# THE DYNAMICS OF GROWTH AND INCOME INEQUALITY UNDER PROGRESSIVE TAXATION

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## Abstract

This paper develops an endogenous growth model having a progressive income tax structure in which heterogeneous agents, who differ in terms of their rates of time preference, supply labor elastically. We analyze the dynamic adjustment to an increase in progressivity and show that the economy will converge to an equilibrium growth path with nondegenerate distributions of both income and wealth. The role of the endogeneity of labor supply is emphasized and shown to have a major impact on the nature of the transitional path, as a result of the impact of the progressive tax on agents' work incentives. Our theoretical analysis is supplemented with, and supported by, numerical simulations, which generally match the empirical evidence rather closely. We also show that the responses of the different income groups contrast sharply from one another so that focusing on the economy-wide average provides an incomplete picture.

## 1. Introduction

The effects of taxes on the growth rate and distribution of income is an important and widely discussed topic. Most of the literature addressing this issue adopts the conventional assumption of flat tax rates (see, e.g., Stokey and Rebelo 1991; Mendoza, Razin, and Tesar 1994; Domeij and Heathcote 2004; García-Peñalosa and Turnovsky 2007, 2011). Since in practice most tax systems in advanced economies exhibit some degree

of progressivity, it is important to reassess these issues in the context of a progressive tax structure.

Two approaches to incorporating progressive income taxation into macrodynamic models can be identified. One strand of literature adopts the incomplete asset markets framework. In this approach to life cycle economies, heterogeneity arises from uninsurable idiosyncratic productivity shocks against which progressive taxes act as a partial insurance mechanism. One important contribution is Castañeda, Díaz-Giménez, and Ríos-Rull (2003), who conclude that endogenous labor supply and progressive income taxation is crucial in order to account properly for wealth inequality. Several studies adopt this approach to focus on income tax reforms. More specifically, they assess the gains in switching from progressive to flat taxes and they evaluate the quantitative effects of such reforms on the U.S. economy (see, e.g., Caselli and Ventura 2000; Altig *et al.* 2001; Caucutt, İmrohoroglu, and Kumar 2003; Conesa and Krueger 2006; İmrohoroglu, İmrohoroglu, and Fuster 2008; Kitao 2010).

An alternative approach, adopted by Sarte (1997) and Li and Sarte (2004), assumes complete asset markets. By introducing a progressive income tax structure, together with heterogeneous rates of time discount, they avoid the degenerate equilibrium distribution of income that would otherwise occur with heterogeneous time preferences alone.[1] Li and Sarte employ an endogenous growth framework and make two restrictive assumptions: first that labor is supplied inelastically, and second, they restrict attention to the equilibrium-balanced growth path. While this provides a natural benchmark, it clearly does not offer a complete picture of the role of tax progressivity in determining the growth–inequality relationship.

This study belongs to this second strand of the literature. Its contribution is threefold. First, we focus on the more general setup where labor is supplied elastically. This generalization is important, since the adjustment of labor has been shown elsewhere to play a crucial role in determining the impact of structural changes on both growth and inequality, even in the absence of taxes.[2] But it becomes an even more crucial part of this process in the presence of a progressive tax structure, with the potentially significant impact on work incentives. Indeed, the fact that an increase in the progressivity of the tax structure has an adverse effect on labor supply is well established empirically, and confirms the relevance of endogenizing the labor supply.[3]

Second, we consider the entire dynamic adjustment path, highlighting how its response to structural changes is highly dependent upon the flexibility of labor supply, particularly in the short run. For example, we find that assuming labor to be supplied inelastically seriously understates the adverse short-run effect of an increase in tax progressivity on the growth rate of output. Moreover, as our analysis suggests, for plausible

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[1] This degenerate equilibrium in which the most patient agent owns all the wealth was first identified by Becker (1980).

[2] For example, Turnovsky and García-Peñalosa (2008) using a neoclassical framework show how endogenizing the labor supply can reverse the long-run growth–inequality response to a productivity increase from that obtained when labor is supplied inelastically. In this case, ignoring the response of labor supply in the adjustment process may be not only quantitatively but also qualitatively misleading!

[3] For example, Hausman (1981) using U.S. data and Blomquist (1983) and Blomquist, Eklöf, and Newey (2001) using Swedish data find that higher progressivity reduces hours of work in their studies. Guvenen, Kuruscu, and Ozkan (2009) reach a similar conclusion in a cross-country study. Koyuncu (2011) documents how the reduction in progressivity in the United States between the periods 1971 and 1974 and 1986 and 1989 and the increase in progressivity in Germany over the same time period were associated with an increase in hours worked in the United States and a decline in hours worked in Germany.

parameterization the transitional adjustment is rather slow. This means that the economy spends the overwhelming fraction of its time away from its balanced growth path, and underscores the need to consider the entire transitional path rather than just the steady state. Endogenizing labor supply speeds up the rate of convergence to an overall rate that is more consistent with empirical evidence, lending further support to its significance in the adjustment process. Last, our results highlight the divergent effects on the time paths of labor supply of different groups in response to policy shocks. This heterogeneity can provide useful insights to policymakers who may seek to use the tax structure to target the work effort of specific groups. This potentially important policy instrument is obscured if labor supply is fixed inelastically.

An important issue to arise with progressive taxes is its interaction with agents' rate of time preference. Interest in this issue dates back to Ramsey (1928), who formulated a model of stationary equilibrium with heterogeneous agents differentiated by their rates of time preference. He conjectured that the long-run equilibrium would be one in which all the capital would be held by the most patient household and the remaining—poor—households would consume at a subsistence level.[4] This idea was later formalized by Becker (1980) for an economy with borrowing constraints, and by Bewley (1982) for the case without borrowing constraints, where the consumption level of the impatient households is at zero. In contrast, Sarte (1997) using a stationary Ramsey model, and Li and Sarte (2004) for an endogenous growth model with inelastic labor, showed that if the tax structure is progressive the long-run distributions of capital and income will be nondegenerate. A progressive income tax breaks the link between the most patient household's preference rate and the return on capital, since each agent faces different after-tax returns. Hence, a unique nondegenerate distribution of income, in which households are ranked by their discount factors, is obtained. The most patient households are still the rich ones, but they do not own all the capital.[5] Li and Sarte (2004) also show that the degree of progressivity yields a negative relationship between the growth rate and income inequality when the labor supply is fixed.[6]

The framework we employ is an endogenous growth model of the generic Romer (1986) type with heterogeneous agents, who differ in terms of their rates of time preference, and supply labor elastically. A major complication introduced by having a progressive tax structure is that aggregate quantities and their distributions across individuals become simultaneously determined. In this case, one can no longer employ the Gorman (1953) aggregation procedure—otherwise possible under uniform taxes—in

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[4] In fact, this idea actually predates Ramsey. Rae (1834) was the first to argue that individuals who have less desire to accumulate than others would gradually fall into poverty, while others acquired assets. Fisher (1907) formulated this "desire" as the rate of time preference. See Becker (2006) for a detailed literature review including these early studies.

[5] Using a somewhat different approach, Carroll and Young (2009) show how a progressive tax structure can also lead to a long-run nondegenerate wealth distribution if agents have a common rate of time preference but are heterogeneous in their labor productivity. In a subsequent paper, Carroll and Young (2011) analyze the long-run effects of changes in tax progressivity on a number of aggregates, including distributional measures. Koyuncu (2011) studies working hour patterns in the United States and Germany using a model that includes heterogeneity in both labor productivity and rates of time preference. Recently, Angyridis (2015) develops a similar endogenous growth model to that presented here to analyze the impact of public investment under progressive taxation on growth and inequality.

[6] There is also a literature that focuses on more abstract aspects of the long-run distribution of wealth, distant from the focus of this paper. For example, using a Ramsey model, Sorger (2002) shows how there may in fact be infinitely many nondegenerate stationary distributions if the marginal tax rate is constant on nontrivial intervals. Bosi and Seegmuller (2010) draw attention to steady-state distributions in which the patient supply capital and the impatient supply labor.

which case a general analysis of the transitional dynamics becomes intractable.[7] We are, however, able to characterize the steady state, and in particular establish that the ranking of wealth and income across agents obtained under inelastic labor supply extends to the case where labor is supplied endogenously. To explore further the dynamics of the inequality–growth relationship, we limit the number of agents with different rates of time preference to just two, which we label as "Poor" and "Rich." In this case, we can describe the transitional dynamics explicitly, and further characterize some of the steady-state responses.[8]

These analytical results are supplemented with numerical simulations, parameterized to approximate a typical OECD economy. These simulations are employed to assess the quantitative characteristics of the dynamic responses to an increase in the degree of tax progressivity and to address the role of the labor supply in this process.[9] The key role played by the endogeneity of labor supply arises from the fact that it is free to jump on impact in response to any structural change, thereby generating immediate short-run consequences that continue throughout the subsequent transition. These turn out to be important in reconciling the simulation results with available empirical evidence.

Thus, an increase in the progressivity of the tax structure causes an immediate discrete decline in overall employment, leading to an immediate increase in the wage rate and decline in the return to capital. Together with the relative tax rates paid by the Poor and the Rich arising from the progressive tax structure, this causes the labor supply of the Poor to immediately increase and that of the Rich to decline. This in turn leads to a significant initial reduction in income inequality, which accounts for almost one third of the subsequent long-run response.

The decline in the return to capital, which is sustained throughout the transition, has a more adverse effect on the Rich, and in particular on their relative rates of wealth accumulation, so that wealth inequality and income inequality continue to decline throughout the transition. But as the relative income of the Poor rises, their tax rate increases, causing them to gradually reduce their labor supply and their rate of capital accumulation. These responses are almost precisely offset by those of the Rich who increase both over time. As a result, despite the sharply contrasting transitional paths followed by the two groups, the average economy-wide employment and growth rate both complete the adjustments to their respective new long-run equilibriums almost immediately.

A consequence of these transitional adjustments by the two groups is that the endogeneity of labor supply has a relatively minor influence insofar as the long-run impact of an increase in progressivity on income inequality is concerned. Under both inelastic and elastic labor supply, a more progressive tax structure reduces long-run income inequality by comparable amounts. This is because the adjustment in factor returns

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[7] This aggregation also depends upon other aspects, such as homogeneity of the preference function, the source of the heterogeneity, and the perfect markets, in which all individuals face the same rates of return on all productive factors. The difficulty introduced by the progressive tax structure is that each individual faces his own after-tax rate of return, depending upon his position in the income scale.

[8] We recognize that limiting ourselves to just two groups is restrictive. But it is inevitable to render the dynamic analysis tractable, given the complexity of the system involving the interaction between many individuals and aggregates.

[9] In an expanded version of this paper, we also conduct simulations to address how the presence of a progressive tax structure influences the responses to other structural changes, particularly an increase in productivity. There we show that it can potentially lead to different implications with respect to the growth–income inequality tradeoff from that obtained in models having flat tax rates, but are otherwise similar.

resulting from the labor supply responses in response to the increasing tax progressivity has largely offsetting effects on the relative incomes of the Rich and the Poor.[10] However, the increase in leisure and the accompanying increase in consumption by the Rich during the early phases of the transition mean substantially less savings, and thus the labor supply effect leads to a significant reduction in long-run wealth inequality.

