

Estimating the Dynamics of Price Discovery*

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Abstract

In this paper we propose a new approach for the econometric analysis of the dynamics of price discovery using a structural cointegration model for the price changes in arbitrage linked markets. Our methodology characterizes the dynamics of price discovery based on the impulse response functions from an identified structural cointegration model, and we measure the efficiency of a market's price discovery by the absolute magnitude of cumulative pricing errors in the price discovery process. We apply our methodology to investigate the extent to which the US dollar contributes to the price discovery of the yen/euro exchange rate. Our results show that substantial price discovery of JPY/EUR occurs through the dollar, and that the efficiency of the dollar's price discovery is positively related to the relative liquidity of the dollar markets versus the cross rate market.

Key words: cointegration, permanent-transitory decomposition, price discovery, structural model.

1 Introduction

Price discovery is one of the central functions of financial markets. In the market microstructure literature, it has been variously interpreted as, “the search for an equilibrium price” (Schreiber and

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Schwartz, 1986), “gathering and interpreting news” (Baillie et al., 2002), “the incorporation of the information implicit in investor trading into market prices” (Lehmann, 2002). These interpretations suggest that price discovery is dynamic in nature, and an efficient price discovery process is characterized by the fast adjustment of market prices from the old equilibrium to the new equilibrium with the arrival of new information. In particular, Madhavan (2002) distinguishes dynamic price discovery issues from static issues such as trading cost determination.

One notable institutional trend of financial markets is the trading of identical or closely related assets in multiple market places. This trend has raised a number of important questions. Does the proliferation of alternative trading venues and the resulting market fragmentation adversely affect the price discovery process? How do the dynamics of price discovery of an asset depend on market characteristics, such as transaction costs and liquidity? What institutional structures and trading protocols facilitate the information aggregation and price discovery process? Answering these questions requires an econometric methodology for measuring price discovery dynamics. In contrast to the wide literature on transaction costs, however, the studies on price discovery are relatively limited. In a recent survey of market microstructure studies, Madhavan (2002) remarks, “The studies surveyed above can be viewed as analyzing the influence of structure on the magnitude of the friction variable. What is presently lacking is a deep understanding of how structure affects return dynamics, in particular, the *speed* (italics as cited) of price discovery.” In this paper, we propose an approach to directly characterize the speed of price discovery in the context of an asset trading in multiple markets.

At the most general modeling perspective, each of the observable prices of an asset in multiple markets can be decomposed into two components: one reflecting the common efficient (full-information) price shared by all these markets (Garbade and Silber, 1979); and one reflecting the transitory frictions that arise from the trading mechanism, such as the bid-ask bounce, liquidity effects, and rounding errors. Evolving as a random walk, the common efficient price captures the fundamental value of the financial asset and its innovation impounds the expectation revisions of investors (thus new information) about the asset payoffs. How observed prices respond to the common efficient price innovation characterizes the dynamics of price discovery. Unfortunately, as emphasized by Hasbrouck (2002), the common efficient price (and its innovation) is generally unobservable. Therefore, identifying the common efficient price innovations is a necessary step before

any meaningful measure of price discovery can be constructed.

In the multiple markets context, the common random walk efficient price implies that the observable prices in different markets for the same asset are cointegrated with a known cointegrating vector and a known long-run impact of the efficient price innovation. In this paper, we show that the structure of the cointegration model allows for the identification of the efficient price innovation with minimal restrictions using a modification of the permanent and transitory (P-T) decomposition of Gonzalo and Ng (2001). With an identified structural cointegration model in hand, we propose new measures of price discovery based on impulse response functions to characterize the dynamics of a market's price discovery process. In this framework, we measure one market's contribution to price discovery by the relative speed to which its observed price moves to the new fundamental value following a shock to the efficient price, and by the magnitude of cumulative pricing errors in the adjustment to the new fundamental value. As a result, our methodology establishes a framework to directly quantify the dynamics of price discovery.

We apply our methodology to investigate the extent to which the US dollar contributes to the price discovery of the yen/euro exchange rate. Our results show that substantial price discovery of JPY/EUR occurs through the dollar, and that the efficiency of the dollar's price discovery is positively related to the relative liquidity of the dollar markets versus the cross rate market.

The rest of the paper is organized as follows. In section 2, we develop a dynamic structural cointegration model for the price changes in two arbitrage linked markets and we propose new dynamic measures of price discovery based on structural impulse response functions. We discuss identification and estimation in section 3, and also perform some simulation experiments to evaluate the finite sample performance of our proposed methodology. In section 4, we present an empirical application of our methodology using ultra high frequency foreign exchange data. Section 5 contains our concluding remarks. Some technical results are given in two short appendices.

2 Structural Price Discovery Cointegration Model

As emphasized by Lehmann (2002), because standard measures of price discovery (e.g., the information share measure of Hasbrouck, 1995) are based on the residuals from a reduced form vector error correction model (VECM) their interpretation is not always clear. A clear interpretation of

price discovery is only possible in a structural model; i.e., a model in which the sources of shocks are identified. In this section, we propose a structural cointegration model for prices in arbitrated linked markets that identifies permanent information and transitory liquidity shocks with minimal restrictions. Our model is motivated by the structural vector autoregressive (SVAR) models widely used in empirical macroeconomics (e.g. Bernanke, 1986, Blanchard and Quah, 1989, King et al., 1991). An excellent survey of these models, from which we draw heavily, is given in Levtchenkova, Pagan, and Robertson (1999).

2.1 Structural Moving Average Representation

Consider a single asset traded in two distinct markets. The generalization to n markets is discussed in Appendix B. Let $\mathbf{p}_t = (p_{1,t}, p_{2,t})'$ denote a 2×1 vector of log prices for the asset from the two markets. In a multiple-trading environment, these prices may be the trade prices or quotes from different trading venues. More generally, the prices may be an asset's cash market price and the price of its derivatives, or the observed price of an asset and its price synthetically constructed from other financial assets. As a result, these prices are closely linked by arbitrage. We assume that the efficient price of the asset follows a random walk shared by each of these observed prices so that \mathbf{p}_t is integrated of order 1, or $I(1)$, and the price change, $\Delta \mathbf{p}_t$, is integrated of order zero, or $I(0)$. Because the prices in \mathbf{p}_t are for the same underlying asset we assume that \mathbf{p}_t is cointegrated with known cointegrating vector $\boldsymbol{\beta} = (1, -1)'$ so that $\boldsymbol{\beta}'\mathbf{p}_t = p_{1,t} - p_{2,t}$ is $I(0)$.

We assume that $\Delta \mathbf{p}_t$ has a structural moving average (SMA) representation of the form

$$\Delta \mathbf{p}_t = \mathbf{D}(L)\boldsymbol{\eta}_t = \mathbf{D}_0\boldsymbol{\eta}_t + \mathbf{D}_1\boldsymbol{\eta}_{t-1} + \mathbf{D}_2\boldsymbol{\eta}_{t-2} + \cdots, \quad (1)$$

where $\mathbf{D}(L) = \sum_{k=0}^{\infty} \mathbf{D}_k L^k$, $\mathbf{D}_0 \neq \mathbf{I}_2$, the elements of $\{\mathbf{D}_k\}_{k=0}^{\infty}$ are 1-summable, and \mathbf{D}_0 is invertible. We omit any deterministic terms in (1) for ease of exposition. We assume that the number of structural shocks is equal to the number of observed prices, so that $\mathbf{D}(L)$ is invertible. We relax this assumption in sub-section 3.2.2. The innovation to the efficient price of the asset, η_t^P , is labeled permanent and the noise innovation, η_t^T , is labeled transitory so that $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$. These structural shocks are assumed to be serially and mutually uncorrelated with diagonal covariance matrix $\mathbf{C} = \text{diag}(\sigma_P^2, \sigma_T^2)$. In price discovery models, the permanent shock is the innovation to

the fundamental value reflecting new information and economic considerations suggest that this innovation should be uncorrelated with the transitory microstructure shocks. The matrix \mathbf{D}_0 contains the initial impacts of the structural shocks on $\Delta \mathbf{p}_t$, and defines the contemporaneous correlation structure of $\Delta \mathbf{p}_t$. Given the dichotomy into permanent and transitory shocks, the SMA model may be re-expressed equation-by-equation as

$$\begin{pmatrix} \Delta p_{1,t} \\ \Delta p_{2,t} \end{pmatrix} = \begin{pmatrix} d_1^P(L) & d_1^T(L) \\ d_2^P(L) & d_2^T(L) \end{pmatrix} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix},$$

where $d_i^P(L)$ and $d_i^T(L)$ ($i = 1, 2$) are lag polynomials describing the dynamic responses to the permanent and transitory shocks, respectively.

The permanent innovation η_t^P carries new information on the fundamental value of the asset, and permanently moves the market prices. The defining characteristic of η_t^P is that it has a one-to-one long-run impact on the expected price levels for each market:

$$\lim_{k \rightarrow \infty} \frac{\partial E_t [\mathbf{p}_{t+k}]}{\partial \eta_t^P} = \lim_{k \rightarrow \infty} \sum_{l=0}^k \frac{\partial E_t [\Delta \mathbf{p}_{t+l}]}{\partial \eta_t^P} = \lim_{k \rightarrow \infty} \sum_{l=0}^k \mathbf{D}_l^P = \mathbf{D}^P(1) = \mathbf{1}, \quad (2)$$

where \mathbf{D}_k^P and $\mathbf{D}^P(1)$ are the first column of the dynamic multiplier matrix \mathbf{D}_k and the long-run impact matrix $\mathbf{D}(1) = \sum_{k=0}^{\infty} \mathbf{D}_k$ that corresponds to η_t^P , respectively, and $\mathbf{1} = (1, 1)'$.

The transitory innovation η_t^T summarizes non-information related shocks, such as the trading by uninformed or liquidity traders and market microstructure effects. The defining characteristic of η_t^T is that it is uncorrelated with the informational innovation η_t^P , and has no long-run effect on the expected price levels:

$$\lim_{k \rightarrow \infty} \frac{\partial E_t [\mathbf{p}_{t+k}]}{\partial \eta_t^T} = \lim_{k \rightarrow \infty} \sum_{l=0}^k \frac{\partial E_t [\Delta \mathbf{p}_{t+l}]}{\partial \eta_t^T} = \lim_{k \rightarrow \infty} \sum_{l=0}^k \mathbf{D}_l^T = \mathbf{D}^T(1) = \mathbf{0}, \quad (3)$$

where \mathbf{D}_k^T and $\mathbf{D}^T(1)$ are the second column of the dynamic multiplier matrix \mathbf{D}_k and the long-run impact matrix $\mathbf{D}(1)$ that corresponds to η_t^T , respectively, and $\mathbf{0} = (0, 0)'$. Hence, the long-run

impact matrix of the structural innovations $\boldsymbol{\eta}_t$ has the form

$$\mathbf{D}(1) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = [\mathbf{1} : \mathbf{0}]. \quad (4)$$

Using (4), the Beveridge-Nelson (BN) decomposition (Beveridge and Nelson, 1981) of the SMA model in (1) is

$$\mathbf{p}_t = \mathbf{p}_0 + \mathbf{D}(1) \sum_{j=1}^t \boldsymbol{\eta}_j + \mathbf{s}_t = \mathbf{p}_0 + \mathbf{1}m_t + \mathbf{s}_t, \quad (5)$$

where $\mathbf{s}_t = \mathbf{D}^*(L)\boldsymbol{\eta}_t$, $\mathbf{D}_k^* = -\sum_{j=k+1}^{\infty} \mathbf{D}_j$, $k = 0, \dots, \infty$, and $m_t = m_{t-1} + \eta_t^P$. Similar to the stylized microstructure models of market prices (e.g., Glosten, 1987), equation (5) shows that each of the market prices for the asset is composed of an unobservable common efficient price, a transitory pricing error $s_{i,t}$ in market i , and a constant. The common efficient price is the driving force of the cointegrated prices. The transitory nature of $s_{i,t}$ implies that $p_{i,t}$ will adjust to the efficient price m_t over time. The remaining constant reflects any nonstochastic difference between the market price and its efficient price; e.g., the average (half) bid-ask spread or the initial value. For simplicity, in what follows assume \mathbf{p}_0 is equal to the zero vector.

Unlike the stylized models commonly used in previous price discovery studies, as summarized by Lehmann (2002), the model (5) clearly identifies how the prices move in response to new information or liquidity shocks. To see this more clearly, rewriting (5) equation-by-equation gives:

$$\begin{pmatrix} p_{1,t} \\ p_{2,t} \end{pmatrix} = \begin{pmatrix} m_t \\ m_t \end{pmatrix} + \begin{pmatrix} d_1^{*P}(L) & d_1^{*T}(L) \\ d_2^{*P}(L) & d_2^{*T}(L) \end{pmatrix} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix}, \quad (6)$$

where $d_i^{*P}(L)$ and $d_i^{*T}(L)$ ($i = 1, 2$) are lag polynomials describing pricing error responses to new information and liquidity shocks, respectively. Price responses to new information involve a permanent change in the efficient price, and the transitory adjustments toward the new equilibrium are captured by $d_i^{*P}(L)$ as the markets work out various market imperfections. The latter dynamics characterize the workings of the price discovery process. In contrast, price responses to liquidity shocks only involve transitory price fluctuations around a fixed efficient price level captured by $d_i^{*T}(L)$.

We emphasize that minimal assumptions, except for the assumption that $\mathbf{D}(L)$ is invertible, have been imposed on the SMA model specification (6). Only the long-run impacts of the structural innovations are specified. How the market prices respond overtime to new information and liquidity shocks is left unrestricted. In particular, the lag lengths of $d_i^{*P}(L)$ and $d_i^{*T}(L)$ are unrestricted and the prices are not required to “fully adjust” over some prespecified time intervals. Therefore, the model is general enough to accommodate various complex market microstructure effects at various stages of the price discovery process. In sub-section 3.2.2, we relax the assumption that $\mathbf{D}(L)$ is invertible and discuss conditions under which our main results remain unchanged.

2.2 Measuring Price Discovery Dynamics

We define price discovery as the dynamic process by which a market incorporates new fundamental information about an asset’s value into the asset’s price. A market is more efficient in the price discovery process than another market if it incorporates a larger amount of the new information more quickly. In general, the dynamic adjustment process involves reactions to both permanent and transitory shocks, and what matters for price discovery is the dynamic adjustment to permanent shocks.

