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Economic growth and inequality tradeoffs under progressive taxation[☆]



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ABSTRACT

This paper examines the sensitivity of the growth-inequality tradeoff to the progressivity of the tax structure. Using an endogenous growth model calibrated to approximate the US tax structure, we simulate a 5% reduction in the economy-wide average tax rate attained by alternative combinations of a reduction in the base tax rate and an increase in progressivity. In all cases the dynamic responses across income classes are diverse. With fixed progressivity the tax benefits favor the poorer quintiles, but the richest quintile is the only group that increases its relative share of capital and income. The net long-run effect is a mild increase in the overall growth rate, accompanied by a substantial increase in inequality. Restructuring the tax cut by allowing a larger reduction in the base tax rate coupled with a large increase in progressivity, designed to maintain the tax burden on the richest quintile unchanged, the exact opposite responses emerge; growth and inequality both decline. By judicious choice of the crucial fiscal parameters, the tax cut can be structured so that the growth rate increases while inequality simultaneously declines. The required structural changes are small, highlighting the extreme sensitivity of the tradeoff to the degree of tax progressivity.

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1. Introduction

The influence of an economy's tax structure on its growth rate, the associated distribution of income across heterogeneous agents, and potential tradeoffs has been widely discussed. This has given rise to an extensive literature, most of which adopts the conventional assumption of flat tax rates; see e.g. [Stokey and Rebelo \(1995\)](#), [Baier and Glomm \(2001\)](#), [Domeij and Heathcote \(2004\)](#), [García-Peñalosa and Turnovsky \(2011\)](#). But in fact tax systems in most advanced economies exhibit vary-

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ing degrees of progressivity, so that it is important to reassess these issues in the context of a more representative tax structure. To do so is the objective of the present paper.

A key issue endemic to this literature concerns the source of the underlying heterogeneity generating the inequality across agents. Ramsey (1928) originally conjectured, and Becker (1980) later formalized, that the long-run distribution of wealth in an economy with flat taxes, and in which agents have differential rates of time preference (RTP) degenerates; all the capital will ultimately be owned by the most patient household. More recently, Sarte (1997), Li and Sarte (2004), and others have shown that this extreme result breaks down with a progressive tax structure. This is because a progressive income tax balances an agent's RTP against his increasing marginal tax rate, thereby ensuring non-degenerate long-run distributions of wealth and income.

A major complication introduced by a progressive tax structure is that aggregate quantities and their distributions across individuals become simultaneously determined. As a result, one can no longer employ the Gorman (1959) aggregation procedure – possible under uniform taxes – in which case a general formal analysis of the transitional dynamics becomes intractable.¹ Koyuncu and Turnovsky (2016) circumvent this difficulty by focusing on just two income classes – the ‘rich’ and ‘poor’. While this offers the advantage of tractability, it yields a limited picture of the internal dynamics generated by tax and other structural changes. It fails to capture adequately the diverse impacts on the different income classes, due to the differential tax rates they face, and how their responses evolve over time as they pass through different tax brackets during the transition.

In this paper we adopt a computational approach by extending the generic Romer (1986) endogenous growth model to evaluate the distributional consequences of tax policy across broader income classes that we shall identify by quintiles. We calibrate the model to reflect the broad features of the US tax structure, and proceed numerically by considering the growth and distributional effects of the Tax Cut and Jobs Act enacted in 2017. Since our model is stylized it obviously cannot capture the details of complex tax legislation.² Rather, its intent is primarily motivational, to provide a concrete context in which to evaluate the distributional consequences of tax policy across broad income classes, highlighting the role of tax progressivity in this process.

Focusing on quintiles, offers several important advantages. First, it provides a much richer picture of the transitional dynamics experienced by the different income groups as they move along the income distribution in response to the evolving tax rates. Moreover, quintile data are widely available, and are sufficiently disaggregate to enable us to calculate a reasonable approximation to GINI coefficients, which conveniently summarizes the overall economy-wide distributional responses. A third reason for disaggregating at the level of quintiles is that for taxation studies the number of income classes should, ideally, be consistent with (or at least closely match) the income brackets of the tax code. This condition is approximately met in the case of the US. Finally, working with 5 income classes is computationally manageable.

In applying this framework we introduce several important modifications to existing related studies. Most importantly, we replace the RTP as the source of heterogeneity by the elasticity of intertemporal substitution (EIS). While variations in the RTP succeed in yielding a non-degenerate long-run distribution of income, in reality the differences across agents are small, certainly no more than two percentage points between the top and bottom income classes, and likely to be insignificantly different from a statistical standpoint.³ Accordingly, as we shall demonstrate in the Appendix, variations in RTP alone are almost certainly insufficient to reconcile the observed differential marginal tax rates with maintaining equilibrium in a conventional growth model. In contrast, the EIS, which plays an identical role to that of the RTP in reconciling tax rates with preferences, is extensively documented to exhibit substantial variation across income classes. These observed differentials are entirely compatible with observed differentials in the marginal tax rates across the range of income levels, and accordingly we view the EIS as a much more compelling source of agent heterogeneity.

We also emphasize two other aspects that assume particular importance in the context of progressive taxation. First, labor supply is assumed to be endogenous. This is particularly relevant in the presence of a progressive tax structure, with its potentially significant impact on work incentives. The fact that the progressivity of the tax structure has an adverse effect on labor supply is well established empirically.⁴ The other issue is the importance of considering the entire dynamic transitional path and not just the steady state, as some studies do. This is because as our analysis suggests, for plausible parameterization the transitional adjustment is slow. This means that the economy spends the overwhelming fraction of its time away from its balanced growth path, underscoring the need to consider the entire transitional path. But, our simulations also highlight the diverse effects on the time paths followed by different income classes in their responses to tax policy, with a tendency, in some cases for initial responses to be reversed over time, as agents' marginal tax rates adjust during the transition.

¹ This aggregation also depends upon other aspects, such as homogeneity of the preference function, and complete markets, with all individuals facing the same rates of return on productive factors. The difficulty introduced by the progressive tax structure is that each individual faces his own net rate of return, depending upon his tax rate.

² For an extensive study of the macroeconomic effects of the 2017 tax reform see Barro and Furman (2018).

³ Empirical estimates of RTP are sparse, but the little available evidence confirms the small variation across income classes and assumed in calibrations. For example, recent estimates obtained by de Lipsis (2021) across 6 affluent Western European countries typically find less than 1 p.p. difference between the RTP for the top quintile and the bottom quintile. A similar difference is obtained by Lawrance (1991) in an earlier study for the US.

⁴ The theoretical importance of labor supply has been emphasized in a variety of contexts pertaining to inequality; see e.g. Carroll and Young (2011), Krueger and Ludwig (2013), Koyuncu and Turnovsky (2016), and Chen (2020). Supporting empirical evidence includes Hausman (1981) using United States data, Blomquist et al. (2001) using Swedish data, and the cross-country study of Guvenen et al. (2014). Guner et al. (2012) evaluate the impact of reforms to the US tax system on labor supply in a life-cycle setup with heterogeneous households.

To analyze the model we perform three sets of numerical simulations. The first is a basic 5% across-the-board cut in the level of the tax rate, with the degree of progressivity unchanged. The other two simulations then restructure the 5% cut in the average tax rate by combining the reduction in the base rate with increases in the degree of progressivity of varying degrees. The point of these comparisons is to highlight just how extremely sensitive the impact of tax policy on the growth-inequality tradeoff is to its progressivity.

The numerical simulations offer several insights into the distributional consequences of tax reductions and how they are impacted by the degree of tax progressivity. First, a seemingly neutral cut in the tax base, alone, has sharply contrasting consequences for the different income classes. In the short run, the rate of asset accumulation of the richest quintile increases most rapidly, providing the main driving force for the initial increase in the economy-wide growth rate. As a result, over time this class increases its shares of output and capital, while those of poorer quintiles decline. As these relative reallocations occur during the transition, the tax rate on the richest quintile increases, while that of all other classes declines. As a result, the growth rates of all quintiles converge to a slightly higher common, growth rate, while the increased prosperity of the rich leads to substantial long-run increases in wealth and income inequality.

Second, if to avoid this increase in inequality the 5% tax cut is restructured to include an increase in progressivity so that the rich do not enjoy an initial tax cut, essentially the opposite adjustment occurs. The growth rate of the rich declines, driving down the overall growth rate of the economy, their overall share of income and capital, and leading to a substantial decline in inequality. Neither tradeoff is appealing, and we show that reducing the overall tax rate by an appropriate balanced reduction in the base tax rate and modest increase in progressivity can achieve a long-run increase in the growth rate accompanied by a reduction in inequality. But the changes in the tax parameters generating these alternative scenarios are small, confirming that the growth-inequality tradeoff is highly sensitive to the degree of progressivity, a warning that care must be taken when adjusting the tax structure to avoid undesired and unexpected consequences with respect to the growth-distribution tradeoffs.

Several papers study the macroeconomic consequences of progressive taxes. Most closely related to the present paper include [Sarte \(1997\)](#), [Li and Sarte \(2004\)](#), [Koyuncu and Turnovsky \(2016\)](#), and [Chen \(2020\)](#) all of which focus on the growth-inequality tradeoff. But in addition to our use of the EIS rather than the RTP as the source of heterogeneity, other important differences exist in each case. Both [Sarte \(1997\)](#) and [Li and Sarte \(2004\)](#) treat labor supply as inelastic, and while they both consider quintiles, most of their focus is on the steady state; to the extent they consider the transitional dynamics, they address the aggregate responses. In contrast, [Koyuncu and Turnovsky \(2016\)](#) and [Chen \(2020\)](#) focus more on the transitional dynamics, but restrict their attention to just two income groups. Other papers address the impact of the progressive structure on the aggregate economy. Thus, [Guo and Lansing \(1998\)](#) and [Chen and Guo \(2013\)](#) discuss how a progressive tax structure can eliminate potential problems of indeterminacy, and influence aggregate stability. [Heathcote et al. \(2017\)](#) determine the optimal degree of progressivity, while [Guner et al. \(2014\)](#) provide alternative parametric estimates of alternative tax functions and their implications for income distribution as quantified by quintiles.

The remainder of the paper is structured as follows. [Section 2](#) sets out the analytical framework, while [Sections 3](#) and [4](#) characterize the macroeconomic equilibrium and steady state, respectively. [Section 5](#) discusses the calibration of the model, while [Section 6](#) then numerically simulates the growth and distributional effects of changes in the tax rates. This section briefly summarizes some simulations in which labor and income are subject to differential tax rates. The final section briefly draws conclusions.

2. The analytical framework

We adapt the endogenous growth model of progressive taxation of income and elastic labor supply developed by [Koyuncu and Turnovsky \(2016\)](#), modifying it in two important respects. The first, is to characterize agents' heterogeneity in terms of their EIS, rather than the more widely used RTP. The second is to replace their specification of the progressive tax structure employed by [Li and Sarte \(2004\)](#) with an alternative adopted by [Guo and Lansing \(1998\)](#). This is because a serious problem with the former is that the progressivity of the tax rates is overly steep. It implies that at some point high income earners (those in the top quintile) would reach a point where not only their marginal tax rate but also their average tax rate would reach 100%. This is clearly unrealistic and unsatisfactory in terms of tracking the US economy, in which the marginal tax rate flattens out at something below 40%. In contrast, full taxation of marginal income in the Guo-Lansing specification occurs only in the limit as the agent's income approaches infinity.

