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Social security, growth, and welfare in overlapping generations economies with or without annuities ☆

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A R T I C L E   I N F O

A B S T R A C T

We examine the impact of a stylized pay-as-you-go (PAYGO) Social Security program in an economy of overlapping generations with equilibrium growth. We adopt realistic mortality and other demographic assumptions and allow for the presence or absence of life annuities. In all cases steady-state economies with PAYGO Social Security programs grow more slowly than those without. Also, we find that lifetime expected utilities are lower for existing and future households in steady-state economies with Social Security. We also report the effect of Social Security on the age profile of consumption and explore the effects of longer life expectancy, compensating Social Security program changes, and capital subsidies.

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1. Introduction

The impact of pay-as-you-go (PAYGO) Social Security programs on growth and welfare, as well as on the program structure necessary to maintain their solvency, is of perennial policy interest. Because such programs substitute intergenerational tax-transfers for saving and investment in productive capital, they are presumed to be detrimental to long-term productivity growth, barring other impacts they may have, such as on fertility and retirement decisions. At the same time, by providing a life annuity of fixed purchasing power, Social Security programs provide consumption insurance to elderly households, and should therefore be expected to enhance lifetime welfare in economies with annuities market failure.

By their nature, the terms of PAYGO Social Insurance programs depend on demographic aspects, such as life expectancy and fertility, as well as labor market characteristics, such as retirement plans and Social Security claiming ages. Changes in these structural features have important implications for the impact of PAYGO Social Security programs and their sustainability. In this paper we examine the impacts of a stylized PAYGO Social Security program in a model of equilibrium growth with overlapping generations and realistic household mortality. We derive quantitative estimates of the effects of such programs on long-term growth and welfare, and numerically simulate the effects of different demographic and labor supply conditions on the terms required for the long-run solvency of PAYGO Social Security.

The use of overlapping generations (OLG) models to analyze the effects of PAYGO Social Security has a long history dating back to the pioneering work of Samuelson (1958) and Diamond (1965). The discrete time, two-period canonical models they employed have been extended to include many periods, and other generalizations. In addition, there is a literature analyzing PAYGO Social Security using the continuous time models of Blanchard (1985) and Weil (1989). Assuming a constant mortality hazard rate (an exponential survival function), these models have proven to be highly tractable and informative. In an important paper in this literature, Saint-Paul (1992) derives analytical results describing the impact of Social Security and other government policies for the Blanchard–Weil model. But the Blanchard–Weil model suffers from a serious shortcoming, in that it has an unrealistic demographic structure, implying the existence of an excessively long tail of very old households. In order to provide a realistic assessment of the impact of Social Security on the performance of an economy, it is critical to embed within the OLG model a realistic demographic structure. To do so is our main objective in this paper.

During the last several years, substantial progress has been made in incorporating more realistic demographic structures in OLG macroeconomic models, and in particular in generalizing the Blanchard–Weil framework. For example, Bommier and Lee (2003), d'Albis (2007), Lau (2009), Gan and Lau (2010) employ very general mortality structures to study the existence and uniqueness of the steady-state equilibrium of a macro-dynamic model with finite-lived agents. Boucekkine et al. (2002), Faruqee (2003), Heijdra and Romp (2008), Heijdra and Mierau (2012), Mierau and Turnovsky (2012), and Bruce and Turnovsky (in press) specify and calibrate empirically plausible mortality functions in their macro-dynamic models.[1]

Regardless of the models used, the consensus is that the direct effect of a PAYGO Social Security system is to reduce the economic growth rate; see e.g. Gertler (1999), Ehrlich and Kim (2005), and Wigger (1999), a result that can be dated back to the early two-period overlapping generations model by Samuelson (1975).[2] However, some authors, such as Zhang (1995), have shown that once one introduces the effects of Social Security on fertility and family size, it is possible for an unfunded social security system to increase the growth rate through indirect effects.

In this paper we introduce a PAYGO Social Security system into the computational demographic endogenous growth model developed by Bruce and Turnovsky (in press). We utilize the survival function introduced by Boucekkine et al. (2002), henceforth denoted as the BCL survival function.[3] Because closed-form analytical results are intractable, we conduct our analysis numerically, with preference and production parameters assigned values consistent with those characterizing the US and other Western economies. The Blanchard–Weil exponential survival function employed by Saint-Paul (1992) is a special case of the BCL function, so we also compute values for this specification for comparison purposes.

In order to focus attention on the demographic aspects, we assume output is produced using a Romer (1986) technology augmented with government infrastructure spending as in Barro (1990). The production technology we adopt ensures that the economy is always on its balanced growth path. We do not address transitional dynamics, such as would arise when a Social Security program is introduced or modified in an economy. Such events have differential effects on different cohorts, depending upon their ages, and generate a transitory period of dynamic adjustment. For this reason, the comparisons we undertake with regard to the effects of Social Security should be viewed as pertaining to two steady-state economies after any transitional effects have been fully completed.

A key element of the mortality structure introduced in the Blanchard model is the existence of actually-fair life annuities, which provide a mechanism whereby the financial wealth of decedents is recycled to the survivors in the economy. To preserve comparability with the existing literature, we begin by maintaining this assumption. However, it is useful, particularly in assessing the welfare impact of Social Security, to consider a more realistic situation of annuities market failure.[4] We consider an economy that lacks annuities markets entirely, so households fully invest their financial wealth in capital, and when they die they leave accumulated financial capital as unintended bequests that are recycled as lump-sum transfers across the surviving population.

Using this framework we obtain the following results. We emphasize that while they are obtained for a plausible calibration, we have conducted extensive sensitivity analysis and find them to be robust across substantial variations in key parameters. We begin by comparing a benchmark steady-state economy having full actuarially-fair life annuities and a Social Security program comparable to that in the United States, to a steady-state economy with no Social Security. We find that labor productivity in the latter economy grows about .72 percentage points (pps) faster than in the economy with Social Security, at 1.91% rather than 1.19%. Although this agrees qualitatively with other studies that adopt less comprehensive demographic structures, and is not surprising because the PAYGO system reduces saving in productive capital, the magnitude of the effect is much greater – about 2.5 times greater – than we find in the Blanchard–Weil model, underscoring the need to analyze Social Security within a more plausible demographic framework. We also find that the lifetime expected utilities of existing and future households are higher in steady-state economies without Social Security, with the welfare of a newborn household (that is, a household entering the economy) at least 11.2% higher without Social Security.

