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Government Spending, Debt Management, and Wealth and Income Inequality in a Growing Monetary Economy*

This paper compares the impact of government investment and government consumption on macroeconomic aggregates and inequality when the government deficit is money-financed while maintaining a fixed debt-money ratio. Real aggregate quantities are independent of the debt-money ratio, as is wealth inequality, but income inequality is impacted. We also investigate the impact of these two forms of government expenditure on the macroeconomic aggregates and distributions, illustrating their sharply contrasting effects on the tradeoffs they entail. While government investment is more effective in increasing the growth rate and moderating inflation, it has a more adverse effect on long-run income inequality.

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THE POTENTIAL PROBLEMS PERTAINING TO THE long-run sustainability of deficit financing have a long and continuing history dating back to the seminal contribution of Sargent and Wallace (1981). Recent papers espouse these concerns from a variety of standpoints and include Chalk (2000), Aguiar, Amador, and Gopinath (2009), Aguiar and Amador (2011), and Kobayashi and Ueda (2022). As economies have engaged in aggressive fiscal spending to mitigate the adverse effects of the COVID-19 pandemic, this issue remains one of ongoing concern. In this paper, we address the consequences of alternative forms of government expenditure within a general macrodynamic growth framework. Our focus is on the potential

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long-run tradeoffs between growth and inequality, and for this purpose, the Sidrauski (1967) monetary growth model serves as a convenient vehicle.¹ We assume that the government deficit is financed by a combination of money and bonds, with debt policy being to maintain a fixed ratio between debt and money—a policy that we suggest is a reasonable approximation to recent U.S. policy over periods of extended financial stability.

Modifying the Sidrauski technology to generate long-run endogenous growth, we address a number of related issues. First, we consider the long-term viability of this form of deficit financing. Second, we consider the long-run consequences of the specified debt policy for key aggregate variables, such as the growth rates of output and capital, the output-capital ratio, and employment. We show that redeeming government debt and thereby increasing the degree of money-financing, while maintaining tax rates and expenditures unchanged, is irrelevant insofar as these variables are concerned, both along a transition path and in a steady state. We further show that the effect of an increase in government expenditure on these same variables is also independent of the chosen debt-money ratio. In contrast, debt policy is shown to impact inflation, and since inflation has potentially important consequences for inequality, we also address the implications of debt policy for various inequality measures. While it turns out to have no effect on wealth inequality, it does have a substantial impact on income inequality.

Taking the specification of debt policy as given, we compare the effects of an increase in government investment and government consumption on the transitional and long-run effects of key aggregate and distributional measures. We pursue this aspect numerically, by calibrating the model and policies to reflect recent U.S. experience, suggesting several conclusions. First, while the two forms of government spending share some similarities, they also generate starkly different time profiles for key macroeconomic variables, including income inequality. Second, the implications contrast somewhat with those of the existing literature that focuses separately on either the monetary or the fiscal aspects of government policy. The interaction of the two elements of government policymaking seems to be important, particularly with respect to the consequences for income inequality. Finally, the paper also shows that the numerical simulations conducted using parameters that approximate the U.S. economy suggest that the implications of the model are broadly consistent with the relationship between U.S. fiscal policy growth, inflation, and income inequality, over the last two decades.

While the vast majority of recent endogenous growth models abstract from monetary aspects, there are several existing studies that have addressed money-finance as the primary means of deficit financing, and compared it with other modes of finance. These studies include van der Ploeg and Alogoskoufis (1994), Gokan (2002),

1. To analyze the short-term impact of inflation, as has been suggested by Gali (2015) and others, it would be more appropriate to use some version of a sticky price macro model. But since this paper is concerned with issues pertaining to long-term fiscal sustainability, the Sidrauski growth model with flexible prices provides a reasonable framework.

and Palivos and Yip (1995). While they differ in terms of their specific frameworks, they all share the following characteristics: (i) Consequences for wealth and income inequality are not addressed; (ii) as in the seminal Romer (1986) model, the economy is always on its balanced growth path; (iii) labor is supplied inelastically; (iv) only government consumption is addressed. All of these are important limitations that the present paper seeks to redress.

At the same time, there are other studies using endogenous growth models that focus explicitly on the impact of fiscal policy on wealth and income inequality. Much of this literature employs the “representative consumer theory of distribution” (RCTD), initiated by Caselli and Ventura (2000), as does the present paper. The key element of this approach is that assuming homogeneous preferences and complete markets enables one to exploit the aggregation procedures due to Gorman (1959), which renders the analysis highly tractable, enabling one to obtain insights that otherwise would be obscured. For example, Chatterjee and Turnovsky (2012) and Turnovsky (2015) focus on tax-financed public investment, emphasizing how differences in the mode of financing are critical in determining the impact on wealth and income inequality.² In contrast to the present paper, this literature excludes the role of money and the issues pertaining to money/debt financing.

One exception to this is the recent paper by Chang et al. Using a monetary growth model with goods purchased by cash and credit, they consider the effects of inflation caused by an increase in the growth rate of money supply on alternative distributional measures. Their concern is on the contrasting consequences for income and consumption inequality, rather than the more general interactions between government financial and fiscal policies, that is the focus of the present paper.³

1. ANALYTICAL FRAMEWORK

This section sets out the structure of the economy.

1.1 Firms

The economy consists of a number of firms, indexed by j , that combine labor, L_j , private capital, K_j , and infrastructure, X , to produce an aggregate good, Y_j , that can be used for private consumption, private investment, or purchased by the government

2. Other examples of this general approach include Chatterjee (1994), Sorger (2002), and Maliar and Maliar (2003).

3. Recently, Gokan and Turnovsky (2021) extend the Sidrauski (1967) model demonstrating that the super-neutrality of money growth associated with that model does not extend to inequality measures. However, that paper does not address the link between government expenditures and debt policy that is the focal point of the present study. Another strand of literature analyzes the aggregate effect of monetary policy on wealth and income distributions using the New Keynesian model with price rigidity. This has a different focus from that presented here; see, for example, Auclert (2019) and Kaplan, Moll, and Violante (2018) for important contributions.

for public consumption or investment. To generate sustained growth, we assume a Cobb–Douglas production function that is constant returns to scale in both private capital and labor, as well as in private capital and infrastructure:

$$Y_j = E \cdot (X \cdot L_j)^s (K_j)^{1-s} \quad 0 < s < 1, \quad (1)$$

where E denotes the productivity level (as measured by TFP). We specify infrastructure as a geometric weighted average of public capital, K_g , and aggregate private capital, K , $X \equiv K^\varepsilon (K_g)^{1-\varepsilon}$, ($0 < \varepsilon < 1$). In effect, the production technology is a combination of the Romer (1986) technology ($\varepsilon = 1$) and the Barro (1990) technology ($\varepsilon = 0$).⁴ Not only is this a plausible representation of infrastructure, which frequently is financed by a mixture of public and private resources, but as Chatterjee and Turnovsky (2012) note, and as we shall show below, it facilitates a more plausible calibration.

All firms employ the same technology, and hence all demand identical quantities of capital and labor; that is, $K_j = K$, $L_j = L$. Aggregate (average) economy-wide output can, therefore, be expressed as:

$$Y = E \cdot (X \cdot L)^s (K)^{1-s} = E \cdot L^s (K_g/K)^{s(1-\varepsilon)} K. \quad (1a)$$

Assuming that firms maximize profit, the rental rate, r , and the real wage rate, w , are determined by the respective private marginal products of capital and labor:

$$r = (1 - s) E \cdot L^s \left(\frac{K_g}{K} \right)^{s(1-\varepsilon)} = (1 - s) Y/K, \quad (2a)$$

$$w = sE \cdot L^{s-1} \left(\frac{K_g}{K} \right)^{s(1-\varepsilon)} K \equiv \tilde{w} \cdot K = sY/L. \quad (2b)$$

From (1a) and (2), we see that public capital raises the productivity of both private factors of production, highlighting its role as key driving forces of growth.

1.2 Heterogeneous Households

We let \mathcal{N} denote the number of households, equivalent to the total population of the economy, which remains constant over time. Individual (household) i owns $A_i(t)$ units of real wealth at time t , and the economy-wide average of total wealth is $A(t) = \frac{1}{\mathcal{N}} \int_0^{\mathcal{N}} A_i(t) di$. The relative share of total wealth owned by individual i is $a_i(t) \equiv A_i(t)/A(t)$, the mean of which is one. There are three sources of private real

4. Our introduction of productive government expenditure as public capital (rather than as a current flow) into an endogenous growth model follows Futagami et al. (1993) and earlier seminal work of Arrow and Kurz (1970).

wealth: private capital, $K(t)$, real money balances, $M(t)$, and real government bonds, $B(t)$, with public capital being a pure public good. Agent i 's holdings of each source of private wealth is denoted by the subscript i , with his relative share of each being defined analogously to $a_i(t)$.⁵

As the economy transitions, the evolution of individuals' relative wealth, $a_i(t)$, traces out the distribution of wealth across agents, the standard deviation of which, $\sigma_a(t)$, serves as a convenient measure of wealth inequality. With the initial distribution of capital endowments being predetermined, and capital being constrained to be accumulated gradually, $\sigma_k(0) \equiv \sigma_{k,0}$ is given and is one key underlying source of heterogeneity. At the same time, while the agent's initial holdings of *nominal* money supply and bonds are also predetermined, the initial distribution of *real* wealth, $\sigma_a(0)$, is endogenous. This is because of the initial jump in the price level following any structural or policy change, and discussed in detail in Appendix E. Over time, the accumulation of the individual's relative wealth drives his relative income, thereby generating the overall income distribution across agents.

Each individual is endowed with one unit of time that can be allocated between leisure, $l_i(t)$, and labor, $L_i(t) = 1 - l_i(t)$, so that the average economy-wide labor and leisure can be expressed as: $L = 1 - l = 1 - \frac{1}{N} \int_0^N l_i di$. The agent maximizes lifetime utility, specified as a constant elasticity function of consumption, $C_i(t)$, leisure, $l_i(t)$, real money balances, $M_i(t)$, as well as government consumption, G_C , the latter being taken as given:

$$\int_0^{\infty} \frac{1}{\gamma} (C_i(t) l_i(t)^\beta M_i(t)^\eta G_C(t)^\phi)^\gamma e^{-\rho t} dt - \infty < \gamma < 1, \quad \beta > 0, \quad \eta > 0, \quad \phi > 0. \quad (3)$$

The first inequality in (3) is required for the intertemporal elasticity of substitution (IES), $\kappa \equiv (1 - \gamma)^{-1} > 0$. Empirical evidence overwhelmingly supports $0 < \kappa < 1$, in which case $\gamma < 0$, ensuring that the utility function is concave in all its arguments. In the less prevalent case $\kappa > 1$, the utility function will be concave with respect to the three choice variables, $C_i(t)$, $l_i(t)$, and $M_i(t)$, if and only if $1 > \gamma(1 + \beta + \eta)$. Strengthening this latter condition mildly to $1 > \gamma(1 + \beta + \eta + \phi)$ ensures that the concavity of the utility function extends to all four variables.