2.1. Production technology

We assume there are J identical firms indexed by j . The representative firm produces output, Y_j , in accordance with the Cobb-Douglas production function

$$Y_j = A(L_j K)^{\alpha} K_j^{1-\alpha} \quad (1a)$$

where K_j and L_j denote the individual firm's capital stock and employment of labor, K is the average economy-wide capital stock, so that $(L_j K)$ specifies the efficiency units of labor employed by the firm, and A represents the level of technology, as reflected in TFP. The production function thus displays constant returns to scale with respect to both labor, L_j , and the firm's capital stock, K_j , as well as in the accumulating factor, K_j and K , and is therefore potentially capable of sustaining

endogenous growth, as in Romer (1986). With firms being identical, they all choose the same levels of capital and labor, producing the same level of output; i.e. $K_j = K$, $L_j = L$, $Y_j = Y$, where analogously to K , L and Y denote the average economy-wide levels of employment and output, respectively. Hence summing over firms, aggregate output is linear in the aggregate capital stock

$$F(K, L) = AL^\alpha K. \tag{1b}$$

Firms are competitive, so that the wage rate, (w), and the gross return to capital, (r), are determined by their respective marginal products. Letting l denote the average economy-wide leisure time, so $L = 1 - l$, the wage rate and the return to capital can be expressed as:

$$w = \alpha AL^{\alpha-1} K = \alpha A(1 - l)^{\alpha-1} K \equiv \omega(l)K \tag{2a}$$

$$r = (1 - \alpha)AL^\alpha K^\alpha K_j^{-\alpha} = (1 - \alpha)A(1 - l)^\alpha \tag{2b}$$

As has long been recognized, and as we discuss in Section 5.1, the calibration of this Romer-type production function requires that capital must be interpreted broadly as an amalgam of human and physical capital. Labor income is the payment for raw labor (L_l) and does not include the returns associated with skills, which are incorporated in the return to capital; see Rebelo (1991).

2.2. Households

The economy comprises N households indexed by i , each of which is endowed with a unit of time that it can allocate either to leisure, l_i , or to providing labor, $L_i = 1 - l_i$. It is also endowed with $K_{i,0}$ units of capital, with the relative share of capital owned by household i at time t being $k_i(t) \equiv K_i(t)/K(t)$. The distribution of relative capital thus has mean 1 and coefficient of variation, $\sigma(t)$, the initial endowed distribution of which is σ_0 .

The representative household maximizes its lifetime utility, assumed to be a function of its consumption and leisure, as specified by the isoelastic utility function

$$\int_0^\infty \frac{1}{\gamma_i} (C_i^t)^{\gamma_i} e^{-\beta t} dt \quad -\infty < \gamma_i < 1; \quad \eta > 0; \quad 1 > \gamma_i \cdot (1 + \eta) \tag{3}$$

where the values of the parameters satisfy the above inequalities, to ensure: (i) that the EIS $\vartheta_i = (1 - \gamma_i)^{-1} > 0$, (ii) consumption and leisure are complements in utility, and (iii) that the utility function is concave in consumption and leisure.

In addition to their initial endowments of capital ($K_{i,0}$), households also differ with respect to their EIS.⁵ Smaller values of the EIS imply that the household is less willing to substitute future for present consumption. Note that the RTP, β , is constant across households, highlighting the contrast between this paper and previous literature, where households have varying RTPs, but share a common EIS. The parameter η is also common across households, so they all display the same tradeoff between consumption and leisure; see Eq. (9).

Household i 's gross income comprises their income from capital plus their income from "raw labor" and is $Y_i = rK_i + \omega K(1 - l_i)$, where the wage rate and the return to capital are given by (2a) and (2b). Aggregating over all households indicates that the average economy-wide household income is: $Y = rK + \omega K(1 - l)$. We assume that all income is taxed at a common rate regardless of its source, i.e. whether it is levied on income from labor or capital.⁶ We specify household i 's average tax rate (τ_i) in accordance with the following schedule:

$$\tau_i \equiv 1 - \zeta (y_i)^{-\phi} \tag{4a}$$

where $y_i \equiv Y_i/Y$ is household i income relative to the economy-wide average income. Note that the average y_i is 1, and that $\sum_1^N y_i = N$. The scale parameter (ζ) drives the general level of the tax rate, and the parameter ϕ determines the degree of progressivity. If $\phi = 0$ the tax schedule is flat, i.e. the tax rate is independent of the agent's relative income ($\tau_i \equiv 1 - \zeta$). If $\phi > 0$, the tax rate is an increasing function of the household's relative income, although this increase occurs at a decreasing rate. The tax rate on marginal income (τ_i^m) is:

$$\tau_i^m \equiv \frac{\partial(\tau_i Y_i)}{\partial Y_i} = 1 - (1 - \phi)\zeta (y_i)^{-\phi} = \tau_i + \phi(1 - \tau_i) \tag{4b}$$

This specification of a progressive tax schedule is the form adopted by Guo and Lansing (1998) and is also employed by Chen and Guo (2013). Provided $0 < \zeta < 1$ and $0 \leq \phi < 1$, the average and marginal tax rates satisfy the conditions $1 > \tau_i^m > \tau_i$ with the ratio of the marginal to the average tax rate, $\tau_i^m/\tau_i = (1 - \phi) + \phi/\tau_i$, declining with the average tax rate,

⁵ Given our inclusive definition of capital, $K_i(0)$ also incorporates the heterogeneity of human capital, which has been shown elsewhere to generate potentially large growth effects; see Tamura, (1992, 1996, 2006).

⁶ See Chen (2020) for the case that differentiates between tax on capital and tax on wages. Section. 6.3 briefly summarizes some extensions of the present model that introduce differential tax structures on these two income classes.

all of which are consistent with the US tax rates. It is virtually identical to the more familiar specification proposed by Benabou (2002), Guner et al. (2014), and Heathcote et al. (2017). The difference is that Guo-Lansing normalize income for tax purposes by the average income, $y_i \equiv Y_i/Y$, while these other studies normalize by the income level corresponding to zero average tax rate.⁷

Households choose their time paths of consumption, leisure, and capital to maximize (3), subject to the following capital accumulation constraint.

$$\dot{K}_i = (1 - \tau_i)[rK_i + \omega K(1 - l_i)] - \delta K_i - C_i + G_i^{TR} \tag{5}$$

This includes two slight modifications to Koyuncu and Turnovsky, namely the constant rate of depreciation of capital, δ , and the presence of the public transfers G_i^{TR} received by agent i . While these are unimportant from a conceptual standpoint, they are helpful for purposes of calibration and for approximating the tax structure of the US economy.

Performing the optimization yields the following first order optimality conditions:

$$C_i^{\gamma_i-1} l_i^{\eta\gamma_i} = \lambda_i \tag{6a}$$

$$\eta C_i^{\gamma_i} l_i^{\eta\gamma_i-1} = (1 - \tau_i^m)\omega K \lambda_i \tag{6b}$$

$$(1 - \tau_i^m) r - \delta = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

where λ_i is the household i 's shadow value of capital, together with the transversality condition,

$$\lim_{t \rightarrow \infty} \lambda_i K_i e^{-\beta t} = 0 \tag{6d}$$

The key point of these equilibrium conditions is that the agent, being subject to a progressive tax structure, takes account of the fact that as he chooses to work or save, he influences his *marginal* tax rate, which therefore becomes the relevant tax rate in driving these decisions.

2.3. Government

For simplicity we abstract from public debt, so the government uses its tax revenues to finance its endogenously determined expenditures, thereby maintaining a balanced budget:

$$G^c + G^{TR} = (g^c + g^{TR}) \cdot F(K, L) = \frac{1}{N} \sum_{i=1}^N \tau_i Y_i \tag{7}$$

where g^c and g^{TR} are, respectively, the share of government consumption and of public transfers in total output. With regard to transfers, we assume that the proportion received by each quintile is as specified by the data so that $(G_i^{TR} = \rho_i G^{TR})$, and $(1/N) \sum_i \rho_i = 1$.⁸

3. Macroeconomic equilibrium

To derive the macroeconomic equilibrium, we first divide (6b) by (6a) and obtain

$$C_i = (1 - \tau_i^m) \frac{\omega(l)}{\eta} K l_i \tag{8}$$

Taking the time derivative of (8), while taking into account that τ_i^m depends on y_i , via Eq. (4b), yields the relation between the rates of growth of consumption and effective leisure:

$$\frac{\dot{C}_i}{C_i} - \left(\frac{\dot{l}_i}{l_i} + \frac{\dot{K}}{K} \right) = \frac{\omega_l \dot{l}}{\omega} - \phi \frac{\dot{y}_i}{y_i} \tag{9}$$

where $\omega_l = \partial\omega/\partial l$. Next, taking the time derivative of (6a) and combining with 6(a) yields:

$$(\gamma_i - 1) \frac{\dot{C}_i}{C_i} + \eta \gamma_i \frac{\dot{l}_i}{l_i} = \beta - (1 - \tau_i^m)r + \delta \tag{10}$$

⁷ Using the notation of this paper, the relationship between these two specifications of the progressive tax structure is as follows. Average tax rate paid by agent earning income Y_i as specified by Heathcote et al. (2017) is $\tau_i = 1 - \lambda(Y_i)^{-\phi}$ where $\lambda = (Y^0)^\phi$ and Y^0 is the income level at which the average tax is zero. Thus, $\tau_i = 1 - (Y_i/Y^0)^{-\phi} = 1 - (Y^0/Y)^\phi (Y_i/Y)^{-\phi}$ which is identical to (4a) with $\xi \equiv (Y^0/Y)^\phi$. Taking our estimate of $\phi = 0.092$, $Y^0 = 9350$, $Y = 86,180$ we obtain $\xi = 0.815$, very close to our calculated value.

⁸ For simplicity we assume that government consumption impacts utility additively. Since it is determined residually and has no impact on consumer behavior, without loss of generality we have set its utility level to zero in Eq. (3). But we should note that the allocation of tax revenues is potentially an important issue and may have far-reaching consequences. For example, Akgigit et al. (2022) find that higher taxes negatively impact the quantity and location of innovation across US states.

and substituting (8) into the household capital accumulation relationship (5) implies:

$$\dot{K}_i = (1 - \tau_i)Y_i - (1 - \tau_i^m)\frac{\omega}{\eta}Kl_i - \delta K_i + \rho_i G^{TR} \tag{11}$$

Eq. (11) clearly indicates the nature of the complications introduced by specifying a progressive tax structure. With a flat tax rate schedule ($\phi = 0$) $\tau_i = \tau_i^m = 1 - \zeta$, and (11) is linear across individuals. Aggregation over agents is straightforward, leading to a macroeconomic equilibrium that is independent of the distribution across agents. The dynamics can be studied recursively by first deriving the aggregate equilibrium dynamics, and then determining how the factor returns it generates determine the various distributional measures. But if the tax structure is progressive this simple recursive solution procedure breaks down. This is because the individual tax rates depend upon both the individual's income and the aggregates, as a result of which the equilibrium aggregates and their distribution among households become jointly determined.