In economies lacking annuities markets, the equilibrium depends upon how unintended bequests are recycled across the surviving population. We consider a natural scheme in which survivors receive, at each age, lump-sum transfers equal to the mortality interest payment they would have received if life annuities existed. In this case, the growth rate is significantly lower than in an economy with annuities, but Social Security reduces the growth rate by about the same amount.[5] We also find that welfare is reduced by the presence of Social Security despite the absence of life annuities.[6] While this welfare result could seemingly be attributed to the capital production externality incorporated in the Romer technology, we find the result continues to apply, even when "corrective" capital subsidies are applied.

The absence of annuities does, however, have important consequences for the lifetime consumption profile of households. In contrast to the annuities economy, which implies that consumption grows indefinitely with age, an economy without annuities exhibits a more empirically plausible hump-shaped longitudinal consumption-age profile. Agents increase consumption during the early phase of their life cycle and reduce it later, consistent with the previous findings of Bütler (2001), Hansen and İmrohoroğlu (2008) and Heijdra and

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[1] Boucekkine et al. (2002) adopt a generalization of the Blanchard mortality function, thereby embedding the latter as a special case. This formulation is also adopted by Heijdra and Mierau (2012) and Mierau and Turnovsky (2012). Heijdra and Romp (2008) use the Gompertz (1825) exponential mortality hazard function in a small open-economy overlapping generations model. Faruqee (2003) approximates the Gompertz function with an estimated hyperbolic function, which he introduces into the Blanchard (1985) model. Finally, Bruce and Turnovsky (in press) compute growth rates using a survival function based on de Moivre's Law, which has the advantage of including the Samuelson-Diamond and the Blanchard models as polar cases. However, this survival function does not track the data as well as does the Boucekkine et al function.

[2] In an important contribution, Gertler (1999) modifies the Blanchard-Weil approach by introducing two stages of life, work and retirement, with constant probabilities of transitioning from work to retirement and from retirement to death. To render this analytically tractable he employs a class of non-expected utility preferences. In a similar model, Bruce and Turnovsky (2007) introduce a decomposition of lifetime into work and retirement and preserve tractability by adopting a CARA utility function. However, this specification does not sustain an equilibrium of constant exponential growth.

[3] The BCL function is not only tractable but also matches well the empirical data on survival across age. The reason for this, as we show later, is that the BCL function is a first-order approximation to the Gompertz (1825) survival function, which is known to fit the human mortality data very well.

[4] Several authors have addressed the role of annuities markets in addressing issues pertaining to life-cycle consumption and growth, though focusing on different aspects. For example, Bütler (2001) develops a stylized partial-equilibrium model of life cycle consumption and shows how the absence of annuities generates hump-saved consumption behavior. Hansen and İmrohoroğlu (2008) analyze the consequences of the absence of an annuities market for life-cycle consumption behavior in a general equilibrium framework. While they incorporate a simple unfunded Social Security system, their equilibrium is one of exogenous growth. Heijdra and Mierau (2012) introduce annuity market imperfections in an endogenous growth framework, but do not incorporate Social Security. The latter two papers also demonstrate how the lack of annuities markets can generate hump-shaped consumption behavior over the life-cycle.

[5] In an earlier version of the paper we also considered an alternative allocation scheme, where the wealth of decedents is distributed to newborn households as initial financial wealth, as a proxy for unintended bequests, with generally similar results being obtained.

[6] The result that the growth rate is reduced in the absence of annuities is consistent with that of Heijdra and Mierau (2012), who reach the same conclusion with their specification of annuity market imperfections.

Mierau (2012). The introduction of Social Security is shown to reduce consumption at every age in the longitudinal profile, implying an unambiguous reduction in utility. However, at any given point in time, Social Security has a significant impact on the cross section age profile of consumption. Specifically, Social Security is found to lower the consumption of younger households in the cross section, while increasing the consumption of older households.

In recent decades, life expectancy at age 65 has been increasing at about one year per decade. This represents a substantial increase, and it is therefore important to understand its consequences for economic growth, as well as for the solvency of PAYGO Social Security. We find that an increase in longevity at age 65 by one year raises the growth rate modestly, with or without Social Security, and that the tax rate needed to fund PAYGO Social Security benefits increases by .33 pps. Also, with enhanced longevity, the presence of Social Security has a marginally greater negative impact on growth. The increase in longevity also raises the questions of reducing the program benefit rate or increasing the retirement and claiming age. We show that a decrease in the benefit rate of .93 pps or, alternatively, an increase in the average claiming age of 0.43 years is required to compensate for a one year increase in life expectancy, with tax rates remaining unchanged.

The remainder of the paper is structured as follows. Section 2 sets out the demographic structure and its impact on the behavior of the individual household in both the presence and absence of life annuities. Section 3 describes the aggregate economy while Section 4 closes the system and discusses the parameterizations and calibrations. Section 5 computes the effects of Social Security on the growth rate, welfare, and the age-consumption profiles under alternative specifications, while Section 6 considers the impact of ageing on PAYGO solvency and the effects of the alternative compensatory policy changes available. Our main results are summarized and assessed in Section 7.

2. The utility-maximizing household

In this section, we develop the life-cycle consumption-saving behavior of a mortal household. We shall assume that the survival function is independent of calendar time and let $S(z)$ denote the probability at birth of the household surviving to age z. Because survival declines with age, $S'(z) < 0$, $0 < z < \omega$, with $S(z) = 0$, for $z \geq \omega$, where ω denotes the maximum attainable age. Using this notation, $S(z)/S(x)$ is the probability of surviving to age z, conditional on surviving to age x, while $- S'(z)/S(z)$ is the mortality hazard rate at age z.

