Competitive Procurement with Ex Post Moral Hazard[1]

Indranil Chakraborty
indro@nus.edu.sg
National University of Singapore

Fahad Khalil
khalil@uw.edu
University of Washington and ERG, Dhaka

Jacques Lawarree
lawarree@uw.edu
University of Washington and ECARES, Brussels

August 5, 2020

Abstract: Unlike standard auctions, we show that competitive procurement may optimally limit competition or use inefficient allocation rules that award the project to a less efficient firm with positive probability. Procurement projects often involve ex post moral hazard after the competitive process is over. A procurement mechanism must combine an incentive scheme with the auction to guard against firms bidding low to win the contract and then cutting back on effort. While competition helps reduce the rent of efficient firms, it exacerbates the problem due to moral hazard. If allocative efficiency is a requirement, limiting the number of participants may be optimal. Alternatively, the same incentives can be optimally provided using inefficient allocation rules.

[1]We thank Pat Bajari, Dirk Bergemann, Roberto Burguet, Yeon-Koo Che, Hae Yun Choung, Jacques Crémer, Dong-Jae Eun, Andrew Foster, Marina Halac, Do-Shin Jeon, George Mailath, Claudio Mezzetti, Preston McAfee, Alessandro Pavan, Patrick Rey, David Sappington, Guofu Tan, Xu Tan, Jean Tirole, Richard Van Weelden and John Wooders for many helpful comments. The paper benefitted greatly from detailed comments by Mark Armstrong and two anonymous referees. Chakraborty acknolowdges financial support from Singapore MOE research grants numbers R-122-000-254-115 and R-122-000-283-115 and generous hospitality from Department of Economics, University of Washington.

1 Introduction

Government agencies and private firms routinely rely on competitive procurement to obtain goods, services, or to complete projects. The OECD estimates that its members spend 12.1% of their GDP on public procurement.[2] It has long been known that the benefit of competitive procurement is to ensure productive efficiency and low cost by selecting the most efficient firm and reducing information rent (see, e.g., Demsetz (1968)). While it is particularly effective for standard items like office supplies, many procurement projects also involve additional work after the competitive process is over. This is the case for construction projects like highway procurement. Such expenditures form a large part of procurement spending.[3] Procurers often use incentive schemes to improve ex post performance of selected firms. For example, in highway construction, incentives schemes are used to motivate timely completion of projects.[4] Firms must then estimate the value of these schemes when competing for the procurement project. Optimal incentive schemes typically require offering ex post rent to the winning firm. However, this begs the question whether ex ante competitive procurement leaves enough rent to ensure ex post performance by the selected firm.[5]

In this article, we study how the ex ante competitive process interferes with the ex post moral hazard problem.[6] We find that competition can be a mixed blessing for the procurer who insists on allocative efficiency. Indeed, allocative efficiency, where the

[2] The number is even higher for developing countries and the World Bank estimates it to be at 14.5% of the GDP for low-income countries (Djankov, Saliola and Islam (2016)).

[3] Expenditure on service and construction contracts often constitute a large fraction of government procurement, accounting for 273 out of 277 billion dollars spent on the top 10 non-defense spending categories in the 2013 U.S. federal budget. http://www.govexec.com/contracting/2015/01/10-categories-where-federal-agencies-spend-most-contracting/102498/

[4] See Lewis and Bajari (2011, 2014).

[5] There is evidence that it may not. In the U.S., the introduction of an experimental competitive bidding program by Medicare had a negative impact on the quality of distribution service for diabetic products (Puckrein et al. (2016)). Reporting on competitive procurement for elderly care by the U.K. National Health Service, The Guardian newspaper notes that companies “bid low to win contracts and then cut back on quality to meet their profit targets.” (Leys, “NHS contracting has been a disaster,” April 22, 2014, *The Guardian.*)

[6] Surprisingly, this crucial aspect of procurement has not been received much attention in the literature. In their survey on public contracting, Armstrong and Sappington (2007) stress the importance of unobservable quality when they discuss competitive procurement and note that relatively little work has been done in the topic. We discuss the literature in more detail below.

most efficient firm must be selected, is a key requirement in many procurement settings.[7] Increased competition may hurt a procurer if allocative efficiency is a requirement. The optimal mechanism features allocative efficiency with a limit on the number of bidders. If the procurer cannot limit the number of bidders, she must give up on allocative efficiency to neutralize the negative impact of competition by relying on a scheme that randomly allocates the project to a less efficient firm.

There is evidence of such procedures in the U.S. and abroad. The EU Commission (2015) recommends using a "restricted procedure" with only a subset of potential providers invited to submit tenders, "where there is a high degree of competition (several potential tenderers) in the marketplace." Bajari et al. (2014) found that contracts were not allocated to the lowest bidder in nearly 4% of the California Department of Transportation first-price auctions.[8]

To study the tension between ex ante rent extraction and ex post performance, we consider a model of competitive procurement with ex post moral hazard in an optimal auction framework (Myerson (1981)). In the initial stage, each agent or firm is asked to report its cost of production (cost of effort in our model). Suppose that the procurer then selects the most efficient firm based on the reports. The selected firm must then exert costly effort to complete the project. Both effort and the cost of effort are private information of the firm. Thus, we study a mixed model with both moral hazard and adverse selection.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition. Both downward and upward incentive constraints can be binding: instead of the standard problem of firms wanting to overstate cost, firms may now also want to understate cost and shirk. While competition is known to be an effective tool to

[7] For instance, a key goal of the FCC is to promote efficient access to and use of the radio spectrum (FCC Spectrum Policy Task Force, https://www.fcc.gov/sptf/files/SEWGFinalReport_1.doc). FCC Chairman William E. Kennard (1999) notes that efficiency in the FCC spectrum auctions means that spectrum ends up in the hands of those who value it most highly.

[8] Citing examples from various countries, Eun (2019) studies a Korean procurement mechanism that has a stochastic cutoff rule to eliminate the lowest bids. With a counterfactual analysis, he also shows that this rule lowers social cost by 7% relative to a standard first-price auction. Split award auctions may allocate part of the project to a less efficient firm to retain future suppliers or promote ex ante investments (Anton and Yao (1989, 1992)). In our model, we assume that the project is allocated to one firm so split awards cannot be used to address moral hazard.

address the problem of overstating cost, we show that it can exacerbate the problem of understating cost while shirking.[9] Because his chance of being selected decreases with competition, a less efficient firm will have to be given a higher information rent. This negative effect of competition can overtake its benefits if the impact of moral hazard is strong enough. We find that high levels of competition may not be beneficial to the principal because the benefit from rent extraction is dominated by the cost of sustaining high ex post effort.

Limiting the number of competing firms can then be optimal for the procurer. When that is not possible,[10] we show that the procurer can effectively neutralize the negative impact of competition with an inefficient allocation rule. That is, the procurer may not allocate the project to the most efficient firm with probability one. We find that giving up on allocative efficiency relaxes the incentive compatibility constraints of less efficient firms who now have a higher chance to win the project by telling the truth. Because those constraints are what restricts the effectiveness of competition in extracting rent, an inefficient allocation rule can remove the negative impact of competition. Specifically, we show that an inefficient allocation rule is optimal and it allows the principal to mimic a mechanism with an efficient allocation rule where she can choose the number of firms. Then, increased competition no longer affects her payoff, and the procurer cannot reap any further benefit from competition even with an inefficient allocation rule.[11]

The literature on competitive bidding for procurement contracts goes back to the late eighties when a set of influential articles analyzed properties of incentive schemes that governed ex post incentives of the selected firm.[12] Highlighting a separation property, they showed how standard auction formats can be used to extract rent while providing second-best incentives at the same time. In these models, competition has no negative effect. The driving force behind the results in these articles is adverse selection rather than moral hazard. In Riordan-Sappington (1987), the quality is observable, so

[9] This is reminiscent of Hart-Schleifer-Vishny (1997), who argue that a private contractor's incentive to engage in cost reduction may be too strong because of the negative impact on noncontractible quality.

[10] For example, because it may appear as corruption.

[11] Thus, when competition is beneficial, we show that restricting attention to allocative efficiency is without loss of generality.

[12] See, e.g., Riordan-Sappington (1987), McAfee-McMillan (1986, 1987) or Laffont-Tirole (1987).

there is no moral hazard. While McAfee-McMillan (1987) and Laffont-Tirole (1987) have an unobservable effort, agents are risk neutral with unlimited liability, and the principal can deduce the effort once the agent has revealed his type. In the terminology of Laffont-Martimort (2002), these are "false moral hazard" models, where upward incentive constraints are not binding: high-cost firms do not want to pretend to be low cost. An increase in competition can only benefit the principal.

Like us, McAfee-McMillan (1986) have a true mixed model with both adverse selection and moral hazard. However, they do not study the optimal contract but rather a linear contract that balances the cost-plus contract and the fixed-price contract. The linear cost-sharing parameter is assumed to be independent of the agent's type. Thus, the optimal choice of effort is independent of types, which implies that the upward incentive constraints are not binding. Our contribution is to solve the optimal auction mechanism in a tractable mixed model, and to show that insisting on allocative efficiency can lead to competition being harmful.[13]

In a recent article, but in a contest setting with an all-pay feature, Che-Iossa-Rey (2017) find that it may not always be optimal to allocate the project to the most efficient firm ex post in order to provide an incentive to exert effort ex ante. Firms have private information on implementation cost and effort comes *before* being selected.[14] In a procurement setting, others have also highlighted allocative inefficiency in models where quality is exogenous. In Manelli-Vincent (1995), firms have private information about the exogenous quality they can produce, leading to a lemons problem where the least cost agent also generates the least value to the principal. Burguet-Ganuza-Hauk (2012) provide a related analysis where firms have private information about their

[13] Piccione and Tan (1996) have shown the separation property, mentioned above, may not hold when bidders make an ex ante investment. Their focus was on the role of the R&D technology (in particular whether it exhibits decreasing returns to scale or not) in the implementation of the project.

Kogan and Morgan (2010) compare, both theoretically and experimentally, debt and equity auctions followed by moral hazard. They don't study the impact of competition. They find allocative efficiency to be optimal in both auctions.

[14] Taylor (1995) and Fullerton-McAfee (1999) have shown that admitting too many contestants in a research tournament reduces the ex ante effort of each contestant because their probability of winning becomes too low. See Bénabou-Tirole (2016) for the effect of competition on ex post incentive in a Hotelling model. Montagnes-Van Weelden (2019) discuss the benefit of curtailing competition in the presence of polarized bidders.

financial status. In Chillemi-Mezzetti (2014), the winning bidder privately discovers the value of the cost overruns during the project's completion. All these articles find allocative inefficiency to be optimal in models with exogenous quality or ex ante effort. Our objective, instead, is to focus on the effect of competition in the presence of ex post moral hazard that are observed in many private and public procurement situations.

The literature on scoring auctions is also relevant but, again, it typically assumes that quality is observable (see, e.g., Che (1993), and Asker-Cantillon (2010)) and there is no moral hazard. Contractual externality across agency problems also plays an important role in dynamic procurement models such as Arve and Martimort (2016). Our article is also related to the literature that combines adverse selection and moral hazard (such as Picard (1987), Ollier-Thomas (2013), and Gottlieb-Moreira (2019)). Those articles are single agent model without an optimal auction. Recently, the empirical literature has also stressed the importance of moral hazard in procurement auctions (see for instance Lewis-Bajari (2011, 2014)).

The article is organized as follows. We present the model in Section 2 and the principal's problem in Section 3. In Sections 4 and 5, we present our main results. In Section 4, we derive the impact of competition assuming allocative efficiency. In Section 5, we derive the optimal mechanism without the restriction of allocative efficiency, and we find conditions when an inefficient allocation rule is optimal. In Section 6, we study the case of continuous effort and show that our key results continue to hold when the principal can screen using different efforts for each type.

2 The Model

A principal (she) must select one of n agents (he) bidding to complete an indivisible project. The success of the project depends on the selected agent's effort, and agents have different costs of effort. The selected agent privately chooses effort $e \in \{0, 1\}$. With probability π_e, the output is high, h, and the principal receives $V > 0$, while, with probability $(1 - \pi_e)$, the output is low, l, and she receives zero. The output is publicly observed. The low output may capture a variety of outcomes such as costly delays. For example, incentive schemes based on time of completion are used in highway projects (Lewis-Bajari (2011, 2014)). While the time of completion is observable, it depends on the contractor's unobservable effort and random shocks. This corresponds well to a moral hazard framework where contractors can put in extra effort to reduce the chance of negative shocks (for instance, better planning and maintenance to prevent equipment failure). We assume that $0 < \pi_0 < \pi_1 \leq 1$. We also assume type-independent probabilities to focus on the effect of competition to screen the agents and extract rent.[15]

The cost of effort is privately known to the agent, and an agent can be one of two types, $x \in \{g, b\}$.[16] It is commonly known that the probability that an agent is of type g is given by $q \in (0, 1)$. With both effort and cost of effort private information of an agent, we have a model with both moral hazard and adverse selection.

Denoting the cost of effort by ψ_e^x, we assume the following conditions about the cost of effort.

Condition L. (i) $\psi_1^b > \psi_1^g$ and $\psi_0^b > \psi_0^g \geq 0$, (ii) $\psi_1^b - \psi_0^b > \psi_1^g - \psi_0^g > 0$, and (iii) $\psi_1^g \geq \frac{\pi_1}{\pi_0}\psi_0^b$.