To proceed further, we define the following aggregates. First, summing over income, capital, and leisure, and imposing clearance of the aggregate labor and capital markets, we obtain:

$$\frac{1}{N} \sum_{i=1}^N \left(\frac{Y_i}{Y}\right) = \frac{1}{N} \sum_{i=1}^N y_i = 1; \quad \frac{1}{N} \sum_{i=1}^N \left(\frac{K_i}{K}\right) = \frac{1}{N} \sum_{i=1}^N k_i = 1; \quad \frac{1}{N} \sum_{i=1}^N \left(\frac{l_i}{l}\right) = 1 \tag{12a}$$

$$\frac{1}{J} \sum_{j=1}^J L_j = L = 1 - l = \frac{1}{N} \sum_{i=1}^N (1 - l_i) = \frac{1}{N} \sum_{i=1}^N L_i \tag{12b}$$

$$\frac{1}{J} \sum_{j=1}^J K_j = K = \frac{1}{N} \sum_{i=1}^N K_i \tag{12c}$$

Corresponding to these aggregates we define the average and marginal economy-wide tax rates as weighted averages of the individual household rates, where the weights are, respectively, their relative income and their relative leisure:

$$\bar{\tau} = \frac{1}{N} \sum_{i=1}^N \tau_i \cdot \left(\frac{Y_i}{Y}\right) = \frac{1}{N} \sum_{i=1}^N \tau_i \cdot y_i = \frac{1}{N} \sum_{i=1}^N y_i [1 - \zeta (y_i)^{-\phi}] \tag{13a}$$

$$\bar{\tau}^m = \frac{1}{N} \sum_{i=1}^N \tau_i^m \cdot \left(\frac{l_i}{l}\right) = \frac{1}{N} \sum_{i=1}^N [1 - (1 - \phi)\zeta (y_i)^{-\phi}] \cdot \left(\frac{l_i}{l}\right) \tag{13b}$$

Aggregating (11) across households, and using the definitions of $\bar{\tau}$ and $\bar{\tau}^m$ in (13a) and (13b), together with the government budget constraint (7), yields the aggregate economy-wide capital accumulation equation:

$$\dot{K} = (1 - \bar{\tau})Y - (1 - \bar{\tau}^m)\frac{\omega}{\eta}Kl - \delta K + G^{TR} \tag{14}$$

To complete the macroeconomic equilibrium we express household i 's relative gross income y_i before tax and transfers as follows:

$$y_i \equiv \frac{Y_i}{Y} = \frac{rk_i + \omega(1 - l_i)}{r + \omega(1 - l)} = (1 - \alpha)k_i + \alpha \left(\frac{1 - l_i}{1 - l}\right) \quad i = 1, \dots, N \tag{15a}$$

where in deriving (15a) we use the definitions of r and ω , from (2a) to (2b). Taking the time derivative of (15a) yields the dynamic constraint:

$$\dot{y}_i = (1 - \alpha)\dot{k}_i - \alpha \left(\frac{1}{1 - l}\right)\dot{l}_i + \alpha \frac{(1 - l_i)}{(1 - l)^2} \dot{l} \quad i = 1, \dots, N \tag{15b}$$

Combining Eqs. (11) and (14) enables us to express the evolution of the agent's relative share of capital, $k_i = K_i/K$:

$$\dot{k}_i = \left\{ (1 - \tau_i)[r(l)k_i + (1 - l_i)\omega(l)] - (1 - \tau_i^m(y_i))\frac{l_i}{\eta}\omega(l) + \rho_i g^{TR}A(1 - l)^\alpha \right\} - \left\{ (1 - \bar{\tau})[r(l) + (1 - l)\omega(l)] - (1 - \bar{\tau}^m)\frac{l}{\eta}\omega(l) + g^{TR}A(1 - l)^\alpha \right\} \quad i = 1, \dots, N \tag{16}$$

Because of the aggregation conditions (12), only $N-1$ of Eqs. (15a), (15b), and (16) are independent. The final step is to use (10) and (14) to substitute for C_i/C_i and K/K , respectively, in Eq. (9) to obtain:

$$\begin{aligned} & \phi \frac{\dot{y}_i}{y_i} - \left(\frac{1 - \gamma_i(1 + \eta)}{1 - \gamma_i}\right) \frac{\dot{l}_i}{l_i} - \frac{w_l l}{w} \\ & = (1 - \bar{\tau})[r + w(1 - l)] - \delta - (1 - \bar{\tau}^m)l \frac{w}{\eta} + \frac{\beta + \delta - (1 - \tau_i^m)r}{1 - \gamma_i} \quad i = 1, \dots, N \end{aligned} \tag{17}$$

Thus the macroeconomic equilibrium can be summarized by the following system of dynamic equations: (i) the $N-1$ equations of (15b), (ii) the $N-1$ independent equations of (16), and (iii) the N independent equations of (17). Together they yield $3N-2$ independent dynamic equations in $y_1, \dots, y_{N-1}, k_1, \dots, k_{N-1}, l_1, \dots, l_{N-1}, l$. Recalling the definitions of $\tau_i, \tau_i^m, \tau, \bar{\tau}^m$, it is clear that the dynamics of the aggregates and the individuals are interdependent, rendering a general analysis intractable.⁹ Section 6 discusses the numerical solution of the system of the calibrated version of the model, which gives the transition path of the economy to the steady state equilibrium in the case where there are five classes of income corresponding to quintiles.

From these core dynamic equations the rest of the macroeconomic equilibrium dynamics can be derived. Thus, for example, household i 's relative income net of taxes, but excluding transfers, y_i^a , is given by $y_i^a = (1 - \tau_i(y_i))y_i$. Also, the resulting individual and aggregate growth rates of capital (wealth) are:

$$\psi_i(t) \equiv \frac{\dot{K}_i}{K_i} = \frac{A(1-l)^{\alpha-1}}{k_i} \left[(1 - \tau_i)(1-l)y_i - (1 - \tau_i^m)\alpha \frac{l_i}{\eta} + \rho_i g^{TR}(1-l) \right] - \delta \tag{18a}$$

$$\psi(t) \equiv \frac{\dot{K}}{K} = A(1-l)^{\alpha-1} \left[(1 - \bar{\tau})(1-l) - (1 - \bar{\tau}^m)\alpha \frac{l}{\eta} + g^{TR}(1-l) \right] - \delta \tag{18b}$$

which can be shown to converge to a common long-run growth rate, $\tilde{\psi}$, defined in Eq. (19a) below. Combining (17a) with (1b), the growth of aggregate output is $\psi_y(t) = \psi(t) - \alpha \dot{l}(t)/(1-l(t))$, which also converges to $\tilde{\psi}$, so that the transversality condition (6d) reduces to $(1 - \tau_i^m)r - \delta > \tilde{\psi}$.

4. Balanced growth path (steady-state)

Assuming that the underlying dynamic system is stable, the economy will converge to a balanced growth path. Along this balanced growth path each individual's quantities of output, capital, and consumption will grow at the same endogenously determined constant rate, while their allocation of time is constant. This implies that agents' relative quantities will be constant and so this long-run equilibrium obtained by setting $\dot{k}_i = \dot{y}_i = \dot{l}_i = \dot{l} = 0$ in (15b), (16), and (17), together with (15a) and can be summarized by the following relationships:

$$\begin{aligned} \tilde{\psi} &= \frac{(1 - \tau_i^m(\tilde{y}_i))r(\tilde{l}) - \beta - \delta}{(1 - \gamma_i)} \\ &= (1 - \bar{\tau})[r(\tilde{l}) + (1 - \tilde{l})\omega(\tilde{l})] - \delta - (1 - \bar{\tau}^m)\tilde{l} \frac{\omega(\tilde{l})}{\eta} + g^{TR}A(1 - \tilde{l})^\alpha \quad i = 1, \dots, N \end{aligned} \tag{19a}$$

$$\begin{aligned} &\left\{ (1 - \tau_i(\tilde{y}_i))[r(\tilde{l})\tilde{k}_i + \omega(\tilde{l})(1 - \tilde{l}_i)] - (1 - \tau_i^m(\tilde{y}_i))\tilde{l}_i \frac{\omega(\tilde{l})}{\eta} + \rho_i g^{TR}A(1 - \tilde{l})^\alpha \right\} \\ &= \tilde{k}_i \left\{ (1 - \bar{\tau})[r(\tilde{l}) + (1 - \tilde{l})\omega(\tilde{l})] - (1 - \bar{\tau}^m)\tilde{l} \frac{\omega(\tilde{l})}{\eta} + g^{TR}A(1 - \tilde{l})^\alpha \right\} \quad i = 1, \dots, N - 1 \end{aligned} \tag{19b}$$

$$\tilde{y}_i = (1 - \alpha)\tilde{k}_i + \alpha \frac{(1 - \tilde{l}_i)}{(1 - \tilde{l})} \quad i = 1, \dots, N - 1 \tag{19c}$$

$$\frac{1}{N} \sum_{i=1}^N \tilde{y}_i = 1 \quad \frac{1}{N} \sum_{i=1}^N \tilde{l}_i = \tilde{l} \quad \frac{1}{N} \sum_{i=1}^N \tilde{k}_i = 1 \tag{19d}$$

Apart from the inclusion of δ and g^{TR} , these steady-state conditions are identical to those obtained by Koyuncu and Turnovsky (2016) with the varying EIS, $\vartheta_i \equiv (1 - \gamma_i)^{-1}$ replacing, and playing precisely the same role, as does the varying RTP, in the earlier analysis.

To see this we compare (19a) for two individuals, i and j , and rewrite it in the form:

$$\frac{\vartheta_i}{\vartheta_j} = \frac{\left[(1 - \phi)\zeta \tilde{y}_j^{-\phi} r(\tilde{l}) - \beta - \delta \right]}{\left[(1 - \phi)\zeta \tilde{y}_i^{-\phi} r(\tilde{l}) - \beta - \delta \right]} \tag{20}$$

⁹ In the case of two income classes, Koyuncu and Turnovsky (2016), show that the core equilibrium is reduced to three independent dynamic equations in y_i, l_i, l . In our numerical simulations for the five income classes we always obtain a unique stable transitional path leading to a unique long-run balanced growth path.

It is clear that a relative increase in the EIS ϑ_i/ϑ_j between the two individuals has equivalent qualitative distributional consequences as does a decrease in their relative rates of time preference. Eq. (20) brings out how the underlying source of heterogeneity (in either case) interacts with the progressive tax system to generate long-run income inequality. Thus we can state the following proposition, originally expressed in terms of RTP, and now in terms of the EIS.

Proposition 1. Suppose that the tax rate is progressive, $\phi > 0$.

- (i) If all households share a common EIS, $\vartheta_i = \vartheta$, then $\tilde{y}_i = \tilde{y}$, $\tilde{l}_i = \tilde{l}$, $\tilde{k}_i = \tilde{k}$ for all i ; the steady-state distributions of wealth and income degenerate to zero.
- (ii) If for any pair of households, $\vartheta_i > \vartheta_j$, then $\tilde{y}_i > \tilde{y}_j$, $\tilde{l}_i > \tilde{l}_j$, $\tilde{k}_i > \tilde{k}_j$; that is the household with the larger EIS will have a higher income (both before-tax and after-tax), enjoy more leisure, and have more wealth.

We may also note that, as in the case considered by Becker (1980), where with flat (or zero) taxes the most patient individual will accumulate capital at the fastest rate and end up owning all the capital, the analogous proposition applies here. In that circumstance, the household having the largest EIS would in steady state own all the capital. Since empirically there is much more variation in the EIS than in the RTP this further suggests that it is much more effective in explaining the larger degree of wealth and income inequality.

5. Numerical simulations

To analyze further this breakdown of income into five income classes it is necessary to proceed numerically, beginning by describing the calibration and the initial steady state. We then discuss the quantitative effects of a cut in the tax rates on the equilibrium trajectory of the main macroeconomic aggregates, as well as on the distributions of wealth and income.

5.1. Baseline calibration and initial steady-state

Table 2 summarizes the values of the calibrated the parameters for the US economy.