2.1. The first-order conditions

A household of age x, at time t, maximizes its expected utility over the remainder of its life:

$$u_t(x) = \int_{z=x}^{z=\omega} \frac{S(z)}{S(x)} \cdot e^{-\rho \cdot (z-x)} \cdot u\big(c_{t+z-x}(z)\big) \cdot dz \tag{1a}$$

where ρ is the pure time discount rate, $u(\cdot)$ is the instantaneous utility function, and $c_{t+z-x}(z)$ is its planned consumption for age z which occurs at time $t + z - x$.[7] This maximization is subject to the flow budget constraint at age z and time t[8]

$$\frac{df_t(z)}{dz} \equiv f'_t(z) = i_t(z) \cdot f_t(z) + w_t(z) \cdot (1-\tau) \cdot L(z) + b_t(z) + y_t(z) - c_t(z) \tag{1b}$$

where, for a household of age z at time t, $w_t(z)$ is the market wage facing the household, $f_t(z)$ is the household's financial wealth, $i_t(z)$ is the interest rate on financial wealth, τ is the total tax rate on labor income used to fund Social Security and other government spending, which will be shown to be time-independent, $b_t(z)$ is the household's age and (possibly) time-dependent Social Security benefit, $y_t(z)$ denotes any other age- and time-dependent transfers, and $L(z) \leq 1$ is the fraction of the household's unit time endowment supplied as labor. Typically, households reduce the fraction of time spent working as they age, thus $L'(z) \leq 0$.[9]

Performing the maximization with respect to $c_t(z)$ and $f_t(z)$ yields the optimality conditions

$$u'(c_t(z)) = \phi_t(z) \tag{2a}$$

$$\rho - \frac{\phi'_t(z)}{\phi_t(z)} - \frac{S'(z)}{S(z)} = i(z). \tag{2b}$$

Eq. (2a) equates the marginal utility of consumption to the household's shadow value of financial wealth, $\phi_t(z)$, while Eq. (2b) equates the rate of return on consumption, modified by the mortality hazard rate, to the rate of return on financial assets. Following much of contemporary growth literature, we assume an iso-elastic instantaneous utility function of the form $u(c_t(z)) = c_t(z)^\varepsilon/\varepsilon$ ($\varepsilon \leq 1$), where $1/(1 - \varepsilon)$ is the inter-temporal elasticity of substitution. This enables us to rewrite Eq. (2b), describing how consumption changes with age, as

$$\frac{c'_t(z)}{c_t(z)} = \frac{1}{(1-\varepsilon)} \cdot \left(i(z) - \rho + \frac{S'(z)}{S(z)} \right). \tag{3}$$

In addition to Eq. (3), the optimality conditions of the household include a transversality condition specified in Eq. (5) below.

Initially, we follow Blanchard (1985) and Yaari (1965) by assuming that households invest wholly in actuarially-fair life annuities. We then consider the case in which there is no annuities market. Under the production conditions introduced in Section 3.3, the risk-free rate of return on physical capital is fixed at r. In the case of life annuities, the interest rate on the financial asset depends on the household's age, but not on time, so $i_t(z) = i(z)$. With uncertain mortality, households may die holding financial wealth. For actuarially fair annuities, $i(z) = r - S'(z)/S(z)$, where r is the risk-free rate of return on capital and $- S'(z)/S(z)$ is the annuity mortality hazard premium for a household at age z that ensures that the wealth of the dying is fully recycled to the surviving. Given the above, we now write

$$- \frac{\partial R(z,x)/\partial z}{R(z,x)} = i(z) = r - \frac{S'(z)}{S(z)} \tag{4}$$

which we may solve to obtain

$$R(z,x) \equiv e^{-\int_{y=x}^{y=z} i(y) \cdot dy} = e^{-r(z-x)} \cdot \left(\frac{S(z)}{S(x)} \right) \tag{4'}$$

implying that $R(z,x)$ is the discount factor for a flow at age z to a household at age x.

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[7] In introducing a demographic structure, it is important to distinguish calendar time, which is denoted by the time subscript, from age, which is indexed in parentheses.

[8] Two points regarding the budget constraint (1b) merit comment. First, as will be made clear in introducing the government budget constraint in Section 3.4 below, we will decompose the income tax rate τ into components allocated to financing different components of government expenditure. Second, the transfer term $y_t(z)$ is introduced in order to present the two cases of annuities and no annuities in a unified way.

[9] We assume that $L(z)$ is specified exogenously, as do Blanchard and Fischer (1989), who assume that $L(z)$ declines at an exponential rate. While the function, $L(z)$, can be quite general, allowing for either abrupt or gradual "retirement", we assume that it does not vary with (calendar) time. By treating labor supply as exogenous we are unable to address the impact of social security on retirement, an important issue addressed in early papers by Feldstein (1974), Sheshinski (1978), and Crawford and Lilien (1981), and more recently by Cremer et al. (2004, 2008).

Combining Eqs. (3) and (4) yields

$$\frac{c'_t(z)}{c_t(z)} = \frac{1}{1-\varepsilon} \cdot (r-\rho) \qquad (3')$$

Eq. (3′) indicates that when a household fully invests in actuarially fair annuities, consumption increases with age at a rate that is constant, and independent of both age and wealth. We can now express the transversality condition for the agent having a maximum lifespan of ω as

$$R(\omega, x) \cdot f_t(\omega) = 0 \qquad (5)$$

2.2. The household's consumption plan with actuarially fair annuities

To derive the household's consumption plan, we begin by integrating (3′) to obtain

$$c_{t+z-x}(z) = c_t(x) \cdot e^{\frac{r-\rho}{1-\varepsilon} \cdot (z-x)}. \qquad (6)$$

We then express the budget constraint (1b) in the equivalent form

$$\frac{d}{dz}[R(z,x) \cdot f_t(z)] \qquad (1b')$$
$$= R(z,x) \cdot \{w_t(z) \cdot (1-\tau) \cdot L(z) + y_t(z) + b_t(z) - c_t(z)\}.$$

