Competitive Procurement with Ex Post Moral Hazard[1]

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Abstract: Unlike a standard auction, we find that competitive procurement may optimally limit competition or rely on an inefficient allocation rule. Competitive procurement differs from a standard auction as many procurement projects also involve additional work after the competitive process is over. When this work is subject to ex post moral hazard, we identify a tension between ex ante rent extraction and ex post effort. In an optimal auction mechanism, the procurement contract must combine an incentive scheme with the auction to guard against firms bidding low to win the contract and then cutting back on effort. Firms must then estimate the value of the incentive scheme (i.e., rent) when competing for the contract. While competition helps reduce the rent of efficient firms, it exacerbates the problem due to moral hazard. We find that competition can reduce the principal's payoff if she insists on allocative efficiency. The principal is better off limiting the number of firms participating in the procurement auction. If limiting the number of firms is not possible, the principal can optimally use an inefficient allocation rule that awards the contract to a less efficient firm with positive probability even when a more efficient firm is present.

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1 Introduction

Government agencies and private firms routinely rely on competitive procurement to obtain goods, services, or to complete projects. The OECD estimates that its members spend 12.1% of their GDP on public procurement.[2] It has long been known that the benefit of competitive procurement is to ensure productive efficiency and low cost by selecting the most efficient firm and reducing information rent (see, e.g., Demsetz (1968)). While it is particularly effective for standard items like office supplies, many procurement projects also involve additional work after the competitive process is over. This is the case for construction projects like highway procurement. Such expenditures form a large part of procurement spending.[3] Procurers often use incentive schemes to improve ex post performance of selected firms. For example, in highway construction, incentives schemes are used to motivate timely completion of projects.[4] Firms must then estimate the value of these schemes when competing for the procurement project. Optimal incentive schemes typically require offering ex post rent to the winning firm. However, this begs the question whether ex ante competitive procurement leaves enough rent to ensure ex post performance by the selected firm.[5]

In this paper, we study how the ex ante competitive process interferes with the ex post moral hazard problem.[6] We find that competition can be a mixed blessing for the procurer who insists on allocative efficiency. Indeed, allocative efficiency, where the

[2]The number is even higher for developing countries and the World Bank estimates it to be at 14.5% of the GDP for low-income countries (Djankov, Saliola and Islam (2016)).

[3]Expenditure on service and construction contracts often constitute a large fraction of government procurement, accounting for 273 out of 277 billion dollars spent on the top 10 non-defense spending categories in the 2013 U.S. federal budget. http://www.govexec.com/contracting/2015/01/10-categories-where-federal-agencies-spend-most-contracting/102498/

[4]See Lewis and Bajari (2011, 2014).

[5]There is evidence that it may not. In the U.S., the introduction of an experimental competitive bidding program by Medicare had a negative impact on the quality of distribution service for diabetic products (Puckrein et al. (2015)). Reporting on competitive procurement for elderly care by the U.K. National Health Service, The Guardian newspaper notes that companies "bid low to win contracts and then cut back on quality to meet their profit targets." (Leys, "NHS contracting has been a disaster," April 22, 2014, *The Guardian*.)

[6]Surprisingly, this crucial aspect of procurement has not been received much attention in the literature. In their survey on public contracting, Armstrong and Sappington (2007) stress the importance of unobservable quality when they discuss competitive procurement and note that relatively little work has been done in the topic. We discuss the literature in more detail below.

most efficient firm must be selected, is a key feature in many procurement settings.[7] Increased competition may hurt a procurer if allocative efficiency is a requirement. If the procurer cannot limit the number of bidders, she must give up on allocative efficiency to neutralize the negative impact of competition by relying on a scheme that randomly allocates the project to a less efficient firm.

There is evidence of such procedures in the U.S. and abroad. The EU Commission (2015) recommends using a "restricted procedure" with only a subset of potential providers invited to submit tenders, "where there is a high degree of competition (several potential tenderers) in the marketplace." Bajari et al. (2014) found that contracts were not allocated to the lowest bidder in nearly 4% of the California Department of Transportation first-price auctions.[8]

To study the tension between ex ante rent extraction and ex post performance, we consider a model of competitive procurement with ex post moral hazard in an optimal auction framework (Myerson (1981). In the initial stage, each agent or firm is asked to report its cost of production (cost of effort in our model). Suppose that the procurer then selects the most efficient firm based on the reports. The selected firm must then exert costly effort to complete the project. Both effort and the cost of effort are private information of the firm. Thus, we study a mixed model with both adverse selection and moral hazard.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition. Both downward and upward incentive constraints can be binding: instead of the standard problem of firms wanting to overstate cost, firms may now also want to understate cost and shirk.[9] While competition is known to be an effective tool to

[7]For instance, a key goal of the FCC is to promote efficient access to and use of the radio spectrum (FCC Spectrum Policy Task Force, https://www.fcc.gov/sptf/files/SEWGFinalReport_1.doc). FCC Chairman William E. Kennard (1999) notes that efficiency in the FCC spectrum auctions means that spectrum ends up in the hands of those who value it most highly.

[8]Citing examples from various countries, Eun (2019) studies a Korean procurement mechanism that has a stochastic cutoff rule to eliminate the lowest bids. With a counterfactual analysis, he also shows that this rule lowers social cost by 7% relative to a standard first-price auction. Split award auctions may allocate part of the project to a less efficient firm to retain future suppliers or promote ex ante investments (Anton and Yao (1987, 1992)). In our model, we assume that the project is allocated to one firm so split awards cannot be used to address moral hazard.

[9]With a continuum of types types, in section 7, the global incentive constraint preventing the highest

address the problem of overstating cost, we show that it can exacerbate the problem of understating cost while shirking. Because his chance of being selected decreases with competition, a low type will have to be given a higher information rent. This negative effect of competition can overtake its benefits if the impact of moral hazard is strong enough. We find that high levels of competition may not be beneficial because the benefit from rent extraction is dominated by the cost of sustaining high ex post effort.

Limiting the number of competing firms can then be optimal for the procurer. When that is not possible,[10] we show that the procurer can effectively neutralize the negative impact of competition with an inefficient allocation rule. That is, the procurer may not allocate the project to the most efficient firm with probability one. We find that giving up on allocative efficiency relaxes the incentive compatibility constraints of less efficient firms who now have a higher chance to win the project by telling the truth.[11] Since those constraints are what restricts the effectiveness of competition in extracting rent, an inefficient allocation rule can remove the negative impact of competition. Specifically, we show that an inefficient allocation rule is optimal and it allows the principal to mimic a mechanism with an efficient allocation rule where she can choose the number of firms. Then, increased competition ceases to affect her payoff, and the procurer cannot reap any further benefit from competition even with an inefficient allocation rule.[12]

In sections 7 and 8, we show that our key ideas hold under continuous efforts or types. We find that when the principal can screen with effort, some amount of competition is always beneficial even under the efficient allocation rule. With continuum of types we find that local incentive constraints are no longer sufficient to characterize the optimal mechanism. The global incentive constraint preventing the highest cost-type from mimicking the least cost-type and shirking is binding along with standard constraints to prevent over–stating cost.

The literature on competitive bidding for procurement contracts goes back to the

cost-type from mimicking the least cost-type and shirking is binding along with standard constraints to prevent over–stating cost. In our base-case binary-type setting, both incentive constraints are binding.

[10]For example, because it may appear as corruption.

[11]This is reminiscent of Hart-Schleifer-Vishny (1997), who argue that a private contractor's incentive to engage in cost reduction may be too strong because of the negative impact on noncontractible quality.

[12]Thus, when competition is beneficial, we show that restricting attention to allocative efficiency is without loss of generality.

late eighties when a set of influential papers analyzed properties of incentive schemes that governed ex post incentives of the selected firm.[13] Highlighting a separation property, they showed how standard auction formats can be used to extract rent while providing second-best incentives at the same time. In these models, competition has no negative effect. The driving force behind the results in these papers is adverse selection rather than moral hazard. In Riordan-Sappington (1987), the quality is observable, so there is no moral hazard. While McAfee-McMillan (1987) and Laffont-Tirole (1987) have an unobservable effort, agents are risk neutral with unlimited liability, and the principal can deduce the effort once the agent has revealed his type. In the terminology of Laffont-Martimort (2002), these are "false moral hazard" models, where upward incentive constraints are not binding: high-cost firms do not want to pretend to be low cost. An increase in competition can only benefit the principal.

Like us, McAfee-McMillan (1986) have a true mixed model with both adverse selection and moral hazard. However, they do not study the optimal contract but rather a linear contract that balances the cost-plus contract and the fixed-price contract. The linear cost-sharing parameter is assumed to be independent of the agent's type. Thus, the optimal choice of effort is independent of types, which implies that the upward incentive constraints are not binding. Our contribution is to solve the optimal auction mechanism in a tractable mixed model, and to show that insisting on allocative efficiency can lead to competition being harmful.[14]

In a recent paper, but in a contest setting, Che-Iossa-Rey (2017) find that it may not always be optimal to allocate the project to the most efficient firm ex post in order to provide incentive to exert effort ex ante. Firms have private information on implementation cost and effort comes *before* being selected.[15] In a procurement setting,

[13] See, e.g., Riordan-Sappington (1987), McAfee-McMillan (1986, 1987) or Laffont-Tirole (1987).

[14] Piccione and Tan (1996) have shown the separation property, mentioned above, may not hold when bidders make an ex ante investment. Their focus was on the role of the R&D technology (in particular whether it exhibits decreasing returns to scale or not) in the implementation of the project.

Kogan and Morgan (2010) compare, both theoretically and experimentally, debt and equity auctions followed by moral hazard. They don't study the impact of competition. They find allocative efficiency to be optimal in both auctions.

[15] Taylor (1995) and Fullerton-McAfee (1999) have shown that admitting too many contestants in a research tournament reduces the ex ante effort of each contestant because their probability of winning becomes too low. See Bénabou-Tirole (2016) for the effect of competition on ex post incentive in a

others have also highlighted allocative inefficiency in models where quality is exogenous. In Manelli-Vincent (1995), firms have private information about the exogenous quality they can produce, leading to a lemons problem where the least cost agent also generates the least value to the principal. Burguet-Ganuza-Hauk (2012) provide a related analysis where firms have private information about their financial status. In Chillemi-Mezzetti (2014), the winning bidder privately discovers the value of the cost overruns during the project's completion. All these papers find allocative inefficiency to be optimal in models with exogenous quality or ex ante effort. Our objective, instead, is to focus on the effect of competition in the presence of ex post moral hazard that are observed in many private and public procurement situations.

The literature on scoring auctions is also relevant but, again, it typically assumes that quality is observable (see, e.g., Che (1993), and Asker-Cantillon (2010)) and there is no moral hazard. Contractual externality across agency problems also plays an important role in dynamic procurement models such as Arve and Martimort (2016). Our paper is also related to the literature that combines adverse selection and moral hazard (such as Picard (1987), Ollier-Thomas (2013), and Gottlieb-Moreira (2017)). Those papers are single agent model without an optimal auction. Recently, the empirical literature has also stressed the importance of moral hazard in procurement auctions (see for instance Lewis-Bajari (2011, 2014)).