5.1.1. Technology parameters

The parameterization of the Romer-type production function is a subtle endeavor. This stems from the requirement of ensuring constant returns to scale in both the private factors K, L , as well as in the accumulating factors, K, \bar{K} , while imposing factor productivity elasticities that are consistent with empirical evidence. We set $\alpha = 0.28$, which although being around half of the labor share in production, we justify by interpreting L as being “raw labor”. The balance of the return to labor, which is attributed to “skills”, is incorporated in capital, which we interpret broadly as being an amalgam of physical capital and human capital. The value $\alpha = 0.28$ is between 0.34 used by Koyuncu and Turnovsky (2016) and 0.20 used by Barro et al. (1995), which in turn is close to the empirical estimates of Mankiw et al. (1992).¹⁰ The scale parameter in the production function was calculated as $A = 0.446$, to yield an equilibrium growth rate of about 2.52% per year, which is approximately equal to the recently observed historical growth rate in the US economy.¹¹

In light of our broad interpretation of capital, we rely on the estimates of the depreciation rates for both physical and knowledge capital for the US economy provided by Nadiri and Prucha (1996), and set the capital depreciation rate (δ) to 0.10, as in Chen (2020). In general, macrodynamic models such as this are robust with respect to values of δ , consistently set in the range between 0.05 and 0.10, their primary impact being on the rate of convergence.

5.1.2. Preference parameters

The RTP (β) is set at 0.038, which is close to the midpoint of the (small) range assumed by Li and Sarte (2004) and Koyuncu and Turnovsky (2016), and is generally conventional. The elasticity of leisure in the utility function (η), which drives how households allocate their time, is set at 0.675. This yields an overall allocation of time to raw labor of around 36% which is generally consistent with the empirical estimates and observed behavior.¹² This parameter also generates a value of the aggregate Frisch elasticity of around 1.3, well within the range typically adopted in macroeconomic simulations; see Keane and Rogerson (2012).

The key preference parameter for the calibration is the EIS which has been extensively estimated in a wide range of contexts. The most comprehensive compilation of empirical evidence is the mega study by Havranek et al. (2015), which reports 2735 estimates covering 104 countries over varying time periods. They find that estimates vary substantially between countries, and datasets, with the overwhelming majority lying between 0 and 1, with a mean of around 0.5. Most relevant

¹⁰ Mankiw et al. (1992) estimate a Cobb-Douglas production function in which output depends upon labor, human capital, and physical capital, and obtain approximately equal exponents on all three. Barro et al. (1995) assume elasticities of 0.3 on physical capital and 0.5 on human capital.

¹¹ The implied equilibrium net rate of return to capital is $r - \delta = 14.3\%$. Recalling our interpretation of capital as a composite of human capital, H , and physical capital, Z , and assuming e.g. $K = H^{1-\theta}Z^\theta$, the net return to physical capital, r_z , is $r_z = r\theta(H/Z)^{1-\theta} - \delta_z$. Assuming e.g. $\theta = 0.5$, $H/Z \approx 2$ and depreciation rate $\delta_z = \delta = 0.10$, r_z is reduced to the more plausible rate of 7.2%.

¹² In 2019, the average hours worked per week in the USA by persons who usually work full time was 42.5, according to U.S. Bureau of Labor Statistics data, available at <https://www.bls.gov/cps/cpsaat22.pdf>. Assuming that they work on average 50 weeks per year and their time endowment available for allocation is 2/3 per day implies an overall allocation of time to labor of around 36.4%.

Table 1
Pre-reform tax parameters by quintiles.

Pre-TCJA head of households' tax codes (Standard Deduction: \$ 9,350.00)						
Quintile	Mean Income*	Marginal Tax Rate	Taxes	Tax Rate	ϕ_i	ζ_i
First	13,258 (0.154)	10%	390.80	0.029	0.073	0.847
Second	35,401 (0.411)	15%	3,240.10	0.092	0.064	0.858
Third	61,564 (0.714)	25%	7,306.00	0.119	0.149	0.838
Fourth	99,030 (1.149)	25%	16,672.50	0.168	0.098	0.843
Highest	221,646 (2.572)	33%	49,759.38	0.224	0.136	0.882

* Mean income received in 2017 (CENSUS Data - available at <https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html>).

for our calibration are the micro studies of [Blundell et al. \(1994\)](#) and [Atanasio and Browning \(1995\)](#) who find that rich households tend to have a larger EIS than poorer households. This pattern tends to be at least indirectly supported by several other studies. [Vissing-Jørgensen \(2002\)](#) obtains a larger EIS for stockholders than for non-stockholders, while at the other extreme, [Bayoumi \(1993\)](#) indicates that liquidity-constrained households have a smaller EIS. [Atkeson and Ogaki \(1996\)](#), using data for India, argue that the EIS differs between rich and poor households, being smaller for the latter, because they spend a larger share of their budget on the consumption of subsistence goods which are harder to reallocate over time.

To calibrate the EIS in our model with income classes, based on quintiles of the income distribution, we set the EIS for each quintile, taking into account the evidence on the variation of the EIS with income as reported above. This leads us to set $\gamma_1 = -0.25$, which implies $EIS = 0.8$, for the highest income quintile, Q5, and to adopt progressively smaller values for the other quintiles, declining to $\gamma_5 = -2.03$, which implies $EIS = 0.330$, for the lowest income class, Q1. The implied economy-wide average EIS is 0.52, close to the consensus estimate. While these parameters have not been derived from formal econometric estimates, they all lie well within the range of previous estimates, and informally may be justified as approximately matching the income shares of their respective income class.

5.1.3. Fiscal parameters

The specification of the parameters ζ , ϕ characterizing the progressive tax structure is critical. While estimates of these parameters have been obtained previously (e.g. [Guo and Chen, 2013](#); [Heathcote et al. 2017](#)), these apply to much earlier periods. As noted, the change in tax structure we consider is motivated by the (TCJA) enacted in 2017. The existing estimates, based on much earlier information are therefore likely to provide an inappropriate summary of the tax structure for 2017, especially given the changes in the tax structure during the intervening years.

Accordingly, we derive our own estimates using the following procedure, based on information contained in [Table 1](#). Hence, we begin by referring to the first column that describes the mean income by quintiles for 2017, provided by the Census data. Using those data we calculate the mean household income to be \$86,180. Next, dividing the mean income for each quintile by \$86,180 we derive the mean *relative* income for each quintile, y_1, \dots, y_5 as reported in parentheses.

We then refer to the tax code that prevailed prior to the implementation of the TCJA 2017, focusing on the “Head of Household” tax brackets. After applying the standard deduction to the household, we apply this schedule to the household income, to determine in which federal income tax bracket it falls, and finally estimate the amount of taxes paid by each quintile.¹³

Thus, for example, consider the middle quintile, Q3, the mean income of which is \$61,564. In the pre-reform scenario we first subtract the standard deduction (\$9350), reducing its taxable income to \$52,214. The first \$13,350 is taxed at 10% (first tax bracket), the next \$37,450 at 15% (second bracket), with the balance (\$1414) taxed at 25% (third bracket). Knowing the marginal tax rate for each quintile, and having calculated its average tax rate, we then apply [Eq. \(4b\)](#) to estimate the degree of progressivity, ϕ_i , pertinent to that quintile. Then, having determined ϕ_i , and previously y_i , we then apply [Eq. \(4a\)](#) to derive the “level” parameter, ζ_i , with these two parameters being reported in the final two columns.

The next step is to transform these quintile estimates into summary economy-wide average estimates, ϕ and ζ , that can then be applied to the formal model. In principle there are several ways this might be done. We proceed by minimizing the sum of squared residuals between the quintile rate calculated by the tax functions ([Eqs. 4a](#) and [4b](#)) and both the corresponding marginal tax rate and the rate in the column “Tax Rate” of [Table 1](#). The range of the parameters ϕ_i and ζ_i constitute the upper and lower constraints to estimate these average fiscal parameters. This results in the economy-wide averages of $\zeta = 0.838$ and $\phi = 0.092$, reported in [Table 2](#).

The final step is to apply these estimated parameters to the steady-state relationships for the underlying model (for $N = 5$) summarized in [Eq. \(19\)](#). The implied initial steady-state values for the quintile income shares are reported in the first column of [Table 3](#).¹⁴ Overall, the estimates are close to those summarized by the Census data, suggesting that our estimates of ϕ and ζ seem very plausible. The parameterization described above produces an average tax rate of 18.4%,

¹³ The table displaying head of household tax brackets and rates is available at <https://taxfoundation.org/2017-tax-brackets/>.

¹⁴ As an alternative, we also estimated ϕ and ζ as simple averages of the ϕ_i and ζ_i reported in [Table 1](#), to obtain estimates $\zeta = 0.854$, $\phi = 0.104$, which, however, do not do as well in terms of implying a steady state distribution for y_i which is close to the economy-wide estimates provided by the Census data.

Table 2
Parameter values.

<i>Technology parameters</i>	
Scale Parameter (A)	0.446
Elasticity of labor in production (α)	0.28
Capital Depreciation Rate (δ)	0.100
<i>Preference parameters</i>	
Elasticity of leisure (η)	0.675
Rate of Time Preference (β)	0.038
Individual Intertemporal Elasticity of Substitution ($1/(1 - \gamma_1)$)	
First quintile: Q1	0.330
Second quintile: Q2	0.410
Third quintile: Q3	0.475
Fourth quintile: Q4	0.565
Highest quintile: Q5	0.800
<i>Policy parameters and taxes</i>	
Degree of progressivity of tax rate ($\hat{\phi}$)	0.092
Level of tax schedule ($\hat{\zeta}$)	0.838
Share of public transfers in total output (g^{TR})	0.11
Percentages of public transfers allocated to quintiles	Q1 = 52 Q2 = 26 Q3 = 12 Q4 = 6 Q5 = 4

Table 3
Initial steady-state equilibrium.

	Growth rate	Agg. Labor share	Av. Econ tax rate	Marg. Econ tax rate	Income GINI	Wealth GINI
Aggregate Variables	2.52%	0.368	18.41%	22.24%	43.1%	66.4%
	Income Shares	Capital Shares		Labor Allocation	Average Tax Rate	Marginal Tax Rate
Q1	0.199 (4.0%)	-0.196 (-3.92%)		0.4459	2.76%	11.71%
Q2	0.433 (8.7%)	0.109 (2.18%)		0.4662	9.50%	17.83%
Q3	0.693 (13.9%)	0.485 (9.70%)		0.4517	13.33%	21.30%
Q4	1.133 (22.6%)	1.180 (23.60%)		0.3717	17.15%	24.78%
Q5	2.542 (50.8%)	3.422 (68.44%)		0.1032	23.09%	30.17%

which is close to the average Federal tax rate that prevailed in the period between 2000 and 2017 according to CBO (20.3%). The individual tax rates are also similar to the average rates reported by CBO for each income quintile over that same period.¹⁶

Finally, we set the government transfers rate at $g^{TR} = 0.11$, which is the average ratio to GDP in US from 2000 to 2017 according to the Federal Reserve Economic Data. The ratio of public transfers rebated to each quintile relative to the average transfers (ρ_i) was defined according to the average transfers from 1976 to 2016, according to CBO data. The government, then, uses the fiscal slack of its revenues from taxes to finance public consumption (g^c) and, in order to maintain its balanced budget, adjusts this expenditure in response to the changes in the overall tax revenues.

5.1.4. Initial long-run equilibrium

Table 3 reports the initial equilibrium. The long-run growth rate of 2.52% matches the GDP annual growth of the US economy in the last 30 years, as reported by the World Bank.

The average allocation of time to labor varies dramatically across the income classes. While our numerical results suggest that the highest quintile allocates only 10.3% of their time to supplying rate labor, one must keep in mind that this refers to “raw labor” and our assumption that a significant portion of the total time this class devotes to labor is associated with their skills and is therefore reflected in the return to capital, as broadly defined. The labor allocation then jumps dramatically to 37% for the next quintile. At the other extreme of the distribution, the poorest quintile, Q1, consume more leisure than do the second and third quintiles, a possible explanation being that they receive the major portion of the government transfers, over 50%, as well as the inclusion of unemployed in that income class.

