

TAXATION AND PUBLIC HEALTH INVESTMENT: POLICY CHOICES AND TRADEOFFS

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In recent years, there has been an explosive increase in the demand for health products and services by people all around the globe, and particularly in advanced economies. Aiming to enhance longevity and also to improve quality of life, individual consumption of pharmaceutical products and services has risen exponentially since the early 1980s. This paper develops a model in which agents invest part of their resources in medical products and time in physical exercise to enhance their health status. In the first part of the paper, we study the steady state and transitional dynamics of the model with special emphasis on the effects of health decisions on aggregate outcomes. In the second part, we explore how public health policies may alter private economic decisions that promote healthier and more productive lives.

Keywords: Private and Public Health Investments, Endogenous Discounting of the Rate of Time Preference, Neoclassical Growth Model, Health Policy

1. INTRODUCTION

Motivated by recent literature that finds significant economic returns from investment in health, this paper incorporates individual agents' health decisions in the neoclassical growth model. But unlike most of the existing literature that focuses attention on the effects of health status on economic development—and more specifically the link between mortality and growth in developing countries—this paper is concerned with health decisions by rational agents living in relatively

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wealthy/healthy societies.¹ Although there is a clear increase in demand for health goods (e.g., medicines, vitamins, and vaccines) and services (e.g., surgery, psychotherapy, and health clubs), coupled with a lively debate regarding the relative merits of private vs. public provision of health in the USA and Europe, there is surprisingly little research on how rational health investment behavior may affect economic aggregates and what, if any, should be the government's role in this process.

This is an important issue, and in this paper we address it by introducing health decisions into an otherwise standard dynamic general equilibrium framework. We identify three channels whereby health status impinges on the aggregate economy. These include: (i) its direct impact on well-being and the consequences for consumption and leisure in utility, (ii) its impact on productivity, and (iii) its impact on the rate of time discount via longevity, as in Blanchard (1985) and the subsequent demographic literature. Once the long-run and transitional dynamic characteristics of the macroeconomic equilibrium are established, we address the key policy question, namely, the effects of government health investment on productivity, growth, and welfare. Underinvestment in health by individuals may require proactive government intervention to subsidize the purchase of medical goods and services as well as memberships to health clubs and other designated forms of physical activities. Our analysis, based on numerical simulations, assesses how government intervention may impact agents' health, productivity, and subsequently aggregate growth and well-being.

The policy experiments we perform suggest that there are significant welfare gains associated with movement from the current tax structure and rate of public health investment (which we take to be typical of a developed economy) to the first best optimum where the government controls all resources, including health, directly. For the benchmark calibration of our model, with a tax structure and other structural characteristics that approximates the US economy, welfare would increase by 24% across steady states (measured in terms of equivalent variation of per-capita consumption). Taking account of the adjustment along the transitional path reduces the welfare gains to 14.3%. Moreover, for a relatively modest increase in the productivity effect of health (channel ii above) the steady-state welfare benefits increase to around 34%.²

However, a large proportion of these gains arise from the dramatic changes in the tax structure necessary to achieve the first best optimum. As is evident from Table 3, these changes are completely implausible and can be dismissed as impractical. Accordingly, to examine the potential of health investment to raise welfare, we consider more modest (and more realistic) policy experiments. Specifically, we compare an increase in health investment expenditure of one percentage point of Gross Domestic Product (GDP) under alternative modes of finance (Table 4). For benchmark parameterization, a lump-sum tax-financed health investment yields more modest (but still quite significant) welfare gains of 2.71%, somewhat larger than the 2.45% increase obtained by employing a consumption tax. However, if the investment is financed by a tax on capital income,

the resulting decline in output and consumption is sufficient for the net effect to reduce welfare by 1.35%. But increasing the productive elasticity of health modestly enhances all these welfare gains, in which case even employing capital tax financing can generate a modest overall improvement in welfare.

These policy experiments highlight the fact that increasing public investment in health by 1 percentage point of GDP—which represents a substantial increase in the *level* of investment in health—with the economy's tax structure virtually unchanged nevertheless leaves the economy far from the social optimum. This raises the question of whether it is possible to realize a substantial portion of the potential welfare gains by combining the government investment in health with additional, moderate tax changes that move at least partially in the direction of the first best policy. Thus, we find that about one-third of the welfare gains can be realized by combining the 1 percentage point increase in health investment with an approximate one-third reduction of the income tax rates from their respective initial benchmark rates, and an approximate halving of the subsidy, all compensated for by a 3.75 percentage point increase in consumption tax (Table 5). While these represent substantial changes in the US tax structure, they are not implausible, and indeed reflect changes in the tax structure that have been advocated by conservative policy-makers over the years.³ Furthermore, eliminating income tax entirely in favor of consumption tax, while doubling investment in health (from 3% to 6% of GDP), would increase the welfare gains realized to more than 75% of the potential gains associated with the first best.

Our results include some interesting implications for health investment policy. These pertain to the robustness of welfare gains from increases in government investment in health relative to the initial tax structure. Specifically, we find that an increase in the share of government investment in health of GDP from 3% to 4% results in an additional welfare gain of around 2%, irrespective of the initial tax structure. However, a further increase from 4% to 6% yields a further welfare gain of something over 1%, suggesting sharply diminishing returns to health investment, for a given tax structure. In this regard, we also find a sharp tradeoff between subsidizing health care and investment in health infrastructure. Specifically, a modest reduction in the subsidy from 0.64 to 0.60 could finance an increase in g from 3% to 4%, yielding a 2.4% increase in welfare. This suggests that the subsidy is too large and the health infrastructure is undersupplied from the standpoint of social welfare.

There is an emerging interest in the potential effects of health on development [see, e.g., Strauss and Thomas (1998), Deaton (2003), Chakraborty (2004), Bloom and Canning (2005), Soares (2005), Lorentzen et al. (2006), Weil (2007), Birchenall and Soares (2009), Chakraborty et al. (2013), and Bhattacharya and Chakraborty (2016)]. This literature focuses primarily on investigating the hypothesis that health status (measured as positively related to life expectancy, or inversely related to mortality or diseases) is a key determinant in explaining cross-country income differences via its direct or indirect effect on individuals' productivity and savings behavior. Similar to this literature, in our work, health

status is an important determinant of productivity of the individuals. However, as our focus is on the potential effects of health investment on the productivity and welfare of agents living in relatively wealthy countries, our work also relates closely to a smaller set of theoretical contributions focusing on the effects of the decisions of individual agents to maximize, in addition to consumption, also their life expectancy (time horizon). These include Ehrlich and Chuma (1990), de la Croix and Licandro (1999), Aísa and Pueyo (2004a, 2004b), Finlay (2006), Agénor (2008, 2010), Hosoya (2009), Halliday et al. (2019), Suen (2017), and Allen and Chakraborty (2018). Our approach is closely related to Blanchard's (1985) seminal contribution, and more recently Faruqee (2003), in which agents' decisions impact the time discount rate.

The remainder of the paper is organized as follows. Section 2 discusses recent trends in health investment, including both increased spending in medical goods and services and a very large increase in health club membership. Section 3 develops and describes in detail the basic elements of the model, and Section 4 studies the dynamic behavior and steady state for a decentralized economy. Section 5 extends the analysis to a centrally planned economy. Section 6 describes the explicit function forms and calibrations employed in the numerical simulations presented for the set of policy experiments in Section 7. Section 8 concludes, while technical details are relegated to the Appendix.

2. MORE PILLS, MORE THERAPY, MORE HEALTH CLUBS

Even a few decades ago the average citizen in a relatively wealthy nation like the USA or the UK would not make active decisions regarding health issues, other than perhaps consulting the medical profession when necessary. This was partly due to a lack of sufficient income to engage more actively in health investment decisions, such as those that would increase one's lifespan and/or quality of life. But mostly it was due to treating health, and especially life expectancy, as largely exogenous—functions of genes, luck, and perhaps the level of development in the community.

This passive outlook on how much control individuals have over their health seems to have reversed over the last few decades, where we now observe a huge emerging demand for health products and services. It is argued that this behavioral change is motivated by life-saving medical technological advances that have contributed to the development of the mass production of new medical products at reasonable prices. Another reason for the drastic increase in health decisions is the structural transformation from agriculture and manufacturing to services. While agricultural production and manufacturing require physical labor, services depend primarily on human capital rather than on raw labor. Therefore, much of the physical activity necessary for a healthy existence now needs to be pursued outside one's workplace.

Recent studies confirm that benefits of regular physical activity include a reduced risk of premature mortality and reduced risks of coronary heart disease,

diabetes, colon cancer, hypertension, and osteoporosis [US Department of Health and Human Services (1996)]. Regular physical activity also improves symptoms associated with musculoskeletal conditions and mental health conditions such as depression and anxiety. Physical activity, along with a healthy diet, plays an important role in the prevention of overweight and obesity, which have increased at alarming rates in the recent years.

In 1995, the Centers for Disease Control and Prevention (CDC) issued a public health recommendation that every US adult should accumulate 30 minutes or more of moderate-intensity physical activity on most, preferably all, days of the week. In 2007, the American Heart Association issued updated recommendations on the types and amounts of physical activity needed by healthy adults to improve and maintain health [Haskell et al. (2007)]. In October 2008, the US Department of Health and Human Services released guidance to help Americans of age 6 and older improve their health through appropriate physical activity [US Department of Health and Human Services (2008)].

The World Health Organization (WHO) reports that health expenditure in OECD countries increased on average by 24% during the period 1995–2012. The USA was at the top of the list of countries in terms of the percentage of health expenditure to GDP of over 18%, with France and the Netherlands distant second with 12%. Between 1999 and 2009, US health care spending nearly doubled, from \$1.3 trillion to \$2.5 trillion. Much of the health expenditure came from government, albeit with diverse experiences across countries. The largest and smallest contributions of government to total health spending in 2012, among OECD countries, were by Denmark (86%) and the USA (46%), respectively, while the governments of 11 OECD countries contributed more than 80% of total health spending. But individuals these days invest not only in improving longevity but also in the quality of life. As is well known, there is a positive and very robust relationship between health spending and income levels [WHO (2017)].

The upper panel of Figure 1 shows this relationship at the cross-country level (this relationship has also been established at the individual/consumer level). The lower panel of Figure 1 depicts another well-established relationship (similar to the Preston curve (1975) relating life expectancy and per capita income) illustrating a strong and positive relationship between health spending and life expectancy early on, but a leveling-off (indeed a flattening) of this relationship at older ages (over-80s). Combining the two figures implies that most of the health spending in developed countries is not aimed at increasing longevity but rather at improving the quality of life.

Indeed, investing in preventive medicine and improved quality of life, especially in advanced economies, is clearly demonstrated by the drastic increase in the fitness and health clubs industry. According to the International Health, Racquet and Sportsclub Association statistics, the number of health club memberships has increased from 36.3 million in 2002 to approximately 58.5 million in January 2013. The Physical Activity Council's Participation Report (2013)

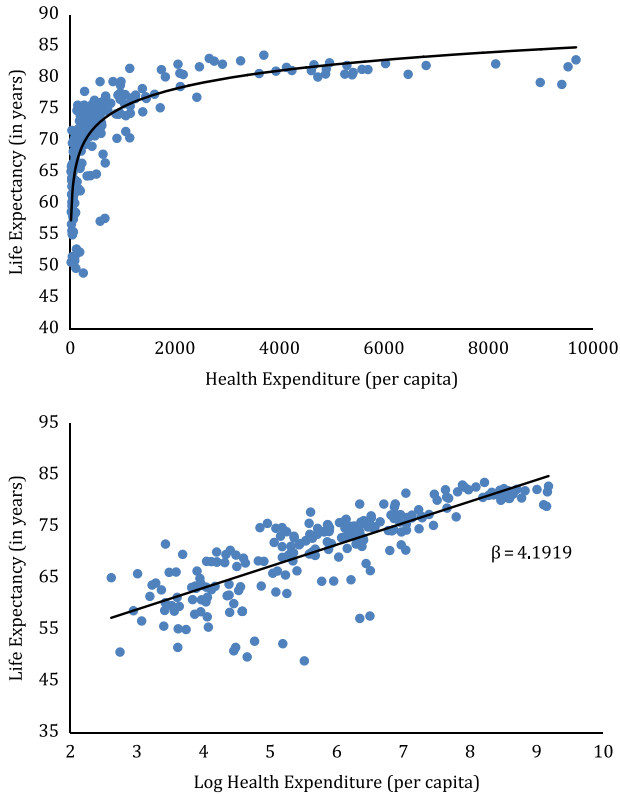


FIGURE 1. Per-capita health expenditures and life expectancy (2015).

revealed that over 60% of Americans regularly participate in fitness sports as of 2012; this is the fifth consecutive year in which these numbers have remained stable at 60% or more. The US Bureau of Labor Statistics (BLS) suggests that the number of jobs in the health club industry is expected to increase by more than 23% over the next 10 years. This goes to show that investing in preventive health and fitness is only showing a potential for stronger growth in the USA, with this phenomenon catching fast throughout the globe.