Recalling that $M_i(t)$ and $B_i(t)$, respectively, represent the *real* quantities of money and bonds held by agent i , his rate of wealth accumulation is:

$$\begin{aligned} \dot{K}_i(t) + \dot{M}_i(t) + \dot{B}_i(t) &= (1 - \tau_K) r(t) K_i(t) + (i(t) - \pi(t)) B_i(t) \\ &\quad - \pi(t) M_i(t) + (1 - \tau_w)(1 - l_i(t)) w(t) - (1 + \tau_C) C_i(t), \end{aligned} \quad (4)$$

5. We note that while the RCTD approach can incorporate heterogeneous labor productivity, we abstract from it here. García-Peñalosa and Turnovsky (2015) introduce heterogeneous labor into a nonmonetary model, showing that it has little effect on the overall structure of the macrodistributional equilibrium. Thus, it would not affect our results describing how debt policy impacts the macro economy. But they also show that it influences the evolution of relative wealth and income and has distributional consequences, though we would not expect them to substantially affect our main results.

where $i(t)$ is the nominal interest rate on government bonds, $\pi(t)$ is the rate of inflation, τ_K , τ_w , and τ_C are, respectively, fixed tax rates on capital income, labor income, and consumption that agent i takes as given. For simplicity, and without any loss of generality, capital is assumed not to depreciate. Performing the optimization yields the following standard first-order conditions:

$$C_i^{\gamma-1} l_i^{\beta\gamma} M_i^{\eta\gamma} G_C^{\phi\gamma} = \lambda_i, \quad (5a)$$

$$\beta \frac{C_i}{l_i} = \frac{1 - \tau_w}{1 + \tau_C} w(K_g, K, l), \quad (5b)$$

$$\eta \frac{C_i}{M_i} = \frac{i}{1 + \tau_C}, \quad (5c)$$

$$(1 - \tau_K)r = i - \pi, \quad (5d)$$

$$\rho - \frac{\dot{\lambda}_i}{\lambda_i} = (1 - \tau_K)r(K_g, K, l), \quad (5e)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i K_i = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i M_i = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} e^{-\rho t} \lambda_i B_i = 0, \quad (5f)$$

where λ_i is agent i 's shadow value of wealth, $A_i(t)$. Using (2) and $l = 1 - L$, we can express the real wage, w , and interest rate, r , as functions of average leisure l .

Equation (5a) equates the agent's marginal utility of consumption to his shadow value of wealth. Equation (5b) equates the marginal rate of substitution between consumption and leisure to the real wage rate, adjusted by the effective tax rate on labor income $\frac{1-\tau_w}{1+\tau_C}$, while (5c) equates the marginal rate of substitution between consumption and real balances to the nominal interest rate which reflects the opportunity cost of holding money.⁶ Equation (5d) is the arbitrage condition equating the real return on bonds to the after-tax return on capital, while equation (5e) is the Euler equation that equates these rates of return to the rate of return on consumption. This equation implies that each agent chooses the same growth rate for the shadow value of wealth, irrespective of his wealth. Finally, (5f) are the transversality conditions that ensure the intertemporal viability of the household's decisions.

6. Excluding interest income from taxation is inconsequential since it is reflected in $i(t)$ via the arbitrage condition (5d).

Using (4), (5), and $A_i(t) \equiv K_i(t) + M_i(t) + B_i(t)$, agent i 's rate of wealth accumulation is:

$$\frac{\dot{A}_i}{A_i} = (1 - \tau_K) r(K_g, K, l) + (1 - \tau_w) \frac{w(K_g, K, l)}{A_i} \left(1 - \frac{1 + \beta + \eta}{\beta} l \right), \quad (6)$$

which, in conjunction with the aggregate wealth accumulation, will be used to determine the evolution of the relative share of wealth owned by an agent i , $a_i(t)$.

We define the economy-wide averages $X = \frac{1}{N} \int_0^N X_i di$ for all relevant variables, $X = K, M, B, l$. A key consequence of the optimality conditions (5b) and (5c) is that since all agents face the same real wage, and same interest rate, the marginal rates of substitution between consumption and leisure and between consumption and real money balances are common to all agents, implying:

$$\frac{C_i(t)}{C(t)} = \frac{M_i(t)}{M(t)} = \frac{l_i(t)}{l(t)} \equiv \zeta_i \quad (7a)$$

so that agent i 's share of the three averages maintains the same constant value, ζ_i , over time. In each case, $\zeta_i \geq 1$ specifies whether agent i 's position is above or below the economy-wide average. Equation (7a) further implies that all agents choose the same growth rates for consumption, leisure, and real money balances (although their levels differ):

$$\frac{\dot{C}_i(t)}{C_i(t)} = \frac{\dot{C}(t)}{C(t)}; \quad \frac{\dot{M}_i(t)}{M_i(t)} = \frac{\dot{M}(t)}{M(t)}; \quad \frac{\dot{l}_i(t)}{l_i(t)} = \frac{\dot{l}(t)}{l(t)}, \quad (7b)$$

which in turn implies that the aggregates grow at the same respective rates.

1.3 Government

We specify the consolidated government budget constraint in *real* terms by:

$$\dot{M} + \dot{B} = G + (i - \pi)B - \pi M - \tau_K rK - \tau_w w(1 - l) - \tau_C C, \quad (8)$$

where G denotes total government expenditure, which takes one of two forms. The first is utility-enhancing government consumption, $G_C(t)$, already introduced; the second is investment, $G_I(t)$ that enhances the stock of public capital, so that $G = G_C + G_I$. To sustain an equilibrium of endogenous growth, the government fixes its expenditures as constant fractions of current output:

$$G_C(t) = g_C Y(t) \quad 0 < g_C < 1, \quad (9a)$$

$$\dot{K}_G(t) = G_I(t) = g_I Y(t) \quad 0 < g_I < 1, \quad (9b)$$

where, like private capital, public capital is assumed not to depreciate. Finally, we assume that the tax rates τ_K , τ_w , τ_C are set exogenously, implying that the burden of ensuring that the government budget is intertemporally sustainable falls upon its debt and money financing policy.

A key element of our setup is the specification of debt policy. The choice of a benchmark formulation involves trading off tractability, compatibility with the endogenous growth framework, and its plausibility relative to actual policy. Taking these issues into account, we assume that the government adopts the following debt policy, of maintaining a constant ratio of debt to money:

$$B(t)/M(t) = \xi, \quad \xi \text{ is constant.} \quad (10)$$

Although B and M are *real* variables, equation (10) also implies that the government maintains the ratio of the corresponding *nominal* quantities constant. A reduction in the ratio, ξ , describes a policy of debt reduction directed at increasing the proportion of money used to finance the current deficit.

In terms of the above criteria, this specification of debt policy offers several advantages. First, it has a well-established tradition in monetary growth models, dating back to Foley and Sidrauski (1970, 1971) and others. It is compatible with the present endogenous growth setup, which requires that to sustain a long-run balanced growth equilibrium, all quantities must ultimately grow at the same constant rate. Thus, (10) implies $\dot{B}(t)/B(t) = \dot{M}(t)/M(t)$ and will track the growth rate followed by the real money supply, ultimately converging to a long-run economy-wide balanced growth rate.

Empirically, the constant ratio policy, (10), seems to be a reasonable approximation to recent U.S. monetary policy particularly during periods of relative stability. To support this view, it is useful to break down the period 1995–2019 into the subperiods 1995–2008 (prefinancial crisis) and 2009–2019 (postfinancial crisis and pre-COVID-19), both of which are periods of relative stability. During the latter period 2009–19, the average ratio of government bonds to M3 (a broad definition of money that we feel is appropriate to reflect the utility benefits it provides) is 1.35, with a standard deviation of only 0.058. In the earlier period, the figures are 1.14 and 0.098, respectively, which still suggest a rather stable relationship over substantial periods of time; see Figure O.1 in Online Appendix.

With minor modification, (10) could be adapted to specify financial policy in terms of targeting debt to output, in which case $\dot{B}(t)/B(t) = \dot{Y}(t)/Y(t)$. With all real variables converging to the identical long-run growth rate, which as will be shown below is independent of debt policy, the main difference would be along the transitional path, with little added additional insight. The neutrality characteristics, described below, associated with debt policy would continue to apply. However, empirically, the Federal debt-GDP ratio is much less stable than our proposed B/M3 ratio. Between 1995 and 2008, the ratio of Federal debt held by the public as a percent of GDP fell

steadily from 48% to 36%, while between 2009 and 2019, it increased gradually from around 50% to 76%.⁷

One final point: by identifying money with M3 we are abstracting from the financial system, since central banks exercise monetary policy by operating on base money. While we recognize this as a (routinely adopted) simplification, over extended periods, the ratio of M3/base money has been pretty stable, although there has been much more variation during recent years.⁸ But with this caveat, we view our specification of the constant debt ratio as providing a plausible benchmark for the purpose of specifying government financial policy in a stylized long-run monetary growth model such as this, in effect viewing it as a “reduced form” summary of central bank policy.⁹

Substituting (10) into (8), using the optimality conditions (5c) and (5d), and the aggregation condition (7), we may rewrite (8) in the form:

$$\begin{aligned} \dot{M}(t) = & r(1 - \tau_K)M(t) + \frac{1}{1 + \xi} (G(t) - \tau_K r(t)K(t) - \tau_w w(t)(1 - l(t)) \\ & - [\tau_C + \eta(1 + \tau_C)]C(t)). \end{aligned} \quad (11)$$

Solving (11) and invoking (5f) yields the intertemporal government budget constraint:

$$\begin{aligned} M_0 + \left(\frac{1}{1 + \xi} \right) \int_0^\infty [G(t) - \tau_K r(t)K(t) - \tau_w w(t)L(t) - \tau_C C(t) \\ - (1 + \tau_C)\eta C(t)] e^{-\int_0^t (1 - \tau_K)r(s)ds} dt = 0. \end{aligned} \quad (11a)$$

Any combination of fiscal and debt policy consistent with (11a) is intertemporally sustainable.

7. See <http://fred.stlouisfed.org/series/FYFGDQ188S>. We should stress that many plausible debt policies are incompatible with the endogenous growth setup. For example, the policy assumed by Gali (2020) of holding real debt constant, ($\dot{B} = 0$), is incompatible with sustaining a long-run balanced growth path, since with growth, the relative stock of bonds would be steadily declining. Likewise, the rule of maintaining the deficit to GDP ratio (\dot{B}/Y) constant, (Minea and Villieu, 2012) is also incompatible, since with ongoing GDP growth, the growth rate of the deficit would have to be ever-increasing.

8. For example over the period 1994–2008, prior to the financial crisis, the ratio of M3/Monetary base was 8.33 with a standard deviation of only 0.20. The relative constancy of this ratio over a substantial time period suggests that little is lost by following standard practice and abstracting from a financial sector, although this merits further study.

9. Being long-run, it obviously misses the short-run fluctuations and should not be interpreted as a precise statement of U.S. monetary policy. In this regard, there is a literature examining the long-run relationship between government debt and the monetary base. While some studies find evidence of a stable relationship, reflecting the outcome as the Fed and Treasury pursue their separate objectives, they do not interpret this as reflecting any significant causal relationship; see, for example, Ahmed (2020). Since our primary objective is to address how alternative forms of government spending impact long-run inequality in a growing monetary economy, this more flexible interpretation suffices for our purposes.

2. MACROECONOMIC EQUILIBRIUM

2.1 Market Equilibrium

With the underlying production function being constant returns to scale, in a steady state, all the average (aggregate) variables, Y , K , K_g , M , and A , grow at the same endogenously determined, constant rate, while the allocation of time to leisure, $l (= 1 - L)$, is constant. In Appendix A, we show that the transitional dynamics, characterizing the macroequilibrium, can be summarized by the evolution of the three stationary variables, $z \equiv K_g/K$, $m \equiv M/K$, and l :

$$\frac{\dot{z}}{z} = \frac{\dot{K}_g}{K_g} - \frac{\dot{K}}{K} \equiv g_l \frac{y(z, l)}{z} - (1 - g_l - g_C)y(z, l) - c(z, l), \quad (12a)$$

$$\begin{aligned} \frac{\dot{m}}{m} = \frac{\dot{M}}{M} - \frac{\dot{K}}{K} = (1 - \tau_K)r(z, l) + \frac{1}{m(1 + \xi)} \{ [g_l + g_C - \tau_K(1 - s) \\ - \tau_w s] y(z, l) - [\tau_C + \eta(1 + \tau_C)] c(z, l) \} - \frac{\dot{K}}{K}, \end{aligned} \quad (12b)$$

$$\frac{\dot{l}}{l} = \frac{1}{\Delta} \left[(1 - \tau_K)r(z, l) - \rho + \eta\gamma \frac{\dot{m}}{m} - s(1 - \varepsilon)(1 - \gamma - \phi\gamma) \frac{\dot{z}}{z} - \Pi \frac{\dot{K}}{K} \right], \quad (12c)$$

where

$$y(z, l) \equiv E(1 - l)^s z^{s(1 - \varepsilon)} (= Y/K); r(z, l) = (1 - s)y(z, l),$$

$$c(z, l) \equiv \left[\frac{1 - \tau_w}{1 - \tau_C} \right] \left(\frac{1}{\beta} \right) sE(1 - l)^{s-1} l z^{s(1 - \varepsilon)} (= C/K),$$

$$\Delta \equiv [(1 - s)(1 - \gamma(1 + \phi)) + \phi\gamma] \frac{l}{1 - l} + (1 - \gamma(1 + \beta));$$

$$\Pi \equiv 1 - \gamma(1 + \eta + \phi).$$

The assumption of concave utility in all four variables ensures that $\Pi > 0$, while very weak conditions ensure $\Delta > 0$, as well. Specifically, $s < 1/[1 + (1 - \kappa)\phi]$ suffices to ensure $\Delta > 0$, which with $s < 1$ always holds, provided the intertemporal elasticity of substitution $\kappa \leq 1$, as empirical evidence strongly suggests. The condition is also clearly met for the calibrated parameter values $s = 0.6$, $\kappa = 0.4$, $\phi = 0.22$, that we employ in our simulations.