The endogeneity of labor has more significant consequences for the long-run growth rate of capital, in that treating labor supply as fixed understates the long-run negative impact of progressivity on this growth rate by more than 25%. In the short run, due to the initial decline in the aggregate labor supply, the response in the GDP growth rate is much larger. The finding that an increase in progressivity is associated with a reduction in both inequality and the growth rate is supported by extensive empirical evidence.[11] But ignoring the adjustment of labor supply in this process severely understates the magnitude of the growth–inequality tradeoff.

The remainder of the paper is organized as follows. Section 2 sets out the analytical framework. Section 3 provides a general characterization of the steady state, while Section 4 provides some specific results for the two-class version of the model. Section 5 reports the simulations, Section 6 relates our numerical results to available empirical evidence, clearly establishing their relevance, and Section 7 concludes. A technical Appendix provides the detailed results for the transitional dynamics in the case of elastic labor supply, as well as the formal proofs of some of the propositions.[12]

## 2. The Analytical Framework

In this section, we develop an endogenous growth model with heterogeneous households subject to progressive taxation.

### 2.1. Technology and Factor Payments

The economy is comprised of $J$ identical firms indexed by $j$. We assume that the representative firm produces output in accordance with the Cobb-Douglas production function

$$Y_j = A(L_j K)^\alpha K_j^{1-\alpha} \quad 0 < \alpha < 1 \tag{1a}$$

where $K_j$ and $L_j$ represent individual firm's capital stock and employment of labor, respectively. $K$ is the average economy-wide capital stock, so that $L_j K$ measures the efficiency units of labor employed by the firm. The production function thus exhibits constant returns to scale in both labor and the firm's own capital stock $(L_j, K_j)$, as well

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[10] In fact, the net effect of the endogeneity of labor supply is to reduce slightly the increase in the relative income of the Poor, thereby reducing the impact of the progressive tax structure on income inequality.

[11] For example, Blomquist *et al.* (2001) find that in Sweden the decrease in the marginal tax rate is associated with an increase in inequality. In their cross-country study, Guvenen *et al.* (2009) find that progressivity reduces wage inequality, whereas Padovano and Galli (2002), using a panel of 25 industrialized countries, show that higher progressivity reduces long-run growth rates. In his documentation of the United States and Germany, Koyuncu (2011) shows that the decrease in progressivity in the United States between 1971 and 1974 and 1986 and 1989 is associated with an increase in the growth rate and in inequality, while the increase in progressivity in Germany during that period is associated with a decline in the growth rate and inequality.

[12] The transitional dynamics in the case of inelastic labor supply are straightforward and are not reported. Details are available from the authors.

as in the accumulating factor, capital $(K_j, K)$, and is therefore capable of sustaining endogenous growth. All firms are assumed to face identical production conditions, and thus they all choose the same levels of employment and capital stock. That is, $K_j = K$ and $L_j = L$ for all $j$, where $L$ is the average economy-wide employment. Hence, the aggregate production function is linear in the aggregate capital stock as in Romer (1986), namely

$$Y = AL^{\alpha}K. \tag{1b}$$

We assume that the wage rate, $\omega$, and the return to capital, $r$, are determined by the respective factor's marginal physical product. Letting $l$ denote the average economy-wide leisure time, so that $L = 1 - l$, we can express the equilibrium factor prices as a function of leisure time,

$$\omega = \alpha AL^{\alpha-1}K = \alpha A(1-l)^{\alpha-1}K \equiv w(l)K \tag{2a}$$

$$r = (1-\alpha)AL^{\alpha} = (1-\alpha)A(1-l)^{\alpha}. \tag{2b}$$

## 2.2. Consumers

The economy is populated by $N$ individuals (households), each indexed by $i$. Agents are identical in all respects except for their initial endowments of capital, $K_{i,0}$, and their rates of time preferences, $\beta_i$. We define the relative share of capital owned by agent $i$ as $k_i \equiv K_i/K$, so that relative capital has mean 1. Each agent is endowed with a unit of time that can be allocated either to leisure, $l_i$ or to work, $L_i = 1 - l_i$.

The representative consumer maximizes his lifetime utility, which is assumed to be a function of his consumption and leisure as specified by the isoelastic utility function, discounted by the individual-specific rate of time preference, $\beta_i$,

$$\max \int_0^{\infty} \frac{1}{\gamma}(C_i l_i^{\eta})^{\gamma} e^{-\beta_i t}dt, \tag{3}$$

where $-\infty < \gamma < 1, \eta > 0, 1 > \gamma(1+\eta)$ to ensure concavity of the utility function. We specify agent $i$'s average tax rate by

$$\tau_i \equiv \zeta(Y_i/Y)^{\phi} \tag{4a}$$

where individual $i$'s gross income is $Y_i = [rK_i + wK(1 - l_i)]$ and $Y = rK + wK(1 - l)$ is the economy-wide average level of income. If $\phi > 0$, this tax rate is an increasing function of the agent's relative income, $y_i \equiv Y_i/Y$, and increases at an increasing or decreasing rate according to whether $\phi \gtrless 1$. The marginal tax rate for such a tax schedule is thus

$$\tau_i^m \equiv \frac{\partial(\tau_i Y_i)}{\partial Y_i} = (1+\phi)\zeta\left(\frac{Y_i}{Y}\right)^{\phi} = (1+\phi)\zeta(y_i)^{\phi} = (1+\phi)\tau_i \tag{4b}$$

where $\phi$ represents the degree of progressivity, and $\zeta$ determines the level of the tax schedule.[13] For $\phi = 0, \tau_i = \zeta$ for all individuals; in this case the tax rate is flat and independent of the agent's relative income.[14] The agent chooses consumption, leisure, and

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[13] Thus, $\zeta$ is the tax rate imposed on the economy-wide average level of income.
[14] The specification we adopt has been previously employed by Li and Sarte (2004) and Lloyd-Braga, Modesto, and Seegmuller (2008). It is similar to the formulation $\tau_i \equiv 1 - \zeta(Y_i/Y)^{-\phi}$ previously

rate of capital accumulation to maximize utility, (3), subject to the capital accumulation constraint

$$\dot{K}_i = (1 - \zeta (Y_i / Y)^{\phi})[rK_i + wK(1 - l_i)] - C_i \tag{5}$$

where we have substituted for the tax rate (4a). Performing the optimization yields the corresponding first-order equations

$$C_i^{\gamma-1} l_i^{\eta\gamma} = \lambda_i \tag{6a}$$

$$\eta C_i^{\gamma} l_i^{\eta\gamma-1} = (1 - \tau_i^{m}) wK\lambda_i \tag{6b}$$

$$(1 - \tau_i^{m})r = \beta_i - \frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

where $\lambda_i$ is agent $i$'s shadow value of capital, together with the transversality condition

$$\lim_{t \to \infty} \lambda_i K_i e^{-\beta_i t} = 0. \tag{6d}$$

In deriving the optimality conditions, the agent, being subject to a progressive tax structure, takes account of the fact that, as he chooses to work or accumulate capital, he influences his marginal tax rate, which thereby becomes the relevant rate to the decisions reflected in (6b) and (6c). As we shall demonstrate below, each agent's capital stock converges to a balanced growth path, having the common growth rate, $\tilde{\psi}$, and hence the transversality condition for each agent reduces to

$$r\left(1 - \tau_i^{m}\right) > \tilde{\psi}. \tag{6e}$$

The aggregate tax revenues raised are used to finance government expenditures

$$G = \sum_{i=1}^{N} \tau_i Y_i = gYN \tag{7}$$

where the second equation in (7) expresses the endogenously determined expenditures as a share, $g$, of aggregate output.

## 3. Macroeconomic Equilibrium

To derive the macroeconomic equilibrium, we first divide (6b) by (6a) to obtain the following relationship between agent $i$'s consumption and his leisure[15]

$$C_i = \left(1 - \tau_i^{m}\right) l_i \frac{wK}{\eta}. \tag{8}$$

Taking the time derivative of (8) leads to

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{w_l \dot{l}}{w} + \frac{\dot{K}}{K} - \phi \frac{\tau_i^{m}}{1 - \tau_i^{m}} \frac{\dot{y}_i}{y_i}. \tag{9}$$

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introduced by Guo and Lansing (1998). Both formulations impose restrictions on the parameters $\zeta, \phi$ to ensure that $0 \le \tau_i, \tau_i^{m} < 1$, which we assume are met and certainly hold in our equilibrium and along the transitional path.

[15] We assume clearance of the aggregate labor and capital markets, which are described by $(1/J) \sum_{j=1}^{J} L_j = L = 1 - l = (1/N) \sum_{i=1}^{N} (1 - l_i) = (1/N) \sum_{i=1}^{N} L_i$ and $(1/J) \sum_{j=1}^{J} K_j = K = (1/N) \sum_{i=1}^{N} K_i$, respectively.

Next, taking the time derivative of (6a) and combining with (6c) gives

$$(\gamma - 1)\frac{\dot{C_i}}{C_i} + \eta\gamma\frac{\dot{l_i}}{l_i} = \beta_i - \left(1 - \tau_i^m\right)r \qquad (10)$$

and substituting for (8) into the individual's capital accumulation constraint (4) implies

$$\dot{K_i} = (1 - \tau_i)Y_i - \left(1 - \tau_i^m\right)l_i\frac{wK}{\eta}. \qquad (10a)$$

Equations (8) and (10a) highlight the complications introduced by a progressive tax structure. In the previous work of García-Peñalosa and Turnovsky (2007, 2011), which assumed a constant flat tax rate so that $\tau_i = \tau_i^m = \tau$, both (8) and (10a) are linear across individuals. This enables straightforward aggregation over the agents, leading to a macroeconomic equilibrium in which aggregates are independent of the distribution. As a result, the analysis of growth and distributional dynamics can be studied recursively, making it highly tractable. One first derives the aggregate equilibrium dynamics and then, having obtained that, determines how the factor returns it generates impact the distributional measures.[16]

But with a progressive tax rate, this recursive structure breaks down. This is because $\tau_i, \tau_i^m$ now vary across agents, so that exact aggregation is no longer possible. Individual tax rates depend both on the individual's quantities and on the aggregates, as a result of which the equilibrium of the aggregates and their distribution across agents become jointly determined. This renders the problem analytically intractable, except for the case of two agents (classes) that we shall discuss below.

To proceed further, we define

$$\bar{\tau} \equiv \frac{1}{N}\sum_{i=1}^{N}\tau_i \cdot \left(\frac{Y_i}{Y}\right) = \frac{\zeta}{N}\sum_{i=1}^{N}(y_i)^{1+\phi} \qquad \frac{1}{N}\sum_{i=1}^{N}\left(\frac{Y_i}{Y}\right) = \frac{1}{N}\sum_{i=1}^{N}y_i = 1 \quad (11a)$$

$$\bar{\tau}^m \equiv \frac{1}{N}\sum_{i=1}^{N}\tau_i^m \cdot \left(\frac{l_i}{l}\right) = \frac{\zeta(1+\phi)}{N}\sum_{i=1}^{N}(y_i)^\phi \cdot \left(\frac{l_i}{l}\right) \qquad \frac{1}{N}\sum_{i=1}^{N}\left(\frac{l_i}{l}\right) = 1 \quad (11b)$$

so that (11a) is a weighted average of the average tax rates paid by individuals, the weights being their respective relative incomes. Analogously, (11b) is a weighted average of the corresponding marginal tax rates, the weights being their respective relative leisure. With this notation, the economy-wide average tax rate $\bar{\tau}$ is equal to the share of government expenditures in aggregate output, $g$, as defined in (7), namely

$$\bar{\tau}YN = \sum_{i=1}^{N}\tau_i Y_i = gYN. \qquad (11c)$$

Using the definitions of the weighted average tax rates enables us to aggregate (10a) over $i$ and write the aggregate capital accumulation equation in the form

$$\dot{K} = (1 - \bar{\tau})Y - (1 - \bar{\tau}^m)l\frac{wK}{\eta}. \qquad (12)$$

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[16] This characteristic prompted Caselli and Ventura (2000) to refer to this as the "representative consumer theory of distribution."