In summary, the health investment incorporated in our model is motivated by recent trends, in developed countries, reflecting individuals trying to enhance their longevity and quality of life by purchasing medical goods and services and/or devoting some of their time to physical activities. It is also motivated by the very large share of GDP allocated to public health expenditure, particularly in the USA. However, the apparent differences in the impact of health spending on health-related outcomes in developing and developed countries (Figure 1) implies that different mechanisms may be at work in the two cases. Therefore, we restrict ourselves to the case of developed countries.

3. BASELINE MODEL

The notion that health should be treated as a durable capital good originated with the seminal contribution of Grossman (1972). Once that is recognized, this raises the issue of whether it should be introduced as a form of private capital, or as public capital. Empirically, the relative importance of these two forms of capital varies dramatically across the OECD economies. With the exception of a few, including most notably the USA, the balance would tend to favor public capital as the dominant source of health capital; see OECD (2015). The formal analytical literature is also somewhat divided. In his early contribution, Grossman treated health as a private capital good, an approach that is also adopted in a recent contribution by Kelly (2017). In contrast, Hosoya (2009) and Agénor (2008) introduce health investment as a form of public capital, very much akin to public infrastructure.⁴

A similar theme emerges from the extensive coverage of the question of public vs. private health systems in the public health literature [see, e.g., McLachlan and Maynard (1982) and Navarro (1985)]. The USA is known to differ from Canada and Europe because it is at the private end of the spectrum. Compared with Canada, France and the UK, the USA has the highest health care bill (private and public combined), but it has also the lowest share of public expenditure on health as a share of GDP. At the opposite end, the UK has the most public health system based on a National Health Service. Canada and France fall midway between the extremes of the USA and the UK. Whether a more public system, with more government involvement, is better than a more private system has been debated extensively on the merits of whether public spending is efficient and whether resorting to a more private system may render a significant fraction of the population uninsured. The hybrid model that we propose below relates to this issue of private vs. public health care provision in that it assesses further government involvement in health care under recent shifts in the well-being behavior of people, particularly in advanced economies.

In our hybrid framework, health services are produced in its own productive sector (rather than as a homegrown activity) using both a private input (labor) and a public input (infrastructure, i.e., medical equipment and hospitals). These health services may then be purchased by private agents at a market-determined price, which may be partially or totally subsidized by the government, characteristic common to most advanced economies. The use of health services impinges on the performance of the economy in three dimensions. First, it directly enhances the welfare of individual agents, raising the marginal utility of consumption, as Finkelstein et al. (2013) have recently emphasized. Second, the aggregate level of health services also enhances productive efficiency in the economy—an externality that is not internalized by firms and individuals. Third, by increasing longevity and quality of life, better health reduces agents' rate of time preference, making them more patient; see, for example, Cutler and Richardson (1997) and Hall and Jones (2007). Thus, our formulation is directed at capturing the role of health in a developed economy.⁵ We begin by describing the model,

emphasizing modifications and additions made to the standard neoclassical model to incorporate public and private health investments.

3.1. The Representative Consumer

We assume that all individuals are identical and that population grows at a constant rate n . The identical consumers in the economy maximize utility specified as a positive function of consumption (c), leisure (l), and health (h). In addition, as noted, since health increases longevity, the rate of time preference $\theta(h)$ is decreasing in health, that is, $\theta_h(h) < 0$. Thus, agents maximize the concave intertemporal utility function⁶

$$\int_0^\infty U(c, l, h) e^{-\int_0^t \theta(h(s)) ds} dt, \quad U_c > 0, \quad U_l > 0, \quad U_h > 0 \tag{1a}$$

subject to the budget constraint

$$\dot{k} = [(1 - \tau_k)r - (n + \delta_k)]k + (1 - \tau_w)w(1 - l) - (1 + \tau_c)c - p(1 - s)h - T, \tag{1b}$$

where all quantities are in per-capita terms: k is capital, r is return to capital, and δ_k is depreciation rate of capital, w is the wage rate, τ_k , τ_w , and τ_c are rates of capital, labor, and consumption taxes, and T denotes the lump-sum tax. Equation (1b) also shows that the agent purchases health services at a price p , which may be subsidized by the government at rate s . These health services are broadly defined to include medical services, pills, and even subscriptions to health clubs. For simplicity, we identify the purchase of these health services as being identical to health itself.⁷ In general, s is unrelated to any tax rate, but if, for example, $s = \tau_w$, this implies that individuals may exactly deduct their health costs from their tax paid on labor income. We also assume that the household may work either in the final output sector or in the health sector, with each sector paying the same wage.

To solve the consumer’s optimization problem, it is convenient to define $z(t)$, the cumulative rate of time preference over the time interval $(0, t)$, by

$$z(t) \equiv \int_0^t \theta[h(s)] ds, \tag{2}$$

which implies

$$\dot{z}(t) \equiv \theta[h(t)]. \tag{3}$$

Thus, the agent’s optimization problem can be re-expressed as to maximize

$$\int_0^\infty U(c, l, h) e^{-z(t)} dt, \tag{1a'}$$

subject to (1b) and (3). Performing the optimization yields the following first-order conditions:

$$U_c(c, l, h) = \lambda(1 + \tau_c), \tag{4a}$$

$$U_l(c, l, h) = w(1 - \tau_w)\lambda, \tag{4b}$$

$$U_h(c, l, h) + \mu\theta_h = p\lambda(1 - s), \tag{4c}$$

$$r(1 - \tau_k) - n - \delta_k = \dot{z}(t) - \frac{\dot{\lambda}}{\lambda}, \tag{4d}$$

$$-\frac{U(c, l, h)}{\mu} = \dot{z}(t) - \frac{\dot{\mu}}{\mu}, \tag{4e}$$

together with the transversality conditions $\lim_{t \rightarrow \infty} \lambda ke^{-z(t)} = \lim_{t \rightarrow \infty} \mu ze^{-z(t)} = 0$, where λ, μ are the co-state variables associated with the dynamic equations (1b) and (3), respectively.

The optimality conditions (4a) and (4b) equate the marginal utility of consumption and leisure to their respective tax-adjusted marginal costs expressed in utility units, and are standard. Equations (4d) and (4e) are arbitrage conditions. The first equates the rate of return on capital to the utility rate of return on consumption measured in terms of units of output, and is also standard. Integrating (4e) yields

$$\mu(t) = - \int_t^\infty U(s)e^{-[z(s)-z(t)]} ds,$$

indicating that $\mu(t)$ reflects the discounted losses in utility resulting from a higher discount rate. Hence, (4c) equates the marginal costs of health services (net of subsidy) to the benefits, which include the benefits from increased longevity, in addition to the direct utility benefits. By combining (4a) and (4c), we obtain

$$\frac{U_c(c, l, h)}{1 + \tau_c} = \frac{U_h(c, l, h) + \mu\theta_h}{(1 - s)p}$$

Thus, the optimal spending decision equates the tax-adjusted marginal utility of consumption to the subsidy-adjusted marginal utility of health, which, as just noted, includes the direct increase in marginal utility plus the indirect marginal benefits, resulting from the increase in longevity.

3.2. The Production Process

Production takes place in two sectors: a conventional final output sector, with each firm owned by a private individual, and a health sector, owned by the government. The representative firm in the final output sector produces in accordance with the conventional production function

$$y = f(k, L, h), \quad f_k > 0, f_L > 0, f_h > 0, \tag{5a}$$

which is homogeneous of degree 1 in k and labor, L . In addition, in light of the empirical evidence provided by Bloom et al. (2004), factor productivity is assumed to increase with the level of health, (h), which agents take as given. Firms choose k, L , to maximize profit:

$$f(k, L, h) - wL - rk. \tag{5b}$$

so that equilibrium factor returns are given by the usual marginal conditions

$$f_k(k, L, h) = r, \quad (6a)$$

$$f_L(k, L, h) = w. \quad (6b)$$

Health services are produced in accordance with the production function

$$h = h(m, e), \quad h_m > 0, \quad h_e > 0, \quad (7a)$$

which is also homogeneous of degree 1, in m and e , where m is the per-capita health infrastructure/capital provided by the government, and e is the labor employed in the health sector.⁸ Thus (private) physical capital is specific to final goods production, while (public) health capital is specific to health services production. The health sector firm chooses employment, e , to maximize

$$ph(m, e) - we, \quad (7b)$$

leading to the optimality condition

$$ph_e(m, e) = w. \quad (7c)$$

The homogeneity of the health production function means that the government earns profit, $p(h - h_e e)$, which contributes to its revenue.

3.3. Government

Expressed in per-capita terms, the government's budget constraint is

$$T = \dot{m} + (n + \delta_m)m + sph - \tau_k rk - \tau_w w(L + e) - \tau_c c - p(h - h_e e). \quad (8)$$

According to (8) current government expenditures include its increase in health capital, both per capita and for the growing population plus depreciation $(\dot{m} + (n + \delta_m)m)$ and its subsidy to health expenditures (sph) . Its revenues include the total tax collected $(\tau_k rk + \tau_w w(L + e) + \tau_c c)$, as well as profit earned by the health sector, $p(h - h_e e)$. To the extent that these items are not balanced, it finances the difference with lump-sum taxes. Since we are focusing on a growing economy, we assume that the government devotes a fraction, g , of aggregate output to augment the aggregate stock of public health capital. Thus, in per-capita terms, we have

$$\dot{m} = gf(k, L, h) - (n + \delta_m)m, \quad (9)$$

which using (7a) and (7b) enables us to rewrite the government budget constraint (8) in the form

$$T = gf(k, L, h) + sph - \tau_k f_k k - \tau_w f_L(L + e) - \tau_c c - p(h - h_e e). \quad (8')$$

3.4. Market Clearance

Labor is assumed to be both fully employed and to enjoy free sectoral mobility, implying:

$$L + e = 1 - l. \tag{10}$$

Combining the consumers' budget constraint (1b), and government's (per-capita) budget constraint (8'), recalling (7c) and utilizing (10) yields the final goods market clearing condition

$$\dot{k} = (1 - g)f(k, L, h) - c - (n + \delta_k)k, \tag{11}$$

which determines the evolution of the capital in the economy.

As noted in the introduction, there is a significant variance across countries in the extent to which health is publicly or privately provided. This model is a hybrid in that the government provides the capital employed in the health sector, while the labor is private individuals.⁹ Thus, the total expenditure on health is $gf + ph$.

4. SOLVING THE BASELINE MODEL

To solve for the macroeconomic equilibrium, we reduce it to a core dynamic system. It is evident that as the model is set out, it is already specified in terms of stationary variables. This is because we assume zero growth in factor productivities in both sectors. Thus, the growth in the aggregates of this economy along the balanced growth path is driven solely by population growth.

To derive the core dynamic system we first equate (7b) and (7c) to obtain

$$ph_e(m, e) = f_L(k, L, h). \tag{12}$$

Using (12), (7a) and (7b) to eliminate r , w , and p from (4b) and (4c), we can reduce the short-run equilibrium conditions to (4a), (5b), (10), and

$$U_l(c, l, h) = \lambda(1 - \tau_w)f_L(k, L, h), \tag{4b'}$$

$$U_l(c, l, h) = \left(\frac{1 - \tau_w}{1 - s}\right) [U_h(c, l, h) + \mu\theta_h(h)]h_e(m, e). \tag{4c'}$$

Equations (4b') and (4c') imply that time should be allocated such that the marginal utility of leisure, the tax-adjusted marginal utility of wages foregone in the final output sector, and the marginal utility of wages foregone in the health sector should all be equalized.

These five equations determine c, h, l, L, e as functions of m, k, λ, μ , the dynamics of which are driven by the system consisting of (11), (9), and

$$\frac{\dot{\lambda}}{\lambda} = \theta(h) + n + \delta_k - f_k(k, L, h)(1 - \tau_k), \tag{4d'}$$

$$\frac{\dot{\mu}}{\mu} = \theta(h) + \frac{U(c, l, h)}{\mu}, \tag{4e'}$$

given the time paths of the exogenous policy variables $g, \tau_k, \tau_w, \tau_c,$ and $s,$ with lump-sum taxes adjusting residually to satisfy the government’s budget constraint (8’).