The paper is organized as follows. We present the model in section 2 and the principal's problem in section 3. We derive the optimal mechanism in section 4, assuming allocative efficiency, and our main result on the impact of competition in section 5. In section 6, we show that, if allocative efficiency is not required, an inefficient allocation rule is optimal. We show our key results hold when effort or types are continuous in sections 7 and 8.

Hotelling model.

2 The Model

A principal (she) must select one of n agents (he) to complete an indivisible project. The success of the project depends on the selected agent's effort, and agents have different costs of effort. The selected agent privately chooses effort $e \in \{0, 1\}$. With probability π_e, the output is high, h, and the principal receives $V > 0$, while, with probability $(1 - \pi_e)$, the output is low, l, and she receives zero. The output is publicly observed. The low output may capture a variety of outcomes such as costly delays. For example, incentive schemes based on time of completion are used in highway projects (Lewis-Bajari (2011, 2014)). While the time of completion is observable, it depends on the contractor's unobservable effort and random shocks. This corresponds well to a moral hazard framework where contractors can put in extra effort to reduce the chance of negative shocks (for instance, better planning and maintenance to prevent equipment failure). We assume that $0 < \pi_0 < \pi_1 \leq 1$. We also assume type-independent probabilities to focus on the effect of competition to screen the agents and extract rent.[16]

Cost of effort is privately known to the agent, and an agent can be one of two types, $x \in \{g, b\}$. It is commonly known that the probability that an agent is of type g is given by $q \in (0, 1)$. With both effort and cost of effort private information of an agent, we have a model with both moral hazard and adverse selection. While our base model has binary efforts and types for ease of exposition, we later show our key results hold if effort or types are continuous.

Denoting the cost of effort by ψ_e^x, we assume the following conditions about the cost of effort.

Condition L. (i) $\psi_1^b > \psi_1^g$ and $\psi_0^b > \psi_0^g > 0$, (ii) $\psi_1^b - \psi_0^b > \psi_1^g - \psi_0^g > 0$, and (iii) $\psi_1^g \geq \frac{\pi_1}{\pi_0}\psi_0^b$.

The first condition ranks the cost of effort and determines that "g" is a good type with a lower cost of effort. The second condition is akin to a standard single-crossing property that the marginal cost of effort is higher for the bad type. The third condition simplifies the exposition and captures the intensity with which ex post moral hazard

[16] As we will show, if the principal wants to induce high effort by both types, she would not be able to screen agents unless there is competition.

interferes with rent extraction. Specifically, it creates an incentive for a bad type to mimic a good type by exerting low effort ($e = 0$). The condition $L(iii)$ requires that $\pi_0 > 0$, which captures that the agent has a chance to succeed even when supplying low effort. The larger the π_0, the more serious the moral hazard problem. We discuss the implications of relaxing condition L after Proposition 2.

Agents are assumed to have a zero outside option and also limited liability, such that the transfers from the principal are non-negative.[17] The principal's ex post payoff is the output net of paid transfers. The ex post payoff for an agent is the transfer from the principal net of effort cost.

When we derive the optimal procurement mechanism, we assume that agents with identical costs of effort are treated symmetrically in terms of transfers and probability of being selected. By the Revelation Principle (see Myerson (1981)), we can restrict ourselves to truth-telling direct mechanisms. In the game that follows, we restrict ourselves to symmetric Perfect Bayes-Nash equilibria. The mechanism proceeds along the following timeline:

Stage 1. The principal announces the mechanism, for type $x \in \{g, b\}$:

$$\left\{t_l^x(n), t_h^x(n), \phi_r^x(n), e_x(n)\right\},$$

where, t_l^x, t_h^x and e_x are the output-based transfers and efforts for type x, and $\phi_r^x(n)$ is the allocation rule, i.e., the probability of allocating the contract to a type x agent if r agents report type x. Since the principal must allocate the contract to one of the n agents, we have $\phi_r^g(n) + \phi_r^b(n) = 1$. To save on notation, we will suppress n when presenting terms in the mechanism.

Stage 2. The agents report their types and the contract is allocated to an agent according to the allocation rules set in stage 1.

Stage 3. The selected agent chooses the privately observable effort.

Stage 4. The output is realized and payments are made accordingly.

[17] Limited liability makes our moral hazard problem relevant with risk neutral agents. Alternatively, we could have assumed risk aversion without limited liability.

It is without loss of generality to (i) restrict the transfers and efforts to depend only on an agent's own report; (ii) assume that only the winning bidder is paid in the mechanism. The first point follows from the types being independent. In the Appendix, we prove the second point as a preliminary claim 1 for the principal's problem of section 3.

The mechanism has two parts: (i) first an allocation rule that selects the agent based on the type announcements; (ii) an incentive contract that gives incentives to the selected agent to exert effort in completing the project.

Allocation rule: When agents report their types truthfully, the allocation rule pins down the probability that an agent will win the contract upon reporting type x, which we denote by γ_n^x. As is well known, it is convenient to write and solve the principal's problem in terms of γ_n^x. To see how γ_n^x is computed, consider the case when all agents report truthfully, and r agents report type g. Then, we have

$$\gamma_n^g = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{\phi_r^g}{r}. \tag{1}$$

For the solution (γ_n^b, γ_n^g) to correspond to a well defined (implementable) allocation rule (ϕ_r^g, ϕ_r^b), it must satisfy certain conditions. A necessary restriction on γ_n^x is that the probability that the contract is allocated to a type x is no greater than the probability that there is a type x. Also, the probability that the contract is allocated at all can be no greater than one.[18] Specifically,

$$qn\gamma_n^g \leq 1 - (1-q)^n \tag{2}$$

$$(1-q)\, n\gamma_n^b \leq 1 - q^n \tag{3}$$

$$qn\gamma_n^g + (1-q)\, n\gamma_n^b \leq 1 \tag{4}$$

Allocative efficiency: In our base model, we assume allocative efficiency, which requires that the principal allocate the contract to an agent who reports to be a good type. If r agents report type g, allocative efficiency requires that $\phi_r^g = 1$ for $r \geq 1$. Therefore, we

[18]Border (1991) proves that these constraints are also sufficient.

have:

$$\begin{aligned}
\gamma_n^g &= \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \\
&= \frac{1}{nq} \sum_{r=1}^{n} \frac{n!}{(n-r)!r!} q^r (1-q)^{n-r} \\
&= \frac{1}{nq} \left(1-(1-q)^n\right) \qquad (5)
\end{aligned}$$

$$\gamma_n^b = \frac{1}{n} (1-q)^{n-1}.$$

Thus, under an efficient allocation rule, condition (2) is binding and (3) is slack. However, as we will show later, an efficient allocation rule may not be optimal. We study the case of possibly inefficient allocation rules in section 6 by allowing for $\phi_r^g \in [0,1]$ and show when $\phi_r^g < 1$ can be optimal for at least some $r \geq 1$.

Often, we will use the ratio γ_n^g/γ_n^b of the two probabilities and denote it by γ_n. We can write

$$\gamma_n \equiv \frac{\gamma_n^g}{\gamma_n^b} = \frac{\left(1-(1-q)^n\right)}{q(1-q)^{n-1}} = \frac{1}{q(1-q)^{n-1}} - \frac{1}{q} + 1.$$

We denote by δ_n the probability that a good type is selected. For instance, if agents report their types truthfully and a good type is always selected whenever there is one,

$$\begin{aligned}
\delta_n &= 1-(1-q)^n \\
&= nq\gamma_n^g.
\end{aligned}$$

Note that, under such a rule, both γ_n and δ_n increase as n increases.

3 The principal's problem

Our goal is to explore the effect of competition between agents on the principal's payoff, and we start by assuming that the principal wants to implement high effort levels for both types of the agent ($e_g = 1, e_b = 1$) for expositional reasons. Later we specify conditions under which inducing high efforts $e_g = 1, e_b = 1$ are indeed optimal. In the section on screening with effort, we study the case of continuous effort and show that our key results continue to hold when the principal can induce different efforts for each type.[19]

We begin our analysis by clarifying the feasible set of contracts starting with the constraints that induce high effort by the selected agent. Given truthful reports of types, the optimal contract has to satisfy the following moral hazard constraints for each type of agent:

$$\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0) t_l^g - \psi_0^g \tag{MH_g}$$

$$\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0) t_l^b - \psi_0^b. \tag{MH_b}$$

To induce truth telling, the optimal contract must satisfy incentive compatibility constraints that account for the possibility of each type working (high effort) or shirking (low effort) if they misreport their type. Thus, we write two incentive compatibility constraints for each type of agent:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^g\right) \tag{IC_1^g}$$

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_0 t_h^b + (1-\pi_0) t_l^b - \psi_0^g\right) \tag{IC_0^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^b\right) \tag{IC_1^b}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_0 t_h^g + (1-\pi_0) t_l^g - \psi_0^b\right), \tag{IC_0^b}$$

where (IC_1^x) prevents misreporting while working and (IC_0^x) prevents misreporting while shirking, with $x \in \{g, b\}$. Note that the level of competition, n, affects payoffs on each side of the constraints through γ_n^g and γ_n^b. Recall that the ratio $\gamma_n = \frac{\gamma_n^g}{\gamma_n^b}$ increases with n, which means that the relative probability of being selected increases with competition if an agent claims to be a good type.

Finally, the optimal contract must satisfy the following IR constraints to induce each type of agent to participate:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1) t_l^g - \psi_1^g\right) \geq 0 \tag{IR^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1) t_l^b - \psi_1^b\right) \geq 0. \tag{IR^b}$$

[19] In that extension section, we also note that our results hold in the binary case when inducing efforts $e_g = 1, e_b = 0$.

The principal's problem in this case is to choose the contract $\{t_0^b, t_1^b, t_0^g, t_1^g\}$ to maximize the expected payoff,

$$\Pi_{11}(n) = \delta_n \left(\pi_1 V - \pi_1 t_h^g - (1-\pi_1)\, t_l^g\right) + (1-\delta_n)\left(\pi_1 V - \pi_1 t_h^b - (1-\pi_1)\, t_l^b\right)$$

subject to the above eight constraints and the non-negativity conditions on all four transfers.

Next, we simplify the principal's problem by first proving that we can set $t_l^g = t_l^b = 0$ without loss of generality Since high effort is induced for both types, it is not optimal to reward either type after a low outcome. Suppose t_l^x are strictly positive for $x = \{b, g\}$. Then, lower t_l^x to zero and raise t_h^x to keep $\pi_1 t_h^x + (1-\pi_1)t_l^x$ constant. The two (IR), the (IC_1^g) and (IC_1^b), and the principal's payoff are unaffected. Since $\pi_1 > \pi_0$, the constraints (IC_0^g), (IC_0^b), (MH_b) and (MH_g) are relaxed. Thus, we have proved the following lemma.

Lemma. Given any transfer vector $(t_h^g, t_l^g, t_h^b, t_l^b)$ that satisfy the IC, MH and IR constraints, there is a transfer vector $(\bar{t}_h^g, \bar{t}_l^g, \bar{t}_h^b, \bar{t}_l^b)$ with $\bar{t}_l^g = 0, \bar{t}_l^b = 0$ that gives the same payoff to the principal and satisfies the IC, MH and IR constraints.