Equations (12) determine the evolution of the macrodynamic equilibrium, as driven by $z(t)$, $m(t)$, and $l(t)$, independently of wealth and income distributions. This is a manifestation of the RCTD upon which our setup is based. Equation (12c), describing the evolution of leisure (or labor), is derived from the Euler equation, spelled out in more detail in Appendix A. Two further points: (i) the growth rate of private capital, \dot{K}/K , reflects goods market equilibrium, and (ii) the equilibrium inflation rate, $\pi(t)$

$$\pi(t) = (1 + \tau_C) \frac{\eta^c(z, l)}{m} - (1 - \tau_K)r(z, l) \quad (13)$$

is obtained from the optimality conditions (5c) and (5d). As we show in Appendix E, following a policy change, the price level undergoes an initial jump to ensure that the real money supply is on the stable path (16c) and thereafter the price level evolves continuously in accordance with (13).¹⁰

B: Steady-State Equilibrium

During the transition, the aggregate variables grow at differential rates, ultimately converging to a common balanced growth rate, with the allocation of time, l and the ratio of public to private capital, z being constant and independent of debt policy, ξ .

With all aggregate quantities growing at the same rate, the implied steady-state growth rate can be expressed in alternative equivalent ways. One convenient and familiar form is to use (12c), in effect the growth rate of consumption, and writing it as:

$$\Psi^* = \frac{(1 - \tau_K)r(z^*, l^*) - \rho}{\Pi} = \frac{(1 - \tau_K)(1 - s)E(1 - I^*)^s(z^*)^{s(1-\varepsilon)} - \rho}{\Pi}, \quad (14)$$

where $*$ denotes steady state. Equation (14) expresses the steady-state growth rate as being proportional to the difference between the after-tax return on capital (which increases with z^* and decreases with l^*) and the given rate of time preference, as long as $\Pi > 0$ is satisfied, as the assumption of concavity assures. It is also independent of debt policy, ξ . Moreover, the transversality conditions (5f) are met if and only if $(1 - \tau_K)r(z^*, l^*)(\Pi - 1) + \rho > 0$, a weak sufficient condition for which is $\gamma < 0$.

Appendix B reports how the government expenditure-GDP ratio influences the balanced growth rate. There we show that an increase in the government investment-GDP ratio, g_I , by increasing the productivity of labor, raises the wage rate, causing agents to substitute labor supply for leisure. It also raises the productivity of private capital inducing a decrease in z . Its direct effect is to raise the public-private capital ratio, which almost certainly dominates the indirect effect, and on balance, z will

10. Equation (13) implies a negative partial relationship between real money balances and inflation. This is endemic to all monetary growth models, dating back to Sidrauski (1967). It is a reflection of the arbitrage condition (5d) and the fact that an increase in inflation raises the nominal interest rate and thus reduces the demand for real money balances.

almost certainly increase. The increase in labor supply and the probable increase in the public-private capital ratio are mutually reinforcing and will cause the balanced growth rate to increase.

An increase in the government consumption-GDP ratio, g_C , operates through a different mechanism. An increase in g_C raises the marginal utility of wealth, inducing people to work more, thereby reducing leisure. The increase in labor increases the productivity of private capital relative to public capital, causing z^* to decline. On balance, the labor supply effect dominates and the equilibrium growth rate increases, although less so than it would for an equivalent increase in g_I .

In effect, debt policy, as specified by the ratio of government debt to money, is neutral in its impact on the aggregate real variables, as summarized in the following proposition:

PROPOSITION 1. (I). *An increase in the debt-money ratio ξ has no impact on (i) steady-state leisure l^* , (ii) steady-state public-private capital ratio, z^* , or (iii) steady-state ratio of the sum of money plus bonds to capital $(1 + \xi)m^* [= m^* + b^*]$. However, it causes the ratio of money to capital to decline and inflation to increase.*

(II). *An increase in either the rate of government investment, g_I , or consumption, g_C , increases the steady-state growth rate, the former being more potent than the latter.*

(III). *The debt-money ratio has no effect on the steady-state growth rate, or how the growth rate is affected by an increase in either government investment or consumption.*

C: Local Dynamics

To see how the inequality of wealth and income evolve, we must first determine how the aggregate macroeconomic variables develop over time. Linearizing equations (12a)–(12c) around the steady state (l^*, z^*, m^*) , we can express the local dynamics of l , z and m as:

$$\begin{bmatrix} \dot{l} \\ \dot{z} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ z^* \Omega_l(l^*, z^*) & z^* \Omega_z(l^*, z^*) & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} l - l^* \\ z - z^* \\ m - m^* \end{bmatrix}, \quad (15)$$

where the elements of the matrix in (15) are defined in Appendix C. By direct calculation, one can show that its determinant, D , is negative if and only if $\Delta > 0$. This ensures that there are either one, or three, negative eigenvalues. While, the dynamic system is too complex to determine analytically which of these two cases prevails, in all our simulations, we obtain just one negative eigenvalue, $\mu < 0$. Since $z(t)$ is predetermined at time t , while the two variables $l(t)$ and $m(t)$ are free to adjust instantaneously at time t (the latter via a jump in the price level), the steady state (l^*, z^*, m^*)

is locally a saddlepoint. Accordingly, the aggregate variables, $z(t)$, $l(t)$, $m(t)$ can be shown to converge to the steady state at the asymptotic speed $|\mu|$:¹¹

$$z(t) = z^* + (z_0 - z^*) e^{\mu t}, \quad (16a)$$

$$l(t) = l^* + \left(\frac{\mu - z^* \Omega_z^*}{z^* \Omega_l^*} \right) (z(t) - z^*), \quad (16b)$$

$$m(t) = m^* + \frac{a_{32} + a_{31} \left((\mu - z^* \Omega_z^*) / z^* \Omega_l^* \right)}{\mu - a_{33}} (z(t) - z^*). \quad (16c)$$

Equations (15), (16a)–(16c), and Proposition 1 enable us to establish the proposition that the bond-money ratio ξ has no effect on the aggregate real variables during the transition to steady state:

PROPOSITION 2. *The debt-money ratio ξ has no impact on the rate of convergence $|\mu|$. In addition, ξ has no effect on the transitional paths of the economy-wide average real variables $z(t)$, $l(t)$ and $(1 + \xi)m(t)[= m(t) + b(t)]$, or on the growth rates $\dot{K}(t)/K(t)$, \dot{K}_g/K_g , and $\dot{Y}(t)/Y(t)$ that are functions of $z(t)$ and $l(t)$.*

The formal proof is provided in the Online Appendix. The significance of Proposition 2 is to show that reducing ξ to redeem accumulated government bonds by issuing money has no effect on the real aggregate economy, either in the short run or over time. It is straightforward to show that the impact of increases in either form of government expenditure on the transitional dynamics of the real aggregate economy are independent of the debt policy employed to finance the fiscal expansions, as long as they ensure that the sustainability condition (11') is met.

3. WEALTH AND INCOME INEQUALITY

We now turn to the implications for the distributions of wealth and income. Since this procedure is not new, details are relegated to Appendix D; only the key steps are noted here.

3.1 Dynamics of Relative Wealth

The evolution of the relative wealth of individual i , described by equations (D.3) and (D.4), reflects two underlying sources of dynamics. First are those of the aggre-

11. Because our benchmark calibration provides plausible rate of convergence ($\mu < 0$) and economic growth ($\Psi^* > 0$) from $z^* \Omega_z^* = -s\Psi^* < 0$, the condition $\mu - z^* \Omega_z^* < 0$ can readily be shown to be satisfied. In that case, combining the condition with $z^* \Omega_l^* > 0$ implies that the slope of (16b) is negative.

12 Summing (5b) over all individuals implies the aggregate relationship $(1 + \tau_c)C = (1 - \tau_w)w/\beta$. Combining this with the assumption $(1 + \tau_c)C > (1 - \tau_w)w(1 - l)$ immediately yields $l^* > \beta/(1 + \beta)$.

gate variables, $z(t)$, $m(t)$, $l(t)$, described by (16), that generate the returns to capital and labor. Second are the internal dynamics of relative wealth, $a_i(t)$, which depend upon the coefficient of a_i in (D.1) in the neighborhood of steady state. Under weak conditions, this coefficient can be shown to be positive. Specifically, assuming that private consumption, inclusive of the consumption tax exceeds labor income, then aggregating the optimality condition, (5b), over individuals implies $l^* > \beta/(1 + \beta)$ which in turn ensures:¹²

$$l^* > \frac{\beta}{1 + \beta + \eta} \equiv v. \quad (17)$$

Imposing (17), (D.2) then implies that the greater is an agent's steady-state relative wealth, the more leisure he enjoys and the less labor he supplies. This reflects the fact that wealthier agents have a lower marginal utility of wealth.¹³

In general, the individual's initial relative wealth, $a_i(0)$, is endogenous. This is because bonds and money, being denominated in nominal terms, are subject to a real shock due to the jump in the price level, $p(0)$, at the time a policy change or structural change occurs. Equation (E.2) sets out the relationship between the jump in $p(0)$, and initial relative wealth, showing that the impact of $p(0)$ on $a_i(0)$ depends upon the difference between the individual's portfolio allocation between the real asset (capital) and the nominal assets (money and bonds) and that of the economy-wide portfolio.

Relative income is shown to be driven by relative wealth. To describe this, we need to determine the agent's relative position in real money balances, M , and income-yielding assets, $K + B$, to his relative wealth. These are given by equations (D.5) and (D.6), respectively. In the case of the latter, with capital and bonds yielding the same rate of return, and in the absence of risk, investors view these two assets as perfect substitutes, enabling us to characterize only their *composite* distribution. Comparing (D.5) and (D.6), we see that the relative asset holdings of an agent who has above-average wealth, $a_i(t) > 1$ are: $v_i(t) - 1 > a_i(t) - 1 > M_i/M - 1$, meaning that his portfolio comprises disproportionately more of the income-earning assets, and less money than his overall relative wealth position represents. An increase in debt, ξ , increases the total proportion of income-earning assets in wealth, while reducing that of real money balances. For a poor agent, that is, $a_i(t) < 1$, the opposite applies, thus reducing the differential impact between the two sets of agents.

12. Summing (5b) over all individuals implies the aggregate relationship $(1 + \tau_c)C = (1 - \tau_w)wl/\beta$. Combining this with the assumption $(1 + \tau_c)C > (1 - \tau_w)w(1 - l)$ immediately yields $l^* > \beta/(1 + \beta)$.