In a typical multiple market trading scenario, when the fundamental price an asset increases by \$1, the traded or quoted prices of the asset in all markets will eventually reflect the \$1 increase in the efficient price. However, the prices from the different markets may converge to the new equilibrium price with different speeds. A leading market’s price may converge very quickly (impounding the new information), and pull other markets’ prices toward the new equilibrium (through arbitrage). The leading market thus has a greater contribution to price discovery. From this dynamic perspective, the relative speed of multiple markets converging toward the new efficient price reveals each market’s contribution to the price discovery process.

The structural cointegration model in (1) offers a convenient tool to directly characterize how multiple market prices for the same underlying asset discover the new equilibrium price following the arrival of new information. The tool is based on the impulse response function (IRF) of a market’s price to the permanent innovation of the common efficient price implied by the cointegration model. From (1), the expected price response in market i , k periods after a one unit increase to the

permanent shock η_t^P , is given by:

$$f_{i,k} = \frac{\partial E_t [p_{i,t+k}]}{\partial \eta_t^P} = \sum_{l=0}^k \frac{\partial E_t [\Delta p_{i,t+l}]}{\partial \eta_t^P} = \sum_{l=0}^k d_{i,l}^P, \quad k = 0, 1, \dots; i = 1, 2, \quad (7)$$

where $d_{i,l}^P$ is the coefficient on the l th lag of $d_i^P(L)$. An important feature of the price responses in (7) is that all market prices have the same long-run response to a one unit increase in the common efficient price. A market with a more efficient price discovery process is one with a faster convergence of the price responses toward the new equilibrium. Hereafter, we refer the impulse response functions of the market prices to a one unit common efficient price innovation as the *price discovery impulse response functions* (PDIRFs).

The PDIRFs in (7) are different from the impulse response functions used in the existing price discovery literature (e.g., Hasbrouck, 1995, 2003). These IRFs measure the market price responses to the orthogonalized reduced form forecasting errors based on the Cholesky factorization, which are, in general, linear combinations of the underlying structural innovations. Only in a special case can these reduced form IRFs be interpreted as capturing the underlying structural innovations. The Cholesky factorization of the VECM residual covariance matrix places zero restrictions, representing a recursive causal ordering, on the contemporary impact matrix \mathbf{D}_0 to identify the structural shocks. As discussed by Levchenkova, Pagan, and Robertson (1999), Ribba (1997), and Fisher and Huh (2007), the Cholesky decomposition by itself does not identify the permanent and transitory shocks in a cointegrated VECM because it does not guarantee a long-run impact matrix of the form (4). The additional restriction that one of the variables is weakly exogenous, and this variable is ordered first in $\Delta \mathbf{p}_t$, is also required. In this case, Fisher and Huh (2007) showed that the Cholesky factorization produces the Gonzalo-Ng (2001) permanent-transitory decomposition we use to identify the structural shocks.

The plots of the PDIRFs are a useful way to graphically characterize and compare the price discovery dynamics between the multiple markets. However, it is also desirable to have a numerical summary of the dynamic efficiency of price discovery for each market. We measure the dynamic efficiency of market i at a given horizon k in response to a one unit permanent shock by the difference between the PDIRF and the long-run response of one unit, $f_{i,k} - 1$. In particular, given a non-negative loss function L , we define the *price discovery efficiency loss* (PDEL) for market i

as the accumulated loss:

$$\text{PDEL}_i(K^*) = \sum_{k=0}^{K^*} L(f_{i,k} - 1), \quad i = 1, 2, \quad (8)$$

where K^* is a truncation lag chosen such that $f_{i,K^*} \approx 1$. Natural symmetric loss functions are the absolute value loss and the squared loss. The PDEL measures each market's efficiency in terms of the magnitude of total information related mispricing errors - deviations of the market price from the new equilibrium - during the process of impounding new information. A highly persistent pricing error process in one market suggests price discovery inefficiency and inflates the PDEL. The smaller the pricing error loss when one market impounds new information, the smaller the PDEL for this market, and the more efficient this market's price discovery process. In the extreme case of a perfectly efficient price discovery process, the PDIRF will reflect an immediate incorporation of new information and the PDEL will be zero.

2.3 Example: Partial Adjustment Model

Consider a partial price adjustment model similar to that used in Amihud and Mendelson (1987) and Hasbrouck and Ho (1987):

$$\begin{aligned} p_{i,t} &= p_{it-1} + \delta_i(m_t - p_{it-1}) + b_{i,0}^T \eta_t^T, \quad i = 1, 2, \\ m_t &= m_{t-1} + \eta_t^P, \end{aligned} \quad (9)$$

where $0 \leq \delta_i \leq 2$. Solving for $\Delta p_{i,t}$ gives $\Delta p_{i,t} = d_i^P(L)\eta_t^P + d_i^T(L)\eta_t^T$, where $d_i^P(L) = [1 - (1 - \delta_i)L]^{-1}\delta_i$ and $d_i^T(L) = [1 - (1 - \delta_i)L]^{-1}(1 - L)b_{i,0}^T$. The SMA representation (1) is determined from the appropriate elements of the lag polynomials $d_i^P(L)$ and $d_i^T(L)$. In particular, the initial impact and long-run impact matrices are given by

$$\mathbf{D}_0 = \begin{pmatrix} d_1^P(0) & d_1^T(0) \\ d_2^P(0) & d_1^T(0) \end{pmatrix} = \begin{pmatrix} \delta_1 & b_{1,0}^T \\ \delta_2 & b_{2,0}^T \end{pmatrix}, \quad \mathbf{D}(1) = \begin{pmatrix} d_1^P(1) & d_1^T(1) \\ d_2^P(1) & d_1^T(1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

Amihud and Mendelson (1987) used (9) with $b_{i,0}^T = 1$ to model the dynamics of price adjustment to changes in fundamental value for a single security. Hasbrouck and Ho (1987) used this model with $b_{i,0}^T = 0$ to explain positive autocorrelations in stock returns. In the price discovery context,

δ_i captures the speed of price discovery. Following a one unit change to the efficient price, in each period market i 's price will move toward, or discover, the new efficient price at rate of δ_i . A value of δ_i closer to one implies a more efficient price discovery process. If $\delta_i = 1$ and $b_{i,0}^T = 0$, then new information is immediately incorporated and the market price will be equal to the efficient price. If $\delta_i > 1$ then overshooting of traders to new information occurs.

The PDIRFs and PDELs for markets with different price discovery efficiency can be illustrated with the partial price adjustment model (9). In this model, the PDIRFs are determined by the coefficients of $d_i^P(L) = [1 - (1 - \delta_i)L]^{-1}\delta_i = \delta_i \sum_{k=0}^{\infty} (1 - \delta_i)^k L^k$. Figure (1) shows the PDIRF plots implied by (9) under different price discovery speed parameters. The price response plot in the middle panel corresponds to a leading market with a fast price discovery speed, $\delta_1 = 0.8$, and the plot in the lower panel is associated with a lagged market with a slow price discovery speed, $\delta_2 = 0.2$. The two plots differ in both the magnitude of the initial impact of new information (informational lag) and the persistence of the pricing error (the difference between the price and the efficient price), which are both captured by the parameter δ_i . The top panel with $\delta = 1.6$ illustrates a market with moderately fast price discovery that exhibits overshooting behavior. The PDELs, computed using an absolute value loss function, associated with the PDIRFs in Figure (1) are 1.5, 0.25 and 4, respectively.

3 Identification and Estimation of the Structural Cointegration Model

In this section we show how to identify and estimate the parameters of the SMA model (1) that are used to construct our dynamic price discovery measures. We also provide simulation examples to illustrate the finite sample properties of our dynamic measures of price discovery.

3.1 Identification and Estimation

The starting point for identification of the SMA model parameters and innovations in (1) is the empirical VECM($K - 1$):

$$\Delta \mathbf{p}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}' \mathbf{p}_{t-1} - \mu) + \sum_{k=1}^{K-1} \boldsymbol{\Gamma}_k \Delta \mathbf{p}_{t-k} + \mathbf{e}_t, \quad (10)$$

where $\boldsymbol{\beta} = (1, -1)'$ is the known cointegrating vector, μ is a scalar capturing systematic differences in the two prices (e.g., the mean bid-ask spread, or the risk free return between the spot and futures prices), and \mathbf{e}_t is a 2×1 vector satisfying $E[\mathbf{e}_t] = \mathbf{0}$ with $E[\mathbf{e}_t \mathbf{e}_s'] = \mathbf{0}$ for $t \neq s$, and $E[\mathbf{e}_t \mathbf{e}_s'] = \boldsymbol{\Sigma}$ for $t = s$. The 2×1 vector $\boldsymbol{\alpha}$ contains the error correction coefficients that measure each price's expected speed in eliminating the price difference. The constant μ can be consistently estimated as the sample mean of $\boldsymbol{\beta}' \mathbf{p}_{t-1}$, and the remaining parameters can be consistently and efficiently estimated by least squares equation by equation.

Following Johansen (1991), (10) can be inverted to obtain a reduced-form moving average model for $\Delta \mathbf{p}_t$ of the form:

$$\Delta \mathbf{p}_t = \boldsymbol{\Psi}(L) \mathbf{e}_t = \mathbf{e}_t + \boldsymbol{\Psi}_1 \mathbf{e}_{t-1} + \boldsymbol{\Psi}_2 \mathbf{e}_{t-2} + \dots, \quad (11)$$

where $\boldsymbol{\Psi}(L) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k L^k$ and $\boldsymbol{\Psi}_0 = \mathbf{I}_2$. The long-run impact matrix $\boldsymbol{\Psi}(1) = \sum_{k=0}^{\infty} \boldsymbol{\Psi}_k$ has rank 1 and can be represented as:

$$\boldsymbol{\Psi}(1) = \boldsymbol{\beta}_{\perp} (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Gamma}(1) \boldsymbol{\beta}_{\perp})^{-1} \boldsymbol{\alpha}'_{\perp} = \boldsymbol{\xi} \boldsymbol{\alpha}'_{\perp}, \quad (12)$$

where $\boldsymbol{\beta}_{\perp}$ and $\boldsymbol{\alpha}_{\perp}$ are 2×1 vectors satisfying $\boldsymbol{\beta}' \boldsymbol{\beta}_{\perp} = \mathbf{0}$ and $\boldsymbol{\alpha}' \boldsymbol{\alpha}_{\perp} = 0$, respectively, $\boldsymbol{\Gamma}(1) = \mathbf{I}_2 - \sum_{k=1}^{K-1} \boldsymbol{\Gamma}_k$, and $\boldsymbol{\xi} = \boldsymbol{\beta}_{\perp} (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Gamma}(1) \boldsymbol{\beta}_{\perp})^{-1}$. As shown in Appendix A, the reduced-form matrices $\boldsymbol{\Psi}_k$ ($k = 1, 2, \dots$) and $\boldsymbol{\Psi}(1)$ can be determined from the VECM($K - 1$) by casting it state-space form and applying a recursive algorithm.

We uniquely identify the structural shocks, η_t^P and η_t^T , and the SMA coefficients in (1) from the residuals and coefficients in (10) using a modification of the P-T decomposition of Gonzalo and Ng (2001). Gonzalo and Ng (2001) defined the permanent and transitory (P-T) innovations from

the reduced-form VECM($K - 1$) residuals as:

$$\boldsymbol{\epsilon}_t = \begin{bmatrix} \epsilon_t^P \\ \epsilon_t^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}'_{\perp} \mathbf{e}_t \\ \boldsymbol{\beta}' \mathbf{e}_t \end{bmatrix} = \mathbf{G} \mathbf{e}_t, \quad (13)$$

where $\mathbf{G} = [\boldsymbol{\alpha}_{\perp} \ ; \ \boldsymbol{\beta}]'$ is a 2×2 transformation matrix assumed to be nonsingular. The nonsingularity of \mathbf{G} is not guaranteed. For example, with $\boldsymbol{\beta} = (1, -1)'$ the matrix \mathbf{G} will be singular if $\boldsymbol{\alpha} = (\alpha_1, \alpha_1)'$. Gonzalo and Ng (2001) showed that the permanent and transitory innovations, ϵ_t^P and ϵ_t^T , satisfy $\lim_{k \rightarrow \infty} \frac{\partial E_t[\mathbf{p}_{t+k}]}{\partial \epsilon_t^P} \neq 0$ and $\lim_{k \rightarrow \infty} \frac{\partial E_t[\mathbf{p}_{t+k}]}{\partial \epsilon_t^T} = 0$, respectively.

As shown in Hasbrouck (1995), the condition $\boldsymbol{\beta}' \boldsymbol{\Psi}(1) = \mathbf{0}$ together with $\boldsymbol{\beta} = (1, -1)'$ implies that the rows of $\boldsymbol{\Psi}(1)$ are identical so that $\boldsymbol{\Psi}(1) = \mathbf{1} \boldsymbol{\psi}'$ where $\boldsymbol{\psi} = (\psi_1, \psi_2)'$. Combining this result with the Johansen factorization (12) implies that $\boldsymbol{\Psi}(1) = \mathbf{1} \boldsymbol{\psi}' = \boldsymbol{\xi} \boldsymbol{\alpha}'_{\perp}$. Now, in the price discovery model, a one unit innovation in the permanent shock has a one unit long-run impact on all price variables, see (2). This restriction implies that $\boldsymbol{\xi} = \mathbf{1}$, $\boldsymbol{\alpha}_{\perp} = \boldsymbol{\psi}$, and $\epsilon_t^P = \boldsymbol{\psi}' \mathbf{e}_t$ as in Hasbrouck (1995). The linking of $\boldsymbol{\alpha}_{\perp}$ with $\boldsymbol{\psi}$ removes a source of uncertainty that is associated with the Gonzalo-Ng P-T decomposition. In their decomposition, $\boldsymbol{\alpha}_{\perp}$ must be estimated directly from $\boldsymbol{\alpha}$ and is not unique. With $\boldsymbol{\psi}$ replacing $\boldsymbol{\alpha}_{\perp}$ in (13) and $\boldsymbol{\beta} = (1, -1)'$, the long-run impact matrix of the P-T shocks is:

$$\boldsymbol{\Psi}(1) \mathbf{G}^{-1} = \frac{1}{-\psi_1 - \psi_2} \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1 & \psi_2 \end{pmatrix} \begin{pmatrix} -1 & -\psi_2 \\ -1 & \psi_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}.$$

In general, the permanent and transitory innovations, ϵ_t^P and ϵ_t^T , are correlated and so they need to be made orthogonal to identify the structural shocks, η_t^P and η_t^T . Gonzalo and Ng (2001) constructed the orthogonalized P-T innovations using $\boldsymbol{\eta}_t = \mathbf{P} \boldsymbol{\epsilon}_t$, where \mathbf{P} is the Choleski factor of $\text{var}(\boldsymbol{\epsilon}_t) = \text{var}(\mathbf{G} \mathbf{e}_t) = \mathbf{G} \boldsymbol{\Sigma} \mathbf{G}' = \boldsymbol{\Sigma}_{\epsilon}$. In their decomposition, the elements in the first column of the long-run impact matrix $\mathbf{D}(1)$ are unrestricted and the structural shocks are normalized to have unit variances to achieve identification. In our framework, however, $\mathbf{D}(1)$ is restricted by (4) which allows $\text{var}(\boldsymbol{\eta}_t) = \text{diag}(\sigma_P^2, \sigma_T^2)$. Since the variances of $\boldsymbol{\eta}_t$ are unrestricted, we use the triangular

factorization $\Sigma_\epsilon = \mathbf{HCH}'$, where

$$\mathbf{H} = \begin{pmatrix} 1 & 0 \\ h_{21} & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_T^2 \end{pmatrix},$$

and define the orthogonal structural shocks as:

$$\boldsymbol{\eta}_t = \mathbf{H}^{-1}\boldsymbol{\epsilon}_t = \mathbf{H}^{-1}\mathbf{G}\mathbf{e}_t.$$

The elements of $\boldsymbol{\eta}_t$ are uncorrelated and the variances of the structural innovations η_t^P and η_t^T are given by the diagonal elements of the matrix \mathbf{C} .