We can simplify the problem further by eliminating some constraints. First, the (MH_g) and (MH_b), coupled with $L(iii)$, make the (IR_g) and (IR_b) redundant. Second, (IC_0^g) is implied by (IC_1^g) and (MH_b), i.e., the moral hazard rent given to the bad type induces the good type to work rather than shirk when misreporting. The bad type's incentive constraint cannot be ignored, which is unlike what is standard in models of contracting under adverse selection. In our setting, we anticipate that the bad type will have an incentive to claim to be a good type, to increase his chance of being selected, and he will pursue this option by putting in low effort rather than high effort, i.e., we expect (IC_0^b) to be relevant. Indeed, (IC_0^b) will play an important role in our analysis. Finally, we will ignore (IC_1^b) and can verify later that the optimal contract satisfies this constraint.

The Reduced Problem

Defining the principal's payoff when she induces high effort by both types by

$$\Pi_{11}(n) = \pi_1 \left[V - \left(\delta_n t_h^g + (1-\delta_n)\, t_h^b\right)\right]$$

we can rewrite the principal's problem in a simpler form, where she chooses the two transfers $\{t_h^b, t_h^g\}$ to solve:[20]

$$\max \Pi_{11}(n) \tag{6}$$

subject to

$$\gamma_n \left(\pi_1 t_h^g - \psi_1^g\right) \geq \pi_1 t_h^b - \psi_1^g \tag{IC_1^g}$$

$$t_h^g \geq \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{MH_g}$$

$$\pi_1 t_h^b - \psi_1^b \geq \gamma_n \left(\pi_0 t_h^g - \psi_0^b\right) \tag{IC_0^b}$$

$$t_h^b \geq \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}. \tag{MH_b}$$

It is useful to briefly consider a benchmark case of contracting with a single agent ($n = 1$, $\gamma_n^g = 1 = \gamma_n^b$), which is depicted in Figure 1 below. Since the principal wants to induce $e_g = e_b = 1$, the pair of transfers (t_h^g, t_h^b) must satisfy the moral hazard constraint for each type. However, there can be no screening since each type will claim the higher transfer given type-independent probabilities of success. Technically, we can see that $t_h^g = t_h^b$ from (IC_1^g) and (IC_0^b). Then (IC_0^b) reduces to (MH_b), so the optimal transfers are given by $t_h^g = t_h^b = \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}$, making (MH_g) slack. In sum, when $n = 1$, the constraints (IC_1^g), (IC_0^b) and (MH_b) all hold as equalities. Thus, the bad type receives a moral hazard rent but no adverse selection rent. The good type earns an adverse selection rent which is strictly higher than the rent needed to induce high effort.

Intuitively, given limited liability, both types can command a moral hazard rent in this model. However, the transfer required to induce the bad type to work t_h^b is strictly higher than that required to induce the good type to work. Thus, the good type has an

[20]Note that maximizing $\pi_{11}(n)$ is equivalent to minimizing the expected payment.

incentive to pretend to be a bad type and earns an adverse selection rent that is larger than his moral hazard rent.

Our model allows us to focus on the effect of competition to screen the agents, which we study next.

4 Competitive Procurement with Multiple Agents ($n > 1$)

Competition is a critical part of the incentive mechanism as the principal uses it to screen the agents. This is reflected in the presence of γ_n^g and γ_n^b in the (IC) constraints. On the other hand, the (MH) constraints are not affected directly by competition because the effort decision occurs once the agent has been selected.

A standard effect of competition is that it increases the cost of lying for a good type, which allows the principal to reduce his rent. By favoring a good type in the allocation rule, she gives him a greater chance of being selected if he tells the truth, $\gamma_n^g > \gamma_n^b$ when $n > 1$. This relaxes the good type's incentive constraint (IC_1^g) and allows the principal to reduce the transfer to the good type.[21] We call this the *good-type transfer effect* of increased competition. This is a standard effect of competition in adverse selection models with many agents.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition as the bad type's incentive constraint (IC_0^b) is also binding. This constraint is typically not binding in pure adverse selection settings.[22] With unobservable effort, a bad type can misreport his type *and* exert low effort. This ability to shirk makes it profitable for the bad type to pretend to be a good type, which yields an additional adverse selection rent to the bad type. Competition exacerbates this problem.[23] In other words, to induce truth-telling, the bad type has to be given a higher transfer as his chance of being selected decreases with competition. We refer to this as the *bad-type*

[21] As n increases, the ratio γ_n^g/γ_n^b increases and this continues to relax (IC_1^g).

[22] This effect of shirking is absent in earlier models of competitive procurement, with false moral hazard as in Laffont-Tirole (1987) and McAfee-McMillan (1987), or if effort is observable as in Riordan-Sappington (1987).

[23] Again, since the ratio γ_n^g/γ_n^b increases with n, it tightens (IC_0^b).

transfer effect of increased competition, which is a new cost of competition due to the ex post moral hazard problem.[24]

Two possible cases emerge depending on the intensity of competition. For small n, we are in case (I). The good type's rent is reduced but it is still large enough to induce high effort by the good type, and the good type's moral hazard constraint (MH_g) is still slack. In proposition 1 below, for low n, the solution is given by the binding (IC_0^b) and (IC_1^g) and we call this case I:

$$
\begin{aligned}
t_h^g &= \frac{\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b}{(\pi_1 - \pi_0)\,\gamma} \\
t_h^b &= \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 (\pi_1 - \pi_0)}.
\end{aligned}
\tag{I}
$$

For larger n, we are in case II. As the principal decreases t_h^g, the good type's moral hazard constraint (MH_g) will bind eventually. At that point, a larger n does not allow the principal to decrease t_h^g further, and (IC_1^g) becomes slack – the moral hazard rent is high enough to induce truth-telling by the good type. In proposition 1 below, for large enough n, the solution is given by the binding (IC_0^b) and (MH_g) and we call this case II:

$$
\begin{aligned}
t_h^g &= \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \\
t_h^b &= \frac{\gamma_n \pi_0}{\pi_1} \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1}.
\end{aligned}
\tag{II}
$$

These results are illustrated in Figure 1 and summarized in the following proposition. In Figure 1, as n increases from $n = 1$, the solution moves north-west as t_h^g decreases and t_h^b increases. This is case I. Once the solution reaches the (MH_g) line, t_h^g cannot

[24]Technically, it is useful to first recall that both (IC_1^g) and (IC_0^b) were satisfied as equalities when $n = 1$. With multiple agents, the (IC_1^g) will become slack and (IC_0^b) will be violated unless the transfers are adjusted. In the optimal contract, the principal adjusts the transfers to both types. With (IC_1^g) slack, she definitely reduces t_h^g. This is the benefit of increased competition in reducing the good type's rent. However, the decrease in t_h^g is not enough to satisfy (IC_0^b) unless t_h^b is increased to remove the bad type's incentive to pretend to be good. More precisely, the fact that t_h^b increases with n is implied by condition $L(iii)$, which reflects a strong moral hazard problem. Later, we relax condition $L(iii)$ to show that (IC_0^b) can still be binding and competition can hurt even under a less restrictive condition.

be further reduced. This is case II. The solution is given by the intersection of (MH_g) and (IC_0^b) and it moves up vertically with n. It is important to note that, in both cases (I) and (II), the bad-type transfer effect is present, i.e., t_h^b keeps increasing with n.

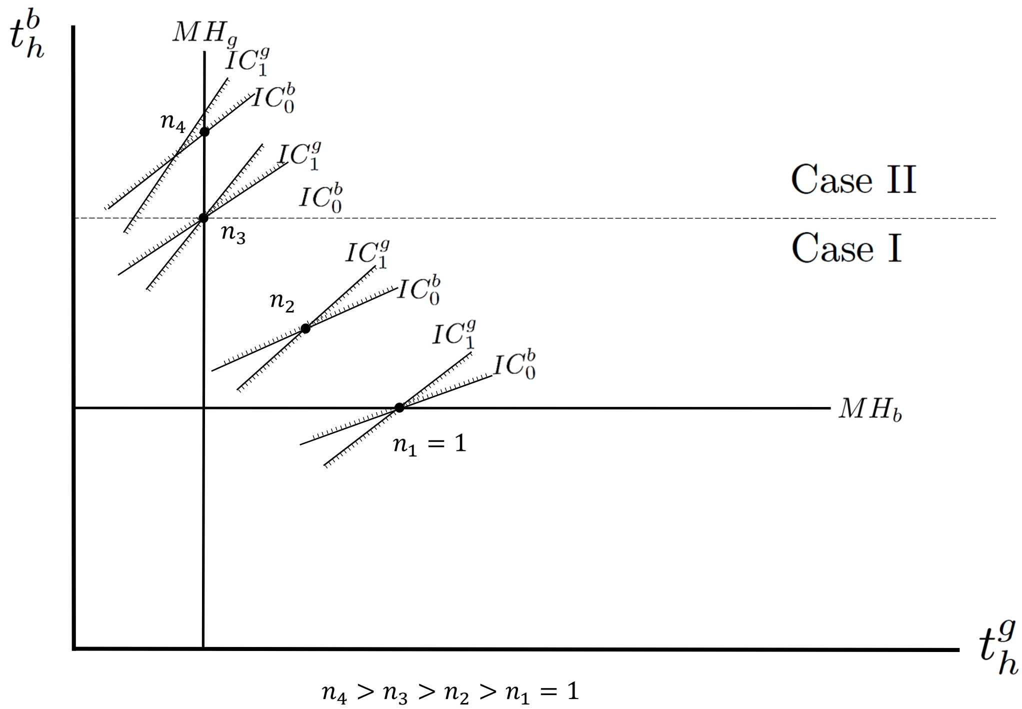

Figure 1: Optimal mechanism

Proposition 1 *The solution to the principal's problem entails:*

(i) the constraint (IC_0^b) is binding for all γ_n,
ii) for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$, the constraint (IC_1^g) is binding and the transfers are given by (I), and
(iii) for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$, the constraint (MH_g) is also binding, and the transfers are given by (II).

Proof. See Appendix I. ■

Recall that we have assumed that it is optimal to induce both types of agents to choose the high effort. Intuitively, the principal will always want to induce high effort if the project is very valuable, i.e., V is large enough. Our next proposition gives a formal proof. We discuss in section 5 the case when the principal may want to induce different efforts for different types.

Proposition 2 *If V is high enough, $e_g = e_b = 1$ is optimal.*

Proof. See Appendix I. ■

Before discussing whether competition hurts or helps the principal, we briefly discuss the role of the assumptions $L(ii)$ and $L(iii)$. First consider $L(iii)$ $\psi_1^g \geq \frac{\pi_1}{\pi_0}\psi_0^b$. The main reason for imposing $L(iii)$ is that it simplifies the analysis. Indeed, condition $L(iii)$ implies that (IC_0^b) is binding for all n, i.e., for both cases I and II of proposition 1. While a binding (IC_0^b) in itself does not guarantee that t_h^b will increase with n, condition $L(iii)$ is necessary and sufficient to ensure that t_h^b increases in case I, and sufficient for t_h^b to increase in n for case II.[25] If condition $L(iii)$ does not hold, there are several cases to consider, but the bad-type transfer effect will continue to hold under weaker conditions.[26]

Next consider condition $L(ii)$ $\psi_1^g - \psi_0^g < \psi_1^b - \psi_0^b$. Without $L(ii)$, the good-type transfer effect is absent, and the bad-type transfer effect comes into play immediately (for any $n > 1$).[27] Since this condition is akin to a standard single-crossing property, the opposite of $L(ii)$ would imply that the moral hazard of the good type is now more serious than the moral hazard of the bad type.