13. Empirical evidence in support of this negative relationship between wealth and labor supply (given the wage rate) is available from a variety of sources; see, for example, MaCurdy (1981), Holtz-Eakin et al. (1993), and Coronado and Perozek (2003). If labor productivity is heterogeneous, then labor supply involves a tradeoff between the agent's relative wealth and his differential wage rate and this will impact wealth and income inequality; see García-Peñalosa and Turnovsky (2015).

3.2 Dynamics of Relative Income

With several alternative income measures, there are several potential measures of income inequality. We shall focus on before-tax personal income, defined as income from the income-earning assets (capital and bonds), plus income from labor. Equations (D.7a) and (D.7b) show that the relative before-tax income of agent i , $y_i(t) \equiv Y_i(t)/Y(t)$, can be expressed as:

$$y_i(t) - 1 = \varphi(t) [a_i(t) - 1], \quad (18a)$$

where

$$\varphi(t) = \left\{ \frac{r(1 + \xi m)}{r(1 + \xi m) + \tilde{w}(1 - l)} \left(1 + \frac{m}{1 + \xi m} \left(1 - \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha} \right) \right) - \frac{\tilde{w}l}{r(1 + \xi m) + \tilde{w}(1 - l)} \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha} \right\}. \quad (18b)$$

The term $\varphi(t)$ comprises two components: (i) the share of personal income from the income-generating assets, bonds plus capital, and (ii) relative labor income, reflected in the second term, capturing the fact that more (less) wealthy agents supply less (more) labor. From (18b), the second term is negative. In our benchmark calibration, it is dominated by the first term, implying $0 < \varphi(t) < 1$.

It is also seen from $\varphi(t)$ that relative income is impacted by debt policy, ξ . This is because personal income includes interest income from government bonds. However, the effect of ξ is not clearcut, being a reflection of the fact that while an increase in ξ increases the share of income derived from the income-earning assets, it raises their share in the agent's overall wealth portfolio.¹⁴

3.3 Wealth and Income Inequality

Because of the linearity of (D.3), (18a), and (18b), we can readily transform these equations into corresponding relationships for the standard deviations of wealth and income across agents. These serve as convenient and tractable measures of wealth and income inequality.

Aggregating (D.3) across agents and using (D.4), wealth inequality at time t , $\sigma_a(t)$, is:

$$\sigma_a(t) = \frac{\alpha(t)}{\alpha(0)} \sigma_a(0), \quad (19)$$

which converges in steady state to $\sigma_a^* = \frac{1}{\alpha(0)} \cdot \sigma_a(0)$. Equation (19) highlights how the evolution of $\sigma_a(t)$ is driven by two factors, $\sigma_a(0)$, and $\alpha(t)$. As noted in equation (D.4b), each individual's initial relative wealth, $a_i(0)$, is endogenous, reflecting the

14. The former effect is described by the term $d((1 + \xi m))/d\xi > 0$ and the latter by $d(m/(1 + \xi m))/d\xi < 0$ in (18b).

effect of the shock on the initial price level and its impact on the preshock real endowments of the nominal assets (bonds and money), N_0 . In Appendix E, we show that the initial postshock wealth inequality across individuals is:

$$\sigma_a(0) = [(1 - \chi)^2 \sigma_{k,0}^2 + 2(1 - \chi) \chi \sigma_{k,n,0} + \chi^2 \sigma_{n,0}^2]^{1/2}, \tag{20}$$

where $\chi \equiv N_0/(p(0)K_0 + N_0)$, is the ratio of nominal assets to total wealth, $\sigma_{k,0}$, $\sigma_{n,0}$ are standard deviations of the initial endowments, and $\sigma_{k,n,0}$ is their covariance. We show that while $\sigma_a(0)$ is independent of debt policy, ξ , it is impacted by other real shocks, such as government expenditure, with the qualitative effects in this case depending upon the original endowment pattern; see equation (E.5).

The other driving force, $\alpha(t)$, summarizes the evolution of relative wealth over time and is defined by (D.3). Recalling Proposition 1, we know that $\partial l(t)/\partial \xi = 0$, $\partial z(t)/\partial \xi = 0$ and $\partial[(1 + \xi)m(t)]/\partial \xi = 0$ for $t \in [0, \infty)$. Thus, from (D.3), we see that $\alpha(t)$ is independent of debt policy, ξ , which, therefore, has *no* effect on wealth inequality at *any* stage during the transition. In addition, we see that $\alpha(t)$ declines as leisure increases over time. Thus, in conjunction with the aggregate dynamics and with the observation that $\dot{l}(t)$ is negatively related to $\dot{z}(t)$ (footnote 11), we see that an increase in the ratio of government capital to private capital will be associated with an increase in the relative wealth of the affluent. Furthermore, relative wealth accumulation occurs only during the transition and is sensitive to the time path of leisure (labor). If $l(t)$ jumps instantaneously to its new steady state, there is no transition, $\alpha(t) \equiv 1$, and there is no relative wealth accumulation.

We may summarize these results in conjunction with (D.5) and (D.6) in the proposition:

PROPOSITION 3. (I). *An increase in the debt-money ratio, ξ , has no impact on initial wealth inequality, $\sigma_a(0)$. With $\alpha(t)$ also independent of ξ , this implies further that debt policy has no impact on the subsequent transitional paths of either wealth inequality, $\sigma_a(t)$, or that of real balances, $\sigma_M(t) = (1 - \nu/l^*)(\sigma_a(t)/\alpha(t))$.*

(II). *In contrast, an increase in ξ does impact agent i 's relative holding of the composite income-earning asset, $v_i(t)$. By increasing the relative supply of income-earning assets, it causes the relative affluent (poor) to reduce (increase) their relative holdings, $v_i(t)$, causing $\sigma_v(t) = [1 + m(t)/(1 + \xi m(t))[1 - (1 - \nu/l^*)(1/\alpha(t))]]\sigma_a(t)$ to decline.*

Similarly, from (18), we can express the relationships between relative income in terms of corresponding standard deviations of their respective distributions,

$$\sigma_y(t) = \varphi(t) \cdot \sigma_a(t), \tag{21}$$

which correspondingly converge to $\sigma_y^* = \varphi^* \cdot \sigma_a^*$.¹⁵ Although the time path of wealth inequality is independent of debt policy, ξ nevertheless affects income inequality,

15. $\varphi^* = [r^*(1 + \xi m^*) + \bar{w}^*(1 - l^*)]^{-1} \{r^*(1 + (1 + \xi)m^*) - (r^*m^* + \bar{w}^*l^*)[1 - \nu l^{*-1}]\}$

TABLE 1
BASIC PARAMETERS

Production parameters	$s = 0.6, E = 0.9$
Geometric weight of average externality	$\varepsilon = 0.6$
Taste parameters	$\beta = 1.4, \eta = 0.12, \phi = 0.22, \rho = 0.04, \gamma = -1.5$
Tax rates	$\tau_k = 0.276, \tau_w = 0.224, \tau_c = 0.08$
Government expenditure ratio	$g_c = 0.150, g_l = 0.035$
Transfer-wealth ratio	$\tau = 0.0137$
Debt policy	$\xi = 1.35$
Initial public-private-capital ratio	$z_0 = 0.5759$

Note: Initial value of z is set at $z_0 = z^*$.

$\sigma_y(t)$, through its effects on the term $\varphi(t)$. Accordingly, we may summarize these results as follows:

PROPOSITION 1. *Debt policy, as specified by the bond-money ratio, ξ , impacts income inequality through the income generated by assets and labor (reflected in $\varphi(t)$), though this depends upon the income measure including interest earned on government bonds. If it is restricted to income from productive factors, it will have no effect.*

4. CALIBRATION

Henceforth, we analyze the effects of increases in the two forms of government expenditure on the aggregate and inequality variables, assuming that the fiscal expansion is financed by an increase in money coupled with the specified debt policy, in accordance with (11) and (11b). Given the complexity of the setup, we calibrate the model and simulate it numerically. In doing so, we wish to clarify two aspects. First, in virtually all cases, our parameters are chosen as consensus values, typically drawn from empirical estimates, or in some cases, averages of underlying data, rather than by targeting some specific moment. In this respect, the parameters pertaining to production and utility are standard and noncontroversial. Parameters pertaining to monetary and debt policy are less available. Since our focus is on long-run relationships, insofar as possible, we base our parameters on averages over periods of stability, rather than on some common historical period, which in some cases may be associated with financial turbulence and be misleading for our purpose of characterizing a long-run stable relationship.

The parameter values we employ are reported in Table 1, with the resulting benchmark equilibrium values for both key aggregate quantities and inequality measures being shown in Table 2.¹⁶ With the labor elasticity in the Cobb–Douglas production

16. The benchmark calibration also includes a small lump-sum transfer $\tau = 0.0137$ the sole purpose of which is to yield benchmark values of the inflation and growth rates closer to that of the United States over the period 2000–20.

TABLE 2

BENCHMARK STEADY-STATE EQUILIBRIUM: $g_C = 0.150$ AND $g_I = 0.035$

z^*	l^*	y^*	m^*	π^*	Ψ^*	σ_v^*	σ_d^*	σ_y^*
0.5759	0.7101	0.3750	0.2908	0.0174	0.0228	1.1634	1.00	0.2837

function set at its conventional value, $s = 0.6$, setting the production externality parameter at the polar values $\varepsilon = 1$ (Romer) or $\varepsilon = 0$ (Barro) are both unsatisfactory. The former implies that the external effect of the aggregate capital stock is significantly more productive than is the firm's own capital; the latter implies an unrealistically large productive elasticity for public capital. Setting $\varepsilon = 0.6$ helps resolve both these issues, and reduces the productive elasticity of public capital to 0.24, close to the plausible range reported by Bom and Ligthart's (2014) extensive meta-study. The remaining productive parameter, $E = 0.9$, reflects TFP and is important in calibrating the equilibrium growth rate.

Regarding the utility parameters, the rate of time preference, $\rho = 0.04$, is standard, while the IES corresponding to $\gamma = -1.5$ (0.4) is well within the estimated range reported by Guvenen (2006) and others. The elasticity of leisure in utility is critical in determining the agent's allocation of time, and setting $\beta = 1.4$ yields the steady-state allocation of time to leisure of $l^* = 0.710$, comparable to the evidence provided by Cooley (1995) and the real business cycle literature in general.

Information on η and ϕ is less direct. In the case of real money balances, our choice of $\eta = 0.12$ is justified as follows. First, it depends upon how broadly one views money with respect to the utility benefits it provides. The implied value of the real money balance-capital ratio is $m^* = 0.2908$, so that the real money balances to consumption ratio is $m^*/c^* = 1.028$. The average value of the real M3-consumption ratio for the period 2002–20 in the United States is 0.88, suggesting that the parameter choices pertaining to money are broadly consistent with the data.

In Section 1.3, in justifying our specification of the constant debt-money policy (10), we appealed to the extremely small variation in the ratio of government bonds held by the public to M3 over the period 2009–19 (mean 1.35, s.d. 0.058) on the basis of which we set the government bond-M3 ratio at 1.35 as our benchmark. The implied equilibrium inflation rate is $\pi^* = 1.74\%$ which is also close to the U.S. average of 1.70% over the extended period 2000–20. We further noted that over the period 1995–2008, the B/M3 ratio was also stable (mean 1.14, s.d. 0.098), and accordingly, we shall also consider the change between 1.1 and 1.35 to assess the impact of financial policy on inequality.