The SMA representation in (1) is identified as:

$$\Delta \mathbf{p}_t = \boldsymbol{\Psi}(L)\mathbf{G}^{-1}\mathbf{H}\mathbf{H}^{-1}\mathbf{G}\mathbf{e}_t = \mathbf{D}(L)\boldsymbol{\eta}_t = \mathbf{D}_P(L)\eta_t^P + \mathbf{D}_T(L)\eta_t^T,$$

where $\mathbf{D}(L) = \boldsymbol{\Psi}(L)\mathbf{D}_0$, $\mathbf{D}_0 = \mathbf{G}^{-1}\mathbf{H}$, $\boldsymbol{\eta}_t = \mathbf{D}_0^{-1}\mathbf{e}_t$, and $\mathbf{D}_P(L)$ and $\mathbf{D}_T(L)$ are the columns of $\mathbf{D}(L)$ corresponding to η_t^P and η_t^T , respectively. The long-run impact matrix of the structural shocks $\boldsymbol{\eta}_t$ is $\boldsymbol{\Psi}(1)\mathbf{G}^{-1}\mathbf{H} = \mathbf{D}(1) = [\mathbf{1} : \mathbf{0}]$ as required. As noted by Gonzalo and Ng (2001), given the structure of \mathbf{G} , \mathbf{H} is invariant to the ordering of the prices in $\Delta \mathbf{p}_t$ which implies that the SMA coefficients in (1) are exactly identified and the structural shocks $\boldsymbol{\eta}_t$ are uniquely determined.

The main practical issue in estimating the structural cointegration model is the choice of lag length for the fitted VECM($K - 1$). The estimated value of $\boldsymbol{\Psi}(1)$, and hence $\boldsymbol{\psi}$, is often sensitive to the chosen lag length. This can be explained from the results from Faust and Leeper (1997) who showed that if the underlying VAR model is of infinite order, then the estimate of $\boldsymbol{\Psi}(1)$ has a variance that diverges as the sample size goes to infinity. With high frequency data, very large lag lengths can be required to explain the data and estimates of $\boldsymbol{\psi}$ can be imprecise and quite sensitive to the chosen lag length.

Because the estimated SMA model parameters and our dynamic measures of price discovery are complicated nonlinear functions of the estimated VECM($K - 1$) parameters, we follow Gonzalo and Ng (2001) and use a bootstrap procedure to assess their sampling variability. We first determine the lag length $K - 1$ and estimate the VECM($K - 1$) (10) giving parameter estimates $\hat{\boldsymbol{\alpha}}$, $\hat{\boldsymbol{\Gamma}}(L)$ and

residuals $\hat{\mathbf{e}}_t$. We then generate a bootstrap sample by random sampling of $\hat{\mathbf{e}}_t$ with replacement and then reconstruct \mathbf{p}_t using the initial estimates $\hat{\boldsymbol{\alpha}}$ and $\hat{\boldsymbol{\Gamma}}(L)$. We then re-estimate the VECM($K - 1$) parameters, form the SMA model estimates from the modified P-T decomposition and construct the price discovery measures (7) and (8). We repeat this procedure 1000 times and use the bootstrap distribution to evaluate the sampling uncertainty of the SMA model parameters and price discovery measures.

3.2 Simulation Examples

In this sub-section we examine the finite sample performance of our proposed methodology for measuring the dynamics of price discovery using simulated data from some stylized dynamic structural models.

3.2.1 Just Identified Bivariate Model

Consider simulated data from the stylized dynamic structural model (9) where the true dynamics of price discovery between markets are known. Our interest centers on the accuracy of the PDIRFs and the PDEL, and the testing of the null hypothesis $H_0 : g(\text{PDEL}_1, \text{PDEL}_2) = \ln(\text{PDEL}_1/\text{PDEL}_2) = 0$ using bootstrap calculations from the estimated reduced form VECM($K - 1$).

The simulation model used is (9) with $\delta_1 = 0.8, \delta_2 = 0.2, b_{0,1}^T = 0.5$ and $b_{0,2}^T = -0.5$. Market 1 has a greater speed of price discovery dynamics than market 2 and both markets respond equally, in absolute value, to transitory shocks. Setting $\sigma_P^2 = 1$ and $\sigma_T^2 = 0.64$ removes the correlation between the reduced form residuals. Since $d_i^P(L) = \delta_i \sum_{k=0}^{\infty} (1 - \delta_i)^k L^k$ it is straightforward to compute analytic values for the PDIRFs and the PDEL, for a given loss function, for any truncation lag K^* .

For the specified parameters, we generate artificial samples of size 500, 1000, 5000 and 10000 observations for the bivariate price system assuming normally distributed errors. To mimic what a researcher would do in practice, we fit the VECM($K - 1$) (10) with the lag order of the system determined by minimizing the Bayesian Information Criterion (BIC). We then estimate the PDIRFs and PDELs and evaluate their sampling uncertainty using the bootstrap.

Figure 2 shows the estimated PDIRFs, (solid blue dots) with 95% bootstrap confidence intervals (open red squares), along with the true PDIRFs (solid black lines) for the two markets. The plots show that the estimated PDIRFs track the true PDIRFs well, especially for very large samples that

are common with high frequency data sets. Table 1 gives the estimated PDEL using the absolute value loss function with $K^* = 30$, along with 95% bootstrap confidence intervals. For smaller samples, there is considerable uncertainty in the individual estimates. However, when $N = 10000$ the estimates are quite precise. For all sample sizes, the 95% percent bootstrap confidence interval for $\ln(\text{PDEL}_1/\text{PDEL}_2)$ excludes zero.

3.2.2 Noninvertible Bivariate Model

A key assumption of the structural cointegration model (1) is that the number of structural shocks is equal to the number of observed prices. As a result, with the same asset trading in n markets there is one permanent shock and $n - 1$ transitory shocks. For example, with two markets all permanent sources of new information are lumped into a single structural permanent shock and all transitory frictions are lumped into a single structural transitory shock. In reality, there may be multiple sources of structural permanent and transitory shocks.

To illustrate the basic issues, consider the stylized microstructure model (9) modified to have two sources of structural transitory shocks:

$$\begin{aligned} p_{i,t} &= p_{i,t-1} + \delta_i(m_t - p_{it-1}) + b_{i,0}^T \eta_{it}^T, \quad i = 1, 2, \\ m_t &= m_{t-1} + \eta_t^P, \end{aligned} \tag{14}$$

where $0 \leq \delta_i \leq 2$. Models similar to (14) have been used by Harris, McInish, and Wood (2002) and Hasbrouck (2002). In (14), the structural errors $\boldsymbol{\eta}_t = (\eta_t^P, \eta_{1t}^T, \eta_{2t}^T)'$ have zero means, diagonal covariance matrix $\text{diag}(\sigma_T^2, \sigma_{1T}^2, \sigma_{2T}^2)$, and are mutually uncorrelated at all leads and lags. The transitory shocks for each market are intended to capture local liquidity-based motives for trade that are uncorrelated with the permanent news innovation. With two prices and three structural errors, the structural moving average representation for $\Delta \mathbf{p}_t$ has the form (1) with

$$\mathbf{D}(L) = \begin{pmatrix} d_1^P(L) & d_1^T(L) & 0 \\ d_2^P(L) & 0 & d_2^T(L) \end{pmatrix},$$

where $d_i^P(L) = [1 - (1 - \delta_i)L]^{-1}\delta_i$ and $d_i^T(L) = [1 - (1 - \delta_i)L]^{-1}(1 - L)b_{i,0}^T$. Since \mathbf{D}_0 is not invertible

there is no longer a unique mapping between the reduced-form moving average representation and the structural moving average representation and so all of the structural shocks cannot be recovered from the observed data.

If the data are generated by (14) and the structural cointegration model with two identified structural shocks is estimated what do the structural impulse response functions represent? A technical result from the appendix to Blanchard and Quah (1989) provides an answer. They considered a bivariate structural model for which there are more than two sources of structural shocks. In particular, they assumed that the economy is driven by m shocks, but each shock is either a permanent or a transitory shock. This assumption is not sufficient to prevent the commingling of shocks (i.e., identified shocks are likely to be a mixture of both types of shocks). However, they proved that commingling of shocks is avoided when the dynamic relationship between the observed variables remains the same across different permanent shocks, with the same result holding for all transitory shocks. This result suggests that the bivariate structural cointegration model should correctly identify the dynamic responses to the permanent shock when prices respond to the different transitory shocks in the same way.

To illustrate the impact of multiple structural transitory shocks on the estimated PDIRFs, we generated data from (14) with $\delta_1 = 0.8, \delta_2 = 0.2, b_{0,1}^T = 0.5, b_{0,2}^T = 0.5, \sigma_P^2 = 1$ and $\sigma_{T,1}^2 = \sigma_{T,2}^2 = 0.64$ assuming normally distributed errors. The true PDIRF is the same as in the previous simulation example with one transitory shock. We then computed the PDIRFs and PDEL values as if there were only one permanent and one transitory shock. Figure 3 shows the estimated PDIRFs, along with the true PDIRFs, for samples of size 500, 1000, 5000 and 10000. For sample sizes greater than 500, the estimated PDIRFs track the true PDIRFs quite well. Table 2 gives the estimated PDEL using the absolute value loss function and $K^* = 30$, along with 95% bootstrap confidence intervals. For larger sample sizes the PDEL estimates are reasonable and the bootstrap confidence intervals contain the true values.

4 Empirical Example: Price Discovery in Foreign Exchange Markets

In this section we illustrate our methodology for characterizing price discovery dynamics using a data set of ultra high frequency foreign exchange (Fx) rate quotes. We first give some background on price discovery using Fx data. We then describe our data set and the variables used for analysis, and follow this by the estimation of the structural cointegration model and price discovery measures.

4.1 Vehicle Currency and Price Discovery

The US dollar (USD) has been the dominant international currency since World War II. One important role of the dollar is to act as a vehicle currency, or medium of exchange, through which transactions between other currencies are made. Such indirect transactions are attractive because the cost of two transactions against the dollar are usually lower than the cost in the direct exchanges of non-dollar currencies. Because of this special role, the dollar markets, e.g. dollar/euro (USD/EUR) and Japanese yen/dollar (JPY/USD), are the largest and most liquid in foreign exchange (Fx) transactions. Even the introduction of the euro in 1999, a perceived challenger against the dollar, has not so far changed the dollar's dominant roll. In fact, the most recent central bank survey by the Bank of International Settlements (BIS) in 2007 reveals that the dollar entered on one side of 86% of 3.1 trillion dollars average daily turnover, with a decreased share from 89% in 2004 (BIS, 2007). By currency pairs, 52% of total trading volume occurs in three dollar markets: USD/EUR (27%), JPY/USD (13%), and USD/British pound (GBP) (12%), while the largest market share for cross rates is only 2% for JPY/EUR, EUR/GB and EUR/CHF.

The dollar's medium-of-exchange role has inspired many studies attempting to explain the rise and evolution of vehicle currencies, including Krugman (1980), Black (1991), Hartmann (1994), and Rey (2001). The common foundation of these studies is the inverse relationship between transaction costs and (expected) trading volumes. Market participants tend to choose the exchange or transaction medium with lower transaction costs and higher market liquidity. In addition, transaction costs may be further lowered as market making (e.g., order processing) costs are spread over large trading volumes. This interaction between transactions costs and trading volume implies a persistent role of any established vehicle currency. The international monetary system, however, may

shift from one vehicle currency to another with the arrival of strong shocks; e.g., the replacement of the British sterling by the US dollar as the vehicle currency. With the launch of the euro, the issues of currency competition and the potential challenge against the dollar's hegemonic status by the euro have become heated research and discussions topics. See, for example, Hartmann (1998a) and Portes and Rey (1998).

We focus on another aspect of vehicle currencies: their contribution to the price discovery of the exchange rates between other currencies. Specifically, we examine the extent to which the dollar contributes to the price discovery of JPY/EUR, the cross rate with the largest market share. In contrast to the well studied medium function of trading other currencies at lower costs, the dollar's price discovery contribution to cross rates has not received as much attention. In fact, the market characteristics of vehicle currencies have important implications for the price leadership of the cross rate constructed from the dollar rates; e.g., the dollar implied JPY/EUR rate by the USD/EUR rate and the JPY/USD rate.

First, most nonpublic information about the euro or yen may first be impounded into the dollar prices of these currencies. Lyons (1995, 1997) develops a model in which foreign exchange dealers may extract private information regarding economic fundamentals from their nondealer customer order flows. The optimal strategy for players with superior information is to profit from trading with uninformed or liquidity traders. Liquidity traders (e.g., corporate customers and hedge fund managers) for the euro and yen are attracted to the dollar markets because of the relatively lower trading costs with the vehicle currency than trading directly with other nondollar currencies. According to Admati and Pfleiderer (1988), informed traders prefer to trade at the times when liquidity traders concentrate. Consequently, the dollar prices of the euro and yen may become more informative as informed players trade with liquidity traders and reveal their private information. Another source of nonpublic information in the Fx market is central bank interventions¹. The dollar is widely used as the intervention currency by non-US central banks². As foreign monetary authorities buy or sell the dollar for the home currency, the intervention effects should be first reflected in the dollar rates.