The first condition ranks the cost of effort and determines that "g" is a good type with a lower cost of effort. The second condition is akin to a standard single-crossing property that the marginal cost of effort is higher for the bad type. The third condition simplifies the exposition and captures the intensity with which ex post moral hazard interferes with rent extraction. Specifically, it creates an incentive for a bad type to mimic a good type by exerting low effort ($e = 0$). The condition $L(iii)$ requires that $\pi_0 > 0$, which means that the agent has a chance to succeed even when supplying low effort. The larger the π_0, the more serious the moral hazard problem. We discuss the implications of relaxing condition L after Proposition 1.

Agents are assumed to have a zero outside option, and they also have limited liability, such that the transfers from the principal are non-negative.[17] The principal's ex post

[15] As we will show, if the principal wants to induce high effort by both types, she would not be able to screen agents unless there is competition.

[16] We show in Chakraborty, Khalil, and Lawarree (2019) that our key results continue to hold in a model with a continuum of types.

[17] Limited liability makes our moral hazard problem relevant with risk neutral agents. Alternatively, we could have assumed risk aversion without limited liability.

payoff is the output net of paid transfers. The ex post payoff for an agent is the transfer from the principal net of effort cost.

When we derive the optimal procurement mechanism, we assume that agents with identical costs of effort are treated symmetrically in terms of transfers and probability of being selected. By the Revelation Principle (see Myerson (1981)), we can restrict ourselves to truth-telling direct mechanisms. In the game that follows, we restrict ourselves to symmetric Perfect Bayes-Nash equilibria. The mechanism proceeds along the following timeline:

Stage 1. The principal announces the mechanism, for type $x \in \{g, b\}$:

$$\{t_l^x(n), t_h^x(n), e_x(n), \phi_r^x(n)\},$$

where, t_l^x, t_h^x and e_x are the output-based transfers and efforts for type x, and $\phi_r^x(n)$ is the allocation rule, i.e., the probability of allocating the contract to a type x agent if r agents report type g. Because the principal must allocate the contract to one of the n agents, we have $\phi_r^g(n) + \phi_r^b(n) = 1$. To save on notation, we will suppress n when presenting terms in the mechanism.

Stage 2. The agents report their types and the contract is allocated to an agent according to the allocation rules set in stage 1.

Stage 3. The selected agent chooses the privately observable effort.

Stage 4. The output is realized and payments are made accordingly.

It is without loss of generality to (i) restrict the transfers and efforts to depend only on an agent's own report; (ii) assume that only the winning agent is paid in the mechanism. The first point follows from the types being independent. In the Appendix A, we prove the second point as a preliminary claim.

The mechanism has two parts: (i) an allocation rule that selects an agent based on the type announcements; (ii) an incentive contract that gives incentives to the selected agent to exert effort in completing the project. In Stage 2, when one agent is selected among the n competitors, there is adverse selection as the competing agents' types are unobservable. In Stage 3, there is moral hazard as the winning agent's effort is unobservable.

Allocation rule: The allocation rule pins down the probability that an agent will win the contract upon reporting type x, which we denote by γ_n^x. As is well known, it is convenient to write and solve the principal's problem in terms of γ_n^x.[18] To see how γ_n^x is computed, consider the case when all agents report truthfully, and $r \geq 1$ agents report type g :

$$\gamma_n^g = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{\phi_r^g}{r}. \tag{1}$$

The probability that a good type is selected is $nq\gamma_n^g$. For $r = 0$, we have $\phi_r^g = 0$.

For the solution (γ_n^b, γ_n^g) to correspond to a well defined (implementable) allocation rule (ϕ_r^g, ϕ_r^b), it must satisfy certain conditions:[19]

$$qn\gamma_n^g \leq 1 - (1-q)^n \tag{2}$$

$$(1-q)\, n\gamma_n^b \leq 1 - q^n \tag{3}$$

$$qn\gamma_n^g + (1-q)\, n\gamma_n^b = 1. \tag{4}$$

The first two conditions ensure that the probability that type x is chosen is no greater than the probability that there is a type x. The third condition ensures that the probability the contract is allocated at all can be no greater than one. Because we assume that the contract must be allocated to an agent, it holds as an equality.

Allocative efficiency: As mentioned in the introduction, in many procurement settings, allocative efficiency is a requirement. We impose the restriction that the mechanism is allocatively efficient in Section 4 but remove it in Section 5. Allocative efficiency requires that the principal allocates the contract to an agent who reports to be a good type, i.e., $\phi_r^g = 1$ for $r \geq 1$. Thus, under allocative efficiency, if all agents report

[18] See, e.g., Maskin and Riley (1984), or Fudenberg and Tirole (1991).

[19] Border (1991) proves that these conditions are also sufficient.

truthfully, we can replace $\phi_r^g = 1$ in (1):

$$\begin{aligned}\gamma_n^g &= \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \\ &= \frac{1}{nq} \sum_{r=1}^{n} \frac{n!}{(n-r)!r!} q^r (1-q)^{n-r} \\ &= \frac{1}{nq} \left(1-(1-q)^n\right) \end{aligned} \tag{5}$$

and, we have

$$\gamma_n^b = \frac{1}{n} (1-q)^{n-1}.$$

Thus, for $r \geq 1$, the condition (2) is binding and (3) is slack. However, as we will show in Section 5, an efficient allocation rule may not be optimal. We study the case of possibly inefficient allocation rules in Section 5 by allowing for $\phi_r^g \in [0,1]$ and show that $\phi_r^g < 1$ can be optimal for at least some $r \geq 1$.

3 The Principal's Problem

Our goal is to explore the effect of competition between agents on the principal's payoff, and we start by assuming that the principal wants to implement high effort levels and participation by both types of agent for expositional reasons. Later we specify conditions under which inducing high efforts $e_g = 1, e_b = 1$ are indeed optimal. In Section 6 on screening with effort, we consider the case of continuous effort and show that our key results continue to hold when the principal can induce different efforts for each type.[20]

We begin our analysis by clarifying the feasible set of contracts starting with the constraints that induce high effort by the selected agent. Given truthful reports of types, the optimal contract has to satisfy the following moral hazard constraints for

[20] In that extension section, we also note that our results hold in the binary case when inducing efforts $e_g = 1, e_b = 0$.

each type of agent:

$$\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0)t_l^g - \psi_0^g \tag{MH_g}$$

$$\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0)t_l^b - \psi_0^b. \tag{MH_b}$$

To induce truth telling, the optimal contract must satisfy incentive compatibility constraints that account for the possibility of each type working (high effort) or shirking (low effort) if they misreport their type. Thus, moral hazard plays a critical role even in the truth-telling constraints, and we write two incentive compatibility constraints for each type of agent:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^g\right) \tag{IC_1^g}$$

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq \gamma_n^b \left(\pi_0 t_h^b + (1-\pi_0)t_l^b - \psi_0^g\right) \tag{IC_0^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^b\right) \tag{IC_1^b}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq \gamma_n^g \left(\pi_0 t_h^g + (1-\pi_0)t_l^g - \psi_0^b\right), \tag{IC_0^b}$$

where (IC_1^x) prevents misreporting while working, and (IC_0^x) prevents *misreporting and shirking*, with $x \in \{g, b\}$. Indeed, we will find that the constraint (IC_0^b) plays a central role in capturing the impact of ex post moral hazard in our analysis.

Note that the level of competition, n, affects payoffs on each side of the (IC) constraints through γ_n^g and γ_n^b. Note also that if types were observable, the (IC) constraints would be absent and only the (MH) constraints would remain. Because those constraints do not depend on γ_n^g and γ_n^b, competition would have no impact, as is expected in a pure moral hazard model. If efforts were observable, the (MH) constraints would be absent and only (IC_1^g) and (IC_1^b) would remain, with only (IC_1^g) binding. Competition would help the principal as is expected in a pure adverse selection procurement model similar to Laffont-Tirole (1987).

Finally, the optimal contract must satisfy the following IR constraints to induce each type of agent to participate:

$$\gamma_n^g \left(\pi_1 t_h^g + (1-\pi_1)t_l^g - \psi_1^g\right) \geq 0 \tag{IR^g}$$

$$\gamma_n^b \left(\pi_1 t_h^b + (1-\pi_1)t_l^b - \psi_1^b\right) \geq 0. \tag{IR^b}$$

Recalling that the probability of allocating the contract to a good type is $nq\gamma_n^g$, the optimal mechanism is the solution to the following problem.

The principal's problem. The principal maximizes the expected payoff,

$$\Pi(n) = nq\gamma_n^g\left(\pi_1 V - \pi_1 t_h^g - (1-\pi_1)\, t_l^g\right) + (1 - nq\gamma_n^g)\left(\pi_1 V - \pi_1 t_h^b - (1-\pi_1)\, t_l^b\right),$$

subject to the above eight constraints and the non-negativity conditions on all four transfers.

Next, we simplify the principal's problem by first proving that we can set $t_l^g = t_l^b = 0$ without loss of generality. As high effort is induced for both types, it is not optimal to reward either type after a low outcome. Suppose t_l^x are strictly positive for $x = \{b, g\}$. Then, lower t_l^x to zero and raise t_h^x to keep $\pi_1 t_h^x + (1-\pi_1)t_l^x$ constant. The two (IR), the (IC_1^g) and (IC_1^b), and the principal's payoff are unaffected. Because $\pi_1 > \pi_0$, the constraints (IC_0^g), (IC_0^b), (MH_b) and (MH_g) are relaxed. Thus, we have proved the following lemma.

Lemma. Given any transfer vector $(t_h^g, t_l^g, t_h^b, t_l^b)$ that satisfy the (IC), (MH) and (IR) constraints, there is a transfer vector $(\bar{t}_h^g, \bar{t}_l^g, \bar{t}_h^b, \bar{t}_l^b)$ with $\bar{t}_l^g = 0, \bar{t}_l^b = 0$ that gives the same payoff to the principal and satisfies the (IC), (MH) and (IR) constraints.

We can simplify the problem further by eliminating some constraints. First, the (MH_g) and (MH_b), coupled with $L(iii)$, make the (IR_g) and (IR_b) redundant. Second, (IC_0^g) is implied by (IC_1^g) and (MH_b), i.e., the rent given to the bad type to satisfy (MH_b) also induces the good type to work rather than shirk when misreporting. The bad type's incentive constraint cannot be ignored, which is unlike what is standard in models of contracting under adverse selection. In our setting, we anticipate that the bad type will have an incentive to claim to be a good type. To increase his chance of being selected, he will pursue this option by putting in low effort rather than high effort, i.e., we expect (IC_0^b) to be relevant. Finally, we will ignore (IC_1^b) and can verify later that the optimal contract satisfies this constraint. The surviving constraints describe the reduced problem below.

The reduced problem. We use the following notation for the ratio of the two probabilities:

$$\gamma_n \equiv \frac{\gamma_n^g}{\gamma_n^b}.$$

The principal's payoff when she induces high effort by both types is given by

$$\Pi(n) = \pi_1 \left[V - \left(nq\gamma_n^g t_h^g + (1 - nq\gamma_n^g)\, t_h^b \right) \right] \tag{6}$$

we can rewrite the principal's problem in a simpler form:[21]

$$\max \Pi(n)$$

subject to[22]

$$\gamma_n \left(\pi_1 t_h^g - \psi_1^g \right) \geq \pi_1 t_h^b - \psi_1^g \tag{IC_1^g}$$

$$t_h^g \geq \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{MH_g}$$

$$\pi_1 t_h^b - \psi_1^b \geq \gamma_n \left(\pi_0 t_h^g - \psi_0^b \right) \tag{IC_0^b}$$

$$t_h^b \geq \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}. \tag{MH_b}$$

It is useful to briefly consider a benchmark case of contracting with a single agent ($n = 1$, $\gamma_n^g = 1 = \gamma_n^b$). Also, it will be shown below that it is sometimes optimal to interact with only one agent. The pair of transfers (t_h^g, t_h^b) must satisfy the moral hazard constraint for each type. However, there can be no screening as each type will claim the higher transfer given type-independent probabilities of success. The solution is depicted in Figure 1 below. Technically, we can see that $t_h^g = t_h^b$ from (IC_1^g) and (IC_1^b). Then (IC_0^b) reduces to (MH_b), so the optimal transfers are given by $t_h^g = t_h^b = \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0}$, making (MH_g) slack. In sum, when $n = 1$, the constraints (IC_1^g), (IC_0^b) and (MH_b) all hold as equalities.

[21] Note that maximizing $\Pi(n)$ is equivalent to minimizing the expected payment. Also, under allocative efficiency, γ_n^g is given, and the principal only chooses the two transfers t_h^g and t_h^b.

[22] When discussing $(IC_1^g), (MH_g), (IC_0^b)$ and (MH_b) in the rest of the paper, we will be referring to these simpler inequalities instead of their equivalent forms mentioned earlier.

Intuitively, given limited liability, both types can command a moral hazard rent in this model. However, the transfer required to induce the bad type to work, t_h^b, is strictly higher than that required to induce the good type to work, t_h^g. Thus, the good type has an incentive to pretend to be a bad type and earns a rent that is strictly larger than what he would earn in a problem with pure moral hazard. The bad type only receives a rent due to moral hazard, i.e., the amount needed to just satisfy (MH_b).[23]

Our model allows us to focus on the effect of competition to screen the agents and extract the good-type's rent, which we study next.