Then, substituting (12) and (9) into (10), we can eliminate $\dot{C}_i/C_i$ to obtain

$$\phi \frac{\tau_i^m}{1-\tau_i^m} \frac{\dot{y}_i}{y_i} - \left(\frac{1-\gamma(1+\eta)}{1-\gamma}\right) \frac{\dot{l}_i}{l_i} - \frac{w_l}{w}\dot{l}$$

$$= (1-\bar{\tau})[r + w(1-l)] - (1-\bar{\tau}^m)l\frac{w}{\eta} + \frac{\beta_i - (1-\tau_i^m)r}{1-\gamma} \quad i = 1, \ldots, N. \quad (13)$$

Furthermore, combining (4) with (12), and recalling the definitions of factor returns in (2), enables us to express the evolution of agent $i$'s relative share of capital, $k_i \equiv K_i/K$ by

$$\dot{k}_i = \left\{(1-\tau_i(y_i))\left[r(l)k_i + w(l)(1-l_i)\right] - (1-\tau_i^m(y_i))\frac{l_i}{\eta}w(l)\right\}$$

$$- \left\{(1-\bar{\tau})[r(l) + w(l)(1-l)] - (1-\bar{\tau}^m)\frac{l}{\eta}w(l)\right\}k_i \quad i = 1, \ldots, N. \quad (14)$$

To complete the macroeconomic equilibrium, we write relative (gross) income, defined above, in the form

$$y_i = \frac{Y_i}{Y} = \frac{rK_i + w(1-l_i)K}{rK + w(1-l)K} = \frac{rk_i + w(1-l_i)}{r + w(1-l)}, \quad (15)$$

which, using the definitions of $r$ and $w$ in (2), can be expressed as

$$y_i = (1-\alpha)k_i + \alpha\left(\frac{1-l_i}{1-l}\right). \quad (16)$$

In conjunction with (11), (16) implies that $(1/N)\sum_{i=1}^{N} k_i = 1$, so that only $N-1$ of the accumulation Equations (14) are independent, with the dynamics being constrained by

$$\dot{y}_i = (1-\alpha)\dot{k}_i - \alpha\left(\frac{1}{1-l}\right)\dot{l}_i + \alpha\frac{(1-l_i)}{(1-l)^2}\dot{l}. \quad (16a)$$

Thus, the macrodynamic equilibrium can be summarized by the following system: (i) the $N$ independent equations of (13), (ii) the $N-1$ independent equations of (14), and (iii) the $N-1$ independent equations of (16). Together they yield three $N-2$ independent dynamic equations in $y_1, \ldots, y_{N-1}, k_1, \ldots, k_{N-1}, l_1, \ldots, l_{N-1}, l$. Comparing (10a) and (9), and recalling the definitions of $\tau_i, \tau_i^m, \tau, \bar{\tau}^m$, it is clear that the dynamics of the aggregates and the individuals are interdependent. While in general explicit analysis of this system is intractable, in the Appendix we set out the equilibrium for the case of two individuals which we identify as "classes."

The core dynamics for the two class case—which we identify as "Rich" and "Poor"—are expressed as a system of three independent differential equations in $y_1, l_1, l$, that are linked to $k_1$ by (16). Because $k_1$ is constrained to evolve sluggishly from a given initial condition, $k_{1,0}$, the stable dynamics consist of a one-dimensional transitional locus.[17] Under plausible conditions, this converges to a balanced growth path, along which all

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[17] In the Appendix, the critical matrix summarizing the local transitional dynamics, denoted by $A$, involves a high degree of interaction between the three dynamically evolving variables, $y_1, l_1, l$. While in principle one could set out formal conditions to ensure that the pattern of eigenvalues of $A$ imply a unique stable transitional path, for a system of this complexity such conditions are unwieldy and totally

individual and aggregate quantities grow at the same constant rate, except for the fraction of time allocated to leisure which remains constant.

From these core dynamics, the rest of the macroequilibrium dynamics can be derived. Thus, from (11a, 11b), $y_2(t) = 2 - y_1(t)$, $l_2(t) = 2l(t) - l_1(t)$, with $k_2(t) = 2 - k_1(t)$ then implied by (16). Also, the resulting individual and aggregate growth rates of capital can then be derived as follows:

$$\psi_i(t) \equiv \frac{\dot{K}_i}{K_i} = \frac{A(1-l)^{\alpha-1}}{k_i}\left[(1-\tau_i)\,y_i(1-l) - \left(1 - \tau_i^m\right)\alpha\frac{l_i}{\eta}\right]. \tag{17}$$

$$\psi(t) \equiv \frac{\dot{K}}{K} = A(1-l)^{\alpha-1}\left[(1-\bar{\tau})(1-l) - (1 - \bar{\tau}^m)\alpha\frac{l}{\eta}\right] \tag{17a}$$

which converge to a common long-run growth rate, $\tilde{\psi}$. Combining (17a) with (1b), the growth of output is given by $\psi_y(t) = \psi(t) - \alpha\dot{l}(t)/(1-l(t))$, which also converges to $\tilde{\psi}$. The main difference in the dynamics when labor is supplied inelastically is that with income being tied directly to capital, which evolves sluggishly, relative income and its distribution across agents are thereby constrained to evolve gradually. This has significant consequences for the dynamics of the growth–inequality relationship, particularly in the short run.

## 4. Characterization of Steady State

Setting $\dot{k}_i = \dot{y}_i = \dot{l}_i = \dot{l} = 0$ in the dynamic Equations (13) and (14), and using (16), the steady state can be summarized by the following relationships:

$$\tilde{\psi} = \frac{[1 - \tau_i^m(\tilde{y}_i)]\,r(\tilde{l}) - \beta_i}{1-\gamma}$$

$$= (1-\bar{\tau})[r(\tilde{l}) + w(\tilde{l})(1-\tilde{l})] - (1-\bar{\tau}^m)\tilde{l}\frac{w(\tilde{l})}{\eta}. \quad i = 1,\ldots,N \tag{18a}$$

$$\left\{(1-\tau_i(\tilde{y}_i))\left[r(\tilde{l})\tilde{k}_i + w(\tilde{l})(1-\tilde{l}_i)\right] - (1-\tau_i^m(\tilde{y}_i))\frac{\tilde{l}_i}{\eta}w(\tilde{l})\right\}$$

$$= \left\{(1-\bar{\tau})[r(\tilde{l}) + w(\tilde{l})(1-\tilde{l})] - (1-\bar{\tau}^m)\frac{\tilde{l}}{\eta}w(\tilde{l})\right\}\tilde{k}_i \quad i = 1,\ldots,N-1 \tag{18b}$$

$$\tilde{y}_i = (1-\alpha)\tilde{k}_i + \alpha\left(\frac{1-\tilde{l}_i}{1-\tilde{l}}\right) \quad i = 1,\ldots,N-1 \tag{18c}$$

$$(1/N)\sum_{i=1}^{N}\tilde{y}_i = 1, \quad (1/N)\sum_{i=1}^{N}\tilde{l}_i = \tilde{l}, \quad (1/N)\sum_{i=1}^{N}\tilde{k}_i = 1 \tag{18d}$$

---

uninformative. Instead, we rely on extensive numerical simulations and find that in all cases a unique stable transitional path is indeed obtained.

where tildes denote steady-state values, $r(\tilde{l})$, $w(\tilde{l})$ are defined in (2), and the various individual and average tax rates are defined above. Given these quantities, these equations jointly determine each individual's relative levels of income, capital, and leisure, the common growth rate, and the average leisure. The complex structure of the equation system due to the progressive tax structure prevents us from solving for the equilibrium values of these variables analytically.

We may observe, however, that in the conventional case of flat constant tax rates, when $\tau_i = \tau_i{}^m = \bar{\tau} = \bar{\tau}^m = \zeta$, the set of Equations (18b) simplifies drastically to the labor supply relationship derived by García-Peñalosa and Turnovsky (2007):

$$\tilde{l}_i - \tilde{l} = \left(\tilde{l} - \frac{\eta}{1+\eta}\right)(\tilde{k}_i - 1). \tag{18e}$$

In that case, the transversality condition (6e) simplifies to $\tilde{l} > \eta/(1+\eta)$ and (18e) asserts that agent $i$'s relative leisure increases with his relative wealth. But when the tax rate varies with relative income, it is convenient to substitute for the factor returns and rewrite (18b) in the form

$$\tilde{k}_i = \frac{[1-\tau_i(\tilde{y}_i)]\tilde{y}_i(1-\tilde{l}) - [1-\tau_i^m(\tilde{y}_i)]\alpha\tilde{l}_i/\eta}{[1-\bar{\tau}](1-\tilde{l}) - [1-\bar{\tau}^m]\alpha\tilde{l}/\eta}$$

which, combined with (18c), yields the following relationship between $\tilde{l}_i$ and $\tilde{y}_i$:

$$\frac{\tilde{y}_i}{1-\alpha} - \frac{\alpha(1-\tilde{l}_i)}{(1-\alpha)(1-\tilde{l})} = \frac{[1-\tau_i(\tilde{y}_i)]\tilde{y}_i(1-\tilde{l}) - [1-\tau_i^m(\tilde{y}_i)]\alpha\tilde{l}_i/\eta}{[1-\bar{\tau}](1-\tilde{l}) - [1-\bar{\tau}^m]\alpha\tilde{l}/\eta}. \tag{19}$$

Using (18a), (18c), and (19), we can establish the following proposition:

PROPOSITION 1: *Assume that taxes are progressive, $\phi > 0$.*

*A. If all agents share a common rate of time preference, $\beta_i = \beta$, then $\tilde{y}_i = \tilde{y}$, $\tilde{l}_i = \tilde{l}$, $\tilde{k}_i = \tilde{k}$ for all i, i.e., the steady-state distributions of income and wealth collapse to zero.*

*B. If for any pair of individuals i and j, $\beta_j > \beta_i$, then $\tilde{y}_i > \tilde{y}_j$, $\tilde{l}_i > \tilde{l}_j$, $\tilde{k}_i > \tilde{k}_j$; that is the more patient individual will have a higher income, (both before-tax and after-tax), enjoy more leisure, and have more wealth.*

*Proof:* See Appendix.                                                              ■

Intuitively, part A of Proposition 1 asserts that, because in equilibrium all agents experience the same long-run growth rate if they have the same rate of time preference, they must face the same marginal tax rate and therefore have the same relative income. In the long run, there is no income inequality. In their analysis of the progressivity of the tax structure, Li and Sarte (2004) note that the more patient individual will enjoy a higher level of income. Part B of Proposition 1 generalizes this pattern, previously obtained for just two individuals and inelastic labor, to an arbitrary number of individuals whose labor is supplied elastically. This proposition immediately leads to:

