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Demographic structures, savings, and international capital flows ☆

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ABSTRACT

This paper investigates how the different demographic characteristics of two countries will drive the flow of capital between them, as inhabitants of the longer-lived nation have a higher savings rate than do those with the shorter lifespan. It is motivated by the recent US experience during which its net foreign asset position declined dramatically, while simultaneously its longevity has fallen increasingly below that of other developed G7 economies. Our contribution is to address the issue by introducing empirically based survival (mortality) functions into a traditional two country macrodynamic framework. In doing so, we eliminate some of the unsatisfactory aspects associated with the traditional two country representative agent model. The sensitivity of the capital flows and the resulting net foreign assets of the two economies to key structural characteristics, and their consequences for savings are emphasized.

1. Introduction

The traditional representative agent model analyzing international capital flows under perfect capital mobility runs into serious problems. In the case of a small open economy, if one combines the standard assumptions of a constant rate of time preference together with a fixed given world interest rate, an interior equilibrium in a stationary economy can be sustained if and only if these two quantities are set equal to one another. This imposes a so-called 'knife-edge' condition, since these two exogenous measures are determined by entirely independent considerations, so that there is no compelling reason for the required equality to hold. Moreover, imposing the condition nonetheless imposes serious constraints on the implied equilibrium dynamics; see Turnovsky (1997). Several modifications have been proposed to circumvent these unsatisfactory elements.[1] One of the most satisfactory is to replace the representative agent assumption with some form of overlapping generations (OLG) structure. This approach was initially adopted by Blanchard (1985) using his "perpetual youth" specification, and more recently under more general demographic conditions by Oxborrow and Turnovsky (2017).

A similar problem confronts the two-country representative agent model of international capital flows. For example, the well known Obstfeld and Rogoff (1995) model, which abstracts from physical capital, is unable to determine a unique endogenous steady state. In a series of models incorporating the international accumulation of physical capital, Turnovsky (1997) showed how the attainment of an interior steady state requires the rates of time preference in the two

☆ I am grateful to David Oxborrow for assistance during the early stages of this study. Unfortunately, his career path prevented him from continuing with the project. I also wish to acknowledge the helpful discussions with Neil Bruce on this general topic over the years.

[1] See Turnovsky (1997) and Schmitt-Grohé and Uribe (2003).

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economies to be equal, otherwise the more patient country will end up owning all the capital.[2] Imposing the equality of rates of time discount plays the same role as does the knife-edge condition with respect to the small open economy. It is equally unsatisfactory, since there is no reason for sovereign nations to have a common time discount rate, a view supported by empirical evidence.[3]

As for the small open economy, introducing an OLG structure can resolve this situation. Ghironi (2006) and Ghironi et al. (2008) develop two models in which the representative agent assumption is replaced by the Blanchard (1985)-Weil (1989) OLG specification to yield a well-defined macrodynamic equilibrium. The first model assumes equal rates of time preference across the two economies, and leads to a steady-state equilibrium in which the net foreign assets of both economies are zero. The second model generalizes this to allow for unequal rates of time preference, which yields a long-run equilibrium in which the less patient economy ends up being a (finite) debtor to the more patient economy. While the Blanchard demographic assumption has the advantage of tractability, its "perpetual youth" characteristic renders it misleading as a description of life cycle behavior (as Blanchard himself originally acknowledged). Consequently, its dynamics deviate substantially from that associated with a more realistic mortality function, as illustrated for the small open economy by Oxborrow and Turnovsky (2017).

In this paper we build on these contributions in several directions. First, and most importantly, we introduce more general demographic structures for the two economies, ones based on actuarial information that also reflect the reality of differential mortality structures across the two nations. Our analysis incorporates the feature that savings are largely driven by the interaction between the nation's demographic characteristics and key preference parameters, most notably the intertemporal elasticity of substitution (IES), as well as the rate of time preference. Thus, in contrast to much of the literature that employs logarithmic utility, we allow the IES both to deviate from unity and to vary between the two economies. This generalization is important for two reasons. First, empirical estimates of the IES vary dramatically across economies and over time, exhibiting far more variation than do the rates of time preference.[4] Second, as the results of our study will highlight, the flow of international capital is extremely sensitive to even small differences in the IES between nations, differences well within the margin of statistical error. In contrast, the international allocation of capital, as a productive factor, is driven by international productivity differentials, with each country's net foreign asset position (NFA) being determined by the net result of these two forces.

We begin by deriving the macrodynamic equilibrium of this two country world economy, in which agents in one country can directly own physical capital located in the other. But even the simple canonical model we develop leads to a high order dynamic system that we analyze using numerical simulations. Since, our objective is to focus on the significance of a realistic demographic structure, in parameterizing the model, particular attention is paid to calibrating the survival functions. To do so we employ the survival function introduced by Boucekkine et al. (2002). This BCL function is highly tractable and tracks the empirical data on survival for most Western economies remarkably successfully.[5] To apply this we proceed as follows. First, we identify the domestic economy as the United States and the foreign economy as a population-weighted average of its main G7 trading partners, Canada, France, Germany, Japan, and the United Kingdom, which we shall refer to as "G5". We estimate this BCL function for the various countries using nonlinear least squares, and aggregating over the G5 group of economies we can summarize the differential demographic transitions of the United States relative to that of its main trading partners. By constructing the foreign economy as an amalgam of the developed countries comprising the G5, with their tight integration of both financial and goods markets, we justify our specification of the cross-country interdependence in terms of a perfect world capital market and a single-traded final output good.

Before proceeding, we should emphasize two points. First, we view the contribution of the paper to be primarily theoretical rather than empirical. Our concern is to determine how symmetries or asymmetries in two developed economies' savings propensities arising from their respective demographic structures and consumption preferences impact international capital flows and their respective net asset positions. Hence we do not attempt to calibrate other aspects of the two economies in any detailed way, but rather do so generically. In this regard, our focus on the US vs the G5 is intended in part to be illustrative. Second, with detailed information on mortality readily available, many studies employ such observed data directly. For several reasons we do not follow this approach. In the first place, representing survival functionally enables us to represent decreases in mortality much more parsimoniously and transparently, which, given our focus on expositing

[2] In the case where capital income is taxed, the condition for an interior steady state requires an appropriate adjustment for tax rates, which then imposes constraints on the tax rates themselves; see Frenkel et al. (1991). Slemrod (1988) has suggested that the stringent conditions for a viable interior solution are essentially an artifact of the absence of risk. Indeed it is true that incorporating risk introduces flexibility, thereby avoiding corner solutions, although this raises other issues; see Stulz, (1988), Turnovsky (1993), and Obstfeld (1994) for examples of stochastic general equilibrium representative agent models of open economies.

[3] See e.g. Wang et al. (2016) for empirical evidence supporting the disparity of rates of time preference across nations.

[4] Extensive empirical evidence highlighting the extreme variation of estimates of the IES across countries is provided by Havranek et al. (2015). For the six countries we consider the averages over the many studies they summarize vary between −0.03 for France and 0.89 for Japan, with substantial standard deviations in each case. In contrast, the IES being dimensionally equivalent to the interest rate, as a practical matter cannot vary by more than a couple of percentage points.

[5] One of the reasons for its success is that it turns out to be very good first order approximation to the Gompertz (1825) which demographers find to be the most accurate representation of mortality, but being a double exponential function, its dynamics are computationally intractable in a general equilibrium framework. Other specifications of survival functions have been employed; see e.g. Mierau and Turnovsky (2014b) for a brief review.

the mechanism, seems appropriate. Furthermore, the BCL function in fact fits the data extremely well in all cases and provides an efficient and accurate summary representation.

Nevertheless interest in the nexus between demographic structure and international capital flows is motivated by the experience of the United States, the NFA position of which, as measured by NFA/GDP ratio, declined dramatically by around 24% over the period 1980–2010. During the same period most countries (including developing countries) experienced significant demographic change. In 1980 life expectancy at birth in the US was around 73.6 and averaged around 74.6 in the G5 economies. By 2010 these had increased to 78.5 and 81.2, respectively [Table 1]. Since life expectancy is a key determinant of savings behavior, the question arises of the impact of the longer, and the increasingly lengthier, lifespan in the G5 relative to the US on the US NFA position.

In addressing the role of the relative demographic structures in the US and its G5 trading partners in determining the capital flows between them, we focus on two main aspects. First, we determine the extent to which the pre-existing difference in mortality in 1980 accounted for the NFA position of the US at that time. Second, we consider the extent to which the growing differences in mortality between the US and G5 over the period 1980–2010 account for the dramatic reduction in the US NFA/GDP ratio that occurred over that period.

As a benchmark, we assume that the two economies are initially identical with the exception of their mortality rates that are set at their respective 1980 rates. We also assume, as is customary in demographic models with uncertain mortality, the existence of an actuarially fair annuities market whereby assets of the deceased are recycled back to the economy.[6] In that case with common rates of time preference and IES, our results suggest that the higher mortality rate in the US accounts for around 5 percentage points decline in its NFA/GDP ratio, relative to an equilibrium with common mortality. With asymmetries in the preference parameters, the respective NFA positions of the two economies vary dramatically, but in all cases the decline in the US NFA/GDP ratio due to the differential mortality rates is consistently around 5 percentage points.[7] In the absence of the annuities market, the decline in the NFA/GDP ratio is larger and more sensitive to variations in the preference parameters.

The ability of a stylized model such as this to account for the dramatic decline in the US NFA/GDP ratio over the period 1980–2010, needs to be kept in perspective. First, the international capital flows reflected in the model are in the nature of FDI, whereas most of the decline in the US NFA was due to the accumulation of net debt, which is not incorporated in our analysis. Also, it seems somewhat implausible to expect the smaller increase in life expectancy of 1.7 years (4.9 vs. 6.6) that occurred in the US relative to the G5 over that period to account for such a substantial change in its NFA position. Other factors not present in this framework must surely have been playing a role.

Nevertheless, our model is consistent with this trend, at least qualitatively. Our benchmark parameterization suggests that the relative declines in mortality in the US and the G5 over the period 1980–2010 causes a 0.3% decline in the US NFA/GDP ratio, the presence of the annuities market and 0.6% in its absence. The decline increases further to 1.6% if the rate of time preference of the G5 is 3.7% while that of the US is 4.0%. On the other hand, if the change in mortality over the period is accompanied by a concurrent modest increase in US impatience, say from 4.0% to 4.2%, then the combined changes yield around a 10% decline in NFA/GDP. If further, the IES also declines modestly from 0.40 to 0.38 – well within statistical sampling error – then the total predicted decline is almost 20%. The key point here is not that the change in mortality alone necessarily accounts for much of the decline in NFA, but rather that the flexibility it introduces into the equilibrium enables one to explain much of the decline in terms of modest, realistic changes in other key preference parameters.

We also briefly address the extent to which demographic differences affect the impact of increases in productivity on the net foreign asset position. The most striking finding is the following. Under the assumption that the demography is described by the BCL survival function, the NFA/GDP ratio remains unchanged in response to proportional increases in productivity of the two economies. This statement applies irrespective of the specific parameterization of the survival functions and of the economies' preference parameters.

Over the past several years, a number of papers have been written introducing demographic aspects into models of international capital flows. Of these studies, this paper is closest to the important contributions of Ghironi (2006) and Ghironi et al. (2008), as discussed, in the sense that both focus on drawing out the implications of the demographic aspects within the context of a dynamic two country equilibrium. In this respect these papers all have the common objective of embedding demographic structures into the traditional two country dynamic general equilibrium models such as those developed by Lipton and Sachs (1983), Frenkel and Razin (1987), Obstfeld (1989), Devereux and Shi (1991), and Turnovsky (1997).

But it is also close in spirit to Backus et al. (2014) who develop a multi-country open economy model looking at the impact of simulated demographic trends on the capital flows across countries. They calibrate the model to match the following countries: China, Germany, Japan, and the US. In contrast to what we have called as the "traditional" two country models, which focus on characterizing the entire equilibrium, Backus et al. develop a parsimonious model and focus only on the implications of the cross-country demographic differences on the international capital flows.

The idea of linking demographic factors to international capital flows is well established. Indeed, Backus et al. (2014) follow in the tradition of this earlier literature in using an aggregate model to draw out the implications of changes in the demographic structure for international capital flows. Studies in this mode include Attanasio and Violante (2000), Feroli

[6] This originated with Yaari (1965) and is a central element of the Blanchard (1985) model.

[7] As a specific example, consider the case where both US rate of time preference and its IES exceed those of the G5. In that case the US has a positive NFA. For the parameters in Table 6, the NFA/GDP ratio declines from 0.564 for common mortality rates to 0.510, for differential mortality rates, again a decline of around 5.4 percentage points.

Table 1
Cross-country demographic characteristics.

Country	Birth life expectancy		Pop. growth rate		Fertility rate	
	1980	2010	1980	2010	1980	2010
Canada	75.1	80.9	1.3	1.1	1.7	1.6
France	74.1	81.7	0.4	0.5	1.9	2.0
Germany	72.7	80.0	0.2	-0.2	1.4	1.4
Japan	76.1	82.8	0.8	0.0	1.8	1.4
UK	73.7	80.4	0.1	0.8	1.9	1.9
US	73.6	78.5	1.0	0.8	1.8	1.9

Data retrieved from the World Bank World Development Indicators.

Table 2
Population aged 18–90 (in millions).

Country	1980	2010	1980 Fraction	2010 Fraction
Canada	17.3	26.7	0.04	0.05
France	38.9	48.6	0.10	0.09
Germany	46.8	67.9	0.12	0.13
Japan	83.2	104.9	0.21	0.20
UK	41.4	48.8	0.11	0.09
US	161.6	232.8	0.42	0.44
Total	389.2	529.7	1	1

Data from the Human Mortality Database.

(2003), Domeij and Floden (2006), and Ferrero (2010), although they vary in terms of the economies upon which they focus and their representation of mortality. Our paper differs in its key objective, namely the introduction of tightly estimated, realistic survival functions into a dynamic two-country general equilibrium model.

The remainder of the paper is structured as follows. Section 2 sets out the basic analytical framework, while Sections 3 and 4 describe the aggregate equilibrium including the steady state. Section 5 describes the simulation approach. Section 6 describes the long-run effect of key structural changes while Section 7 discusses the transitional dynamics. Section 8 briefly discusses an important but typically neglected aspect, namely how the existence of cohorts of different stages in their respective life cycles naturally impacts the degree of wealth inequality. Section 9 concludes, with technical details being relegated to the Appendix.