Another possible explanation for the substantial reduction in labor allocation between Q4 and Q5 is the dramatic differential in average shares of wealth between these two groups. A general characteristic of the representative consumer approach to income distribution is that there is a positive relationship between an agent’s relative wealth and his allocation of time to leisure (Turnovsky and García-Peñalosa, 2008). This relationship is strongly supported by empirical evi-

¹⁶ According to CBO Data reported by quintiles, the average Federal Taxes from 2000 to 2017 in ascending order was: 3.15%, 9.59%, 13.97%, 17.60%, and 25.37%.

dence drawn from a variety of sources (Holtz-Eakin et al., 1993; Cheng and French, 2000; Coronado and Perozek, 2003, and Algan et al. 2003). Intuitively, the allocation of time to labor reflects the marginal utility of wealth, which declines with wealth with the result that wealthier agents, increase their consumption of all goods, including leisure.

The initial equilibrium shares of income are reported in the first column of Table 3. These show that the income shares of the lowest and highest income quintiles are, respectively, 4.0% and 50.8%, which compare to the observed values of income before tax and transfers in 2017 provided by CBO of 3.8% and 54%, respectively.¹⁷ The ratios of the income share of the highest quintile to that of the fourth and third quintile are 2.24 and 3.67, respectively, which are close to the observed ratios (2.6 and 3.9, respectively). The after-tax income shares across the five quintiles of 4.8%, 9.6%, 14.7%, 23.0%, and 47.9% are also generally consistent with the shares of disposable income 5.4%, 10.8%, 16.0%, 22.7%, and 45.1% reported by the OECD.¹⁸

The conventional empirical measures of overall income inequality is the GINI coefficient which for N income classes can be conveniently approximated by:

$$GINI = \frac{N + 1}{N} - \frac{2 \sum_{i=1}^N (N + 1 - i)y_i}{N \sum_{i=1}^N y_i} \tag{21}$$

with analogous expressions applying to after-tax income and wealth. For the benchmark year of 2017 we estimate the GINI coefficient of income to be 43.1, which is almost identical to the OECD estimate for the US for that year (43.4). The corresponding measure of after-tax income inequality is 39.8, which is also close to the OECD estimate based on household disposable income (39.3).

The second column of the table shows that the share of capital of the highest income quintile in total wealth is $k_5/5 = 68.4\%$, while the share of the lowest income is actually negative. The fact that the poorest income class has negative wealth is not unprecedented and unfortunately information on the distribution of wealth is sparse so it difficult to assess the plausibility of -3.92% . One of the few studies to provide estimates of quartile shares of household wealth is Davies et al. (2011). In the case of the US their estimated quartile shares are -0.1% , 1.2% , 4.5% , 11.8% , and 82.6% . But despite the differences across the quintile distributions our estimate of the overall GINI wealth coefficient of 66.4% is not too distant from the Davies et al. estimate of 70.4% .¹⁹

Overall, we view this calibration as providing a plausible benchmark for determining the distributional consequences of a progressive tax structure. In particular, in contrast to the difficulty of spanning the range of US marginal tax rates with variations in the RTP, this can be readily accomplished with the observed variations in the EIS. We should also note that estimates of the income GINI coefficient vary, according to sources and measurement and the average for the US in 2017, as reported in the comprehensive “All the Ginis” database is around 48.²⁰ To the extent that the benchmark calibration may slightly understate the measure of inequality, we should keep in mind the abstractions of the model, which excludes many factors, such as demographic characteristics that impact inequality. But as long as such omitted structural factors can be assumed to remain largely independent of the fiscal structure, we feel that the marginal effects on growth and distribution of the simulated tax policy should not be severely affected.

6. The dynamic effects of a tax cut

We now use the benchmark calibration to simulate the macrodynamic effects of a tax cut. As indicated, our motivation is in part to assess the growth and distributional consequences of the TCJA tax cuts introduced in 2017. Despite the hype of politicians and pundits, the overall tax cut was in fact quite modest, with the overall initial reduction in tax collection being estimated at about 0.9% of GDP.²¹ Moreover, again despite the concern expressed by some that it favored the rich, the overall view is that in fact it has been, if anything, slightly progressive. With tax revenues approximately 20% of GDP, this suggests that we can map the essential characteristic of the TCJA tax cut by considering a reduction of 5% in the average tax rate $\bar{\tau}$ on impact, namely a 5% reduction in $\bar{\tau}$ from its benchmark share of 18.41% reported in Table 3.²² In light of the broad nature of the cut, we assume that this is accomplished by a slight increase in the “level” ζ , from 0.838 to 0.8475. Also, since there is no obvious progressivity, we initially assume that ϕ remains unchanged at its initial value 0.092. Two anecdotal pieces of evidence support this assumption. First, the 2017 tax cut was characterized by a near doubling of the standard deduction. Second, we have recomputed Table 1 for 2018, the year following the tax change and find that there is little change in the average ϕ across quintiles.

But the nature of the progressivity of the tax structure is an issue, both to politicians and policymakers, and to examine the potential impact of progressivity, on the growth-inequality tradeoff, we perform two other simulations in which the

¹⁷ Available at: <https://www.cbo.gov/publication/57061>.

¹⁸ Available at: <https://www.oecd.org/social/soc/IDD-Key-Indicators.xlsx>.

¹⁹ One must keep in mind that the broader interpretation of capital in our model includes the skill component of labor income which is almost certainly distributed more equally across agents than are other forms of wealth.

²⁰ Detailed description of this data set is available at <https://stonecenter.gc.cuny.edu/files/2019/02/Milanovic-all-the-ginis-dataset-description.pdf>.

²¹ This estimate is provided by the Committee for a Responsible Federal Budget. They further suggest that it is only the 8th largest tax cut since 1918, being even smaller than the tax cuts in 2010 and 2012 of 1.31% and 1.78% of GDP, respectively passed by the Obama administration.

²² We also concede that the stylized framework cannot accommodate the real extreme levels of income and wealth that largely exploit the loopholes in the tax system, features that are not incorporated into our setup.

Table 45% Average tax cut: No change in progressivity Positive long-run growth and inequality ($\zeta = 0.8475$, $\phi = 0.092$).

	Impact	After 10 periods	Long-run
Δ[Growth rate, ψ]			
Aggregate	0.123	0.126	0.135
Q1	0.520	0.419	0.135
Q2	-0.426	-0.366	0.135
Q3	0.035	0.045	0.135
Q4	0.114	0.115	0.135
Q5	0.179	0.172	0.135
Δ[Labor Allocation, L]			
Aggregate	0.0003	0.0004	0.0012
Q1	0.0005	0.0022	0.0079
Q2	-0.0002	0.0008	0.0065
Q3	-0.0006	0.0000	0.0047
Q4	-0.0006	-0.0006	0.0011
Q5	0.0024	-0.0002	-0.0143
Average Tax Rate, τ			
Aggregate	17.49%	17.51%	17.64%
Q1	1.67%	1.51%	1.00%
Q2	8.47%	8.41%	8.09%
Q3	12.34%	12.31%	12.12%
Q4	16.21%	16.21%	16.15%
Q5	22.23%	22.26%	22.40%
Marginal Tax Rate, τ^m			
Aggregate	21.35%	21.34%	21.27%
Q1	10.71%	10.57%	10.10%
Q2	16.89%	16.84%	16.55%
Q3	20.40%	20.38%	20.20%
Q4	23.92%	23.92%	23.86%
Q5	29.38%	29.41%	29.54%
Δ[Income GINI]	0.025	0.246	1.307
Δ[Wealth GINI]	0	0.387	2.214
Δ[Income Share y_i]			
Q1	0.000	-0.003	-0.014
Q2	0.000	-0.003	-0.020
Q3	-0.001	-0.003	-0.019
Q4	-0.001	-0.002	-0.010
Q5	0.002	0.011	0.063
Δ[After-tax Inc. Share y_i^a]			
Q1	0.001	-0.002	-0.013
Q2	0.001	-0.002	-0.017
Q3	0.000	-0.001	-0.016
Q4	-0.003	0.006	0.053
Q5			
Δ[Wealth Share, k_i]			
Q1	0.000	-0.006	-0.026
Q2	0.000	-0.005	-0.032
Q3	0.000	-0.004	-0.030
Q4	0.000	-0.001	-0.014
Q5	0.000	0.016	0.103

increase in ζ is accompanied by varying increases in ϕ . Finally, we shall assume that the tax cut is to be permanent, so that the economy will converge over time to a new steady state.

6.1. Reduction in "level" of tax rate

Table 4 summarizes the changes in key variables at the following stages of the transition: (i) on impact, (ii) in the medium term (after 10 years) and (iii) in the long-run (new steady state).²³ Fig. 1 displays the trajectories of the main endogenous variables for each quintile. There it is seen that the transitional paths of the different variables vary substantially across the income groups, even for what is a relatively modest, seemingly 'neutral', reduction in the tax rate.

²³ The discrete time version of the model is programmed with the software GAMS (General Algebraic Modeling System) described in GAMS (2013). As emphasized by Wang (1996), a discrete-time model can be easier to run numerical experiments and the results are usually satisfactory if the model is run for a large enough number of periods to approximate satisfactorily the results of the infinite horizon of the theoretical model. We used a horizon of $T = 480$ years, and, as the long run results are observed around $t = 200$, the figures display the dynamic effects until this period.

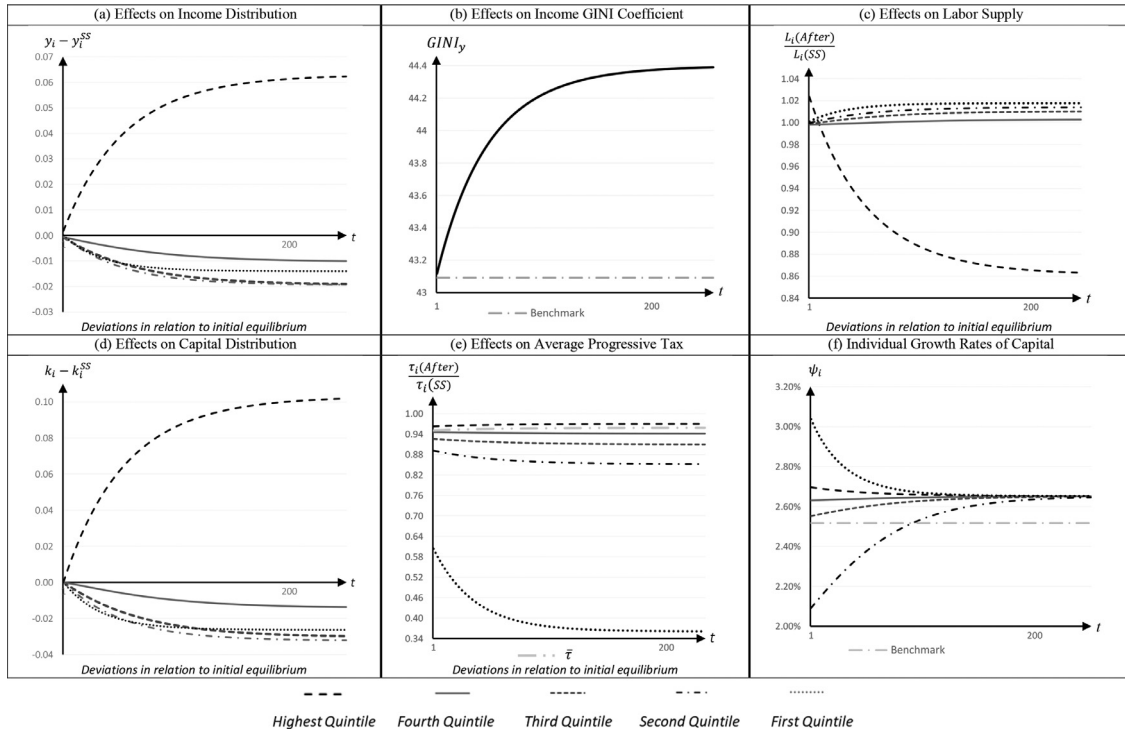


Fig. 1. Dynamic Effects of 5% tax cut ($\zeta = 0.8474, \phi = 0.092$).