[25]In case II, a weaker condition $L(iv)$ is necessary and sufficient for t_h^b to increase with n.
Condition $L(iv) : \psi_1^g > \frac{\pi_1}{\pi_0}\psi_0^b - \left(\psi_0^b - \psi_0^g\right)$.

[26]Without $L(iii)$, we cannot ignore (IR^g) since it is no longer implied by (MH^g). If we assume that (MH^g) implies (IR^g), and $L(iv)$ holds, the solution will be given by (IC_0^b) and (MH^g) and t_h^b is increasing in n for a high enough n. If (IR^g) is more restrictive than (MH^g) when lowering t_h^g, then a weaker condition than $L(iv)$ would be enough to obtain the bad-type transfer effect for high enough n.

[27]For $n = 1$, we still have $t_h^b = t_h^g$ but it is the good type's moral hazard constraint (MH_g) that is binding. As n increases, because the (MH_g) remains binding, t_h^g is constant at (MH_g).

5 Does competition help?

We say that a given level of competition n hurts the principal if her expected payoff is higher with fewer bidders.[28] The main finding from our analysis so far is that increased competition generates a trade-off between the rents of the good type and the bad type, and we now show that there may indeed be too much competition. Recall from proposition 1 that the optimal mechanism moves from case I to case II as n increases. We now discuss how the trade-off determines whether competition helps or hurts the principal in each of the two cases. Note that along with the good and bad-type transfer effects of increased competition mentioned already, there is also a *selection effect* of increased competition due to a change in δ_n in the principal's payoff $\Pi_{11}(n)$. Thus, there are three potential effects of increased competition:

- *selection effect*: an increase in n increases the probability, δ_n, of awarding the contract to a good type. This effect on the principal's payoff (6) is positive since $t_h^g < t_h^b$.
- *good-type transfer effect*: an increase in n decreases the transfer t_h^g. This effect is again positive for the principal.
- *bad-type transfer effect*: an increase in n increases the transfer t_h^b. This effect is negative for the principal.

In case I, all three effects are present. The first two are standard effects due to adverse selection, while the third one, the negative effect, is new due to ex post moral hazard. The net effect of competition on the principal's payoff depends on the combined impact of the three effects. The principal benefits from competition when the two positive effects (the selection effect and the good-type transfer effect) dominate the negative effect (the bad-type transfer effect).

The good-type transfer effect is limited by the need to provide the good type with a moral hazard rent. Hence, as competition increases the good type's transfer cannot be reduced indefinitely: t_h^g cannot be reduced further when (MH_g) is binding, and

[28] We say that increasing competition always helps if her expected payoff increases as n increases.

we are in case II. In that case the good-type transfer effect, which is one of the positive effects, vanishes and only the selection and bad-type transfer effects remain. The principal benefits from competition when the remaining positive effect (the selection effect) dominates the negative effect (the bad-type transfer effect).

Therefore, it is possible that competition (i) never hurts, (ii) hurts in cases I and II, or (iii) hurts only in case II. This last case (iii) is particularly interesting. Intuitively, as competition intensifies, the principal lowers the transfer to the good type, but at some point the transfer becomes so low that the agent would no longer exert effort if the transfer kept decreasing. The principal cannot lower the transfer to the good type any further. Thus, the good-type transfer effect disappears, which weakens the overall benefit of competition. The precise conditions when competition hurts the principal in each case are given in the next proposition.

Proposition 3 *Competition hurts if and only if*

$$\textit{Case } I: \quad \pi_1 q\left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] \qquad \textit{when } \gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right],$$

$$\begin{aligned} \textit{Case } II: \quad & \pi_1 q\left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] \\ & + (1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0\psi_1^b - \pi_1\psi_0^g\right) \quad \textit{when } \gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right). \end{aligned}$$

Proof. See Appendix I. ■

The proposition formalizes the relative impact of the positive and negative effects of competition for the principal. Consider case I first. Given condition $L(iii)$, the right hand side of the first inequality is positive. However, for competition to hurt in case I, we also need a relatively large bad-type transfer effect. For instance, if ψ_1^b is close to ψ_1^g, adverse selection is not a serious problem. However, the moral hazard concern remains. We still have the issue of the bad type mimicking the high type while shirking. So the bad type transfer effect remains and, if it dominates, the competition will hurt. In case II, the good type transfer effect is absent, which implies that the condition for competition to hurt is less strict. This can be seen by the two extra positive terms on the right hand side of the inequality, making it easier to satisfy than the condition for case I. Therefore, if competition hurts the principal for small n it cannot help her when n is large. This is reflected in the following Corollary.

Corollary 1 *If $\Pi_{11}(n)$ increases for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ then it also increases for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$.*

Proof. See Appendix I. ■

Choosing the number of participants n

Up to now we assumed that the principal takes n as given. We see that competition may not hurt when the number of participants is low but it may when the number is large. Therefore, we examine next the case when the principal could choose the number of participants.

Denoting by n^* the number of participants for which the principal's payoff is the highest, the proposition and the corollary imply that there are three possible outcomes of competition depending on the parameter values:

(i) $n^* = \infty$. Competition always helps the principal, in which case the principal would like as many participants as possible. This is the standard result due to competition.

(ii) $n^* = 1$. Competition always hurts the principal and contracting with a single agent is better. This would be the extreme implication of ex post moral hazard in a competitive setting.

(iii) $1 < n^* < \infty$. Low levels of competition help the principal, but higher levels of competition hurt the principal.[29] There is an optimal number of participants, and the principal would prefer to limit the number of participants to that level. The optimal n^* is given by $\gamma_{n^*} \approx \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$.[30] This may explain why some existing procurement mechanisms try to restrict the number of participants (see Eun (2019) for examples). This may also explain the criticisms in the earlier examples of NHS and Medicare procurement that too much competition adversely affects ex post quality of service. Next,

[29] Note that if we assumed $\psi_0^g = \psi_0^b$, case II disappears and MH_g never binds. In that case, either competition always helps or competition never helps.

[30] We write $\gamma_n \approx \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ instead of $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ because n is discrete and there need not be an n for which the equality holds.

we study an alternative to restricting the number of participants by using inefficient allocation mechanisms.

6 An inefficient allocation rule can be optimal

So far we have restricted attention to an efficient allocation rule, where the principal committed to allocating the contract to a good type with probability one when at least one agent reported to be a good type. This implied that condition (2) is binding. Here, we remove the requirement of allocative efficiency and consider inefficient allocation rules, which give a chance for a bad type to be selected even when another agent has reported to be a good type.

We will show that whenever competition hurts under an efficient allocation rule, the optimal contract requires an inefficient allocation rule. Thus, if a given level of competition hurts her payoff under an efficient allocation rule, i.e., $n > n^*$, optimality requires that she give up on allocative efficiency. Furthermore, we show that the optimal (inefficient) rule with n agents gives the principal the same payoff as she would obtain with n^* agents under an efficient allocation rule .[31] This means that the principal does not gain from additional competition when $n > n^*$. Thus, by giving up on allocative efficiency, the principal can in fact entirely neutralize the negative effect of competition on her payoff even when she cannot reduce the number of agents competing for the contract.

Technically, to allow for inefficient allocation rules, we remove the restriction that the first constraint (2) must hold as an equality. Indeed, if the solution involves $qn\gamma_n^g < 1-(1-q)^n$, the optimal allocation rule must necessarily be inefficient.[32] In the appendix, we show that the principal's objective function and constraints depend on n only through γ_n^g. Whenever competition hurts under an efficient allocation rule, we show that the principal's payoff is decreasing in γ_n^g and condition (2) is slack, which implies

[31]When $1 < n^* < \infty$ and $\frac{1-(1-q)^{n^*}}{(1-q)^{n^*-1}q} < \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}$ the optimal expected payoff is in fact higher than the payoff at n^* since the payoff is continuous in γ_n and n^* is an integer. We ignore this point in the statement and the rest of the proof for expositional ease.

[32]In that case, it cannot be that $\phi_r^g = 1$ for all r, i.e., a bad type must have a chance of being awarded the contract even when a good type is present.

that the optimal allocation rule inefficient. The opposite is true when competition helps under an efficient allocation rule. The constraint is binding and an efficient allocation rule is optimal.

Proposition 4 *Whenever competition hurts under an efficient allocation rule (i.e., $n > n^*$), the optimal allocation rule is inefficient, and it makes the principal's expected payoff equal to her payoff at n^* under the efficient allocation rule.*

Proof. See Appendix I. ■

To see the intuition for this result, note that the bad type's incentive to mimic the good type with zero effort can be lowered in two different ways. We have already discussed one method, increasing his transfer t_h^b, which leads to the the bad-type transfer effect. An alternative approach is to use an inefficient allocation rule that gives the bad type a positive probability of being selected despite the presence of good types. By doing so, the principal increases the probability for a bad type to be selected, which used to be zero under an efficient allocation rule even if there were only one good type present. This relaxes (IC_0^b) and benefits the principal.[33]

Suppose competition hurts for all n under an efficient allocation rule. This occurs when the case I condition holds in proposition 3. Then, the principal wants to choose the smallest possible γ_n^g, making the allocation rule inefficient and the principal's payoff identical to her payoff when there is only one agent ($n^* = 1$).[34] If competition helps for all n under efficient allocation, then the allocative efficiency is optimal for all n.

Finally, we consider the interesting case when competition helps for small n but hurts for large n under an efficient allocation rule. This case occurs when the case II condition holds in proposition 3, but not the case I condition. For small n, the optimal

[33] Strausz (2006) shows that a stochastic contract can also relax the upward binding constraint in a single-agent model if there is bunchiing and the agent's types differ in the degree of risk aversion. See Kadan et al (2017) for a recent discussion on the role of randomization in principal agent models.

[34] In the appendix, we show that the smallest value of γ_n^g makes $\gamma_n = 1$, which is the lower bound given by the monotocity condition $\gamma_n \geq 1$ implied by the incentive constraints.

allocation rule is efficient. For large n, the optimal allocation rule is inefficient, again making the principal's payoff identical to that at n^*.[35]

To characterize the allocation rules that implement the optimal γ_n, consider first the case where competition always helps under an efficient allocation rule. In this case, the principal allocates the contract to a good type whenever there is one. If competition hurts for all n under an efficient allocation rule, then the principal allocates the contract randomly to an agent regardless of their announcement. Finally, consider the interesting case when competition helps for small n but hurts for large n under an efficient allocation rule. For small n, the principal allocates the contract to a good type whenever there is one. For large n, she allocates the contract to a bad type with a high enough probability such that $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_1^g}$. We prove the following Corollary in Appendix 1:

Corollary 2 *When $n^* = 1$, it is optimal to choose $\phi_r^g = \frac{r}{n}$ for all r and $n > 1$. When $n^* = \infty$, it is optimal to choose $\phi_r^g = 1$ for $r \geq 1$ and $\phi_0^g = 0$ for all n. When $1 < n^* < \infty$, for all r, it is optimal to choose $\phi_r^g = 1$ for $n \leq n^*$ and $\phi_r^g = \left(\frac{nq}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ for $n > n^*$.*

7 Screening with effort

So far we have considered the case where effort was binary and the principal found it optimal to induce high effort by both types ($e_b = e_g = 1$). As a result, the principal did not use effort as a screening instrument and the agents were restricted to only one alternative effort level when shirking. In this section, we relax that restriction by letting the agent adjust effort $e \geq 0$ continuously. There is a trade-off for the principal. On the one hand, by adjusting effort continuously, the principal can screen the agent better. On the other hand, the bad type agent now has more options in choosing effort when mimicking the good type.