The long-run equilibrium (steady state) of the system is described by the above set of equations when dynamic variables are time-invariant, that is, by setting $\dot{k} = \dot{m} = \dot{\lambda} = \dot{\mu} = 0.$ This yields:

$$U_c(\tilde{c}, \tilde{l}, \tilde{h}) = \tilde{\lambda}(1 + \tau_c), \tag{13a}$$

$$U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \tilde{\lambda}(1 - \tau_w)f_L(\tilde{k}, \tilde{L}, \tilde{h}), \tag{13b}$$

$$\left(\frac{1-s}{1-\tau_w}\right) U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \left[U_h(\tilde{c}, \tilde{l}, \tilde{h}) + \tilde{\mu}\theta_h(\tilde{h})\right] h_e(\tilde{m}, \tilde{e}), \tag{13c}$$

$$\tilde{h} = h(\tilde{m}, \tilde{e}), \tag{13d}$$

$$\tilde{L} + \tilde{l} + \tilde{e} = 1, \tag{13e}$$

$$(1 - g)f(\tilde{k}, \tilde{L}, \tilde{h}) = \tilde{c} + (n + \delta_k)\tilde{k}, \tag{13f}$$

$$gf(\tilde{k}, \tilde{L}, \tilde{h}) = (n + \delta_m)\tilde{m}, \tag{13g}$$

$$f_k(\tilde{k}, \tilde{L}, \tilde{h})(1 - \tau_k) = \theta(\tilde{h}) + n + \delta_k, \tag{13h}$$

$$\tilde{\mu}\theta(\tilde{h}) + U(\tilde{c}, \tilde{l}, \tilde{h}) = 0. \tag{13i}$$

These equations then can be solved for $(\tilde{c}, \tilde{l}, \tilde{L}, \tilde{e}, \tilde{h}, \tilde{k}, \tilde{m}, \tilde{\lambda}, \tilde{\mu}),$ where the tilde denotes the steady-state value of the corresponding variable. In particular, (13h) corresponds to the modified golden rule equilibrium of the standard neoclassical growth model. As in that model, increases in capital–labor ratio arise from factors that raise marginal product of capital (such as growth in total factor productivity). While this also applies to h in this model (as long as $f_{kh} > 0)$, this equation shows that, by reducing the rate of time preference and stimulating savings, an increase in health status has an additional long-run stimulative effect on the capital–labor ratio.

From these solutions one can derive the equilibrium relative price of health services from the relationship $\tilde{p} = f_L(\tilde{k}, \tilde{L}, \tilde{h})/h_e(\tilde{m}, \tilde{e}).$ The steady-state government budget constraint is (in per-capita terms) given by

$$\tilde{T} = g\tilde{f} - \tau_k\tilde{f}_k\tilde{k} - \tau_w\tilde{f}_L(\tilde{L} + \tilde{e}) - \tau_c\tilde{c} + s\tilde{p}\tilde{h} - \tilde{p}(\tilde{h} - h_e\tilde{e}). \tag{13j}$$

To solve for the transitional dynamics, we substitute the solutions for $c, h, l, e,$ and L obtained from (4a), (4b’), (4c’), (7a), and (10) into the dynamic equations (11), (9), (4d’), and (4e’). Linearizing the resulting expressions around the steady state (13a)–(13i) yields the local dynamics. The partial derivatives required for this process are reported in Appendix A.1, enabling us to approximate the transitional dynamics by the linearized system (Appendix A.2). We assume that this

system has saddle point structure, yielding a unique stable transitional path, an assumption that is unambiguously supported by the simulations that we discuss below.

5. THE SOCIAL PLANNER’S PROBLEM

By its very nature, health has many of the characteristics of a public good. In terms of our formal model, it appears as an externality in the production of the physical good as the health status of the worker is a non-market or non-remunerated input into the production function. It is thus natural to determine the socially optimal resource allocation in such an economy, and to investigate the extent to which the government in a decentralized economy can use the fiscal instruments at its disposal to replicate the first best outcome, as would be chosen by a central planner.

Specifically, we assume that the social/central planner chooses the resource allocations directly, namely, c, l, e, L, h, g , and the rates of accumulation, $\dot{k}, \dot{z}, \dot{m}$, to maximize the representative agent’s intertemporal utility (1a) subject to the resource constraints (3), (7a), (9), (10), and (11). By performing optimization, the macroeconomic equilibrium in the centrally planned economy can be reduced to the following static equations:

$$U_c(c, l, h) = \lambda, \tag{14a}$$

$$U_l(c, l, h) = \lambda f_L(k, L, h), \tag{14b}$$

$$U_l(c, l, h) = [U_h(c, l, h) + \lambda f_h + \mu \theta_h(h)] h_e(m, e), \tag{14c}$$

$$f_k(k, L, h) - \delta_k = \frac{h_m(m, e)}{h_e(m, e)} f_L(k, L, h) - \delta_m, \tag{14d}$$

$$h = h(m, e), \tag{7a}$$

$$l + L + e = 1. \tag{10}$$

together with the dynamic equations:

$$\dot{k} = (1 - g)f(k, L, h) - c - (n + \delta_k)k, \tag{11}$$

$$\dot{m} = g f(k, L, h) - (n + \delta_m)m, \tag{9}$$

$$\frac{\dot{\lambda}}{\lambda} = \theta(h) + n + \delta_k - f_k, \tag{15a}$$

$$\frac{\dot{\mu}}{\mu} = \theta(h) + \frac{U(c, l, h)}{\mu}. \tag{15b}$$

It is important to note that this macrodynamic equilibrium implicitly generates a time path for the optimal health investment, g . To determine this, we view the set of static equations (14a–14d), (7a), and (10) as a subsystem that can be solved for

c, l, L, e, h, μ as functions of k, m, λ . In particular, consider $\mu = \mu(k, m, \lambda)$. Taking the time derivative of this expression yields $\dot{\mu} = \mu_k \dot{k} + \mu_m \dot{m} + \mu_\lambda \dot{\lambda}$, implying that the four dynamic equations, (11), (9), (15a), (15b), are not all independent. Substituting for these equations yields

$$\begin{aligned} \mu\theta(h) + U &= \mu_k [(1 - g)f - c - (n + \delta_k)k] + \mu_m [gf - (n + \delta_m)m] \\ &+ \mu_\lambda \lambda [(\theta + n + \delta_k) - f_k]. \end{aligned}$$

This equation can now be solved explicitly for the time path of g , which, through this relationship, is also of the form

$$g = g(k, m, \lambda). \tag{16}$$

The steady state of a centrally planned economy, denoted by bars, consists of

$$U_c(\bar{c}, \bar{l}, \bar{h}) = \bar{\lambda}, \tag{17a}$$

$$U_l(\bar{c}, \bar{l}, \bar{h}) = \bar{\lambda} f_L(\bar{k}, \bar{L}, \bar{h}), \tag{17b}$$

$$U_h(\bar{c}, \bar{l}, \bar{h}) = [U_h(\bar{c}, \bar{l}, \bar{h}) + \bar{\lambda} \bar{f}_h + \bar{\mu} \theta_h(\bar{h})] h_e(\bar{m}, \bar{e}), \tag{17c}$$

$$f_k(\bar{k}, \bar{L}, \bar{h}) - \delta_k = \frac{h_m(\bar{m}, \bar{e})}{h_e(\bar{m}, \bar{e})} f_L(\bar{k}, \bar{L}, \bar{h}) - \delta_m, \tag{17d}$$

$$\bar{h} = h(\bar{m}, \bar{e}), \tag{17e}$$

$$\bar{l} + \bar{L} + \bar{e} = 1, \tag{17f}$$

$$(1 - \bar{g})f(\bar{k}, \bar{L}, \bar{h}) = \bar{c} + (n + \delta_k)\bar{k}, \tag{17g}$$

$$\bar{g}f(\bar{k}, \bar{L}, \bar{h}) = (n + \delta_m)\bar{m}, \tag{17h}$$

$$f_k(\bar{k}, \bar{L}, \bar{h}) = [\theta(\bar{h}) + n + \delta_k], \tag{17i}$$

$$\bar{\mu} \theta(\bar{h}) + U(\bar{c}, \bar{l}, \bar{h}) = 0. \tag{17j}$$

These 10 equations determine the steady-state values of $\bar{c}, \bar{l}, \bar{L}, \bar{e}, \bar{h}, \bar{k}, \bar{m}, \bar{\lambda}, \bar{\mu}, \bar{g}$.

5.1. Replication of the Steady State of the First Best Equilibrium

We now turn to the question posed earlier: can government use its instruments (tax and subsidy rates) to replicate the first best outcome in the decentralized market equilibrium? For convenience, we focus on the steady state. To achieve this, we need to determine if we can find $\tau_w, \tau_c, \tau_k, s$ so that the allocation of (13a)–(13i) mimics that of (17a)–(17j). Our objective is to replicate the first best allocation of real quantities. To do this we eliminate shadow values and prices, which only serve as signals for resource allocation.

PROPOSITION 1. (*Replication of the steady state of the first best equilibrium*). *The first best optimum will be attained if and only if the following three conditions hold:*

$$\tau_k = 0. \tag{18a}$$

$$\tau_w + \tau_c = 0. \tag{18b}$$

$$\frac{1 - s}{1 - \tau_w} = \frac{U_h - U\theta_h\theta^{-1}}{U_h - U\theta_h\theta^{-1} + U_l f_h(f_L)^{-1}} = 1 - \frac{f_h}{f_L} h_e < 1. \tag{18c}$$

For proof, see Appendix A.2.

Setting the long-run optimal tax on capital, $\hat{\tau}_k = 0$, is consistent with the celebrated proposition introduced by Chamley (1986) and Judd (1985). However, here a different mechanism is at play. The Chamley–Judd proposition is based on the assumption that government expenditure is fixed in *levels*. If, instead, it were set as an optimal fraction of income, \hat{g} say, then the corresponding optimal tax on capital would be set correspondingly as $\hat{\tau}_k = \hat{g}$. By contrast, in the present context, if the fraction of government expenditure, g , is set arbitrarily, then one can show $\hat{\tau}_k = g(1 - \phi/\lambda)$ where ϕ is the shadow value of public capital (health).¹⁰ With private capital, k , being costless to transfer to public capital, m (and vice versa), the optimal policy is to set the allocation between the two capital goods so as to equate their respective shadow prices, implying $\phi = \lambda$ and hence $\hat{\tau}_k = 0$.

The condition (18b) is also familiar.¹¹ Since there are no imperfections in either the labor market or the final goods market, the distortions introduced by taxing labor income and consumption must be exactly offsetting. An important consequence of this constraint is that the ability to tax consumption introduces limited flexibility with respect to taxing income from labor. This is important in generating the tax revenues to finance its expenditures. With τ_c and τ_w working against each other in terms of revenue generation, the effective tax base for the government is the difference between labor income and consumption. Given the difference between the two is almost surely less than one-fifth of GDP for a typical country, raising realistic amount of revenue (even without considering subsidies for health good) would require very high tax rates. Moreover, the tax/subsidy rate would be highly sensitive to the revenue needs.

Equation (18c) is new and arises specifically through the introduction of health. It implies that a necessary condition to replicate the first best optimum is that $s > \tau_w$; that is, the subsidy to health costs must exceed the rate of income tax (on labor income). The reason is as follows. The purpose of the subsidy is to offset the undersupply of health services in the market equilibrium arising from externality in final goods production. However, a tax on labor income by reducing reward to working, and hence reducing overall labor supply and production in both sectors, counteracts the effectiveness of subsidy. s must exceed τ_w to address the equilibrium under provisioning of health services. Finally (18c) intuitively implies that,

the higher the indirect marginal benefit of labor through employment in health sector ($f_h h_e$) in terms of output relative to its direct benefit through employment in goods sector, the higher is the subsidy.

Substituting (18a)–(18c) into the government budget constraint (13j) and using the steady-state conditions (13a)–(13i) and (17a)–(17j), one can show that the following tradeoffs between the optimal tax on labor income $\hat{\tau}_w$, health subsidy \hat{s} , and lump-sum taxes \hat{T} ensure that the steady-state equilibrium in the decentralized economy will replicate the first best optimum. To simplify expressions and facilitate interpretation, we shall assume that production in the final output and health sectors are specified by the following Cobb–Douglas functions that we shall employ in subsequent simulations:

$$y = Ak^\alpha L^{1-\alpha} h^\beta, \tag{19a}$$

$$h = Bm^\varphi e^{1-\varphi}. \tag{19b}$$

Substituting the optimal tax rates (18a)–(18c) into the government budget constraint (8') yields

$$\hat{\tau}_w + \frac{\hat{T}/y}{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}} - \left(\frac{\theta(h)}{\theta(h)+n+\delta_k}\right) \alpha} = \frac{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}}}{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}} - \left(\frac{\theta(h)}{\theta(h)+n+\delta_k}\right) \alpha} \tag{20a}$$

$$\hat{s} = \hat{\tau}_w + (1 - \hat{\tau}_w) \beta \frac{(1 - \varphi) \bar{L}}{(1 - \alpha) \bar{e}}. \tag{20b}$$

Two special cases naturally arise. The first is to set $\hat{\tau}_w = \hat{\tau}_c = 0$, in which case (20a) and (20b) reduce to

$$\hat{T}/y = \hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}}, \tag{21a}$$

$$\hat{s} = \beta \frac{(1 - \varphi) \bar{L}}{(1 - \alpha) \bar{e}}. \tag{21b}$$

Thus, one option for achieving the first best optimum is to subsidize health in accordance with (21b) and to finance all expenditures with lump-sum taxes. The health subsidy increases with the importance of health, β , in the production of final output and labor productivity in the health sector, $(1 - \varphi)/\bar{e}$, but decreases with labor productivity in the final output sector, $(1 - \alpha)/\bar{L}$.