2. Basic analytical framework

We consider a two-country, one-good model of a decentralized world economy, inhabited by households, characterized by cohorts of overlapping generations, and infinitely-lived firms. Both countries accumulate capital gradually over time and the world market for capital is perfectly integrated. Each agent supplies a unit of labor inelastically. To ensure a balanced-growth equilibrium the long-run labor supply in each country grows at the same exogenous rate, n, and is assumed to be perfectly immobile across international borders. Denoting the domestic and foreign labor supplies at any instant of time t by $L(t), L^*(t)$, respectively, $\theta \equiv L^*(t)/L(t)$ parameterizes the relative sizes of the two economies, which with a common secular growth rate, remains constant in the long run. It does, however, permit us to incorporate temporary deviations in the two growth rates, consistent with maintaining a long-run balanced growth. In expositing the model we shall focus primarily on the domestic economy. Variables pertaining to that economy (referred to as "Home") are unstarred, and the corresponding foreign economy ("Foreign") are starred. The subscript d refers to the holdings of domestic residents and f to the holdings of foreign agents.

With overlapping generations, it is important to distinguish between calendar time and age. In general, a variable $X(v, t)$ pertains to an agent born at time v, when he has attained the age $(t - v)$. The partial derivative with respect to calendar time, $\partial X(v, t)/\partial t \equiv X_t(v, t)$, describes the change in X over time for a given cohort, as it ages.

2.1. Individual demographic structure

The probability of an individual born at time v surviving until age $(t - v)$ is described by the general survival function $S(t - v)$, where $S'(s) \equiv dS(s)/ds < 0$, declines with age. The corresponding mortality rate, (instantaneous probability of death) is $\mu(t - v) = -S'(t - v)/S(t - v) > 0$. The probability that the agent dies before reaching age $(t - v)$ is determined by the cumulative mortality rate: $M(t - v) = \int_0^{t-v} \mu(\tau)d\tau$, implying that the survival and the mortality functions are related by

$S(t-v) = e^{-M(t-v)}$. We assume $S(0) = e^{-M(0)} = 1,\ S(D) = e^{-M(D)} = 0$, implying that the probability of survival of a newborn is 1, while D defines the maximum age than an individual can attain.[8]

Specifying the mortality function as above enables us to express the discounted expected lifetime utility of a Home agent born at time v, and having an isoelastic function, by

$$E(U(v)) = \int_v^{v+D} \frac{C(v,t)^{1-1/\sigma} - 1}{1-1/\sigma} e^{-\rho(t-v)-M(t-v)} dt \tag{1}$$

where $C(v,t)$ denotes consumption of the traded good, σ is the intertemporal elasticity of substitution, ρ is the pure rate of time discount, while $\rho + \mu(t-v)$ is the overall rate of time discount at age $(t-v)$.[9]

The Home household has direct access to both the Home and Foreign capital markets, the latter enabling it to own Foreign capital. Its holdings of these two assets are $K_d(v,t)$ and $K_d^*(v,t)$, respectively, with $r(t)$ denoting the return on Home capital, and $r^*(t)$ the return on Foreign capital. Individuals are born without assets, have no bequest motive, and do not wish to leave unexpected assets should they reach the maximum attainable age, D. These conditions at the beginning and end of life imply $K_d(v,v) = K_d^*(v,v) = K_d(v, v+D) = K_d^*(v, v+D) = 0$.

Since agents are subject to lifetime uncertainty and have no bequest motives, we follow Yaari (1965) and Blanchard (1985) and assume the existence of an annuities market which enables capital from the dying to be recycled back to survivors in the economy. In this regard, the bulk of the literature adopts the Yaari-Blanchard assumption that the market is actuarially fair, so that the agent receives a premium on the rate of return of their capital equal to their probability of death, $\mu(t-v)$. While this assumption is convenient, it is extreme. Accordingly, we also follow Heijdra and Mierau (2012) and Bruce and Turnovsky (2013) and allow for an imperfect annuities market where the premium is reduced to $\phi\mu(\tau - v)$, where $0 \leqslant \phi \leqslant 1$, with $\phi = 1$ reducing to the traditional case. As Heijdra and Mierau note this can be justified in several ways, one of which is that the annuities firms enjoy monopoly power, yielding profits, enabling them to offer a less than actuarially fair annuity rate. These profits are then redistributed back to the agents as lump-sum transfers, $X(v,t)$. In a two-country framework, there exists a separate annuities market in each country. The agent of one country will hold capital domiciled in both regions, but the insurance company located in that agent's country will reallocate the assets acquired upon his death only among individuals in his country of origin. Thus, no international movement of assets is involved in this process.

The Home household thus chooses its consumption level, and its holdings of Home and Foreign capital to maximize (1) subject to its budget constraint

$$\begin{aligned} &\tfrac{\partial K_d(v,t)}{\partial t} + \tfrac{\partial K_d^*(v,t)}{\partial t} \\ &= (r(t) + \phi\mu(t-v))K_d(v,t) + (r^*(t) + \phi\mu(t-v))K_d^*(v,t) + w(t) + X(v,t) - C(v,t) \end{aligned} \tag{2}$$

where $w(t)$ denotes the Home (real) wage rate. Performing the optimization with respect to $C(v,t),\ K_d(v,t),\ K_d^*(v,t)$ yields:

$$C(v,t)^{-1/\sigma} = \lambda(v,t) \tag{3a}$$

$$\rho - \frac{\lambda_t(v,t)}{\lambda(v,t)} + \mu(t-v) = r(t) + \phi\mu(t-v) \tag{3b}$$

$$\rho - \frac{\lambda_t(v,t)}{\lambda(v,t)} + \mu(t-v) = r^*(t) + \phi\mu(t-v) \tag{3c}$$

where $\lambda(v,t)$ denotes the shadow value of wealth of cohort v at time t. Eqs. (3b) and (3c) we immediately see that with perfect capital mobility the rates of return on capital in the two countries must be equalized, i.e. $r(t) = r^*(t)$. Taking the time derivative of (3a) and combining with (3b) we obtain

$$\frac{C_t(v,t)}{C(v,t)} = \sigma(r(t) - \rho - (1-\phi)\mu(t-v)) \equiv \psi(t,v) \tag{4}$$

where $\psi(t,v)$ denotes the individual's growth rate of consumption. Under the assumption of perfect and competitive annuities markets, $\phi = 1$, each cohort will experience a common consumption growth rate, regardless of age, which adjusts with the endogenously determined international return on capital. In the more general case, the increasing mortality with age will cause each cohort's consumption growth rate to decline with age.

Solving (4) we can obtain the cohort's consumption level at any arbitrary point in time, τ, relative to some prior date, t:

$$C(v,\tau) = C(v,t)e^{\sigma(R(t,\tau) - \rho(\tau - t) - (1-\phi)[M(\tau - v) - M(t-v)])} \tag{5}$$

where $R(t,\tau) \equiv \int_t^\tau r(s)ds$.

[8] We shall treat D as being finite, though the extension to D being infinite (as in Blanchard, 1985) is straightforward.

[9] From the specification of the survival function, the mortality rate and therefore the discount rate increases with age if and only if $SS'' < (S')^2$, which is certainly the case if the survival function is concave.

To express the agent's consumption in terms of his financial resources we integrate the cohort's instantaneous budget constraint, (2), forward from time t and impose the transversality condition. To do this we first define nonhuman wealth of the Home agent of cohort v, at time t, as his total holdings of capital from the two regions:

$$A(v,t) \equiv K_d(v,t) + K_d^*(v,t) \tag{6}$$

thus enabling us to express the transversality conditions more compactly: $A(v,v) = A(v, v+D) = 0$.

Imposing the capital international arbitrage condition, and the transversality condition, yields the cohort's intertemporal budget constraint:

$$\begin{aligned} A(v,t) &+ \int_t^{v+D}[w(\tau) + X(v,\tau)]e^{-R(t,\tau)-\phi[M(\tau-v)-M(t-v)]}d\tau \\ &= \int_t^{v+D}C(v,\tau)e^{-R(t,\tau)-\phi[M(\tau-v)-M(t-v)]}d\tau \end{aligned} \tag{7}$$

which asserts that the present value of the agent's consumption, discounted for both time and survival, equals his current wealth held as capital plus the present discounted value of expected future wages, and lump-sum transfers also discounted for survival. For simplicity we shall assume that the surviving households born at time v receive a lump-sum transfer so that the total income from capital equals the mortality interest premium that they would receive in a perfect annuities market. That is, $X(v,\tau) = (1-\phi)\mu(\tau - v)A(v,\tau)$. This ensures that the financial wealth of the dying is fully cycled among the survivors.[10] Substituting for $X(v,\tau)$, the intertemporal budget constraint can be written as

$$A(v,t) + \int_t^{v+D} w(\tau)e^{-R(t,\tau)-[M(\tau-v)-M(t-v)]}d\tau \ = \int_t^{v+D} C(v,\tau)e^{-R(t,\tau)-[M(\tau-v)-M(t-v)]}d\tau \tag{7'}$$

Substituting the individual's consumption path, (5), into the intertemporal budget constraint, (7), and solving for consumption, yields the following expression for the consumption of an agent born at time v, at calendar time $t, C(v,t)$

$$C(v,t) = \frac{A(v,t) + H(v,t)}{\Delta(v,t)} \tag{8}$$

where:

$$H(v,t) \equiv \int_t^{v+D} w(\tau)e^{-R(t,\tau)-M(\tau-v)+M(t-v)}d\tau \tag{9a}$$

denotes the discounted future labor income (human wealth) at time t for cohort of vintage v, and

$$\Delta(v,t) \equiv \int_t^{v+D} e^{(\sigma-1)R(t,\tau)-\sigma\rho(\tau-t)+[\sigma(\phi-1)-1][M(\tau-v)-M(t-v)]}d\tau \tag{9b}$$

is the inverse of the marginal propensity to consume out of total wealth (capital pus human wealth).

As will become evident, the aggregate dynamics can be fully characterized by considering aggregate per capita quantities together with just one cohort, for which purpose the newborns are most convenient. Thus, setting $v = t$, in (8) and (9), with no bequests, $A(t,t) = 0$, and the consumption of newborns is $C(t,t) = H(t,t)/\Delta(t,t)$.

2.2. Aggregate demography

At each instant, a cohort of size $P(v,v) = \varphi P(v)$ is born, where $P(v,v)$ is the size of the cohort, $P(v)$ is the size of the total population at time v, and φ is the (crude) birth rate, as measured by the average number of births per population size, and taken as constant.[11] The number of individuals of cohort v alive at time t is $P(v,t) = \varphi P(v)e^{-M(t-v)}$. Summing over all surviving cohort members, the population at time t is $P(t) = \varphi \int_{t-D}^t P(v)e^{-M(t-v)}dv$. Alternatively, knowing $P(v)$, the population alive at time v, the population at time t is equal to $P(t) = P(v)e^{n(t-v)}$, where n is the population growth rate, assumed to be constant. Equating the two expressions for $P(t)$ leads to the following relationship linking the mortality rate, the birth rate, and the population growth rate:

$$\frac{1}{\varphi} = \int_{t-D}^t e^{-n(t-v)-M(t-v)}dv. \tag{10}$$

Under our assumptions (i) φ and n are constants and (ii) mortality depends only on age and is independent of calendar time, we can eliminate t from (10), and rewrite it in the form:

[10] Although households receive at each age the same transfer as they would if the annuities market were perfect, a fraction of the transfers are lump-sum and do not depend on the individual households' savings behavior. That is, savings decisions are made on the basis of $r - (1-\phi)\mu$, rather than r in the perfect annuities case.

[11] It is straightforward to modify the specification of the birth rate to allow the child-bearing population to be some subset of the overall population.

$$\frac{1}{\varphi} = \int_0^D e^{-ns-M(s)} ds \tag{10'}$$

where s indexes age; (10′) is commonly referred to as the demographic steady state (Lotka, 1998, p.60). Dividing $P(v,t)$ by $P(t)$, the relative size of each cohort is:

$$p(t-v) \equiv \frac{P(v,t)}{P(t)} = \varphi e^{-n(t-v)-M(t-v)} = \varphi e^{-ns-M(s)}, \tag{11}$$

which in the demographic steady state also depends only on age $s \equiv t - v$, but not on calendar time t. The dynamics of (11) are given by

$$\frac{p_t(t-v)}{p(t-v)} = -[n + \mu(t-v)] \tag{12}$$

so that the decline in the relative size of each cohort over time reflects both its mortality rate and the overall population growth rate.

To obtain aggregate per capita values, we sum across cohorts by employing the following generic aggregator function

$$x(t) \equiv \int_{t-D}^{t} p(t-v)X(v,t)dv = \varphi \int_{t-D}^{t} e^{-n(t-v)-M(t-v)} X(v,t)dv \tag{13}$$

Using this function, aggregate per capita consumption is:

$$c(t) \equiv \int_{t-D}^{t} p(t-v)C(v,t)dv = \varphi \int_{t-D}^{t} e^{-n(t-v)-M(t-v)} C(v,t)dv. \tag{14}$$

Taking the time derivative of this expression and using (5) and (12), the dynamics of aggregate per capita consumption can be expressed in the form:

$$\dot{c}(t) \equiv \varphi C(t,t) + \left[\sigma(r(t)-\rho) - [1+\sigma(1-\phi)]\mu_c(t-v_1) - n\right]c(t) \tag{15}$$

where we define:[12]

$$\mu_c(t-v_1) \equiv \frac{\int_{t-D}^{t} \mu(t-v)p(t-v)C(v,t)dv}{\int_{t-D}^{t} p(t-v)C(v,t)dv} = \frac{1}{c(t)} \int_{t-D}^{t} \mu(t-v)p(t-v)C(v,t)dv \quad v_1 \in (t-D,t) \tag{16}$$

From (16) we see that $\mu_c(t-v_1)$ is the ratio of the consumption given up by the dying to aggregate consumption and can be interpreted as providing an estimate of average mortality over the period $(t-D,t)$, from the consumption profile across the cohorts.[13] For the general mortality function, $\mu_c(t-v_1)$ varies with time, although in the special case of the Blanchard (1985) survival function it is time-invariant.