We find that, although some amount of competition is always beneficial, being able to adjust effort continuously is not enough for the principal to guarantee that high levels

[35]In the appendix, we show that the smallest value of γ_n^g makes $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ which is the smallest value of γ_n consistent with case II, propsition 1.

of competition are beneficial. We again find that (IC^b) is binding for all n. Except that competition is always beneficial when n is small, our results are largely analogous to those from the binary model. We relegate details to Appendix II and outline the model and key arguments here

We consider a model where the agent privately chooses effort $e \in [0,1]$. We focus on interior solutions such that effort adjusts continuously as n increases. Accordingly, we will assume that V is not too large.[36] Let $\psi_x(e) = xe^2$ be the cost of effort to type $x \in \{b, g\}$, where $0 < g < b$. We assume that the probability of high outcome given effort e is $\pi(e)$, and $\pi(e) = e$.[37] We restrict attention to interior solutions for effort on and off the equilibrium path and provide a sufficient condition for that in Appendix II. As in the main section we first consider efficient allocation rules where the contract is awarded to the good type whenever there is one, i.e., $\phi_r^g = 1$ for all $r \geq 1$, and then we consider inefficient allocation rules by allowing $\phi_r^g \leq 1$.

We present some preliminary steps and notation before writing the (IC) constraints. The optimal effort by a type x agent who reports type y is denoted by

$$e_{xy} \in \arg\max_e \left\{\pi(e)\, t_h^y + (1-\pi(e))\, t_l^y - \psi_x(e)\right\}, \tag{7}$$

and $e_x \equiv e_{xx}$. The first order conditions determining effort e_{xy} in an interior solution are:

$$t_h^y - t_l^y = 2xe_{xy}.$$

We see that rent extraction can be limited by moral hazard. If the principal tries to extract rent from the good type by lowering t_h^g, it will result in a lower equilibrium effort by the good type. The two (IC) constraints are:

$$\gamma_n\left(e_g t_h^g + (1-e_g)\, t_l^g - \psi_g(e_g)\right) \geq e_{gb} t_h^b + (1-e_{gb})\, t_l^b - \psi_g(e_{gb}) \tag{IC^g}$$

$$e_b t_h^b + (1-e_b)\, t_l^b - \psi_b(e_b) \geq \gamma_n\left(e_{bg} t_h^g + (1-e_{bg})\, t_l^g - \psi_b(e_{bg})\right). \tag{IC^b}$$

[36] Our binary model presents a corner solution where V is so large that $e = 1$ is always optimal. When effort is binary a smaller V would result in effort choices $(e_g = 1, e_b = 0)$ or $(e_g = 0, e_b = 0)$. In the former case the solution is given by (IC_0^b) and (MH_g), but our key results continue to hold.

[37] Alternatively, we could assume the agent chooses the probability of high outcome π at a cost $x\pi^2$, with $\pi \in [0,1]$.

The principal's problem is to choose the transfers $\{t_l^b, t_h^b, t_l^g, t_h^g\}$ to maximize her objective function:

$$\left(\delta_n e_g + (1-\delta_n) e_b\right) V - \delta_n \left(e_g t_h^g + (1-e_g) t_l^g\right) - (1-\delta_n) \left(e_b t_h^b + (1-e_b) t_l^b\right),$$

subject to $(IC^g), (IC^b)$, and $t_h^x, t_l^x \geq 0$. The participation constraints are automatically satisfied as $t_h^x - t_l^x \geq 0$.

We again find that (IC^b) is binding for all n, and that competition will hurt the principal if the ex post moral hazard problem plays a significant enough role. As in the binary effort model, we again have two cases I and II depending on whether (IC^g) is binding. The (IC^g) is binding for low n, i.e., for $\gamma_n < \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$, when rent extraction from the good type is important. For high n, i.e., for $\gamma_n \geq \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$, the (IC^g) ceases to bind. As before, we denote by n^* the number of participants for which the principal's payoff is the highest.

Proposition 5 *When effort is continuous,*

(i) The bad type's incentive constraint (IC^b) is binding for all γ_n, but the good type's incentive constraint (IC^g) binds if and only if $\gamma_n < \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$.

(ii) If the good type does not have a large cost advantage over the bad type, i.e., if $(b-g)^2/g^2 < 1/q$, competition hurts the principal under efficient allocation if and only if $\gamma_n > \left(\frac{b}{g}\right)^2$, i.e., $n > n^*$. Otherwise, competition always helps, and $n^* = \infty$.

Proof. See Appendix II. ■

With continuous effort the agent has more options when shirking, and the (IC^b) is binding for all n without the need for parameter restrictions. However, for competition to hurt, we do need restrictions on parameters that are reminiscent of results from the binary model. Part (ii) of the proposition determines when competition hurts. For competition to hurt, the cost of (high) effort for the good type cannot be too small (similarly to condition $L(iii)$), and the good type cannot have a large cost advantage over the bad type such that adverse selection is not a serious problem (similarly to propositions 1 and 3). Under the condition $(b-g)^2/g^2 < 1/q$, which is equivalent to $\left(\frac{b}{g}\right)^2 < \left(2+\frac{g}{b}\frac{(1-q)}{q}\right)^2$, competition hurts when $\gamma_n > \left(\frac{b}{g}\right)^2$. So, the optimal n^* is given by $\gamma_n \approx \left(\frac{b}{g}\right)^2$.[38] Since $\frac{b}{g} > 1$, this suggests that some degree of competition is always beneficial.[39] On the other hand, if $(b-g)^2/g^2 > 1/q$, competition can only help.

As in the binary effort case, we can again show that an inefficient allocation rule with $\phi_r^g \leq 1$, with strict inequality for some r, is optimal whenever the principal's payoff is decreasing in n under an efficient allocation rule. Furthermore, we can clearly show that the principal prefers to have some competition. That is, if he can choose the number of participants, he will pick at least two agents to participate in the mechanism. For example, if the solution to the optimality condition $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ implies that $n^* \in (1,2)$, then the principal would prefer to have two participants and use an inefficient allocation rule ($\phi_r^g < 1$) to obtain her maximum payoff.[40]

Proposition 6 *With continuous effort, whenever competition hurts under an efficient allocation rule (i.e.,* $(b-g)^2/g^2 < 1/q$ *and* $n > n^*$*), the optimal allocation rule is inefficient, and it makes the principal's expected payoff equal to her payoff at* n^* *under the efficient allocation rule.*

Proof. See Appendix II. ■

Two other notable results in the continuous effort case are for high n : (i) the bad type's effort can be higher than the good type's effort ($e_b > e_g$) and (ii) the bad type

[38] We write $\gamma_{n^*} \approx \left(\frac{b}{g}\right)^2$ instead of $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ because with discrete n there need not be a n for which the equality holds.

Furthermore, in case I, it is shown in the appendix that competition hurts the principal if and only if $\gamma_n > \left(\frac{b}{g}\right)^2$. This means that if competition hurts in case I, it must also hurt in case II. In case I, there is a possibility of competition to be helpful for low levels of n but harmful for larger n. In that case, the optimal level of competition is determined by n^* such that $\gamma_{n^*} \approx \left(\frac{b}{g}\right)^2$ if competition hurts. If $q > \frac{g^2}{(b-g)^2}$, increased competition can only help even though (IC^b) is binding.

[39] Note that, since n is an integer, the optimal n could turn out to be 1 if $\gamma_{n^*} = \left(\frac{b}{g}\right)^2$ implies that $n^* \in (1,2)$.

[40] Again, the incentive constraints depend on n only through γ_n and the principal's payoff depends on n only through δ_n.

can receive a positive transfer even when the outcome is low ($t_l^b > 0$). To keep inducing truth-telling by the bad type as his chance of winning decreases due to an increase in n, he must be given a higher rent conditional on winning. This can push up e_b above e_g. The principal can also make $t_l^b > 0$, which occurs in case II when only the (IC^b) is binding. These results are formally derived in Appendix II.

8 Continuum of Types

In this section, we consider a model with a continuum of types to show that our key results do not depend on the binary type modeling choice. Increasing the number of types offers more options for each type to misreport and shirk, which greatly complicates the model, but our key ideas remain intact. We again show that competition increases the incentive to understate cost, but there is a notable technical difference: we find that local ICs do not imply the global ones given ex post moral hazard. Under an efficient allocation rule, we show that the principal is always worse off if the level of competition is sufficiently high. Also, using a uniform distribution example, we identify conditions similar to those derived above for competition to be harmful. We discuss these further after presenting the model, but proofs and detailed arguments are relegated to Appendix III.

We assume that the agent's type θ is distributed over $[g, b]$, where we now refer to type-g as the best and type-b as the worst type. The distribution and density functions are given by $F(\theta)$ and $f(\theta)$ respectively. As in the main section, suppose that the agent chooses one of two effort levels $e \in \{0, 1\}$, and the probability of high output from effort e is π_e, with $0 < \pi_0 < \pi_1 \leq 1$. To keep our analysis simple, we assume that, for each type θ, the cost of zero effort is 0 and the cost of effort 1 is $\psi(\theta) > 0$, with $\psi'(\theta) > 0$, and that the principal uses an efficient allocation rule and induces $e(\theta) = 1$.

The transfers to the selected agent if his type is θ is $t_h(\theta)$ when the output is high. Again, without loss of generality, we can set low-output transfer $t_l(\theta) = 0$. The probability that an agent will win the contract upon reporting type θ in an efficient mechanism is $\gamma_n(\theta) = (1 - F(\theta))^{n-1}$. For notational convenience, define an "expected transfer function" $T(\theta)$ for each type as follows:

$$T(\theta) \equiv \gamma_n(\theta)\, t_h(\theta).$$

Then, the incentive constraints are given by

$$\pi_1 T(\theta) - \gamma_n(\theta)\,\psi(\theta) \geq \pi_1 T\left(\hat{\theta}\right) - \gamma_n\left(\hat{\theta}\right)\psi(\theta) \qquad (IC_1)$$

$$\pi_1 T(\theta) - \gamma_n(\theta)\,\psi(\theta) \geq \pi_0 T\left(\hat{\theta}\right). \qquad (IC_0)$$

where $\hat{\theta}$ is the agent's report in $[g, b]$. The (IC_1) and (IC_0) ensure that no type has an incentive to misreport while working or shirking, respectively. Note that (IC_0) is identical to the moral hazard constraints when $\theta = \hat{\theta}$, and it also ensures that the participation constraint can be ignored.