In a more plausible case where lump-sum taxation is unavailable, the optimal policy is

$$-\hat{\tau}_c = \hat{\tau}_w = \frac{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}}}{\hat{g} + \beta - \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{\bar{L}} - \left(\frac{\theta(h)}{\theta(h)+n+\delta_k}\right) \alpha} \equiv \tau^*, \tag{22}$$

with \hat{s} set correspondingly by (20b). In order to be feasible $0 < \hat{\tau}_w < 1$, two cases shall arise:

- (i) $\hat{g} + \beta < \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{L}$, which implies $0 < \hat{\tau}_w = -\hat{\tau}_c < 1$;
- (ii) $\left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{L} < \hat{g} + \beta < \left(\frac{\varphi}{1-\varphi}\right) \frac{\bar{e}(1-\alpha)}{L} + \left(\frac{\theta(h)}{\theta(h)+n+\delta_k}\right) \alpha$ which implies $\hat{\tau}_w = -\hat{\tau}_c < 0$.¹²

Case (i) immediately ensures that $\hat{s} > 0$. Case (ii) implies $\hat{s} > 0$ if and only if $\tau^* > -\beta(1-\varphi)\bar{L}[(1-\alpha)\bar{e} - \beta(1-\varphi)\bar{L}]^{-1}$, a condition that most of our simulations satisfy. Of these two cases, (ii) is more plausible suggesting that, for most parameterizations, replication of the first best optimum may be attained by taxing consumption while subsidizing labor income and health costs. However, as our simulations below illustrate, the required tax and subsidy rates may be unrealistically high, and therefore, attainment of the first optimum may in fact be impractical.

Further intuition underlying the optimal tax policies summarized by (21a)–(21b) and (22) is obtained by realizing that $\varphi(1-\varphi)^{-1}\bar{e}(1-\alpha)(\bar{L})^{-1}$ equals the profit per unit of output earned by the government from providing health, π_h/y .¹³ Thus, (21a) asserts that if the profit earned by the government from running health is more than sufficient to cover its expenditure commitments (investment plus health subsidy), it can finance a general subsidy $T/y < 0$. Alternatively, it becomes feasible to tax labor income and subsidize consumption in accordance with (22). The other extreme case identified in footnote 12, which would require $\hat{\tau}_w > 1$ and is therefore infeasible, corresponds to the situation where the profit from providing health is extremely small.

6. NUMERICAL SIMULATIONS

This section lays the groundwork for a further analysis of the model using numerical methods. To this end, it begins by specifying the functional forms and the values of various parameters of the model to match the salient relevant features of the US economy. This is followed by the computation of the long-run socially optimal allocations, in particular, government expenditure on health and the assessment of feasibility of its decentralization using various taxes and subsidies. As we shall see, this exercise verifies our conjecture from the previous section about the infeasibility of the required optimal taxes and subsidies. It also forms a basis of policy experiments implementing more practical tax policies to get closer to the optimal government spending on health in the next section—with the idea of capturing at least some of the gains associated with increased investment in health to address health externality in the production process.

In addition to the two Cobb–Douglas production functions specified in (19a) and (19b) given above, we specify utility by the conventional constant elasticity function

$$U = \frac{1}{\gamma} (c l^n h^\omega)^\gamma, \tag{19c}$$

TABLE 1. Baseline parameter values

Utility	$\gamma = -1.5$ (i.e., IES 0.4), $\eta = 1.5$, $\omega = 0.15$
Final output	$A = 1$, $\alpha = 0.36$, $\beta = 0.05$ (0.10)
Health production	$B = 0.4$, $\varphi = 0.55$
Rate of time preference	$\theta = 0.045$, $\sigma = 4$ (0, 8)*
Government policy parameters	$g = 0.03$, $\tau_k = 0.276$, $\tau_w = 0.224$, $\tau_c = 0.08$, $s = 0.64$
Population growth rate	$n = 0.01$
Depreciation rate	$\delta_k = 0.08$, $\delta_m = 0.04$

*This yields an equilibrium rate of time preference that varies approximately between 0.045 and 0.072.

and the rate of time preference by the exponential function

$$\theta(h) = \theta e^{-\sigma h}. \quad (19d)$$

The functional form of the rate of time preference, $\theta(h)$, is motivated by the non-linear empirical relationship between life expectancy and health expenditure, as illustrated in Figure 1 (upper panel), together with the fact that increasing life expectancy is reflected as a decreasing rate of time discount; see, for example, Boucekkine et al. (2002). Together these suggest that increasing health expenditures will reduce the rate of time preference, doing so at a decreasing rate.

6.1. The Parameter Values

While many key parameters, particularly those pertaining to aggregate consumption, preferences, and aggregate output, are well documented, direct information on other relevant parameters and measures, particularly related to health, is limited, although other available information provides helpful guidance as to their plausible magnitudes. Table 1 reports the assigned parameter values for what we shall characterize as the baseline scenario. These have been chosen to match the key characteristics of the US economy.

Referring to Table 1, the first row reports the selected utility parameters. Setting $\gamma = -1.5$ implies an intertemporal elasticity of substitution of 0.4, which is well within the reported range documented by Guvenen (2006). The elasticity of leisure $\eta = -1.5$ implies an aggregate Frisch elasticity of around 1.35, well within the range (1–2) adopted in macroeconomic simulations; see Keane and Rogerson (2012).¹⁴ The elasticity of health expenditures in utility is critical in determining the share of consumption expenditure devoted to health. Setting $\omega = 0.15$ (in conjunction with other parameters, most notably the subsidy, s) implies that around 6.5% of consumption is spent on health, which is close to the average in recent years.¹⁵

Setting the productive elasticity of private capital in the final output $\alpha = 0.36$ is standard. However, information on the productive elasticity of health is sparse, and our choice of benchmark $\beta = 0.05$ is based on the following considerations. Most directly, using life expectancy as a proxy for health, Bloom et al. (2004)

estimated the productive elasticity of health as 0.04. In addition they cited several other studies yielding slightly higher estimates of elasticity. Finally, to the extent that one might view health as a component of public infrastructure, we may draw on the comprehensive evidence provided by Bom and Ligthart (2014) whose meta-regression analysis yielded an estimate of elasticity of public capital in aggregate output to be of the order of 0.10.¹⁶ In the extreme case that health capital is a constant fraction of the overall public infrastructure, this would imply $\beta = 0.10$, which we also consider as part of the sensitivity analysis.

Setting $B = 0.4$, $\varphi = 0.55$ in the production function for the health sector, in conjunction with specifying the final output sector, yields a macroeconomic equilibrium in which the health sector is almost 16% of GDP, close to the recent US experience. In addition, it implies that around 70% of the total available time is devoted to leisure, very much a consensus estimate. In addition, approximately 10.5% of the time devoted to labor is allocated to the health sector. This is generally consistent with the USA where 9% of total employment is in the health care sector, especially taking account of the fact that health workers typically work somewhat longer hours.¹⁷

To parameterize σ , the impact of health on longevity and therefore on the rate of time discount, we use the quality-adjusted life year (QALY) weights estimated by Cutler and Richardson (1997). Specifically, they showed that due to declining health the QALY weight of a 65-year-old relative to that of a 20-year-old is approximately 0.776, meaning that the quality of a 65-year-old's life declines by about 22.6% relative to a 20-year-old, who we assume may be in perfect health. Setting $\sigma = 4$, and with the equilibrium $h = 0.0321$, implies that in equilibrium the raw discount rate $\theta = 0.045$, associated with an economy that does not invest in health, is reduced to 0.0396. This means that in the 45 years, after reaching the age of 20, the discount factor applied to utility will be 0.132 in the absence of health, as compared to 0.168 with health, the ratio of which is 0.786, approximately that of the QALY weights. This suggests that with $\sigma = 4$ the equilibrium investment in health enables a 65-year-old to enjoy a quality of life comparable to that of a healthy 20-year-old.

The choice of tax rates is less straightforward and has generated debate, due to the difficulty of mapping the complexities of the real-world tax structure into a simple one-sector growth model. We use the effective tax rates on consumption, labor, and capital constructed by McDaniel (2007), following the methodology proposed by Mendoza et al. (1994). The tax rates listed in Table 1 are the US averages for the decade 1991–2000. A key feature is that for the US economy $\tau_k > \tau_w$, a characteristic that holds uniformly since 1953. The subsidy rate $s = 0.64$ is taken from a recent study by Himmelstein and Woolhandler (2016).¹⁸

The rate of population growth, $n = 0.01$, and the rate of depreciation of private capital, $\delta_k = 0.08$, are non-controversial. Information on the depreciation of public health capital is sparse. Several studies estimated the rate of depreciation of the health stock to be of the order of 3%–4%; see Kelly (2017), while IMF studies on the rate of public investment yielded a similar value for the depreciation of public

TABLE 2. Equilibrium ratios

Consumption–GDP ratio	0.79
Capital–output ratio	2.01
Allocation of time to leisure	0.699
Allocation of labor to final output production	0.272
Allocation of labor to health production	0.0291
Equilibrium rate of time discount	0.0396
Percentage of consumption devoted to health	6.5%
Public health as a percentage of total health	62.4%
Total health as a percentage of GDP	15.8%

capital for advanced economies, leading us to set $\delta_m = 0.04$ as a plausible benchmark.¹⁹ Finally, our choice $g = 0.03$, while arbitrary, is consistent with the gross rate of investment in infrastructure of 5%, typical for most advanced economies, including the USA.²⁰

Table 2 reports the corresponding benchmark steady-state equilibrium for our chosen parameters. In addition to the health ratios, we note that all of the implied ratios appear to be plausible and to reasonably reflect closely those of the US economy.

6.2. The Long-Run Social Optimum

Table 3 characterizes the tax structures and expenditures that enable the decentralized economy to achieve the first best allocation as set out in Section 5.1. We present results for two values of $\beta = 0.05, 0.10$. For each value of β , we consider three values of $\sigma = 0, 4, 8$, which parameterizes the effect of health on longevity and, thus, on the discount rate. With respect to the configuration of government policy we consider the two alternatives summarized in (21) and (22). In the first, where the government has only distortionary taxes available, $T/f = 0, \hat{\tau}_k = 0, \hat{\tau}_w = -\hat{\tau}_c$; the second where all distortionary taxes are set to zero and only lump-sum taxation is employed. All other parameters are set at their benchmark levels identified in Table 1.

Table 3 highlights several elements pertaining to the optimal policy. First, in all cases the economy would experience dramatic improvements in long-run consumption, capital stock, and output, if the economy were in steady state corresponding to the first best optimum. We have also reported changes in welfare, measured in terms of equivalent variations in per-capita consumption. Thus, W_{SS} suggests a 24% increase in welfare across steady states for $\beta = 0.05$ (and $\sigma = 4$), which increases to over 40% if the productivity of health in producing the final output is increased to $\beta = 0.10$. The corresponding measure, W_{INT} , approximates the present value of gains, considering the entire transitional path. These too are significant.

TABLE 3. Policies to attain a long-run social optimum

	Gains from attaining long-run optimum					\bar{g}	$\frac{T}{f} = 0, \tau_k = 0$		$\tau_k = \tau_w = \tau_c = 0$	
	$\frac{\Delta y}{y}$ (%)	$\frac{\Delta k}{k}$ (%)	$\frac{\Delta c}{c}$ (%)	ΔW_{SS} (%)	ΔW_{INT} (%)		$\hat{\tau}_w = -\hat{\tau}_c$	\hat{s}	$\frac{T}{f}$	\hat{s}
$\beta = 0.05; \sigma = 0$	69.6	134	52.2	33.8	28.5	0.0455	-0.0816	0.2625	0.0091	0.3181
$\beta = \mathbf{0.05}; \sigma = \mathbf{4}$	62.1	125	48.2	24.0	14.3	0.0276	-0.35503	0.4045	0.0285	0.5605
$\beta = 0.05; \sigma = 8$	59.7	122	47.3	21.9	7.19	0.0196	-0.5157	0.7092	0.0356	0.8081
$\beta = 0.10; \sigma = 0$	81.9	151	60.1	42.9	39.2	0.0594	-0.6342	0.1626	0.0466	0.4876
$\beta = 0.10; \sigma = 4$	74.4	145	56.3	34.5	27.0	0.0385	-2.0079	0.4461	0.0710	0.8144
$\beta = 0.10; \sigma = 8$	71.6	143	55.4	32.5	19.7	0.0283	-3.8972	1.7053	0.0803	1.1440

Boldface refers to baseline case.