Focusing initially on economies with actuarially fair annuities, we set the lump-sum transfer $y_t(z) = 0$ at all points of time. Integrating Eq. (1b′) forward at age z and using the transversality condition, Eq. (5), yields the agent's inter-temporal budget constraint applicable from age x at time t as

$$\int_{z=x}^{z=\omega} e^{-r(z-x)} \left(\frac{S(z)}{S(x)}\right) \cdot c_{t+z-x}(z) dz = f_t(x) + h_t(x) \qquad (7)$$

where $f_t(x)$ is the household's financial wealth and $h_t(x)$ is the household's non-financial wealth, including both its human and Social Security wealth, more specifically,

$$h_t(x) = \int_{z=x}^{z=\omega} e^{-r(z-x)} \cdot \frac{S(z)}{S(x)} \cdot [(1-\tau)w_{t+z-x}(z) \cdot L(z) + b_{t+z-x}(z)] \cdot dz. \qquad (8)$$

We will call the sum of the household's financial and non-financial wealth its "all-inclusive" wealth. Substituting Eq. (6) for $c_{t+z-x}(z)$ in Eq. (7), the household's consumption at age x can be expressed as

$$c_t(x) = m(x) \cdot [f_t(x) + h_t(x)] \qquad (9a)$$

where $m(x)$ denotes the household's time independent marginal (and average) propensity to consume out of its all-inclusive wealth at age x, and is defined as[10]

$$m(x) = \left[\int_{z=x}^{z=\omega} e^{\frac{\varepsilon \cdot r - \rho}{1-\varepsilon} \cdot (z-x)} \cdot \left(\frac{S(z)}{S(x)}\right) \cdot dz\right]^{-1}. \qquad (9b)$$

That is, at time t a household of age x consumes a fraction $m(x)$ of its all-inclusive wealth.

We posit that the productivity of labor increases over calendar time at a constant rate g (to be determined in equilibrium as the "economic growth rate" or "productivity growth rate"). This market wage is economy-wide and common to all households, regardless of their birth dates. Thus the market wage at time t can be expressed $w_t = w_{t-x} \cdot e^{g \cdot x}$ where w_{t-x} is the wage rate prevailing in the economy at the time a household of age x is born (enters the economy).

To preserve tractability, we adopt a stylized form of Social Security benefit, namely $b_t(z) = \beta \cdot w_t \cdot (1 - L(z))$. That is, a household's Social Security benefit is equal to a time-independent fraction β (the benefit, or earnings replacement rate) of the earnings foregone as a result of the reduced labor supply due to "retirement".[11] The assumption that the program parameters β and τ_s, the payroll tax funding the program, are time-independent for a PAYGO Social Security program is validated later. Substituting for $b_t(z)$ into the expression for $h_t(x)$, defined in Eq. (8), the non-financial wealth of a household of age x at time t can be expressed as $h_t(x) = w_t \cdot h(x)$ where $h(x)$, a time-independent present value factor, is

$$h(x) = \int_{z=x}^{z=\omega} e^{-(r-g) \cdot (z-x)} \cdot \left(\frac{S(z)}{S(x)}\right) \cdot [(1-\tau-\beta)L(z) + \beta] \cdot dz. \qquad (10)$$

This present value factor, specific to agents of age x, reflects the Social Security benefit rate, the tax rate on labor income, the growth rate of wages, the return on capital, the age-dependent labor supply function, and the age dependent survival function.

We assume a household enters the economy with no financial wealth. Using $c_t(x) = c_{t-x}(0) \cdot e^{\left(\frac{r-\rho}{1-\varepsilon}\right) \cdot x}$, $c_{t-x}(0) = m(0) \cdot h_{t-x}(0)$, and $w_{t-x} = w_t \cdot e^{-gx}$ we can write

$$c_t(x) = w_t \cdot e^{\left(\frac{r-\rho}{1-\varepsilon}-g\right) \cdot x} \cdot m(0) \cdot h(0). \qquad (11)$$

Substituting Eq. (6) into Eq. (1a), and using the iso-elastic form of the instantaneous utility function, we can express the expected lifetime utility of a new-born household at time t as a multiple u of the economy-wide wage rate at time t, that is,

$$u_t(0) = u \cdot w_t^\varepsilon \quad \text{where} \quad u = \frac{[m(0) \cdot h(0)]^\varepsilon}{\varepsilon} \cdot \int_{z=0}^{z=\omega} e^{\left(\frac{\varepsilon \cdot r - \rho}{1-\varepsilon}\right) \cdot z} \cdot S(z) \, dz \qquad (12)$$

We use the utility multiplier, u, to analyze the welfare effects of Social Security and capital subsidies.

2.3. The household's consumption plan without annuities

Because the welfare basis of Social Security is often justified as a response to market failure in annuities markets, we also consider economies in which households are unable to purchase annuities and must therefore fully invest their financial wealth in direct claims on physical capital, so that $i(z) = r$ for all z. As a result, consumption evolves over age according to

$$\frac{c'_t(z)}{c_t(z)} = \frac{1}{(1-\varepsilon)} \cdot \left(r - \rho + \frac{S'(z)}{S(z)}\right) \qquad (3'')$$

which can be integrated to obtain

$$c_{t+z-x}(z) = c_t(x) \cdot e^{\frac{r-\rho}{1-\varepsilon} \cdot (z-x)} \cdot \left(\frac{S(z)}{S(x)}\right)^{\frac{1}{1-\varepsilon}}. \qquad (6')$$

Comparing Eqs. (3″) and (6′) with Eqs. (3′) and (6), we see that in the absence of annuities, households choose a different consumption

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[10] Note that for the infinitely-lived household with logarithmic utility ($S(z) \to 1$, $\omega \to \infty$, $\varepsilon \to 0$), the marginal propensity to consume wealth in (9b) reduces to the familiar rate of time preference, ρ.