Setting $\phi = 0.22$ implies the optimal ratio of government consumption to private consumption is 0.22, equal to the U.S. average over the period 2000–19. Total government expenditure, $G = G_I + G_C$, accounts for about 18.85% of GDP. With G_I comprising about 19% of total government expenditure, the government consumption-GDP ratio is set at $g_C = 15.0\%$, and the government investment-GDP ratio is set at

$g_I = 3.5\%$; see Figure 2. For tax rates on consumption, labor, and capital income, we use estimates of the effective tax rates provided by McDaniel (2007) as shown in Table 1, which are consistent with the U.S. tax rates for the 1990–2000 period, a period of stable tax rates. Taken together, these parameter choices imply an equilibrium ratio of public to private capital of $z^* = 0.575$, a GDP-private capital ratio of $y^* = 0.375$, and a steady-state GDP growth rate of $\Psi^* = 2.28\%$, all of which are relatively close to their respective empirical magnitudes over extended periods of time.¹⁷

5. AGGREGATE AND DISTRIBUTIONAL EFFECTS OF GOVERNMENT EXPENDITURE POLICIES

This section examines the dynamic effects of one percentage point (p.p.) increases in the two forms of government expenditure, based on the above benchmark calibration. The short-run and long-run effects of g_I and g_C are summarized in Tables 3 and 4, respectively. Figure 1 illustrates the corresponding transitional dynamics for key variables, while Figure 2 illustrates the variations in the long-run impacts as the two types of government expenditure vary over more extensive ranges.

5.1 Increase in Government Investment-GDP Ratio, g_I

We begin by considering an unanticipated permanent increase in g_I from its benchmark level of 3.5% to 4.5%, with the debt-money ratio fixed at $\xi = 1.35$.

5.1.1 Aggregate effects. Table 3A summarizes the instantaneous and the long-run effects on the aggregate real quantities (all known to be independent of ξ), while the aggregate monetary variables are reported in 3B. With z being constrained to adjust only gradually, on impact the ratio of government-private capital remains unchanged. The increase in demand due to government investment causes an instantaneous increase in the price level of 6.43%, and real money balances immediately decline proportionately. The increase in government investment raises the productivity of both private capital and labor. By impacting public capital directly and private capital indirectly, the ratio z begins to increase, doing so steadily until the new steady-state $z^* = 0.7164$ is reached which represents an approximate 24% increase. On impact, labor increases by 0.18 p.p., and continues to increase further by 0.33 p.p., as the accumulating private capital enhances its productivity. The initial increase in demand generated by the additional public investment, coupled with an initial modest increase in output causes the inflation rate to immediately increase by 0.76 p.p., though in the long run, this is moderated to just 0.48 p.p., as the increases in public and private capital enhance the economy's productive capacity (Figure 1.1). Reflective of this, the growth rate of fi-

17. The data on the government consumption-GDP ratio are extracted from the United Nations, and the data on the government investment-GDP ratio and the growth rate of real GDP are extracted from the IMF. All other data are extracted from FRED (<https://fred.stlouisfed.org/>).

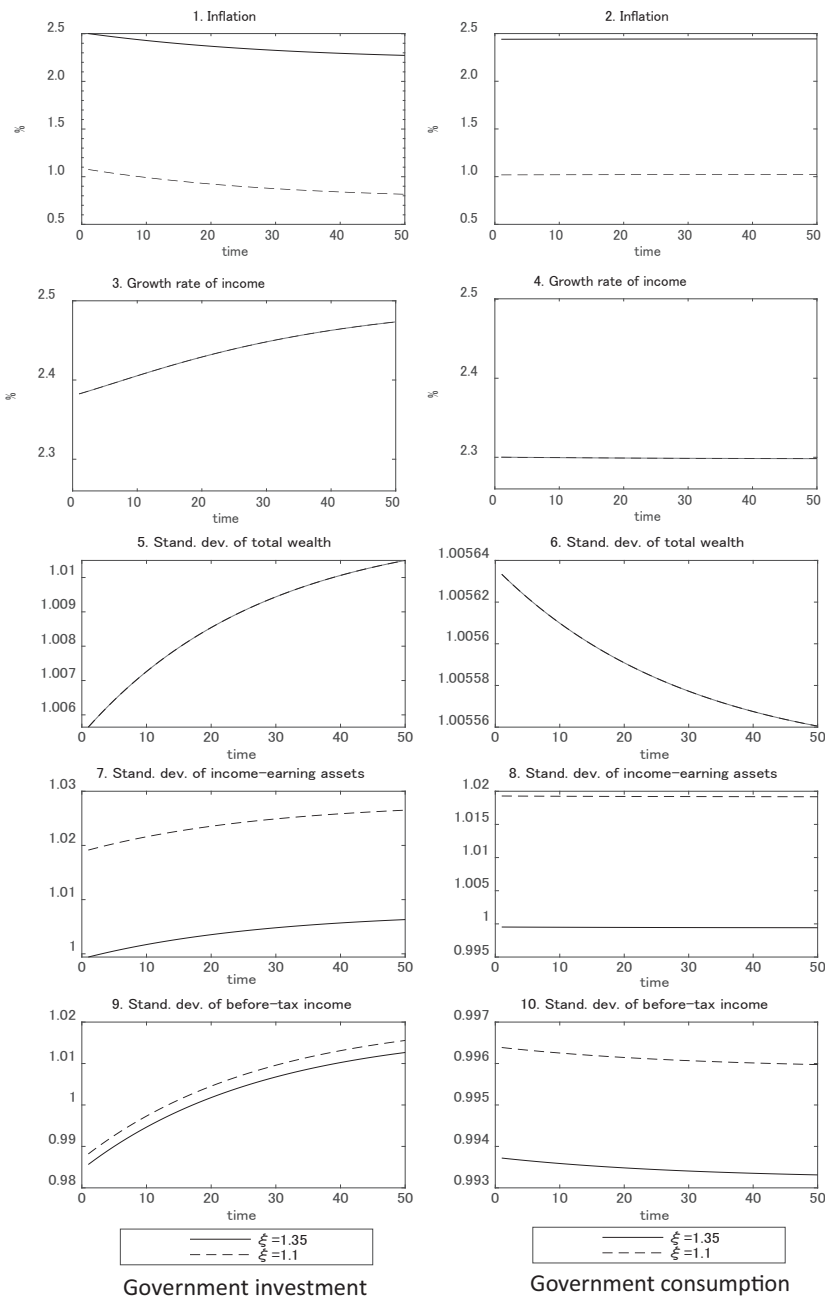


FIGURE 1. Dynamic Responses to Government Expenditure. *The values on the vertical axis for the distribution indicate the ratio to each benchmark value in Table 2.

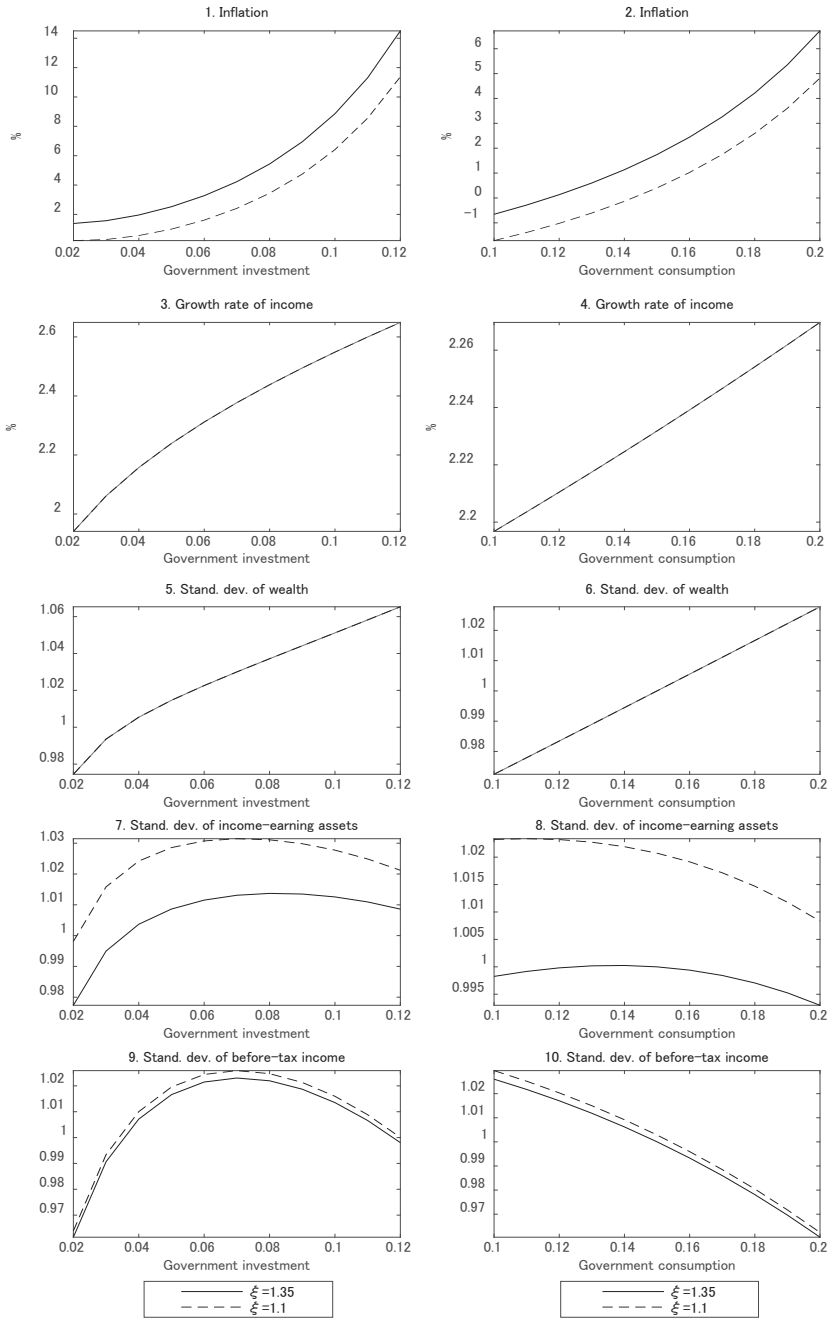


FIGURE 2. Variations in Long-run Impacts of Government Expenditures. *The values on the vertical axis for the distribution indicate the ratio to each benchmark value in Table 2.

TABLE 3
INITIAL AND LONG-RUN EFFECTS OF INCREASE IN g_I FROM 3.5% TO 4.5%

A: Aggregate real variables						
	z		l		\dot{Y}/Y	
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	0.5759		0.7101		2.28%	
$g_I = 0.045, g_C = 0.150,$ $\xi = 1.35, \xi = 1.10$	$z(0)$	z^*	$l(0)$	l^*	$\dot{Y}(0)/Y(0)$	\dot{Y}^*/Y^*
	0.5759 (0%)	0.7164 (24.39%)	0.7083 (−0.18%pt)	0.7068 (−0.33%pt)	2.36% (0.08%pt)	2.50% (0.22%pt)
B: Aggregate monetary variables						
	m		π			
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	0.2908		1.74%			
$g_I = 0.045, g_C = 0.150,$ $\xi = 1.35$	$m(0)$	m^*	$\pi(0)$	π^*		
	0.2721 (−6.43%)	0.2785 (−4.24%)	2.50% (0.76%pt)	2.22% (0.48%pt)		
$g_I = 0.045, g_C = 0.150,$ $\xi = 1.10$	0.3045 (11.9%)	0.3116 (11.9%)	1.08% (−1.42%pt)	0.76% (−1.46%pt)		
C: Distributions ^a						
	σ_a		σ_v		σ_y	
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	1		1.1634		0.2837	
$g_I = 0.045, g_C = 0.150,$ $\xi = 1.35$	$\sigma_a(0)$	σ_a^*	$\sigma_v(0)$	σ_v^*	$\sigma_y(0)$	σ_y^*
	1.0056 (0.56%)	1.0115 (1.15%)	0.9994 (−0.06%)	1.0078 (0.78%)	0.9856 (−1.44%)	1.0183 (1.83%)
$g_I = 0.045, g_C = 0.150,$ $\xi = 1.10$	1.0056 (0%)	1.0115 (0%)	1.0191 (1.97%)	1.0280 (2.00%)	0.9882 (0.26%)	1.0213 (0.30%)

Note: Changes in Tables 3B and 3C are between $\xi = 1.1$ and $\xi = 1.35$, given g_I, g_C .
^aNumbers indicate the ratio of the standard deviation to its benchmark value.

nal output increases from 2.36%, immediately following the expenditure increase, to 2.50% in the new steady state (Figure 1.3), while the initial 6.43% decline in the real money balances-capital ratio is moderated to 4.24% in the long run.