However, the first best optimum is really not practical to accomplish. For example, to achieve the first best optimum in the benchmark case ($\beta = 0.05$, $\sigma = 4$) would require taxing consumption and subsidizing labor income at 35.5%, while subsidizing health expenditure at 40.5%. Alternatively, it could be attained by imposing a lump-sum tax of 2.9% while subsidizing health at 57.1%. While these are, in principle, feasible policies, they are dramatically different from any currently existing tax structure and would involve a revolution in tax policy to implement. Other optimal policies are equally, or even more, extreme, and one overriding characteristic is the sensitivity of the optimal tax structure to variations in σ , which impacts the rate of time preference. We find this to be somewhat surprising since the implied impact on $\theta(h)$ is relatively small. For example, increasing σ from 0 to 4 reduces $\theta(h)$ from 0.045 to 0.0396.

Finally, we see that for baseline parameterization in Table 3 the government never makes sufficient profit to finance a general subsidy T/y in accordance with (21a). However, we may note that this does become possible when $\beta = 0.05$, $\sigma = 0$ and the depreciation rate of health capital is reduced to $\delta_m = 0.02$. In that case it can achieve the first best optimum by taxing labor at 1.95%, subsidizing general consumption at that same rate, and healthcare at 32.8%.

7. POLICY EXPERIMENTS

The above discussion of the potential to attain the first best optimum is useful in terms of providing a benchmark but—for reasons just alluded to regarding the necessarily massive adjustments of tax policy to implement—is of little more than academic interest. In this section, we, therefore, focus on more pragmatic policy issues.

7.1. Tax Policy and Welfare Benefits of Increased Health Investment

It is clear that the tax rates can generate sufficient revenues for the governments to increase the rate of investment in health if they so choose. Thus, we consider the impact of increasing the rate of investment, g , by 1 percentage point from its base level of 0.03 to 0.04. We compare the financing of this increase in health investment in five different ways: (i) lump-sum taxes (bond financing), (ii) increase in τ_w , (iii) increase in τ_k , (iv) increase in τ_c , and (v) reduction in subsidy to health expenditure, s . In cases (ii)–(v) the taxes are adjusted so as to maintain the government deficit at its initial level. Table 4 reports the long-run responses for baseline parameterization ($\sigma = 4$, $\beta = 0.05$), as well as for the case where the productive elasticity of health is increased to 0.10. All changes are across steady states. In particular, the change in welfare reported in the final column is measured in terms of equivalent variations in steady-state per-capita consumption, but does not reflect welfare changes incurred during the transition.

The following features in Table 4 can be identified. Unsurprisingly, the overall ranking places lump-sum tax financing best from a welfare standpoint. This

TABLE 4. Macroeconomic and welfare effects of increase in government investment

A. Baseline ($\sigma = 4, \beta = 0.05$)										
	l	L	e	y	k	c	h	$\frac{p(1-s)h}{p(1-s)h+c}$ (%)	$\frac{ph+mn}{y+ph}$ (%)	ΔW_{SS} (%)
Baseline	0.6992	0.2717	0.0291	0.3077	0.6191	0.2428	0.0321	6.49	15.81	–
$\Delta(T/y) = 0.0111$	0.6995	0.2756	0.0249	0.3154	0.6371	0.2454	0.0356	5.64	14.93	
	0.03%pts	0.39%pts	–0.42%pts	2.48%	2.91%	1.01%	10.76%	–0.85%pts	–0.88%pts	2.71
$\Delta\tau_c = 0.0074$	0.7008	0.2742	0.0250	0.3138	0.6338	0.2442	0.0356	5.66	15.02	
	0.16%pts	0.25%pts	–0.41%pts	1.95%	2.38%	0.56%	10.64%	–0.85%	–0.79%pts	2.45
$\Delta\tau_w = 0.0102$	0.7021	0.2729	0.0249	0.3122	0.6305	0.2430	0.0354	5.66	15.03	
	0.29%pts	0.12%pts	–0.42%pts	1.44%	1.85%	0.07%	10.14%	–0.83%pts	–0.78%pts	2.16
$\Delta\tau_k = 0.0573$	0.7025	0.2717	0.0258	0.2963	0.5507	0.2349	0.0350	5.78	15.42	
	0.33%pts	–0.00%pts	–0.33%pts	–3.72%	–11.0%	–3.27%	8.73%	–0.71%pts	–0.39%pts	–1.35
$\Delta s = -0.0403$	0.7006	0.2759	0.0235	0.3149	0.6354	0.2451	0.0346	5.86	14.36	
	0.14%pts	0.42%pts	–0.56%pts	2.32%	2.63%	0.95%	7.76%	–0.63%pts	–1.45%pts	2.39
B. Alternative ($\beta = 0.10$)										
	l	L	e	y	k	c	h	$\frac{p(1-s)h}{p(1-s)h+c}$ (%)	$\frac{ph+mn}{y+ph}$ (%)	$\frac{\Delta W}{W}$
Baseline	0.6967	0.2705	0.0328	0.2299	0.4605	0.1815	0.0289	7.30	17.28	–
$\Delta(T/y) = 0.0111$	0.6970	0.2744	0.0286	0.2380	0.4789	0.1854	0.0325	6.46	16.40	
	0.03%pts	0.39%pts	–0.42%pts	3.53%	3.97%	2.11%	12.21%	–0.84%pts	–0.88%pts	3.98%
$\Delta\tau_c = 0.0028$	0.6975	0.2738	0.0286	0.2375	0.4779	0.1846	0.0324	6.43	16.43	
	0.08%pts	0.33%pts	–0.42%pts	3.33%	3.77%	1.71%	12.16%	–0.87%pts	–0.85%pts	3.87%
$\Delta\tau_w = 0.0039$	0.6980	0.2734	0.0286	0.2370	0.4769	0.1846	0.0324	6.43	16.43	
	0.13%pts	0.29%pts	–0.42%pts	3.12%	3.56%	1.71%	11.95%	–0.87%pts	–0.85%pts	3.75%
$\Delta\tau_k = 0.0199$	0.6981	0.2730	0.0289	0.2328	0.4555	0.1825	0.0322	6.47	16.57	
	0.14%pts	0.25%pts	–0.39%pts	1.29%	–1.09%	0.55%	11.43%	–0.83%pts	–0.71%pts	2.50%
$\Delta s = -0.0142$	0.6975	0.2745	0.0280	0.2376	0.4780	0.1851	0.0321	6.50	16.14	
	0.08%pts	0.40%pts	–0.48%pts	3.39%	3.79%	1.99%	10.97%	–0.80%pts	–1.14%pts	3.78%

is followed, in turn, by consumption tax financing, reduction of health subsidy, labor income tax, with the worst being financing by a tax on capital income. In the case of $\beta = 0.05$, financing the investment in health by taxing capital leads to a contraction in activity and reduces welfare by 1.35%. Financing by a tax on labor income reduces per-capita consumption, but the offsetting increase in health ensures an overall increase in welfare. The same ranking results if the productive elasticity of health is increased to $\beta = 0.10$. In all cases the welfare gains are enhanced, and even taxing capital, although still associated with an albeit minor decline in activity, nevertheless yields an overall modest welfare improvement. In terms of quantities, the rankings are generally similar, with the exception that, in most cases, financing by reducing the subsidy has a greater effect than does financing by a consumption tax. Finally, in all cases we see that increasing government investment in health leads to a significant reduction in the share of total private consumption devoted to health services.

7.2. Transitional Dynamics

Figure 2 illustrates the transitional dynamics followed by key variables in response to an increase in government investment in health under the five alternative modes of finance. Because the adjustments under four of the five modes of financing track each other closely, the figures are actually drawn for $\beta = 0.075$, which is a little more discriminatory.²¹ With the exception of health capital, all quantities converge rather rapidly to their respective new steady-state equilibria. Also, the magnitudes of the relative responses are remarkably uniform along the transition.

To indicate how the transition proceeds we suppose that the increase in health investment, g , is financed by a lump-sum tax. With forward-looking agents, the knowledge that this will increase longevity immediately reduces μ , the losses in utility from a higher discount rate. The increase in g also reduces λ , the shadow value of wealth. The net effect of these initial responses is to cause labor to move to the health sector, primarily by reducing leisure, but also from the final output sector. This leads to an immediate increase in health services [Figure 2(d)], which, with capital fixed instantaneously, will cause output to immediately rise [Figure 2(b)]. The net effect of the decline in λ is for consumption to immediately increase slightly [Figure 2(e)], but with increase in health causing an expansion in output to dominate, capital starts to accumulate [Figure 2(a)]. In addition, the increase in health tied to the increase in output will generate an increase in health capital [Figure 2(c)]. Over time, as capital is accumulated, the marginal utility of wealth continues to decline and the economy converges monotonically to its new steady state. In this process, labor migrates back to the final output sector, so that during the transition health overshoots its long-run increase.

These expansionary responses are dampened when the health investment is financed by higher taxation. In the case of τ_w , τ_c , or the subsidy, s , these offsetting effects are minor and the expansionary effect dominates. However, a sharp

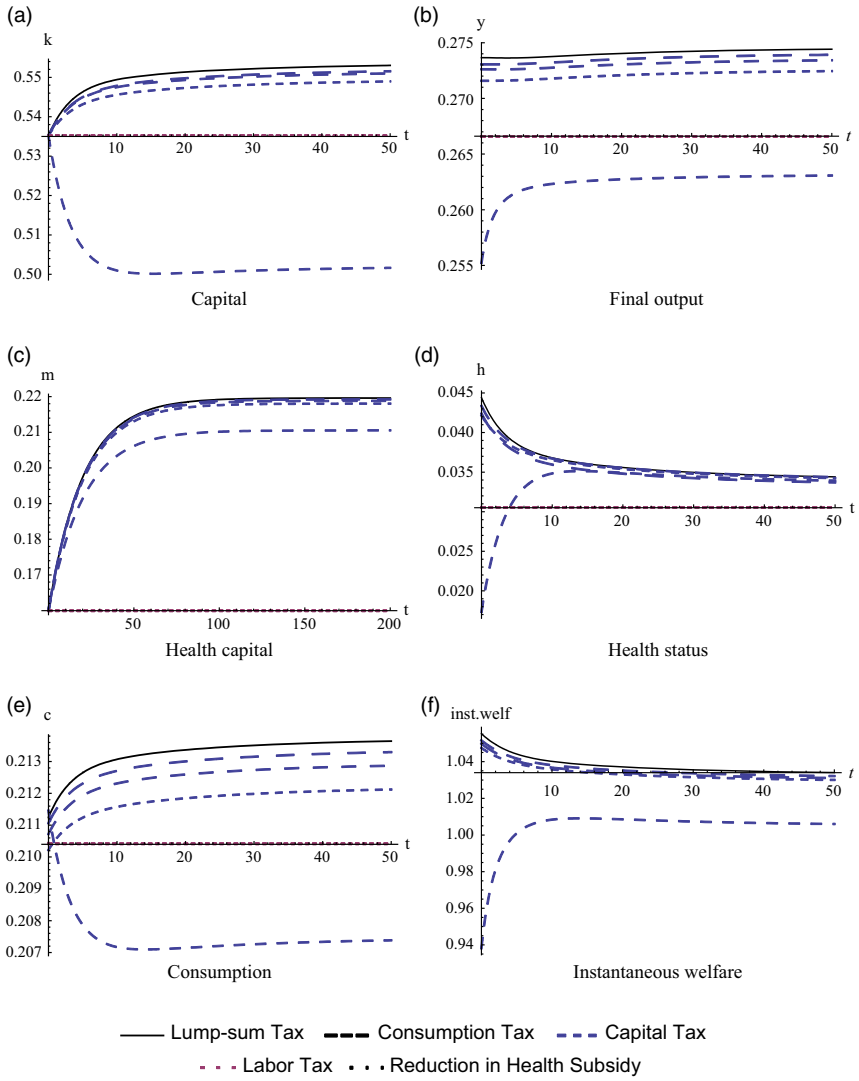


FIGURE 2. Dynamic response to increased public investment in health.

contrast is obtained in the case of financing by taxing capital, which is associated with a long-run decline in economic activity and a loss in welfare. On being impacted, it is actually associated with an increase in consumption, in excess of that obtained with lump-sum tax financing. This is because financing public investment with a tax on private capital requires a large increase in tax rate, τ_k , leading to an immediate substantial reduction in the rate of private investment. This in turn, reduces the productivity of labor and the wage rate, inducing workers to supply less labor in both sectors instead of allocating more time to leisure