4 Competitive Procurement under Allocative Efficiency

Competition is a critical part of the incentive mechanism as the principal uses it to screen the agents. This is reflected by the presence of γ_n^g and γ_n^b in the (IC) constraints. However, the (MH) constraints are not affected directly by competition because the effort decision occurs once the agent has been selected. We assume allocative efficiency for this section, which is a requirement in many procurement settings. It implies that the contract is allocated to the most efficient firm. Technically, this means that condition (2) is binding and γ_n^g is given by (5).

There are two standard effects of increased competition under allocative efficiency. First, there is a *selection effect* because an increase in n raises the principal's payoff as the probability of awarding the contract to a good type increases. Second, competition relaxes a good type's incentive constraint (IC_1^g) by increasing his cost of lying, which allows the principal to reduce his rent. By favoring a good type in the allocation rule, she gives him a greater chance of being selected if he tells the truth. This is reflected in $\gamma_n^g > \gamma_n^b$ when $n > 1$. As n increases, the ratio γ_n increases and relaxes (IC_1^g). We call this the *good-type transfer effect* of increased competition. These are two standard, positive effects of competition in adverse selection models with several agents.

[23] See Picard (1987), Ollier-Thomas (2013), and Gottlieb-Moreira (2017) for single-agent models with both moral hazard and adverse selection.

Ex post moral hazard introduces a new element that restricts the effectiveness of competition in extracting a good-type's rent as a bad type's incentive constraint (IC_0^b) is also binding. This constraint is typically not binding in pure adverse selection settings.[24] With unobservable effort, a bad type can misreport his type *and* exert low effort – a double-deviation. This ability to shirk makes it profitable for a bad type to pretend to be a good type unless the principal gives a rent to a bad type. Competition exacerbates this problem.[25] In other words, to induce truth-telling, a bad type has to be given a higher transfer as his chance of being selected decreases with competition. We refer to this as the *bad-type transfer effect* of increased competition, which is a new cost of competition due to the ex post moral hazard problem. The principal pays a rent to a bad type in order to extract rent from a good type as n increases.

To precisely demonstrate the combined impact of these three effects on the principal's payoff, we derive the optimal contract and then explain the impact of competition in the following two subsections.

□ The Optimal Contract under Allocative Efficiency

As n increases from 1, the ratio γ_n increases and two possible cases emerge depending on which constraints are binding. For small n, we are in Case I. The good type's transfer t_h^g is smaller compared to when $n = 1$. His rent is reduced, but it is still large enough to induce high effort. So, his moral hazard constraint (MH_g) remains slack. The solution in this case is given by the binding (IC_0^b) and (IC_1^g):

Case I

$$\begin{aligned} t_h^g &= \frac{\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b}{(\pi_1 - \pi_0)\,\gamma_n} \\ t_h^b &= \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 (\pi_1 - \pi_0)}. \end{aligned} \tag{7}$$

For larger n, we are in Case II, where the principal can no longer reduce t_h^g as the good type's moral hazard constraint (MH_g) is binding. Constraint (IC_1^g) becomes

[24] This effect of shirking is absent in earlier models of competitive procurement, with false moral hazard as in Laffont-Tirole (1987) and McAfee-McMillan (1987), or with observable effort as in Riordan-Sappington (1987).

[25] Again, since the ratio γ_n increases with n, it tightens (IC_0^b).

slack as the rent due to moral hazard is now larger than the rent required to induce truth-telling by the good type. The solution in this case is given by the binding (IC_0^b) and (MH_g):

Case II

$$t_h^g = \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \tag{8}$$
$$t_h^b = \frac{\gamma_n \pi_0}{\pi_1} \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\left(\gamma_n \psi_0^b - \psi_1^b\right)}{\pi_1}.$$

These results are illustrated in Figure 1 and summarized in Proposition 1 and Corollary 1. In Figure 1, as n increases from $n = 1$, the solution moves north-west as t_h^g decreases and t_h^b increases. This is Case I. Despite the increase in t_h^b, the selection effect is strong enough to help the principal reduce t_h^g. However, this reduction in t_h^g is less than that under a procurement problem with pure adverse selection and no ex post moral hazard. Once the solution reaches the (MH_g) line, t_h^g cannot be further reduced. This is Case II. The solution moves up vertically with n along (MH_g).

INSERT FIGURE 1 ABOUT HERE

Proposition 1 *The solution to the principal's problem entails:*

(i) the constraint (IC_0^b) is binding for all γ_n,
(ii) for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$, the constraint (IC_1^g) is binding, and the Case I transfers given by (7) are optimal,
(iii) for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$, the constraint (MH_g) is binding, and the Case II transfers given by (8) are optimal.

Proof. See the Appendix A. ■

Taking a derivative of the transfers above with respect to γ_n gives the impact of increased competition on the bad- and good-type transfers:

Corollary 1 *As γ_n increases, the bad type's transfer t_h^b increases. The good type's transfer t_h^g decreases with γ_n if $\gamma_n \leq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ and remains constant for $\gamma_n > \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$.*

Technically, it is useful to first recall that both (IC_1^g) and (IC_0^b) are satisfied as equalities when $n = 1$. With multiple agents, the (IC_1^g) will become slack and (IC_0^b) will be violated unless the transfers are adjusted. In the optimal contract, the principal adjusts the transfers to both types. With (IC_1^g) slack, she definitely reduces t_h^g. However, the decrease in t_h^g is not enough to satisfy (IC_0^b) unless t_h^b is increased to remove the bad type's incentive to pretend to be good. More precisely, the fact that t_h^b increases with n is implied by condition $L(iii)$, which reflects a strong moral hazard problem. The main reason for imposing $L(iii)$ is that it simplifies the analysis. Indeed, condition $L(iii)$ implies that (IC_0^b) is binding for all n, i.e., for both Cases I and II of Proposition 1. While a binding (IC_0^b) in itself does not guarantee that t_h^b will increase with n, condition $L(iii)$ is necessary and sufficient to ensure that t_h^b increases in Case I, and sufficient for t_h^b to increase in n for Case II.[26] If condition $L(iii)$ does not hold, there are several cases to consider, but the bad-type transfer effect will continue to hold under weaker conditions.[27]

It is also useful to discuss condition $L(ii)$ $\psi_1^g - \psi_0^g < \psi_1^b - \psi_0^b$. Without $L(ii)$, the good-type transfer effect is absent, and the bad-type transfer effect comes into play immediately (for any $n > 1$).[28] Because this condition is akin to a standard single-crossing property, the opposite of $L(ii)$ would imply that the moral hazard of the good type is now more serious than the moral hazard of the bad type.

□ Does Competition help under Allocative Efficiency?

We say that a given level of competition n hurts the principal if her expected payoff is higher with fewer agents competing for the project.[29] Recall that there are three potential effects of increased competition:

[26] In Case II, a weaker condition $L(iv)$ is necessary and sufficient for t_h^b to increase with n.
Condition $L(iv) : \psi_1^g > \frac{\pi_1}{\pi_0}\psi_0^b - \left(\psi_0^b - \psi_0^g\right)$.

[27] Without $L(iii)$, we cannot ignore (IR^g) since it is no longer implied by (MH_g). If we assume that (MH_g) implies (IR^g), and $L(iv)$ (see footnote 26) holds, the solution will be given by (IC_0^b) and (MH_g) and t_h^b is increasing in n for a high enough n. If (IR^g) is more restrictive than (MH_g) when lowering t_h^g, then a weaker condition than $L(iv)$ would be enough to obtain the bad-type transfer effect for high enough n.

[28] For $n = 1$, we still have $t_h^b = t_h^g$ but it is the good type's moral hazard constraint (MH_g) that is binding. As n increases, because the (MH_g) remains binding, t_h^g is constant at (MH_g).

[29] We say that increasing competition always helps if her expected payoff increases as n increases.

- *selection effect*: an increase in n increases the probability of awarding the contract to a good type. This effect is positive for the principal as $t_h^g < t_h^b$ (see the principal's payoff (6)).

- *good-type transfer effect*: from Corollary 1, an increase in n weakly decreases the transfer t_h^g. This effect is again positive for the principal.

- *bad-type transfer effect*: from Corollary 1, an increase in n increases the transfer t_h^b. This effect is negative for the principal.

In Case I, all three effects are present. The first two are standard effects due to adverse selection. The third one, the negative effect, is new and arises from the interaction of ex post moral hazard and adverse selection. With increased competition, the bad type is less likely to be selected and has to be compensated more for not mimicking the good type and then shirking. The third effect would be absent without moral hazard, which indicates that ex-post moral hazard (or precisely its interaction with the adverse selection problem) is the key reason for why competition may hurt the principal.

The net effect of competition on the principal's payoff depends on the combined impact of the three effects. Competition helps when the two positive effects (the selection effect and the good-type transfer effect) dominate the negative effect (the bad-type transfer effect). In the proof of Proposition 2, we show that competition hurts in Case I, when $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$, if and only if

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right]. \tag{9}$$

Extracting the good type's adverse-selection rent is the *raison d'être* of increased competition. We interpret the difference in cost of high effort between types, $\psi_1^b - \psi_1^g$, as the strength of the adverse selection problem. When ψ_1^g is very close to ψ_1^b, the two types of agents are very similar, and rent extraction through the selection and the good-type transfer effects has low significance. However, the moral hazard problem must still be addressed, and increased competition hurts the principal through the bad-type transfer effect. When condition (9) is violated, the adverse selection problem is severe and competition helps.

As competition intensifies, the good-type transfer effect is limited by the need to provide the good type with a rent to satisfy (MH_g). Thus, the good type's transfer t_h^g cannot be reduced indefinitely or the transfer would be so low that the agent would no longer exert effort. Constraint (MH_g) is now binding, and we are in Case II.

In Case II, the good-type transfer effect, one of the two positive effects, vanishes. Only the selection and bad-type transfer effects remain. Competition helps when the remaining positive effect (the selection effect) dominates the negative effect (the bad-type transfer effect). In the proof of Proposition 2, we show that competition hurts in Case II, when $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$, if and only if

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right] + (1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0 \psi_1^b - \pi_1 \psi_0^g\right). \quad (10)$$

Because the good-type transfer effect is now absent, the condition for competition to hurt is less strict than condition (9). This can be seen by the two extra positive terms on the right hand side of (10), making it easier to satisfy than the condition (9) for Case I. Thus, competition can help for small n but hurt for large n.

In addition, even when competition helps, it is less effective as n becomes larger as we prove that $\Pi(n+1) - \Pi(n)$ is decreasing in n . Furthermore, we also prove that when n and $n+1$ are in case I and II, respectively, competition will always help if (10) is violated. In the following proposition, we present our key result about the net impact of competition in the two cases discussed above.

Proposition 2 *Under allocative efficiency, competition helps if and only if the difference in the cost of high effort between types is large enough, i.e. the adverse selection problem is strong enough. Competition is less effective as n becomes larger.*

Proof. See the Appendix A. ■

Given conditions (9) and (10), we can also infer that if competition hurts the principal for small n, it cannot help her when n is large. This is reflected in the following corollary, which also raises the question of an optimal n discussed next.

Corollary 2 *If $\Pi(n)$ decreases with n for $\gamma_n \in \left[1, \frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}\right]$, then it also decreases for $\gamma_n \in \left[\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}, \infty\right)$.*

Proof. See the Appendix A. ■

□ **Choosing the number of agents n under allocative efficiency**

Up to now we have assumed that the principal takes n as given. We have seen that the interaction of the three effects can lead to increased competition hurting the principal's payoff. In this section, we ask what is the optimal number of agents for the principal if n were a choice for the principal.

Denoting by n^* the number of agents for which the principal's payoff is the highest, Proposition 2 and Corollary 2 imply that there are three possible solutions to n^* depending on the strength of the adverse selection problem. Defining the opposite of condition (10) by:

$$\pi_1 q\left(\psi_1^b-\psi_1^g\right) > (1-q)\left[\pi_0\psi_1^g-\pi_1\psi_0^b\right]+(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right) \quad (11)$$

and an intermediate condition by:

$$\begin{aligned}(1-q)\,\pi_0\psi_1^g-\pi_1\psi_0^b &\leq \pi_1 q\left(\psi_1^b-\psi_1^g\right) \\ &\leq (1-q)\left[\pi_0\psi_1^g-\pi_1\psi_0^b\right]+(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right)\end{aligned} \quad (12)$$

we collect the three solutions in the following Proposition:

Proposition 3 *Under allocative efficiency, the optimal number of agents for the principal is either (i) $n^*=1$, (ii) $n^*=\infty$, or (iii) $1<n^*<\infty$, depending on the strength of the adverse selection problem represented by the value of $\pi_1 q\left(\psi_1^b-\psi_1^g\right)$ given by (9), (11), and (12).*

The first possible solution ($n^*=1$) is the extreme where competition always hurts the principal. This is obtained under the condition (9). This case occurs when the adverse selection problem is relatively minor compared to the moral hazard problem.

For instance, if ψ_1^b is close to ψ_1^g, the two types of agents are very similar and the principal would not benefit much from increased competition through the selection and the good-type transfer effects. However, the moral hazard constraints must still be satisfied, and competition hurts the principal through the bad-type transfer effect. Thus, the principal does not benefit from competition between agents who are very similar in their cost of effort.