PROPOSITION 2: *If an agent who is more patient than average becomes more patient (less patient, but remains above average), the distributions of income (both pretax and posttax) and wealth will become more (less) unequal; the opposite applies to a more impatient individual.*

It is of interest to compare these results to the more familiar case of flat taxes, $\phi = 0$, when we obtain the following. As in Becker (1980) and Li and Sarte (2004), with

differential rates of time preference, the most patient individual will accumulate capital at the fastest rate, with the consequence that he will eventually own all the capital.[18] If, on the other hand, all agents have the same rate of time preference, the model reduces to that of García-Peñalosa and Turnovsky (2007) in which the long-run distributions of wealth and income are nondegenerate. But in contrast to the steady state summarized by (18) and (19), the distributions in this latter case will depend upon the initial endowments of capital, $k_{i,0}$. This is because in these circumstances all agents obtain identical net rates of return on both capital and labor. This makes the ratio of $l_i/l$ constant, introducing a zero root into the distributional dynamics, the effect of which is to render the long-run equilibrium distribution time-dependent (see Atolia, Chatterjee, and Turnovsky 2012).[19]

A key element in establishing these propositions is the existence of a large number of atomistic agents, each of whom has negligible effects on the aggregate quantities $\tilde{l}, \tilde{\psi}$. This enables us to determine the impact of the characteristics, such as $\beta_i$, of one individual taken in isolation. The responses to other characteristics, such as the tax structure parameterized by $(\zeta, \phi)$, will generate responses by all agents, the collective decisions of whom will influence the aggregate equilibrium quantities and are more difficult to determine. We shall characterize some of them in Section 5 for a two-class economy comprised of "Rich" and "Poor" agents.

## 5. Further Characterization of the Equilibrium Income Distribution

We now focus on the two-class case, where $\beta_1 > \beta_2$. That is, Class 1 is less patient and will end up poorer than Class 2. Restricting the number of heterogeneous agents enables us to characterize important features of the model pertaining to the structure of the tax system and its impact on income distribution and growth. Setting $N = 2$, we may rewrite the equilibrium growth equations in (18a) for both classes

$$\tilde{\psi} = \frac{\left[1 - (1 + \phi)\zeta \tilde{y}_1{}^\phi\right] r(\tilde{l}) - \beta_1}{1 - \gamma} \tag{20a}$$

$$\tilde{\psi} = \frac{\left[1 - (1 + \phi)\zeta \tilde{y}_2{}^\phi\right] r(\tilde{l}) - \beta_2}{1 - \gamma} \tag{20b}$$

from which we obtain

$$\tilde{y}_2{}^\phi - \tilde{y}_1{}^\phi = \frac{\beta_1 - \beta_2}{\zeta (1 + \phi) r(\tilde{l})} \tag{21}$$

where $\tilde{y}_1 + \tilde{y}_2 = 2$. With agent $i$ being more impatient, (21) implies $\tilde{y}_2 > 1 > \tilde{y}_1$.

From (20) and (21), we can determine the impact of changes in the base tax rate, $\zeta$, and the progressivity $\phi$ on relative income and growth.

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[18] One further point is worth noting. Like most of the literature, we assume that wage income and capital income are taxed at a common rate. If these two classes of income are subject to differential rate structures, the relevant progressivity in Proposition 1 would refer to that of the *capital* income tax. In general, the tax on capital income is less progressive than is the tax on labor income. In the limiting case that it is flat, the extreme result that the most patient individual ends up owning all the capital would again obtain, irrespective of the progressivity of the tax on labor income.

[19] In this respect, we note that Becker (1980) suggests that under these conditions the steady-state distributions are indeterminate. This is because he does not consider the hysteresis associated with the transitional dynamics, which adds the extra condition that provides the determinacy. Moreover, the same determinacy is obtained with inelastic labor supply (see García-Peñalosa and Turnovsky 2009).

(i) Increase in base tax rate:

$$\frac{\partial \tilde{y}_1}{\partial \zeta} = \frac{\tilde{y}_2{}^\phi - \tilde{y}_1{}^\phi}{\zeta \phi \left( \tilde{y}_1{}^{\phi-1} + \tilde{y}_2{}^{\phi-1} \right)} \left[ 1 + \alpha E_\zeta^L \right] \tag{22a}$$

$$\frac{\partial \tilde{y}_2}{\partial \zeta} - \frac{\partial \tilde{y}_1}{\partial \zeta} = -2 \frac{\partial \tilde{y}_1}{\partial \zeta} \tag{22b}$$

$$\frac{\partial \tilde{y}_2^a}{\partial \zeta} - \frac{\partial \tilde{y}_1^a}{\partial \zeta} = -\left[ 2 - \left( \tilde{\tau}_1^m + \tilde{\tau}_2^m \right) \right] \frac{\partial \tilde{y}_1}{\partial \zeta} - \left[ \tilde{y}_2{}^{1+\phi} - \tilde{y}_1{}^{1+\phi} \right] \tag{22c}$$

$$\frac{\partial \tilde{\psi}}{\partial \zeta} = -\frac{(1+\phi)\tilde{r}\tilde{y}_1{}^{\phi-1}}{1-\gamma} \left[ \tilde{y}_1 + \zeta \phi \frac{\partial \tilde{y}_1}{\partial \zeta} \right] + \frac{\left[ 1 - (1+\phi)\zeta \tilde{y}_1{}^\phi \right] \tilde{r}}{1-\gamma} \frac{\alpha E_\zeta^L}{\zeta}. \tag{22d}$$

(ii) Increase in progressivity:

$$\frac{\partial \tilde{y}_1}{\partial \phi} = \frac{1}{\phi \left( \tilde{y}_1{}^{\phi-1} + \tilde{y}_2{}^{\phi-1} \right)} \left[ \left( \tilde{y}_2{}^\phi - \tilde{y}_1{}^\phi \right) \left( \frac{1}{1+\phi} + \alpha E_\phi^L \right) + \left( \tilde{y}_2{}^\phi \ln \tilde{y}_2 - \tilde{y}_1{}^\phi \ln \tilde{y}_1 \right) \right] \tag{23a}$$

$$\frac{\partial \tilde{y}_2}{\partial \phi} - \frac{\partial \tilde{y}_1}{\partial \phi} = -2 \frac{\partial \tilde{y}_1}{\partial \phi} \tag{23b}$$

$$\frac{\partial \tilde{y}_2^a}{\partial \phi} - \frac{\partial \tilde{y}_1^a}{\partial \phi} = -\left[ 2 - \left( \tilde{\tau}_1^m + \tilde{\tau}_2^m \right) \right] \frac{\partial \tilde{y}_1}{\partial \phi} - \zeta \left[ \tilde{y}_2{}^{1+\phi} \ln \tilde{y}_2 - \tilde{y}_1{}^{1+\phi} \ln \tilde{y}_1 \right] \tag{23c}$$

$$\frac{\partial \tilde{\psi}}{\partial \phi} = -\frac{\zeta \tilde{r} \tilde{y}_1{}^{\phi-1}}{1-\gamma} \left[ \tilde{y}_1 + (1+\phi) \left( \tilde{y}_1 \ln \tilde{y}_1 + \phi \frac{\partial \tilde{y}_1}{\partial \phi} \right) \right] + \frac{\left[ 1 - (1+\phi)\zeta \tilde{y}_1{}^\phi \right] \tilde{r}}{1-\gamma} \frac{\alpha E_\phi^L}{\phi} \tag{23d}$$

where $y_i^a$ is the after-tax income of agent $i$, and $E_\zeta^L$, $E_\phi^L$ denote the elasticity of aggregate labor supply with respect to the base tax rate, $\zeta$, and the progressivity, $\phi$, respectively. The responses are summarized in the following proposition:[20]

PROPOSITION 3:
   *A. If labor is supplied inelastically so that the return to capital is fixed, an increase in either the base tax rate or its progressivity will (i) raise the relative steady-state income of the less patient class, (ii) reduce the relative steady-state income of the more patient individual, (iii) reduce both before-tax and after-tax income inequality, and (iv) reduce the long-run equilibrium growth rate.*
   *B. These qualitative effects continue to apply if labor is supplied elastically and the elasticities of aggregate labor supply with respect to the base tax rate, $\zeta$, and the progressivity, $\phi$, satisfy $E_\zeta^L > -\alpha^{-1}$, $E_\phi^L > -[\alpha(1+\phi)]^{-1}$.*

From (20) and (21) it is seen that the endogeneity of labor supply operates through its impact on the return to capital. If the labor supply declines when either the base tax rate or its progressivity increases, as the theory of labor supply suggests, then the return

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[20] These include responses noted by Li and Sarte (2004) for an increase in tax progressivity under inelastic labor supply. Part A of the proposition immediately generalizes to the case of elastic labor supply with no further restrictions if, as do Li and Sarte (2004), one employs the two-sector production technology of Rebelo (1991), for which the rate of return is constant, independent of the labor supply.

to capital, $r$, would also decline since it is increasing in labor (see (2b)).[21] This tends to reinforce the negative effects on the growth rate obtained under inelastic labor supply. The distributional effects are offsetting. This is because for both agents to experience the same long-run equilibrium growth rate, given the differential rates of time preference, $\beta_1 - \beta_2$, $\tilde{y}_1$ must decline relative to $\tilde{y}_2$. This is necessary to compensate for the reduction in the return to capital and thereby maintain a differential between the agents' after-tax rates of return consistent with their differential rates of time preference.

The inequalities in Part B of Proposition 3 pertain to general equilibrium elasticities of aggregate labor supply with respect to the base tax rate, $\zeta$, and its progressivity, $\phi$. For the calibration described in Section 5, the responses under inelastic labor supply will continue to hold provided these elasticities satisfy $E_\zeta^L > -2.94$, $E_\phi^L > -1.68$, respectively. Empirical information on such elasticities is sparse. However, a comprehensive study by the Congressional Budget Office (2007) on the effects of tax changes on labor supply suggests that such labor supply elasticities are small, certainly in the range $(-1, 0)$. This evidence suggests that the responses under inelastic labor supply will certainly dominate and continue to apply. In this case, the net effect of endogenous labor supply is to reduce income inequality.

## 6. Numerical Analysis

To obtain further insight, we shall proceed numerically, focusing both on steady-state responses and on the transitional dynamics. In conducting the numerical analysis, we continue to focus on the two-class economy. This has the advantage that it enables us to intuit the workings of the model—particularly the dynamic adjustments—which is clearly infeasible when an arbitrary number of agents are included. As already identified, Class 1 contains the impatient households indexed with time preference $\beta_1$ while Class 2 is comprised of the patient households having a discount rate $\beta_2 < \beta_1$. As we have seen, the patient households will have relatively more income in the long run. Accordingly, we have characterized the two groups as "Poor" and "Rich," respectively. One can view these as two broad classes in a society, the Poor representing the individuals with income levels below the national average and the Rich representing those earning above the average.

### 6.1. Baseline Calibration

We begin by setting $N = 2$ in the steady-state system (18) to characterize an initial baseline equilibrium. In order to solve this system numerically, we set the standard parameter values as in Table 1, which yields the initial steady-state equilibrium reported in Table 2.