¹Most central bank interventions, except in the U.S., are not released to the news media simultaneously with the operation. See Lyons (2001) and references therein.

²For example, the dollar operation accounts for the majority of Bank of Japan's (BOJ) interventions. BOJ seldom uses the euro at much smaller magnitudes. For more information, see the quarterly intervention reports at the website of Japan's Ministry of Finance.

Second, consider the effects of public information in the Fx market through macroeconomic news announcements. Harris and Raviv (1993), and Kandel and Pearson (1995) suggest that market participants may have differential interpretations of such public signals, and it is the trading process that aggregates heterogeneous beliefs of investors and produces price discovery. Evans and Lyons (2002), and Love and Payne (2002) find evidence that order flows transmit significant shares of macroeconomic news into exchange rates. As the dollar markets are the most liquid, macroeconomic news releases from Japan (Europe) may be more efficiently assimilated into the dollar price of the yen (euro), rather than the cross rate.

In summary, the special role of the dollar as a vehicle currency suggests that substantial price discovery of JPY/EUR may occur through the dollar. The more liquid are the dollar markets relative to the cross rate market, the more informative are the dollar market prices and the stronger is the price leadership of the dollar implied JPY/EUR rate.

Our analysis provides several important contributions to the Fx price discovery literature. First, the Fx market has seen substantial structural changes in the past ten years, notably the replacement of the legacy EMS currencies by the euro (for other structural changes, see Galati 2001, BIS 2002, and ECB 2003). Most price discovery studies examined a market environment that no longer exists. Using more recent data, we measure the relative price discovery contributions of major currencies during the beginning of the so-called “euro era”. Second, we apply our newly developed measures of price discovery dynamics. Finally, we relate measures of relative liquidity and transaction costs, the critical attributes for a vehicle currency, to the price discovery contribution of an international currency³.

Our analysis is closely related to the analysis in De Jong et al. (1998). Using one year (1992-1993) of indicative quotes data, they find that the direct JPY/Deutschemarks (DM) exchange rate and the dollar implied JPY/DM rate (constructed from DM/USD and JPY/USD) are roughly equally important for the price discovery of the direct JPY/DM rate, and the direct JPY/DM rate obtains its strongest price discovery role during the European and American trading hours.

³These factors have been examined in the price discovery studies of equity markets. Hasbrouck (1995) and Huang (2002) look at the trading volume shares, and Eun and Sabherwal (2003) use the ratio of bid-ask spreads.

4.2 Data Description and Variable Construction

Our analysis of the dynamics of price discovery is based on the bid-ask quotes of the spot Fx rates for three currency pairs: USD/EUR, JPY/USD, and JPY/EUR. The quotes are from Electronic Brokerage Service (EBS) and obtained from Lehman Bros. The data is taken from the interdealer electronic currency exchanges, which currently accounts for roughly one third of overall spot Fx trading volume⁴. The data sample covers a period from 22:00 Greenwich Mean Time (GMT) July 6, 2003 to 22:00 GMT September 26, 2003. Each observation is time-stamped up to the millisecond from midnight GMT, and contains the bid and ask quotes. Unlike the Reuters FxFx indicative quotes widely used in empirical studies⁵, the bid-ask quotes in our data are tradeable or firm. The data is delivered to us with a proprietary outlier filter applied. We further visually screen the data by plotting the quotes by days, and find no significant outliers. The quote observations with non-positive spreads (ask quote minus bid quote) are removed.

The Fx market operates on a 24-hour basis, and only turns quiet over the weekend periods. Our analysis focuses on the business week when Fx trading is especially active. All observations with time stamps from Friday 22:00 GMT to Sunday 22:00 GMT are excluded, which leaves us with 12 business weeks, or 60 trading days, of quotes data. Each trading day is defined as from 22:00 GMT of the previous day to 22:00 GMT of that day. A similar “business weekend” definition is also used in Andersen and Bollerslev (1998).

One distinguishing feature of the Fx market is that the Fx trading activity systematically varies across a 24-hour trading day as the earth sequentially passes through the business hours of the major geographical financial centers: Tokyo, London, and New York. Identifying the portions of a trading day that correspond to the business hours of these financial centers is important for price discovery studies, since the relative (currency pair-wise) importance and market liquidity of a particular currency may change across a day. Generally, during the business hours of the home market, the trading of currency pairs involving the home currency are more active than others. For example, the market for JPY/USD is more active than that for USD/EUR during Tokyo’s business hours even though USD/EUR accounts for a larger share of overall turnover. Accordingly,

⁴Lyons (2001) reports that the spot Fx trades can be classified into three types based on the involved counter parties: dealer-customer trades, direct inter-dealer trades, and brokered inter-dealer trades, each of which accounts for one third of the overall volume.

⁵See Goodhart and O’Hara (1997) for a survey of the studies using the FxFx data.

we break a 24-hour trading day into four sessions - Asian, European, American, and post American - based on the local business hours of the three major geographical financial centers⁶. Table 3 lists the timing definition for the four sessions. Notice that the correspondence between the GMT hours and local business hours is based on the daylight saving time (DST) that prevails during our sample period. Furthermore, the starting time of each intraday session is based on the conventional business starting hour of that center. Because the business hours of consecutive financial centers may be overlapping, the traders from the previous center may still be present in the market in the following session. This is especially true for the American session. When New York traders start trading, the traders in London still have the whole afternoon to go through. The American session ends earlier in the local time because New York traders quit from active trading when their European counterparts quit from the market (Goodhart and Demos, 1991). The post American session intends to capture the quiet interim period after the active trading in North America, but before the start of trading in Asia. This naming convention of intraday trading sessions is used throughout our empirical investigation.

The price variables for exchange rates are measured by logarithmic (log) mid quotes. Log mid quotes are further multiplied by 10,000 so that price changes are in basis points. The raw quotes are unequally spaced in time and are aligned to an equally spaced time clock using the “previous tick” method for statistical analysis. The aligned observations thus measure the prevailing market price (midquote) levels. After alignment the three exchange rate series have the same number of observations. The direct JPY/EUR rate is defined as log mid quote of JPY/EUR_t , while the US dollar implied rate is defined as log mid quote of $\text{JPY}/\text{USD}_t + \log \text{mid quote of } \text{USD}/\text{EUR}_t$. One advantage of quotes data is that it is free of the infrequent trading problem. Infrequent trading arises as a measurement problem when investors’ opinions can not be reflected in the market price until a trade occurs and the transaction price is observed (e.g. see Lo and MacKinlay, 1990). Quotes are different from trades in that quotes are valid until they are revised, and quote revision can occur in the absence of trades. Because the quote changes over weekends are quite different from those within normal trading intervals, all quote changes over weekends are excluded from the sample.

⁶The Fx market is a decentralized market without specific market open and close hours for each geographical financial center.

The spreads are defined in terms of the differences of the paired ask and bid quotes, and are expressed in units of the minimum grid on which each exchange rate can move, so called pips. The USD/EUR rate is conventionally quoted as how many dollars per euro, and one pip is 0.0001 USD. The JPY/USD and JPY/EUR rates are quoted as how many yen per dollar and yen per euro, respectively. One pip for these two exchange rates is 0.01 JPY. Because the price of USD/EUR is in the 1 dollar range and the prices of JPY/USD and JPY/EUR are in the range of 100 JPY, the spreads measured in pips are approximately the percentage spreads. Accordingly, the pip spreads may be compared across these three exchange rates. We use the pip spreads as the measure of transaction costs in each market. There are other potential transaction costs to consider, such as commissions. Our communication with Fx practitioners suggests that the interdealer commissions are about \$15 per trade. Goodhart et al. (1996) find that there is little variation in the size of interdealer Fx trades and the typical trade size is \$1 million worth. This amounts to a \$100 spread cost and \$30 commissions for a typical round-trip transaction assuming a one pip spread. Therefore, the bid-ask spreads make up the majority of actual transaction costs. To characterize the transaction costs in each market at a particular time of day, we further compute the hourly mean spreads in pips by averaging all spreads of each exchange rate within a particular GMT hour across the 60 trading days in the sample.

We use the ratio of the sum of hourly mean spreads of USD/EUR and JPY/USD to the hourly mean spreads of JPY/EUR (hereafter, the spread ratio) to measure the hourly relative liquidity of the dollar markets against the cross rate market. Transaction costs and liquidity in individual markets measured by bid-ask spreads have been examined in many previous vehicle currency and more general Fx studies; e.g., Black (1991), Hartmann (1998b), and Huang and Masulis (1998). However, whether liquidity traders for the euro or yen use the dollar as the transaction medium, and consequently where informed traders may reveal their nonpublic information, ultimately depends on the relative transaction costs in the dollar markets versus in the cross rate market. The lower the spread ratio is from one, the more cost-attractive it is to trade the yen and the euro through the dollar and the more liquid are the dollar markets relative to the cross rate market. Measuring relative liquidity with the spread ratio allows for a rich characterization of the relative importance/attractiveness of the dollar throughout the intraday Fx market evolution.

4.3 Preliminary Data Analysis

We start our data analysis by looking at the market trading activity approximated by the hourly tick frequency of quote entries. The use of quote tick frequency as a proxy for trading volume and market activity in the Fx studies can be traced back to Goodhart and Demos (1991) and Bollerslev and Domowitz (1993). For each exchange rate, we compute the average hourly frequency of quote entries by averaging the quote counts of the exchange rate within each intraday hour across the 60 days in our sample. The resulting trading activity measures are plotted in Figure 4, with the upper, middle, and lower panels for USD/EUR, JPY/USD, and JPY/EUR, respectively.

A first look at the plots suggests that there are similar intraday activity patterns across the three exchange rate markets. The intraday patterns reflect three activity peaks corresponding to the business hours of three world's financial centers, and are largely consistent with the earlier findings in Goodhart and Demos (1991)⁷. Interestingly, although the trading restrictions over the Tokyo lunch hours have been removed since 1994 (see Ito et. al., 1998), a market activity trough is still observed during this period (3:00 - 4:00 GMT).

A closer examination of the activity patterns of three markets reveals more interesting differences. First, consistent with the BIS survey data, the overall activity of USD/EUR is highest, followed by JPY/USD and JPY/EUR. This ordering is expected given the dollar's dominance in the Fx market. The market activity differences, however, become more subtle during individual geographical trading sessions. During the Asian trading hours, the market activity of JPY/USD is highest, followed by USD/EUR, reflecting the home market effect. But the home market effect only applies to JPY/USD, not to JPY/EUR, which indicates the dollar's dominance and vehicle currency role in the Asian session.

When European markets are open, which includes both the European and American trading sessions, the market for USD/EUR is naturally most active. However, the market activity of JPY/EUR elevates to a comparable level to that of JPY/USD. This makes the size of the euro markets, measured by the sum of the activity of JPY/EUR and USD/EUR, very competitive against the size of the dollar markets, measured by the sum of the activity of JPY/USD and USD/EUR. After the European markets and American session close, the dollar regains its dominance in the

⁷Goodhart and Demos (1991) did not examine the cross rate market.

post American session.

The hourly mean spreads in pips are plotted in Figure 5. The intraday spread variations generally reverse the pattern found in the market activity plots. This is due to lower order processing costs as expected trading volumes increase (Black, 1991, Hartmann, 1998b) and/or the effects from increased dealer competition (Huang and Masulis, 1998). The spread level is lowest for USD/EUR, followed by JPY/USD, and is highest for JPY/EUR.

4.4 Estimation Details

There is little formal guidance for the choice of the sampling interval for the statistical analysis, and the results of the price discovery analysis may be sensitive to the chosen sampling frequency. For the highly liquid Fx market, determining which price leads and which price follows requires sampling at very high time resolutions. To illustrate, Figure 6 presents one episode of exchange rate movements in our sample around 23:50 GMT on August 11, 2003, at which Japan released the first GDP estimates for the second quarter of 2003. The line with squares depicts the movement of the dollar implied JPY/EUR price (midquote) and the line with triangles traces the direct JPY/EUR price (data are in their original scale). The economic recovery data in Japan's GDP release caused appreciation of Japanese yen from 134.63 yen/euro to 134.50 yen/euro. Both the direct and implied JPY/EUR rates moved toward the new price level following the announcement, with the direct rate lagging behind by about three minutes. The figure clearly indicates an incidence of how the dollar contributes to the price discovery of JPY/EUR. For uncovering the price discovery dynamics, the choice of sampling frequency is crucial. The price discovery dynamics of the two rates, from the original price to the new price, occurs within a 5-minute time resolution. Accordingly, we initially choose a 15-second sampling interval for our analysis. We also provide results based on a 5-minute sampling interval for comparison purposes.

Starting with 15-second exchange rate quotes within each intraday GMT hour, or geographical trading session, we fit the $VEC(K - 1)$ model in (10) with K chosen to minimize the Bayesian Information Criterion (BIC). The PDIRFs and PDEL for each market are then computed from the estimated $VEC(K - 1)$ coefficients following the procedure outlined in section 4.

A stylized fact of the Fx market that impacts our bootstrapping procedure to assess the sampling variability of the estimated PDIRFs and PDEL is the systematic intraday patterns of exchange rate

volatility. Dacorogna et al. (1993) show that the intraday volatility seasonality in the Fx market results from the sequential alternation of geographical trading centers. Andersen and Bollerslev (1998) identify calendar features, such as sequential trading centers, holidays of major trading centers, and day light savings time, as well as effects of prescheduled macroeconomic announcements, in the intraday volatility pattern of DM/USD rate. Because the analysis of price discovery mainly focuses on modeling the means of exchange rate movements, volatility patterns are expected to remain in estimated residuals. When computing the empirical distribution of the price discovery measures through bootstrapping, the systematic features of the residual distribution can be preserved by sampling deseasonalized residuals and then restoring the seasonalities. Together with the exchange rate quotes data, the investment company also provides an estimated volatility scaling (multiplicative) factor for the JPY/EUR rate over the sample period, accounting for the seasonal calendar effects and macroeconomic announcement effects. The scaling factor is constructed using the approach outlined in Andersen and Bollerslev (1998) with three years (2000 - 2002) of 5-minute historical data. For residuals at the 15-second resolution, each of the 5-minute scaling factors is repeated 20 times to obtain the 15-second scaling factors. Our modified bootstrapping procedure is as follows: deseasonalize the estimated residuals with the volatility scaling factors; sample the deseasonalized residual pairs with replacement; generate the bootstrapped data sample with the state-space representation of the estimated model; refit the model with the bootstrapped data and compute the PDIRFs and PDEL. In this way, we construct a bootstrap distribution with 1,000 samples for each of the price discovery measures.