Focusing first on the changes in the relative tax burdens resulting from the reduced tax rate, it is clear from the specification of (4a) and (4b) that the immediate effect of an increase in ζ is to reduce both the average and the marginal tax rates for all income groups, with the declines being proportionately larger for the poorer quintiles. Thus, for the highest quintile, Q5, the average tax rate initially decreases by 0.86 p.p., from 23.09% to 22.23%, while for the lowest quintile, Q1, the average tax rate initially falls from 2.76% to 1.67%, a 1.09 p.p. decline. But despite this slight advantage to the poorer groups, the savings of Q1–Q4 decline, while those of Q5 increase. To see why, we recall (19a) and (6b). Ignoring the initial impact of the tax cut on labor supply (which is negligible), we see that the immediate effect of the tax cut on the rate of capital accumulation of quintile i is

$$d\dot{K}_i(0) \approx A(1-l)^{\alpha-1}K_0 \left[(1-l)y_i - (1-\phi)\alpha \frac{l_i}{\eta} \right] (-d\tau_i) \tag{22}$$

The first term in parentheses is the positive effect of the reduction in the *average* tax rate on the quintile’s income, while the second term is the offsetting effect of the reduction in the *marginal* tax rate on consumption. For Q5 the first effect dominates, so that its rate of capital (wealth) accumulation increases. For Q4 and Q3 the consumption effect is stronger, but they also accumulate capital at a faster rate, although below the economy-wide growth rate. For the other quintiles, however, the consumption effect prevails and their rates of capital accumulation immediately drop. For the lowest income class, Q1, the fact that it owns negative capital means that its *growth rate*, $d\dot{K}_i(0)/K_{i,0}$ actually increases; (see Fig. 1f).

During the subsequent transition, the decline in Q5’s average tax rate is partially reversed, reverting in the long run to 22.40% (see Fig. 1e), as their relative income (y_5) increases and they are subjected to a progressively higher marginal tax rate (see Fig. 1a). In contrast, the average tax rate of Q1 continues to decline during the transition, decreasing in the long run to 1.00% (see Fig. 1e), as their relative income steadily falls (see Fig. 1a). Comparing the reductions in the average tax rates of the 5 income classes, reported in Table 4, reveals a uniform pattern, with the percentage decline increasing by 0.69 p.p. for the top quintile to 1.76 p.p. for the poorest.²⁴ At the same time, with the increase in relative income of Q5 and the decrease of the other income classes all slowing over time, their growth rates converge to the common rate of 2.66%.

The labor supply of Q5 initially increases, but then declines during the transition (see Fig. 1c), although the amount of actual time reallocated by this quintile is small. Recalling Eq. (8), which relates individual consumption to leisure, one can note that this increase in labor supply in the short run is accompanied by a reduction in consumption, which means more savings. Thus, individuals in the highest quintile significantly increase their rate of capital accumulation while gradually increasing their consumption of goods and leisure in the early stages of the transitional path. The relative labor supply of

²⁴ A similar pattern is observed for the marginal tax rates which decline by 0.63 p.p. and 1.61 p.p., respectively.

the fourth quintile is stable at the benchmark level throughout the transition, and the increase in the labor supply of the three poorest quintiles more than offsets the reduction in highest quintile, so there is a slight rise in the average hours worked.

Somewhat unexpectedly, the simulations suggest that what one might characterize as a “neutral” tax cut in fact has a progressive impact in the sense that the tax relief it affords to the alternative income classes varies inversely with their income level. But, at the same time, the tax cuts lead to an increase in inequality, since long-run relative pre-tax income of the highest quintile rises by 0.063 (from 50.8% to 52.1%), while it declines for all the other quintiles, as shown in the fifth panel of Table 4. The same pattern applies to after-tax income as well as to capital.

To see the intuition underlying the long-run relationships between the tax cut, the resulting impact on the tax burdens borne by the different income classes, and their relative income we recall the equilibrium conditions (20), together with the definition of the average tax rate, (4a). From these two relationships one can derive the steady-state responses:

$$\frac{d\tilde{y}_i}{\tilde{y}_i} = \frac{1}{\phi} \left[\frac{d\zeta}{\zeta} + \frac{d\tilde{r}(l)}{\tilde{r}(l)} - \frac{d\tilde{\psi}}{\tilde{\psi} + \vartheta_i(\beta + \delta)} \right] \tag{23a}$$

$$\frac{d\tilde{\tau}_i}{(1 - \tilde{\tau}_i)} = \phi \frac{d\tilde{y}_i}{\tilde{y}_i} - \frac{d\zeta}{\zeta} \tag{23b}$$

From (23a) we see that the direct impact of the reduction in the base tax rate, accomplished by the increase in ζ is to raise the income of all groups equi-proportionately. Likewise, the response of aggregate labor supply and its impact on the rate of return has a similar common effect, which turns out to be negligible, with the decrease in employment of the top quintile more or less offsetting that of the other four income groups. However, it is impossible for all groups to experience an increase in their relative income, since $\sum_i d\tilde{y}_i = 0$. The third term corrects for this, and the fact that the term $d\zeta/\zeta$ overstates the growth in income due to the fact some of it is allocated to consumption. Since the fraction of income spent on consumption varies inversely with the EIS, the downward correction reflected in this term is largest for the poorest quintile, Q1. For the benchmark calibration this dominates the direct effect and on balance $d\tilde{y}_1/\tilde{y}_1 < 0$. The same applies to the lowest four quintiles and since, ϑ_i increases as we move up the income scale this effect weakens and we obtain:

$$\frac{d\tilde{y}_5}{\tilde{y}_5} > 0 > \frac{d\tilde{y}_4}{\tilde{y}_4} > \frac{d\tilde{y}_3}{\tilde{y}_3} > \frac{d\tilde{y}_2}{\tilde{y}_2} > \frac{d\tilde{y}_1}{\tilde{y}_1} \tag{24}$$

In the long run the only group to increase its relative income share in response to the tax cut is the top quintile, Q5, with the percentage decline in relative income increasing as we move down the quintiles. The reason why this is so is because Q5 has a significantly larger EIS than do any of the other quintiles; see Table 2. If, for example, the EIS for Q4 was substantially larger than those below, then its relative income share would also likely increase.

With the direct effect of the increase in $d\zeta/\zeta$ reducing the average tax rates of all quintiles equi-proportionately, the fact that the richer income classes benefit more in accordance with (24), (23b) implies that the decrease in the tax burden increases as we move down the income scale. As a result, the top quintile, which is the only one to increase its relative income position, also has the smallest proportional reduction in its average tax rate.

Regarding the long-run wealth distribution, following the tax cut, the highest quintile increases its share of ownership of capital to 70.5% ($k_1 = 3.525$), 2.06 p.p. more than prior to the tax cut. The lowest quintile, in contrast, reduces its share of capital from -3.92% in the prior equilibrium to -4.44%, as shown in Fig. 1(d).

Finally, the overall effect on inequality is summarized by the evolution of the GINI coefficient for income, which increases by 0.25 p.p. after 10 years and by 1.31 p.p. in the long run, as shown in Fig. 1b. A similar adjustment pattern is exhibited by the GINI coefficient for wealth. Recalling that in the post-tax cut steady state the common growth rate increases by 0.135 p.p., reveals a tradeoff between growth and inequality that matches the empirical evidence (Blomquist et al., 2001; Koyuncu, 2011). Indeed, if one assesses the long-run tradeoff between growth and income inequality in terms of the ratio ($\Delta \psi / \Delta GINI = 0.103$), it is evident that this modest 5% tax reduction is very costly from a distributional perspective; increasing the growth rate by just 0.1 p.p. raises income inequality by 1 p.p.

6.2. Tax cut coupled with increase in progressivity

One of the striking features of the 5% across-the-board tax cut is that while the existing (fixed) progressivity of the tax structure reduces the tax benefits received by the wealthiest quintile, nevertheless both income and wealth inequality increase quite significantly over time. This somewhat paradoxical outcome is generated by the internal dynamics in response to the tax cut. It reflects the fact that, given the initial unequal income distribution, the highest quintile can allocate more of their tax reduction to savings, while the poorer groups, being subject to less regressivity, allocate it to consumption, with the net effect being to exacerbate the problem of inequality. To the extent this is deemed to be an undesirable outcome, it raises the question of restructuring the 5% overall tax cut. Specifically, we consider a scenario where the 5% average tax cut is achieved by further increasing ζ to 0.8506, while simultaneously raising the progressivity ϕ to 0.1095, the combined effect of which is to maintain the average tax rate of the highest quintile approximately unchanged at its prior rate of 23.1%. The results of this are reported in Table 5, with the transitional dynamics illustrated in Fig. 2.

Table 55% Average Tax cut: Substantial change in progressivity Negative long-run growth and inequality ($\zeta = 0.8506, \phi = 0.1095$).

	Impact	After 10 periods	Long-run
Δ[Growth rate, ψ]			
Aggregate	-0.141	-0.149	-0.180
Q1	-2.250	-2.099	-0.180
Q2	3.091	1.977	-0.180
Q3	0.404	0.300	-0.180
Q4	-0.074	-0.090	-0.180
Q5	-0.464	-0.422	-0.180
Δ[Labor Allocation, L]			
Aggregate	-0.0057	-0.0066	-0.0100
Q1	0.0102	0.0008	-0.0298
Q2	0.0035	-0.0027	-0.0330
Q3	-0.0016	-0.0055	-0.0288
Q4	-0.0078	-0.0084	-0.0137
Q5	-0.0327	-0.0170	0.0555
Average Tax Rate, τ			
Aggregate	17.47%	17.31%	16.69%
Q1	-0.82%	0.05%	2.62%
Q2	6.98%	7.37%	9.05%
Q3	11.52%	11.71%	12.70%
Q4	16.08%	16.12%	16.35%
Q5	23.12%	22.94%	22.02%
Marginal Tax Rate, τ^m			
Aggregate	22.48%	22.57%	13.28%
Q1	10.22%	10.99%	19.01%
Q2	17.17%	17.51%	22.26%
Q3	21.21%	21.37%	25.51%
Q4	25.27%	25.30%	30.56%
Q5	31.54%	31.38%	22.88%
Δ[Income GINI]			
	-0.675	-1.907	-7.367
Δ[Wealth GINI]			
	0.000	-2.175	-11.700
Δ[Income Share y_i]			
Q1	0.013	0.031	0.092
Q2	0.008	0.025	0.109
Q3	0.004	0.018	0.096
Q4	-0.002	0.003	0.033
Q5	-0.024	-0.078	-0.330
Δ[After-tax Inc. Share$_i^a$]			
	0.022	0.040	0.103
Q1	0.017	0.033	0.112
Q2	0.011	0.023	0.090
Q3	0.000	0.002	0.020
Q4	-0.051	-0.099	-0.326
Q5			
Δ[Wealth Share, k_i]			
	0.000	0.033	0.147
Q1	0.000	0.029	0.174
Q2	0.000	0.022	0.151
Q3	0.000	0.007	0.050
Q4	0.000	-0.091	-0.522
Q5			

The impact on the economy, particularly over time, is substantially different from that when only the base tax rate, ζ is changed. In the case of Q5, the increase in ϕ offsets the increase in ζ , leaving their initial average tax rates essentially unchanged. For the lower income quintiles the increase in ζ dominates the increase in ϕ , so that their initial average tax rates are all reduced increasingly, to the point where Q1 is subject to a *negative* income tax. The increase in progressivity raises the short-run marginal tax rates on quintiles Q2–Q5, while reducing it on Q1.

