/acre).

Note that in the model wages are equalized across sectors, but it is not the case in the data. We use the real wage $w_{M,t}$ in the estimation equation (22) and the real wage $w_{A,t}$ in the estimation equations (23) and (24).

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