The parameterization of a Romer-type production function is generically problematic. The value chosen for the production elasticity of labor, $\alpha$, implies that 34% of output accrues to labor. This is consistent with the empirical evidence suggested by Mankiw, Romer, and Weil (1992) and Barro, Mankiw, and Sala-i-Martin (1995), provided one

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[21] These theories usually assume that the substitution effect dominates the income effect. Because the income distribution is simultaneously changing in our model as tax rates change, this may not be true at all times. In a static model, Sandmo (1983) shows that income effect dominates under certain conditions if income redistribution is taken into account.

Table 1: Parameter values

| Technology parameters | |
|---|---|
| Scale parameter, $A$ | 0.22 |
| Elasticity of labor in production, $\alpha$ | 0.34 |
| Preference parameters | |
| Elasticity of leisure, $\eta$ | 0.85 |
| Intertemporal elast. of subst. $1/(1-\gamma)$ | 0.5 |
| Individual rate of time preference, $\beta_i$ | $\beta_1 = 0.042$, $\beta_2 = 0.032$ |
| Tax schedule parameters | |
| Economy-wide average tax rate, $\bar{\tau}$ (%) | 12.4 |
| Marginal tax rate (%) | 21.4 |
| Degree of progressivity, $1+\phi$ | 1.75 |
| Level of tax schedule, $\zeta$ | 0.115 |

Table 2: Two-class economy baseline calibration results

| | Model |
|---|---|
| Growth rate (%), $\psi$ | 2.01 |
| Labor supply | |
| Poor, $1 - l_1 \equiv L_1$ | 0.49 |
| Rich, $1 - l_2 \equiv L_2$ | 0.107 |
| Average, $1 - l \equiv L$ | 0.299 |
| Income shares, $y_i \equiv Y_i/Y$ | |
| Poor, $y_1$ | 0.66 |
| Rich, $y_2$ | 1.34 |
| Capital shares, $k_i \equiv K_i/K$ | |
| Poor, $k_1$ | 0.15 |
| Rich, $k_2$ | 1.85 |

interprets capital broadly to be an amalgam of physical capital and human capital.[22] With this interpretation, labor in the production function represents "raw labor," while the return to labor associated with skills is incorporated in the return to capital. As in all endogenous growth models, the equilibrium growth rate is highly sensitive to the scale parameter, $A$, which is set to 0.22 to target an equilibrium growth rate of 2%, typical of the United States and other OECD countries. It also depends upon the intertemporal elasticity of substitution (IES) and setting $\gamma = -1$ implies an IES $= 0.5$, which is well within the consensus range.

The elasticity of leisure in utility, $\eta$, is set so as to yield an overall allocation of time to raw labor of around 30%, which is consistent with empirical estimates.[23] Subjective

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[22] Mankiw *et al.* (1992) estimate a Cobb-Douglas production function in which output depends upon labor, human capital, and physical capital and obtain approximately equal exponents on all three arguments.

[23] It implies an aggregate Frisch elasticity of around 1.5, well within the range (1–2) adopted in macroeconomic simulations (see Keane and Rogerson 2012). The inconsistency between these aggregate values and the smaller estimates obtained from microdata is an issue currently occupying the attention of labor economists. Keane and Rogerson (2012) offer a reconciliation that credibly supports the range typically adopted in macroeconomic simulations.

discount rates of the two classes, $\beta_1 = 0.042$, $\beta_2 = 0.032$, are chosen to obtain plausible levels of steady-state wealth and income inequality. In this respect, they imply that the poorer half of the population owns around 7.5% of total wealth, while their average income is around 49% of the average of the rich group. This distribution of capital between the lower and upper 50% is well within the range of estimates for most developed countries (see Davies *et al.* 2007), and in particular for the United States, where it is around 8%. While the income allocation is also a reasonable representation of an economy with relatively unequal income distribution, it is somewhat more equitable than in fact it is in the United States, where the ratio is about 33%.[24] These rates of time preference together with the choice of $\eta$ imply that the Poor and the Rich allocate 49% and 11%, respectively, of their time to providing raw labor. While this surely understates the amount of total time devoted to labor by the Rich, we must bear in mind that some of that allocation is reflected, through skills, in the return to capital, which we are interpreting as a broad measure.

The parameterization of the two tax parameters $\zeta, \phi$ are set to approximate the effective U.S. Federal tax schedule as reported by Feenberg and Coutts (1993). They imply an economy-wide average tax rate of 12.4% with an average marginal rate of 21.4%, very close to the economy-wide estimates of 12.9% and 22.6%, respectively, for 1991 .

These figures constitute our baseline calibration for the hypothetical two-class economy. Our objective is to assess the significance of the response of the labor supply through incentives to structural changes, specifically the progressivity of the tax rate. To preserve comparability and isolate the role of the flexibility of labor supply, in the case of fixed labor supply we set the labor supply of the two groups at the same levels as in the baseline scenario, i.e., $\bar{L}_2 = 0.49$, $\bar{L}_2 = 0.107$ and $\bar{L} = 0.299$.

## 6.2. Increase in Progressivity

In this section, we examine the dynamic responses of inequality and growth to a 50% increase in the progressivity of the tax structure, $\phi$, from its benchmark level of 0.75 to 1.125, contrasting both the cases of elastic and inelastic labor supply. By normalizing the initial labor supply, in both cases the economy starts out from the initial steady state as reported in Table 2.

An important characteristic of the specification of the progressive tax system is that because it is expressed in terms of income *relative* to the mean, the direct effect of an increase in $\phi$ is that the average tax rate for the poor class actually declines, while that for the rich increases. This is evident from $\partial \tau_i / \partial \phi = \xi y_i^{\phi}$. ln $y_i$ which for the poor class, $y_1 < 1$, is negative, for the rich class, $y_2 > 1$, it is positive (see Figures 1b and 1c). We assume that in increasing the progressivity of the tax structure the government seeks to maintain the long-run steady-state fraction of income that is taxed, so that it can maintain its share of government expenditure. As our simulations reveal, over time the declining growth rate coupled with the declining tax base provided by the Poor, leads to a slight decline in the overall tax revenues. Thus, in order to maintain its long-run share of output, we assume that the government supplements its increase in the progressivity $\phi$, with a slight increase in the base rate, $\xi$, from 0.115 to 0.119.[25]

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[24] For many OECD countries such as Sweden, Denmark, the Netherlands, and the Czech Republic, the percentage share of income of the poorer half is around 50% of that of the richer half (see World Development Indicators 2013).

[25] Whether the government actually adjusts the base tax rate in this case has virtually no bearing on the results.
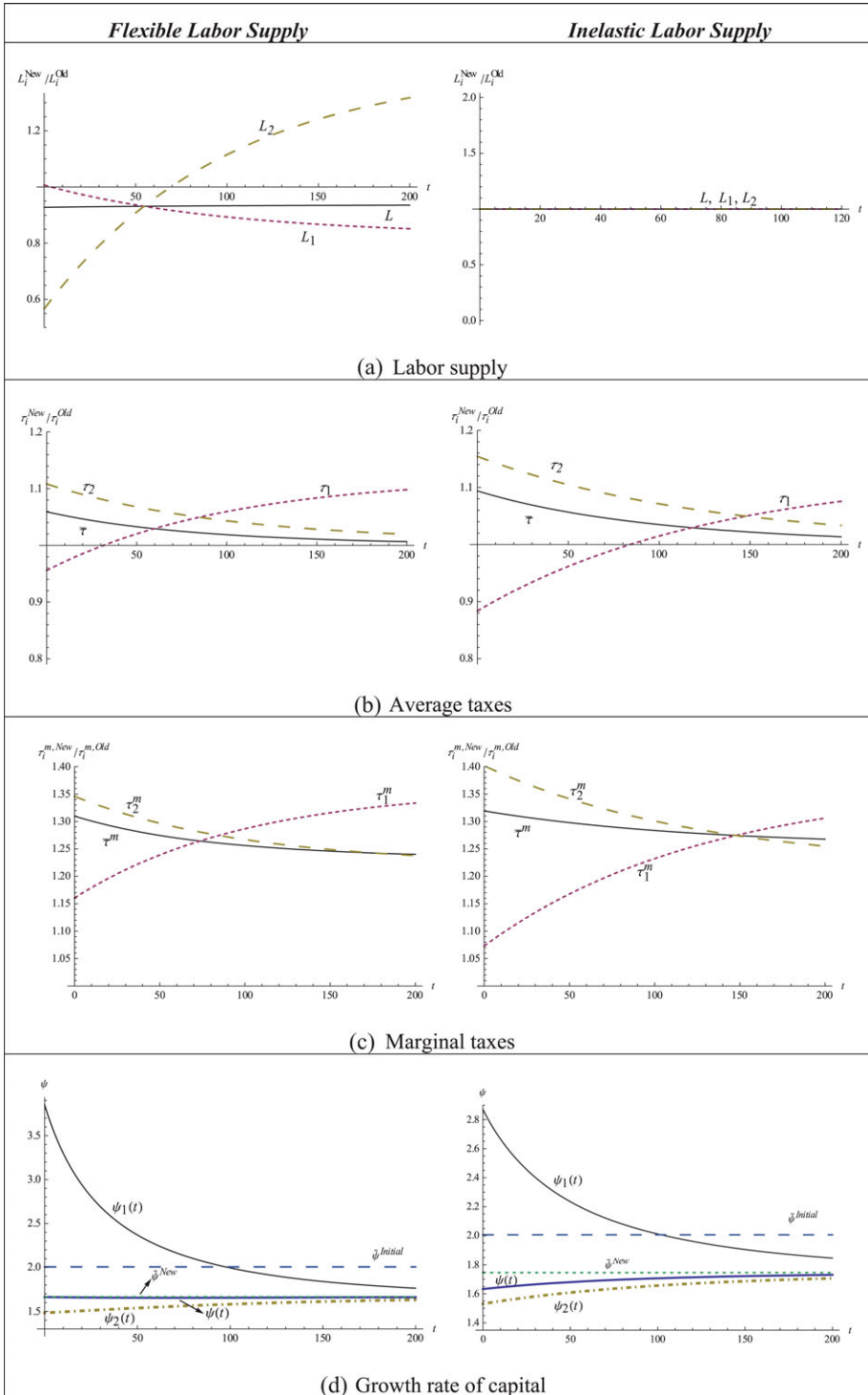
Figure 1: Dynamic effects of increase in progressivity.