The second possible solution ($n^* = \infty$) is the standard case that is also obtained in adverse selection models without moral hazard. Here it is obtained under the condition (11), which is the opposite of condition (10). Increased competition always helps the principal by extracting the rent of the good type through the selection and the good type transfer effects. This occurs when the difference in cost of high effort between types is large, i.e., when the adverse selection problem is severe. In this case, the principal would never want to limit the number of agents.

The third possible solution ($1 < n^* < \infty$) has low levels of competition helping the principal, but higher levels of competition hurting the principal.[30] This intermediate solution is obtained under the condition (12). The optimal number of agents, n^*, is the solution to $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$.[31] This corresponds to the situation when the principal first benefits from competition for low values of n but is hurt by increased competition for high values of n. This is because the good-type transfer effect has vanished and the bad-type transfer effect dominates the selection effect. Technically, condition (10) holds but condition (9) does not. The principal would prefer to limit the number of agents to the optimal level in this case.

Our analysis may explain why some existing procurement mechanisms try to restrict the number of agents (see Eun (2019) for examples). This may also explain the criticisms in the earlier examples of NHS and Medicare procurement that too much competition adversely affects ex post quality of service. Next, we study an alternative to restricting the number of agents by using inefficient allocation mechanisms.

[30] Note that if we assumed $\psi_0^g = \psi_0^b$, case II disappears and (MH_g) never binds. In that case, either competition always helps or competition never helps.

[31] If there is no integer that satisfies the equality, consider n satisfying $\gamma_{n-1} < \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g} < \gamma_n$, and define n^* to be $n-1$ or n depending on which results in a higher expected payoff for the principal.

5 The Optimal Mechanism: an inefficient allocation rule can be optimal

So far we have considered the optimal mechanism under efficient allocation rules only, where the principal committed to allocating the contract to a good type with probability one when at least one agent reported to be a good type. Here, we remove the requirement of allocative efficiency and consider inefficient allocation rules, which give a chance for a bad type to be selected even when another agent has reported to be a good type.

To see the intuition why an inefficient allocation rule may help, note that the bad type's incentive to mimic the good type with zero effort can be lowered in two different ways. We have already discussed one method, increasing his transfer t_h^b, which leads to the the bad-type transfer effect. An alternative approach is to use an inefficient allocation rule that gives the bad type a positive probability of being selected despite the presence of good types. By doing so, the principal increases the probability for a bad type to be selected, which used to be zero under an efficient allocation rule even if there were only one good type present. This relaxes (IC_0^b) and benefits the principal.[32]

We will show that whenever competition hurts under an efficient allocation rule, the optimal mechanism requires an inefficient allocation rule. Thus, if a given level of competition hurts her payoff under an efficient allocation rule, i.e., $n > n^*$, optimality requires that she give up on allocative efficiency. Furthermore, we show that the optimal (inefficient) rule with n agents gives the principal the same payoff as she would obtain with n^* agents under an efficient allocation rule.[33] This means that the principal does not gain from additional competition when $n > n^*$. Thus, by giving up on allocative efficiency, the principal can in fact entirely neutralize the negative effect of competition on her payoff even when she cannot reduce the number of agents competing for the contract.

[32]Strausz (2006) shows that a stochastic contract can also relax the upward binding constraint in a single-agent model if there is bunching and the agent's types differ in the degree of risk aversion. See Kadan et al. (2017) for a recent discussion on the role of randomization in principal agent models.

[33]The optimality is of course subject to the caveat that n is an integer as discussed at end of Section 4 in footnote 31. For expositional ease, we will ignore this issue for the rest of the paper.

Technically, besides the transfers, the principal can now also choose γ_n^b and γ_n^g. These choices were determined by the binding (2) and (4) under allocative efficiency. To allow for inefficient allocation rules, we remove the restriction that the first constraint (2) must hold as an equality. Given (4), it is sufficient to consider her choice of γ_n^g. Indeed, if the solution involves $qn\gamma_n^g < 1-(1-q)^n$, the optimal allocation rule must necessarily be inefficient, i.e., a bad type must have a chance of being awarded the contract even when a good type is present.[34] Whenever competition hurts under an efficient allocation rule, we show that the principal's payoff is decreasing in γ_n^g and constraint (2) is slack, which implies that the optimal allocation rule is inefficient. The opposite is true when competition helps under an efficient allocation rule. The constraint (2) is binding and an efficient allocation rule is optimal.

Proposition 4 *Whenever competition hurts under an efficient allocation rule (i.e., $n > n^*$), the optimal allocation rule is inefficient, and it makes the principal's expected payoff equal to her payoff when $n = n^*$ under the efficient allocation rule. When competition helps under an efficient allocation rule, the optimal allocation is efficient.*

Proof. See the Appendix A. ■

The proposition implies that the mechanism where the allocation is efficient but the number of agents is potentially restricted is optimal among all mechanisms. For $n > n^*$, competition ceases to be useful even if the principal can use an inefficient allocation rule.

We now explain how the principal optimally chooses γ_n^g when competition helps or hurts under allocative efficiency. Suppose competition hurts for all n under an efficient allocation rule. This occurs when condition (9) holds. Then, the principal wants to choose the smallest possible γ_n^g, making the allocation rule inefficient and the principal's payoff identical to her payoff when there is only one agent ($n^* = 1$).[35] If competition helps for all n under efficient allocation, then the allocative efficiency is optimal for all n. This occurs when condition (10) is violated.

[34] In that case, it cannot be that $\phi_r^g = 1$ for all r, given (1) and (5). We discuss allocation rules to implement the optimal γ_n below.

[35] In the appendix, we show that the smallest value of γ_n^g makes $\gamma_n = 1$, which is the lower bound given by the monotocity condition $\gamma_n \geq 1$ implied by the incentive constraints.

Finally, we consider the interesting case when competition helps for small n but hurts for large n under an efficient allocation rule. This case occurs when condition (10) holds but not (9). For small n, the optimal allocation rule is efficient. For large n, the optimal allocation rule is inefficient, such that the optimal γ_n is given by $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$. This makes the principal's payoff identical to that for $n = n^*$ under allocative efficiency.[36]

□ **Allocation rule ϕ_r^g to implement the optimal γ_n**

To characterize this allocation rule, consider first the case where competition always helps under an efficient allocation rule. In this case, the principal allocates the contract to a good type whenever there is one. If competition hurts for all n under an efficient allocation rule, then the principal allocates the contract randomly to an agent regardless of their announcement. Finally, consider the interesting case when competition helps for small n but hurts for large n under an efficient allocation rule. For small n, the principal allocates the contract to a good type whenever there is one. For large n, she allocates the contract to a bad type with a high enough probability such that $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_1^g}$. We prove the following corollary in the Appendix A:

Corollary 3 *When competition helps under allocative efficiency for all n, i.e., $n^* = \infty$, it is optimal to choose $\phi_r^g = 1$ for $r \geq 1$ for all n. When competition hurts under allocative efficiency for all n, i.e., $n^* = 1$, it is optimal to choose $\phi_r^g = \frac{r}{n}$ for all r and $n > 1$. When competition helps for small n but hurts for large n, i.e., $1 < n^* < \infty$, then, for $r \geq 1$, it is optimal to choose $\phi_r^g = 1$ when $n \leq n^*$ and $\phi_r^g = \left(\frac{q}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ when $n > n^*$. Finally, $\phi_0^g = 0$ for all n.*

Proof. See the Appendix A. ■

For implementation, we can follow Myerson (1981) or Riordan-Sappington (1987), where the principal announces the mechanism and asks each competing agent to report their cost of production. Under an efficient allocation rule, the principal allocates the contract to a lowest cost agent. Under an inefficient allocation rule, the principal follows the random allocation rule committed to as part of the mechanism.

[36] In the appendix, we show that the smallest value of γ_n^g makes $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ which is the smallest value of γ_n consistent with case II, Proposition 1.

There are examples of auctions with inefficient allocations, where the most efficient agent is not guaranteed to win. In the Korean example mentioned above (Eun (2019)), the procurer uses a publicly drawn random variable as a cut-off for acceptable bids. In an Average Price Auction (Decarolis (2018)), the winning bid is the average of submitted bids. Ariely-Ockenfels-Roth (2005) report on online auctions used by eBay and Amazon in which the participant with the highest value does not necessarily win. These mechanisms are motivated by the desire to curtail overly aggressive bidding.

We conclude the discussion on the optimal mechanism by arguing that the principal will always want to induce high effort if the project is very valuable, i.e., V is large enough. Our next proposition gives a formal proof. We discuss in Section 6 the case when the principal may want to induce different efforts for different types.

Proposition 5 *If V is high enough, $e_g = e_b = 1$ is optimal.*

Proof. See the Appendix A. ■

6 Screening with Effort

So far we have considered the case where effort was binary and the principal found it optimal to induce high effort by both types ($e_b = e_g = 1$). As a result, the principal did not use effort as a screening instrument and the agents were restricted to only one alternative effort level when shirking. In this section, we relax that restriction by letting the agent adjust effort $e \geq 0$ continuously. There is a trade-off for the principal. On the one hand, by adjusting effort continuously, the principal can screen the agents better. On the other hand, the bad type agent now has more options in choosing effort when mimicking the good type.

Except that competition is beneficial under allocative efficiency when n is small, our results are largely analogous to those from the binary model. We again find that, under allocative efficiency, the bad type's incentive constraint (IC^b) is binding for all n, and that the need to sustain effort makes (IC^g) slack for high n. Also, being able to adjust effort continuously is not enough for the principal to guarantee that high levels of competition are beneficial under efficient allocation. We provide details in the Appendix B.

7 Conclusion

There is widespread concern that competitive bidding can lead to poor quality ex post. This connection has been largely ignored in the theoretical literature that has focused on the adverse selection problem to emphasize ex ante rent extraction. Procuring a project requires not only to select the most efficient firm (adverse selection), but also to make sure that the selected firm has the correct incentives to implement the project (moral hazard). Our analysis highlights the interaction between the two and explains how competition for the project results in a trade-off that may hurt the procurer. While competition is typically expected to be beneficial in reducing rent due to adverse selection, the presence of moral hazard can significantly interfere with rent extraction. Introducing the option to shirk allows a high-cost firm to mimic a low-cost firm and put in low effort, which results in an additional rent for the high-cost firm. Attempts to use increased competition to extract a low-cost firm's rent may lead to an increased rent to a high-cost firm. As a consequence, the procurer may find it optimal to limit the number of potential firms.

We show that insisting on allocative efficiency is costly from an incentive point of view and gives a strong incentive for a bad type to claim to be a good type as it lowers the probability that a bad type will be awarded the contract – it is zero as soon as there is only one good type present. By randomly assigning the contract to a bad type even when a good type is present, the procurer can lower the cost of inducing truth-telling significantly. Remarkably, we show that the procurer can use inefficient allocations to mimic her payoff from an allocatively efficient mechanism with an optimal number of agents without actually limiting the number of agents.

These results are in line with practical attempts to address well-publicized concerns about bidding low and shirking mentioned in the introduction. Such concerns have led to various random (inefficient) allocation rules in practice. For example, the Korean government has uses a publicly drawn random variable as a cut-off for acceptable bids to award road construction contracts (Eun (2019)). There are many examples of procurers using an Average Price Auction (Decarolis (2018)), where the winning bid is the average of submitted bids.

The framework presented here captures an important element of procurement auctions. At the same time, it is highly tractable which should allow exploration of a variety of relevant interesting questions in competitive procurement. For instance, suppose that the procurer can use an audit technology to verify ex post the efficiency of the selected firm (adverse selection) or its effort (moral hazard). Which one should she concentrate her resources on? We have also ignored the cost of suppliers to participate in the procurement process. What if it is costly to prepare a submission? Similarly, the procurer could also decide to impose a fee to participate in the mechanism. We could study the role of endogenous entry instead of assuming that there is a fixed number of agents. The model presented here is simple enough that it would be possible to explore these and other related questions in procurement with moral hazard.