4.5 Results

Figure 8 through Figure 11 show the PDIRF plots of the dollar implied and direct JPY/EUR prices in the four trading sessions. In each figure, the plots depict the impulse responses of the dollar implied (upper panel) and direct (lower panel) JPY/EUR price levels following a one unit (basis point) innovation to the JPY/EUR efficient price. In all figures, the responses of both the implied and direct JPY/EUR price levels converge toward the one basis point permanent level change in the efficient price of JPY/EUR which reflects the fact that both the dollar implied and direct JPY/EUR rates represent the same fundamental asset and share the common efficient price. Furthermore, in all trading sessions, the plots suggest that the dollar implied rate discovers the

new efficient price at a faster pace than the direct JPY/EUR rate. In particular, the PDIRFs of the direct JPY/EUR rate in all trading sessions display the dynamics characterized by the lagged price discovery model in Figure 1. There are, however, significant differences in the price discovery dynamics of the two JPY/EUR prices across trading sessions. In the Asian and post American trading sessions, the dollar implied JPY/EUR rate quickly converges to the new equilibrium while the pricing error of the direct JPY/EUR price is highly persistent. In contrast, the price discovery dynamics of the two prices during the European and American trading sessions are more similar.

To examine the sensitivity of the results to the chosen sampling frequency we re-do the PDIRF analysis using quotes aligned to a 5-minute time clock. Figures 12 through 15 present the corresponding PDIRF plots. For all but the post-American session, the plots suggest little differences in the price discovery dynamics between the two rates.

Table 4 gives the estimated PDEL ratio, $PDEL_{implied}/PDEL_{direct}$, and the spread ratio for each intraday hour. All point estimates of the PDEL ratio are less than .55, and only five of the upper confidence bounds are greater than 0.76, which suggests that the dollar markets indeed contribute substantial price discovery for JPY/EUR. The spread ratio is lower than 1 across the day implying the expected costs of exchanging the euro and yen through two round transactions against the dollar is generally lower than the costs of directly trading the two currencies. However, even though the spread ratio is lower than 1, some transactions between the euro and yen may still be carried out directly because of the “double coincidence of wants problem”⁸.

Table 5 summarizes a regression of the PDEL ratio on the spread ratio and a constant. The coefficient on the spread ratio variable is 0.40, with a robust standard error of 0.14, indicating that the dollar’s contribution to the price discovery of JPY/EUR is negatively (positively) related to the spread ratio (relative liquidity of the dollar markets). From Table 5 it can be seen that the dollar’s PDEL ratio varies from roughly 0 to .50 as the spread ratio varies from 0 to 1. The regression results support the argument that as the dollar markets become more liquid compared to the cross rate market, more nonpublic information will be revealed in the dollar markets, and the effects of public announcements (e.g. macroeconomic releases) from Japan or Europe will be more efficiently impounded into the dollar prices of the yen or the euro. Consequently, the prices in the dollar markets will be more informative. On the other hand, if the direct exchange of the euro and yen

⁸For example, see the partial indirect exchange case in Krugman (1980)).

in the cross rate market becomes relatively more liquid, there will be lower volume in the dollar markets and less price discovery of JPY/EUR occurring through the dollar.

The intraday evolution of the dollar's PDEL ratio and the spread ratio are plotted in Figure 7. Consistent with the regression analysis, there is a clear positive relationship between the relative liquidity of the dollar markets and the dollar's price discovery contribution across the day. In particular, the spread ratio is lower in the Asian and post American trading sessions, while it is higher and reaches a peak during the European and American sessions when the European markets are open. This intraday pattern of the spread ratio suggests that the spreads in the cross rate market decrease more progressively than the spreads in the two dollar markets during this period of the day even though each exchange rate market experiences lower spreads in the European business hours (see Figure 5).

This relative liquidity change between the dollar markets and the cross rate market coincides with the intraday market activity variations in Figure 4. Unlike the sharp contrast of market activity in the Asian and post American sessions, the JPY/USD market and the JPY/EUR market have more comparable trading activities during the European business hours. We interpret the non-proportional changes of market activity and spreads between the three markets as the enhanced competition of the euro against the dollar for transactions with the yen. In the Asian and post American trading sessions, the dollar dominates and most transactions between the euro and yen are intermediated by the dollar. In contrast, a more significant fraction of transactions between the euro and yen is carried out directly in the cross rate market. The euro's competition weakens the dollar's role as vehicle currency and lowers trading volume concentration in the dollar markets. As a result, the dollar's contribution to the price discovery of JPY/EUR drops and reaches a minimum during the European business hours.

Similar findings have been documented in an earlier study of the price discovery of JPY/DM by De Jong et al (1998). They find that the direct JPY/DM rate obtains its strongest price discovery role in the European and American trading hours. However, they offer no explanation for this result. The close linkage between relative liquidity and price discovery contribution identified in our analysis suggests that their findings may be explained by the enhanced liquidity in the cross rate market during the European business hours.

5 Conclusion

In this paper we propose a new approach for the econometric analysis of the dynamics of price discovery using a structural cointegration model for the price changes in arbitrage linked markets. The structural model not only characterizes the common efficient price shared by the multiple markets, but also quantifies the dynamic process by which market prices converge to, or discover, the new equilibrium efficient price. Our methodology characterizes the dynamics of price discovery based on the impulse response functions from an identified structural cointegration model, and we measure the efficiency of a market's price discovery by the absolute magnitude of cumulative pricing errors in the price discovery process. We apply our methodology to investigate the extent to which the US dollar contributes to the price discovery of the yen/euro exchange rate. Our results show that substantial price discovery of JPY/EUR occurs through the dollar. The efficiency of the dollar's price discovery is positively related to the relative liquidity of the dollar markets versus the cross rate market. This suggests that the relative liquidity and lower transaction costs in the dollar markets are attractive to profit-taking trading by informed agents, and are conducive to efficient assimilation of dispersed economy-wide information. We find that the relative liquidity of the dollar markets and the dollar's price discovery efficiency are particularly low during the European business hours, and we attribute this to the enhanced competition of the euro against the dollar for transactions with the yen.

In Yan and Zivot (2006), we used the structural price discovery model (1) to investigate the structural determinants of the information share (Hasbrouck, 1995) and the component share (Booth et al., 1999, Chu et al., 1999, and Harris et al., 2002) - two widely used price discovery measures. We found that the component share does not reflect a market's price responses to new information at all, and the information share cannot be interpreted unambiguously even when the cross-market innovations are uncorrelated. More importantly, we showed that the component share and the information share are static measures of price discovery since they only account for contemporaneous price responses to the underlying structural innovations. Hence, they say very little about the dynamics of price discovery.

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Appendix A: State Space Representation

The reduced form VECM($K - 1$) (10) may be re-expressed as

$$\begin{aligned}\beta' \mathbf{p}_t - \mu &= \beta'(\Delta \mathbf{p}_t + \mathbf{p}_{t-1}) - \mu \\ &= \beta' \sum_{k=1}^{K-1} \Gamma_k \Delta \mathbf{p}_{t-k} + (\beta' \boldsymbol{\alpha} + \mathbf{I}_2)(\beta' \mathbf{p}_{t-1} - \mu) + \beta' \mathbf{e}_t\end{aligned}\quad (15)$$

where \mathbf{I}_2 is a 2×2 identity matrix. The state space representation of (15) and (10) is:

$$\begin{bmatrix} \Delta \mathbf{p}_t \\ \Delta \mathbf{p}_{t-1} \\ \Delta \mathbf{p}_{t-2} \\ \vdots \\ \Delta \mathbf{p}_{t-K+2} \\ \beta' \mathbf{p}_t - \mu \end{bmatrix} = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \cdots & \Gamma_{K-2} & \Gamma_{K-1} & \boldsymbol{\alpha} \\ \mathbf{I}_2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_2 & \mathbf{0} & \mathbf{0} \\ \beta' \Gamma_1 & \beta' \Gamma_2 & \cdots & \beta' \Gamma_{K-2} & \beta' \Gamma_{K-1} & \beta' \boldsymbol{\alpha} + \mathbf{I}_2 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{p}_{t-1} \\ \Delta \mathbf{p}_{t-2} \\ \Delta \mathbf{p}_{t-3} \\ \vdots \\ \Delta \mathbf{p}_{t-K+1} \\ \beta' \mathbf{p}_{t-1} - \mu \end{bmatrix} + \begin{bmatrix} \mathbf{e}_t \\ 0 \\ 0 \\ \vdots \\ 0 \\ \beta' \mathbf{e}_t \end{bmatrix}, \quad (16)$$

or

$$\Delta \tilde{\mathbf{p}}_t = \mathbf{F} \Delta \tilde{\mathbf{p}}_{t-1} + \tilde{\mathbf{e}}_t,$$

where \mathbf{F} is a square matrix of dimension $l = 2 \times (K - 1) + 1$. By recursive substitution, the state vector at $t + j$ periods ahead, $\Delta \tilde{\mathbf{p}}_{t+j}$, can be written as:

$$\Delta \tilde{\mathbf{p}}_{t+j} = \tilde{\mathbf{e}}_{t+j} + \mathbf{F} \tilde{\mathbf{e}}_{t+j-1} + \mathbf{F}^2 \tilde{\mathbf{e}}_{t+j-2} + \cdots + \mathbf{F}^j \tilde{\mathbf{e}}_t + \mathbf{F}^{j+1} \Delta \tilde{\mathbf{p}}_{t-1}.$$

The dynamic multiplier matrix of $\tilde{\mathbf{e}}_t$ on $\Delta \tilde{\mathbf{p}}_{t+j}$ is:

$$\frac{\partial \Delta \tilde{\mathbf{p}}_{t+j}}{\partial \tilde{\mathbf{e}}_t} = \mathbf{F}^j.$$

The dynamic multiplier matrix of \mathbf{e}_t on $\Delta \mathbf{p}_{t+j}$, the $\boldsymbol{\Psi}_j$ matrix in (11), is:

$$\boldsymbol{\Psi}_j = \frac{\partial \Delta \mathbf{p}_{t+j}}{\partial \mathbf{e}_t} = \mathbf{F}_{[1:2, 1:2]}^j + \mathbf{F}_{[1:2, l]}^j \beta'. \quad (17)$$

The matrix $\mathbf{F}_{[1:2, 1:2]}^j$ denotes the submatrix consisting of the first 2 rows and first 2 columns of the matrix \mathbf{F}^j , and $\mathbf{F}_{[1:2, \iota]}^j$ is the submatrix consisting of the first two rows and the last column of \mathbf{F}^j . Using (16), the long-run impact matrix $\Psi(1)$ can be computed exactly as follows:

$$\begin{aligned} \Psi(1) &= \sum_{j=0}^{\infty} \Psi_j = \left(\sum_{j=0}^{\infty} \mathbf{F}^j \right)_{[1:2, 1:2]} + \left(\sum_{j=0}^{\infty} \mathbf{F}^j \right)_{[1:2, \iota]} \boldsymbol{\beta}' \\ &= \left((\mathbf{I}_2 - \mathbf{F})^{-1} \right)_{[1:2, 1:2]} + \left((\mathbf{I}_2 - \mathbf{F})^{-1} \right)_{[1:2, \iota]} \boldsymbol{\beta}'. \end{aligned} \quad (18)$$

The result is numerically equivalent Johansen's factorization (12).

The state-space approach substantially reduces the computational burden relative to the simulation approaches used in Cochrane (1994) and Hasbrouck (1995), especially in a large multi-variate system. It is also convenient for generating artificial data when bootstrapping the estimated VECM.

6 Appendix B: Extension of Structural Cointegration Model to n Markets

The extension of the structural cointegration model and the dynamic measures of price discovery to a single asset trading in n markets is straightforward. Let $\mathbf{p}_t = (p_{1,t}, \dots, p_{n,t})'$ denote a $n \times 1$ vector of $I(1)$ prices of a single asset trading in n markets linked by arbitrage. Since there is a single $I(1)$ fundamental value, there are $n - 1$ cointegrating vectors β_i such that $\beta_i' \mathbf{p}_t \sim I(0)$. Furthermore, since the difference between any two prices is $I(0)$ it is convenient to use as a basis for the cointegrating space the following $(n - 1) \times n$ matrix of rank $n - 1$:

$$\mathbf{B}' = \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \dots & \vdots \\ 1 & 0 & \dots & \dots & -1 \end{pmatrix} = (\mathbf{1}_{n-1} \dot{\vdash} -\mathbf{I}_{n-1}),$$

where $\mathbf{1}_{n-1}$ is an $(n - 1) \times 1$ vector of ones. As in the two asset case, the restriction $\mathbf{B}'\Psi(1) = \mathbf{0}$ implies that the $n \times n$ matrix $\Psi(1)$ has rank one and can be expressed as $\mathbf{1}_n \psi'$ where $\psi' = (\psi_1, \dots, \psi_n)$ is the $n \times 1$ common row vector of $\Psi(1)$.

In the SMA representation (1), the structural shocks are now $\boldsymbol{\eta}_t = (\eta_t^P, \boldsymbol{\eta}_t^{T'})'$ where $\boldsymbol{\eta}_t^{T'}$ is an $(n-1) \times 1$ vector of transitory shocks. The elements of $\boldsymbol{\eta}_t$ have mean zero, are mutually uncorrelated at all leads and lags and have diagonal covariance matrix $\text{diag}(\sigma_P^2, \sigma_{T,1}^2, \dots, \sigma_{T,n-1}^2)$. The permanent shock satisfies (2), the $n - 1$ transitory shocks satisfy (3) and so the $n \times n$ long-run impact matrix has the form $\mathbf{D}(1) = [\mathbf{1}_n \dot{\vdash} \mathbf{0}_{n \times (n-1)}]$.