[11] In fact, a household cannot receive a Social Security benefit until it reaches a certain age of eligibility (in United States, 62 for a reduced benefit). However, households can and do receive Social Security Disability benefits at any age after age 30. The disability benefit is determined according to a modified version of the usual Social Security benefit formula. Also, unlike the stylized Social Security benefit in this paper, actual benefits are not indexed to wage growth after the household claims. However, this complication can be translated into a lower benefit rate for our stylized program.

profile than in the fully annuitized case. In the latter case, household consumption increases exponentially with age provided $r > \rho$ as is conventionally assumed. From Eq. (3″) it is evident that the rate of consumption increase, as the household ages, is reduced by the mortality hazard rate $(-S'(z)/S(z))$. Moreover, since the mortality hazard rate increases with age, in fact and according to the survival function used in this paper, household consumption reaches a maximum, and then declines for households of very old age. We discuss the effect of Social Security on the age-consumption profile further in Section 5.4.[12]

In the absence of annuities, households leave unintended bequests and it is necessary to specify how this decedent wealth is recycled. To simplify, we assume that surviving households of age z receive a *lump-sum* transfer equal to the mortality interest premium that they would have received on life annuities. That is, $y_t(z) = -(S'(x)/S(x)) \cdot f_t(z)$. In Section 3.2 we show that demographic constraints ensure that decedent financial wealth is fully recycled under this scheme.[13]

This scheme has the added advantage of equalizing the present value of resources of a new born household to that of the annuities economy, facilitating comparisons. In particular, setting $y_t(z) = -(S'(z)/S(z)) \cdot f_t(z)$ in the intertemporal budget constraint, Eq. (1b), solving the resulting equation, and substituting Eq. (6′), we again obtain Eq. (9a), but where with no annuities we now have

$$m(x) = \left[\int_{z=x}^{z=\omega} e^{\frac{\varepsilon \cdot r - \rho}{1-\varepsilon} \cdot (z-x)} \cdot \left(\frac{S(z)}{S(x)} \right)^{\frac{2-\varepsilon}{1-\varepsilon}} \cdot dz \right]^{-1}. \tag{9b′}$$

$h_t(x) = w_t \cdot h(x)$, and $h(x)$ remains as defined in Eq. (10). Using $c_t(x) = c_{t-x}(0) \cdot e^{\left(\frac{r-\rho}{1-\varepsilon}\right) \cdot x} \cdot S(x)^{\frac{1}{1-\varepsilon}}$, $c_{t-x}(0) = m(0) \cdot h_{t-x}(0)$, and $w_{t-x} = w_t \cdot e^{-gx}$ we can write

$$c_t(x) = w_t \cdot e^{\left(\frac{r-\rho}{1-\varepsilon} - g\right) \cdot x} \cdot S(x)^{\frac{1}{1-\varepsilon}} \cdot m(0) \cdot h(0). \tag{11′}$$

Substituting Eq. (6′) into Eq. (1a), we can express the expected lifetime utility of a household newly entering an economy without annuities at time t as a multiple u of the economy-wide wage rate at time t. That is,

$$u_t(0) = u \cdot w_t{}^{\varepsilon} \quad \text{where} \quad u = \frac{[m(0) \cdot h(0)]^{\varepsilon}}{\varepsilon} \cdot \int_{z=0}^{z=\omega} e^{\left(\frac{\varepsilon \cdot r - \rho}{1-\varepsilon}\right) \cdot z} \cdot S(z)^{\frac{1}{1-\varepsilon}} \cdot dz \tag{12′}$$

We now aggregate and parameterize the two economies with, and without, annuities.

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[12] Another concern is the age-financial wealth profile of the household. Using the Yaari (1965) model, Leung (1994) proves that it is possible for household financial wealth to become negative in late life, and indeed that this must occur if the interest rate function facing the agent is *bounded* [1994, p.1236], in which case a non-negativity constraint on the agent's financial wealth must be imposed. This implies that a mortality model such as ours need not necessarily have an interior solution lasting until the maximum survival age, as we have assumed, because lenders would not be willing to lend to such households. However, in our annuities model, because the BCL mortality function we adopt implies that the age-dependent interest rate function is *unbounded*, with $i(z) \to \infty$ as $z \to \omega$, Leung's proposition does not apply. On the other hand, under the no annuities assumption, such a problem could arise, since $i(z)$ remains bounded in that case. Consequently we have plotted the age-financial wealth profile both with, and without, annuities. As expected, we find that the problem does not occur in the annuities model. Moreover, it occurs only to an insignificant degree in the model without annuities, in that the small fraction of agents living to the maximum attainable age, ω, will find that their wealth will become slightly negative in the last year of their life. This is barely perceptible, and has a negligible bearing on our results. We assume that the few individuals fortunate enough to live that long can borrow against their social security (as necessary) in the last year of their life.

[13] Although households receive at each age the same transfer that they do in the life annuities case, the transfers are lump-sum and do not depend on the household's individual saving behavior. Under this scheme households perceive and make saving decision on the basis of an interest rate of r rather than $r - S'(x)/S(x)$, as in the annuities economy.

3. The aggregate economy

To derive aggregate variables we sum over the cohorts in the economy. Let B_t denote the size of the population cohort born at time t. Given the survival function, $S(x)$, the size of that cohort at time t (now of age x) is $N_t(x) = B_{t-x} \cdot S(x)$. Assuming that birth cohorts grow at the rate n, $B_t = B_{t-x} \cdot e^{n \cdot x}$ and we may express $N_t(x)$ in terms of the size of the current birth cohort as $N_t(x) = B_t \cdot e^{-n \cdot x} \cdot S(x)$. Aggregating over all cohorts, the total population size at time t is $N_t = \int_{x=0}^{x=\omega} N_t(x) \cdot dx = B_t \cdot \int_{x=0}^{x=\omega} e^{-n \cdot x} \cdot S(x) \cdot dx$. Given the time-invariance of the survival function, the total population also grows at rate n.

3.1. Aggregate household variables

We now use the demographic structure to derive the two key aggregate economic variables: aggregate labor income and aggregate consumption.