8 Appendix A

Claim. *The principal cannot be better off making a payment to a losing agent.*

Proof of Claim. For a given allocation rule, suppose the principal paid $f^g \geq 0$ and $f^b \geq 0$ to losing good and bad type agents, respectively. Then, the transfers $\left\{\left(f^g, t_h^g, t_l^g\right), \left(f^b, t_h^b, t_l^b\right)\right\}$ would have to satisfy the incentive and participation constraints:

$$\pi_1 t_h^g + (1-\pi_1)\, t_l^g - \psi_1^g \geq \pi_0 t_h^g + (1-\pi_0)\, t_l^g - \psi_0^g \qquad (MH_g)$$

$$\pi_1 t_h^b + (1-\pi_1)\, t_l^b - \psi_1^b \geq \pi_0 t_h^b + (1-\pi_0)\, t_l^b - \psi_0^b \qquad (MH_b)$$

$$\begin{aligned} &\left(1-\gamma_n^g\right) f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g \left(1-\pi_1\right) t_l^g - \gamma_n^g \psi_1^g && (IC_1^g) \\ &\geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_1 t_h^b + \gamma_n^b \left(1-\pi_1\right) t_l^b - \gamma_n^b \psi_1^g \\ &\left(1-\gamma_n^g\right) f^g + \gamma_n^g \pi_1 t_h^g + \gamma_n^g \left(1-\pi_1\right) t_l^g - \gamma_n^g \psi_1^g && (IC_0^g) \\ &\geq \left(1-\gamma_n^b\right) f^b + \gamma_n^b \pi_0 t_h^b + \gamma_n^b \left(1-\pi_0\right) t_l^b - \gamma_n^b \psi_0^g \end{aligned}$$

$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \qquad (IC_1^b)$$
$$\geq\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g-\gamma_n^g \psi_1^b$$
$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \qquad (IC_0^b)$$
$$\geq\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_0 t_h^g+\gamma_n^g\left(1-\pi_0\right) t_l^g-\gamma_n^g \psi_0^b$$

$$\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g-\gamma_n^g \psi_1^g \geq 0 \qquad (IR^g)$$
$$\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b-\gamma_n^b \psi_1^b \geq 0 \qquad (IR^b)$$

Suppose f^g and f^b are strictly positive. Now construct payments

$$\hat{t}_h^g=\frac{\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g}{\gamma_n^g \pi_1}, \hat{t}_l^g=0$$
$$\hat{t}_h^b=\frac{\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b}{\gamma_n^b \pi_1}, \hat{t}_l^b=0$$

and consider the transfers $\left\{\left(0, \hat{t}_h^g, \hat{t}_l^g\right),\left(0, \hat{t}_h^b, \hat{t}_l^b\right)\right\}$, instead. The two IR, the (IC_1^g) and (IC_1^b) are satisfied under these transfers. Because $\pi_1>\pi_0$, the constraints (IC_0^g), (IC_0^b), (MH_b) and (MH_g) are relaxed.

Furthermore, the expected payment by the principal to a typical agent i under the constructed transfers is

$$(1-q) \gamma_n^b \pi_1 \hat{t}_h^b+q \gamma_n^g \pi_1 \hat{t}_h^g .$$

Substituting the expressions for $\hat{t}_h^b$ and $\hat{t}_h^g$ we have the expected payment by the principal to a typical agent:

$$(1-q)\left[\left(1-\gamma_n^b\right) f^b+\gamma_n^b \pi_1 t_h^b+\gamma_n^b\left(1-\pi_1\right) t_l^b\right]$$
$$+q\left[\left(1-\gamma_n^g\right) f^g+\gamma_n^g \pi_1 t_h^g+\gamma_n^g\left(1-\pi_1\right) t_l^g\right]$$

which is the same as that with the transfers $\left\{\left(f^g, t_h^g, t_l^g\right),\left(f^b, t_h^b, t_l^b\right)\right\}$. Thus, the principal is not worse off under the constructed transfers that pays only the winning agent. This completes the proof of the Claim. ■

Proof of Proposition 1. First observe that IC_1^g and IC_1^b can be rewritten as

$$IC_1^g: \gamma_n \pi_1 t_h^g-\gamma_n \psi_1^g+\psi_1^g \geq \pi_1 t_h^b$$
$$IC_1^b: \pi_1 t_h^b \geq \gamma_n \pi_1 t_h^g-\gamma_n \psi_1^b+\psi_1^b$$

which imply

$$\gamma_n\left(\psi_1^b-\psi_1^g\right)\geq\psi_1^b-\psi_1^g$$

or,

$$\gamma_n\geq 1$$

We have $\gamma_1=1$ and γ_n is increasing in n when the allocation rule is efficient.

Now consider the reduced problem given in Section 3. The principal chooses the two transfers $\{t_h^b,t_h^g\}$ to solve

$$\max\Pi(n)=\pi_1\left[V-\left(nq\gamma_n^g t_h^g+\left(1-nq\gamma_n^g\right)t_h^b\right)\right]$$

subject to,

$$\gamma_n\left(\pi_1 t_h^g-\psi_1^g\right)\geq\pi_1 t_h^b-\psi_1^g \qquad (IC_1^g)$$

$$t_h^g\geq\frac{\psi_1^g-\psi_0^g}{\pi_1-\pi_0} \qquad (MH_g)$$

$$\pi_1 t_h^b-\psi_1^b\geq\gamma_n\left(\pi_0 t_h^g-\psi_0^b\right) \qquad (IC_0^b)$$

$$t_h^b\geq\frac{\psi_1^b-\psi_0^b}{\pi_1-\pi_0}. \qquad (MH_b)$$

To start, consider the problem of maximizing

$$\pi_1 V-\left[\left(1-\left(1-q\right)^n\right)\pi_1 t_h^g+\left(1-q\right)^n\pi_1 t_h^b\right]$$

subject only to the IC_1^g and IC_0^b constraints. Because the principal's payoff is decreasing in t_h^g and t_h^b, the inequality and the fact that the LHS of IC_1^g and the RHS of IC_0^b above are both increasing in t_h^g together imply that the solution of the reduced problem is given by

$$t_h^g=\frac{\gamma_n\left(\psi_1^g-\psi_0^b\right)+\left(\psi_1^b-\psi_1^g\right)}{\gamma_n\left(\pi_1-\pi_0\right)}$$

$$t_h^b=\frac{\gamma_n\pi_0}{\pi_1}t_h^g-\frac{\left(\gamma_n\psi_0^b-\psi_1^b\right)}{\pi_1}=\frac{\gamma_n\left(\pi_0\psi_1^g-\pi_1\psi_0^b\right)+\pi_1\psi_1^b-\pi_0\psi_1^g}{\pi_1\left(\pi_1-\pi_0\right)}$$

We will now show that for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the remaining constraints are satisfied. Substituting for t_h^g we have

$$
\begin{aligned}
MH_g : t_h^g - \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} &= \frac{\psi_1^b - \psi_1^g - \gamma_n \left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right)\gamma_n} \\
&\geq \frac{\psi_1^b - \psi_1^g - \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\left(\psi_0^b - \psi_0^g\right)}{\left(\pi_1 - \pi_0\right)\gamma_n} \\
&= 0
\end{aligned}
$$

Substituting for t_h^b

$$
MH_b : t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} = \left(\gamma_n - 1\right)\frac{\pi_0 \psi_1^g - \pi_1 \psi_0^b}{\pi_1\left(\pi_1 - \pi_0\right)} > 0
$$

and under our assumption

$$
IC_1^b : \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n\left(\pi_1 t_h^g - \psi_1^b\right) = \left(\gamma_n - 1\right)\left(\psi_1^b - \psi_1^g\right) > 0.
$$

Next for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ consider the restricted problem of maximizing $\pi_1 V - \left[\left(1 - (1-q)^n\right)\pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]$ subject only to the MH_g and IC_0^b constraints. Following similar arguments as above, the solution is given by

$$
\begin{aligned}
t_h^g &= \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} \\
t_h^b &= \frac{\gamma_n \pi_0}{\pi_1}\frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \frac{\gamma_n \psi_0^b - \psi_1^b}{\pi_1}
\end{aligned}
$$

We will now show that all the remaining constraints are satisfied by this solution:

$$
\begin{aligned}
IC_1^g : \frac{1}{\pi_1}\left(\gamma_n\left(\pi_1 t_h^g - \psi_1^g\right) + \psi_1^g\right) - t_h^b & \\
&= \frac{\psi_1^g - \psi_1^b + \gamma_n\left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\
&> \frac{\psi_1^g - \psi_1^b + \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\left(\psi_0^b - \psi_0^g\right)}{\pi_1} \\
&= \frac{\psi_1^g - \psi_1^b + \psi_1^b - \psi_1^g}{\pi_1} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
IC_1^b &: \left(\pi_1 t_h^b - \psi_1^b\right) - \gamma_n \left(\pi_1 t_h^g - \psi_1^b\right) \\
&= \gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b - \gamma_n \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \gamma_n \psi_1^b \\
&= \frac{\gamma_n}{\pi_1 - \pi_0} \left(\pi_1 - \pi_0\right) \left[\left(\psi_1^b - \psi_0^b\right) - \left(\psi_1^g - \psi_0^g\right)\right] \\
&> 0 \text{ by L(ii)}
\end{aligned}
$$

$$
\begin{aligned}
&MH_b : t_h^b - \frac{\psi_1^b - \psi_0^b}{\pi_1 - \pi_0} \\
&\quad = \frac{1}{\pi_1(\pi_1 - \pi_0)} \left[\gamma_n \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b + \pi_0 \left(\psi_0^b - \psi_0^g\right)\right] + \psi_1^b \left(\pi_1 - \pi_0\right) - \pi_1 \left(\psi_1^b - \psi_0^b\right)\right]
\end{aligned}
$$

(using L(iii) the coefficient of γ_n is positive, hence replacing γ_n by its minimum value)

$$
\begin{aligned}
&\geq \frac{1}{\pi_1 \left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right)} \left[\begin{array}{c} \left(\psi_1^b - \psi_1^g\right) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b + \pi_0 \left(\psi_0^b - \psi_0^g\right)\right] \\ + \psi_1^b \left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right) - \pi_1 \left(\psi_1^b - \psi_0^b\right) \left(\psi_0^b - \psi_0^g\right) \end{array} \right] \\
&= \frac{1}{\pi_1 \left(\pi_1 - \pi_0\right) \left(\psi_0^b - \psi_0^g\right)} \left[\left(\psi_1^b - \psi_0^b\right) - \left(\psi_1^g - \psi_0^g\right)\right] \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) \\
&> 0 \text{ by Condition } L.
\end{aligned}
$$

This completes the proof of Proposition 1. ∎

Proof of Proposition 2. The expected payoff to the principal is

$$
\begin{aligned}
\Pi(n) &= \left(1 - (1-q)^n\right) \pi_1 \left(V - t_h^g\right) + (1-q)^n \pi_1 \left(V - t_h^b\right) \\
&= \pi_1 V - \left[\left(1 - (1-q)^n\right) \pi_1 t_h^g + (1-q)^n \pi_1 t_h^b\right]
\end{aligned}
$$

For $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$ the payoff at the optimal transfers is

$$
\Pi(n) = \pi_1 V - \left(\begin{array}{c} \left(1 - (1-q)^n\right) \pi_1 \frac{\left(\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b\right)}{(\pi_1 - \pi_0)\gamma_n} \\ + (1-q)^n \pi_1 \frac{\gamma_n \left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 (\pi_1 - \pi_0)} \end{array} \right)
$$

and for $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ it is

$$
\Pi(n) = \pi_1 V - \left(\left(1 - (1-q)^n\right) \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + (1-q)^n \left(\gamma_n \pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \gamma_n \psi_0^b + \psi_1^b \right) \right)
$$

Substituting the expression for $\gamma_n = \frac{(1-(1-q)^n)}{q(1-q)^{n-1}}$ under efficient allocation we have for $\gamma_n \in \left[1, \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}\right]$

$$\begin{aligned}\Pi(n) &= \pi_1 V - \pi_1 \frac{\left(\psi_1^g - \psi_0^b\right)}{(\pi_1 - \pi_0)} - \left(\frac{1-q}{q}\right) \pi_1 \frac{\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right)}{\pi_1 (\pi_1 - \pi_0)} \\ &+ \left[\begin{array}{c} \left(\pi_1 \frac{\left(\psi_1^g - \psi_0^b\right)}{(\pi_1 - \pi_0)} + \left(\frac{1-q}{q}\right) \frac{\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right)}{(\pi_1 - \pi_0)} - \frac{\pi_1 \psi_1^b - \pi_0 \psi_1^g}{(\pi_1 - \pi_0)} \right) (1-q) \\ - \pi_1 \frac{\left(\psi_1^b - \psi_1^g\right) q}{(\pi_1 - \pi_0)} \end{array} \right] (1-q)^{n-1}.\end{aligned}$$

It is *increasing* in n if and only if

$$\left[\begin{array}{c} \left(\pi_1 \frac{\left(\psi_1^g - \psi_0^b\right)}{(\pi_1 - \pi_0)} + \left(\frac{1-q}{q}\right) \frac{\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right)}{(\pi_1 - \pi_0)} - \frac{\pi_1 \psi_1^b - \pi_0 \psi_1^g}{(\pi_1 - \pi_0)} \right) (1-q) \\ - \pi_1 \frac{\left(\psi_1^b - \psi_1^g\right) q}{(\pi_1 - \pi_0)} \end{array} \right] < 0$$

or, simplifying,

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) > (1-q) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right], \tag{13}$$

which is the opposite of (9) in the main text. Alternatively, we could have obtained the same condition by computing:

$$\Pi(n+1) - \Pi(n) = \frac{(1-q)^{n-1}}{\pi_1 - \pi_0} \left(\pi_1 \left(\psi_0^b (1-q) + \left(\psi_1^b - \psi_1^g\right) q\right) - \pi_0 \psi_1^g (1-q)\right).$$

Therefore, competition helps if and only if the difference in the cost of high effort between types is large enough.