5.1.2 Distributional effects. Table 3C summarizes the effects of the increase in government investment on several measures of inequality, including total wealth inequality, “income-earning asset” inequality, and pretax income inequality.¹⁸ Normalizing

18. We also consider (but do not report) after-tax income inequality and welfare inequality. The former broadly reflects $\sigma_y(t)$, with differences being entirely due to the differential tax rates $\tau_K - \tau_w$, as in García-

TABLE 4
INITIAL AND LONG-RUN EFFECTS OF g_C FROM 15.0% TO 16.0%

A: Aggregate real variables						
	z		l		\dot{Y}/Y	
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	0.5759		0.7101		2.28%	
$g_I = 0.035, g_C = 0.160,$ $\xi = 1.35, \xi = 1.10$	$z(0)$	z^*	$l(0)$	l^*	$\dot{Y}(0)/Y(0)$	\dot{Y}^*/Y^*
	0.5759	0.5742	0.7073	0.7073	2.30%	2.30%
	(0%)	(-0.30%)	(-0.28%pt)	(-0.28%pt)	(0.02%pt)	(0.02%pt)
B: Aggregate monetary variables						
	m		π			
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	0.2908		1.74%			
$g_I = 0.035, g_C = 0.160,$ $\xi = 1.35$	$m(0)$	m^*	$\pi(0)$	π^*		
	0.2721	0.2720	2.44%	2.44%		
	(-6.42%)	(-6.45%)	(0.70%pt)	(0.70%pt)		
$g_I = 0.035, g_C = 0.160,$ $\xi = 1.10$	0.3045	0.3044	1.02%	1.02%		
	(9.11%)	(9.11%)	(-1.42%pt)	(-1.42%pt)		
C: Inequality measures ^a						
	σ_a		σ_v		σ_y	
Benchmark $g_I = 0.035, g_C = 0.150,$ $\xi = 1.35$	1		1.1634		0.2837	
$g_I = 0.035, g_C = 0.160,$ $\xi = 1.35$	$\sigma_a(0)$	σ_a^*	$\sigma_v(0)$	σ_v^*	$\sigma_y(0)$	σ_y^*
	1.0056	1.0055	1.1628	1.1627	0.9937	0.9933
	(0.56%)	(0.55%)	(-0.05%)	(-0.06%)	(-0.63%)	(-0.68%)
$g_I = 0.035, g_C = 0.160,$ $\xi = 1.10$	1.0056	1.0055	1.1858	1.1857	0.9964	0.9959
	(0%)	(0%)	(1.98%)	(1.98%)	(0.27%)	(0.26%)

Note: Changes in Tables 4B and 4C are between $\xi = 1.1$ and $\xi = 1.35$, given g_I, g_C .
^aNumbers indicate the ratio of the standard deviation to its benchmark value.

the prior equilibrium wealth inequality at $\sigma_{a,0} = 1$, and focusing on the case where the initial inequality in endowments of the real and nominal assets are equal and are uncorrelated across agents, the impact of the initial price increase is to raise wealth inequality by 0.56%. Since the initial 6.43% decrease in the real money balances-capital ratio dominates the 0.56% increase in wealth inequality, the inequality associated with the income-earning assets, σ_v , declines slightly from its prior equilibrium

Peñalosa and Turnovsky (2011). Welfare inequality is shown to be proportional to steady-state wealth inequality and is constant over time.

by 0.06% in accordance with (D.6). This decline in σ_v causes an initial decline in pretax income inequality of 1.44% (Figures 1.5, 1.7, and 1.9).

During the ensuing transition, the increase in labor, coupled with the increase in productive capacity from the accumulation of public capital, leads to a steady increase in the growth rate, as well as increases in both the return to capital and the wage rate. However, the increase in labor is modest (0.33 p.p. from its initial steady state), and since wealth inequality occurs only during the transition, the impact on wealth inequality is small. Its overall increase across steady states is just 1.15%, half of which occurs on impact. The increase in the return to capital during the transition dominates the increase in the wage rate which favors the relatively wealthy. Hence, following the initial decline, income inequality increases by 3.27% during the transition with a net long-run increase of 1.83%.

The last two rows of Tables 3B and 3C also report the impact of pure financial policy by comparing the effects of increasing the debt ratio ξ from 1.1 to 1.35, with g_I and g_C held constant. Although in isolation increasing ξ has no impact on aggregate real variables, it raises inflation by approximately 1.42 p.p. in the short run and by 1.46 p.p. in the steady state. Also, while wealth inequality has been shown to be independent of debt policy, increasing ξ tends to decrease income inequality by around 0.26 p.p., on impact, and by 0.30 p.p. in steady state.¹⁹

5.2 Increase in Government Consumption-GDP Ratio, g_C

Table 4 reports the analogous effects of an unanticipated permanent increase in g_C from its benchmark level of 15% to 16%.

5.2.1 Aggregate effects. The increase in demand due to government consumption raises the price level by 6.43%, causing real money balances to decline by that amount. The increase in demand also increases the demand for labor, raising employment by 0.28 p.p. The increase in employment enhances the productivity of capital, raising the initial return on capital by 0.58% and reducing the wage rate by 0.39%. In contrast to the increase in g_I , the stimulus to the public to private capital ratio occurs via private capital, which starts to increase, causing z to begin to decline, which continues until it has fallen by 0.30%. The initial increase in the return to capital resulting from the increase in labor leads to a small initial increase in the growth rate of 0.02 p.p. Along the stable transitional path, the continuing decline in z is associated with a decline in labor supply. These two effects are approximately offsetting and the 0.02 p.p. increase in the growth rate is maintained throughout the transition (Figure 1.4). Since the increase in the growth rate is extremely small, the increase in productive capacity is significant. With the additional government expenditure directed toward consumption rather than production, the inflation rate increases by 0.70 p.p. and this is sustained as well (Figure 1.2).

19. The effects of the increase in ξ from 1.1 to 1.35 are seen by comparing the dashed and solid lines in Figures 1 and 2.

5.2.2 Distributional effects. With the increase in government consumption from 15% to 16% increasing the price level by 6.43%, initial wealth inequality increases by 0.56%, which is comparable to the corresponding effect of government investment. Since a 6.43% decrease in the real money balances-capital ratio dominates a 0.56% increase in wealth inequality, the inequality associated with the interest-earning assets, σ_v , declines slightly by 0.05%. This decline in σ_v causes an initial drop in income inequality of 0.63%, which in the long run is increased to 0.68% as z declines (Figures 1.6, 1.8, 1.10).

6. COMPARING THE TWO FORMS OF GOVERNMENT EXPENDITURE

In comparing the consequences of the two forms of government spending, we see that while their short-run effects are rather similar, over time they diverge substantially. On impact, a 1 p.p. increase in g_C by crowding out private consumption leads to a slightly greater reduction in leisure than does the comparable increase in g_I (-0.28 vs. -0.18 p.p.). These relative reductions in consumption are offset by the relative adjustment in the return to capital so that in both cases the 1 p.p. increase in government expenditure leads to approximately equal increases in the inflation rate of around 0.76% and a comparable decline in the real money balances-capital ratio. In both cases, the price increase necessary to drive the aggregate real money stock onto the stable transitional adjustment path, (16c), is also comparable leading to similar initial increases in wealth inequality of 0.56%. The small differential in the initial income inequality reflects the differential in the impact on leisure (labor).

But major differences accumulate over time, as is apparent by comparing the dynamics illustrated in Figure 1. This is because a 1 p.p. increase in g_I leads to a substantial (24.4%) increase in the ratio of public to private capital, z , which takes time to absorb and become productive. In contrast, a comparable increase in g_C leads to a small decline in z (-0.30%) and most of the adjustment is completed on impact. These sharp differences in the evolution of the ratio of public to private capital have important consequences for the changing labor supply, which together have implications for the wage rate, the return to capital, and inequality measures. In the case of g_I , this is associated with an increasing ratio that favors capital and increases wealth inequality. In contrast, an increase in g_C has an almost negligible negative effect on labor supply over time, reflecting the fact that labor almost completes its adjustment instantaneously. Since the impact of government investment leads to an ongoing increase in wealth inequality, its long-run effect on wealth inequality exceeds that of government consumption, though in both cases, the long-run effects are small.

The adjustments in wealth inequality impact income inequality, which for both forms of expansion decline on impact. In the case of g_I , the steady increase in z over time favors the return to capital, and income inequality increases over time, reversing the initial decline. In the case of g_C , there is no transition and the initial decline in income inequality is effectively permanent.

The numerical simulations also suggest that the two forms of government expenditure involve sharp contrasts in the tradeoffs they impose on various other key measures of economic activity. For example, a 1 p.p. increase in government investment will raise the long-run growth rate by 0.22 p.p., inflation by 0.48 p.p., and overall income inequality by 1.83%. In contrast, a 1 p.p. increase in government consumption has virtually no impact on the long-run growth rate, while raising inflation by 0.70 p.p., and reducing overall income inequality by around 0.70%. To dramatize the sharply contrasting long-run effects of these two forms of government spending, we note that the *difference* between allocating a 1 p.p. increase in government spending to investment rather than to consumption is (i) a 0.22 p.p. higher growth rate, (ii) a 0.22 p.p. lower inflation rate, together with (iii) a 2.51% long-run difference in income inequality. Clearly, the allocation of only a marginal *increase* in government expenditure has profound long-run macroeconomic and distributional consequences!

To provide further perspective on the long-run consequences of government expenditure policies, Figure 2 illustrates the long-run aggregate and distributional consequences, as the ranges over which g_I and g_C vary are expanded drastically. Several observations merit comment.

First, most of the impacts are monotonic, meaning that the changes obtained from the small increases we have considered extrapolate qualitatively to larger changes, but there are two exceptions. While increasing g_I from 3.5% to 4.5% leads to a small long-run increase in the inequality, σ_v^* associated with “income-earning” assets, as g_I increases further, σ_v^* declines. This reflects the fact that it incorporates two offsetting influences. On the one hand, as g_I increases, σ_a^* increases, while at the same time, the decrease in m^* leads to a decline in the overall share of wealth that V comprises and lowers σ_v^* for a given magnitude of σ_a^* ; see Proposition 3. This in turn impacts the relative importance of V in income, and as a result, both income inequality measures share the same characteristic.

Second, for the fixed debt ratio policy, in conjunction with the specified tax rates, more substantial increases in government expenditure do not appear to lead to any dire consequences for the economy. Figure 2 suggests that the effects of doubling g_I from around 3.5% to 7.0% of GDP—a huge increase in infrastructure investment—would not be overly dramatic. The long-run growth rate and inflation rate would increase by around 1 p.p., wealth inequality could increase by something less, and income inequality by even less. These relatively moderate impacts may provide encouragement for the view that the economy could absorb substantial increases in public investment, without any significant increase in tax rates and no dramatic adverse distributional effects.

Finally, we briefly examine the compatibility of the numerical simulations with the observed time paths of government expenditure and the evolution of income inequality experienced in the U.S. economy since 2000. We emphasize that our intent (given the stylized model) is modest. It is simply to see the extent to which g_I and g_C may have potentially contributed to the time path of income inequality. Figure A2 suggests that since 2000 income inequality has increased very modestly, with the Gini coefficient increasing by little over 1 p.p. As is apparent from Figure A1, the two forms of

government spending have followed parallel time paths, increasing between 2000 and 2009 and then declining sharply after 2010. During both phases, the changes in g_C significantly exceed those of g_I . Also, Tables 3C and 4C indicate that while g_I increases long-run inequality, g_C decreases it, with g_I having the significantly stronger impact. Taking into account these contrasting effects and their relative intensities raises the possibility that the impacts of the two forms of government expenditure on income inequality have tended to be mutually offsetting, contributing to the gentle slope of the GINI coefficient. Moreover, the larger decline in g_C relative to g_I over the latter period (3 vs. 1 p.p.) could have contributed to an increase in income inequality, but this was offset by the increase in ξ from 1.1 to 1.35 that occurred between the two subperiods.