Aggregate labor income at time t is equal to the aggregate labor supply L_t times the economy-wide wage rate or:

$$w_t \cdot L_t = w_t \cdot \int_{x=0}^{x=\omega} N_t(x) L(x) dx = w_t \cdot B_t \cdot \Sigma^L \tag{13a}$$

where

$$\Sigma^L \equiv \int_{x=0}^{x=\omega} e^{-n \cdot x} \cdot S(x) \cdot L(x) \cdot dx. \tag{13b}$$

is a time-independent aggregator, taken over all living cohorts.

Aggregate consumption at time t is

$$C_t = \int_{x=0}^{x=\omega} N_t(x) \cdot c_t(x) \cdot dx = w_t \cdot B_t \cdot \Sigma^C \tag{14a}$$

where, using Eqs. (11) and Eq. (11′), we obtain, respectively, the time-independent aggregator in the economy with annuities

$$\Sigma^C = m(0) \cdot h(0) \cdot \int_{x=0}^{x=\omega} e^{\left(\frac{r-\rho}{1-\varepsilon} - g - n\right) \cdot x} S(x) \cdot dx \tag{14b}$$

and in the economy without annuities

$$\Sigma^C = m(0) \cdot h(0) \cdot \int_{x=0}^{x=\omega} e^{\left(\frac{r-\rho}{1-\varepsilon} - g - n\right) \cdot x} S(x)^{\frac{2-\varepsilon}{1-\varepsilon}} \cdot dx. \tag{14b′}$$

The time-independent aggregator Σ^L depends on the population growth rate and demographic functions $S(x)$ and $L(x)$, while Σ^C depends additionally on the taste parameters ε and ρ, the productivity growth rate g, the rate of return on capital r, and (through $m(0)$ and $h(0)$) the Social Security benefit rate β and the labor income tax rate τ. Both aggregate labor income and consumption grow at the rate $g + n$.

3.2. Demographic constraints

The number of deaths at time t of persons of age x is $D_t(x) = -(S'(x)/S(x)) \cdot N_t(x)$ so that the total number of deaths equals $D_t = -\int_{x=0}^{x=\omega} S'(x) \cdot B_{t-x} \cdot dx$. Let $\pi(x)$ denote the fertility or birth rate by individuals of age x, so the number of births at time t to persons of age x is $\pi(x) \cdot N_t(x)$. Then, the total number of births

equals $B_t = \int\limits_{x=0}^{x=\omega} \pi(x) \cdot S(x) \cdot B_{t-x} \cdot dx$. Since $n \cdot N_t = B_t - D_t$ and $B_{t-x} = B_t \cdot e^{-n \cdot x}$, we can substitute, rearrange, and integrate by parts, to obtain the following constraint on the chosen demographic functions:

$$\int\limits_{x=0}^{x=\omega} \pi(x) \cdot S(x) \cdot e^{-n \cdot x} dx = 1. \tag{15}$$

Eq. (15) represents a demographic constraint linking the birth rate, the parameters of the survival function, and the overall population growth rate in such a way that the economy's demographic composition remains stable over time. In assessing the impact of Social Security under different scenarios, we utilize this equation to ensure that the parameters are chosen consistently.

With uncertain mortality, households may die with positive financial wealth. Decedent wealth at time t is equal to $\int_{x=0}^{x=\omega} D_t(x) \cdot f_t(x) \cdot dx = - \int_{x=0}^{x=\omega} N_t(x) \cdot (S'(x)/S(x)) \cdot f_t(x) \cdot dx$. With fair annuities, decedent wealth is 'recycled' in the form of the *mortality interest rate premium* $(i(x) - r) \cdot f(x) = - (S'(x)/S(x)) \cdot f_t(x)$ for a household of age x. In the economy without annuities we assume surviving households of age x receive a *lump-sum* transfer of equal amount. That is, $y_t(x) = - (S'(x)/S(x)) \cdot f_t(x)$. In both economies, aggregate transfers are $\int\limits_{x=0}^{x=\omega} N_t(x) \cdot y_t(x) \cdot dx$, so it is apparent that the financial wealth of decedents is fully recycled to survivors in both cases.

3.3. The aggregate production sector

In deriving the behavior of the household, we assume that the rate of return on capital and the growth rate of labor productivity are constant over time, and together with the prevailing wage rate, are exogenous to the household. To complete the model and determine the equilibrium, the values of w_t, r, and g must be derived. These values depend on the underlying production technology. We adopt a Romer (1986) production function, augmented with government infrastructure spending as in Barro (1990). With the implied constant productivity of capital (AK) technology, r and g will indeed be constant, consistent with our maintained assumptions.[14]

Specifically, we assume that there are L_t identical firms, and each hires one unit of labor and k_t units of capital. The representative firm's output net of capital replacement, q_t, is produced in accordance with the Cobb-Douglas production function

$$q_t = \hat{A} \cdot k_t^{\alpha} \cdot (X_t)^{1-\alpha} - \delta \cdot k_t \tag{16}$$

where $\hat{A}$ is the total factor productivity term, k_t denotes the firm's capital stock, X_t denotes a productivity externality and δ is the capital depreciation rate. We assume that the production externality arises from the interaction between the aggregate capital-labor ratio K_t/L_t and government infrastructure spending G_t, according to the function $X_t = (G_t/L_t)^{\xi}(K_t/L_t)^{1-\xi}$. This specification of the production externality ensures that the equilibrium productivity of capital remains constant, enabling the economy to sustain a constant equilibrium growth rate.[15]

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[14] If the underlying production function is neoclassical, these quantities would be time varying and this would need to be taken into account by the household sector in its decision-making process.

[15] This specification of the externality will be recognized as being a combination of the original Romer and Barro models. The reason for adopting it is that it facilitates the calibration; see Chatterjee and Turnovsky (2012). We should also point out that, as in the original Barro model, we are expressing productive government expenditure as a flow. If, as is frequently argued, government infrastructure expenditure is more appropriately introduced as a stock (government capital), this will generate an equilibrium in which the economy only gradually converges to its balanced growth path, as was originally demonstrated by Futagami et al. (1993).