The procedure to identify the permanent and transitory shocks remains essentially the same. The $n \times n$ matrix to rotate the reduced form errors to (correlated) permanent and transitory shocks is $\mathbf{G} = [\boldsymbol{\psi} \dot{\vdash} \mathbf{B}]'$ so that the single permanent shock is $\epsilon_t^P = \boldsymbol{\psi}' \mathbf{e}_t$ and the $(n - 1) \times 1$ vector of transitory shocks is $\boldsymbol{\epsilon}_t^T = \mathbf{B}' \mathbf{e}_t$. The ordering of the variables in $\Delta \mathbf{p}_t$ does not alter ϵ_t^P but does alter $\boldsymbol{\epsilon}_t^T$. The rotated errors $\boldsymbol{\epsilon}_t = (\epsilon_t^P, \boldsymbol{\epsilon}_t^{T'})' = \mathbf{G} \mathbf{e}_t$ are orthogonalized using the triangular factorization matrix \mathbf{H} to give the orthogonalized structural errors $\boldsymbol{\eta}_t = \mathbf{H}^{-1} \boldsymbol{\epsilon}_t$. Because \mathbf{H} is lower triangular with ones along the diagonal, $\eta_t^P = \epsilon_t^P$ and is not affected by the ordering of the variables in $\Delta \mathbf{p}_t$ but $\boldsymbol{\eta}_t^{T'}$ is. As a result, the PDIRFs and the PDELs are not influenced by the ordering of the variables

in $\Delta \mathbf{p}_t$.

Table 1: PDEL Estimates from Simulated Prices

This table reports the price discovery efficiency loss (PDEL) estimates from the price data simulated from the following 2-market model:

$$\begin{aligned}
 p_{i,t} &= p_{i,t-1} + \delta_i(m_t - p_{i,t-1}) + b_{i,0}^T \eta_t^T \\
 m_t &= m_{t-1} + \eta_t^P
 \end{aligned}$$

where $i = 1, 2$ for two markets, and the structural errors $\boldsymbol{\eta}_t = (\eta_t^P, \eta_t^T)'$ are normally distributed with zero means, and diagonal covariance matrix $diag(\sigma_P^2, \sigma_T^2)$, and are mutually uncorrelated at all leads and lags. The simulation parameterization is $\delta_1 = 0.8, \delta_2 = 0.2, b_{0,1}^T = 0.5, b_{0,2}^T = -0.5, \sigma_P^2 = 1$ and $\sigma_T^2 = 0.64$. The second column gives the true PDELs implied by the specified parametrization. The last four columns of the table report the estimated PDELs from artificial samples of size 500, 1000, 5000, and 10000 observations. For each sample, the VEC model is fitted with the BIC optimal lag length and the PDIRFs are estimated. The PDELs are computed with the absolute value loss function and $K^* = 30$.

	True Values	N = 500	N = 1000	N = 5000	N = 10000
PDEL ₁	0.250	0.999 (0.367, 1.515)	0.380 (0.226, 1.094)	0.326 (0.211, 0.808)	0.243 (0.219, 0.496)
PDEL ₂	3.995	2.518 (1.557, 3.954)	3.745 (2.633, 4.997)	4.092 (3.551, 4.666)	3.971 (3.596, 4.339)
$\log(\frac{PDEL_1}{PDEL_2})$	-2.771	-0.925 (-2.280, -0.155)	-2.288 (-2.876, -1.094)	-2.531 (-2.916, -1.761)	-2.793 (-2.912, -2.100)

Notes: 95% bootstrap confidence interval in parenthesis

Table 2: PDEL Estimates from Simulated Prices: Two Transitory Innovations

This table reports the price discovery efficiency loss (PDEL) estimates from the price data simulated from the following 2-market model:

$$\begin{aligned}
 p_{i,t} &= p_{i,t-1} + \delta_i(m_t - p_{it-1}) + b_{i,0}^T \eta_{it}^T \\
 m_t &= m_{t-1} + \eta_t^P
 \end{aligned}$$

where $i = 1, 2$ for two markets, and the structural errors $\boldsymbol{\eta}_t = (\eta_t^P, \eta_{1t}^T, \eta_{2t}^T)'$ are normally distributed with zero means, and diagonal covariance matrix $diag(\sigma_T^2, \sigma_{1T}^2, \sigma_{2T}^2)$, and are mutually uncorrelated at all leads and lags. The simulation parameterization is $\delta_1 = 0.8, \delta_2 = 0.2, b_{0,1}^T = 0.5, b_{0,2}^T = 0.5, \sigma_P^2 = 1, \sigma_{1T}^2 = 0.64, \sigma_{2T}^2 = 0.64$. The second column gives the true PDELs implied by the specified parametrization. The last four columns of the table report the estimated PDELs from artificial samples of size 500, 1000, 5000, and 10000 observations. For each sample, the VEC model is fitted with the BIC optimal lag length and the PDIRFs are estimated. The PDELs are computed with the absolute value loss function and $K^* = 30$.

	True Values	N = 500	N = 1000	N = 5000	N = 10000
PDEL ₁	0.250	0.945 (0.210, 1.700)	0.218 (0.052, 1.120)	0.056 (0.026, 0.488)	0.111 (0.022, 0.424)
PDEL ₂	3.995	2.384 (1.393, 3.705)	3.773 (2.679, 5.057)	3.672 (3.189, 4.185)	3.849 (3.476, 4.236)
$\log(\frac{PDEL_1}{PDEL_2})$	-2.771	-0.925 (-2.773, 0.107)	-2.850 (-4.357, -0.958)	-4.182 (-4.937, -2.018)	-3.543 (-5.152, -2.302)

Notes: 95% bootstrap confidence interval in parenthesis

Table 3: Intraday Geographical Sessions of the Fx Market

The table defines four geographical trading sessions in a 24-hour trading day based on the local business hours of Tokyo, London, and New York. The correspondance of GMT hours and local business hours is based on the daylight savings time.

Geographical Segments	Hours in GMT	Local Business Hours
Asian	22:00 - 06:00 (next day)	07:00 - 15:00 (Tokyo time)
European	06:00 - 12:00	07:00 - 13:00 (London time)
American	12:00 - 18:00	08:00 - 14:00 (New York time)
Post American	18:00 - 22:00	14:00 - 18:00 (New York time)

Table 4: Price Discovery Efficiency Loss (PDEL) Ratio and Spread Ratio

This table reports the price discovery efficiency loss (PDEL) ratio and the spread ratio between the US dollar implied JPY/EUR market and the direct JPY/EUR market for each intraday GMT hour. The PDELs are estimated by the VEC model of the dollar implied and direct JPY/EUR prices at 15-second resolutions with the sample from July to September 2003. The lag length of the VEC model is optimally chosen by the BIC. The PDELs are computed with the absolute loss function and the PDIRF truncation lag of 30 (7 and a half minutes equivalent). Column Qt2.5 and Qt97.5 bracket 95% bootstrap confidence intervals of PDEL estimates. The spread ratio is defined as the ratio of the sum of spreads in the USD/EUR and JPY/USD markets to the spreads in the JPY/EUR market.

GMT Hours	PDEL Ratio			Spread Ratio
	Estimates	Qt2.5	Qt97.5	
22:00 - 23:00	0.08	0.02	0.19	0.42
23:00 - 00:00	0.23	0.07	0.76	0.45
00:00 - 01:00	0.21	0.09	0.47	0.53
01:00 - 02:00	0.26	0.14	0.57	0.56
02:00 - 03:00	0.54	0.32	1.38	0.60
03:00 - 04:00	0.27	0.08	0.98	0.49
04:00 - 05:00	0.13	0.06	0.62	0.53
05:00 - 06:00	0.32	0.15	0.88	0.55
06:00 - 07:00	0.28	0.20	0.46	0.63
07:00 - 08:00	0.21	0.14	0.37	0.76
08:00 - 09:00	0.35	0.24	0.52	0.78
09:00 - 10:00	0.26	0.21	0.43	0.72
10:00 - 11:00	0.22	0.15	0.38	0.73
11:00 - 12:00	0.47	0.32	0.70	0.71
12:00 - 13:00	0.34	0.26	0.52	0.73
13:00 - 14:00	0.29	0.25	0.40	0.76
14:00 - 15:00	0.46	0.36	0.62	0.75
15:00 - 16:00	0.34	0.21	0.63	0.66
16:00 - 17:00	0.21	0.11	0.34	0.55
17:00 - 18:00	0.26	0.11	0.56	0.51
18:00 - 19:00	0.11	0.07	0.25	0.44
19:00 - 20:00	0.24	0.08	0.53	0.45
20:00 - 21:00	0.18	0.07	0.31	0.39
21:00 - 22:00	0.39	0.15	0.77	0.43

Table 5: Regression of The PDEL Ratio and The Spread Ratio

This table presents regression analysis of the PDEL ratio on the spread ratio. The standard errors are autocorrelation-heteroscedasticity consistent.

Variables	Coefficient	Std. Error	p value
Constant	0.04	0.09	0.63
Spread Ratio	0.40	0.14	0.01
AdjR2	0.17		

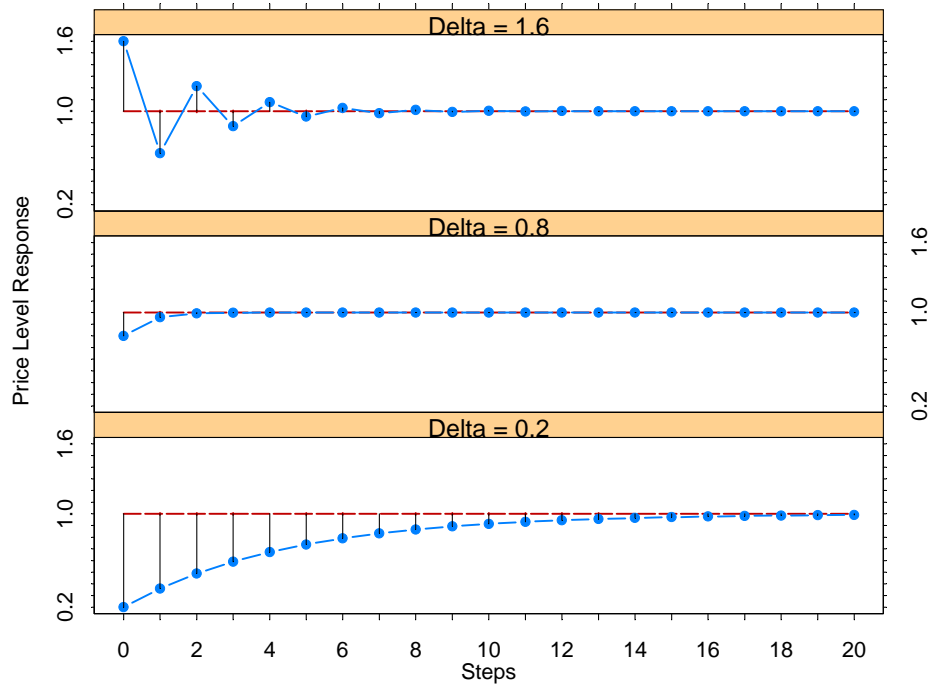


Figure 1: Price discovery impulse response functions. The figure plots the price responses implied by the partial price adjustment model subsequent to one unit innovation in the efficient price.

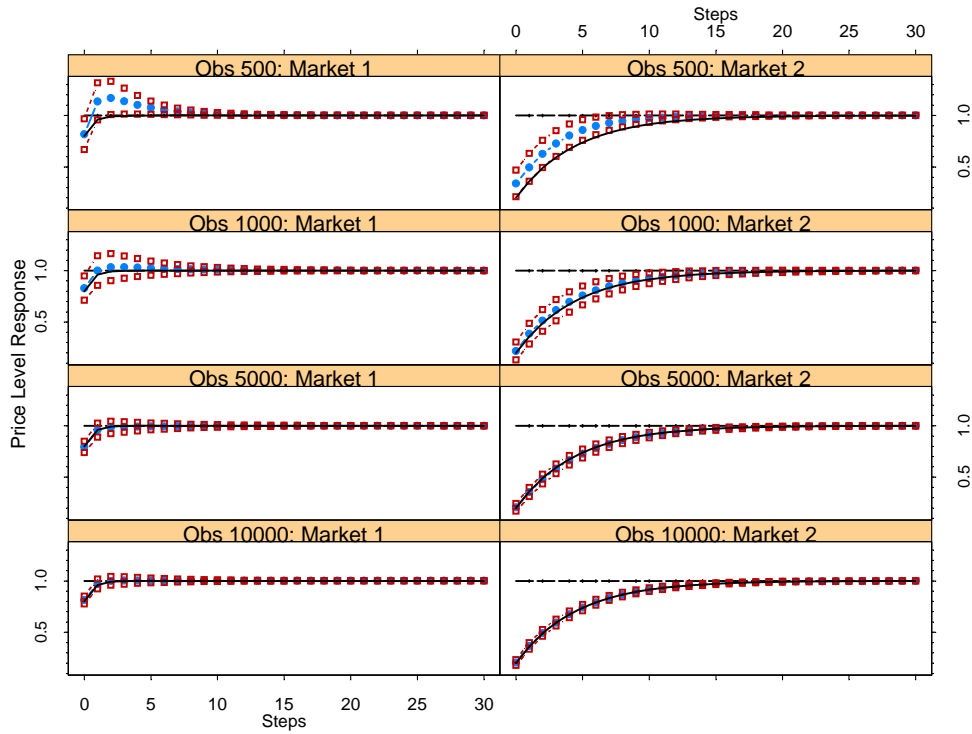


Figure 2: Price discovery impulse response functions (PDIRFs) of simulated prices.

This figure compares the estimated price discovery dynamics of two markets with their true dynamics given by a stylized partial adjustment model. The model is specified as in Table 1, and features a frictional innovation affecting both markets. Four rows of plots corresponds to simulation samples of size 500, 1000, 5000, and 10000 observations. The PDIRFs are estimated using the VEC model with the BIC optimal lag length. In each plot, the solid black line depicts the true PDIRFs; the estimated PDIRFs are given by the blue diamond dotted line; the red square dotted lines bracket the ± 2 bootstrap standard deviation confidence intervals.