TABLE 5. Welfare gains of various policies in the baseline scenario

		$\frac{\Delta y}{y}$ (%)	$\frac{\Delta k}{k}$ (%)	$\frac{\Delta c}{c}$ (%)	ΔW_{SS} (%)
<i>g</i>	First best optimum	62.1	125	48.2	24.0
	(Baseline)				
0.03	$s = 0.64, \tau_w = 0.224, \tau_k = 0.276, \tau_c = 0.08$	–	–	–	–
0.04	$s = 0.64, \tau_w = 0.224, \tau_k = 0.276, \tau_c = 0.0874$	1.95	2.38	0.56	2.45
0.06	$s = 0.64, \tau_w = 0.224, \tau_k = 0.276, \tau_c = 0.1079$	4.51	5.53	0.30	4.59
0.03	$s = 0.30, \tau_w = 0.15, \tau_k = 0.20, \tau_c = 0.1028$	12.2	24.0	10.2	5.40
0.04	$s = 0.30, \tau_w = 0.15, \tau_k = 0.20, \tau_c = 0.1175$	14.3	26.2	10.1	7.43
0.06	$s = 0.30, \tau_w = 0.15, \tau_k = 0.20, \tau_c = 0.1487$	16.4	29.2	8.99	9.04
0.03	$s = 0.20, \tau_w = 0.10, \tau_k = 0.15, \tau_c = 0.1473$	19.6	39.7	15.0	8.48
0.04	$s = 0.20, \tau_w = 0.10, \tau_k = 0.15, \tau_c = 0.1639$	21.1	42.1	14.8	10.5
0.06	$s = 0.20, \tau_w = 0.10, \tau_k = 0.15, \tau_c = 0.1984$	23.2	45.3	13.4	12.0
0.03	$s = 0.20, \tau_w = 0.10, \tau_k = 0, \tau_c = 0.2015$	32.0	81.7	20.6	14.3
0.04	$s = 0.20, \tau_w = 0.10, \tau_k = 0, \tau_c = 0.2203$	33.7	84.7	20.3	16.2
0.06	$s = 0.20, \tau_w = 0.10, \tau_k = 0, \tau_c = 0.2591$	35.9	88.8	18.6	17.4
0.03	$s = \tau_w = \tau_k = 0, \tau_c = 0.2718$	36.3	87.4	24.6	15.2
0.04	$s = \tau_w = \tau_k = 0, \tau_c = 0.2929$	37.9	90.4	24.2	17.1
0.06	$s = \tau_w = \tau_k = 0, \tau_c = 0.3357$	40.1	94.4	22.3	18.3
0.04	$s = 0.600, \tau_w = 0.224, \tau_k = 0.276, \tau_c = 0.08$	2.32	2.63	0.96	2.39
0.06	$s = 0.382, \tau_w = 0.224, \tau_k = 0.276, \tau_c = 0.08$	5.49	5.93	1.32	3.46

and, with it, consumption. With health capital being augmented only slowly, health production declines in the short run. Over time, as the private capital stock continues to decline, consumption declines. In contrast, the accumulation of health capital brought about by government investment brings about a reversal in the decline of health, which eventually increases to above its initial level.

7.3. Tax Policy and Welfare Benefits of Health Investment: Further Analysis

As Table 3 highlights, the increase in economic activity and the resulting welfare improvement associated with the long-run social optimum are substantial. By comparing our preferred parameterization ($\beta = 0.05, \sigma = 4$) we see that the welfare gains (across steady state) are 24% as measured by equivalent variation in consumption units. But as the table also underscores and as already noted, the corresponding tax rates required to achieve these gains are simply unrealistic from a practical point of view. In contrast, the policy experiments summarized in Table 4, starting from the existing tax structure and raising g from 3% to 4%, generate decidedly modest welfare gains. This raises the question of whether it is possible to realize a substantial portion of the benefits associated with the social optimum by combining government investment in health with more moderate and feasible tax changes that move at least partially in the direction of the first best policy. This issue is addressed in Table 5, which focuses only on comparisons across steady states.

Row 1 (of panel 1) of Table 5 summarizes the percentage gains in output, capital, consumption, and welfare associated with the first best optimum. The remaining panels in Table 5 consider realistic tax changes that move successively closer to the optimal tax structure. In particular, in each panel the first row with $g = 0.03$ shows a pure change in tax policy without changing the rate of health investment. Successive reductions in income tax, compensated for by appropriate increases in consumption tax, lead to successively larger welfare gains. Thus, for example, the fourth panel shows that the elimination of capital income tax, as advocated by Chamley (1986) and Judd (1985), coupled with a reduction in subsidy to 20% and labor tax rate to 10%, compensated for by an increase in consumption tax to 20.2%, can capture almost 60% of the welfare gain relative to the optimum (14.3% vs. 24.0%). Eliminating the tax on labor income and subsidy entirely and increasing the consumption tax further to 27.2% increases the welfare gain to 15.2%. And while these represent dramatic changes in the US tax structure, they are not implausible and, indeed, reflect changes in the tax structure that have been advocated by conservative policy-makers over the years.

Subsequent rows introduce increases in health investment in conjunction with the corresponding tax policy. Combining tax and health investment policies can further enhance welfare gains. Thus, for example, reducing the income tax rates τ_w, τ_k by about 12.5 percentage points to 10% and 15%, respectively; reducing the subsidy to 20%, while doubling the consumption tax τ_c from 8% to 16.4%; and simultaneously increasing health investment from 3% to 4% capture almost 44% of welfare benefits associated with the social optimum (10.5% vs. 24.0%). Moreover, as health investment increases further from 3% to 6% and the income tax is increasingly reduced in favor of a consumption tax, it is possible to take the welfare gains to above 65% of those associated with the social optimum, as the penultimate panel indicates.

From the perspective of health investment policy, which is the focus of this paper, a very interesting pattern emerges from Table 5, namely the robustness of welfare gains from increases in government investment in health with respect to the initial tax structure. Specifically, we see that in all panels, which represent very different tax structures, an increase in g from 3% to 4% results in an additional welfare gain of around 2%. A further increase from 4% to 6% yields a further welfare gain of something over 1%, suggesting sharply diminishing returns to health investment, for a given tax structure. In this regard, the final panel illustrates a sharp tradeoff between subsidizing health care and investment in health infrastructure. Specifically a modest reduction in subsidy from 0.64 to 0.60 could finance an increase in g from 3% to 4%, yielding a 2.4% increase in welfare, but with further investment requiring more drastic reductions in subsidy.

7.4. Tax–Health Tradeoffs

Our numerical analysis has so far been restricted primarily to baseline calibration, $\beta = 0.10, \sigma = 4$. However, there is, in fact, a tradeoff between the level of

TABLE 6. Tax–health tradeoff in financing increase in government investment from 0.03 to 0.04

A. Labor income tax								
	$\beta = 0.025$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$	
	τ_w	$\Delta W_{SS} (\%)$	τ_w	$\Delta W_{SS} (\%)$	τ_w	$\Delta W_{SS} (\%)$	τ_w	$\Delta W_{SS} (\%)$
$\sigma = 0$	0.2358	1.84	0.2325	2.69	0.2253	4.64	0.2170	6.98
$\sigma = 4$	0.2370	1.50	0.2342	2.16	0.2279	3.75	0.2202	5.74
$\sigma = 8$	0.2408	0.03	0.2389	0.49	0.2341	1.66	0.2277	3.26

B. Capital income tax								
	$\beta = 0.025$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$	
	τ_k	$\Delta W_{SS} (\%)$	τ_k	$\Delta W_{SS} (\%)$	τ_k	$\Delta W_{SS} (\%)$	τ_k	$\Delta W_{SS} (\%)$
$\sigma = 0$	0.3727	-4.78	0.3377	-1.53	0.2840	4.08	0.2360	9.89
$\sigma = 4$	0.4550	-11.3	0.3333	-1.35	0.2959	2.50	0.2576	6.93
$\sigma = 8$	0.4460	-10.9	0.3578	-4.16	0.3273	-1.28	0.2936	2.23

C. Consumption tax								
	$\beta = 0.025$		$\beta = 0.05$		$\beta = 0.10$		$\beta = 0.15$	
	τ_c	$\Delta W_{SS} (\%)$	τ_c	$\Delta W_{SS} (\%)$	τ_c	$\Delta W_{SS} (\%)$	τ_c	$\Delta W_{SS} (\%)$
$\sigma = 0$	0.0908	1.92	0.0877	2.78	0.0811	4.66	0.0722	6.74
$\sigma = 4$	0.0894	1.83	0.0874	2.45	0.0828	3.87	0.0758	5.52
$\sigma = 8$	0.0913	0.45	0.0900	0.88	0.0869	1.96	0.0812	3.32

economy’s health and the tax rates necessary to support its level of expenditures. This is illustrated in Table 6 where, for each of the three taxes, we present the rates required to finance an increase in government investment in health from 3% of GDP to 4% of GDP. The table presents results for a grid over which the productivity of health varies from $\beta = 0.025$ (low productivity) to $\beta = 0.10$ (high productivity) and its impact on longevity varies from $\sigma = 0$ (no impact) to $\sigma = 8$ (strong impact). The combination corresponding to the baseline summarized in Table 4 ($\beta = 0.05, \sigma = 4$) is indicated in bold face.

Focusing initially on panel A indicates that, in the benchmark situation, financing an increase in the share of government investment in GDP by 1 percentage point using a tax on labor income would require τ_w to increase from 0.224 to 0.234, resulting in an increase in welfare of 2.16%. If the productive elasticity of health were increased from $\beta = 0.05$ to $\beta = 0.10$, this could be financed by a lower tax rate, τ_w , namely 0.228, yielding a larger welfare gain of 3.75%. And if health productivity were higher still, the required tax rate could be reduced further. There is a clear tradeoff between (i) the productive elasticity of health in producing final output and (ii) the tax rate necessary to finance any increase in government investment in health. This tradeoff applies to all three tax rates.

The final column presents the case $\beta = 0.15$. While this might seem extreme, it is well within the range of plausible estimates of productive elasticity of public

capital proposed by Bom and Ligthart (2014).²² What is interesting about this case is that if $\sigma = 0, 4$, so that health has only a limited impact on the rate of time preference, it is possible to finance the increase in government investment out of higher productivity and thereby actually reduce the corresponding tax rate. This is true for each of the tax rates.

But there is also a tradeoff that applies in the opposite direction between the tax rate and the impact of health on longevity and on the rate of time preference. Focusing again on τ_w and benchmark parameterization, we see that if σ were increased to 8, the 1 percentage point investment in health would require a slightly higher tax rate, 0.239, leading to a substantially reduced increase in welfare of only 0.49% per capita. Intuitively, the reduced rate of time preference leads to a slight increase in leisure met by a reduction in employment in both sectors, necessitating an increase in tax rate to finance an increase in government investment.

Looking across the three panels we observe the following. First, the required capital income tax financing is extremely sensitive to the productive elasticity of health. In all cases, it ranges from a high tax rate with welfare losses if β is small to a much reduced rate and the largest welfare gain if health is extremely productive. Second, variations in either labor income tax or consumption tax across the grid are modest and, in all cases, they yield welfare gains. Third, financing with a consumption tax generally yields the greatest welfare gains if β is small, but its superiority over labor income tax financing declines with health productivity, and in the extreme case $\beta = 0.15$, it is the least desirable from the welfare standpoint.

8. CONCLUSION

This paper incorporated individual agents' health decisions into the neoclassical growth model to study the effects of health investment subsidies on productivity, growth, and welfare. The paper first examined the steady state and transitional dynamic properties of the model focusing on the effects of health decisions on aggregate outcomes. Next, it considered how public health policies may alter private economic decisions.

The analysis showed that there is a significant welfare upside from government interventions to subsidize the consumption of medical goods and services and also subsidize physical activities and recreation. The policy experiments we performed showed that there are significant welfare gains associated with moving from the current tax structure and public health investment to the optimal levels identified by the model. We also showed that this result is robust to other initial tax structures that are closer to the first best equilibrium generated by the model. The tradeoff we identified between health status and tax rate leads to an important, but little discussed, policy implication. It suggests that one of the benefits of a healthier, more productive, labor force is that any specified level of government services can be supported by a lower tax rate, enabling consumers to enjoy a higher level of welfare.

Finally, while the model we presented is highly stylized, if one accepts its implications it does point to a certain irony in the policy debate currently taking

place in Washington, DC. While much of the political discussion is framed in terms of *reduced* health expenditures being necessary to finance proposed tax cuts, our analysis suggests precisely the opposite, namely *increased* health investment by leading to more productive labor may permit tax rates to be reduced.

We conclude by suggesting some directions in which this analysis could be usefully extended. While we focused on the nexus between health spending and economic performance in a developed country, it would be of interest to examine analogous links in case of developing economies, modifying the framework to take account of the constraints and impediments they face. Another important extension would be to introduce heterogeneous agents having both differential financial resources, as well as levels of health among its population. To design an appropriate public health system and to analyze its financing in the face of such diversity would be very challenging. But it would increase the insight that a canonical model such as this could play in studying the real-world health issues facing disparate economies.

NOTES

1. Our focus on developed countries stems from the fact that the impact of health spending on various health-related outcomes differs substantially between developed and developing countries, as we detail in Section 2. Some possible avenues of extension of the framework to make it relevant to developing countries are suggested in Section 8.

2. The increase in longevity associated with improvements in health tend to have a negative impact on per-capita welfare, reflecting the fact that with people living longer, the output of the economy must be shared among a larger population.