Applying (13) to the cohort wealth, the aggregate per capita wealth in the Home economy is $a(t) \equiv \int_{t-D}^{t} p(t-v)A(v,t)dv$. Taking the time derivative and using (2b) and (12) together with the fact that $A(t,t) = A(t,t+D) = 0$, yields

$$\dot{a}(t) = [r(t)-n]a(t) + w(t) - c(t) \tag{17}$$

2.3. Foreign household equilibrium

The situation confronting the Foreign household is symmetric. It optimizes a constant elasticity utility function analogous to (1) subject to a budget constraint analogous to (2), where $K_f(v,t)$, $K_f^*(v,t)$ denote the Foreign individual's holdings of Home and Foreign capital. The corresponding equilibrium conditions facing the individual cohort member are:

$$C^*(v,t) = \frac{A^*(v,t) + H^*(v,t)}{\Delta^*(v,t)} \tag{8'}$$

$$H^*(v,t) \equiv \int_t^{v+D^*} w^*(\tau) e^{-R(t,\tau)-M^*(\tau-v)+M^*(t-v)} d\tau \tag{9a'}$$

$$\Delta^*(v,t) \equiv \int_t^{v+D^*} e^{(\sigma^*-1)R(t,\tau)-\sigma^*\rho^*(\tau-t)+[\sigma^*(\phi^*-1)-1][M^*(\tau-v)-M^*(t-v)]} d\tau \tag{9b'}$$

[12] Eq. (16) is a statement of the first mean value theorem of integration. We should note that the intermediate value $v_1 \in (t-D,t)$ will in general be a function of t, which case $\mu_c(t-v_1)$ should be written as $\mu_c(t-v_1(t))$. We refrain from representing this explicitly, so as not to clutter notation.

[13] The quantity $[\mu_c(t-v_1)+n]c(t) - \beta C(t,t)$ reflects the reduction in aggregate per capita consumption growth, below that of each cohort due to the arrival of newborn agents with no accumulated assets and the departure due to death of agents with assets. For this reason Heijdra and van der Ploeg (2002) initially identify this as the "generational turnover term"; see also Heijdra and Romp (2008) and Mierau and Turnovsky (2014a).

Aggregating (8′) over the foreign cohorts leads to:

$$c^*(t) \equiv \int_{t-D^*}^{t} p^*(t-v)C^*(v,t)dv = \varphi^* \int_{t-D^*}^{t} e^{-n(t-v)-M^*(t-v)} C^*(v,t)dv. \tag{14'}$$

together with the dynamics of aggregate per capita consumption and wealth:

$$\dot{c}^*(t) \equiv \varphi^* C^*(t,t) + \left[\sigma^*(r(t)-\rho^*) - [1+\sigma^*(1-\phi^*)]\mu_c^*(t-v_1) - n\right]c^*(t) \tag{15'}$$

$$\dot{a}^*(t) = [r(t)-n]a^*(t) + w^*(t) - c^*(t) \tag{17'}$$

2.4. Firms

Home output is produced by a single representative firm using labor and capital in accordance with a usual neoclassical production function having the usual properties of positive, but diminishing marginal physical product and constant returns to scale:

$$Y(t) = F[K(t), L(t)] \tag{18}$$

With labor being fully employed, and with no migration, $L(t)$ in production coincides with $P(t)$ generated by the demographic structure. The quantity $K(t)$ denotes the *total* capital stock domiciled in the Home country and is owned partially by Home residents and partially by Foreign residents, so that:

$$K(t) = L(t)k_d(t) + L^*(t)k_f(t) \tag{19}$$

where $k_d(t) = \varphi\int_{t-D}^{t} e^{-n(t-v)-M(t-v)}K_d(v,t)dv,\ k_f(t) = \varphi^*\int_{t-D^*}^{t} e^{-n(t-v)-M^*(t-v)}K_f(v,t)dv$ denotes per capita holdings of the domestic capital by Home and Foreign residents, respectively. Using the assumption that $L^*(t) = \theta L(t)$, the capital stock domiciled at Home, per capita of the Home population is

$$k(t) \equiv \frac{K(t)}{L(t)} = k_d(t) + \theta k_f(t) \tag{20a}$$

and per capita output in the Home economy is

$$y(t) \equiv \frac{Y(t)}{L(t)} = F(k,1) \equiv f(k) \tag{20b}$$

With no impediments to factor usage, the optimality conditions for the Home firm are the usual marginal product conditions:

$$F_K(K,L) - \delta = f'(k(t)) - \delta = r(t) \tag{21a}$$

$$F_L(K(t),L(t)) = f(k(t)) - f'(k(t))k(t) = w(t) \tag{21b}$$

where capital depreciates at the rate δ and the wage is determined so as equate the demand for labor to its inelastic supply.

The Foreign firm is symmetric. The total capital stock in Foreign is

$$K^*(t) = L(t)k_d^*(t) + L^*(t)k_f^*(t) \tag{19'}$$

so that the capital stock domiciled in Foreign, per capita of the Foreign population is

$$k^*(t) \equiv \frac{K^*(t)}{L^*(t)} = \frac{k_d^*(t)}{\theta} + k_f^*(t) \tag{20a'}$$

and per capita output in the Foreign economy is

$$y^*(t) \equiv \frac{Y^*(t)}{L^*(t)} = F^*(k^*,1) \equiv f^*(k^*) \tag{20b'}$$

and the usual marginal product conditions:

$$F_K^*(K^*(t),L^*(t)) - \delta = f^{*\prime}(k^*(t)) - \delta = r(t) \tag{21a'}$$

$$F_L^*(K^*(t),L^*(t)) = f^*(k^*(t)) - f^{*\prime}(k^*(t))k^*(t) = w^*(t) \tag{21b'}$$

2.5. World wealth, world goods market equilibrium, and the net foreign asset position

From Eqs. (19) and (19′), the aggregate capital stock in the world economy constitutes aggregate world wealth:

$$K(t) + K^*(t) = L(t)\left[k_d(t) + k_d^*(t)\right] + L^*(t)\left[k_f(t) + k_f^*(t)\right] = A(t) + A^*(t) = L(t)a(t) + L^*(t)a^*(t)$$

Taking the time derivative of this equation and using (17) and (17′) yields:

$$\dot{K}(t) + \dot{K}^{*}(t) = r(t)[A(t) + A^{*}(t)] + w(t)L(t) + w^{*}(t)L^{*}(t) - c(t)L(t) - c^{*}(t)L^{*}(t)$$

which substituting for factor returns reduces to the aggregate product market equilibrium condition

$$\dot{K}(t) + \dot{K}^{*}(t) = F[K(t), L(t)] + F^{*}[K^{*}(t), L^{*}(t)] - \delta[K(t) + K^{*}(t)] - C(t) - C^{*}(t) \tag{22}$$

where $C(t), C^{*}(t)$ denote aggregate Home and Foreign consumption. In per capita terms this reduces to

$$\dot{k}(t) + \theta\dot{k}^{*}(t) = f[k(t)] + \theta f^{*}[k^{*}(t)] - (n+\delta)[k(t) + \theta k^{*}(t)] - [c(t) + \theta c^{*}(t)] \tag{23}$$

The aggregate net foreign asset position of the domestic economy, $Z(t)$, is defined by

$$Z(t) = L(t)k_d^{*} - L^{*}(t)k_f$$

Thus total Home and Foreign wealth can be expressed as $A(t) = K(t) + Z(t)$, $A^{*}(t) = K^{*}(t) - Z(t)$, which in per capita terms become:

$$z^{*}(t) = a^{*}(t) - k^{*}(t) = -\frac{z(t)}{\theta} = -\frac{1}{\theta}(a(t) - k(t)) \tag{24}$$

Thus the evolution of Home's current account in per capita terms follows

$$\dot{z}(t) = \dot{a}(t) - \dot{k}(t) \tag{25}$$

3. Aggregate equilibrium

With the introduction of a general demographic structure, we see from (15) and (15′) that the consumption of newborns in both countries plays a critical role in determining the dynamics of the aggregate consumption and need to be taken into account. In the Appendix we show how the core equilibrium dynamics of the world economy can be expressed by an autonomous system comprising the following variables pertaining to Home and Foreign: (i) per capita consumptions $[c(t), c^{*}(t)]$; (ii) human wealth of newborns $[H(t), H^{*}(t)]$; (iii) inverse of the marginal propensity to consume of newborns $[\Delta(t), \Delta^{*}(t)]$; (iv) per capita wealth $[w(t), w^{*}(t)]$; (v) per capita stock of capital in the Home economy $[k(t)]$. Once these are determined the per capita capital stock in the Foreign economy, as well as the net foreign asset position of the two economies, can be determined. The system set out in the Appendix, which forms the basis for our subsequent simulations, assumes that output in both economies is produced by Cobb-Douglas functions with identical exponents on capital, namely

$$y = Ak^{\alpha}, \; y^{*} = A^{*}k^{*\alpha} \tag{26}$$

It is important to stress that the dynamics described in the Appendix are functions of μ_c, μ_H, and μ_Δ which are defined by the integrals (16), (A.4), and (A.7), and are therefore in general functions of time (and analogously expressions for the Foreign country).[14] However, as Mierau and Turnovsky (2014b) show, given the assumption of the demographic steady state, they in fact vary only slightly over time. Their contribution to the dynamics is of second order and for practical purposes these terms can be treated as constants. Moreover, being estimates of mortality rates they are uniformly small, and can be approximated by their respective steady-state levels, as derived from (30) below. Accordingly, we approximate the equilibrium dynamics, summarized in the Appendix, by assuming that μ_c, μ_H, and μ_Δ remain constant at these values (see Fig. 1).

The linearized system described by (A.10) that we employ to analyze the local dynamics is a 9th order system.[15] We assume that Home and Foreign wealth, $a(t),\ a^{*}(t)$ are accumulated gradually, while all the remaining variables can all adjust instantaneously to any exogenous shock. This includes capital domiciled in either of the countries, which in the absence of installation costs can be obtained instantaneously from abroad. Under these assumptions the dynamic system will have a unique bounded stable transitional path if and only if there are two negative and seven positive eigenvalues, in which case the local stable manifold will be two-dimensional. To establish formal conditions that ensure this required configuration of eigenvalues is impractical, although our numerical simulations indicate that the underlying dynamics does indeed have this required formal structure.[16]

[14] In general, the global dynamics of an OLG model having a realistic demographic structure is represented by a high order transcendental equation and is intractable with a neoclassical production function, such as we are employing here; see d'Albis and Augeraud-Véron (2009). In response to this, Mierau and Turnovsky propose a linear approximation which enables one to represent the aggregate dynamics locally, as we do here. The details of this procedure are spelled out in Mierau and Turnovsky (2014b, pp. 872-874).

[15] In the cases where the shocks generating the transition occur gradually, the order of the dynamics is increased further.

[16] We should also acknowledge that while we take the demographic structure to be given there is an extensive literature that endogenizes this as part of the economic decision. For a range of examples see Becker (1981), Becker and Barro (1988), and Manuelli and Seshadri (2009). While the interdependence between the economic and demographic structures is important, it is tangential to the main issue being addressed in this paper.

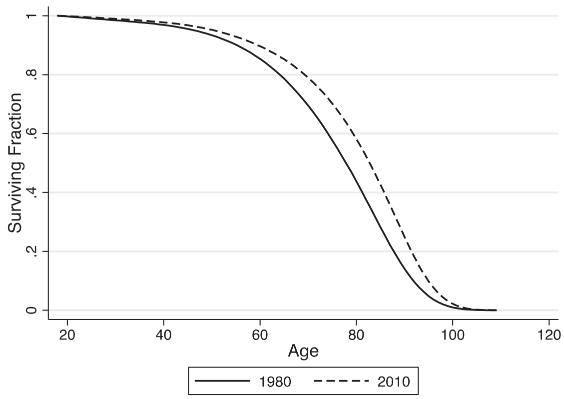

Fig. 1. US observed survival data 1980–2010. Data retrieved from www.mortality.org.

4. Steady state

In steady state, all quantities depend only upon age, $s \equiv t - v$, and not on calendar time. With no long-run per capita growth, per capita consumption, output, capital and its return, and the wage rate all remain constant over time. Hence, recalling (4) each cohort's consumption in Home grows at the rate $\tilde{\psi}(s) \equiv \sigma(\tilde{r} - \rho) - (1 - \phi)\tilde{\mu}(s)$, while in Foreign it grows at $\tilde{\psi}^*(s) \equiv \sigma^*(\tilde{r} - \rho^*) - (1 - \phi^*)\tilde{\mu}^*(s)$, where tildes denote steady states. As long as the annuities market is imperfect, the consumption growth rate declines with age. Thus, the steady state consumption levels of a cohort of age s in the two economies are respectively:

$$\tilde{C}(s) = \tilde{C}_0 e^{\sigma[(\tilde{r}-\rho)s-(1-\phi)M(s)]}; \tag{27}$$

where steady-state consumption at birth, $\tilde{C}_0,\ \tilde{C}_0^*$, are given by

$$\tilde{C}_0 = \frac{\tilde{H}}{\tilde{\Delta}}; \quad \tilde{C}_0^* = \frac{\tilde{H}^*}{\tilde{\Delta}^*} \tag{28a}$$

and

$$\tilde{H} = \tilde{w}\int_0^D e^{-\tilde{r}s-M(s)}ds; \quad \tilde{H}^* = \tilde{w}^*\int_0^{D^*} e^{-\tilde{r}s-M^*(s)}ds \tag{28b}$$

$$\tilde{\Delta} = \int_0^D e^{[(\sigma-1)\tilde{r}-\sigma\rho]s-[1+\sigma(1-\phi)]M(s)}ds; \quad \tilde{\Delta}^* = \int_0^{D^*} e^{[(\sigma^*-1)\tilde{r}-\sigma^*\rho^*]s-[1+\sigma^*(1-\phi^*)]M^*(s)}ds \tag{28c}$$

$$\tilde{c} = \varphi\tilde{C}_0\int_0^D e^{[\sigma(\tilde{r}-\rho)-n)s-[1+\sigma(1-\phi)]M(s)}ds; \quad \tilde{c}^* = \varphi^*\tilde{C}_0^*\int_0^{D^*} e^{[\sigma^*(\tilde{r}-\rho^*)-n)s-[1+\sigma^*(1-\phi^*)]M^*(s)}ds \tag{28d}$$