The incentive constraint while shirking (IC_0) is the more interesting one, and we find that local ICs are not enough to guarantee global incentives. Intuitively, to maximize his chance of being selected, every type will want to claim to be the best type when understating cost. Since this incentive is the greatest for the worst type who has the lowest payoff, we show that (IC_0) can be simply presented as one global constraint which prevents the worst type from mimicking the best type while shirking:[41]

$$\pi_1 T(b) - \gamma_n(b)\,\psi(b) \geq \pi_0 T(g). \qquad (IC_0')$$

Important for our analysis, this global $(IC_0^{'})$ captures the incentive to understate cost while shirking, and we find that it is always binding. If not, the expected transfer $T(\theta)$ can be decreased by the same amount for each θ without violating either IC constraint.[42]

[41] In Appendix III, we use (IC_1) to show that $\pi_1 T^{'}(\theta) = \gamma_n^{'}(\theta)\psi(\theta)$, which implies that $T(\theta)$ is non-increasing. Thus, from the RHS of (IC_0), each type will prefer to claim to be the best type when shirking. Finally, since (IC_1) implies that the agent's payoff is non-increasing in θ, this incentive is the greatest for the worst type.

[42] From (IC_1), the expected transfer function can be characterized up to a constant $T(b)$:

$$T(\theta) = T(b) - \frac{1}{\pi_1}\int_\theta^b \gamma_n'(x)\,\psi(x)\,dx,$$

and $T(b)$ can be decreased if $(IC_0^{'})$ is not binding.

Next, we study the impact of competition on the principal's payoff. The following proposition shows that high levels of competition hurts the principal in general.

Proposition 7 *Given an efficient allocation rule, there can be too much competition. Specifically, there is a* $n^* < \infty$ *such that the principal is worse off with an additional agent whenever* $n \geq n^*$*.*

Proof. See Appendix III. ■

Recall from our previous sections that under the efficient allocation rule the effect of competition on the principal's payoff depended on the extent of cost advantage of the good type over the bad type. Using a uniform distribution for θ for tractability, we find a similar effect with continuous types as well which is relevant for lower levels of competition. In particular, if the best type's cost advantage over the worst type is large, some competition is beneficial. Of course, as noted more generally in the above proposition there can be too much competition and an inefficient allocation rule can be optimal.

Proposition 8 *Given an efficient allocation rule, suppose that the cost type* θ *is distributed uniformly over* $[g, b]$ *where* $b > g > 0$ *and* $\psi(\theta) = \theta$*.*

(i) If the best type's cost advantage over the worst type is not large, i.e., $(b - g)/g < \frac{3\pi_0}{\pi_1 - \pi_0}$*, increased competition always hurts the principal.*

(ii) If the best type's cost advantage over the worst type is large, i.e., $(b - g)/g > \frac{3\pi_0}{\pi_1 - \pi_0}$*, there is an* $n^* \geq 2$ *such that increased competition helps the principal when* $n < n^*$ *and hurts the principal when* $n \geq n^*$*.*

(iii) For n *large enough, an inefficient allocation rule is optimal.*

Proof. See Appendix III. ■

9 Conclusion

There is widespread concern that competitive bidding can lead to poor quality ex post. This connection has been largely ignored in the theoretical literature that has focused on the adverse selection problem to emphasize ex ante rent extraction. Procuring a project requires not only to select the most efficient firm (adverse selection), but also to make sure that the selected firm has the correct incentives to implement the project (moral hazard). Our analysis highlights the interaction between the two and explains how competition for the project results in a trade-off that may hurt the procurer. While competition is typically expected to be beneficial in dissipating the rent due to adverse selection, the presence of moral hazard can significantly interfere with rent extraction. Introducing the option to shirk allows a high-cost firm to mimic a low-cost firm and put in low effort, which results in an additional rent for the high-cost firm. Attempts to use increased competition to extract a low-cost firm's rent may lead to an increased rent to a high-cost firm. As a consequence, the procurer may find it optimal to limit the number of potential firms.

We show that insisting on allocative efficiency is costly from an incentive point of view and gives a strong incentive for a bad type to claim to be a good type as it lowers the probability that a bad type will be awarded the contract – it is zero as soon as there is only one good type present. By randomly assigning the contract to a bad type even when a good type is present, the procurer can lower the cost of inducing truth-telling significantly. Remarkably, we show that the procurer can use an inefficient allocation mechanism to mimic an efficient mechanism with the optimal number of agents without actually limiting the number of agents.

The framework presented here captures an important element of procurement auction. At the same time, it is highly tractable which should allow exploration of a variety of relevant interesting questions in competitive procurement. For instance, suppose that the procurer can use an audit technology to verify ex post the efficiency of the selected firm (adverse selection) or its effort (moral hazard). Which one should she concentrate her resources on? We have also ignored the cost of suppliers to participate in the procurement process. What if it is costly to prepare a submission? Similarly, the procurer could also decide to impose a fee to participate in the mechanism. We could study the role of endogenous entry instead of assuming that there is a fixed number of participants. The model presented here is simple enough that it would be possible to explore these and other related questions in procurement with moral hazard.

10 Appendix I

Claim 1. *The principal cannot be better off making a payment to a losing bidder.*

Proof of Claim 1. Suppose the principal paid $f^g \geq 0$ and $f^b \geq 0$ to losing good and bad type agents, respectively. Then, the transfers $\left\{\left(f^g, t_h^g, t_l^g\right), \left(f^b, t_h^b, t_l^b\right)\right\}$ would have to satisfy the incentive and participation constraints:

$$\pi_1 t_h^g + (1-\pi_1)\, t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0)\, t_l^g - \psi_0^g \tag{MH_g}$$

$$\pi_1 t_h^b + (1-\pi_1)\, t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0)\, t_l^b - \psi_0^b \tag{MH_b}$$

$$\begin{aligned} &(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_1)\, t_l^g - \gamma_n^g \psi_1^g \\ &\geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b (1-\pi_1)\, t_l^b - \gamma_n^b \psi_1^g \end{aligned} \tag{IC_1^g}$$

$$\begin{aligned} &(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_1)\, t_l^g - \gamma_n^g \psi_1^g \\ &\geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_0 t_h^b + \gamma_n^b (1-\pi_0)\, t_l^b - \gamma_n^b \psi_0^g \end{aligned} \tag{IC_0^g}$$

$$\begin{aligned} &\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b (1-\pi_1)\, t_l^b - \gamma_n^b \psi_1^b \\ &\geq (1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_0)\, t_l^g - \gamma_n^g \psi_1^b \end{aligned} \tag{IC_1^b}$$

$$\begin{aligned} &\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b (1-\pi_1)\, t_l^b - \gamma_n^b \psi_1^b \\ &\geq (1-\gamma_n^g)\, f^g + \gamma_n^g \pi_0 t_h^g + \gamma_n^g (1-\pi_0)\, t_l^g - \gamma_n^g \psi_0^b \end{aligned} \tag{IC_0^b}$$

$$(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g (1-\pi_1)\, t_l^g - \gamma_n^g \psi_1^g \geq 0 \tag{IR^g}$$

$$\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b (1-\pi_1)\, t_l^b - \gamma_n^b \psi_1^b \geq 0 \tag{IR^b}$$

Suppose f^g and f^b are strictly positive. Lower f^g to zero and raise t_h^g to keep $(1-\gamma_n^g)\, f^g + \gamma_n^g \pi_1 t_h^g$ constant. Also, Lower f^b to zero and raise t_h^b to keep $\left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b$ constant. The two IR, the (IC_1^g) and (IC_1^b) are unaffected. Since $\pi_1 > \pi_0$, the constraints (IC_0^g), (IC_0^b), (MH_b) and (MH_g) are relaxed.

Furthermore, the expected payment by the principal to a typical bidder i under the constructed transfers is

$$(1-q)\, \gamma_n^b \pi_1 \hat{t}_h^b + q \gamma_n^g \pi_1 \hat{t}_h^g.$$

Substituting the expressions for $\hat{t}_h^b$ and $\hat{t}_h^g$ we have the expected payment by the principal to a typical agent:

$$(1-q)\left[\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g\right]$$
$$+q\left[\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g\right]$$

which is the same as with the transfers $\left\{\left(f^g, t_h^g, t_l^g\right),\left(f^b, t_h^b, t_l^b\right)\right\}$. Thus, the principal is not worse off under the constructed transfers that pays only the winning bidder. This completes the proof of Claim 1. ■

Proof of Proposition 1. First note that under an efficient allocation rule the principal can either (i) allocate the contract to a good type over a bad type whenever there is one, or (ii) allocate the contract to a bad type over a good type whenever there is one. If the contract is awarded to a bad type with probability 1, whenever there is one, for $n>1$, we have $\gamma_n<1$. In this case, the IC_1^g and IC_1^b become

$$IC_1^g: \gamma_n \pi_1 t_h^g-\gamma_n \psi_1^g+\psi_1^g \geq \pi_1 t_h^b$$
$$IC_1^b: \pi_1 t_h^b \geq \gamma_n \pi_1 t_h^g-\gamma_n \psi_1^b+\psi_1^b$$

Putting these together we have

$$\gamma_n\left(\psi_1^b-\psi_1^g\right) \geq \psi_1^b-\psi_1^g.$$

Thus IC_1^g and IC_1^b cannot hold together when $\gamma_n<1$, i.e., the solution necessarily involves awarding the contract to a good type when there is one.

Now consider the reduced problem given in section 3. The principal chooses the two transfers $\{t_h^b, t_h^g\}$ to solve

$$\max \Pi_{11}(n)=\pi_1\left[V-\left(\delta_n t_h^g+\left(1-\delta_n\right) t_h^b\right)\right]$$

subject to,

$$\gamma_n\left(\pi_1 t_h^g-\psi_1^g\right) \geq \pi_1 t_h^b-\psi_1^g \qquad (IC_1^g)$$
$$t_h^g \geq \frac{\psi_1^g-\psi_0^g}{\pi_1-\pi_0} \qquad (MH_g)$$

$$\pi_1 t_h^b - \psi_1^b \geq \gamma_n \left(\pi_0 t_h^g - \psi_0^b\right) \tag{IC_0^b}$$

$$t_h^b \geq \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}. \tag{MH_b}$$

First, consider the restricted problem of maximizing

$$\pi_1 V - \left[\left(1-(1-q)^n\right)\pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]$$

subject only to the IC_1^g and IC_0^b constraints. Since the principal's payoff is decreasing in t_h^g and t_h^b, the inequality and the fact that the LHS of IC_1^g and the RHS of IC_0^b above are both increasing in t_h^g together imply that the solution of the reduced problem is given by

$$\begin{aligned}
t_h^g &= \frac{\gamma_n\left(\psi_1^g - \psi_0^b\right) + \left(\psi_1^b - \psi_1^g\right)}{\gamma_n\left(\pi_1 - \pi_0\right)} \\
t_h^b &= \frac{\gamma_n \pi_0}{\pi_1} t_h^g - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1} = \frac{\gamma_n\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1\left(\pi_1 - \pi_0\right)}
\end{aligned}$$

We will now show that for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the remaining constraints are satisfied: Substituting for t_h^g we have

$$\begin{aligned}
MH_g : t_h^g - \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} &= \frac{\psi_1^b - \psi_1^g - \gamma_n\left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right)\gamma_n} \\
&\geq \frac{\psi_1^b - \psi_1^g - \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right)\gamma_n} \\
&= 0
\end{aligned}$$

Substituting for t_h^b

$$MH_b : t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} = \left(\gamma_n - 1\right)\frac{\pi_0 \psi_1^g - \pi_1 \psi_0^b}{\pi_1\left(\pi_1 - \pi_0\right)} > 0$$

and under our assumption.