3. The Cato Institute in the USA is well known for advocating the elimination of income tax in favor of sales tax.

4. In fact Agénor motivates his study in terms of examining the link between infrastructure and health. In contrast, Hugonnier et al. (2013) analyzed investment in health as part of the joint determination by individual agents of their optimal consumption and portfolio holdings.

5. While the current model does not incorporate health insurance, it could be modified to address issues such as whether health costs ought to be shared between individuals and firms. Recent studies emphasizing the role of health insurance include Jung and Tran (2016) and Pelgrin and St-Amour (2016).

6. We should note that while we focused on the rate of time discount decreasing with health, our approach is essentially equivalent to the demography literature that postulates that it increases with mortality. To see this, suppose $\theta(h) = \theta - h$. In this case the rate of time discount is in agreement with Blanchard (1985) where $-h$ plays the role of mortality rate, assumed to be constant and independent of age.

7. Several modifications to the formulation and interpretation of health services are possible. First, one could specify health as a more general positive concave function of resources devoted to health. Second, we could introduce health services as a stock rather than as a flow. In this respect, by relating h to the stock of public health capital (see 7a), it, in fact, retains much of the characteristics of a stock. Also, health costs are likely to be age-dependent. Since our objective was to develop a simple canonical model, we refrained from introducing these modifications.

8. We recognize that this is a very stylized specification of the production function for health, but one that suffices for our purposes and is consistent with the canonical nature of the model. Basically, production functions for health begin by disaggregating labor (e.g., to physicians and paramedics)

and some measure of medical capital but augment to include other factors such as education, drug expenditures, etc.; see, for example, Auster et al. (1969) for an early seminal formulation.

9. This contrasts with the growth model of Agénor (2008, 2013), for example, in which health is fully provided by the government.

10. This is evident from the aggregate resource constraint: $f(k, L, h) - c - (n + \delta_k)k - (n + \delta_m)m = \dot{k} + \dot{m}$.

11. Conditions similar to (18a) and (18b) are obtained by Turnovsky (2000) in a macro growth model that abstracts from the health sector. As noted there, (18b) is an intertemporal application of the Ramsey principle of optimal taxation; see Deaton (1981). If the utility function is multiplicatively separable in consumption and leisure, as we shall assume, then uniform taxation of leisure and consumption is optimal.

12. The third condition, $\varphi(1 - \varphi)^{-1}\bar{e}(1 - \alpha)\bar{L}^{-1} + \theta(h)[\theta(h) + n + \delta_k]^{-1}\alpha < \hat{g} + \beta$, implies $\hat{\tau}_w > 1$ in which case the first best optimum cannot be replicated by distortionary taxes alone.

13. Recall that profit earned from providing health equals $p(h - h_e e) \equiv \pi_h$. Substituting for p , $p(h - h_e e)/y = (f_i/(yh_e))(h - h_e e)$. For the Cobb–Douglas functions (19a) and (19b), this simplifies further to $\varphi(1 - \varphi)^{-1}\bar{e}(1 - \alpha)(\bar{L})^{-1}$.

14. The inconsistency between these aggregate values and the smaller estimates obtained from micro data is an issue currently occupying the attention of labor economists. Keane and Rogerson (2012) offer a reconciliation that credibly supports the range typically adopted in macroeconomic simulations.

15. The BLS reports that the percentage of consumption expenditure devoted to health in the USA increased from 5% to 8% over the 30-year-period, 1984–2014.

16. As Bom and Ligthart (2014) document, empirical estimates on the productive elasticity of public capital, e , are far ranging. In their comprehensive study, they summarized 578 estimates and found the average productive elasticity of public capital to be around 0.19, while their meta-regression analysis yielded an estimate of around 0.10.

17. The estimate of 9% of labor employed in health care is provided by the Kaiser Family Foundation based on the data obtained from BLS. Since our notion of labor allocated to health also includes time spent by individuals taking care of their individual health needs (such as going to the gym), this provides further justification for our time allocation of 10.5%.

18. The procedure used by Himmelstein and Woolhandler to calculate the subsidy rate can be summarized as follows. First they tabulated Centers for Medicare and Medicaid Services' official figures on direct government spending on health programs and public employees' health benefits for 2013, and projected figures through 2024. They calculated the value of tax subsidies for private spending from official federal budget documents and figures for state and local tax collections. They found that tax-funded health expenditures totaled \$1.877 trillion in 2013, which is projected to increase to \$3.642 trillion in 2024. Government's spending on health was 64.3% of national health expenditures in 2013, which is expected to rise to 67.1% in 2024. They concluded that, contrary to public perceptions and Centers for Medicare and Medicaid Services estimates, government funds most health care in the USA.

19. See, for example, IMF (2015).

20. While we chose parameters to approximate the US economy, we also conducted simulations to reflect a "European" health system. The main differences between "Europe" and the USA are (i) in the relative tax rates, and (ii) in the relative size of the health sector; around 10% of GDP, rather than 17%, with a larger share being provided by the government. Most of the results are similar to those presented here. One difference worth noting is that with τ_w being substantially higher than τ_k in Europe; financing a 1 percentage point increase in health investment using τ_w leads to a reduction in welfare analogous to that obtained using τ_k for US parameterization (see Table 4).

21. The required distortionary tax rates in this case are $\tau_w = 0.2312$, $\tau_k = 0.3146$, $\tau_c = 0.0852$, and subsidy $s = 0.6126$.

22. This still lies within a plausible range for the productive elasticity of public capital estimated by Bom and Ligthart (2014) as noted in footnote 16.

REFERENCES

- Agénor, P.-R. (2008) Health and infrastructure in a model of endogenous growth. *Journal of Macroeconomics* 30, 1407–1422.
- Agénor, P.-R. (2010) A theory of infrastructure-led development. *Journal of Economic Dynamics and Control* 34, 932–950.
- Agénor, P.-R. (2013) *Public Capital, Growth and Welfare: Analytical Foundations for Public Policy*. Princeton, NJ: Princeton University Press.
- Aísa, R. and F. Pueyo (2004a) Endogenous longevity, health and economic growth: A slow growth for a longer life? *Economics Bulletin* 9, 1–10.
- Aísa, R. and F. Pueyo (2004b) Government Health Spending and Growth in a Model of Endogenous longevity. Economic Analysis Department, University of Zaragoza.
- Allen, J. and S. Chakraborty (2018) Aspirations, health and the cost of inequality. *Journal of Economic Dynamics and Control* 86, 144–164.
- Auster, R. D., I. Leveson and D. Sarachek (1969) The production of health: An exploratory study. *Journal of Human Resources* 4, 411–436.
- Bhattacharya, J. and S. Chakraborty (2016) Contraception and the demographic transition. *Economic Journal* 127, 2263–2301.
- Birchenall, J. and R. R. Soares (2009) Altruism, fertility, and the value of children: Health policy evaluation and intergenerational welfare. *Journal of Public Economics* 93, 280–295.
- Blanchard, O. J. (1985) Models with endogenous rate of time preference debt, deficits and finite horizon. *Journal of Political Economy* 93, 223–247.
- Bloom, D. and D. Canning (2005) Population Health and Economic Growth. Commission on Growth and Development, Washington, DC: World Bank, Working Paper No. 24.
- Bloom, D., D. Canning and J. Sevilla (2004) The effect of health on economic growth: A production function approach. *World Development* 32, 1–13.
- Bom, P. D. and J. E. Ligthart (2014) What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys* 28, 889–916.
- Boucekkine, R., D. de la Croix and O. Licandro (2002) Vintage human capital, demographic trends, and endogenous growth. *Journal of Economic Theory* 104, 340–375.
- Bureau of Labor Statistics (2016) Industry–Occupation Matrix Data, by Industry, Table 1.9, 2014–24. http://www.bls.gov/emp/ep_table_109.htm.
- Center for Disease Control and Prevention (2006) Trends in strength training–United States, 1998–2004. *MMWR* 55, 769–772. <http://www.cdc.gov/mmwr/preview/mmwrhtml/mm5528a1.htm>.
- Chakraborty, S. (2004) Endogenous lifetime and economic growth. *Journal of Economic Theory* 116, 119–137.
- Chakraborty, S., C. Papageorgiou and F. P. Sebastian (2013) Health Cycles and Health Transitions, MPRA Paper No. 50588, University Library of Munich, Germany.
- Chamley, C. (1986) Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica* 54, 607–622.
- de la Croix, D. and O. Licandro (1999) Life expectancy and economic growth. *Economics Letters* 65, 255–263.
- Cutler, D. M. and E. Richardson (1997) Measuring the health of the US population. *Brookings Papers on Economic Activity, Microeconomics* 28, 217–282.
- Deaton, A. (1981) Optimal taxes and the structure of preferences. *Econometrica* 49, 1245–1260.
- Deaton, A. (2003) Health, inequality, and economic development. *Journal of Economic Literature* 41, 113–158.
- Ehrlich, I. and H. Chuma (1990) A model of the demand for longevity and the value of life extension. *Journal of Political Economy* 98, 761–782.
- Faruqee, H. (2003) Debt, deficits, and age-specific mortality. *Review of Economic Dynamics* 6, 300–312.

- Finkelstein, A., E. Luttmer and M. J. Notowidigdo (2013) What good is wealth without health? The effect of health on the marginal utility of consumption. *Journal of the European Economic Association* 11, 221–258.
- Finlay, J. (2006) Endogenous Longevity and Economic Growth. School of Economics Faculty, July 24, 2006.
- Grossman, M. (1972) On the concept of health capital and the demand for health. *Journal of Political Economy* 80, 223–255.
- Guvenen, F. (2006) Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective. *Journal of Monetary Economics* 53, 1451–1472.
- Hall, R. E. and C. I. Jones (2007) The value of life and the rise in health spending. *Quarterly Journal of Economics* 122, 39–72.
- Halliday, T., H. He, L. Ning and H. Zhang (2019) Health investment over the life-cycle. *Macroeconomic Dynamics* 23, 178–215.
- Haskell, W. L., I.-M. Lee, R. P. Pate, K. E. Powell, S.N. Blair, B.A. Franklin, C. A. Macera, G. W. Heath, P. D. Thompson and A. Bauman (2007) Physical activity and public health: Updated recommendation for adults from the American College of Sports Medicine and the American Heart Association. *Circulation* 116, 1423–1434.
- Himmelstein, D. U. and S. Woolhandler (2016) The current and projected taxpayer shares of U.S. health costs. *American Journal of Public Health* 106, 449–452.
- Hosoya, K. (2009) Tax financed government health expenditure and growth with capital deepening externality. *Economics Bulletin* 5, 1–10.
- Hugonnier, J., P. St-Amour and F. Pelgrin (2013) Health and (other asset holdings). *Review of Economic Studies* 80, 663–710.
- IHRSA (2016) About the Industry. <http://www.ihrsa.org/about-the-industry/>.
- IMF (2015) Estimating Public, Private, and PPP Capital Stocks, Annex to *Making Public Investment More Efficient*.
- Judd, K. L. (1985) Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics* 28, 59–81.
- Jung, J. and C. Tran (2016) Market inefficiency, insurance mandate and welfare: U.S. health care reform 2010. *Review of Economic Dynamics* 20, 132–159.
- Keane, M. and R. Rogerson (2012) Micro and macro labor supply elasticities: A reassessment of conventional wisdom. *Journal of Economic Literature* 50, 464–476.
- Kelly, M. (2017). Health capital accumulation, health insurance, and aggregate outcomes: A neoclassical approach. *Journal of Macroeconomics* 52, 1–22.
- Lorentzen, P., J. McMillan and R. Wacziarg (2006) Death and Development. 2006 Meeting Papers No. 61, Society for Economic Dynamics.
- McDaniel, C. (2007) Average Tax Rates on Consumption, Investment, Labor and Capital in the OECD 1953–2003. Mimeo, Arizona State University.
- McLachlan, G. and A. Maynard (eds.) (1982) *The Public/Private Mix for Health*. London: The Nuffield Provincial Hospitals Trust.
- Mendoza, E. G., A. Razin and L. L. Tesar (1994) Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics* 34, 297–323.
- Navarro, V. (1985) The public/private mix in the funding and delivery of health services: An international survey. *American Journal of Public Health* 75, 1318–1320.
- OECD (2015) *Health at Glance 2015, OECD Indicators*. Paris: OECD Publishing.
- Pelgrin, F. and P. St-Amour (2016) Life cycle responses to health insurance status. *Journal of Health Economics* 49, 79–96.
- Physical Activity Council (2013) Participation Report.
- Preston, S. H. (1975) The changing relation between mortality and level of economic development. *Population Studies* 29, 231–248.

Rampell, C. (2012) A jobs boom built on sweat in an age of belt-tightening. *The New York Times* [New York] 30 June 2012. http://www.nytimes.com/2012/07/01/business/for-personal-trainers-a-boom-fueled-by-a-belt-tightening-age.html?pagewanted=all&_r=0.