(e) Distribution of capital
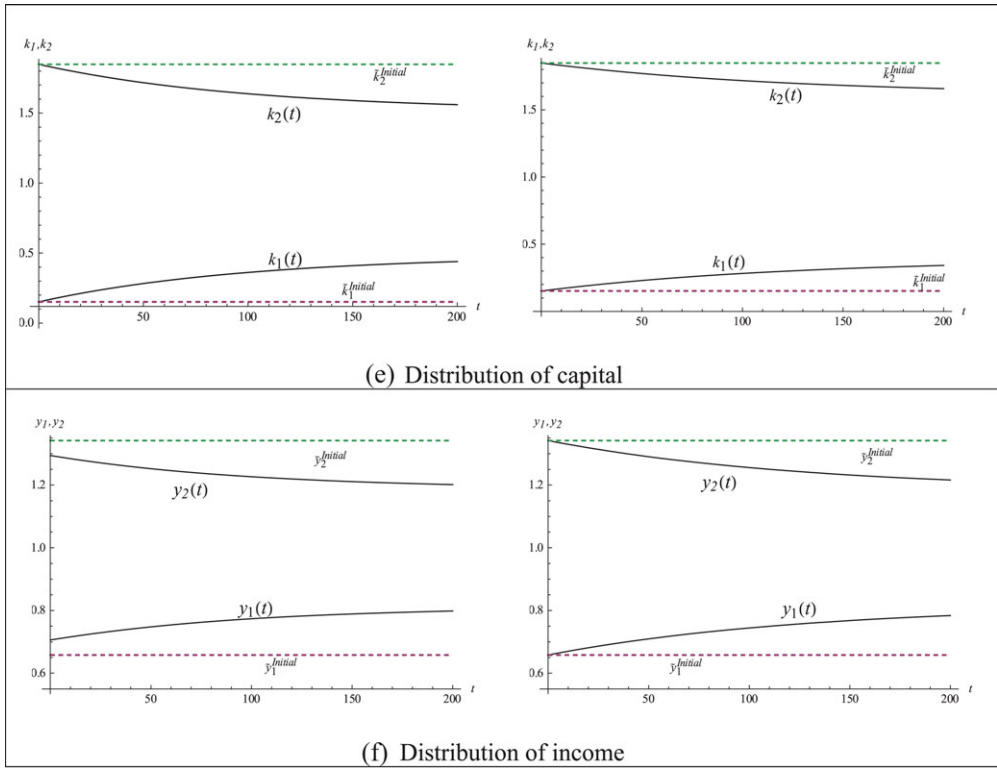
(f) Distribution of income

Figure 1: Continued.

In Table 3, we report the changes on impact (marked SR) and the steady-state changes (marked LR), in both cases, relative to their common initial steady state. Figure 1 illustrates the corresponding transitional dynamics. From Figure 1a, we see that the increase in progressivity is associated with an immediate decline in overall employment, $L$, which remains virtually unchanged throughout the entire subsequent transition. This decline is consistent with empirical evidence (see footnote 3). It is reflected in an immediate increase in the wage rate, $w(l)$, coupled with a decrease in the return to capital, $r(l)$, which also remain essentially constant throughout the transition. Setting $\alpha = 0.34$ implies that the proportionate increase in the wage rate is approximately double that of the decline in the return to capital (see (2a, 2b)) and is a further factor favoring the Poor.

Despite the increase in $w(l)$, the increase in progressivity leads to a reduction in the posttax real wage rate of the Rich, $w(l)(1 - \tau_2)$, while the after-tax wage rate of the Poor increases. Accordingly, on impact, the Poor increase their supply of labor, while the labor supply of the Rich declines, with the latter being more adversely affected by virtue of their higher income. As Table 3 indicates, the initial change in the labor supply comprises a small increase of 0.003 by the Poor, accompanied by a larger decline of 0.048 by the Rich, yielding an overall decrease in labor supply of 0.022 to 0.277. The corresponding impact on relative income is that $y_1$ increases by 0.048, while $y_2$ decreases correspondingly, implying an initial discrete decline in pretax income inequality (see Figure 1f). But with the capital stock fixed instantaneously, wealth inequality remains unchanged.

Table 3: Short- and long-run effects of higher progressivity

|  | Flexible Labor Supply | | Inelastic Labor Supply | |
|---|---|---|---|---|
|  | SR | LR | SR | LR |
| % pt $\Delta\psi_1$ | 1.846 | −0.341 | 0.864 | −0.261 |
| % pt $\Delta\psi_2$ | −0.523 | −0.341 | −0.476 | −0.261 |
| % pt $\Delta\psi$ | −0.343 | −0.341 | −0.374 | −0.261 |
| % pt $\Delta\psi_y$ | −2.901 | −0.341 | −0.377 | −0.261 |
| $\Delta L_1$ | 0.003 | −0.085 | 0 | 0 |
| $\Delta L_2$ | −0.046 | 0.047 | 0 | 0 |
| $\Delta L$ | −0.022 | −0.019 | 0 | 0 |
| $\Delta y_1$ | 0.048 | 0.155 | 0 | 0.159 |
| $\Delta y_2$ | −0.048 | −0.155 | 0 | −0.159 |
| $\Delta|y_1 - y_2|/2$ | −0.048 | −0.155 | 0 | −0.159 |
| $\Delta y_1^a$ | 0.046 | 0.134 | 0.006 | 0.137 |
| $\Delta y_2^a$ | −0.061 | −0.134 | −0.029 | −0.137 |
| $\Delta|y_1^a - y_2^a|/2$ | −0.054 | −0.134 | −0.018 | −0.137 |
| $\Delta k_1$ | 0 | 0.333 | 0 | 0.241 |
| $\Delta k_2$ | 0 | −0.333 | 0 | −0.241 |
| $\Delta|k_1 - k_2|/2$ | 0 | −0.333 | 0 | −0.241 |

The decline in the return to capital has an additional adverse effect on the Rich, both because they own more capital and because they are more negatively affected by the increase in progressivity of the tax rate. Accordingly, they immediately reduce their rate of capital accumulation by around 0.52 percentage points. In contrast, the combination of the lower average tax rate accompanied by the higher marginal tax rate for the Poor causes their income to increase more than their consumption, so that on impact they significantly increase their rate of capital accumulation. On balance, however, with the Rich owning most of the capital the economy-wide growth rate declines by 0.34 percentage points from 2.01% to 1.67%, and wealth inequality begins to decline.

While the growth rate of capital is relevant, the growth rate of greatest interest is that of GDP. From the aggregate production function (1d) this is given by

$$\psi_y(t) = \alpha \frac{\dot{l}(t)}{L(t)} + \psi(t). \tag{25}$$

On impact, the initial downward jump in $L(0)$ introduces an impulse into (25), implying that $\psi_y(0)$ becomes infinite at that instant. In order to "smooth" this effect so that it can be compared to estimates of empirical growth rates obtained over some finite period, we integrate (25) over a unit period implying the following approximation to the short-run growth rate of GDP over the first period following the increase in progressivity:[26]

$$\psi_{y,1} \approx \alpha[\ln L_1 - \ln \tilde{L}_0] + \psi_1. \tag{26}$$

From (26) we see that the initial response in the growth rate of GDP is dominated by the initial decline in employment, which contributes 2.56 percentage points to the

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[26] Our explanation here is heuristic. The calculations can be done more formally using the Dirac delta function.

initial decline in the GDP growth rate, so that in total it drops by 2.90 percentage points. Following the initial jump, with $L(t)$ remaining approximately constant throughout the transition, the growth rate of GDP closely tracks that of capital.

Over time, as the relative share of capital owned by the Poor increases, while that of the Rich declines, wealth inequality and income inequality continue to decline, eventually stabilizing. However, as $y_1$ increases the tax rate of the Poor increases and similarly that of the Rich declines. As a result, with $w(l)$ fixed during the transition, the after-tax wage rate of the Poor declines while that of the Rich rises, leading the former to reduce their labor supply and the latter to increase it. These adjustments are approximately offsetting, so that the overall rate of employment remains constant at 0.277. At the same time, the increase in the rate of capital accumulation by the Poor declines rapidly and is offset by a much more gradual increase in capital accumulation by the Rich such that on balance the overall growth rate of capital also remains approximately constant during the transition. Thus, while the increase in the progressive tax rate causes both average employment and the average economy-wide growth rate of capital to immediately adjust close to their new long-run equilibria, the two groups pursue very different time paths. After their initial responses, the Poor decrease their employment and their growth rate of capital, while the Rich increase both during the transition. Between the initial and final steady state the growth rate declines by 0.34% points, while pretax income inequality measured by $\Delta|(y_1 - y_2)|/2$ declines by 15.5% and wealth inequality by 33.3%.[27] The finding that progressivity has a substantially more adverse effect on wealth inequality than on income inequality is a consequence of the progressivity impacting more heavily on the Rich, via its effect on the return to capital, thereby inhibiting their ability to accumulate capital.

The right-hand panel of Figure 1 illustrates the adjustment in response to the increase in progressivity when labor is supplied inelastically. As in the case of elastically supplied labor, the increase in progressivity, $\phi$, reduces the average tax rate for the Poor and raises it for the Rich. But with labor supply being fixed, $r(l)$, $w(l)$ also remain fixed at all points of time, including the instant at which the increase in $\phi$ occurs. Thus, the relative income of both groups is predetermined, along with their relative capital stock, which it exactly reflects.

Comparing the two cases, we note some similarities, but also several significant differences. First, while the responses of the individual and average growth rates share the same qualitative features, the adjustments are somewhat smaller. This reflects the fact that when the labor supply is fixed, the constancy of the wage rate and the return to capital reduces the increase in income to the Poor, so that their rate of capital accumulation increase by only 0.86%, while an analogous argument applies to the Rich. The constancy of factor returns is also the reason that the reduction in the overall growth rate now exhibits more of a transition and is substantially smaller. This is because when labor is elastically supplied, progressivity induces substitution to leisure, leading to less output and growth. Second, the fact that the decline in $r(l)$ in this case is essentially completed on impact means that the bulk of the adjustment in the growth rate, and a third of the adjustment in income inequality also occur on impact. Third, while the long-run reduction in wealth inequality is larger with elastic labor supply, the reduction in income inequality is slightly smaller, the latter being consistent with Proposition 3. This is a reflection of the fact that the reduction in $r(l)$ which occurs in this case hurts

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[27] The finding that an increase in progressivity of the tax structure leads to a decline in the long-run growth rate accompanied by a decline in inequality is robust across the changes in progressivity, although the declines in inequality decrease with the degree of progressivity.

the Rich (wealthy) more than the Poor, while the accompanying increase in $w(l)$ favors the Poor over the Rich. Overall, our analysis suggests that the flexibility of labor supply is a significant factor in the adjustment process.

One final observation is that with elastic labor supply, the labor supplied by both the Rich and the Poor adjusts nonmonotonically to an increase in progressivity. While the Poor increase their short-run labor supply by 0.003, they reduce it by 0.085 in the long run. Similarly, the Rich reduce their short-run labor supply by 0.046, but increase it by 0.047 over time. The reason is that the declining income inequality that occurs over time raises the marginal tax rate for the Poor, causing them to reduce their work hours, while having the opposite apply to the Rich. In both cases, these intertemporal adjustments are sufficient to reverse the initial responses.

## 7. Empirical Implications

To assess the plausibility of these numerical simulations, we compare their implications to available empirical evidence. We shall focus our attention of the effects of tax progressivity on three aspects, namely (i) the growth rate, (ii) the labor supply, and (iii) income inequality. Although there is a voluminous literature examining the effects of average (or uniform) taxes on these measures, the literature that addresses the corresponding effects of progressive structures is much sparser. We should also note that the empirical literature focuses on the growth rate of GDP (rather than capital). Finally, since the transitions generated by our simulations are extremely slow, it is more relevant to compare the short-run predictions with the evidence, which always pertain to a much shorter time horizon than that implied by the steady-state adjustment.