$$IC_1^b : \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n\left(\pi_1 t_h^g - \psi_1^b\right) = \left(\gamma_n + 1\right)\psi_1^g + \left(\gamma_n - 1\right)\psi_1^b > 0.$$

Next for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ consider the restricted problem of maximizing $\pi_1 V - \left[\left(1-(1-q)^n\right)\pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]$ subject only to the MH_g and IC_0^b constraints. Following similar arguments as above, the solution is given by

$$t_h^g = \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0}$$
$$t_h^b = \frac{\gamma_n \pi_0}{\pi_1} \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\gamma_n \psi_0^b - \psi_1^b}{\pi_1}$$

We will now show that all the remaining constraints are satisfied by this solution:

$$\begin{aligned}
IC_1^g : & \frac{1}{\pi_1} \left(\gamma_n \left(\pi_1 t_h^g - \psi_1^g\right) + \psi_1^g\right) - t_h^b \\
&= \frac{\psi_1^g - \psi_1^b + \gamma_n \left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\
&> \frac{\psi_1^g - \psi_1^b + \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g} \left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\
&= \frac{\psi_1^g - \psi_1^b + \psi_1^b - \psi_1^g}{\pi_1} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
IC_1^b : & \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n \left(\pi_1 t_h^g - \psi_1^b\right) \\
&= \gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b - \gamma_n \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \gamma_n \psi_1^b \\
&= \frac{\gamma_n}{\pi_1 - \pi_0} \left(\pi_1 - \pi_0\right) \left[\left(\psi_1^b - \psi_0^b\right) - \left(\psi_1^g - \psi_0^g\right)\right] \\
&> 0 \text{ by L(ii)}
\end{aligned}$$

$$\begin{aligned}
MH_b : & t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} \\
&= \frac{1}{\pi_1 - \pi_0} \left[\gamma_n \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b + \pi_0 \left(\psi_0^b - \psi_0^g\right)\right] + \psi_1^b \left(\pi_1 - \pi_0\right) - \pi_1 \left(\psi_1^b - \psi_0^b\right)\right]
\end{aligned}$$

(using L(iii) the coefficient of γ_n is positive, hence replacing γ_n by its minimum value)

$$\begin{aligned}
&\geq \frac{1}{\left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right)} \left[\begin{array}{c} \left(\psi_1^b - \psi_1^g\right) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b + \pi_0 \left(\psi_0^b - \psi_0^g\right)\right] \\ + \psi_1^b \left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right) - \pi_1 \left(\psi_1^b - \psi_0^b\right) \left(\psi_0^b - \psi_0^g\right) \end{array}\right] \\
&= \frac{1}{\left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right)} \left[\left(\psi_1^b - \psi_0^b\right) - \left(\psi_1^g - \psi_0^g\right)\right] \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) \\
&> 0 \text{ by L(ii) and L(iii)}
\end{aligned}$$

This completes the proof of Proposition 1. ■

Proof of Proposition 2. The expected payoff to the principal under a given contract is given by

$$\delta_n \left(\pi_{e_g} V - \pi_{e_g} t_h^g - \left(1 - \pi_{e_g}\right) t_l^g\right) + (1 - \delta_n)\, \pi_{e_b} V$$

which can be rewritten as

$$\delta_n \pi_{e_g} V + (1 - \delta_n)\, \pi_{e_b} V - \left[\delta_n \left(\pi_{e_g} t_h^g + \left(1 - \pi_{e_g}\right) t_l^g\right) + (1 - \delta_n) \left(\pi_{e_b} t_h^b + (1 - \pi_{e_b})\, t_l^b\right)\right]$$

First, notice that given a good type is favored in an auction and the effort levels assigned to the two types, the additive separability of V and the transfers imply that the principals expected payoff maximization problem is equivalent to her expected transfer minimization problem. Second in the minimization problem subject to the relevant IC, MH and IR constraints the minimum is well defined. Let the minimum expected transfer be defined for a given effort assignment e_g and e_b by $\tau(e_g, e_b)$.

The principal's problem is then to solve

$$\max_{e_g, e_b} \; \delta_n \pi_{e_g} V + (1 - \delta_n)\, \pi_{e_b} V - \tau(e_g, e_b).$$

Specifically, we need to identify the maximum among

$$\pi_1 V - \tau(1, 1), \;\; (\delta_n \pi_1 + (1 - \delta_n)\, \pi_0)\, V - \tau(1, 0), \text{ and } \pi_0 V - \tau(0, 0)$$

It follows that for V large enough the maximum is given by setting $e_g = 1, e_b = 1$ since $\pi_1 > (\delta_n \pi_1 + (1 - \delta_n)\, \pi_0) > \pi_0$ and for large enough V we have

$$\pi_1 V - (\delta_n \pi_1 + (1 - \delta_n)\, \pi_0)\, V > \tau(1, 1) - \tau(1, 0)$$

and

$$(\delta_n \pi_1 + (1 - \delta_n)\, \pi_0)\, V - \pi_0 V > \tau(1, 0) - \tau(0, 0).$$

This completes the proof of Proposition 2. ■

Proof of Proposition 3. The expected payoff to the principal is

$$\begin{aligned} &(1 - (1 - q)^n)\, \pi_1 \left(V - t_h^g\right) + (1 - q)^n\, \pi_1 \left(V - t_h^b\right) \\ &= \pi_1 V - \left[(1 - (1 - q)^n)\, \pi_1 t_h^g + (1 - q)^n\, \pi_1 t_h^b\right] \end{aligned}$$

For $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the payoff at the optimal transfers is

$$\Pi(n) = \pi_1 V - \left(\begin{array}{c} \left(1-(1-q)^n\right) \pi_1 \frac{\left(\gamma_n \psi_g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b\right)}{(\pi_1 - \pi_0)\gamma_n} \\ +(1-q)^n \pi_1 \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1(\pi_1 - \pi_0)} \end{array} \right)$$

and for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ it is

$$\Pi(n) = \pi_1 V - \left(\left(1-(1-q)^n\right) \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + (1-q)^n \left(\gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b + \psi_1^b \right) \right)$$

We have for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$

$$\Pi(n+1) - \Pi(n) = \frac{(1-q)^{n-1}}{\pi_1 - \pi_0} \left(\pi_1 \left(\psi_0^b (1-q) + \left(\psi_1^b - \psi_1^g\right) q \right) - \pi_0 \psi_1^g (1-q) \right)$$

Thus, $\Pi(n)$ is *decreasing* for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ if and only if

$$\pi_1 \left(\psi_0^b (1-q) + \left(\psi_1^b - \psi_1^g\right) q \right) - \pi_0 \psi_1^g (1-q) < 0$$

or,

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right]$$

For $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ we have

$$\Pi(n+1) - \Pi(n) = \frac{(1-q)^n}{\pi_1 - \pi_0} \left(\begin{array}{c} \pi_1 \left(\psi_0^b - \psi_0^b q + \left(\psi_0^g + \psi_1^b - \psi_1^g\right) q \right) \\ -\pi_0 \left(\psi_0^b + \psi_1^g + \psi_0^g (-1+q) - \psi_0^b q + \psi_1^b q - \psi_1^g q \right) \end{array} \right)$$

In this case $\Pi(n)$ is *decreasing* if and only if

$$\left(\pi_1 \left(\psi_0^b - \psi_0^b q + \left(\psi_0^g + \psi_1^b - \psi_1^g\right) q \right) - \pi_0 \left(\psi_0^b + \psi_1^g + \psi_0^g (-1+q) - \psi_0^b q + \psi_1^b q - \psi_1^g q \right) \right) < 0$$

or, simplifying,

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right] + (1-q) \left(\psi_0^b - \psi_0^g\right) \pi_0 + q \left(\pi_0 \psi_1^b - \pi_1 \psi_0^g\right)$$

This completes the proof of Proposition 3. ■

Proof of Corollary 1. The proof follows from proposition 3 upon observing that condition L implies $\pi_0\psi_1^b - \pi_1\psi_0^g \geq 0$ and $\psi_0^b > \psi_0^g$, so

$$(1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0\psi_1^b - \pi_1\psi_0^g\right) \geq 0.$$

In this case, if

$$\pi_1 q\left(\psi_1^b - \psi_1^g\right) > (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] + (1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0\psi_1^b - \pi_1\psi_0^g\right)$$

holds, so does

$$\pi_1 q\left(\psi_1^b - \psi_1^g\right) > (1-q)\left(\pi_0\psi_1^g - \pi_1\psi_0^b\right).$$

This completes the proof of Corollary 1. ■

Proof of Proposition 4. To find the solution, the principal maximizes her expected payoff subject to the same constraints as in section 3 plus the additional constraints (2), (3), and (4), but choosing not only the transfers but also (γ_n^b, γ_n^g). The main change from the analysis of the problem in section 3 is that the principal is now free to consider inefficient allocation rules. This is done by removing the restriction that $qn\gamma_n^g = 1-(1-q)^n$, i.e., the first constraint (2) must hold as an equality. Indeed, if the solution involves $qn\gamma_n^g < 1-(1-q)^n$, the optimal allocation rule must necessarily be inefficient.[43]

Since the principal must allocate the contract to some agent with probability 1, we have $\delta_n^g + \delta_n^b = 1$, or condition (4) is binding:

$$qn\gamma_n^g + (1-q)\,n\gamma_n^b = 1 \tag{8}$$

In addition, we know IC_1^g and IC_1^b imply the monotonicity condition:

$$\gamma_n \geq 1. \tag{9}$$

Under allocative efficiency, monotonicity is implied and (9) is redundant, but it is relevant when inefficient allocation rules are allowed.

[43]There must be some $\phi_r^g < 1$ for some r, i.e., a bad type would have a chance of being awarded the contract even when a good type is present.

Since the results of proposition 1 do not depend on the restriction $qn\gamma_n^g = 1 - (1-q)^n$, for every feasible $\left(\gamma_n^g, \gamma_n^b\right)$, i.e., those satisfying conditions (2), (3), (8) and (9), the optimal transfers are given as before in cases (I) and (II) by,

$$t_h^g = \frac{\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b}{(\pi_1 - \pi_0)\gamma_n} \tag{I}$$
$$t_h^b = \frac{\gamma_n\left(\pi_0\psi_1^g - \pi_1\psi_0^b\right) + \pi_1\psi_1^b - \pi_0\psi_1^g}{\pi_1(\pi_1 - \pi_0)}.$$

if $\gamma_n \leq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ and

$$t_h^g = \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{II}$$
$$t_h^b = \frac{\gamma_n \pi_0}{\pi_1}\frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1}.$$

if $\gamma_n > \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$.