$$\frac{1}{\varphi} = \int_0^D e^{-ns-M(s)}ds; \quad \frac{1}{\varphi^*} = \int_0^{D^*} e^{-ns-M^*(s)}ds \tag{28e}$$

$$\tilde{y} = A\tilde{k}^{\alpha}; \quad \tilde{y}^* = A^*\tilde{k}^{*\alpha} \tag{28f}$$

$$\tilde{r} = \alpha A\tilde{k}^{\alpha-1} - \delta = \alpha A^*\tilde{k}^{*(\alpha-1)} - \delta \tag{28g}$$

$$\tilde{w} = A(1-\alpha)\tilde{k}^{\alpha}; \quad \tilde{w}^* = A^*(1-\alpha)\tilde{k}^{*\alpha} \tag{28h}$$

$$\tilde{c} = (\tilde{r} - n)\tilde{a} + \tilde{w}; \quad \tilde{c}^* = (\tilde{r} - n)\tilde{a}^* + \tilde{w}^* \tag{28i}$$

$$A\tilde{k}^{\alpha} - (n+\delta)\tilde{k} = \chi[\tilde{c} + \theta\tilde{c}^*] \tag{28j}$$

$$\tilde{z} = \tilde{a} - \tilde{k}; \quad \tilde{z}^* = -\frac{\tilde{z}}{\theta} \tag{28k}$$

where $\chi \equiv \left[1 + \theta(A/A^*)^{\frac{1}{\alpha-1}}\right]^{-1}$

Eq. (28a) follow from (8) and (8′), while (28b)–(28k) follow from (9a) and (9a'), (9b) and (9b'), (14) and (14′), (10′), (26), (21a) and (21a'), (21b) and (21b'), (17), (17′), (A.9), and (24), respectively. Several features of the long-run equilibrium merit mention. First, the steady state has the characteristic that although the consumption growth rates across cohorts reflects the difference between the rate of return on capital and the rate of time preference adjusted for mortality, and therefore will likely differ between the two economies, the aggregate long-run economy-wide growth rates are both equal to the common population growth rate, consistent with the Ramsey technology. Second, steady state is consistent with both countries having finite capital stocks and wealth, despite having different rates of time preference, ρ, ρ^*. This contrasts sharply with the representative agent economy in which the asymptotic distribution of wealth is a corner solution in which all the capital stock is held by the more patient economy having the lower rate of time preference; see Becker (1980). Third, although in order to ensure a steady state in which both economies have finite sizes constrains them to have the same secular growth rate, nevertheless $\theta \equiv L/L^*$ parameterizes population size differentials due some temporary divergence in population growth rates or possibly temporary immigration policy.

Finally, (27) and (28) highlight the structural determinants of the long-run equilibrium. Specifically, the international allocation of capital, relative national wage rates, and relative outputs are determined by relative productivity,

$$\frac{\tilde{k}}{\tilde{k}^*} = \frac{\tilde{w}}{\tilde{w}^*} = \frac{\tilde{y}}{\tilde{y}^*} = \left(\frac{A}{A^*}\right)^{\frac{1}{1-\alpha}} \tag{29}$$

and are independent of the economies' respective demographic conditions and characteristics of the utility function. These impact the long-run equilibrium and the allocation of wealth through savings and consumption. In the extreme case where both countries are subject to the same demographic structure ($e^{-M^*(u)} = e^{-M(u)}$), same preferences ($\rho = \rho^*, \sigma = \sigma^*$), and analogous annuities market ($\phi = \phi^*$), (29) extends to

$$\frac{\tilde{k}}{\tilde{k}^*} = \frac{\tilde{w}}{\tilde{w}^*} = \frac{\tilde{y}}{\tilde{y}^*} = \left(\frac{A}{A^*}\right)^{\frac{1}{1-\alpha}} = \frac{\tilde{H}}{\tilde{H}^*} = \frac{\tilde{c}}{\tilde{c}^*} = \frac{\tilde{a}}{\tilde{a}^*} \tag{29'}$$

implying further that $\tilde{z} = \tilde{z}^* = 0$, so that the long-run net asset positions of both countries are zero. Otherwise, the country with the lower survival rate and/or the higher rate of time preference will in general have a negative long-run net asset position ($\tilde{z} < 0$); cf Ghironi et al. (2008). To see why this is so, suppose that the domestic economy has a lower survival rate. In that case Home has less incentive to save and the proportion of average wealth that is consumed will exceed that in Foreign, so that $\tilde{c}/\tilde{a} > \tilde{c}^*/\tilde{a}^*$. It then follows from (28i) that $\tilde{a}/\tilde{a}^* < \tilde{k}/\tilde{k}^* = (A/A^*)^{(\alpha-1)^{-1}}$, so that the relative wealth of Home to Foreign is less than the ratio of the relative capital stocks domiciled in their respective economies. Since the capital stocks in the two economies comprise the total world wealth ($a + \theta a^* = k + \theta k^*$) it immediately follows that $\tilde{a} < \tilde{k}$ $\left(\tilde{a}^* > \tilde{k}^*\right)$ so that some of the capital in Home is owned by Foreign, so that Home has a negative long-run net asset position ($\tilde{z} < 0\,, \tilde{z}^* > 0$).

A similar argument applies to the case where Home has a higher rate of time preference than does Foreign, $\rho > \rho^*$. However, if Home has a higher elasticity of substitution, $\sigma > \sigma^*$, we can show that for the BCL survival function (assumed in the simulations) that $sgn(\tilde{c}/\tilde{c}^* - \tilde{w}/\tilde{w}^*) = sgn(\sigma - \sigma^*)$. Thus if $\sigma > \sigma^*$, (28i) yields $\tilde{a}/\tilde{a}^* > \tilde{w}/\tilde{w}^* = \tilde{k}/\tilde{k}^*$, which now implies that Home has a positive long-run net asset position ($\tilde{z} > 0\,, \tilde{z}^* < 0$). Intuitively, in order to sustain a higher per capita consumption relative to the ratio of the respective national wage rates Home must own some of Foreign capital.

Having determined the equilibrium quantities in (27) and (28), the corresponding steady-state values of the mortality rates, $\mu_c, \mu_H,$ and μ_Δ are obtained from the steady-state relationships corresponding to the dynamic Eqs. (25) and are respectively[17]

$$\tilde{\mu}_c = \frac{1}{1+\sigma(1-\phi)]}\left\{\varphi\frac{\tilde{H}}{\tilde{\Delta}\tilde{c}} + \sigma(\tilde{r}-\rho) - n\right\}; \quad \tilde{\mu}_c^* = \frac{1}{1+\sigma^*(1-\phi^*)]}\left\{\varphi^*\frac{\tilde{H}^*}{\tilde{\Delta}^*\tilde{c}^*} + \sigma^*(\tilde{r}-\rho^*) - n\right\} \tag{30a}$$

$$\tilde{\mu}_H = \frac{\tilde{w}}{\tilde{H}} - \tilde{r}; \quad \tilde{\mu}_H^* = \frac{\tilde{w}^*}{\tilde{H}^*} - \tilde{r} \tag{30b}$$

$$\tilde{\mu}_\Delta = \frac{1}{1+\sigma(1-\phi)}\left\{\frac{1}{\tilde{\Delta}} - (1-\sigma)\tilde{r} - \sigma\rho\right\}; \quad \tilde{\mu}_\Delta = \frac{1}{1+\sigma^*(1-\phi^*)}\left\{\frac{1}{\tilde{\Delta}^*} - (1-\sigma^*)\tilde{r} - \sigma^*\rho^*\right\} \tag{30c}$$

5. Baseline simulation

To determine the impact of the international demographic trends on the international capital flows, we investigate the transitional dynamics using numerical simulations around the steady state summarized in Section 4. As noted, we view

[17] For the baseline simulations summarized in Section 5 below we obtain: $\tilde{\mu}_c = 0.020$, $\tilde{\mu}_H = 0.003$, $\mu_\Delta = 0.004$, $\tilde{\mu}_c^* = 0.019$, $\tilde{\mu}_H^* = 0.002$, $\tilde{\mu}_\Delta^* = 0.003$, which are very small and their variation over time, which is also small, contributes negligibly to the equilibrium dynamics.

the United States as the Home country and represent a population-weighted average of five of its key trading partners (UK, Germany, France, Japan, Canada) as the Foreign economy.

Output is based on the Cobb-Douglas production functions (28f), with a common productive elasticity of capital set at the conventional value $\alpha = 0.36$. Based on empirical evidence summarized and updated by Islam (1999) we set the benchmark TFP parameters at their relative 1980 values, $A = 1,\ A^* = 0.90$. To consider the impact of productivity changes we allow for increases in these two parameters of 30% and 15%, respectively, which approximate the relative increases in TFP of the US and the weighted average of its five key trading partners between 1980 and 2010 [see Table 3].

Preferences are based on the parameterized version of the utility function (1), which includes the two key parameters, ρ, σ, both of which are crucial determinants of savings and therefore of international capital flows. With the calibration being based on a time unit of one year, and with capital being perfectly mobile internationally, natural benchmark values of the pure rates of time preference are to set $\rho = \rho^* = 0.04$. However, as noted, relative differences between preference rates will generate a nonzero NFA position due to the international disparity in the valuation of current versus future consumption.[18] Empirical cross-country evidence on rates of time preference is sparse, one exception being Wang et al. (2016), who find significant variation across countries.[19] In particular their study suggests that the US is somewhat less patient than other countries, $\rho > \rho^*$, causing us to also consider $\rho^* = 0.037$, as well as briefly to set $\rho = 0.037$, as part of the sensitivity analysis (see Table 4).

Cross-country estimates of the intertemporal elasticity of substitution vary significantly across individual countries, as reported in the meta-study of Havranek et al. (2015), who report 2635 estimates covering 104 countries. With few exceptions, these are well below unity, consistent with the evidence provided by Guvenen (2006) for the US. In light of the variations in the estimates, as reflected in the reported standard errors, a reasonable benchmark is to assume equality of σ for both US and G5, setting $\sigma = \sigma^* = 0.4$. However, since the empirical estimates vary so extensively, including across the G5 economies, and in fact suggest that the likelihood that $\sigma > \sigma^*$, we also briefly consider $\sigma = 0.6$, as well as $\sigma^* = 0.6$, in the course of the sensitivity analysis.

The third, and most critical element determining preferences, is the demographic structure, and for this purpose we employ the parametric survival function proposed by Boucekkine, de la Croix, and Licandro (2002) (BCL):

$$S(t-v) \equiv e^{-M(t-v)} = \frac{\mu_0 - e^{\mu_1 (t-v)}}{\mu_0 - 1} \quad 0 \leqslant t - v \leqslant D, \quad \mu_0 > 1,\ \mu_1 > 0 \tag{31}$$

Survival is characterized by the two parameters, μ_0, μ_1, where the former impacts primarily the younger phases of the life cycle ("youth mortality") and the latter older stages ("old age" mortality). The maximum attainable age, $D = \ln\mu_0/\mu_1$ and is reached when $S(t-v) = 0$. Eq. (31) is a highly tractable and accurate survival function specification. Unlike the widely adopted perpetual youth specification of Blanchard (1985), this function produces an age-varying survival probability that realistically increases with age and therefore captures more adequately life-cycle aspects of consumption and savings behavior.

The BCL survival function parameters, μ_0 and μ_1, have been estimated by nonlinear least squares using age-survival data from the Human Mortality Database for the included countries for the two years 1980 and 2010.[20] The estimated function tracks observed mortality data remarkably well for most Western nations as it does here. The function produces an accurate approximation of the survival data resulting in an adjusted R^2 value of 0.99. As the childhood period is excluded, the estimation is performed for the age interval from 18 to 90.

Fig. 2 exhibits the 2010 age survival data for the US and the estimated parameterized BCL survival function for that year. The figure confirms how the estimated survival function produces an excellent fit for the major part of the individual's life cycle, except for the tail of the distribution. Given that individuals over 90 years old constitute a small percentage of the population and are likely relatively inactive in the economy, we do not view this as a serious shortcoming.

In order to parameterize the survival function for the G5 economies, we estimate the mortality parameters for each trading partner. We then create an overall weighted average utilizing weights associated with the population of each country for the specified age interval. Our estimated BCL functions implies a maximum attainable ages of 91.1 and 91.6 in 1980 for the US and G5, with corresponding life expectancies at age 18 of 74.1 and 76.4, respectively. In both cases we assume an exogenous population growth rate of 1% which, given the survival function, implies birth rates of 1.93% and 1.6%, respectively. By 2010 the life expectancies have increased to 78.6 and 81.5 and the implied birth rates declined to 1.73% and 1.37%, respectively.

It is important to note once again that in order to yield a stable per-capita equilibrium, long-run population growth rates across the two economies must be assumed to be equal. In Table 2 we summarize the relative sizes of the relevant populations of the US and the individual G5 economies over the period 1980–2010. Comparing these, we find that over this period US population grew at the average annual rate of 1.2% while the average of its 5 trading partners was 0.9%. Accordingly, we

[18] See e.g. Buiter (1981) and Ghironi et al. (2008).

[19] Wang et al. (2016) test this by surveying economics students from 53 countries, questioning whether the student would prefer to receive a smaller payment immediately or a larger payment in the future. With the exception of France, which was not included in their sample, approximately 10% more of the students from the remaining four countries included in G5 (Canada, France, Germany, Japan, and the UK) would prefer to wait than would students from the US. We interpret this as being approximately 10% more patient, equivalent to 0.3 percentage point reduction in the rate of time preference from 4% to 3.7%.

[20] Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research, Rostock (Germany). Available at www.mortality.org.

Table 3
Relative productivity.

	TFP	
	1980	2010
Canada	1.00	1.025
France	1.00	1.236
Germany	1.00	1.384
Japan	1.00	0.969
UK	1.00	1.341
G5	1.00	1.173
US	1.00	1.343

Data from FRED, St Louis Fed.

Table 4
Parameter and demographic values.