### 7.1. Effects of Tax Progressivity on the Economic Growth Rate

In our analysis, we have specified an increase in progressivity by a 50% increase in the exponent, $\phi$, holding the average tax rate constant. In our simulated equilibrium, this represents a 5 percentage point increase in the average marginal tax rate (AMTR) from 21.3% to 26.3%, or equivalently a 23.3% increase in the AMTR. Normalizing these changes, the results discussed in Section 5 imply that with elastic labor supply a 1 percentage point increase in the AMTR reduces the short-run growth rate of GDP by about 0.58 percentage points and the long-run growth rate by about 0.07 percentage points. With inelastic labor supply, the corresponding effects are 0.08 percentage points and 0.05 percentage points, respectively.

These implications are broadly consistent with available empirical evidence. They closely match the Barro and Redlick (2011) findings that a 1 percentage point increase in the AMTR reduces the short-run growth rate of GDP by around 0.5 percentage points. The fact that this closeness is due to the elasticity of labor supply is consistent with Rhee's (2013) conclusion that the negative relationship he obtains between tax progressivity and growth can be explained by different supply elasticities across different income groups. The sharp contrast that we obtain between the short run and the long run is consistent with the evidence provided by Padovano and Galli (2002). Using a panel of decade averages based on 25 industrialized countries they find that a 1 percentage point increase in the AMTR reduces the long-run growth rate by about 0.034 percentage points. As one further body of evidence, OECD studies summarized by Arnold (2008) use a measure of progressivity similar to the one we employ. Adapting it our expression and in terms of our notation, they find that a 10% increase in $\phi$ reduces the growth rate

by about 0.25 percentage points, somewhat less than what we find, but still of the same order of magnitude.

## 7.2. Effects of Tax Progressivity on Labor Supply

From Table 3 we see that a 5 percentage point (23.2%) increase in the AMTR leads to an overall reduction in labor supply of around 7.3% in the short run and around 6.3% in the long run. There is a huge disparity between the responses of the two groups. Whereas in the short run the Poor increase their labor supply by 0.6%, in the long run they reduce it by 17.3%. The Rich respond in precisely the opposite way, decreasing it by 43% in the short run, and increasing it by a comparable amount asymptotically. For the reason noted above, the short-run responses are the relevant ones to compare with the evidence. Also, in interpreting the responses of the Rich, we should bear in mind that we have interpreted $L$ as reflecting only their unskilled component of labor. Much of their labor is in the form of human capital and is incorporated in our measure of capital (which remains fixed on impact), so that in fact these numbers substantially overstate their overall percentage response of labor supply.

Nevertheless, these patterns are generally consistent with much of the empirical evidence. For example, the more intensive responses of the richer agents is obtained by Hausman (1981) who, using U.S. data, finds that a 30% tax cut leads to a 1.3% increase in labor supply for low-paid workers and a 4.6% increase for high-paid workers. Moreover, Ashworth and Ulph (1981) using U.K. data obtain similar contrasting responses to those characterizing our simulations. They find that a 15% tax increase leads to a 0.3% increase in labor supply for low-paid workers but a 2.6% reduction for higher paid workers, with an average of a 2.9% reduction. In both studies, the more intensive responses of the higher paid workers are attributed to the progressivity of the tax structure.

Sweden with its extensive tax reforms provides excellent empirical evidence. In this regard, Blomquist and Hansson-Brusewitz (1990) show that replacing progressive taxes by proportional taxes generating similar revenue will raise labor supply of men by 6.6% and women by 10.2%. Blomquist *et al.* (2001) show that the Swedish tax reforms of 1980–1991 led to the AMTR being reduced by around 27%, leading to an average increase in hours worked of 3.9%, with a bigger response by higher paid workers. Finally, Altig and Carlstrom (1999) using U.S. data find that the 25% decrease in the AMTR from 0.32 to 0.24 between 1984 and 1989 increased average weekly hours by around 3.1%.

Although the labor supply responses generated by this model generally exceed those observed empirically, they are generally of similar orders of magnitude and share the qualitative attributes. They certainly underscore the empirical significance of endogenizing the labor supply.

## 7.3. Effects of Tax Progressivity on Income Inequality

Table 3 implies that with elastic labor supply a 5 percentage point increase in the AMTR reduces short-run before-tax income inequality by 4.8% and after-tax income inequality by 5.4%. This compares rather closely with empirical evidence obtained for the United States by Altig and Carlstrom (1999) who find that increasing the AMTR by 8 percentage points from 0.24 to 0.32 reduces the before-tax Gini coefficient for income by 7.8%. It is also clear from Table 2 that this close matching of the data that our model achieves is entirely due to the endogeneity of labor supply. For the Swedish tax reform, Blomquist *et al.* (2001) find that reducing the AMTR increases the after-tax income inequality by

15.4%, closer to our long-run response. This is reasonable, given that the Swedish reform took time.

## 8. Conclusions

The progressivity of the tax structure is an important determinant of income and wealth inequality. In this paper, we have developed an endogenous growth model with a progressive income tax rate in which agents supply labor elastically. We have analyzed the dynamic adjustment to an increase in progressivity and shown that in general the economy will converge to an equilibrium growth path along which the distributions of both income and wealth are nondegenerate, with the individual's share of income and wealth being inversely related to his rate of time preference. While the endogeneity of labor supply has relatively little impact on steady-state income inequality, it plays a more significant role in determining the long-run consequences of the progressive tax structure on wealth inequality and on the long-run growth rate. With inelastic labor supply, an increase in progressivity leads to a gradual decrease in income inequality. When labor is supplied elastically, however, much of the long-run decline in growth and inequality occurs immediately through its impact on factor returns. Indeed, the endogeneity of the labor supply underlying these changes is a crucial element in explaining the short-run responses. It is also the key reason why this parsimonious model is able to approximate the empirical evidence as well as it does.

Thus, whether labor is fixed or endogenous, an increase in progressivity involves a long-run tradeoff between growth and inequality. If the policymaker increases the progressivity in order to reduce inequality, it comes at the price of also reducing the growth rate. However, if he accompanies the increase in $\phi$ from 0.75 to 1.125 that we have considered, with a reduction in the base tax rate $\zeta$ from 11.5% to around 9.1%, he can easily maintain the initial growth rate, but with a somewhat smaller, but nevertheless significant, reduction in long-run inequality.

One final insight offered by our numerical analysis is the observation that while the economy as measured by its average may appear to adjust quickly to a structural change, this may be misleading and conceal the reality that different agents are actually responding slowly and in offsetting ways. This is indeed the case for aggregate employment and growth as they respond to changes in progressivity, obscuring the fact that the Rich and Poor are actually adjusting slowly to the contrasting ways they are being impacted.

## Appendix

### Dynamic Structure of the Equilibrium with Endogenous Labor

We begin by recalling (13) and (14) of the text. Using (16) and (16a), we can eliminate $\dot{k}_i$, $k_i$ to obtain the following differential equations in $y_i$, $l_i$, $l$:

$$\phi \frac{\tau_i^m}{1 - \tau_i^m} \frac{\dot{y}_i}{y_i} - \left( \frac{1 - \gamma(1 + \eta)}{1 - \gamma} \right) \frac{\dot{l}_i}{l_i} - \frac{w_l \dot{l}}{w}$$

$$= (1 - \bar{\tau})[r + w(1 - l)] - (1 - \bar{\tau}^m) l \frac{w}{\eta} - \frac{(1 - \tau_i^m) r - \beta_i}{1 - \gamma} \qquad \text{(A1)}$$

$$\dot{y}_i + \frac{\alpha}{(1-l)}\dot{l}_i - \alpha\frac{(1-l_i)}{(1-l)^2}\dot{l} = (\bar{\tau} - \tau_i)r\left(y_i - \alpha\frac{(1-l_i)}{(1-l)}\right)$$

$$+ (1-\alpha)w\left[(1-\tau_i)(1-l_i) - (1-\tau_i^m)\frac{l_i}{\eta}\right]$$

$$- w\left[(1-\bar{\tau})(1-l) - (1-\bar{\tau}^m)\frac{l}{\eta}\right]\left(y_i - \alpha\frac{(1-l_i)}{(1-l)}\right). \tag{A2}$$

For reasons discussed in the text, (A1) and (A2) comprise $2N-1$ independent equations and when combined with the definitions of average taxes, (11a) and (11b), mean relative income, (18c), and average labor supply, (18d), define the complete dynamic system. Since the equations of motion for each individual's income and labor are functions of all other individuals' respective variables, the analysis of the system, in its general form is intractable. Thus, we focus on the two-class case.

Setting $N = 2$, the dynamic system can be summarized by the following three equations:

$$B\dot{X} = E \tag{A3}$$

where

$$B = (b_{ij})$$

$$\equiv \begin{pmatrix} \phi\left[\dfrac{1}{1-\zeta(1+\phi)y_1^\phi} - 1\right]\dfrac{1}{y_1} & -\left[\dfrac{1-\gamma(1+\eta)}{1-\gamma}\right]\dfrac{1}{l_1} & \dfrac{\alpha-1}{1-l} \\[4mm] -\phi\left[\dfrac{1}{1-\zeta(1+\phi)(2-y_1)^\phi} - 1\right]\dfrac{1}{(2-y_1)} & \left[\dfrac{1-\gamma(1+\eta)}{1-\gamma}\right]\dfrac{1}{2l-l_1} & -\left[\dfrac{1-\gamma(1+\eta)}{1-\gamma}\right]\dfrac{2}{2l-l_1} + \dfrac{\alpha-1}{1-l} \\[4mm] 1 & \alpha(1-l)^{-1} & -\alpha(1-l)^{-2}(1-l_1) \end{pmatrix},$$

$$X \equiv \begin{pmatrix} y_1 \\ l_1 \\ l \end{pmatrix}, \quad E \equiv \begin{pmatrix} E_1(y_1, l_1, l) \\ E_2(y_1, l_1, l) \\ E_3(y_1, l_1, l) \end{pmatrix}$$

$$E_1(y_1, l_1, l) \equiv \left(1 - \zeta\frac{y_1^{\phi+1} + (2-y_1)^{\phi+1}}{2}\right)A(1-l)^\alpha$$

$$- \left(l - \zeta(1+\phi)\frac{y_1^\phi l_1 + (2-y_1)^\phi(2l-l_1)}{2}\right)\frac{\alpha}{\eta}A(1-l)^{\alpha-1}$$

$$- \frac{[1 - \zeta(1+\phi)y_1^\phi](1-\alpha)A(1-l)^\alpha - \beta_1}{1-\gamma}$$

$$E_2(y_1, l_1, l) \equiv \left(1 - \zeta\frac{y_1^{\phi+1} + (2-y_1)^{\phi+1}}{2}\right)A(1-l)^\alpha$$

$$- \left(1 - \zeta(1+\phi)\frac{y_1^\phi l_1 + (2-y_1)^\phi(2l-l_1)}{2l}\right)\frac{\alpha}{\eta}A(1-l)^{\alpha-1}l$$

$$- \frac{[1 - \zeta(1+\phi)(2-y_1)^\phi](1-\alpha)A(1-l)^\alpha - \beta_2}{1-\gamma}$$

$$E_3(y_1, l_1, l) \equiv \zeta \left( \frac{y_1^{\phi+1} + (2-y_1)^{\phi+1}}{2} - y_1^\phi \right)(1-\alpha)A(1-l)^\alpha \left( y_1 - \alpha\frac{1-l_1}{1-l} \right)$$

$$+\alpha A(1-l)^{\alpha-1} \left\{ \begin{array}{l} (1-\alpha)\left[ \left(1-\zeta y_1^\phi\right)(1-l_1) - \left[1 - \zeta(1+\phi)y_1^\phi\right]\frac{l_1}{\eta} \right] \\[2ex] -\left(y_1 - \alpha\frac{1-l_1}{1-l}\right)\left[\left(1 - \zeta\frac{y_1^{\phi+1} + (2-y_1)^{\phi+1}}{2}\right)(1-l)\right] \\[2ex] -\left(1 - \zeta(1+\phi)\frac{y_1^\phi l_1 + (2-y_1)^\phi(2l-l_1)}{2l}\right)\frac{l}{\eta} \end{array} \right\}.$$