We assume each firm is small enough to ignore its own impact on the economy-wide values of K_t and L_t, and because firms are identical and employ one unit of labor, in equilibrium $K_t = k_t \cdot L_t$. We can express aggregate net output (equal to income), $Q_t = q_i \cdot L_t$, as

$$Q_t = \hat{A} \cdot K_t^{1-(1-\alpha) \cdot \xi} \cdot G_t^{(1-\alpha) \cdot \xi} - \delta \cdot K_t. \tag{17a}$$

We assume government infrastructure spending is proportional to gross output, or $G_t = \gamma \cdot (Q_t + \delta \cdot K_t)$ and substitute in (17a) to obtain the "AK" form

$$Q_t = (A - \delta) \cdot K_t \tag{17b}$$

where $A = \left(\hat{A} \cdot \gamma^{(1-\alpha) \cdot \xi}\right)^{\frac{1}{1-(1-\alpha) \cdot \xi}}$. Substituting Eq. (17b) into the expression for G_t yields $G_t = \gamma \cdot A \cdot K_t$.

Differentiating Eq. (16) with respect to k_i, and substituting $k_i = K_t/L_t$ together with $G_t = \gamma \cdot A \cdot K_t$, we obtain:

$$r^p \equiv \frac{\partial q_i}{\partial k_i} = \alpha \cdot A - \delta \tag{18a}$$

Similarly, differentiating Eq. (17a) with respect to K_t and substituting for G_t yields

$$r^s \equiv \frac{\partial Q_t}{\partial K_t} = [1 - \xi \cdot (1-\alpha)] \cdot A - \delta \tag{18b}$$

where r^p and r^s denote the private and social returns to capital respectively.

The aggregate capital production externality measured by the difference, $r^s - r^p$, is thus

$$\chi_K \equiv r^s - r^p = (1-\alpha) \cdot (1-\xi) \cdot A. \tag{18c}$$

If $\xi = 0$, the production externality is due solely to aggregate private capital and $\chi_K = (1 - \alpha) \cdot A$. But in general, the aggregate capital production externality depends on the value of $\xi \cdot (1 - \alpha)$, which is the elasticity of gross output with respect to government infrastructure spending.

The equilibrium rate of return to households on private capital is constant over time and given by $r = r^p + \sigma$ where σ is a government subsidy on ownership of private capital. The equilibrium wage at time t is given by $w_t = q_i - (\alpha \cdot A - \delta) \cdot k_i$. Substituting $k_i = K_t/L_t$ and $G_t = \gamma \cdot A \cdot K_t$ into Eq. (16), we can write labor income as

$$w_t \cdot L_t = (1-\alpha) \cdot A \cdot K_t. \tag{19}$$

The production side of the economy is fully described by Eqs. (16)–(19).

3.4. The government budget constraint

Government spending consists of: (i) Social Security benefits $\int_{x=0}^{x=\omega} N_t(x) \cdot b_t(x) \cdot dx$, where as defined earlier, $b_t(x) = \beta \cdot w_t \cdot (1 - L(x))$; (ii) government infrastructure spending G_t; and (iii) the cost of financing the capital subsidy (if any) which is equal to $\sigma \cdot K_t$. We assume the Social Security program is financed on a "Pay-As-You-Go" basis (PAYGO) and funded by a dedicated tax on the earnings of labor, as is the case in most countries. To simplify, we assume other government spending is also funded by labor income taxes, although alternative tax funding could be considered, such as a consumption tax or a tax on capital income. Let τ_s, τ_i and τ_k denote the tax rates required to finance Social Security benefits, government infrastructure spending, and capital subsidies respectively, so that the total tax rate on labor income is $\tau = \tau_s + \tau_i + \tau_k$.

Under a PAYGO program, the total revenue collected by the Social Security payroll tax at time t, $\tau_s \cdot w_t \cdot \int_{x=0}^{x=\omega} N_t(x) \cdot L(x) \cdot dx =$

$\tau_s \cdot w_t \cdot B_t \cdot \Sigma^L$, must equal Social Security benefits paid out contemporaneously. Thus, the tax rate applied to labor income required for Social Security PAYGO solvency is:

$$\tau_s = \beta \left\{ \frac{\int_{x=0}^{x=\omega} e^{-n\cdot x} \cdot S(x) \cdot dx - \Sigma^L}{\Sigma^L} \right\} \tag{20}$$

This expression defines the social security tax rate τ_s as the product of the benefit or earnings replacement rate β, times the "dependency rate", defined by the term in brackets. This constraint implies that the Social Security parameters are time-independent, validating our earlier assumption, and in applying it we substitute the parameterized survival and retirement functions in Eq. (22) below. Aggregate labor income can also be expressed as $(1 - \alpha) \cdot A \cdot K_t$, and the cost of government infrastructure spending is $G_t = \gamma \cdot A \cdot K_t$. Accordingly, the tax rate on labor income needed to fund this component is $\tau_i = \gamma/(1 - \alpha)$. The cost of the capital subsidy is $\sigma \cdot K_t$ so $\tau_k = \sigma/[(1 - \alpha)A]$.

3.5. Social welfare

Households are heterogeneous by virtue of their birth dates. From Eqs. (12) and Eq. (12′), we see that the lifetime expected utility of a household born at time is given by a utility multiplier u times w_t^ε, where w_t is the economy-wide wage rate at the time the household is born. Although u is the same across existing and future households, the wage rate differs across generations, with later generations born with higher initial wage rates. In our specification of preferences, $\varepsilon < 0$ so u is negative and w_t^ε is decreasing in the wage rate. The lifetime utility of a new born household is higher if u is higher (less negative) or the wage rate is increased.