During the subsequent transition, these short-run responses are reversed. Q5's average tax rate declines, converging to 22.02% (see Fig. 2e), as their relative income (y_5) declines and they are subjected to a progressively lower marginal tax rate. In contrast, the average tax rates of the Q1–Q4 increase as their relative income steadily rises (see Fig. 2a), almost reaching their previous long-run levels. Comparing the long-run reductions in the average tax rates of the 5 income classes, reported in Table 5, reveals a uniform pattern, with the decline increasing from 4.63% (1.07 p.p.) for the top quintile to 5.07% (0.14 p.p.) for the poorest. With the substantial decline in the income share of the highest quintile, Q5, this results in an overall reduction in the average tax rates of 9.34% (1.72 p.p.).²⁵

²⁵ In this case the marginal tax rates of all quintiles will increase, ranging from 0.39 p.p. for Q5 to 1.58 p.p. for Q1, with the economy-wide average increasing by 0.44 p.p.

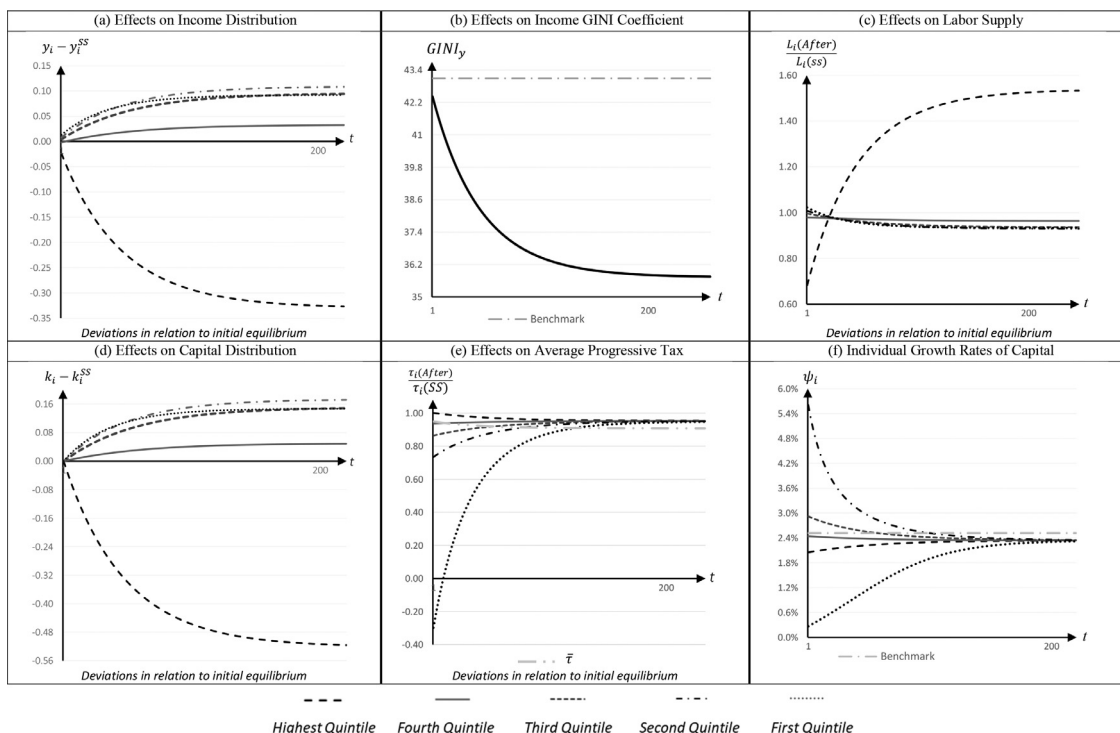


Fig. 2. Tax cut with substantial increase in progressivity ($\zeta = 0.8506, \phi = 0.1095$).

The marginal tax rate follows a similar pattern, gradually declining for Q5, while increasing for Q1–Q4, but in all cases ending higher than prior to the tax cut. Ironically, the increase in progressivity, ϕ , the intent of which is to reduce the tax benefits for the richest has a perverse effect. In the long-run it reduces the average tax rate of Q5 by 1.07 p.p. while raising their marginal tax rate by 0.39 p.p, with the comparable changes for Q1 being a reduction of 0.14 and an increase of 1.57 p.p, clearly favoring the rich. Moreover, comparing Table 5 with Table 4 we see that Q5 is the only group whose average tax rate is actually lower when the 5% overall tax cut is accomplished by increasing both ζ and ϕ . However, this is a result of the reduction in their share of income, which pushes them into lower income tax brackets.

The increase in tax progressivity, ϕ , has substantial consequences for labor supply across the quintiles. The greatest immediate impact is on the highest quintile, Q5, causing those agents to reduce their labor supply substantially, by 3.27 p.p. This response in their labor supply dominates any tax benefits, resulting in an immediate reduction in their relative income share of 0.48 p.p. and in their growth rate of capital of 0.46 p.p. The poorer quintiles are less adversely affected by the increase in ϕ . Apart from Q4, which suffers a slight loss in its income share, all others gain increasingly, with their growth rates gaining correspondingly.²⁶ Quintiles Q1 and Q2 increase their labor supply so that the overall reduction in labor supply is 0.57 p.p. Over time, as the different quintiles adjust their labor supplies in response to the changing tax rates, we see that Q5 increases its labor supply while Q1–Q4 supply less labor. The net impact is a modest reduction in aggregate labor supply of 0.1 p.p, suggesting a reallocation of labor across the quintiles with the incentives of different groups having largely offsetting effects.

The reduction in the accumulation of capital by the wealthier quintiles and the increase by the poorer income classes reduces income inequality. Specifically in the long run the income share of Q5 is reduced to 44.2% while that of Q1 increases to 5.8%, with the overall income GINI coefficient dropping from 43.1 to 35.7. The drop in the wealth GINI is even more dramatic, declining by 11.7 p.p. At the same time, the dramatic drop in the growth rate of the richest quintile dominates the increase in the growth rates of the poorer quintiles causing an immediate short-run decline in the average growth rate of 0.14 p.p. Over time, the declining growth rate of the highest quintile, Q5, is the main driving force behind the contraction of the economy. As this moderates, along with the growth of the poorer quintiles, in the long run the growth rates of all quintiles converge to the common rate of 2.34%.

Thus in the long-run a 5% cut in the average tax rate, in which the reduction in the base rate is accompanied by an increase in progressivity, so that initially the taxes paid by the highest profile remains unchanged, will in the long run lead to an equilibrium in which income and wealth inequality are both drastically reduced, but the growth rate also declines by

²⁶ The reduction in the growth rate of Q1, again reflects the fact that it has negative capital so that its growth rate, $d\hat{k}_i(0)/K_{i,0}$ is actually positive (see Fig. 2f).

Table 65% Average tax cut: Moderate change in progressivity Positive long-run growth and negative inequality ($\zeta = 0.8482, \phi = 0.095$).

	Impact	After 10 periods	Long-run
Δ[Growth rate, ψ]			
Aggregate	0.081	0.081	0.079
Q1	0.043	0.051	0.079
Q2	0.188	0.169	0.079
Q3	0.102	0.099	0.079
Q4	0.085	0.084	0.079
Q5	0.071	0.073	0.079
Δ[Labor Allocation, L]			
Aggregate	-0.0007	-0.0007	-0.0008
Q1	0.0023	0.0021	0.0016
Q2	0.0005	0.0003	-0.0008
Q3	-0.0008	-0.0009	-0.0019
Q4	-0.0019	-0.0019	-0.0023
Q5	-0.0036	-0.0031	-0.0007
Average Tax Rate, τ			
Aggregate	17.47%	17.47%	17.45%
Q1	1.21%	1.23%	1.28%
Q2	8.19%	8.20%	8.26%
Q3	12.18%	12.18%	12.22%
Q4	16.17%	16.17%	16.18%
Q5	22.37%	22.36%	22.34%
Marginal Tax Rate, τ^m			
Aggregate	21.52%	21.53%	10.66%
Q1	10.60%	10.61%	16.97%
Q2	16.91%	16.92%	20.56%
Q3	20.52%	20.53%	24.15%
Q4	24.13%	24.14%	29.71%
Q5	29.74%	29.74%	
Δ[Income GINI]			
	-0.095	-0.129	-0.301
Δ[Wealth GINI]			
	0.000	-0.059	-0.352
Δ[Income Share y_i]			
Q1	0.002	0.002	0.004
Q2	0.001	0.001	0.004
Q3	0.000	-0.001	0.001
Q4	-0.001	-0.004	-0.013
Q5	-0.003		
Δ[After-tax Inc. Share$_i^d$]			
	0.004	0.004	0.005
Q1	0.003	0.003	0.006
Q2	0.001	0.002	0.005
Q3	0.000	0.000	0.001
Q4	-0.007	-0.009	-0.017
Q5			
Δ[Wealth Share, k_i]			
Q1	0.000	0.001	0.003
Q2	0.000	0.001	0.006
Q3	0.000	0.001	0.006
Q4	0.000	0.000	0.003
Q5	0.000	-0.003	-0.018

0.18 percentage points. This stands in sharp contrast to the case where the 5% tax cut is met entirely by reduction in the base tax rate, when both inequality and growth increase over time.

Neither of these outcomes may be deemed as being desirable, raising the question of whether it is feasible to structure the 5% tax cut so that in the long run the growth rate increases, but inequality declines. Table 6 and Fig. 3 illustrate how such an outcome is indeed possible by moderating the increases in ζ and ϕ to 0.8482 and 0.095, respectively. In this case, the average and marginal tax rates for all quintiles decline on impact and remain approximately constant through the transition. There are very little changes in income shares and in the long-run the Gini coefficients for income and wealth are reduced by 0.30 and 0.35 p.p. respectively, while the growth rate increases slightly from 2.52% to 2.60%. These are modest changes, but then the 5% cut in the average tax rate is also quite modest.