For $\gamma_n \in \left[\frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}, \infty\right)$ we can similarly rewrite

$$\begin{aligned}\Pi(n) &= \pi_1 V - \pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \left(\frac{1-q}{q}\right) \left(\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \psi_0^b\right) \\ &+ \left[\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \left(\frac{1-q}{q}\right) \left(\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \psi_0^b\right) - \psi_1^b\right] (1-q)^n.\end{aligned}$$

In this case $\Pi(n)$ is *increasing* if and only if

$$\left[\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + \left(\frac{1-q}{q}\right) \left(\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - \psi_0^b\right) - \psi_1^b\right] < 0,$$

or, simplifying,

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) > (1-q) \left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right] + (1-q) \left(\psi_0^b - \psi_0^g\right) \pi_0 + q \left(\pi_0 \psi_1^b - \pi_1 \psi_0^g\right),$$

which is identical to condition (11) and the opposite of (10) in the main text. Alternatively, we could have obtained the same condition by computing:

$$\Pi(n+1)-\Pi(n)=\frac{(1-q)^n}{\pi_1-\pi_0}\left(\begin{array}{c}\pi_1\left(\psi_0^b-\psi_0^b q+\left(\psi_0^g+\psi_1^b-\psi_1^g\right)q\right)\\ -\pi_0\left(\psi_0^b+\psi_1^g+\psi_0^g\left(-1+q\right)-\psi_0^b q+\psi_1^b q-\psi_1^g q\right)\end{array}\right).$$

Again, competition helps if and only if the difference in the cost of high effort between types is large enough.

For the sake of completeness, we next show that (11) guarantees that competition helps even when n' and $n'+1$ are such that $\gamma_{n'}\leq\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}\leq\gamma_{n'+1}$ (with at least one of the inequalities being strict). As above we can again obtain the necessary and sufficient condition for $\Pi(n')<\Pi(n'+1)$. Let us refer to this condition as Condition C. Now $\Pi(n)$ is continuous in the real number n for each of the cases. Furthermore, $\Pi(n)$ is also continuous across the two cases since $\Pi(n)$ computed for $\gamma_n\in\left[1,\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}\right]$ and that for $\gamma_n\in\left[\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g},\infty\right)$ are identical when $\gamma_n=\frac{\psi_1^b-\psi_1^g}{\psi_0^b-\psi_0^g}$. Suppose that (11) holds. Given that (11) implies (13), and that $\Pi(n)$ is continuous at the boundary, we must have $\Pi(n')<\Pi(n'+1)$. This implies that condition (11) is stronger than Condition C. Suppose that (13) is violated, i.e., (9) holds. Then, (11) is also violated, i.e., (10) also holds. Thus, we must have $\Pi(n')\geq\Pi(n'+1)$, and Condition C is violated. This implies that Condition C is stronger than condition (13).

Finally, we argue that when competition helps, it is less effective as n becomes larger. When competition helps, we know that either (13) is satisfied, or both (13) and (11) are satisfied. Therefore $\Pi(n+1)-\Pi(n)$ is decreasing in n because $\frac{(1-q)^{n-1}}{\pi_1-\pi_0}$ and $\frac{(1-q)^n}{\pi_1-\pi_0}$ are both decreasing in n.

This completes the proof of Proposition 2. ■

Proof of Corollary 2. The proof follows from Proposition 2 upon observing that condition L implies $\pi_0\psi_1^b-\pi_1\psi_0^g\geq 0$ and $\psi_0^b>\psi_0^g$, so

$$(1-q)\left(\psi_0^b-\psi_0^g\right)\pi_0+q\left(\pi_0\psi_1^b-\pi_1\psi_0^g\right)\geq 0.$$

In this case, if

$$\pi_1 q\left(\psi_1^b-\psi_1^g\right)<(1-q)\left[\pi_0\psi_1^g-\pi_1\psi_0^b\right]$$

holds, so does

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) < (1-q)\left[\pi_0 \psi_1^g - \pi_1 \psi_0^b\right] + (1-q)\left(\psi_0^b - \psi_0^g\right)\pi_0 + q\left(\pi_0 \psi_1^b - \pi_1 \psi_0^g\right).$$

This completes the proof of Corollary 2. ■

Proof of Proposition 4. We start by characterizing the optimal mechanism for all n. Recall that in the proof of Proposition 1 we found the optimal transfers to the principal's problem stated in Section 3 for a given γ_n. In this proof we will find the solution to the principal's problem stated in Section 3, where the principal chooses not only the transfers but also (γ_n^b, γ_n^g) subject to the feasibility constraints (2), (3), and (4). In particular, the principal is now free to consider inefficient allocation rules. Thus, it is not required that the constraint (2) must hold as an equality. Indeed, if the solution involves $qn\gamma_n^g < 1 - (1-q)^n$, the optimal allocation rule must necessarily be inefficient.[37]

We know that IC_1^g and IC_1^b imply the monotonicity condition that the probability of winning the contract with a type g report is at least as high as that with a type b report:

$$\gamma_n \geq 1. \tag{14}$$

Under allocative efficiency, monotonicity is implied and (14) is redundant, but it is relevant when inefficient allocation rules are allowed. When determining the solution, we can ignore condition (3) because it is implied by (4) and (14). Using the binding constraint (4), we have $\gamma_n^b = \frac{1-qn\gamma_n^g}{(1-q)n}$.

Because the results of Proposition 1 hold for every feasible $\left(\gamma_n^g, \gamma_n^b\right)$, and in particular do not depend on the restriction $qn\gamma_n^g = 1 - (1-q)^n$, the optimal transfers are given as before in Cases (I) and (II) by (7) if $\gamma_n \leq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$, and (8) if $\gamma_n > \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$.

Because we will substitute these transfers into the principal's payoff, we must pay attention to the conditions under which the optimal transfers change from Case I to

[37] There must be some $\phi_r^g < 1$ for some r, i.e., a bad type would have a chance of being awarded the contract even when a good type is present.

II depending on γ_n according to:

$$\gamma_n \leq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g} \tag{15}$$

$$\gamma_n \geq \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}. \tag{16}$$

Next, we can rewrite (2), (14), (15) and (16) as a function of γ_n^g only:

$$\gamma_n^g \leq \frac{1-(1-q)^n}{nq} \tag{17}$$

$$\gamma_n^g \geq \frac{1}{n} \tag{18}$$

$$\gamma_n^g \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \tag{19}$$

$$\gamma_n^g \geq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \tag{20}$$

We define $\hat{n}$ as the largest integer such that

$$\frac{1-(1-q)^n}{nq} \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}. \tag{21}$$

Multiplying both sides by n, we see that the resulting RHS is independent of n, whereas $\frac{1-(1-q)^n}{q}$ is increasing in n. Thus, we know that the RHS of (17) is smaller than the RHS of (19) when $n \leq \hat{n}$. We will analyze the case $n \leq \hat{n}$, and the case $n > \hat{n}$ in turn next.

Small n $(n \leq \hat{n})$

Given $n \leq \hat{n}$, the upper bound on γ_n^g is $\frac{1-(1-q)^n}{nq} \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ and Case I transfers are optimal. Substituting the optimal Case I transfers, the principal's expected payoff is given by

$$nq\gamma_n^g \pi_1 \left(V - \frac{\gamma_n \psi_1^g - \psi_1^g - \gamma_n \psi_0^b + \psi_1^b}{(\pi_1 - \pi_0)\gamma_n}\right)$$
$$+ n(1-q)\gamma_n^b \pi_1 \left(V - \frac{\gamma_n\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right) + \pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1(\pi_1 - \pi_0)}\right),$$

and, after substituting $\gamma_n^b = \frac{1-qn\gamma_n^g}{(1-q)n}$, her payoff becomes

$$
\begin{aligned}
&n\gamma_n^g \left(q\pi_1 \left(V - \frac{\psi_1^g - \psi_0^b}{(\pi_1 - \pi_0)} \right) - (1-q)\,\pi_1 \frac{\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right)}{\pi_1 \left(\pi_1 - \pi_0\right)} \right) \\
&+ \frac{1 - qn\gamma_n^g}{(1-q)} \left((1-q)\,\pi_1 \left(V - \frac{\pi_1 \psi_1^b - \pi_0 \psi_1^g}{\pi_1 \left(\pi_1 - \pi_0\right)} \right) - q\pi_1 \frac{\psi_1^b - \psi_1^g}{(\pi_1 - \pi_0)} \right).
\end{aligned} \tag{22}
$$

Taking derivative with respect to γ_n^g we have:

$$
\begin{aligned}
&- \frac{nq\pi_1\psi_1^g - n\pi_1\psi_0^b - nq\pi_1\psi_1^b + n\pi_0\psi_1^g}{\pi_1 - \pi_0} \\
&+ \frac{1}{(1-q)} \frac{nq^2\pi_1\psi_1^b - nq^2\pi_1\psi_1^g}{(\pi_1 - \pi_0)},
\end{aligned} \tag{23}
$$

which is positive if and only if,

$$
\pi_1 q \left(\psi_1^b - \psi_1^g\right) > (1-q)\left(\pi_0 \psi_1^g - \pi_1 \psi_0^b\right). \tag{24}
$$

Thus, for small $n \leq \hat{n}$, revenue is increasing in γ_n^g if (24) holds, and the optimal γ_n^g is at the highest extreme given by (17), $\gamma_n^g = \frac{1-(1-q)^n}{nq}$. In this solution, we have allocative efficiency. If the condition (24) does not hold, her payoff is decreasing in γ_n^g and the optimal γ_n^g is given by the binding monotonicity condition at the lowest extreme of (18), where $\gamma_n^g = \gamma_n^b = \frac{1}{n}$. In this case, the optimal allocation rule is inefficient when $n > 1$. When $n = 1$, $\gamma_n^g = 1$, and equation (2) is binding, i.e., the allocation is efficient.

Large n $(n > \hat{n})$

With $n > \hat{n}$, condition (21) no longer holds and we have to consider both Case I and II contracts as γ_n^g can satisfy both (19) and (20).

For $\frac{1}{n} \leq \gamma_n^g \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$, the principal's payoff is given by (22) above with the Case I transfers. For $\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \leq \gamma_n^g \leq \frac{1-(1-q)^n}{nq}$, the principal's payoff is given by

$$\left[q\pi_1 V - q\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - (1-q)\,\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + (1-q)\,\psi_0^b\right] n\gamma_n^g$$
$$+ \left[(1-q)\,\pi_1 V - (1-q)\,\psi_1^b\right] \frac{1 - qn\gamma_n^g}{(1-q)}. \tag{25}$$

with the Case II transfers and after substituting $\gamma_n^b = \frac{1-qn\gamma_n^g}{(1-q)n}$

For $\frac{1}{n} \leq \gamma_n^g \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ (Case I), taking derivative with respect to γ_n^g we have (23) above.

For $\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \leq \gamma_n^g \leq \frac{1-(1-q)^n}{nq}$ (Case II), taking derivative with respect to γ_n^g we have

$$\left[nq\pi_1 V - nq\pi_1 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} - n\,(1-q)\,\pi_0 \frac{\psi_1^g - \psi_0^g}{\pi_1 - \pi_0} + n\,(1-q)\,\psi_0^b\right]$$
$$- \left[n\,(1-q)\,\pi_1 V - n\,(1-q)\,\psi_1^b\right] \frac{qn}{(1-q)\,n}$$

The last derivative is positive if and only if

$$\pi_1 q \left(\psi_1^b - \psi_1^g\right) > (1-q)\left[\pi_0\psi_1^g - \pi_1\psi_0^b\right] + (1-q)\,\pi_0\left(\psi_0^b - \psi_0^g\right) + q\left(\pi_0\psi_1^b - \pi_1\psi_0^g\right). \tag{26}$$

Thus, when (26) holds (24) also holds so the principal's payoff increases with γ_n^g over $\frac{1}{n} \leq \gamma_n^g \leq \frac{1-(1-q)^n}{nq}$ (i.e., the entire range of γ_n^g) and the optimal γ_n^g is given by $\gamma_n^g = \frac{1-(1-q)^n}{nq}$. In this case the optimal allocation rule is efficient. When (24) is violated then (26) is also violated and the objective function decreases with γ_n^g over $\frac{1}{n} \leq \gamma_n^g \leq \frac{1-(1-q)^n}{nq}$. So the optimal $\gamma_n^g = \frac{1}{n}$. In this case the optimal allocation rule is inefficient.

Finally, when (24) holds but (26) is violated, her payoff increases over $\frac{1}{n} \leq \gamma_n^g \leq \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ and decreases over $\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \leq \gamma_n^g \leq \frac{1-(1-q)^n}{nq}$. So the optimal $\gamma_n^g = \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ (i.e., $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$). In this case the optimal allocation rule is inefficient.

Having characterized the optimal mechanism for all n, we now show that whenever competition hurts under an efficient allocation rule (i.e., $n > n^*$), the optimal allocation rule is inefficient. Note that the conditions (24) and (26) for the principal's payoff to increase with γ_n^g under the optimal allocation rule is identical to the conditions (13) and (**??**) for her payoff to increase with n under an efficient allocation rule.

When $n \leq \hat{n}$, i.e., (21) holds, and competition hurts under efficient allocation, i.e., (13) does not hold, then the optimal $\gamma_n^g = \frac{1}{n}$ as we have shown above. Hence, the optimal allocation is inefficient.

When $n > \hat{n}$, and competition hurts under efficient allocation, i.e., (**??**) does not hold, then the optimal $\gamma_n^g = \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ (i.e., $\gamma_n = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$) if (24) holds and $\gamma_n = \frac{1}{n}$ if (24) does not hold. In either case, the optimal allocation is inefficient.

When competition helps under efficient allocation for all n, the optimal $\gamma_n = \frac{1-(1-q)^n}{nq}$ and allocation is efficient for all n, with $n^* = n$.