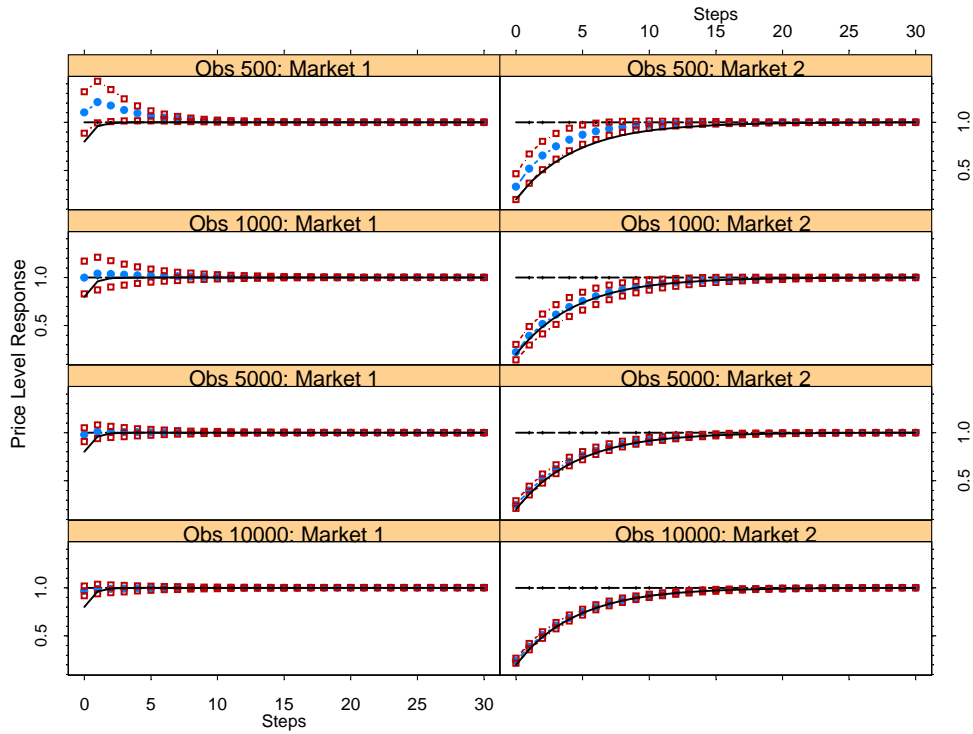


Figure 3: Price discovery impulse response functions (PDIRFs) of simulated prices: separate transitory innovations. This figure compares the estimated price discovery dynamics of two markets with their true dynamics given by a stylized partial adjustment model. The model is specified as in Table 2, and features separate transitory innovations for two markets. Four rows of plots corresponds to simulation samples of size 500, 1000, 5000, and 10000 observations. The PDIRFs are estimated using the VEC model with the BIC optimal lag length. In each plot, the solid black line depicts the true PDIRFs; the estimated PDIRFs are given by the blue diamond dotted line; the red square dotted lines bracket the ± 2 bootstrap standard deviation confidence intervals.

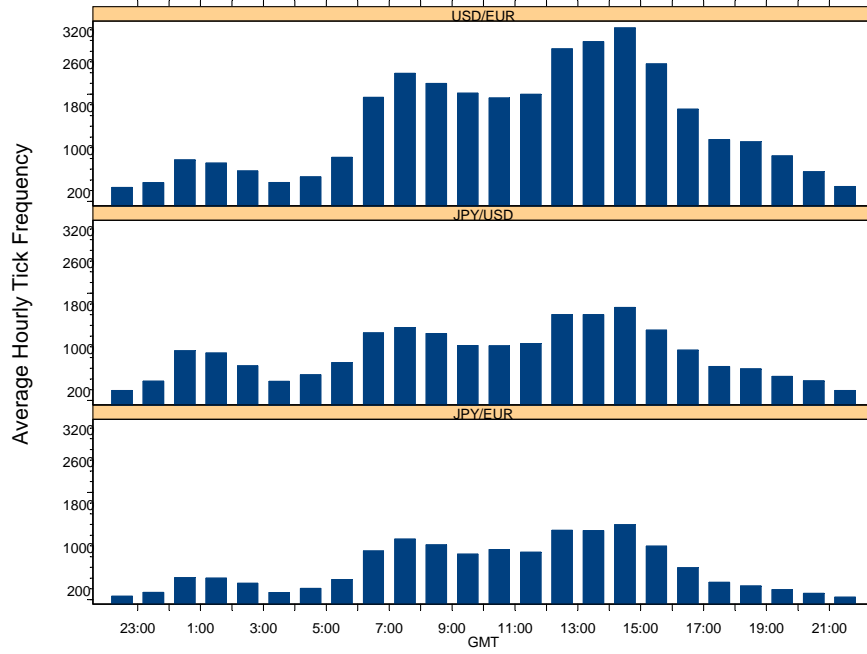


Figure 4: Average hourly tick Frequency of exchange rate quotes. The figure displays the average hourly quote entries of USD/EUR, JPY/USD, and JPY/EUR in the upper, middle, and lower panels respectively. All times are in GMT. The sample period ranges from July 6 to September 26, 2003, 60 trading days in total. The average hourly quote frequency for one exchange rate is obtained by averaging the quote counts for the exchange rate within a particular GMT hour across 60 trading days. The first stackbar is for the hour 22:00 - 23:00, and the last stackbar is for the hour 21:00 - 22:00.

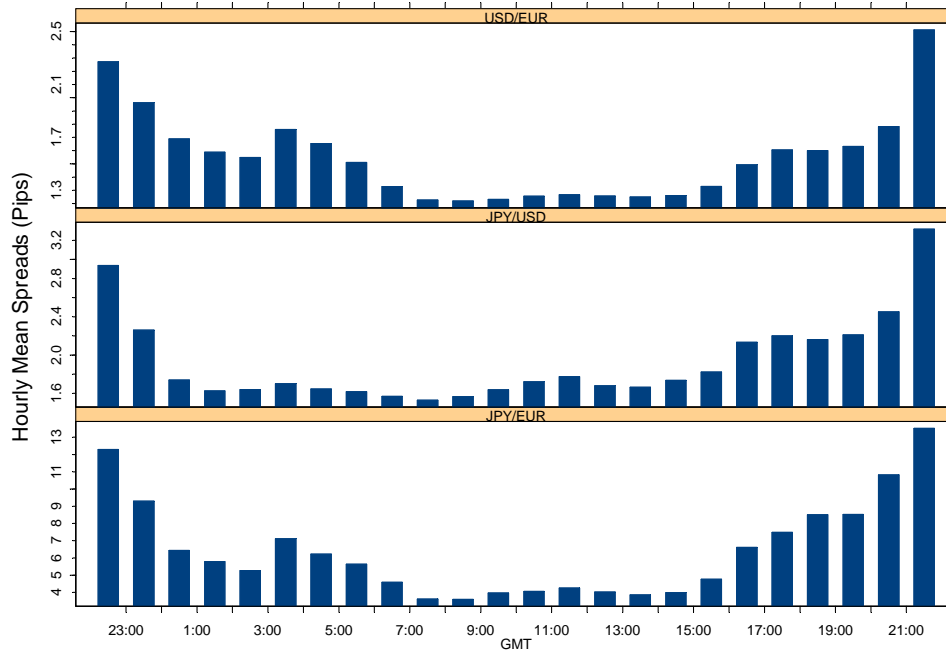


Figure 5: Hourly mean bid-ask spreads of exchange rates. The figure displays the hourly average bid-ask spreads of USD/EUR, JPY/USD, and JPY/EUR in the upper, middle, and lower panels respectively. All times are in GMT. The sample period ranges from July 6 to September 26, 2003, 60 trading days in total. The mean bid-ask spreads are computed by averaging all spreads within a particular GMT hour across 60 trading days in the sample. The first stackbar is for the hour 22:00 - 23:00, and the last stackbar is for the hour 21:00 - 22:00. Spreads are measured in units of pips.

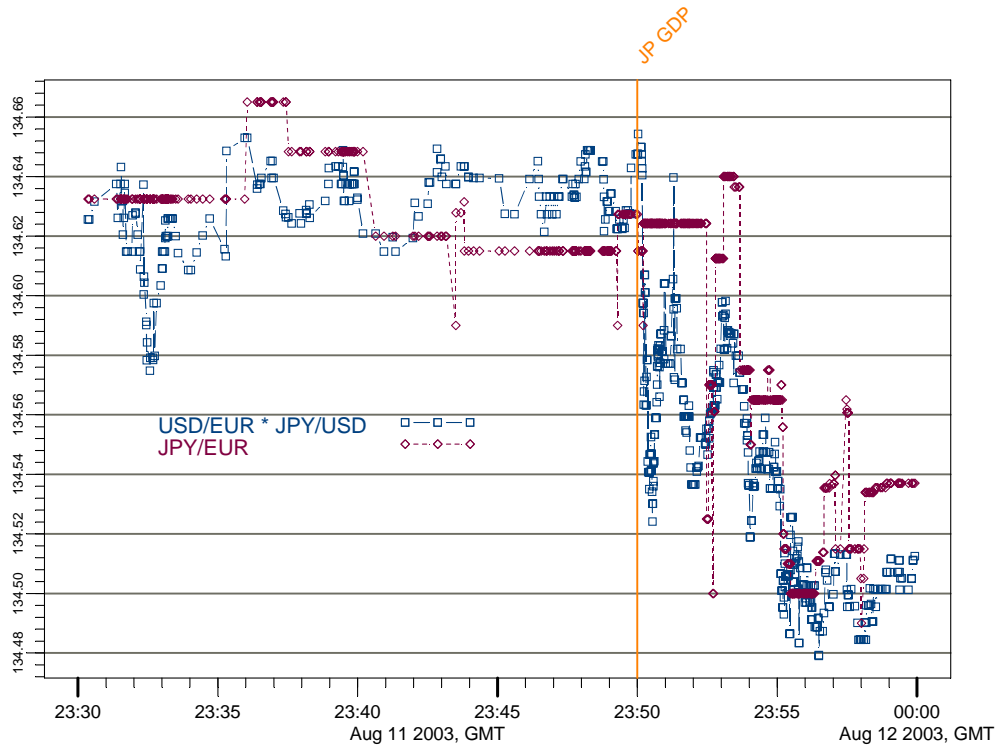


Figure 6: One episode of the Fx market movement. The figure depicts the price movements of the direct and dollar implied JPY/EUR rates around 23:50 GMT, August 11, 2003, at which Japan released the first GDP estimates for the second quarter of 2003. The blue line with squares depicts the movement of the dollar implied JPY/EUR price (midquote) and the red line with triangles traces the direct JPY/EUR price (data are in their original scale).

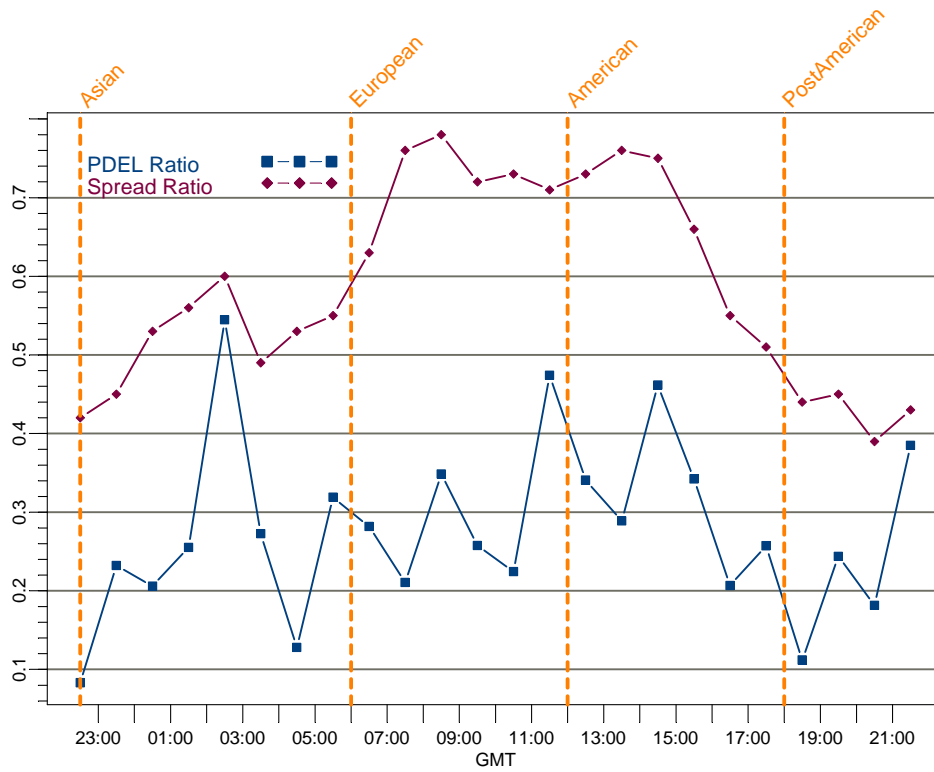


Figure 7: The PDEL ratio and spread ratio, July to September, 2003. The figure plots the price discovery efficiency loss (PDEL) ratio and the spread ratio between the US dollar implied JPY/EUR market and the direct JPY/EUR market for each intraday GMT hour. The PDELs are estimated by a vector error correction model of the dollar implied and direct JPY/EUR prices at 15-second resolutions. The spread ratio is defined as the ratio of the sum of spreads in the USD/EUR and JPY/USD markets to the spreads in the JPY/EUR market.

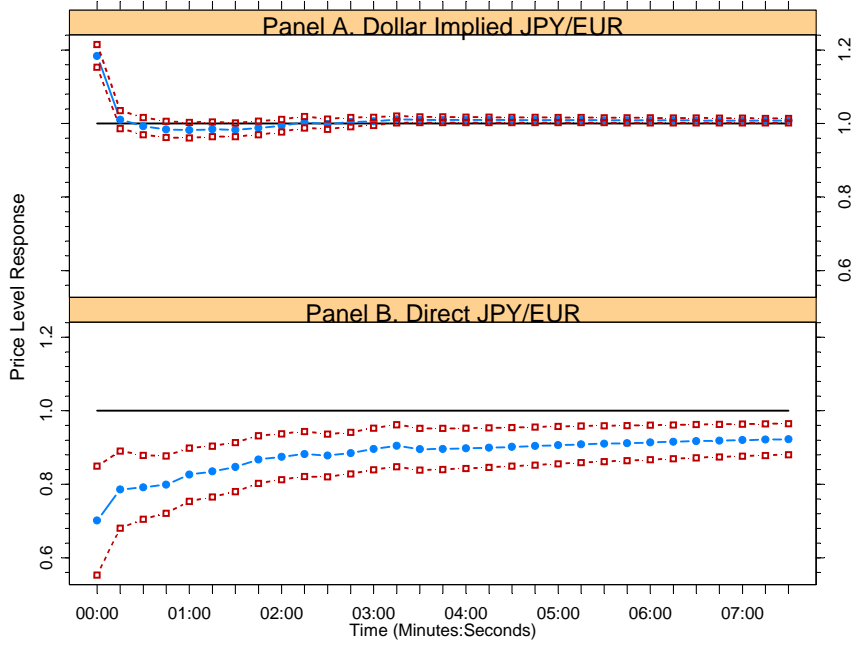


Figure 8: Price discovery impulse response functions: Asian; 15-second. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during Asian trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 15-second resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two- bootstrapping-standard deviations of the impulse responses.