6.3. Differential taxes on labor and capital

A key assumption underlying our analysis is that income from labor and capital are both taxed at the same rate. We have briefly explored the sensitivity of the growth-inequality tradeoff to this assumption by postulating tax function that specify differential tax rates to be applied separately to labor and capital income along the lines addressed by Chen (2020), while

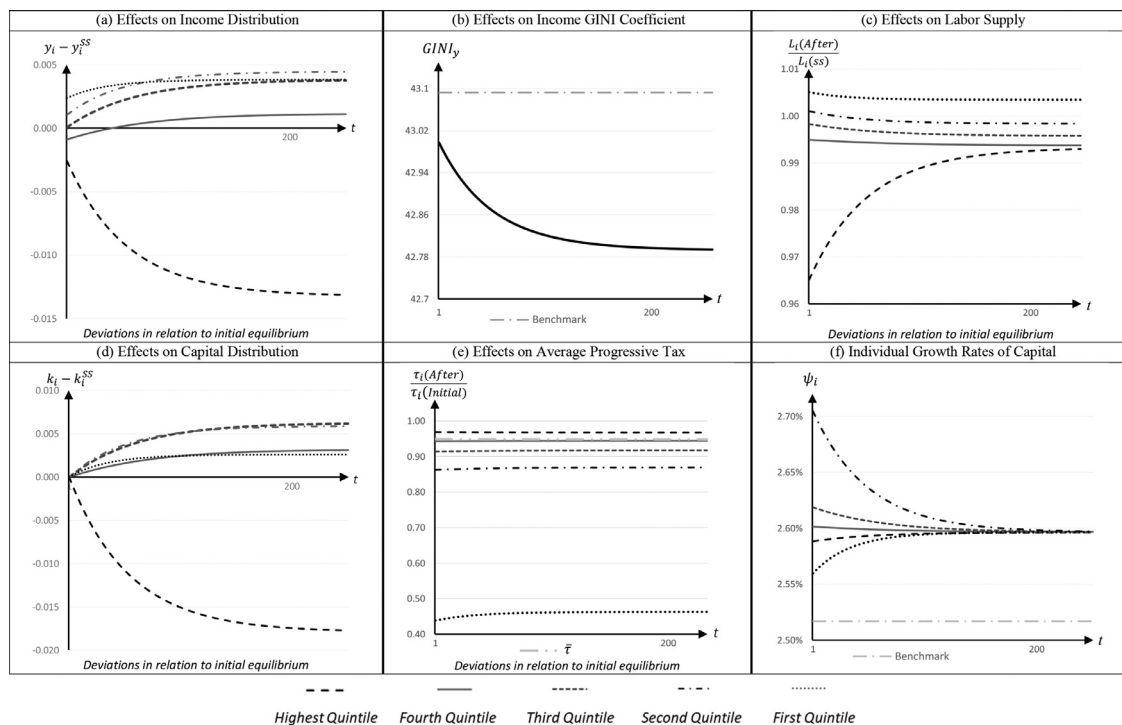


Fig. 3. Dynamic effects of a tax cut with moderate increase in progressivity ($\zeta = 0.8482, \phi = 0.095$).

maintaining all other features and a similar initial calibration of the model to that reported in Table 1.²⁷ Most importantly, labor income and capital income are initially both subject to the same tax structure, described by $\xi = 0.838, \phi = 0.092$. Starting from this equilibrium we carry out two simulations.

The first policy change eliminates the progressivity of the tax on labor income, so that all labor income is taxed at the flat rate of 16.2%. We found that this does not significantly change the aggregate tax revenue.²⁸ Overall, labor responds to the lower tax by increasing aggregate labor supply, while the unchanged taxation on capital income leaves tax revenues from the capital income tax roughly unaffected. The long-run growth rate increases by 0.20 p.p., but this is accompanied by a mild long-run increase in inequality with the income Gini increasing by 0.6 p.p. The latter effect is the consequence of a reduction of the relative wealth and income shares of the middle-income quintiles. The poorest quintile’s relative income increases, but they end up working more hours in the long run. The highest quintile reduces consumption and boost the rate of capital accumulation, increasing their overall long-run share of income and capital. Flattening the tax rate on labor income nevertheless increases income inequality. However, the overall effect on the growth-inequality tradeoff ratio is 0.33, which is more favorable than the across the board tax cut.

The second policy we simulate is one that combines this flat tax rate on labor income with a 25% increase in the progressivity of capital income taxation, increasing ϕ on capital to 0.115. This results in powerful distributive effects that significantly reduce income and wealth inequality. The income and capital GINI coefficients fall by 6.4 p.p. and 9.3 p.p., respectively, at the cost of reducing growth rate by 0.3 p.p. Increasing the progressivity of the tax on capital income, while eliminating it on labor income exacerbates the growth-inequality tradeoff discussed in Section 6.2.

7. Conclusions

The progressivity of the tax structure is an important determinant of the tradeoff between growth and both income and wealth inequality. To be compatible with long-run non-degenerate distributions of wealth and income inequality the agents’ preferences must be characterized by some form of heterogeneity. In contrast to previous literature which focuses on differential rates of time preference, we do so in terms of the elasticity of intertemporal substitution. Two compelling reasons justify this: first empirical evidence on the EIS is much more extensive, and second variations in the EIS across income groups are much more compatible with the range of marginal tax rates necessary to sustain long-run equilibrium

²⁷ The details are available from the authors.

²⁸ This result is consistent with Guner et al. (2016) who find that increasing progressivity of the tax structure generates only small increases in revenue.

across agents. With this modification we obtain a unique nondegenerate distribution of income and wealth in which more affluent households having a higher EIS end up with higher incomes and owning more of the wealth.

Using an endogenous growth model calibrated to the US economy we examine the dynamic responses of inequality and growth to tax changes that are motivated by the recent Tax Cut and Jobs Act enacted in 2017. The numerical implementation of the model considers the income distribution segregated into quintiles. This offers two main advantages: (i) it allows for an assessment of the diverse impact on different income classes in the economy; and (ii) quintiles enable one to derive reasonable estimates of Gini coefficients, thereby enabling one to determine the overall tradeoffs between the effects on income inequality and economic growth. One advantage of the TCJA is that the tax cuts were in fact quite modest, and as an initial benchmark could be reasonably approximated as a 5% reduction in the average tax rates, brought about by an appropriate reduction in the base tax rate, with the underlying progressivity unchanged.

The benchmark simulation of the 5% reduction of the average tax rate, illustrates that the differences in the dynamic responses across the quintiles are indeed substantial. In the presence of progressivity, a reduction in the parameter that drives the level of the tax schedule implies asymmetric reductions in the tax rates that favor the poorer quintiles. Yet the richest quintile is the only group that increases its relative share of capital and income during the transition, and that works less in the new steady-state. With the tax cut reducing the marginal rates of the poorer quintiles, they increase their labor, more than offsetting the response of the highest quintile, and leading to a slight increase in the long-run aggregate labor supply. This simulation also suggests that the 5% average tax cut brought about by the reduction in the base rate will have diverse short-run effects on the different income classes. These are dominated by the positive impact on the richest quintile, ensuring a positive economy-wide impact, which is sustained during the transition, and also increases the long-term growth rate by 0.135 p.p. But this is a modest increase and comes at the expense of substantial long-run increases in both income and wealth inequality.²⁹

These responses are sensitive to how the overall average tax cut is structured. If the 5% average tax cut takes the form of a larger reduction in the base tax rate coupled with a large increase in progressivity, designed to maintain the tax burden on the richest quintile unchanged, the exact opposite results. The long-run responses by agents to the accompanying increase in progressivity, and particularly the reduction in labor supply will generate a long-run reduction in the growth rate. Moreover, since the increase in progressivity impacts the richer groups most heavily, the reduction in growth is accompanied by substantial reductions in both income and wealth inequality.

But by moderating the increases in the two key parameters, ζ and ϕ , determining the tax rate it is possible to structure the 5% average tax cut so that the economy can increase its growth rate while simultaneously reducing inequality. We should stress, however, that the differences in parameter values giving rise to the diverse outcomes identified are quite small, suggesting that any major tax reform must be undertaken with care in order to avoid unintended adverse consequences.

Overall, these simulations suggest that the growth-inequality tradeoff is highly sensitive to the degree of progressivity of the tax structure although this conclusion must be viewed with some caution. Heterogeneity across agents is critical and we have incorporated just one source, albeit it a key one. Other sources of heterogeneity may have moderating effects. In a modified version of the model we have expanded heterogeneity across agents to also include differential rates of time preferences and indeed the growth-inequality tradeoff is moderated. Other sources of heterogeneity, such as differential labor productivity are also potentially important. Clearly, the nature of the progressivity of the tax rate in influencing the growth-inequality tradeoff merits further analysis, not only with respect to changes in tax policy, but also with respect to other structural changes such as technological developments as they impact firms' hiring decisions.

Appendix

Incompatibility of variation of RTP across income classes with progressivity of tax rates

This appendix demonstrates that for the RTP alone to be compatible with the observed progressivity of the tax rates, extreme and implausible differentials in the RTP across the income classes must be imposed. As we established in Section 2, in steady state all individuals or income groups experience the same growth rates of their income and capital. Analogous to (19a), this steady-state equilibrium is described by:

$$\vartheta [(1 - \tau_H^m(y_H))\tilde{r} - \beta_H - \delta] = \vartheta [(1 - \tau_L^m(y_L))\tilde{r} - \beta_L - \delta] = \tilde{\psi} \tag{A.1}$$

where the subscripts "H" and "L" refer to the highest and lowest income groups, ϑ is their common EIS, β_L, β_H their respective RTP, and other quantities are as defined in the text. From (A.1), one can readily derive the following long-run equilibrium relationship, between the rates of times preference and the relative marginal tax rates between these two income groups

$$\beta_L = \beta_H + \left[\frac{\tilde{\psi}}{\vartheta} + (\beta_H + \delta) \right] \left[\frac{\tau_H^m - \tau_L^m}{1 - \tau_H^m} \right] \tag{A.2}$$

²⁹ It would be desirable to validate the model and calibration by comparing its predicted changes in growth and inequality from past tax reforms. To do so, would require the recalibration of the entire model to reflect the general economic conditions and tax structures prevailing at the time and is impractical. In this respect, we are encouraged to see that the estimate of the long-run increase in the growth rate due to the TCJA implied by the model is quite close to 0.2 p.p. long-run increase estimated by Barro and Furman (2018). In addition, its implied short-run increase in the income GINI of 0.246 is comparable to the observed change of 0.2 between 2017 and 2019; see <https://fred.stlouisfed.org/series/SIPOVGINIUSA>.

Table A.1
Calibration of common EIS and individual rate of time preference.

Common Intertemporal Elasticity of Substitution ($1/(1 - \gamma)$)	0.52
Rate of Time Preference (β_i)	
First quintile: Q1	0.048
Second quintile: Q2	0.043
Third quintile: Q3	0.038
Fourth quintile: Q4	0.033
Highest quintile: Q5	0.028

Table A.2
Initial steady-state equilibrium.

	Growth rate	Agg. Labor share	Av. Econ tax rate	Marg. Econ tax rate	Income GINI	Wealth GINI
Aggregate Variables	2.41%	0.358	16.78%	23.63%	22.8%	33.2%
	Income Shares	Capital Shares	Labor Allocation	Average Tax Rate	Marginal Tax Rate	
Q1	0.514 (10.3%)	0.341 (6.8%)	0.3424	10.90%	19.10%	
Q2	0.681 (13.6%)	0.508 (10.1%)	0.4029	13.19%	21.18%	
Q3	0.911 (18.2%)	0.825 (16.5%)	0.4049	15.48%	23.25%	
Q4	1.227 (24.5%)	1.313 (26.3%)	0.3596	17.76%	25.33%	
Q5	1.667 (33.4%)	2.013 (40.3%)	0.2782	20.05%	27.41%	

First, we can illustrate the inability of realistic differentials in the RTP to satisfy this long-run equilibrium relationship by choosing parameter values, typical of those employed in simulations. For example, if we set $\beta_H = 0.03$, $\delta = 0.06$, $\vartheta = 1$, $\bar{\psi} = 0.02$, (Li and Sarte, 2004; Koyuncu and Turnovsky, 2016; Chen, 2020), coupled with $\tau_L^m = 0.10$, $\tau_H^m = 0.35$, the lowest and second highest, marginal Federal tax rates in US in 2017, (A.2) implies $\beta_L = 0.072$. A differential RTP of 4.2 p.p. between the lowest and second highest income groups is clearly an implausible differential and is inconsistent with the estimated differences; see footnote 3. The implausibility increases if ϑ is moderated to 0.5, when the differential is increased to 5 percentage points.

As further verification we ran a simulation that (i) assumes a common EIS of 0.52 across income classes, (ii) varies the RTP across quintiles between 0.028 and 0.048, (iii) retains the technology and policy parameters at the benchmark values as reported in Section 5.1. Table A.1 summarizes this calibration, while Table A.2. reports the main features of the benchmark economy. Even admitting a relatively broad range of the RTP between the top and bottom income classes, this source of heterogeneity alone did not come close to matching the observed wealth or income distribution across quintiles, or to spanning the range of marginal tax rates across the income levels.

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