Parameters	Value	
Constant	US	G5
Time preference rate, ρ	0.04, (0.037)	0.04, (0.037)
Intertemporal Elasticity of Substitution, σ	0.4 (0.6)	
Productive elasticity of capital, α	0.36	
Population growth rate, n	0.01	
Adjustment speeds, a	0.10	
Initial Specification (1980 Estimate):	US	G5
Life expectancy at age 18, L_{18}	74.1	76.4
Implied maximum age, D	91.1	91.6
Youth mortality, μ_0	65.64	118.01
Old age mortality, μ_1	0.05724	0.06482
Birth Rate (Implied), φ	1.93 %	1.6 %
TFP,A	1.0	0.9
Final Specification (2010 Estimate):	USA	Region
Life expectancy at age 18, L_{18}	78.6	81.5
Implied maximum age, D	93.8	94.6
Youth mortality, μ_0	139.86	326.16
Old age mortality, μ_1	0.06518	0.07595
Birth Rate (Implied), φ	1.73 %	1.37 %
TFP, A	1.15, 1.30	1.035

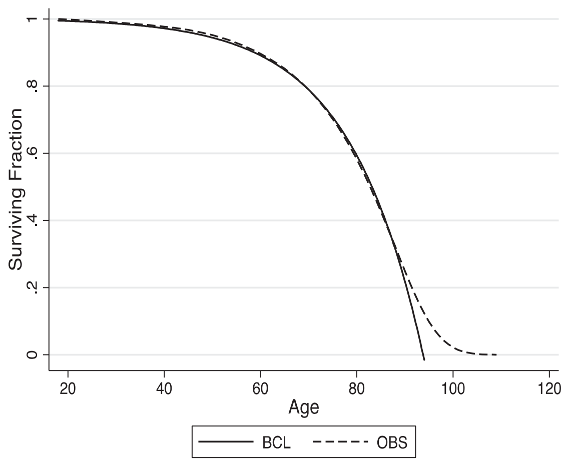

Fig. 2. Estimated BCL function and 2010 US survival data. Data retrieved form: www.mortality.org.

approximate the common secular population growth rate by 1%, and attribute the deviations to country-specific stochastic factors. From the table we see that the relative size of the US population to that of the G5 economies increased from 0.42 to 0.44, implying that the ratio θ declined from 1.38 to 1.27 over that 30 year period.

6. Long-run responses

Table 5a reports the long-run equilibria corresponding to a range of scenarios in the benchmark case, characterized by symmetric preferences, where both countries have the same rates of time preference, and intertemporal elasticity of substitution, $(\rho = \rho^* = 0.04,\ \sigma = \sigma^* = 0.4)$, and perfect annuities markets $(\phi = \phi^* = 1)$. The second panel corresponds to the benchmark parameterization of the remaining parameters. There we see that this benchmark case implies an equilibrium capital output ratio of 2.64 in the two economies and a world-wide rate of return on capital of 7.64%. With the G5 economies on average enjoying a longer lifespan we see that $\tilde{a}/\tilde{a}^* = 1.138 < 1.179 = \tilde{k}/\tilde{k}^*$, implying a negative NFA position for Home, consistent with the steady-state allocations summarized in Section 4.

The first panel in Table 5a reports the long-run wealth/capital allocations in the counter-factual case where Foreign had the same demographic structure as Home. In that case, the international allocation of wealth would be identical to that of capital and the net asset position of both countries would be zero. Comparing the first two panels, our calibration suggests that the longer life expectancy of around 2.3 years in the G5 relative to the US accounts for the US having a negative NFA position of around 5% of its GDP. A further point of some interest is that while the longer life expectancy in G5 increases the capital stock proportionately in both countries, and therefore total world wealth by the same proportion, it has a disproportionately larger effect on wealth in G5, causing wealth in the US (and therefore consumption) to actually decline.

Panel 3 summarizes the combined effects of the decreases in mortality in US and G5 over the period 1980 to 2010. The decline in mortality has no effect on the long-run international location of capital, which depends upon relative TFP. The capital stock in both countries increases by 3%, leading to increases in output of 1.1%. But it slightly favors G5 in terms of relative wealth accumulation so that the NFA/GDP ratio of the US declines by a further 0.30 percentage points over the 30 year period.

Panel 4 reports a 15% increase in US TFP, with the demographic structure remaining unchanged at the 1980 level. This immediately raises US output and increases the desirability of investing in the US. In the long run, total world capital stock and wealth increases by 11.9%. Capital migrates to the US increasing its share from 46.07% to 51.52%. A small component of this comprises capital initially domiciled in G5, so that k^* declines slightly, leading to a negligible decline in foreign output. The NFA of the US therefore declines from -0.0867 to -0.0971, although with the increase in domestic output, the decline in NFA/y is mitigated somewhat.

A uniform increase in world TFP of 15% is reported in panel 5. Total world output and capital increase by 24.4%, with their allocation, as well as that of wealth allocation, across the two economies remaining unchanged from that of the benchmark allocation. With the more favorable demographic structure in G5, the US NFA declines further, although NFA/y remains unchanged from its benchmark level.

The final panel combines all the major changes between 1980 and 2010. These include (i) the demographic changes, (ii) increases in US and G5 TFP of 30% and 15%, respectively, and (iii) a decrease in the relative size of G5 from 1.38 to 1.27. The higher rate of technological change in the US increases the allocation of capital and wealth in its favor. This partially offsets the decline it its NFA due to its less favorable demographic shift so that overall decline in NFA/y is reduced to 4.65%.

Table 5a
$\rho = 0.04 = \rho^*$ (perfect annuities).

	GDP	*Capital*	*Cap-Output*	*cons*	*wealth*	$\frac{y}{y+\theta y^*}$	$a+\theta a^*$	$\frac{a}{a+\theta a^*}$	*NFA*	*NFA/y*	***r***
Equal mortality (US)	***y***	***k***	***k/y***	***c***	***a***	46.07%	9.824	46.07%	0	0	7.70%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.722	4.526	2.628	1.405	4.526						
$\mu_0^* = 65.6, \mu_1^* = 0.05724$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.461	3.839	2.628	1.192	3.839						
Benchmark (1980)	***y***	***k***	***k/y***	***c***	***a***	46.07%	9.885	45.20%	-0.0867	-5.02%	7.64%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.726	4.554	2.639	1.401	4.467						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.464	3.863	2.639	1.198	3.926						
Decrease in mortality	***y***	***k***	***k/y***	*c*	***a***	46.07%	10.18	45.16%	-0.0928	-5.32%	7.39%
$\mu_0 = 139.9, \mu_1 = 0.06518$	1.744	4.690	2.689	1.410	4.597						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	***k*/y****	*c**	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.479	3.978	2.689	1.205	4.045						
15% Increase in A	***y***	***k***	***k/y***	***c***	***a***	51.52%	10.99	50.64%	-0.0971	-4.52%	7.65%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.147	5.662	2.638	1.744	5.565						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 0.9, \theta = 1.38$	1.464	3.860	2.638	1.198	3.931						
15% Increase in A, A*	***y***	***k***	***k/y***	***c***	***a***	46.07%	12.30	45.20%	-0.1078	-5.02%	7.64%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.147	5.665	2.639	1.743	5.558						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 1.035, \theta = 1.38$	1.821	4.805	2.639	1.490	4.884						
Combined changes (2010)	***y***	***k***	***k/y***	***c***	***a***	52.93%	13.34	52.01%	-0.1223	-4.65%	7.40%
$\mu_0 = 139.9, \mu_1 = 0.06518$	2.627	7.060	2.687	2.125	6.938						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.30, A^* = 1.035, \theta = 1.27$	1.840	4.945	2.687	1.500	5.041						

Table 5b repeats the scenarios reported in Table 5a for the other polar case where there are no annuities markets ($\phi = \phi^* = 0$). The lack of such markets leads to an overall contraction in both economies. In the case of output and the capital stock, both economies decline proportionately in per capita terms, so that the international allocations of output and of capital remain unchanged. But, with the lower return on savings, the proportional decline in capital exceeds that of output and the capital-output ratios decline. With Foreign residents enjoying the greater longevity and with diminishing returns to capital, Foreign residents increase their investment abroad and purchase Home capital, causing the NFA position of Home to decline correspondingly. Indeed, the differential mortality associated with the benchmark case now causes a significant increase in the decline in the *NFA/y* ratio of 16.7%, as compared to the more modest decline of 5.02% in the presence of annuities markets.

6.1. Some long-run sensitivity

In light of the importance of the utility characteristics as determinants of savings and international capital flows, Tables 5c and 6 report some sensitivity analysis with respect to the rate of time preference and intertemporal elasticity of substitution. In view of evidence suggesting that the US is less patient than other developed economies, Table 5c reports the same scenarios as does Table 5a, where Foreign has a lower rate of time preference, $\rho^* = 0.037$. There we see that with the lower rate of time preference in the G5 economies, long-run output and capital in both countries is increased in all scenarios, so that world wealth always increases. The allocation of capital and share of world output produced in the two economies is independent of the rate of time preference and thus remains unchanged from that in Table 5a. In contrast, the greater patience of G5 residents has a major impact on the allocation of wealth across the two economies. Thus even in the extreme case of Panel 1 where both economies have the same demographic characteristics, the US has a negative NFA of 14.2% of its GDP. When one accounts for the greater longevity in G5 in the base year 1980, the NFA/y ratio increases by 5.3 percentage points (similar to that in Table 5a). By comparing Table 5c to Table 5a one sees that in each scenario, the lower rate of time preference of G5, by increasing their savings rate causes a reduction in US wealth and therefore in its per capita consumption.

Table 6 reports the steady-state NFA/GDP ratios for Home and Foreign for a grid of values of the key preference parameters (ρ, ρ^*), (σ, σ^*), while also distinguishing between the presence and absence of the perfect annuities market. To isolate the role of the mortality structure, the two parts of Table 6 report the differential and common mortality rates, respectively. The corresponding equilibrium ratios reflect the following characteristics:

(i) A decrease in Home rate of time preference,ρ, causes Home to increase savings, leading to an increase in Home NFA and a corresponding decline in Foreign NFA. The effects of a decline in Foreign rate of time preference are analogous.
(ii) An increase in Home IES,σ, causes Home to increase savings, leading to an increase in Home NFA and a corresponding decline in Foreign NFA. The effects of an increase in Foreign IES, σ^*, are analogous.

Table 5b
$\rho = 0.04 = \rho^*$ (no annuities markets, $\phi = 0$).

	GDP	*Capital*	*Cap-Output*	*cons*	*wealth*	$\frac{y}{y+\theta y^*}$	$a+\theta a^*$	$\frac{a}{a+\theta a^*}$	*NFA*	*NFA/y*	***r***
Equal mortality (US)	***y***	***k***	***k/y***	***c***	***a***	46.07%	8.114	46.07%	0	0	9.48%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.608	3.738	2.325	1.346	3.738						
$\mu_0^* = 65.6, \mu_1^* = 0.05724$	***y****	***k****	***k*/y****	c^*	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.364	3.171	2.325	1.142	3.171						
Benchmark (1980)	***y***	***k***	***k/y***	***c***	***a***	46.07%	8.292	42.82%	−0.2698	−16.7%	9.27%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.620	3.820	2.358	1.330	3.550						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	c^*	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.374	3.240	2.358	1.164	3.436						
Decrease in mortality	***y***	***k***	***k/y***	*c*	***a***	46.07%	8.867	42.83%	−0.2878	−17.3%	8.63%
$\mu_0 = 139.9, \mu_1 = 0.06518$	1.660	4.085	2.461	1.352	3.797						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	***k*/y****	c^*	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.408	3.465	2.461	1.181	3.674						
15% Increase in A	***y***	***k***	***k/y***	***c***	***a***	51.52%	9.204	48.24%	−0.3026	−15.0%	9.29%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.014	4.742	2.358	1.657	4.440						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 0.9, \theta = 1.38$	1.373	3.233	2.358	1.165	3.453						
15% Increase in A, A*	***y***	***k***	***k/y***	***c***	***a***	46.07%	10.32	42.82%	−0.3357	−16.7%	9.27%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.016	4.753	2.358	1.655	4.417						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 1.035, \theta = 1.38$	1.710	4.031	2.358	1.448	4.274						
Combined changes (2010)	***y***	***k***	***k/y***	***c***	***a***	52.93%	11.60	49.65%	−0.3797	−15.2%	8.65%
$\mu_0 = 139.9, \mu_1 = 0.06518$	2.498	6.139	2.457	2.040	5.760						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.30, A^* = 1.035, \theta = 1.27$	1.750	4.300	2.457	1.472	4.599						

Table 5c
$\rho = 0.04,\ \rho = 0.037$.

	GDP	Capital	Cap-Output	Cons	Wealth	$\frac{y}{y+\theta y^*}$	$a+\theta a^*$	$\frac{a}{a+\theta a^*}$	NFA	NFA/y	r
Equal mortality (US)	***y***	***k***	***k/y***	***c***	***a***	46.07%	9.997	43.61%	−0.2460	−14.2%	7.54%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.733	4.606	2.658	1.394	4.360						
$\mu_0^* = 65.6, \mu_1^* = 0.05724$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.470	3.907	2.658	1.208	4.085						
Benchmark (1980)	***y***	***k***	***k/y***	***c***	***a***	46.07%	10.06	42.71%	−0.3382	−19.5%	7.49%
$\mu_0 = 65.6, \mu_1 = 0.05724$	1.737	4.636	2.669	1.391	4.298						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.473	3.932	2.669	1.214	4.178						
Decrease in mortality	***y***	***k***	***k/y***	*c*	***a***	46.07%	10.37	42.50%	−0.3698	−21.1%	7.23%
$\mu_0 = 139.9, \mu_1 = 0.06518$	1.756	4.777	2.720	1.398	4.407						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1, A^* = 0.9, \theta = 1.38$	1.489	4.052	2.720	1.222	4.320						
15% Increase in A	***y***	***k***	***k/y***	***c***	***a***	51.52%	11.17	48.13%	−0.3790	−17.6%	7.51%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.159	5.754	2.665	1.732	5.375						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 0.9, \theta = 1.38$	1.472	3.923	2.665	1.215	4.198						
15% Increase in A, A*	***y***	***k***	***k/y***	***c***	***a***	46.07%	12.52	42.71%	−0.4207	−19.5%	7.49%
$\mu_0 = 65.6, \mu_1 = 0.05724$	2.161	5.768	2.669	1.730	5.347						
$\mu_0^* = 118.0, \mu_1^* = 0.06482$	***y****	***k****	***k*/y****	***c****	***a****						
$A = 1.15, A^* = 1.035, \theta = 1.38$	1.833	4.892	2.669	1.510	5.197						
Combined changes (2010)	***y***	***k***	***k/y***	***c***	***a***	52.9 3%	13.56	49.33%	−0.4877	−18.5%	7.26%
$\mu_0 = 139.9, \mu_1 = 0.06518$	2.643	7.175	2.715	2.110	6.687						
$\mu_0^* = 326.2, \mu_1^* = 0.07595$	***y****	***k****	*k*/y**	***c****	***a****						
$A = 1.30, A^* = 1.035, \theta = 1.27$	1.851	5.025	2.715	1.523	5.409						

Table 6a
Sensitivity of NFA (taking account of differential mortality).