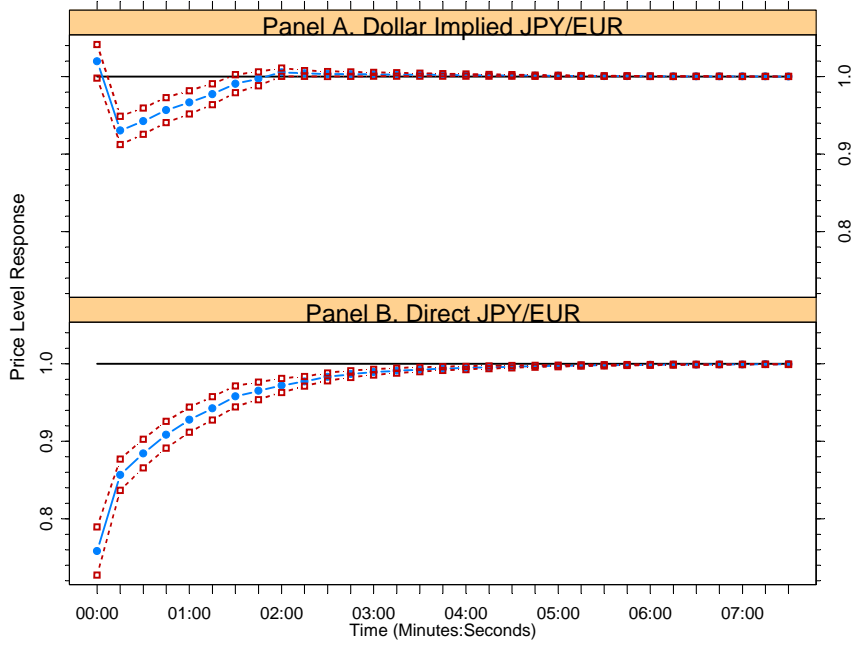


Figure 9: Price discovery impulse response functions: European; 15-second. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during European trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 15-second resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two-bootstrapping-standard deviations of the impulse responses.

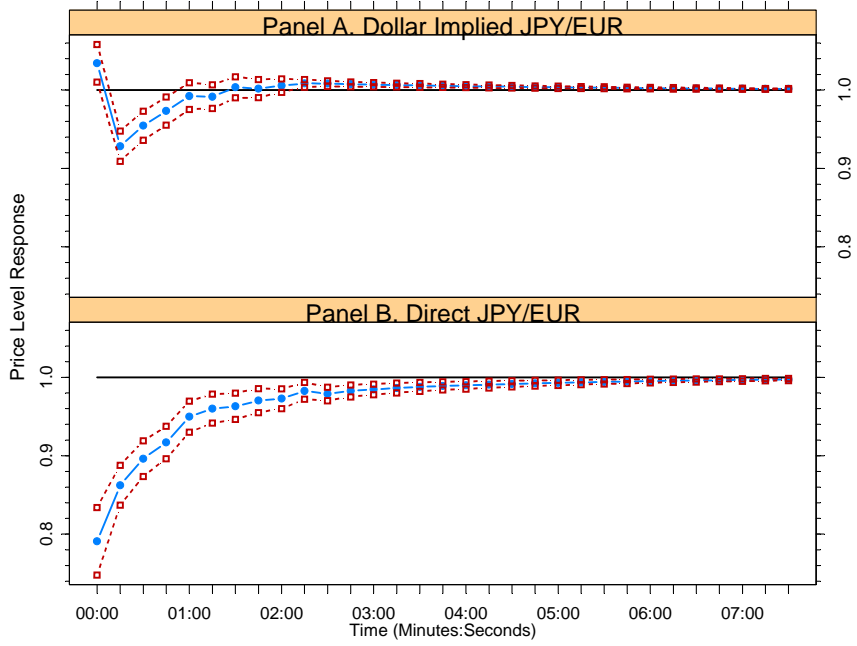


Figure 10: Price discovery impulse response functions: American; 15-second. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during American trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 15-second resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two- bootstrapping-standard deviations of the impulse responses.

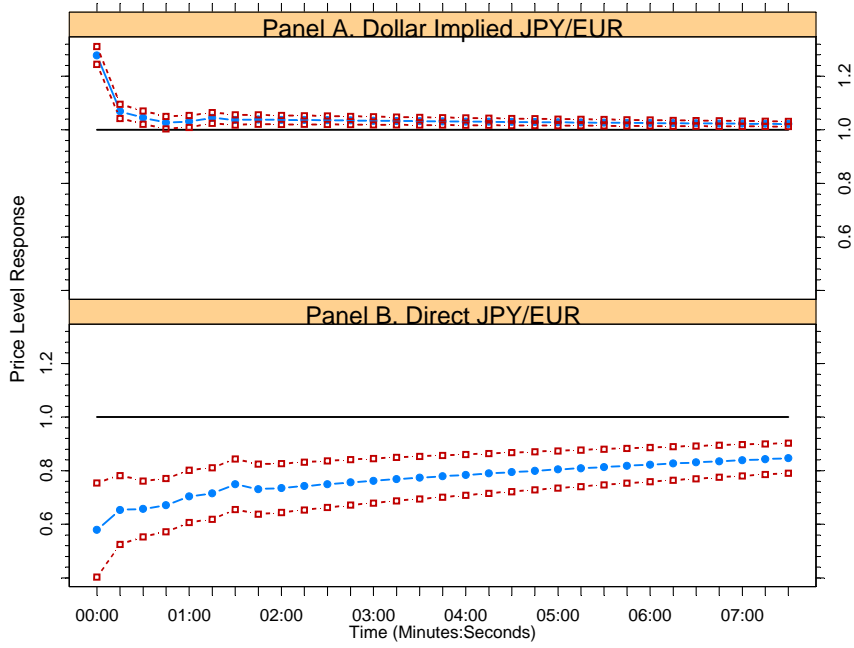


Figure 11: Price discovery impulse response functions: post American; 15-second. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during post American trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 15-second resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two- bootstrapping-standard deviations of the impulse responses.

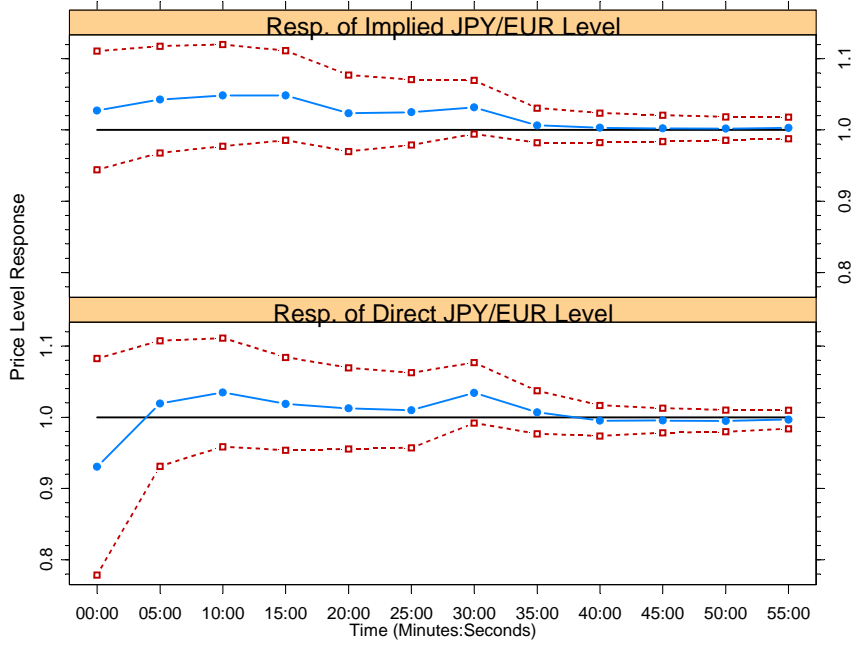


Figure 12: Price discovery impulse response functions: Asian; 5-minute. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during Asian trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 5-minute resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two- bootstrapping-standard deviations of the impulse responses.

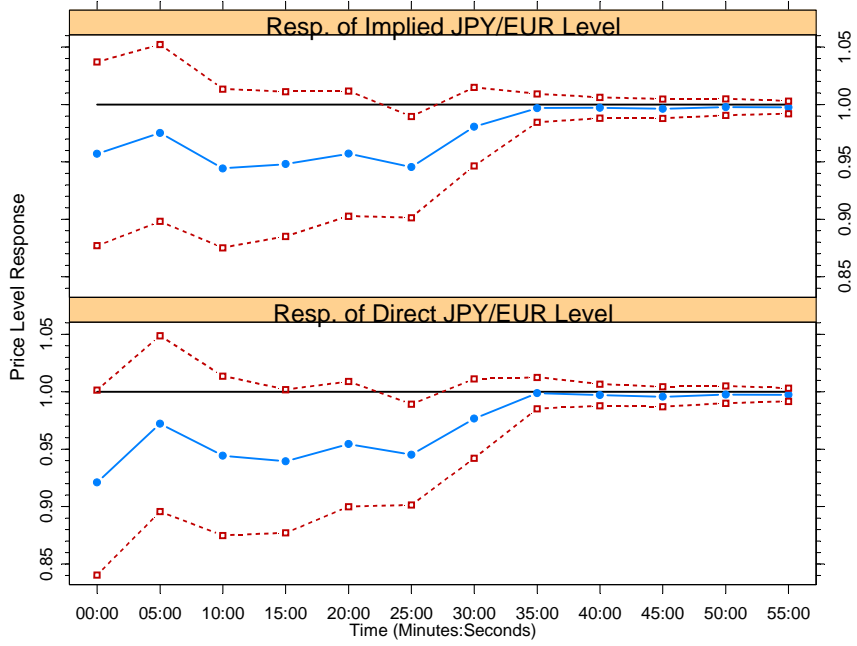


Figure 13: Price discovery impulse response functions: European; 5-minute. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during European trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 5-minute resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two-bootstrapping-standard deviations of the impulse responses.

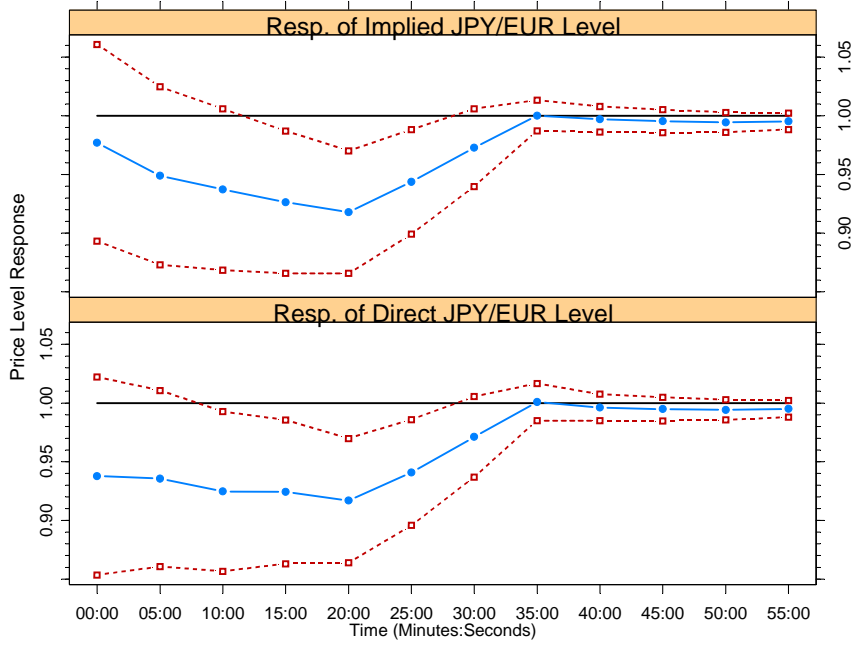


Figure 14: Price discovery impulse response functions: American; 5-minute. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during American trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 5-minute resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two-bootstrapping-standard deviations of the impulse responses.

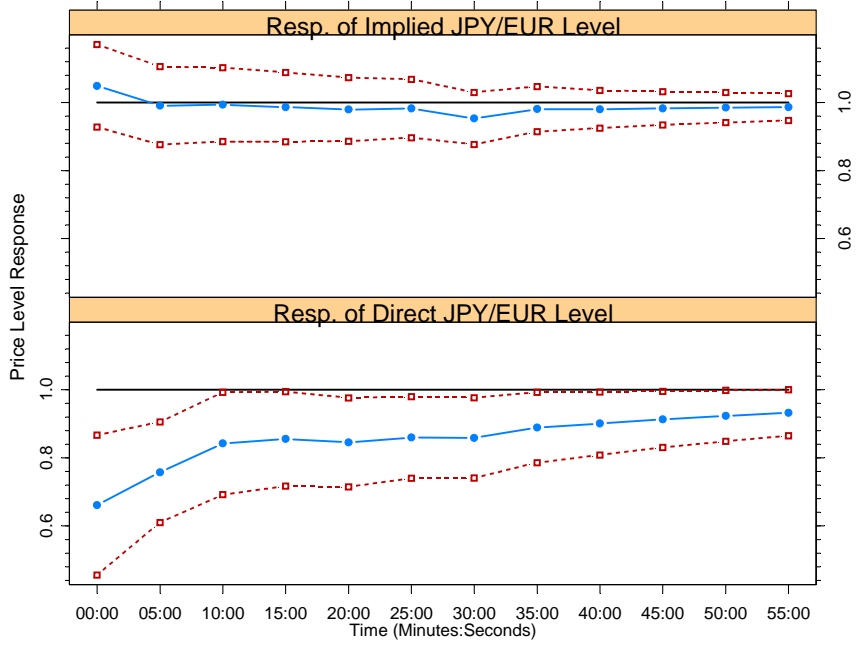


Figure 15: Price discovery impulse response functions: post American; 5-minute. The figure plots the impulse response functions of the dollar implied (upper panel) and direct (lower panel) JPY/EUR prices during post American trading hours subsequent to one unit innovation in the efficient price of JPY/EUR. The estimates are based on the vector error correction model estimated at 5-minute resolution. For trading hours specification, refer to Table 3. Dotted lines bracket the confidence interval constructed by two- bootstrapping-standard deviations of the impulse responses.