When determining the solution, we can ignore condition (3) since it is implied by (8) and (9). However, we must pay attention to the conditions under which the optimal transfers change from case I to II depending on γ_n according to:

$$\gamma_n \leq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g} \tag{10}$$
$$\gamma_n \geq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}. \tag{11}$$

Next, using the binding constraint (8), we have $\gamma_n^b = \frac{1 - qn\gamma_n^g}{(1-q)n}$, and we can rewrite the constraints (2), (9), (10) and (11) as a function of γ_n^g only:

$$\gamma_n^g \leq \frac{1 - (1-q)^n}{nq} \tag{12}$$
$$\gamma_n^g \geq \frac{1}{n} \tag{13}$$

$$\gamma_n^g \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \tag{14}$$
$$\gamma_n^g \geq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \tag{15}$$

It will be convenient to know when the upper bound of γ_n^g is given by (12) rather than (14). We compare the RHS of the two constraints $\frac{1-(1-q)^n}{nq} \leq \frac{\psi_1^b-\psi_1^g}{n\left((1-q)\left(\psi_0^b-\psi_0^g\right)+q\left(\psi_1^b-\psi_1^g\right)\right)}$ and define $\hat{n}$ to be the largest integer such that (12) is more restrictive:[44]

$$\frac{1-(1-q)^n}{(1-q)^{n-1}q} \leq \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}. \tag{16}$$

Thus, for $n \leq \hat{n}$, the upper bound of γ_n^g is given by $\frac{1-(1-q)^n}{nq}$.

Comparing the RHS of (13) and (15), we can verify that the lower bound on γ_n^g is given by the monotonicity condition (13) since:

$$\frac{1}{n} < \frac{\psi_1^b-\psi_1^g}{n\left((1-q)\left(\psi_0^b-\psi_0^g\right)+q\left(\psi_1^b-\psi_1^g\right)\right)},$$

given condition $L(ii)$. However, when the case II contract is optimal, the constraint (13) is redundant given (15).

Small n ($n \leq \hat{n}$)

We start by looking at the case for small n, with $n \leq \hat{n}$, i.e., γ_n^g satisfying (13). We know that the case I contract is optimal, and that constraint (14) is redundant. Consider the principal's expected payoff maximization under (12) and (13), i.e., $\gamma_n^g \in \left[\frac{1}{n}, \frac{1-(1-q)^n}{nq}\right]$. Substituting the optimal case I transfers, the principal's expected payoff is given by

$$\begin{aligned} &nq\gamma_n^g\pi_1\left(V-\frac{\gamma_n\psi_1^g-\psi_1^g-\gamma_n\psi_0^b+\psi_1^b}{(\pi_1-\pi_0)\gamma_n}\right) \\ &+n(1-q)\gamma_n^b\pi_1\left(V-\frac{\gamma_n\left(\pi_0\psi_1^g-\pi_1\psi_0^b\right)+\pi_1\psi_1^b-\pi_0\psi_1^g}{\pi_1(\pi_1-\pi_0)}\right), \end{aligned}$$

and, after substituting $\gamma_n^b = \frac{1-qn\gamma_n^g}{(1-q)n}$, the objective function becomes

$$\begin{aligned} &\gamma_n^g\left(nq\pi_1\left(V-\frac{\psi_1^g-\psi_0^b}{(\pi_1-\pi_0)}\right)-n(1-q)\pi_1\frac{\left(\pi_0\psi_1^g-\pi_1\psi_0^b\right)}{\pi_1(\pi_1-\pi_0)}\right) \\ &+\frac{1-qn\gamma_n^g}{(1-q)n}\left(n(1-q)\pi_1\left(V-\frac{\pi_1\psi_1^b-\pi_0\psi_1^g}{\pi_1(\pi_1-\pi_0)}\right)-nq\pi_1\frac{\psi_1^b-\psi_1^g}{(\pi_1-\pi_0)}\right). \end{aligned}$$

[44] Recall that under efficient allocation γ_n is equal to the LHS of (16). Thus, the inequality corresponds to the inequality that defines Case I in Proposition 1.

Taking derivative with respect to. γ_n^g we have:

$$
\begin{aligned}
& -\frac{nq\pi_1\psi_1^g - n\pi_1\psi_0^b - nq\pi_1\psi_1^b + n\pi_0\psi_1^g}{\pi_1 - \pi_0} \\
& + \frac{1}{(1-q)}\frac{nq^2\pi_1\psi_1^b - nq^2\pi_1\psi_1^g}{(\pi_1 - \pi_0)},
\end{aligned}
$$

which is positive if and only if,

$$
\pi_1 q\left(\psi_1^b - \psi_1^g\right) > (1-q)\left(\pi_0\psi_1^g - \pi_1\psi_0^b\right). \tag{17}
$$

Thus, for small $n \leq \hat{n}$, revenue is increasing in γ_n^g if (17) holds, and the optimal γ_n^g is at the highest extreme given by (12), $\gamma_n^g = \frac{1-(1-q)^n}{nq}$. In this solution, we have allocative efficiency**.** If the condition (17) does not hold, the principal's revenue is decreasing in γ_n^g and the optimal γ_n^g is given by the binding monotonicity condition at the lowest extreme of (13), where $\gamma_n^g = \frac{1}{n}$. The optimal allocation rule is inefficient.

Recalling that under efficient allocation γ_n is equal to the LHS of (16), it should not come as a surprise that the condition (17), for the revenue to be increasing in γ_n^g for $n \leq \hat{n}$, is identical to the condition for the principal's profit to be increasing in n under an efficient allocation rule in case I, in proposition 3. Thus, for $n \leq \hat{n}$, if competition hurts under efficient allocation, the solution involves an inefficient allocation rule setting $\gamma_n^g = \gamma_n^b = \frac{1}{n}$, i.e., to randomly allocate (with equal probability) to one of the n bidders regardless of the announced type.

Large n $(n > \hat{n})$

Consider next the case of large $n > \hat{n}$, where condition (15) is now relevant as it is no longer ruled out by (12). Thus, we have to consider both case I and II contracts as γ_n^g can satisfy both (14) and (15). Note that the case when γ_n^g satisfies (14) is identical to the case of small $n \leq \hat{n}$. Therefore, we now consider the principal's expected payoff maximization under (15), such that the case II contract is optimal, and the lower bound on γ_r^g is given by (12), i.e., $\gamma_n^g \in \left[\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}, \frac{1-(1-q)^n}{nq}\right]$.[45] Substituting the

[45] Recall that under (15) inequality (13) becomes redundant.

optimal case II transfers under the constraint $\gamma_n \geq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ the principal's expected payoff is given by:[46]

$$nq\gamma_n^g \pi_1 \left(V - \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \right) + n(1-q)\gamma_n^b \pi_1 \left(V - \frac{\gamma_n \pi_0}{\pi_1} \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1} \right),$$

and, after substituting $\gamma_n^b = \frac{1 - qn\gamma_n^g}{(1-q)n}$, the expected payoff becomes

$$\left[nq\pi_1 V - nq\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - n(1-q)\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + n(1-q)\psi_0^b \right] \gamma_n^g$$
$$+ \left[n(1-q)\pi_1 V - n(1-q)\psi_1^b \right] \frac{1 - qn\gamma_n^g}{(1-q)n}.$$

Taking derivative with respect to. γ_n^g we have

$$\left[nq\pi_1 V - nq\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - n(1-q)\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + n(1-q)\psi_0^b \right]$$
$$- \left[n(1-q)\pi_1 V - n(1-q)\psi_1^b \right] \frac{qn}{(1-q)n},$$

and, it is positive if and only if

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) > (1-q)\left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right] + (1-q)\pi_0 \left(\psi_0^b - \psi_0^g\right) + q\left(\pi_0 \psi_1^b - \pi_1 \psi_0^g\right). \quad (18)$$

Thus, when (18) holds, revenue increases with γ_n^g, and the optimal γ_n^g is at the highest extreme given by (12), with $\gamma_n^g = \frac{1-(1-q)^n}{nq}$, and we have allocative efficiency. On the other hand, when (18) is violated, revenue decreases with γ_n^g and the optimal γ_n^g is at the lowest extreme given by (14), with $\gamma_n^g = \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ (i.e., $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$). Then, the optimal allocation rule is inefficient.

Again, not surprisingly, the condition (18), for the revenue to increase with γ_n^g, is identical to the condition for revenue to increase in n under efficient allocation in case II, in proposition 3. Also, as noted in section 3, condition (18) implies (17).

Therefore, if (17) is violated, competition hurts under efficient allocation for all n, and the optimal $\gamma_n^g = \frac{1}{n}$, with $n^* = 1$. The principal's payoff is equal to the payoff under a single agent.

[46] We include $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ in both cases so that the maximums are well defined.

If condition (17) is satisfied but not (18), competition helps under efficient allocation for $n \leq \hat{n}$ but hurts for larger n. Then, the optimal γ_n^g is equal to $\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$, with n^* solving $\gamma_n \approx \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$. The allocation rule is efficient for $n \leq \hat{n}$ but inefficient for larger n. In this case $1 < n^* < \infty$. [47]

If condition (18) holds, so does (17). Then, competition helps under efficient allocation for all n, the optimal $\gamma_n = \frac{1-(1-q)^n}{nq}$ and allocation is efficient for all n, with $n^* = n$. Thus, the (asymptotic) maximum payoff is that corresponding to $n^* = \infty$ with an efficient allocation rule.

This completes the proof of Proposition **4**. ■

Proof of Corollary 2: When $n^* = 1$, $n > n^*$ implies $n > \hat{n}$ (where $\hat{n}$ is defined in the proof of proposition 4). By proposition 4 the optimal $\gamma_n^g = \frac{1}{n}$ which corresponds to $\phi_r^g = \frac{r}{n}$.

When $n^* = \infty$ then for all n the efficient allocation is optimal so that the optimal $\phi_r^g = 1$ for $r \geq 1$.

If $1 < n^* < \infty$ and $n \leq n^*$ then the efficient allocation is still optimal by proposition 4 which means the optimal $\phi_r^g = 1$ for $r \geq 1$. If $n > n^*$ then the optimal allocation is inefficient and we want to find ϕ_r^g-s that give

$$\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{\phi_r^g}{r}$$

From efficient allocation rules we have

$$1 = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \left(\frac{nq}{1-(1-q)^n}\right)$$

then multiplying the LHS of the second equality on both sides we have

$$\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \left(\frac{nq}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$$

[47] If $\hat{n}$ satisfies the (16) as equality then the optimal expected payoff is equal to the efficient allocation payoff for $n = n^*$, but if $\hat{n}$ satisfies the (16) as strict inequality then optimal expected payoff is higher than the efficient allocation payoff at n^*.

Let

$$\phi_r^g = \left(\frac{nq}{1-(1-q)^n}\right)\frac{\psi_1^b-\psi_1^g}{n\left((1-q)\left(\psi_0^b-\psi_0^g\right)+q\left(\psi_1^b-\psi_1^g\right)\right)}$$

ϕ_r^g-s are well-defined allocation probabilities if $\phi_r^g \leq 1$. So it is enough to show that

$$\left(\frac{nq}{1-(1-q)^n}\right)\frac{\psi_1^b-\psi_1^g}{n\left((1-q)\left(\psi_0^b-\psi_0^g\right)+q\left(\psi_1^b-\psi_1^g\right)\right)} < 1$$

or,

$$\frac{\psi_1^b-\psi_1^g}{n\left((1-q)\left(\psi_0^b-\psi_0^g\right)+q\left(\psi_1^b-\psi_1^g\right)\right)} < \frac{(1-(1-q)^n)}{nq}$$

which is true for $n > \hat{n}$ by definition of $\hat{n}$. Therefore, ϕ_r^g defined above gives the allocation that implements the allocation probability in this case. This completes the proof of Corollary 2. ■

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