Finally, we show whenever competition hurts under an efficient allocation rule (i.e., $n > n^*$), the optimal allocation rule makes the principal's expected payoff equal to her payoff when $n = n^*$ under the efficient allocation rule. Note that in the optimal mechanism the principal's payoff, (22) and (25), depends on n only through $n\gamma_n^g$. Consider the case when competition hurts under efficient allocation, and therefore the optimal allocation is inefficient. Then, in the optimal mechanism, $n\gamma_n^g = \frac{\psi_1^b - \psi_1^g}{\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$ and $n\gamma_n^g = 1$, are independent of n and equal to the corresponding cases for $1 < n^* < \infty$ and $n^* = 1$ under efficient allocation defined in Section 4.[38]

This completes the proof of Proposition 4. ■

Proof of Corollary 3. When $n^* = 1$, if $n > n^*$, the optimal allocation is inefficient and given by $\gamma_n^g = \frac{1}{n}$. This corresponds to $\phi_r^g = \frac{r}{n}$.

When $n^* = \infty$ then for all n the efficient allocation is optimal so that the optimal $\phi_r^g = 1$ for $r \geq 1$.

If $1 < n^* < \infty$ and $n \leq n^*$ then the efficient allocation is still optimal by Proposition **4** which means the optimal $\phi_r^g = 1$ for $r \geq 1$. If $n > n^*$ then the optimal allocation is

[38] Recalling that $\gamma_n^b = \frac{1-qn\gamma_n^g}{(1-q)n}$ and $\gamma_n^g = \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$, we have the optimal $\gamma_n = \frac{\gamma_n^g}{\gamma_n^b} = \frac{\psi_1^b - \psi_1^g}{\psi_0^b - \psi_0^g}$ when $1 < n^* < \infty$.

inefficient and we want to find the ϕ_r^g that give

$$\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{\phi_r^g}{r}$$

From efficient allocation rules we have

$$1 = \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \left(\frac{nq}{1-(1-q)^n}\right)$$

which, after multiplying both sides, becomes

$$\begin{aligned}
&\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} \\
&= \sum_{r=1}^{n} \binom{n-1}{r-1} q^{r-1} (1-q)^{n-r} \frac{1}{r} \left(\frac{nq}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}
\end{aligned}$$

Let

$$\phi_r^g = \left(\frac{nq}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)}$$

The ϕ_r^g are well-defined allocation probabilities if $\phi_r^g \leq 1$. So it is enough to show that

$$\left(\frac{nq}{1-(1-q)^n}\right) \frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} < 1$$

or,

$$\frac{\psi_1^b - \psi_1^g}{n\left((1-q)\left(\psi_0^b - \psi_0^g\right) + q\left(\psi_1^b - \psi_1^g\right)\right)} < \frac{\left(1-(1-q)^n\right)}{nq}$$

which is true for $n > \hat{n}$ by definition of $\hat{n}$. Therefore, ϕ_r^g defined above gives the allocation that implements the allocation probability in this case. This completes the proof of Corollary 3. ■

Proof of Proposition 5. Denoting the probability of allocating the contract to a good type by $\delta_n \equiv nq\gamma_n^g$, the expected payoff to the principal under a given contract is given by

$$\delta_n \left(\pi_{e_g} V - \pi_{e_g} t_h^g - \left(1-\pi_{e_g}\right) t_l^g\right) + (1-\delta_n)\left(\pi_{e_b} V - \pi_{e_b} t_h^b - \left(1-\pi_{e_b}\right) t_l^b\right)$$

which can be rewritten as

$$\delta_n \pi_{e_g} V + (1-\delta_n)\pi_{e_b} V - \left[\delta_n \left(\pi_{e_g} t_h^g + \left(1-\pi_{e_g}\right) t_l^g\right) + (1-\delta_n)\left(\pi_{e_b} t_h^b + \left(1-\pi_{e_b}\right) t_l^b\right)\right]$$

First, notice that given a good type is favored in an auction and the effort levels assigned to the two types, the additive separability of V and the transfers imply that the principal's expected payoff maximization problem defined in Section 3 is equivalent to her expected transfer minimization problem. Second in the minimization problem subject to the relevant constraints, the minimum is well defined. Let the minimum expected transfer be defined for a given effort assignment e_g and e_b by $\tau(e_g, e_b)$.

The principal's problem is then to solve

$$\max_{e_g, e_b} \delta_n \pi_{e_g} V + (1 - \delta_n) \pi_{e_b} V - \tau(e_g, e_b).$$

Specifically, we need to identify the maximum among

$$\pi_1 V - \tau(1,1), \ \left(\delta_n \pi_1 + (1 - \delta_n) \pi_0\right) V - \tau(1,0), \text{ and } \pi_0 V - \tau(0,0)$$

It follows that for V large enough the maximum is given by setting $e_g = 1, e_b = 1$ because $\pi_1 > \left(\delta_n \pi_1 + (1 - \delta_n) \pi_0\right) > \pi_0$ and for large enough V we have

$$\pi_1 V - \left(\delta_n \pi_1 + (1 - \delta_n) \pi_0\right) V > \tau(1,1) - \tau(1,0)$$

and

$$\left(\delta_n \pi_1 + (1 - \delta_n) \pi_0\right) V - \pi_0 V > \tau(1,0) - \tau(0,0).$$

This completes the proof of Proposition 5. ■

9 Appendix B: Screening with Effort

We consider a model where the agent privately chooses effort $e \in [0,1]$. We focus on interior solutions such that effort adjusts continuously as n increases. Accordingly, we will assume that V is not too large.[39] Let $\psi_x(e) = xe^2$ be the cost of effort to type $x \in \{b, g\}$, where $0 < g < b$. We assume that the probability of high outcome given effort e is $\pi(e)$, and $\pi(e) = e$.

[39] Our binary model presents a corner solution where V is so large that $e = 1$ is always optimal. When effort is binary a smaller V would result in effort choices $(e_g = 1, e_b = 0)$ or $(e_g = 0, e_b = 0)$. In that case, the solution is given by (IC_0^b) and (MH_g), but our key results continue to hold.

With continuous effort the agent has more options when shirking, and the (IC^b) is binding for all n without the need for parameter restrictions. As in the binary effort model, we again have two cases depending on whether (IC^g) is binding. The (IC^g) is binding for $\gamma_n < \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$ but slack for $\gamma_n \geq \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$. As before, we denote by n^* the number of agents for which the principal's payoff is the highest under an efficient allocation rule.

For competition to hurt, under allocative efficiency, we do need restrictions on parameters that are reminiscent of results from the binary model. We show that if

$$(b-g)^2/g^2 < 1/q, \tag{27}$$

which is equivalent to $\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g}{b}\frac{(1-q)}{q}\right)^2$, then n^* is the solution to $\gamma_n = \left(\frac{b}{g}\right)^2$ under allocative efficiency.[40] We define γ_n^* by $\gamma_n^* \equiv \left(\frac{b}{g}\right)^2$. When condition (27) is satisfied, the adverse selection problem is not too strong (b and g are close to each other), and competition hurts for $n > n^*$ under an efficient allocation rule. On the other hand, if $(b-g)^2/g^2 > 1/q$, competition can only help.

We can again show that an inefficient allocation rule with $\phi_r^g \leq 1$, with strict inequality for some $r \geq 1$, is optimal whenever the principal's payoff is decreasing in n under an efficient allocation rule. We can also show that the principal prefers to have some competition. That is, if he can choose the number of agents, he will pick at least two agents to participate in the mechanism. Because the solution under a general allocation rule is $\gamma_n^* = \left(\frac{b}{g}\right)^2 > 1$, and $\gamma_1 = 1$, the principal is better off with at least two agents and an inefficient allocation rule ($\phi_r^g < 1$) than having exactly one agent. Thus, some competition is always helpful.

We summarize these results in the proposition below.

Proposition 6 *With continuous effort,*

(i) under an efficient allocation rule there exists n^* such that competition hurts the principal for $n > n^*$ if and only if $(b-g)^2/g^2 < 1/q$, i.e. the adverse selection problem is not too strong;

[40] If there is no integer that satisfies the equality under allocative efficiency, consider n satisfying $\gamma_{n-1} < \left(\frac{b}{g}\right)^2 < \gamma_n$ under allocative efficiency, and define n^* to be $n-1$ or n depending on which results in a higher expected payoff for the principal.

(ii) whenever competition hurts under an efficient allocation rule (i.e., $n > n^*$), the optimal allocation rule is inefficient, and it makes the principal's expected payoff equal to her payoff when $n = n^*$ under the efficient allocation rule;[41] *when competition helps under an efficient allocation rule, the optimal allocation is efficient;*

(iii) some degree of competition is always helpful.

Proof of Proposition 6 Rewriting the probability of allocating the contract to a good type in terms of γ_n, we have $\delta_n = \frac{q\gamma_n}{1-q+q\gamma_n}$. The principal chooses the transfers to maximize

$$\begin{aligned}&\left(\delta_n \pi\left(e_g\right)+\left(1-\delta_n\right) \pi\left(e_b\right)\right) V-\delta_n\left(\pi\left(e_g\right) t_h^g+\left(1-\pi\left(e_g\right)\right) t_l^g\right)\\&-\left(1-\delta_n\right)\left(\pi\left(e_b\right) t_h^b+\left(1-\pi\left(e_b\right)\right) t_l^b\right)\end{aligned}$$

s.t.

$$\begin{aligned}&IC^g: \gamma_n\left(\pi\left(e_g\right) t_h^g+\left(1-\pi\left(e_g\right)\right) t_l^g-\psi_g\left(e_g\right)\right) \geq \pi\left(e_{gb}\right) t_h^b+\left(1-\pi\left(e_{gb}\right)\right) t_l^b-\psi_g\left(e_{gb}\right)\\&IC^b: \pi\left(e_b\right) t_h^b+\left(1-\pi\left(e_b\right)\right) t_l^b-\psi_b\left(e_b\right) \geq \gamma_n\left(\pi\left(e_{bg}\right) t_h^g+\left(1-\pi\left(e_{bg}\right)\right) t_l^g-\psi_b\left(e_{bg}\right)\right)\end{aligned}$$

where the optimal effort by a type x agent who reports type y is denoted by

$$e_{xy} \in \arg\max_e \left\{\pi(e) t_h^y+(1-\pi(e)) t_l^y-\psi_x(e)\right\}.$$

Note that we write e_{gg} as e_g and e_{bb} as e_b.

We show the following in the online appendix: writing $\Delta_y \equiv t_h^y - t_l^y$ for $y = g, b$, in the optimal mechanism,

(a) the constraint (IC^b) is always binding,

(b) when $\gamma_n < \left(2+\frac{g(1-q)}{qb}\right)^2$, (IC^g) is binding and the solution to the principal's problem is given by:

$$\begin{aligned}&\Delta_g=\frac{\left(\delta_n b+\left(1-\delta_n\right) \sqrt{\gamma_n} g\right)}{2\left(b \delta_n+g \gamma_n\left(1-\delta_n\right)\right)} V,\ \Delta_b=\frac{\sqrt{\gamma_n}\left(\delta_n b+\left(1-\delta_n\right) \sqrt{\gamma_n} g\right)}{2\left(b \delta_n+g \gamma_n\left(1-\delta_n\right)\right)} V\\&t_l^g=0,\ t_l^b=0,\end{aligned}$$

[41] Again, the equality is approximate, as before, since n can take only discrete values.

and, when $\gamma_n \geq \left(2 + \frac{g(1-q)}{qb}\right)^2$, (IC^g) is slack, and the solution to the principal's problem is given by:[42]

$$\Delta_g = \frac{bVq}{2bq + (1-q)\,g}, \Delta_b = V$$
$$t_l^g = 0,\, t_l^b = \frac{1}{4b}\left(\gamma_n\left(\frac{qbV}{2qb + (1-q)\,g}\right)^2 - V^2\right).$$

Thus, we have solved for the optimal transfers given γ_n.We next analyze how the objective function depends on γ_n.

Putting the transfers into the objective function when $\gamma_n < \left(2 + \frac{g(1-q)}{qb}\right)^2$, i.e., (IC^g) is binding, and using $\delta_n = \frac{q\gamma_n}{1-q+q\gamma_n}$ the principal's objective function can be written as

$$\frac{\left(qb\sqrt{\gamma_n} + (1-q)\,g\right)^2}{(1-q+q\gamma_n)}\frac{V^2}{8bg\,(bq + g\,(1-q))}$$

Then taking derivative with respect to $\sqrt{\gamma_n}$ and simplifying we have

$$\frac{b - g\sqrt{\gamma_n}}{(1-q+q\gamma_n)^2}\frac{q\,(1-q)\left(qb\sqrt{\gamma_n} + (1-q)\,g\right)V^2}{4bg\,(bq + g\,(1-q))}$$

which is positive if and only if

$$\gamma_n < \left(\frac{b}{g}\right)^2.$$

Observation 1: This also implies that under allocative efficiency whenever $\left(2 + \frac{g(1-q)}{qb}\right)^2 > \gamma_n > \left(\frac{b}{g}\right)^2$ competition decreases the principal's expected payoff in this case.