	$\sigma = 0.4,\ \sigma^* = 0.4$		$\sigma = 0.4,\ \sigma^* = 0.6$		$\sigma = 0.6,\ \sigma^* = 0.4$		$\sigma = 0.6,\ \sigma^* = 0.6$	
	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$ *NFA/y* *NFA*/y**	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$ *NFA*/y* NFA/y*	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$*NFA/y* *NFA*/y**	$\phi = 1$*NFA/y* *NFA*/y**	$\phi = 0$ *NFA/y* *NFA*/y**
$\rho = 0.037$, $\rho^* = 0.037$	−0.051	−0.171	−0.772	−0.592	0.697	0.137	−0.056	−0.312
	0.044	0.146	0.660	0.506	−0.596	−0.117	0.048	0.266
$\rho = 0.037$, $\rho^* = 0.04$	0.093	−0.052	−0.593	−0.459	0.872	0.271	0.172	−0.155
	−0.079	0.044	0.506	0.392	−0.745	−0.232	−0.147	0.133
$\rho = 0.04$, $\rho^* = 0.037$	−0.195	−0.285	−0.936	−0.719	0.510	0.008	−0.282	−0.459
	0.166	0.243	0.800	0.614	−0.435	−0.007	0.241	0.392
$\rho = 0.04$, $\rho^* = 0.04$	−0.050	−0.167	−0.758	−0.586	0.685	0.144	−0.055	−0.303
	0.043	0.142	0.647	0.501	−0.585	−0.123	0.047	0.259

Table 6b
Sensitivity of NFA (common mortality US rate).

	$\sigma = 0.4,\ \sigma^* = 0.4$		$\sigma = 0.4,\ \sigma^* = 0.6$		$\sigma = 0.6,\ \sigma^* = 0.4$		$\sigma = 0.6,\ \sigma^* = 0.6$	
	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$ *NFA/y* *NFA*/y**	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$ *NFA*/y* NFA/y*	$\phi = 1$ *NFA/y* *NFA*/y**	$\phi = 0$*NFA/y* *NFA*/y**	$\phi = 1$*NFA/y* *NFA*/y**	$\phi = 0$ *NFA/y* *NFA*/y**
$\rho = 0.037$, $\rho^* = 0.037$	0	0	−0.722	−0.318	0.749	0.326	0	0
	0	0	0.617	0.272	−0.640	−0.278	0	0
$\rho = 0.037$, $\rho^* = 0.04$	0.142	0.113	−0.545	−0.195	0.920	0.454	0.224	0.143
	−0.121	−0.096	0.465	0.167	−0.786	−0.388	−0.192	−0.122
$\rho = 0.04$, $\rho^* = 0.037$	−0.142	−0.113	−0.885	−0.443	0.564	0.200	−0.224	−0.143
	0.121	0.096	0.756	0.378	−0.482	−0.171	0.192	0.122
$\rho = 0.04$, $\rho^* = 0.04$	0	0	−0.708	−0.321	0736	0.329	0	0
	0	0	0.605	0.274	−0.628	−0.281	0	0

In both cases (i) and (ii) the described change occurs primarily through the effect on the marginal propensity to consume out of wealth, Δ^{-1}, over the agent's lifetime, aggregating to an increase in Home wealth and NFA.

(iii) With no annuities markets, the magnitudes of the responses described in (i) and (ii) are reduced, the sensitivity being greater for a change in the IES than for the rate of time preference.

The following insights can be gleaned from Table 6a.

(A) Suppose preferences across the two economies are identical with respect to both σ and ρ (the more darkly shaded cells). Then
  (i) With the existence of perfect annuities markets the NFA/GDP ratios are insensitive to variations in both, parameters; the Home and Foreign NFA/GDP ratios ranges between -5.0% and -5.6% and 4.3% and 4.8%, respectively.
  (ii) In the absence of the annuities markets, the NFA/GDP ratios remain insensitive to variations in ρ, ρ^*, but are much more sensitive to variations in the IES, σ, σ^*.
(B) In the case that preferences across the two economies are asymmetric with respect to either or both of ρ, ρ^* and σ, σ^*, the NFA/GDP ratios across the two economies become highly sensitive to these parameters. For example, starting with the benchmark parameterization$\phi = 1, \rho = \rho^* = 0.04,\ \sigma = \sigma^* = 0.4$, then, if σ^* is increased to 0.6, Home NFA/GDP ratio deteriorates dramatically from -5.0% to -75.8%; if σ increases to 0.6, Home GNFA/GDP ratio increases to 68.5%, and it becomes a creditor nation.

Table 6b, which assumes that both economies share the mortality rate of the US exhibits generally similar characteristics. The four more darkly shaded cells now correspond to the case where Home and Foreign have both identical preferences and mortality, confirming that the steady-state NFA position of both countries is zero. Comparing these with the corresponding cells in Table 6a indicates the contribution of the differential mortality rates of the two economies to the equilibrium international allocation of capital. Thus, focusing on the benchmark $\rho = \rho^* = 0.04, \sigma = \sigma^* = 0.4$, this suggests differential mortality accounts for about 5% of the NFA/GDP ratio for Home in the presence of the annuities market which would increase to around 16.7% in its absence.

The same interpretation applies to other cells, corresponding to their respective parameterization of preferences. As alluded to in our calibration, empirical evidence suggests $\rho > \rho^*$ and $\sigma > \sigma^*$. Such a case is identified by the lightly shaded cells in the two parts of Table 6. Taking the differences suggests that the NFA/GDP ratio would decrease by 5.3% and 19.2%, depending upon the existence, or lack thereof, of the annuities market.

7. Transitional dynamics

We now consider the impact of the demographic structures on the transitional dynamics. In doing so we shall focus on three aspects. First, we illustrate the transitional paths followed by a number of key variables in response to a general worldwide increase in TFP. In order to isolate the role of the differential mortality between the two economies, Home and Foreign, we set all other aspects, such as tastes and productivity equal. Also, since structural change does not occur rapidly, we specify the change in Home TFP from A_0 to $\tilde{A}$ by $A(t) = \tilde{A} + \left(A_0 - \tilde{A}\right)e^{-at}$, where a denotes the rate of adjustment, and analogously for Foreign TFP. In our simulations we take the adjustment speed to be 10% per annum. This is illustrated by the solid line in the figures. For comparative purposes, the dashed line illustrates the conventional case where the structural change occurs instantaneously.[21] The second issue we address concerns the sensitivity of the response of the Home NFA position following the worldwide increase in TFP to the differentials in the preference parameters. This is related to the sensitivity analysis reported in Table 6, where we showed that the long-run NFA positions were highly sensitive to these aspects. The third issue we consider is the sensitivity of the time path of Home NFA to the differential decreases in mortality experienced by Home and Foreign between 1980 and 2010, to likely differentials in the key preference parameters.

7.1. 15% increase in home and foreign TFP

Fig. 3 sets out the transition paths for a number of key variables in response to a 15% increase in Home and Foreign TFP. As in the benchmark parameterization, all taste parameters are assumed to be identical, while the respective demographic structures of the two countries are maintained as in 1980. In addition, to fully isolate the role of the differential mortality rates, the TFP are set equal in both countries, increasing from $A = A^* = 1$ by 15%.

With technology and preferences being similar in the two economies, their respective transitional paths for output and capital are identical, while their time paths for consumption and wealth deviate only slightly due to their differential mortality rates. If the increase in technology is completed instantaneously, both countries will immediately be able to increase their output and consumption and to begin to accumulate capital, and wealth. Because of the greater longevity in Foreign, residents of that economy will have a higher savings rate, than do Home causing them to invest abroad. Accordingly, over time Home NFA declines, while Foreign NFA increases correspondingly.

In the case where the increase in productivity occurs gradually, output in both economies can increase only gradually, and therefore remains unchanged at the initial instant. Since consumers are forward-looking and base their consumption decisions on their lifetime wealth, which they know will increase with the technology, they increase their initial consumption, albeit by a lesser amount. Accordingly, with output fixed in the short run wealth and capital initially decline in both economies. Residents of Foreign reduce their savings more and in the short run Home NFA increase, while those of Foreign, decline. After a few years, as the increase in TFP builds up and output increases, and the declining time paths of wealth, capital, and NFA for both economies are reversed.

[21] We also illustrate only the conventional case of the existence of an actuarially fair annuities market.

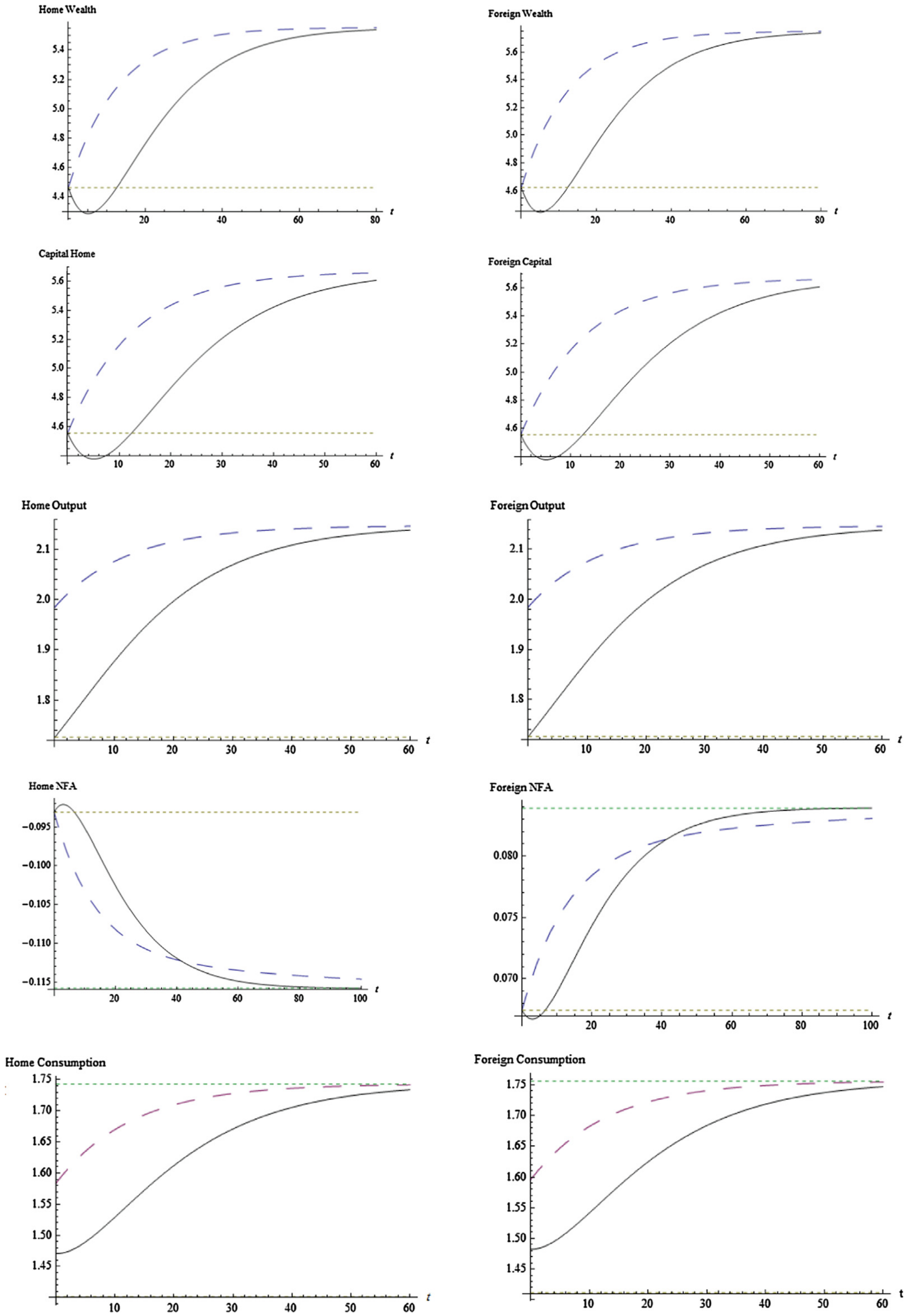

Fig. 3. 15% Increase in World-wide TFP.

7.2. Sensitivity of time paths of Home TFP in preference parameters

Fig. 4 illustrates the sensitivity of the time paths for both Home NFA and NFA/GDP ratio to key preference parameters. With identical proportional increases in Home and Foreign TFP, the long-run NFA/GDP ratio remains unchanged, as shown in Table 5. The preference specifications in panel (i) correspond to Fig. 3. The fact that Home NFA/GDP ratio initially temporarily rises sharply if the increase in TFP increases gradually, is a direct consequence of the initial increase in NFA coupled with the sluggish increase in output. With Foreign agents more patient ($\rho^* = 0.037$) the same pattern emerges, the main difference is a larger foreign debt for the Home economy, and a more substantial decline [panel (ii)].

Adopting the assumption, supported by some empirical evidence that Home has a larger IES ($\sigma = 0.6$), changes the adjustment path of NFA quite substantially [panel (iii)]. In the first place, Home starts out as a creditor, rather than as a debtor nation. This reflects the fact that a larger σ implies a faster growth of per capita consumption, which in steady state requires a larger per capita wealth. On the other hand, the capital stock domiciled in Home is determined primarily by productivity, and is impacted by preferences in only a secondary way. Accordingly, Home attains its equilibrium level of wealth by lending abroad.