7. CONCLUSION

In advanced economies, the implementation of expansive fiscal policies to mitigate the large declines in GDP due to COVID-19 has led to high government debt-GDP ratios. As a result, policymakers hesitate to implement further expansive fiscal policies to return the economy to their pre-COVID stable growth trajectories. Motivated in part by this concern, this paper has adapted the Sidrauski (1967) monetary growth model to compare the impact of government investment and government consumption expenditure on the macroeconomic and distributional characteristics of an economy in which the government finances its deficit using money while maintaining a fixed ratio of debt-money. Using this framework yields the following main conclusions.

First, real aggregate quantities and how they are impacted by fiscal policy are independent of the debt-money ratio chosen by the monetary authority, a proposition that corresponds to the “super-neutrality of money,” associated with Sidrauski (1967), Fischer (1979), and others. It also has no impact on wealth inequality, either in the short run or over time. In contrast, increasing the debt-money ratio tends to lead to a small but sustained decline in income inequality.

Second, calibrating the model to reflect recent U.S. experience, we have investigated numerically the impact of the above two forms of government expenditure on the economy-wide averages and distributions of key economic variables. It is found that while they cause moderate inflation, they have no detrimental effect on the economy, provided the government spending-GDP ratio is within the range that ensures the existence of a sustainable balanced growth.

Finally, the numerical simulations suggest that the two forms of government expenditure involve sharp contrasts in the tradeoffs they impose on various key measures of economic activity, both at the aggregate level as well as their distributional consequences.²⁰ The results suggest that government investment is more effective in raising

20. The Online Appendix offers some conjectural comments regarding the potential contribution of government expenditure to the increase in wealth inequality that has occurred since 2000.

the average growth rate than is government consumption, while potentially exacerbating long-run income inequality. This concern suggests that one should pay careful attention in designing a coherent public expenditure policy to ensure that disparities are minimized and the benefits are widely shared across the economy.

APPENDIX A

Derivation of Macroeconomic Equilibrium

Combining the equations describing the growth rates of public and private capital, \dot{K}/K , \dot{K}_g/K_g together with the time derivative of $z \equiv K_g/K$ yields:

$$\frac{\dot{z}}{z} = g_I \frac{y(z, l)}{z} - (1 - g_I - g_C)y(z, l) - c(z, l). \quad (\text{A.1})$$

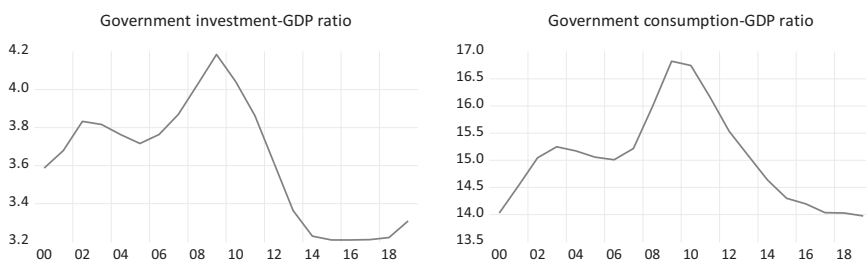


FIGURE A1. Government Expenditure-GDP Ratio in the United States.

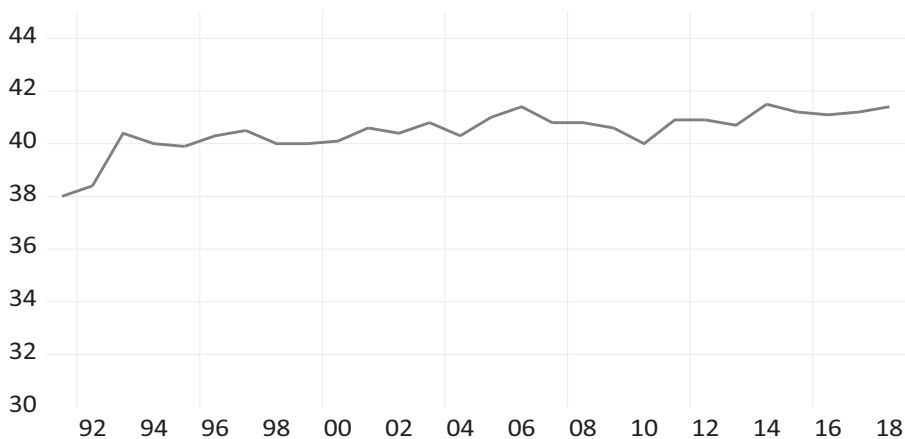


FIGURE A2. Gini Coefficient of Income in the United States.

Differentiating $m \equiv M/K$ with respect to time, and recalling (11), we obtain:

$$\begin{aligned} \frac{\dot{m}}{m} = & \frac{1}{m(1+\xi)} \{ [g_I + g_C - \tau_K(1-s) - \tau_w s] y(z, l) \\ & - [\tau_C + \eta(1 + \tau_C)] c(z, l) \} + (1 - \tau_K)r(z, l) - \frac{\dot{K}}{K}. \end{aligned} \quad (\text{A.2})$$

Differentiating (5a) with respect to t , and combining with (5e), (7b), and (9a), yields:

$$(\gamma - 1)\frac{\dot{C}}{C} + \beta\gamma\frac{\dot{l}}{l} + \eta\gamma\frac{\dot{M}}{M} + \phi\gamma\frac{\dot{Y}}{Y} = \rho - (1 - \tau_K)r(z, l).$$

Next, differentiating the aggregate production function (1a), we obtain:

$$\frac{\dot{Y}}{Y} = - \left(\frac{sl}{1-l} \right) \frac{\dot{l}}{l} + s(1-\varepsilon)\frac{\dot{z}}{z} + \frac{\dot{K}}{K}.$$

Recalling from (7a) that $C_i/l_i = C/l$, substituting into (5b), using (2b), and taking the time derivative,

$$\frac{\dot{C}}{C} = \left(\frac{1-sl}{1-l} \right) \frac{\dot{l}}{l} + s(1-\varepsilon)\frac{\dot{z}}{z} + \frac{\dot{K}}{K}.$$

Substituting for \dot{C}/C , \dot{M}/M , and \dot{Y}/Y into (A.2), and rearranging terms yields (12c):

$$\frac{\dot{l}}{l} = \frac{1}{\Delta} \left[(1 - \tau_K)r(z, l) - \rho + \eta\gamma\frac{\dot{m}}{m} - s(1-\varepsilon)(1-\gamma-\phi\gamma)\frac{\dot{z}}{z} - \Pi\frac{\dot{K}}{K} \right], \quad (\text{A.3})$$

where Δ and Π are defined in the text. Equations (A.1)–(A.3) correspond to equations (12a)–(12c).

APPENDIX B

Steady-State Equilibrium and Long-Run Comparative Statics

Setting $\dot{z} = 0$ in (12a), and substituting for $r(z, l)$, $y(z, l)$, and $c(z, l)$, yields:

$$g_I = z^* \left[(1 - g_I - g_C) - \left[\frac{1 - \tau_w}{1 - \tau_C} \right] \left(\frac{l^*}{1 - l^*} \right) \frac{s}{\beta} \right]. \quad (\text{B.1a})$$

This equation describes the steady-state relationship between z and l , denoted by $*$, that will maintain equality between the long-run growth rates of K and K_g . It can be shown to be upward sloping, reflecting the fact that while an increase in z^* increases the growth rate of private capital, it reduces that of public capital, and requires an increase in l^* to maintain long-run equality.

Similarly, setting $\dot{m} = \dot{z} = \dot{l} = 0$, together with $\dot{K}/K = \dot{K}_g/K_g$ in (12c), leads to:

$$E \cdot (z^*)^{s(1-\varepsilon)-1} \left[\frac{(1 - \tau_K)(1 - s)z^*}{\Pi} - g_I \right] = \frac{\rho}{\Pi(1 - l^*)^s}. \tag{B.1b}$$

This too determines a positive relationship between z and l required to maintain equality between the growth rates of consumption and private capital. In this case, while an increase in z^* increases the growth rate of consumption, it is less than the increase in that of private capital, again requiring an increase in l^* for equality to be maintained. Under weak conditions, hinging on the relative curvatures of the two curves, (B.1a) and (B.1b), one can establish that they intersect in a unique equilibrium (z^*, l^*) which is independent of the debt-policy, ξ .

Setting $\dot{m} = 0$ in (12b) and substituting for \dot{K}/K , one can further show that the ratio of total government liabilities to capital, $(M + B)/K \equiv (1 + \xi)m$, is:

$$(1 + \xi)m^* = \Pi \frac{\Phi y(z^*, l^*) + [(1 + \tau_C)\eta + \tau_C]c(z^*, l^*)}{(\Pi - 1)(1 - \tau_K)r(z^*, l^*) + \rho}, \tag{B.2}$$

where $\Phi \equiv [(1 - s)\tau_K + s\tau_w - g_I - g_C]$ and is also invariant to ξ . With z^* and l^* being jointly determined by (B.1a) and (B.1b), independently of ξ , the result immediately follows. Furthermore, the ratio of total government liabilities to wealth $(M^* + B^*)/A^*$ shares the same property. The fact that $(1 + \xi)m^*$ is independent of ξ implies that an increase in ξ reduces the money-capital ratio, $dm^*/d\xi = -m^*/(1 + \xi) < 0$, which from (13) leads to an increase in the equilibrium inflation rate, π^* .

As noted in the text, all aggregate variables grow at a common rate, Ψ^* , which is driven by the unique steady-state values of z^* and l^* . While these are independent of debt policy, their responsiveness to government expenditure policies is obtained by taking the appropriate derivatives of the two equilibrium relationships (B.1a) and (B.1b). Specifically, we obtain:

$$\begin{aligned} \frac{dl^*}{dg_I} &= -\frac{1}{T} \frac{y^*}{z^*} \left[\Psi^* + s(1 - \varepsilon) \frac{\rho}{\Pi} (1 + z^*) \right] < 0; \\ \frac{dz^*}{dg_I} &= \frac{1}{T} \left(\frac{1}{1 - l^*} \right) \frac{y^*}{z^*} \left[\frac{c^*}{l^*} - s \frac{\rho}{\Pi} (1 + z^*) \right] \end{aligned} \tag{B.3}$$

$$\frac{dl^*}{dg_C} = -\frac{1}{T} \frac{y^*}{z^*} \left[\Psi^* + s(1 - \varepsilon) \frac{\rho}{\Pi} \right] < 0; \quad \frac{dz^*}{dg_C} = -\frac{y^*}{T} \left(\frac{s}{1 - l^*} \right) \frac{\rho}{\Pi} < 0 \tag{B.4}$$

where $T \equiv \frac{\Psi^*}{z^*} \left(\frac{1}{1 - l^*} \right) \left[\frac{c^*}{l^*} - s \frac{\rho}{\Pi} \right] + \frac{c^*}{l^*} \left(\frac{1}{1 - l^*} \right) \frac{s(1 - \varepsilon)}{z^*} \frac{\rho}{\Pi}$. From these equations, the corresponding effect on the equilibrium balanced growth rate Ψ^* is:

$$\frac{d\Psi^*}{dg_I} = \frac{1}{\Pi} (1 - \tau_K) s r^* \frac{1}{T} \frac{y^*}{z^*(1-l^*)} \left[\left(1 + 2s(1-\varepsilon) \frac{1+z^*}{z^*} \right) \Psi^* + \frac{1}{z^*} \frac{c^*}{l^*} \right] > 0, \tag{B.5a}$$

$$\frac{d\Psi^*}{dg_C} = \frac{1}{\Pi} (1 - \tau_K) s r^* \frac{1}{T} \frac{y^*}{z^*(1-l^*)} \Psi^* > 0. \tag{B.5b}$$