Next putting the transfers into the objective function when $\gamma_n \geq \left(2 + \frac{g(1-q)}{qb}\right)^2$, i.e., IC^g is slack, and using $\delta_n = \frac{q\gamma_n}{1-q+q\gamma_n}$ the principal's objective function can be written as

$$\left[\frac{b^2q^2\gamma_n + (1-q)\,g\,(2qb + (1-q)\,g)}{(1-q+q\gamma_n)}\right]\frac{1}{2bg\,(2qb + (1-q)\,g)}V^2$$

The derivative of the bracketed term with respect to γ_n is given by

$$\frac{b^2q - g\,(2qb + (1-q)\,g)}{(1-q+q\gamma_n)^2}\,(1-q)\,q$$

[42]Note that when $g = \frac{\sqrt{\gamma_n}-2}{(1-\delta_n)\gamma_n}\delta_n b$ the solution IC^g holds with equality and the two solutions coincide.

Observation 2: Hence the principal's expected payoff is decreasing with γ_n if and only if

$$\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2 \leq \gamma_n.$$

Proof of Part (i). Under allocative efficiency, when $\left(\frac{b}{g}\right)^2 > \left(2 + \frac{g(1-q)}{qb}\right)^2$, then competition only helps the principal by Observations 1 and 2 above. Again, under allocative efficiency and by Observations 1 and 2 above, if $\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2$ competition helps the principal when $\gamma_n < \left(\frac{b}{g}\right)^2$ and hurts the principal both when $\left(\frac{b}{g}\right)^2 < \gamma_n < \left(2 + \frac{g(1-q)}{qb}\right)^2$ and $\gamma_n \geq \left(2 + \frac{g(1-q)}{qb}\right)^2$.

When $\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2$ is satisfied, define n^* by the n that solves $\gamma_n^* = \left(\frac{b}{g}\right)^2$ if there is an integer that satisfies the equality under allocative efficiency; otherwise consider n satisfying $\gamma_{n-1} < \left(\frac{b}{g}\right)^2 < \gamma_n$ under allocative efficiency, and define n^* to be $n-1$ or n depending on which results in a higher expected payoff for the principal. It follows that when $\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2$ competition hurts the principal if and only if $n > n^*$. When $\left(\frac{b}{g}\right)^2 > \left(2 + \frac{g(1-q)}{qb}\right)^2$, competition always helps the principal.

The part (i) of the proof is completed upon observing the following equivalence

$$\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2 \Longleftrightarrow (b-g)^2/g^2 < 1/q.$$

□

Proof of Part (ii). Suppose competition hurts under efficient allocation rule, i.e., $\left(\frac{b}{g}\right)^2 < \left(2 + \frac{g(1-q)}{qb}\right)^2$ and $n > n^*$. This implies that under efficient allocation rule $\gamma_n = \frac{1-(1-q)^n}{q(1-q)^{n-1}} > \left(\frac{b}{g}\right)^2$ whereas the optimal γ_n is equal to $\gamma_n^* = \left(\frac{b}{g}\right)^2$. Under a general allocation rule the only way to obtain $\gamma_n = \left(\frac{b}{g}\right)^2$ in this case is through choosing $\phi_r < 1$ for at least some r, i.e., by choosing an inefficient (random) allocation rule. □

Proof of Part (iii). Because $\gamma_n^* = \left(\frac{b}{g}\right)^2 > 1$ and $\gamma_1 = 1$, the principal is better off with at least two agents and an inefficient allocation rule than having exactly one agent. Thus, some competition is always helpful. ■

10 References

Anton, James, and Dennis Yao, 1989, "Split Awards, Procurement, and Innovation," *The RAND Journal of Economics*, Vol. 20(4), pp. 538-552.

Anton, James, and Dennis Yao, 1992, "Coordination in Split Award Auctions," *The Quarterly Journal of Economics*, Vol. 107(2), pp. 681-707.

Ariely, Daniel, Axel Ockenfels and Alvin Roth, 2005, "An Experimental Analysis of Ending Rules in Internet Auctions," *The RAND Journal of Economics*, Vol. 36(4), pp. 890-907.

Armstrong, Mark, and David E.M. Sappington, 2007, "Recent Developments in the Theory of Regulation," *Handbook of Industrial Organization* (eds: by M. Armstrong and R. Porter), vol. 3, ch-27.

Arve Malin, and David Martimort, 2016, "Dynamic Procurement under Uncertainty: Optimal Design and Implications for Incomplete Contracts," *The American Economic Review*, vol. 106(11), pp.3238–3274.

Asker, John, and Estelle Cantillon, 2010, "Procurement when price and quality matter," *The RAND Journal of Economics*, Vol. 41(1), pp. 1-34.

Bajari, Patrick, Stephanie Houghton, and Steven Tadelis, 2014, "Bidding for Incomplete Contracts: An Empirical Analysis of Adaptation Costs," *American Economic Review*, 104(4), pp. 1288-1319.

Bénabou, Roland, and Jean Tirole, 2016, "Bonus Culture: Competitive Pay, Screening, and Multitasking," *Journal of Political Economy*, 124(2), pp. 305-370.

Border, Kim, 1991, "Implementation of reduced form auctions: A geometric approach," *Econometrica*, vol. 59, pp. 1175–1187.

Burguet, Roberto, Juan-José Ganuza, Esther Hauk, 2012, "Limited liability and mechanism design in procurement," *Games and Economic Behavior*, vol. 76, pp. 15–25.

Chakraborty, Indranil, Fahad Khalil, and Jacques Lawarree, 2019, "Competitive Procurement with Ex Post Moral Hazard," Manuscript, University of Washington.

Che, Yeon-Koo, 1993, “Design Competition Through Multidimensional Auctions,” *The RAND Journal of Economics*, Vol. 24(4), pp. 668-680 .

Che, Yeon-Koo, Elisabetta Iossa, and Patrick Rey, 2017, “Prizes versus Contracts as Incentives for Innovation,” Manuscript, Toulouse School of Economics.

Chillemi, Ottorino, and Claudio Mezzetti, 2014, “Optimal Procurement Mechanisms: Bidding on Price and Damages for Breach," *Economic Theory*, vol. 55(2), pp. 335-355.

Decarolis Francesco, 2018, "Comparing Public Procurement Auctions, " *International Economic Review*, vol. 59 (2), pp. 391–419.

Djankov, Simeon, Federica Saliola, and Asif Islam, 2016, “Is Public Procurement a Rich Country’s Policy?” *Governance for Development*, The World Bank, 2016.

Demsetz, Harold, 1968, “The costs of transacting,” *Quarterly Journal of Economics*, vol. 82, pp. 33-53.

Eun, Dong-Jae, 2019, “Reining in Ex-post Adjustment: A New Procurement Mechanism from Korea,” Manuscript, University of Washington.

European Commission, 2015, “*Public procurement guidance for practitioners*,” Brussels, Belgium.

Fudenberg, Drew, and Jean Tirole, 1991, "Game Theory, " MIT Press.

Fullerton, Richard, and R. Preston McAfee, 1999, "Auctioning Entry into Tournaments,” *Journal of Political Economy*, vol. 107(3), pp. 573-605.

Gottlieb, Daniel, and Humberto Moreira, 2017, “Simple Contracts with Adverse Selection and Moral Hazard,” Manuscript.

Hart, Oliver, Andrei Shleifer, and Robert W. Vishny, 1997, “The Proper Scope of Government: Theory and an Application to Prisons,” *Quarterly Journal of Economics*, Vol 112(4), pp 1127–1161.

Kadan, Ohad, Philip Reny, and Jeroen Swinkels, 2017, “Existence of optimal mechanisms in principal-agent problems,” *Econometrica*, Vol. 85(3), pp.769–823.

Kennard, William, 1999, *Connecting the Globe: A Regulator’s Guide to Building a*

Global Information Community. Federal Communication Commission.

Kogan Shimon and John Morgan, 2010, "Securities Auctions under Moral Hazard: An Experimental Study," *Review of Finance,* vol. 14(3), pp.477-520.

Laffont, Jean-Jacques, and David Martimort, 2002, *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

Laffont, Jean-Jacques, and Jean Tirole, 1987, "Auctioning Incentive Contracts," *Journal of Political Economy*, vol. 95(5), pp. 921-937.

Lewis, Gregory, and Patrick Bajari, 2011, "Procurement Contracting With Time Incentives: Theory and Evidence," *The Quarterly Journal of Economics*, vol. 126(3), pp. 1173-1211.

Lewis, Gregory, and Patrick Bajari, 2014, "Moral hazard, incentive contracts, and risk: evidence from procurement," *Review of Economic Studies*, vol. 81(3), pp. 1201-1228.

Manelli, Alejandro M., and Daniel R. Vincent, 1995, "Optimal Procurement Mechanisms," *Econometrica*, vol. 63(3), pp. 591-620.

Maskin, Eric, and John Riley, 1984, "Optimal auctions with risk averse buyers," *Econometrica*, vol. 52(6), pp. 1473-1518.

McAfee, R. Preston, and John McMillan, 1986, "Bidding for Contracts: A Principal-Agent Analysis," *The RAND Journal of Economics*, vol. 17(3), pp. 326-338

McAfee, R. Preston, and John McMillan, 1987, "Competition for Agency Contracts," *The RAND Journal of Economics*, Vol. 18(2), pp. 296-307.

Montagnes, Pablo, and Richard Van Weelden, 2019, "On the Incentives to Exacerbate Polarization," Manuscript.

Myerson, Roger B., 1981, "Optimal Auction Design," *Mathematics of Operations Research*, Vol. 6(1), pp. 58-73.

Ollier, Sandrine, and Lionel Thomas, 2013, "Ex post participation constraint in a principal–agent model with adverse selection and moral hazard," *Journal of Economic Theory*, Vol 148 (6), pp. 2383-2403.

Picard, Pierre, 1987, "On the design of incentive schemes under moral hazard and adverse selection." *Journal of Public Economics*, vol. 33(3), pp. 305-331.

Piccione, Michele, and Guofu Tan, 1996, "Cost-Reducing Investment, Optimal Procurement and Implementation by Auctions." *International Economic Review*, vol. 37(3), pp. 663–685.

Puckrein, Gary A., Gail Nunlee-Bland, Farhad Zangeneh, Jaime A. Davidson, Robert A. Vigersky, Liou Xu, Christopher G. Parkin and David G. Marrero, 2016, "Impact of CMS Competitive Bidding Program on Medicare Beneficiary Safety and Access to Diabetes Testing Supplies: A Retrospective, Longitudinal Analysis," *Diabetes Care*, vol. 39(4), 563-571.

Riordan, Michael H., and David E. M. Sappington, 1987, "Awarding Monopoly Franchises," *The American Economic Review*, vol. 77(3), pp. 375-387.

Strausz, Roland, 2006, "Deterministic versus stochastic mechanisms in principal–agent models," *The Journal of Economics Theory*, vol. 128, pp. 306-214.

Taylor, Curtis, 1995, "Digging for Golden Carrots: An Analysis of Research Tournaments," *The American Economic Review*, vol. 85(4), pp. 872-890.

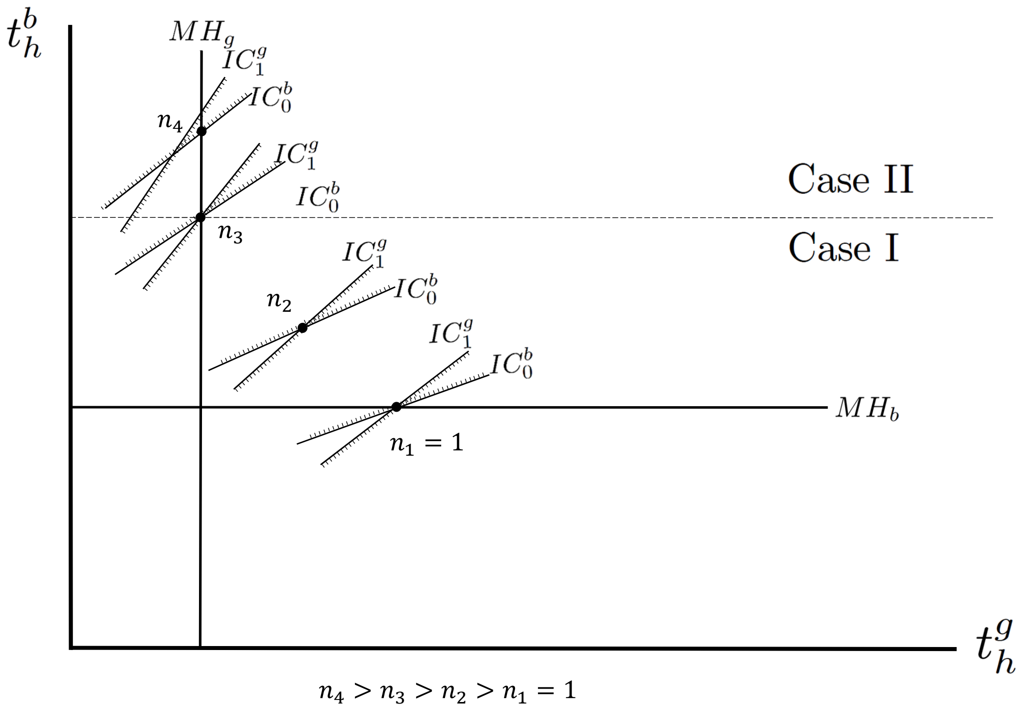

Figure 1: Optimal mechanism