The instantaneous increase in world wide TFP causes Home wealth to start increasing at a faster rate than does Home capital so that Home NFA initially increases. At the same time, the initial increase in Home output causes the Home NFA/GDP ratio to immediately decline. Over early stages of the transition wealth grows faster than capital and NFA increases. However, in the latter stages of the transition, Home wealth accumulation declines relative to capital accumulation and NFA declines to its higher new steady state equilibrium level.

If the increase in world wide TFP occurs gradually, the increase in Home NFA which occurs during the earlier stages is more pronounced. This is because capital accumulation is retarded more than is wealth accumulation, leading to a larger overshooting of the long-run increase in Home NFA during the transition.

Finally, panel (iv) combines the two cases (ii) and (iii). Increasing the patience of Foreign residents moderates the transitional path of (iii), while preserving its main characteristics. In all cases, the time paths followed by Foreign are simply mirror images of those depicted in Fig. 4.

7.3. Impact of decreasing mortality on NFA

Fig. 5 illustrates the sensitivity of the time paths of Home NFA and NFA/GDP resulting from the respective declines in mortality of Home and Foreign over the period 1980–2010. Since demographic change by its nature proceeds slowly over generations, we assume that the change over the period proceeds at the rate of 10% per annum. We also assume that the relative TFP of the two economies remains fixed at their respective initial 1980 levels. Again, Panel (i) corresponds to the initial benchmark case, where both economies have identical preferences. In this case, the relative increase in longevity experienced by Foreign causes it to have a higher savings rate, as a result of which its wealth increases at a faster rate than does its capital stock. Accordingly, it accumulates capital domiciled in Home as a result of which Home's NFA declines. With output growing over the period, Home's NFA/GDP declines at a faster rate, while the accumulation of Foreign's NFA/GDP is correspondingly quicker. In both cases, the slow demographic change ensures that the transitions proceed monotonically.

The same patterns as in Panel (i) prevail as long as both economies have symmetric preferences. Panel (ii) illustrates the case corresponding to Table 5b, where Foreign is more patient than Home, with ρ^* reduced to 0.037. This simply increases the relative amount of savings undertaken by Foreign increasing the reduction in Home NFA, which continues to proceed at a constant steady state.

Panel (iii) illustrates the case where, as some empirical evidence suggests, Home has a higher IES than does Foreign, and σ is increased to 0.6, well within the range of empirical estimates. Now there are two conflicting forces impinging on the international flow of capital. First, the fact that Home has a higher IES than does Foreign implies that it has a lower marginal propensity to consume out of wealth leading to a higher initial steady-state level of wealth. While this is offset partially the increased savings of Foreign stemming from their lower mortality rate in 1980, the first effect dominates and Home is initially a net creditor rather than a net debtor nation. These relative effects continue during the demographic transition extending over 1980–2010, during which period Home NFA and NFA/GDP increase slightly. However, we should emphasize that this is very sensitive to the assumed magnitude of σ. Panel (iv) combines the increase in σ with the decrease in ρ^*. As a result, following an initial increase, Home NFA starts to decline and eventually converges to a level below where it initially began.

8. Demographic structure and natural rate of wealth inequality

In an early contribution, Atkinson (1971) argued that there is an inherent wealth inequality in societies due to the changing savings behavior of agents over their life-cycle. The overlapping generations structure we are employing provides a convenient vehicle for tracing out the accumulation of assets over the individual life-cycle and to see how this is impacted by the international transmission of assets. We restrict our brief analysis to comparing the steady-state wealth distributions for the two economies due to their differential initial and changing mortality rates.

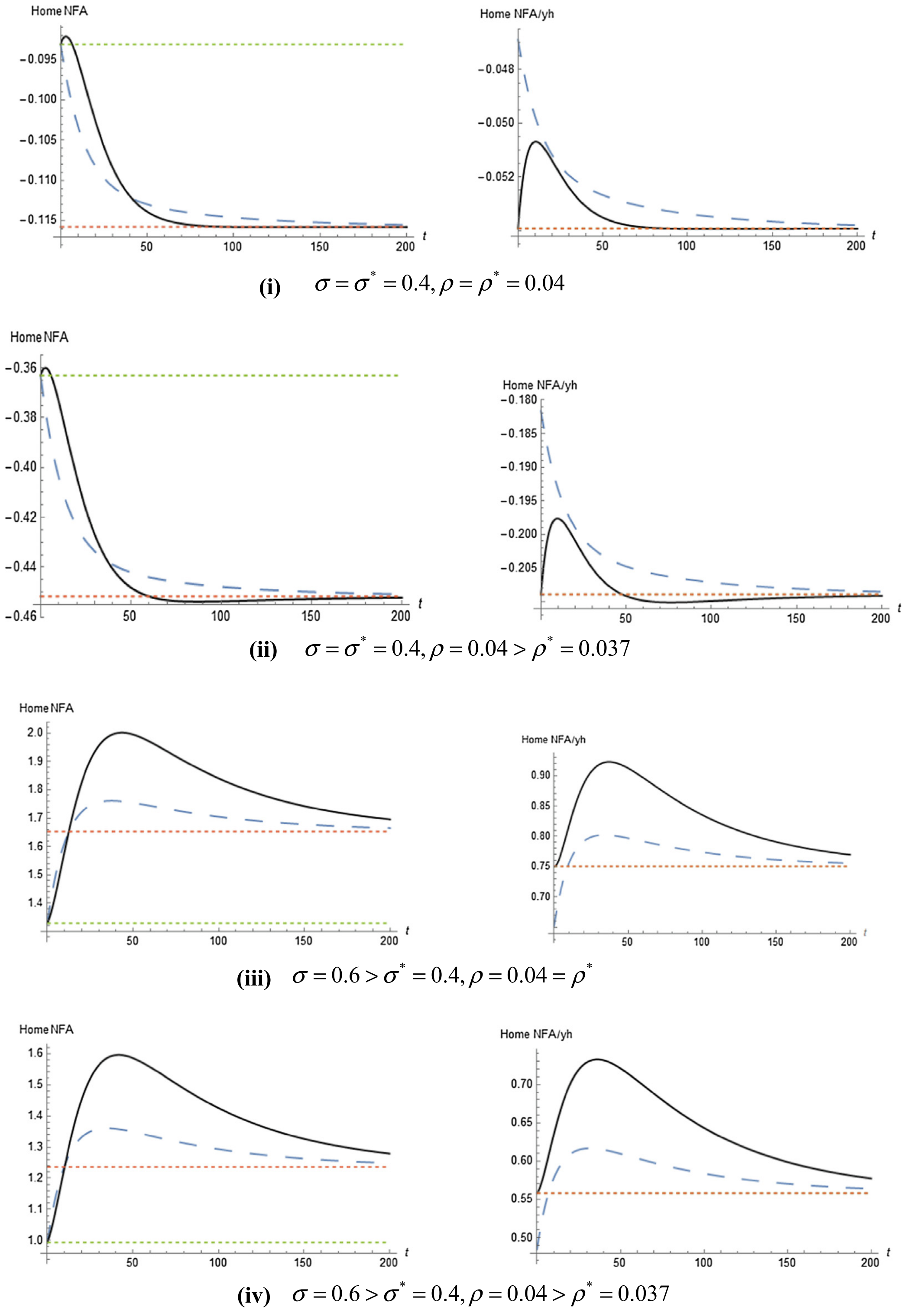

Fig. 4. Time paths for Home NFA and NFA/y following WW 15% increase in TFP.

We focus on the Home economy, although the Foreign economy is entirely analogous. We begin by recalling the individual's accumulation Eq. (2), which in steady state is independent of calendar time and can be written as

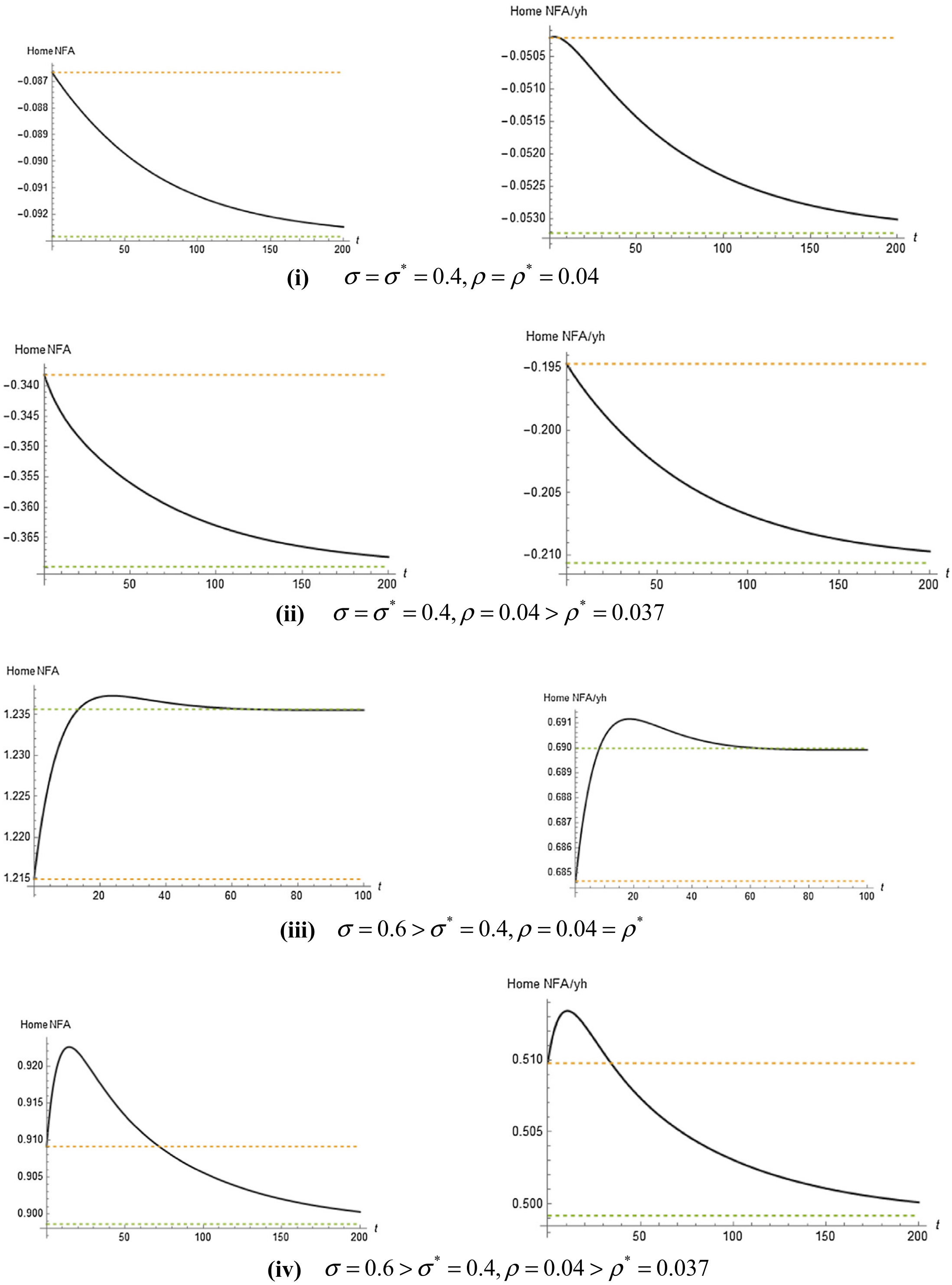

Fig. 5. Time paths for Home NFA and NFA/y following decline in mortality.

$$\dot{A}(u) = (\tilde{r} + \mu(u))\tilde{A}(u) + \tilde{w} + \tilde{Z}(u) - \tilde{C}(u) \tag{32}$$

where u indexes age. Tilde designates steady state so that $\mu(u)$ and therefore $M(\mu)$ denote the steady-state mortality and cumulative mortality rates associated with age u. Recalling (5), (28a), and the assumption of transfers, we have

$$\tilde{C}(u) = \frac{\tilde{H}}{\tilde{\Delta}} e^{\sigma(\tilde{r}-\rho)u-(1-\phi)M(u)}, \quad \tilde{Z}(u) = (1-\phi)\mu\tilde{A}(u)$$

Substituting these expressions into (32) and integrating yields the steady-state distribution of wealth across individuals in the Home economy:

$$\tilde{A}(u) = e^{\tilde{r}u+M(u)} \left\{ \tilde{w} \int_0^u e^{-\tilde{r}s-M(s)} ds - \frac{\tilde{H}}{\tilde{\Delta}} \int_0^u e^{((\sigma-1)\tilde{r}-\sigma\rho)s-\phi M(s)} ds \right\} \tag{33}$$

One can verify that $\tilde{A}(u)$ is "hump-shaped" in that agents start with zero assets $\tilde{A}(0) = 0$, accumulate wealth until a certain age, after which they dissave, with those that survive to the maximal age, D, leaving zero assets $\tilde{A}(D) = 0$.

From (33), we can easily compute the coefficient of variation of the wealth distribution across agents, which serves as a convenient measure of wealth inequality. The resulting expressions are reported in Table 7 for both Home and Foreign, and both in the presence of and in the absence of the annuities market. Table 7 conforms to Table 5, and suggests the following observations:

1. In the benchmark economy, the absence of annuities market increases wealth inequality by around 6.2%. This amount is approximately the same across all the scenarios.
2. In the presence of the annuities market:
   (i) The initial differential in mortality accounts for about a 0.30 percentage points more wealth inequality in the US than in the G5.
   (ii) The decrease in mortality over the period 1980–2010 leads to approximately 0.5 percentage point reduction in wealth inequality in both countries.
   (iii) The combined changes between 1980 and 2010 lead to approximately 0.7 percentage point reduction in US wealth inequality and approximately 0.9 percentage points in G7 wealth inequality.
3. In the absence of the annuities market, these changes are all somewhat larger.

We should emphasize that the coefficients of variation reported in Table 7 reflect differences in inequality due only to the nations' respective demographic structures and abstracts from any within-cohort inequality. In this sense, they serve as measures of the "natural rate of wealth inequality".

With the population growth assumed constant across steady states, (28e) implies that any decline in mortality must be offset by a decline in fertility. This leads to two offsetting effects on wealth inequality. A decrease in the birth rate reduces the number of young individuals but leaves the life-cycle savings pattern of the individuals unchanged. This reduction in the number of young people likely to have far fewer assets than do older agents, who have accumulated over time will tend to reduce wealth inequality. In contrast, a decrease in the mortality rate changes both the relative distribution of cohort sizes

Table 7
Wealth Inequality.