APPENDIX C

Local Stability

The elements of the transitional matrix (15) can be shown to be:

$$a_{11} \equiv \frac{l^*}{\Delta} \left[\eta\gamma \cdot \frac{\partial \dot{m}}{\partial l} \frac{1}{m^*} - \Pi \cdot \frac{\partial(\dot{K}/K)}{\partial l} - s(1-\varepsilon)(1-\gamma-\phi\gamma) \cdot \Omega_l(l^*, z^*) + (1 - \tau_K) r_l(l^*, z^*) \right], \tag{C.1a}$$

$$a_{12} \equiv \frac{l^*}{\Delta} \left[\eta\gamma \cdot \frac{\partial \dot{m}}{\partial z} \frac{1}{m^*} - \Pi \cdot \frac{\partial(\dot{K}/K)}{\partial z} - s(1-\varepsilon)(1-\gamma-\phi\gamma) \cdot \Omega_z(l^*, z^*) + (1 - \tau_K) r_z(l^*, z^*) \right], \tag{C.1b}$$

$$a_{13} \equiv \frac{l^*}{\Delta} \eta\gamma \cdot \frac{\partial \dot{m}}{\partial m} \frac{1}{m^*}. \tag{C.1c}$$

Differentiating (12b) with respect to l, z and m , we can express $\partial \dot{m} / \partial l (\equiv a_{31})$, $\partial \dot{m} / \partial z (\equiv a_{32})$, and $\partial \dot{m} / \partial m (\equiv a_{33})$ in (15) as:

$$a_{31} = \frac{1}{(1+\xi)} [(g_I + g_C - (1-s)\tau_K - s\tau_w) y_l(z^*, l^*) + (1 - \tau_K) r_l(z^*, l^*) (1 + \xi) m^* - (\tau_C + \eta(1 + \tau_C)) c_l(z^*, l^*)] - [(1 - g_I - g_C) y_l(z^*, l^*) - c_l(z^*, l^*)], \tag{C.2a}$$

$$a_{32} = \frac{1}{(1+\xi)} [(g_I + g_C - (1-s)\tau_K - s\tau_w) y_z(z^*, l^*) + (1 - \tau_K) r_z(z^*, l^*) (1 + \xi) m^* - (\tau_C + \eta(1 + \tau_C)) c_z(z^*, l^*)] - [(1 - g_I - g_C) y_z(z^*, l^*) - c_z(z^*, l^*)], \tag{C.2b}$$

$$a_{33} = (1 - \Pi) \Psi^* - \rho. \tag{C.2c}$$

APPENDIX D

The Dynamics of Relative Wealth and Income

To derive the relative dynamics of individual i 's relative wealth, $a_i(t) \equiv A_i(t)/A(t)$, we sum over the agents' individual budget constraints, (6), to obtain an expression for the evolution of aggregate wealth, $A(t)$, which we then combine with the individual wealth accumulation equation (6). This leads to the following relationship determining agent i 's relative rate of wealth accumulation:

$$\dot{a}_i = (1 - \tau_w) \cdot \frac{\tilde{w}(z, l)}{1 + (1 + \xi)m} \left[1 - \frac{1 + \beta + \eta}{\beta} l_i - \left(1 - \frac{1 + \beta + \eta}{\beta} l \right) a_i \right], \quad (D.1)$$

where using (2b) we may write the real wage-capital ratio as $w/K \equiv \tilde{w}(z, l)$. Recalling equation (7a), $l_i/l = \varsigma_i$, a constant over time, that we obtain from the steady state to (D.1):

$$\frac{l_i(t)}{l(t)} = \varsigma_i = 1 + \left(1 - \frac{\beta}{1 + \beta + \eta} \cdot \frac{1}{l^*} \right) (a_i^* - 1). \quad (D.2)$$

To analyze the local dynamics of relative wealth, we linearize (D.1) around the steady state l^* , z^* , and a_i^* . Using (17), the bounded solution for agent i 's relative share of wealth, $a_i(t)$, is:

$$a_i(t) - 1 = \alpha(t) (a_i^* - 1), \quad (D.3)$$

where $\alpha(t) \equiv 1 + \left(\frac{(1 - \tau_w)\tilde{w}(z^*, l^*)}{1 + (1 + \xi)m^*} \left[1 - \frac{l(t)}{l^*} \right] \right) \cdot \left(\frac{(1 - \tau_w)\tilde{w}(z^*, l^*)}{1 + (1 + \xi)m^*} \left[\frac{l^*}{v} - 1 \right] - \mu \right)^{-1}$. Setting $t = 0$ in (D.3) yields $a_i(0) - 1 = \alpha(0)(a_i^* - 1)$, which combined with (D.3), implies:

$$a_i(t) - 1 = \frac{\alpha(t)}{\alpha(0)} (a_i(0) - 1). \quad (D.4)$$

Letting $t \rightarrow \infty$, we see that agent i 's relative wealth in steady state, a_i^* , is:

$$a_i^* - 1 = \frac{1}{\alpha(0)} (a_i(0) - 1), \quad (D.4a)$$

where the individual's initial relative wealth, $a_i(0)$, is endogenous.

As noted, to determine the agent's relative income, we need to determine his relative wealth position in real money balances and income earning assets. Using (7a), (D.2), and (D.3), we can relate agent i 's relative position in real money balances to his relative wealth:

$$\frac{M_i}{M} - 1 = \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha(t)} \cdot [a_i(t) - 1]. \quad (D.5)$$

With capital and bonds yielding the same rate of return, and in the absence of risk, investors view these two assets as perfect substitutes, enabling us to char-

acterize their *composite* distribution. Letting $V_i \equiv B_i + K_i$ and $V \equiv B + K$, so that $v_i - 1 \equiv (V_i/V) - 1$, and using (D.5) in conjunction with the definition of relative wealth, we can show:

$$(v_i(t) - 1) = \left[1 + \frac{m}{(1 + \xi m)} \left[1 - \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha(t)} \right] \right] (a_i(t) - 1), \quad (\text{D.6})$$

which expresses the agent's relative position of the composite income-earning asset, $(B(t) + K(t))$ in terms of his overall relative position in wealth.

With several alternative income measures, there are several potential measures of income inequality. One natural and convenient measure is before-tax personal income, defined as income from the income-earning assets (capital and bonds), plus income from labor. Using this measure, the before-tax income of individual i is:

$$Y_i(t) = r(t) V_i(t) + \tilde{w}(t) K(t) (1 - l_i(t)), \quad (\text{D.7a})$$

while the average economy-wide before-tax personal income is:

$$Y(t) = r(t) V(t) + \tilde{w}(t) K(t) (1 - l(t)). \quad (\text{D.7b})$$

Dividing (D.7a) by (D.7b), and using (D.2) and (D.5), the relative before-tax income of agent i , $y_i(t) \equiv Y_i(t)/Y(t)$, can be expressed as:

$$y_i(t) - 1 = \varphi(t) [a_i(t) - 1], \quad (\text{D.8a})$$

where:

$$\varphi(t) = \left\{ \frac{r(1 + \xi m)}{r(1 + \xi m) + \tilde{w}(1 - l)} \left(1 + \frac{m}{1 + \xi m} \left(1 - \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha} \right) \right) - \frac{\tilde{w}l}{r(1 + \xi m) + \tilde{w}(1 - l)} \left(1 - \frac{v}{l^*} \right) \frac{1}{\alpha} \right\}. \quad (\text{D.8b})$$

APPENDIX E

Determination of Initial Price and Initial Wealth Inequality

In general, following a shock, the initial price level $p(0)$ undergoes an initial jump, $dp(0)$. This is required to ensure that the real money stock, $m(0)$, jumps onto the stable transitional path, (16c), and depends upon the specific shock driving the aggregate dynamics. If the shock is real (e.g., government expenditure), then with z and K being predetermined, this is obtained by solving the pair of equations: (i) $dm(0) = dm^* - \vartheta dz^* + (z_0 - z^*)d\vartheta$ (where ϑ denotes the slope of (16c)) and (ii) $dp(0)/p(0) = -dm(0)/m(0)$, implying:

$$dp(0)/p(0) = -[dm^* - \vartheta dz^* + (z_0 - z^*)d\vartheta]/m(0).$$

If the shock is a change in debt policy, $d\xi$, Proposition 2 implies $d[(1 + \xi)m(0)]/d\xi = 0$. Assuming this occurs by an initial open market operation, $dM'_0 + dB'_0 = 0$, where primes denote *nominal* quantities, $d[(1 + \xi)m(0)]/d\xi = d[N_0/p(0)K_0]$ where $N_0 \equiv M'_0 + B'_0$ denotes the sum of the two nominal assets. But since N_0 is predetermined and, therefore, along with K_0 , is unchanged by the financial trade, $d[(1 + \xi)m(0)]/d\xi = 0$ implies $dp(0) = 0$.

To determine the effect on individual i 's initial relative wealth, $a_i(0)$, and on initial wealth inequality, $d\sigma_a(0)$, we begin by writing:

$$a_i(0) = \frac{p(0)K_0}{p(0)K_0 + N_0} \cdot \frac{K_{i,0}}{K_0} + \frac{N_0}{p(0)K_0 + N_0} \cdot \frac{N_{i,0}}{N_0}, \tag{E.1}$$

where analogously $N_{i,0} \equiv M'_{i,0} + B'_{i,0}$ pertains to individual i . From (E.1), we see that the impact of the initial jump in the price level from its prior level on initial relative wealth is:

$$\frac{da_i(0)}{a_i(0)} = \left[\frac{p(0)K_{i,0}}{p(0)K_{i,0} + N_{i,0}} - \frac{p(0)K_0}{p(0)K_0 + N_0} \right] \frac{dp(0)}{p(0)}. \tag{E.2}$$

If the agent's endowed portfolio of assets coincides with the economy-wide average, the term in parentheses is zero and $da_i(0) = 0$, independent of any real shock and its associated price change. If the shock is due to debt policy when $dp(0) = 0$, $da_i(0) = 0$, irrespective of the agent's portfolio.

To determine the impact on initial wealth inequality, $\sigma_a(0)$, we first rewrite (E.1), as:

$$a_i(0) - 1 = \frac{p(0)K_0}{p(0)K_0 + N_0} (k_{i,0} - 1) + \frac{N_0}{p(0)K_0 + N_0} (n_{i,0} - 1), \tag{E.3}$$

where $n_{i,0} \equiv N_{i,0}/N$. Denoting the ratio of nominal assets to total nominal wealth by $\chi \equiv N_0/(p(0)K_0 + N_0)$, $\sigma_a(0)$ immediately following a shock is:

$$\sigma_a(0) = \left[(1 - \chi)^2 \sigma_{k,0}^2 + 2(1 - \chi)\chi \sigma_{k,n,0} + \chi^2 \sigma_{n,0}^2 \right]^{1/2}. \tag{E.4}$$

Hence, the effect of the initial price change, on initial wealth inequality across agents is:

$$\sigma_a(0)d\sigma_a(0) = \left[-(1 - \chi)\sigma_{k,0}^2 + (1 - 2\chi)\sigma_{k,n,0} + \chi\sigma_{n,0}^2 \right] d\chi,$$

where $d\chi/\chi = -(1 - \chi)dp(0)/(p(0))$, and substituting, yields:

$$\frac{d\sigma_a(0)}{\sigma_a(0)} = \left[(1 - \chi)\sigma_{k,0}^2 - (1 - 2\chi)\sigma_{k,n,0} - \chi\sigma_{n,0}^2 \right] \frac{(1 - \chi)\chi dp(0)}{\sigma_a^2(0)p(0)}. \tag{E.5}$$

From equation (E.5), we see that since a change in debt policy implies $dp(0)/p(0) = 0$, it has no effect on initial wealth inequality. For real shocks,

$d\sigma_a(0)/dp(0)$ will depend upon (i) the total endowment ratio of nominal assets to total nominal wealth, χ , and (ii) the distribution of endowments across agents, $\sigma_{k,0}$, $\sigma_{n,0}$ and their covariance, $\sigma_{k,n,0}$.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Figure O.1: Government bonds-M3 ratio in the United States

Figure O.2: Share of wealth in the United States