Linearizing the dynamic system (A3) around the steady state $(\tilde{y}_1, \tilde{l}_1, \tilde{l})$ yields

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \dot{y}_1 \\ \dot{l}_1 \\ \dot{l} \end{pmatrix} = \begin{pmatrix} \dfrac{\partial E_1}{\partial y_1} & \dfrac{\partial E_1}{\partial l_1} & \dfrac{\partial E_1}{\partial l} \\[2ex] \dfrac{\partial E_2}{\partial y_1} & \dfrac{\partial E_2}{\partial l_1} & \dfrac{\partial E_2}{\partial l} \\[2ex] \dfrac{\partial E_3}{\partial y_1} & \dfrac{\partial E_3}{\partial l_1} & \dfrac{\partial E_3}{\partial l} \end{pmatrix} \begin{pmatrix} y_1 - \tilde{y}_1 \\ l_1 - \tilde{l}_1 \\ l - \tilde{l} \end{pmatrix}$$

where all elements are evaluated at the steady state $(\tilde{y}_1, \tilde{l}_1, \tilde{l})$, determined by $E_j(\tilde{y}_1, \tilde{l}_1, \tilde{l}) = 0$, $j = 1, 2, 3$. The basic dynamic system is thus approximated by

$$\begin{pmatrix} \dot{y}_1 \\ \dot{l}_1 \\ \dot{l} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} y_1 - \tilde{y}_1 \\ l_1 - \tilde{l}_1 \\ l - \tilde{l} \end{pmatrix} \tag{A4}$$

where

$$A = (a_{ij}) \equiv \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \equiv \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{\partial E_1}{\partial y_1} & \dfrac{\partial E_1}{\partial l_1} & \dfrac{\partial E_1}{\partial l} \\[2ex] \dfrac{\partial E_2}{\partial y_1} & \dfrac{\partial E_2}{\partial l_1} & \dfrac{\partial E_2}{\partial l} \\[2ex] \dfrac{\partial E_3}{\partial y_1} & \dfrac{\partial E_3}{\partial l_1} & \dfrac{\partial E_3}{\partial l} \end{pmatrix}.$$

Although the initial values of $y_1, l_1, l$ are in principle "jump" variables, collectively they are constrained by the relationship (16) and the fact that $k_i$ is sluggish, with the given initial value $k_{i,0}$. Setting $t = 0$ in this equation yields the constraint

$$y_i(0) = (1-\alpha)k_{i,0} + \alpha\frac{(1-l_i(0))}{(1-l(0))}, \tag{A5}$$

the effect of which is to constrain the stable adjustment path to a one-dimensional manifold expressed by (16). In order for (A4) to yield a unique stable adjustment path, the matrix $A$ must have two unstable and one stable eigenvalue, the latter being denoted by $\mu < 0$. Imposing the initial constraint (A5), the linear approximation to the stable adjustment path is expressed by

$$y_1(t) = \tilde{y}_1 + (y_1(0) - \tilde{y}_1)e^{\mu t} \tag{A6a}$$

$$l_1(t) - \tilde{l}_1 = v_1(y_1(t) - \tilde{y}_1) \quad t \geq 0 \tag{A6b}$$

$$l(t) - \tilde{l} = v_2(y_1(t) - \tilde{y}_1) \quad t \geq 0 \tag{A6c}$$

where $(1, v_1, v_2)$ is the normalized eigenvector associated with the stable root $\mu < 0$, namely

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 \\ v_1 \\ v_2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ v_1 \\ v_2 \end{pmatrix}, \tag{A6d}$$

and $y_1(0), l_1(0), l(0)$ are constrained by (A5). The dynamic system (A6) determines the evolution of $y_1(t), l_1(t), l(t)$, which in effect are the core dynamics of the system.

Using the fact that for $N = 2$, $y_2(t) = 2 - y_1(t)$, $l_2(t) = 2l(t) - l_1(t)$, these relationships immediately imply the transitional paths for $y_2(t)$, $l_2(t)$. Furthermore, letting $t \to \infty$ in (16) yields the steady-state value $\tilde{k}_1$, the transitional dynamics being then described by $\tilde{k}_1(t) = \tilde{k}_1 + (k_{1,0} - \tilde{k}_1)e^{\mu t}$, with $k_2(t) = 2 - k_1(t)$. The dynamics of all other variables can then be derived. In particular, the growth rate of the aggregate capital stock is obtained by substituting the solutions for the core variables into (12).

## Proof of Proposition 1

We begin with left-hand pair of equations in (18a), which we write as

$$\frac{[1 - \tau_j^m(\tilde{y}_j)]r(\tilde{l}) - \beta_j}{1 - \gamma} = \frac{[1 - \tau_i^m(\tilde{y}_i)]r(\tilde{l}) - \beta_i}{1 - \gamma} \tag{A7}$$

which simplifies to

$$\tau_i^m(\tilde{y}_i) - \tau_j^m(\tilde{y}_j) = \frac{\beta_j - \beta_i}{r(\tilde{l})}. \tag{A8}$$

Next, write (19) as

$$F(\tilde{y}_i, \tilde{l}_i, \tilde{l}, \bar{\tau}, \bar{\tau}^m) \equiv \frac{\tilde{y}_i}{1 - \alpha} - \frac{\alpha(1 - \tilde{l}_i)}{(1 - \alpha)(1 - \tilde{l})}$$

$$- \frac{[1 - \tau_i(\tilde{y}_i)]\tilde{y}_i(1 - \tilde{l}) - [1 - \tau_i^m(\tilde{y}_i)]\alpha\tilde{l}_i/\eta}{[1 - \bar{\tau}](1 - \tilde{l}) - [1 - \bar{\tau}^m]\alpha\tilde{l}/\eta} = 0. \tag{A9}$$

Using the intermediate value theorem, we have

$$F(\tilde{y}_i, \tilde{l}_i, \tilde{l}, \bar{\tau}, \bar{\tau}^m) = 0 = F(\tilde{y}_j, \tilde{l}_j, \tilde{l}, \bar{\tau}, \bar{\tau}^m) = \left.\frac{\partial F}{\partial \tilde{y}_i}\right|_{\tilde{y}_k, \tilde{l}_k} (\tilde{y}_i - \tilde{y}_j) + \left.\frac{\partial F}{\partial \tilde{l}_i}\right|_{\tilde{y}_k, \tilde{l}_k} (\tilde{l}_i - \tilde{l}_j)$$

where $\tilde{y}_k \in (\tilde{y}_i, \tilde{y}_j)$, $\tilde{l}_k \in (\tilde{l}_i, \tilde{l}_j)$ so that

$$(\tilde{y}_i - \tilde{y}_j) = -\frac{\partial F/\partial \tilde{l}_i|_{\tilde{y}_k, \tilde{l}_k}}{\partial F/\partial \tilde{y}_i|_{\tilde{y}_k, \tilde{l}_k}} (\tilde{l}_i - \tilde{l}_j). \tag{A10}$$

Differentiating (A9) yields

$$\frac{\partial F}{\partial \tilde{l}_i} \equiv \frac{\alpha}{(1 - \alpha)} + \frac{[1 - \tau_i^m(\tilde{y}_i)]\alpha/\eta}{D} > 0 \tag{A11a}$$

$$\frac{\partial F}{\partial \tilde{y}_i} \equiv \frac{1}{(1-\alpha)} - \frac{1}{D}\left[(1-\tilde{l})\left[1-\tau_i^m(\tilde{y}_i)\right] + \alpha\phi\frac{\tilde{l}_i}{\eta}\frac{\tau_i^m}{\tilde{y}_i}\right] \tag{A11b}$$

where $D \equiv [1-\bar{\tau}](1-\tilde{l}) - [1-\bar{\tau}^m]\alpha\tilde{l}/\eta$. Now multiplying by $A(1-\tilde{l})^{\alpha-1}$, we obtain, recalling (17a),

$$DA(1-\tilde{l})^{\alpha-1} = A(1-\tilde{l})^{\alpha-1}[(1-\bar{\tau})(1-\tilde{l}) - (1-\bar{\tau}^m)\alpha\tilde{l}/\eta] = \tilde{\psi}$$

and hence, assuming that the equilibrium growth rate is positive, yields

$$D = \frac{\tilde{\psi}}{A(1-\tilde{l})^{\alpha-1}} > 0.$$

Since each agent must satisfy the transversality condition, $\tilde{\psi} < r(\tilde{l})[1-\tau_i^m(\tilde{y}_i)]$

$$\frac{\partial F}{\partial \tilde{y}_i} \equiv \frac{1}{(1-\alpha)\tilde{\psi}}\left\{\tilde{\psi} - r(\tilde{l})\left[1-\tau_i^m(\tilde{y}_i)\right] - (1-\alpha)w(\tilde{l})\phi\frac{\tilde{l}_i}{\eta}\frac{\tau_i^m}{\tilde{y}_i}\right\} < 0. \tag{A11c}$$

Now from (18c) we have

$$\tilde{k}_i - \tilde{k}_j = \frac{\tilde{y}_i - \tilde{y}_j}{1-\alpha} + \frac{\alpha(\tilde{l}_i - \tilde{l}_j)}{(1-\alpha)(1-\tilde{l})}. \tag{A12}$$

The two parts of the proposition now follow directly from (A8), (A10), and (A12). If $\beta_i = \beta_j = \beta$, then these three equations immediately imply $\tilde{y}_i = \tilde{y}$, $\tilde{l}_i = \tilde{l}$, $\tilde{k}_i = \tilde{k}$ for all $i$, establishing part (i). If $\beta_j > \beta_i$, (A8) implies $\tau_i^m(\tilde{y}_i) > \tau_y^m(\tilde{y}_j)$ which under a progressive tax structure implies $\tilde{y}_i > \tilde{y}_j$. Moreover, as long as the marginal tax rate is less than unity, the after-tax income of agent $i$, $y_i(1-\tau_i(y_i))$, exceeds that of agent $j$. Noting (A11a) and (A11c), from (A10) we infer

$$\tilde{y}_i > \tilde{y}_j \Rightarrow \tilde{l}_i > \tilde{l}_j$$

and combining this relationship with (A12) we obtain

$$\beta_j > \beta_i \text{ implies } \tilde{y}_i > \tilde{y}_j, \tilde{l}_i > \tilde{l}_j, \tilde{k}_i > \tilde{k}_j \tag{A13}$$

thus establishing part (ii).                                                          ∎

## Proof of Proposition 2

Suppose agent $i$ is more patient than average, in which case $\tilde{y}_i, \tilde{l}_i, \tilde{k}_i$ for that individual are all above average. If this agent becomes even more patient, then $\tilde{y}_i, \tilde{l}_i, \tilde{k}_i$ for that individual all increase, relative to the average, thereby increasing the degree of inequality.                                                                                          ∎

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