	Perfect annuities markets		*No annuities markets*	
	US	*G5*	*US*	*G5*
Equal mortality (US) $\mu_0 = 65.6, \mu_1 = 0.05724$ $\mu_0^* = 65.6, \mu_1^* = 0.05724$ $A = 1, A^* = 0.9, \theta = 1.38$	0.5616	0.5616	0.6256	0.6256
Benchmark (1980) $\mu_0 = 65.6, \mu_1 = 0.05724$ $\mu_0^* = 118.0, \mu_1^* = 0.06482$ $A = 1, A^* = 0.9, \theta = 1.38$	0.5599	0.5569	0.6227	0.6187
Decrease in mortality $\mu_0 = 139.9, \mu_1 = 0.06518$ $\mu_0^* = 326.2, \mu_1^* = 0.07595$ $A = 1, A^* = 0.9, \theta = 1.38$	0.5546	0.5521	0.6122	0.6085
15% Increase in A $\mu_0 = 65.6, \mu_1 = 0.05724$ $\mu_0^* = 118.0, \mu_1^* = 0.06482$ $A = 1.15, A^* = 0.9, \theta = 1.38$	0.5601	0.5571	0.6231	0.6190
15% Increase in A, A* $\mu_0 = 65.6, \mu_1 = 0.05724$ $\mu_0^* = 118.0, \mu_1^* = 0.06482$ $A = 1.15, A^* = 1.035, \theta = 1.38$	0.5599	0.5569	0.6227	0.6187
Combined changes (2010) $\mu_0 = 139.9, \mu_1 = 0.06518$ $\mu_0^* = 326.2, \mu_1^* = 0.07595$ $A = 1.30, A^* = 1.035, \theta = 1.27$	0.5548	0.5523	0.6126	0.6088

and the life-cycle savings pattern. Due to the longer life-span, individuals will save more for life-cycle purposes, so that the dispersion between asset holdings at different moments of the life-cycle increases substantially. In addition, more agents are alive who are at the top of their life-cycle savings, although offsetting this are an increased number of individuals who are in the dissaving phase of their life cycle. With the relative size of the cohorts decreasing with age (due to both population growth and attrition) the fertility effect dominates and on balance inequality declines, albeit by a small amount.

9. Conclusion

This paper has investigated the extent to which differences between the demographic characteristics of two countries will drive the flow of capital between them, as inhabitants of the longer-lived nation have higher savings rate than do those with the shorter lifespan. It is motivated in part by the recent US experience during which its net foreign asset position declined dramatically, transforming it from a creditor to a debtor nation, while simultaneously its longevity has fallen increasingly below that of other developed G7 economies. This issue has spawned an extensive literature. Our contribution has been to address it by introducing empirically based survival (mortality) functions into a traditional two country macrodynamic framework. In doing so we eliminate some of the pitfalls associated with the traditional two country representative agent model, and further we focus on other aspects of the macroeconomic equilibrium in addition to the capital flows.

In calibrating the model, our major concern has been with the demographic aspects, with other aspects of the parameterization of the US and the G5 economies being generic. As an initial benchmark we have taken the year 1980 and assumed that all parameters pertaining to preferences are identical. Under these conditions, and assuming the presence of a neutral annuities market, the higher mortality in the US accounts for around a 5% reduction in its NFA/GDP ratio relative to if mortality were equal when the NFA position would be zero. This increases to nearly 17% in the absence of such a market.

The long-run net asset position is highly sensitive to differences in the key preference parameters, particularly to the intertemporal elasticity of substitution. Thus, assuming (i) both economies have similar rates of time preference, and (ii) the IES for US is say 0.6, while for the G5 it is 0.4, the US would have a large positive NFA/GDP ratio, of around 0.79 in the presence of the annuities market. However, this would become a large negative ratio, of around -0.76, if these magnitudes for the IES were reversed. In contrast, the differentials in these NFA positions from their respective positions corresponding to identical mortality rates remain robust at around 5%. However, in the absence of annuities the differences are larger and more sensitive to variations in the preference parameters.

The growing differential of mortality between 1980 and 2010 does little to explain the dramatic decline in the US NFA/GDP ratio that occurred during that period. Alone it may account for a 1% reduction. This is not surprising since most of the decline occurred in the form of foreign debt, rather than in FDI, which is what is more accurately reflected in the model. But if one couples the change in mortality with modest plausible changes in preference parameters – well within sampling errors – one can explain a substantial proportion of the decline in the NFA/GDP ratio. Moreover, the time path for mortality does generate a slightly increasing capital output ratio, consistent with what occurred over that period.

These findings relate to the natural wealth inequality for the baseline model as follows. A fall in mortality causes an increase in inequality due to the presence of longer living wealthy individuals; however, with a constant population growth rate, the birth rate will also decline in order to maintain the demographic steady state thereby decreasing inequality. Structural changes that benefit capital holders, such as an increase the share of capital in the production process, will increase inequality. Increasing the pure rate of time preference decreases inequality, due to the fact that a preference towards current consumption limits the saving behavior of an individual, flattening the life-cycle path of asset accumulation.

Finally, the aging of the population has become a serious challenge for the funding of national pension schemes and retirement in many countries. This is addressed by a number of authors; see Aldrich (1982), Attanasio, Kitao, and Violante (2007), Borsch-Supan et al. (2006) and Marchiori (2011). We do not address this issue, but clearly it is important to do so with an empirically based demographic structure, since by interacting with saving it will impact the international capital flow between economies having different demographic characteristics. Indeed, to the extent that the G5 economies may have experienced a decline in their pension benefits, that would have increased their private saving, thereby augmenting the decline in the US NFA/GDP ratio.

Appendix

A.1. Derivation of macrodynamic equilibrium

We begin by noting that with Home and Foreign output produced by Cobb-Douglas production functions as specified in (26), the factor returns, as reported in (21, 21′) are

$$r(t) = A\alpha k^{\alpha-1} - \delta = A^*\alpha(k^*)^{\alpha-1} - \delta; \;\; w(t) = A(1-\alpha)k^{\alpha}; \;\; w^* = A^*(1-\alpha)(k^*)^{\alpha} \tag{A.1}$$

In particular, the rates of return equalized across the economies implies the following equilibrium relationship between foreign and domestic per capita capital stock:

$$k^* = \left(\frac{A}{A^*}\right)^{\frac{1}{\alpha-1}} k \tag{A.2}$$

and hence

$$\dot{k}^* = \left(\frac{A}{A^*}\right)^{\frac{1}{\alpha-1}} \dot{k} \tag{A.3}$$

Setting $v = t$ in (8) and recalling $A(t,t) = 0$, we may write (15) as

$$\dot{c}(t) \equiv \varphi \frac{H(t)}{\Delta(t)} + \left[\sigma\left(\alpha Ak(t)^{\alpha-1} - \delta - \rho\right) - [1+\sigma(1-\phi)]\mu_c(t-v_1) - n\right]c(t) \tag{A.4}$$

where for notational convenience we denote $H(t) \equiv H(t,t),\ \Delta(t) \equiv \Delta(t,t)$. The analogous condition in the Foreign economy can be expressed as

$$\dot{c}^*(t) = \varphi^* \frac{H^*(t)}{\Delta^*(t)} + \left(\sigma^*\left[\alpha Ak(t)^{\alpha-1} - \delta - \rho^*\right] - n - [1+\sigma^*(1-\phi^*)]\mu_c^*(t-v_1^*)\right)c^*(t) \tag{4'}$$

Next, setting $t = v$ in (9a), (9a'), (9b), (9b') and taking derivatives with respect to t, yields

$$\dot{H}(t) = -(1-\alpha)Ak(t)^\alpha + \left[\alpha Ak(t)^{\alpha-1} - \delta + \mu_H(\tau_1 - t)\right]H(t) \tag{A.5}$$

$$\dot{H}^*(t) = -(1-\alpha)\left(\frac{A}{A^*}\right)^{\frac{1}{\alpha-1}} Ak^\alpha + \left[\alpha Ak(t)^{\alpha-1} - \delta + \mu_H^*(\tau_1^* - t)\right]H^*(t) \tag{5'}$$

$$\dot{\Delta}(t) = -1 + \left[(1-\sigma)\left(\alpha Ak(t)^{\alpha-1} - \delta\right) + \sigma\rho + [1+\sigma(1-\phi)]\mu_\Delta(\tau_2 - t)\right]\Delta(t) \tag{A.6}$$

$$\dot{\Delta}^*(t) = -1 + \left[(1-\sigma^*)\left(\alpha Ak(t)^{\alpha-1} - \delta\right) + \sigma^*\rho^* + [1+\sigma^*(1-\phi^*)]\mu_\Delta^*(\tau_2^* - t)\right]\Delta^*(t) \tag{A.6'}$$

where

$$\mu_H(\tau_1 - t) = \frac{1}{H(t)}\int_{t-D}^{t} \mu(t-v)p(t-v)H(v,t)dv \tag{A.7a}$$

$$\mu_\Delta(\tau_2 - t) = \frac{1}{\Delta(t)}\int_{t-D}^{t} \mu(t-v)p(t-v)\Delta(v,t)dv \tag{A.7b}$$

(A.7a) is interpreted similarly as (16) as the ratio of human wealth given up by the dying to the per-capita human wealth level and can be interpreted as providing an estimate of the average mortality over the period $(t-D,t)$ from information on human wealth across the cohorts. Expression (A.7b) has an analogous interpretation and similar comments apply to the corresponding expressions vis a vis the Foreign economy.

Substituting factor returns into (17) and (17′), yields

$$\dot{a}(t) = \left(\alpha Ak(t)^{\alpha-1} - \delta - n\right)a(t) + (1-\alpha)Ak(t)^\alpha - c(t) \tag{A.8}$$

$$\dot{a}^*(t) = \left(\alpha Ak(t)^{\alpha-1} - \delta - n\right)a^*(t) + (1-\alpha)\left(\frac{A}{A^*}\right)^{\frac{1}{\alpha-1}} Ak^\alpha - c^*(t) \tag{8'}$$

Finally, substituting (A.2) and (A.3) into (23)

$$\dot{k}(t) = Ak(t)^\alpha - (n+\delta)k - \chi(c(t) + \theta c^*(t)) \tag{A.9}$$

where $\chi \equiv \left[1 + \theta(A/A^*)^{\frac{1}{\alpha-1}}\right]^{-1}$. Eqs. (A.4), (4′), (A.5), (5′), (A.6), (6′), (A.8), (8′), (A.9) yield autonomous dynamic equations determining the dynamic adjustment of the aggregate quantities, $x(t) \equiv (c(t), c^*(t), H(t), H^*(t), \Delta(t), \Delta^*(t), a(t), a^*(t), k(t))$. Having determined these, the dynamics of $k^*(t)$ are obtained from (A.2) and the net asset positions, $z(t), z^*(t)$, from (24).

A.1.1. Linearized matrix of baseline model

To analyze the local dynamics we linearize this system about its steady state, enabling it to be expressed in the form

$$\dot{x}(t) = \Omega(x(t) - \tilde{x}) \tag{A.10}$$

where Ω is:

$$\begin{pmatrix} -\frac{\varphi\tilde{H}}{\tilde{\Delta}\tilde{c}} & 0 & \frac{\varphi}{\tilde{\Delta}} & 0 & -\frac{\varphi\tilde{H}}{\tilde{\Delta}^2} & 0 & 0 & 0 & \sigma(\alpha-1)\tilde{c}\frac{(\tilde{r}+\delta)}{\tilde{k}} \\ 0 & -\frac{\varphi^*\tilde{H}^*}{\tilde{\Delta}^*\tilde{c}^*} & 0 & \frac{\varphi^*}{\tilde{\Delta}^*} & 0 & -\frac{\varphi^*\tilde{H}^*}{\tilde{\Delta}^{*2}} & 0 & 0 & \sigma^*(\alpha-1)\tilde{c}^*\frac{(\tilde{r}+\delta)}{\tilde{k}} \\ 0 & 0 & \frac{\tilde{w}}{\tilde{H}} & 0 & 0 & 0 & 0 & 0 & (\alpha-1)\frac{(\tilde{r}+\delta)\tilde{H}}{\tilde{k}} - \alpha\frac{\tilde{w}}{\tilde{k}} \\ 0 & 0 & 0 & \frac{\tilde{w}^*}{\tilde{H}^*} & 0 & 0 & 0 & 0 & (\alpha-1)\frac{(\tilde{r}+\delta)\tilde{H}^*}{\tilde{k}} - \alpha\frac{\tilde{w}^*}{\tilde{k}} \\ 0 & 0 & 0 & 0 & \frac{1}{\tilde{\Delta}} & 0 & 0 & 0 & (1-\sigma)(\alpha-1)\frac{(\tilde{r}+\delta)}{\tilde{k}}\tilde{\Delta} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\tilde{\Delta}^*} & 0 & 0 & (1-\sigma^*)(\alpha-1)\frac{(\tilde{r}+\delta)}{\tilde{k}}\tilde{\Delta}^* \\ -1 & 0 & 0 & 0 & 0 & 0 & \tilde{r}-n & 0 & (\alpha-1)\frac{(\tilde{r}+\delta)}{\tilde{k}}\tilde{a} + \alpha\frac{\tilde{w}}{\tilde{k}} \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & \tilde{r}-n & (\alpha-1)\frac{(\tilde{r}+\delta)}{\tilde{k}}\tilde{a}^* + \alpha\frac{\tilde{w}^*}{\tilde{k}} \\ -\chi & -\chi\theta & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{r}-n \end{pmatrix}$$

For the benchmark parameterization the two stable eigenvalues are -0.077, -0.015 in the presence of perfect annuities markets ($\phi = \phi^* = 1$) and -0.085, -0.014 in their absence ($\phi = \phi^* = 0$). The similarity of these roots indicates that the transitional dynamics in the two cases are not too dissimilar.

Appendix B. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jimonfin.2019.102062.

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Further reading

World Bank. World Development Indicators. http://data.worldbank.org.
World Health Organization. Global Health Observatory Data Repository. http://apps.who.int.