Soares, R. R. (2005) Mortality reductions, educational attainment, and fertility choice. *American Economic Review* 95, 580–601.

Strauss, J. and D. Thomas (1998) Health, nutrition, and economic development. *Journal of Economic Literature* 36, 766–817.

Suen, R. M. H. (2017) Technological Advance and the Growth in Health Care Spending. University of Connecticut, Working Paper.

Turnovsky, S. J. (2000) Fiscal policy, elastic labor supply, and endogenous growth. *Journal of Monetary Economics* 45, 185–210.

U.S. Department of Health and Human Services (1996) Physical Activity and Health: A Report of the Surgeon General. Atlanta, GA: CDC. <http://www.cdc.gov/nccdphp/sgr/sgr.htm>.

U.S. Department of Health and Human Services (2008) Physical Activity Guidelines for Americans. Washington, DC: DHHS. <http://www.health.gov/paguidelines/guidelines/default.aspx>.

Weil, D. N. (2007) Accounting for the effect of health on economic growth. *Quarterly Journal of Economics* 122, 1265–1306.

World Health Organization (WHO) (2017) Global Health Expenditure Database. <http://www.who.int/health-accounts/ghed/en/>.

Technical Appendix

A.1: THE LINEARIZED CORE DYNAMIC SYSTEM

To derive the transitional dynamics, we linearize the macrodynamic equilibrium around the steady state (13a)–(13i). As stated in the text, we first solve equations (4a), (4b'), (4c'), (5b), and (10) for c, h, l, L , and e in terms of k, m, λ , and μ . The solution is of the form: $c = c(k, m, \lambda, \mu; \tau_w, \tau_c, s), h = h(k, m, \lambda, \mu; \tau_w, \tau_c, s), l = l(k, m, \lambda, \mu; \tau_w, \tau_c, s), L = L(k, m, \lambda, \mu; \tau_w, \tau_c, s), e = e(k, m, \lambda, \mu; \tau_w, \tau_c, s)$, where the partial derivatives are obtained from the following system:

$$\begin{pmatrix} U_{cc} & U_{ch} & U_{cl} & 0 & 0 \\ U_{lc} & U_{lh} - \lambda(1 - \tau_w)f_{Lh} & U_{ll} & -\lambda(1 - \tau_w)f_{LL} & 0 \\ \left(\frac{1-s}{1-\tau_w}\right)U_{lc} - U_{hc}h_e & \left(\frac{1-s}{1-\tau_w}\right)U_{lh} - [U_{hh} + \mu\theta_{hh}]h_e & \left(\frac{1-s}{1-\tau_w}\right)U_{ll} - U_{hl}h_e & 0 & -[U_h + \mu\theta_h]h_{ee} \\ 0 & 1 & 0 & 0 & -h_e \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dc \\ dh \\ dl \\ dL \\ de \end{pmatrix} = \begin{pmatrix} 0 & 0 & (1 + \tau_c) & 0 \\ \lambda(1 - \tau_w)f_{Lk} & 0 & (1 - \tau_w)f_{Ll} & 0 \\ 0 & [U_h + \mu\theta_h]h_{em} & s_e & \theta_h h_e \\ 0 & h_m & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} dk \\ dm \\ d\lambda \\ d\mu \end{pmatrix} + \begin{pmatrix} 0 & \lambda & 0 \\ -\lambda f_L & 0 & 0 \\ -\frac{1-s}{(1-\tau_w)^2}U_l & 0 & \frac{1}{(1-\tau_w)}U_l \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\tau_w \\ d\tau_c \\ ds \end{pmatrix} \tag{A.1}$$

Linearizing the dynamic system (11), (9), (4d'), and (4e') around the steady state (13) yields

$$\begin{pmatrix} \dot{k} \\ \dot{m} \\ \dot{\lambda} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} k - \bar{k} \\ m - \bar{m} \\ \lambda - \bar{\lambda} \\ \mu - \bar{\mu} \end{pmatrix}, \tag{A.2}$$

where

$$\begin{aligned}
 a_{11} &\equiv (1 - g) [f_k + f_L L_k + f_h h_k] - c_k - (n + \delta_k) & a_{21} &\equiv g [f_k + f_L L_k + f_h h_k] \\
 a_{12} &\equiv (1 - g) [f_L L_m + f_h h_m] - c_m & a_{22} &\equiv g [f_L L_m + f_h h_m] - (n + \delta_m) \\
 a_{13} &\equiv (1 - g) [f_L L_\lambda + f_h h_\lambda] - c_\lambda & a_{23} &\equiv g [f_L L_\lambda + f_h h_\lambda] \\
 a_{14} &\equiv (1 - g) [f_L L_\mu + f_h h_\mu] - c_\mu & a_{24} &\equiv g [f_L L_\mu + f_h h_\mu] \\
 a_{31} &\equiv \lambda [\theta_h h_k - (f_{kk} + f_{kL} L_k + f_{kh} h_k) (1 - \tau_k)] & a_{41} &\equiv \mu \theta_h h_k + U_c c_k + U_l l_k + U_h h_k \\
 a_{32} &\equiv \lambda [\theta_h h_m - (f_{kL} L_m + f_{kh} h_m) (1 - \tau_k)] & a_{42} &\equiv \mu \theta_h h_m + U_c c_m + U_l l_m + U_h h_m \\
 a_{33} &\equiv \lambda [\theta_h h_\lambda - (f_{kL} L_\lambda + f_{kh} h_\lambda) (1 - \tau_k)] & a_{43} &\equiv \mu \theta_h h_\lambda + U_c c_\lambda + U_l l_\lambda + U_h h_\lambda \\
 a_{34} &\equiv \lambda [\theta_h h_\mu - (f_{kL} L_\mu + f_{kh} h_\mu) (1 - \tau_k)] & a_{44} &\equiv \theta(h) + \mu \theta_h h_\mu + U_c c_\mu + U_l l_\mu + U_h h_\mu
 \end{aligned}$$

and the partial derivatives are derived from (A.1). For the functional forms employed in the numerical simulations, (A.1) becomes

$$\begin{aligned}
 &\begin{pmatrix} \frac{\gamma-1}{c} & \frac{\omega\gamma}{h} & \frac{\eta\gamma}{l} & 0 & 0 \\ \frac{\gamma}{c} & \frac{(\omega\gamma-\beta)}{h} & \frac{(\eta\gamma-\beta)}{l} & \frac{\alpha}{L} & 0 \\ \frac{\gamma}{c} & \frac{1}{h} \left(1 - \frac{\Gamma}{\Phi}\right) & \frac{(\omega\gamma-1)}{h} \frac{\Gamma}{\Phi} - \sigma \frac{\Gamma}{\Phi} & \frac{\gamma}{l} \frac{\Gamma}{\Phi} & 0 \\ 0 & \frac{1}{h} & 0 & 0 & -\frac{(1-\varphi)h}{e} \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} dc \\ dh \\ dl \\ dL \\ de \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & \frac{1}{k} & 0 \\ \frac{\alpha}{k} & 0 & \frac{1}{\lambda} & 0 \\ 0 & \frac{\varphi}{m} & 0 & -\frac{1}{\mu} \frac{\Delta}{\Phi} \\ 0 & \frac{\varphi}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} dk \\ dm \\ d\lambda \\ d\mu \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{1+\tau_c} & 0 \\ -\frac{1}{1-\tau_w} & 0 & \frac{1}{1-s} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\tau_w \\ d\tau_c \\ ds \end{pmatrix} \tag{A.1'}
 \end{aligned}$$

where $\Gamma \equiv \omega c^\gamma l^\eta h^{\omega\gamma-1}$; $\Delta \equiv \mu \theta \sigma \text{Exp}(-\sigma h)$; $\Phi \equiv (\Gamma - \Delta)$. Applying these partial derivatives, the corresponding dynamic system (A.2) employed in the simulations has the appropriate saddlepoint structure, yielding a unique stable transitional dynamic time path.

A.2: PROOF OF PROPOSITION 1

We begin by eliminating shadow values, $\tilde{\lambda}, \tilde{\mu}$ from the steady-state allocations for the decentralized/market equilibrium described by (13a)–(13i). This leads to

$$\frac{U_l(\tilde{c}, \tilde{l}, \tilde{h})}{U_c(\tilde{c}, \tilde{l}, \tilde{h})} = \left(\frac{1 - \tau_w}{1 + \tau_c} \right) f_L(\tilde{k}, \tilde{L}, \tilde{h}), \tag{A.3a}$$

$$\left(\frac{1 - s}{1 - \tau_w} \right) U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \left[U_h(\tilde{c}, \tilde{l}, \tilde{h}) - \frac{U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h})}{\theta(\tilde{h})} \right] h_e(\tilde{m}, \tilde{e}), \tag{A.3b}$$

$$\tilde{h} = h(\tilde{m}, \tilde{e}), \tag{A.3c}$$

$$\tilde{L} + \tilde{l} + \tilde{e} = 1, \tag{A.3d}$$

$$(1 - g)f(\tilde{k}, \tilde{L}, \tilde{h}) = \tilde{c} + (n + \delta_k)\tilde{k}, \tag{A.3e}$$

$$gf(\tilde{k}, \tilde{L}, \tilde{h}) = (n + \delta_m)\tilde{m}, \tag{A.3f}$$

$$f_k(\tilde{k}, \tilde{L}, \tilde{h})(1 - \tau_k) = \theta(\tilde{h}) + n + \delta_k. \tag{A.3g}$$

These seven equations now determine the equilibrium allocation of the quantity variables $\tilde{c}, \tilde{l}, \tilde{e}, \tilde{H}, \tilde{k}, \tilde{m}$ for given $\tau_k, \tau_w, \tau_c, s$, and g .

The corresponding equations for the centralized/planner’s problem are

$$\frac{U_l(\tilde{c}, \tilde{l}, \tilde{h})}{U_c(\tilde{c}, \tilde{l}, \tilde{h})} = f_L(\tilde{k}, \tilde{L}, \tilde{h}), \tag{A.4a}$$

$$U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \left[U_h(\tilde{c}, \tilde{l}, \tilde{h}) - \frac{U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h})}{\theta(\tilde{h})} + U_c(\tilde{c}, \tilde{l}, \tilde{h})\tilde{f}_h \right] h_e(\tilde{m}, \tilde{e}), \tag{A.4b}$$

$$f_k(\tilde{k}, \tilde{L}, \tilde{h}) - \delta_k = \frac{h_m(\tilde{m}, \tilde{e})}{h_e(\tilde{m}, \tilde{e})} f_L(\tilde{k}, \tilde{L}, \tilde{h}) - \delta_m, \tag{A.4c}$$

$$\tilde{h} = h(\tilde{m}, \tilde{e}), \tag{A.4d}$$

$$\tilde{l} + \tilde{L} + \tilde{e} = 1, \tag{A.4e}$$

$$(1 - \bar{g})f(\tilde{k}, \tilde{L}, \tilde{h}) = \tilde{c} + (n + \delta_k)\tilde{k}, \tag{A.4f}$$

$$\bar{g}f(\tilde{k}, \tilde{L}, \tilde{h}) = (n + \delta_k)\tilde{m}, \tag{A.4g}$$

$$f_k(\tilde{k}, \tilde{L}, \tilde{h}) = [\theta(\tilde{h}) + n + \delta_k]. \tag{A.4h}$$

Note that four of the seven equations governing the two allocations are identical. The first-best optimum will be attained if and only if the following three conditions hold:

$$\tau_k = 0, \tag{A.5a}$$

$$\frac{1 - \tau_w}{1 + \tau_c} = 1, \tag{A.5b}$$

$$\frac{1 - s}{1 - \tau_w} = \frac{U_h - U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h}) \cdot (\theta(\tilde{h}))^{-1}}{U_h - U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h}) \cdot (\theta(\tilde{h}))^{-1} + U_{f_h} \cdot (f_L)^{-1}}. \tag{A.5c}$$

To further simplify (A.5c), from (A.4a) and (A.4b), we have

$$U_l = (U_h - U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h}) \cdot (\theta(\tilde{h}))^{-1} + U_{f_h} \cdot (f_L)^{-1}) h_e(\tilde{m}, \tilde{e})$$

and hence

$$U_l(\tilde{c}, \tilde{l}, \tilde{h}) = \frac{(U_h - U(\tilde{c}, \tilde{l}, \tilde{h})\theta_h(\tilde{h}) \cdot (\theta(\tilde{h}))^{-1}) h_e(\tilde{m}, \tilde{e})}{1 - f_h \cdot (f_L)^{-1} h_e(\tilde{m}, \tilde{e})}$$

and using these in (A.5c’) implies

$$\frac{1 - s}{1 - \tau_w} = \frac{(U_h - U\theta_h \cdot \theta^{-1}) h_e}{U_l} = 1 - \frac{f_h}{f_L} h_e < 1 \tag{A.5c’}$$