Rather than defining a social welfare function, we evaluate the welfare effects of policy in terms of Pareto improving changes for the expected lifetime utilities of existing and future households. To do this, we compare steady-state economies which differ according to policy, and for which we assume that the wage rate is the same (normalized to unity) at the time the oldest household was born.[16] In this case, the lifetime expected utility of a household currently of age x as of the time of its birth is equal to $U_{t-x}(x) = u \cdot e^{\varepsilon \cdot g \cdot (\omega - x)}$. From this we can conclude that all existing and future households are made better off by a policy difference if u or g are not decreased and one or both of them are increased. In cases where u and g change in the opposite directions, the policy difference has different impacts on lifetime expected utilities across the population according to the household's birth date.

4. Closing the system

The final step in determining the equilibrium growth rate, g, is to require that the aggregate household variables be consistent with the aggregate production variables. The equilibrium growth rate is obtained directly from the goods market equilibrium condition

$$\frac{\dot{K}_t}{K_t} = \frac{Q_t - G_t - C_t}{K_t} = A(1-\gamma) - \delta - \frac{C_t}{K_t}. \tag{21}$$

Substituting Eq. (19) into Eq. (13a) and recalling Eq. (14a), we obtain $C_t/K_t = (1 - \alpha) \cdot A \cdot (\Sigma^C/\Sigma^L)$, and using $\dot{K}_t/K_t = g + n$, we can write Eq. (21) as

$$g = A(1-\gamma) - \delta - (1-\alpha) \cdot A \cdot \frac{\Sigma^C}{\Sigma^L} - n \tag{21′}$$

Eq. (21′), together with definitions of the aggregators, Eqs. (13b) and (14b), and the expressions determining the required tax rates, implicitly solve for the balanced growth rate, where in addition we equate the aggregate productive capital stock K_t to the households' aggregate financial wealth. The demographic constraint, Eq. (15), is used to determine consistent demographic parameters.

4.1. The parameterized demographic functions

In order to evaluate the aggregators $[\Sigma^C, \Sigma^L]$ we parameterize the demographic functions $S(x)$ and $L(x)$ which describe how survival and labor supply decline with a household's age. We assume these functions are exogenous, although in practice $L(x)$ may reflect household choices.[17]

It is well-known that the law of mortality proposed by Benjamin Gompertz in 1825 fits the facts of human mortality remarkably well. Gompertz observed that the logarithm of the mortality hazard rate is approximately linear affine in age, implying the two-parameter Gompertz survival function as $S(z) = \exp\left[(1 - e^{\mu_1 \cdot z})(\mu_0 - 1)^{-1}\right]$. This survival function, while accurate, is doubly exponential and computationally intractable in our model. However, using the first order expansion of the exponential function, we obtain a linear approximation to the Gompertz survival function as $S(z) \approx 1 + (1 - e^{\mu_1 \cdot z})(\mu_0 - 1)^{-1}$.[18] This in turn, can be rearranged as $S(z) = (\mu_0 - e^{\mu_1 \cdot z})(\mu_0 - 1)^{-1}$ for $0 \le x \le \ln[\mu_0]/\mu_1$, zero otherwise. This form can be recognized as the tractable survival function introduced by Boucekkine et al. (2002), and therefore we will refer to it as the BCL survival function. Setting log $[\mu_0] = \omega \cdot \mu_1$, and redefining $\mu_1 = \mu$, we can re-parameterize the BCL function in the form

$$S(z) = \frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1} \quad \text{for } 0 \le z \le \omega \tag{22a}$$

We also assume the labor supply fraction declines in accordance with a BCL form, namely

$$L(z) = \frac{e^{\nu \cdot \ell} - e^{\nu \cdot z}}{e^{\nu \cdot \ell} - 1} \text{ for } 0 \le z \le \ell, L(z) = 0 \text{ for } z > \ell \tag{22b}$$

where ℓ is the oldest age for which a worker remains in the labor force and $L(z) = 0$ for $x > \ell$. Substituting the parametric forms, we summarize the parameterized equations defining the equilibrium in the Appendix A. The equilibrium growth rate g is obtained by substituting Eqs. (A2)–(A5) into the goods market clearing condition, Eq. (A1), and solving for g. Social Security parameters β and tax rates τ_s, τ_i, τ_k must satisfy Eq. (A6) and demographic parameters satisfy Eq. (A7) where, for simplicity, we assume the fertility rate π is independent of age. The utility multiplier is determined by Eq. (A8). In an economy without annuities, Eqs. (A2), (A5) and (A8) are replaced with Eqs. (NA2), (NA5), and (NA8) respectively.

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[16] Because the equilibrium wage rate, being tied to capital via Eq. (19), grows at a constant rate g, and because we assume the economy has been operating indefinitely, its level is indeterminate. Thus we are free to normalize the wage for comparison purposes in the way described. The choice was made to confine the effects of the introduction of the policy to households who are now deceased. Note that our welfare results hold *a fortiori* if we normalize wages for the comparison economies further in the past.

[17] As we acknowledge in our previous paper, Bruce and Turnovsky (in press), there is a literature demonstrating how the demographic structure may be dependent upon the economic conditions.

[18] Moreover, since $(1 - e^{\mu_1 \cdot z})(\mu_0 - 1)^{-1}$ is very small (for our parameterized function <0.015) the BCL function is in fact a very good approximation to the more general Gompertz function.

4.2. Calibration

Because the production and preference characteristics of the economy are standard and well documented in the literature, we maintain the following values throughout our analysis. On the production side we assume $r^p = 0.067$, $\alpha = 0.4$, $\delta = 0.05$, which, using Eq. (18a), implies $A = 0.2925$. We assume government infrastructure spending equals 5% of gross output, so $\gamma = .05$ and we set the output elasticity of government infrastructure spending $(1 - \alpha) \cdot \xi$ equal to 0.25 consistent with many estimates.[19] This implies $\xi = 0.417$.[20] For preference parameters, we assume $\rho = 0.03$ and $\varepsilon = -2/3$ (that is, an inter-temporal elasticity of substitution equal to 0.6, well within the range of estimates discussed by Guvenen, 2006).[21]

We calibrate the two parameters of the BCL survival function using the estimates of Mierau and Turnovsky (2012) who used non-linear least squares to fit the BCL function to US 2006 cohort data for persons over age 18. The parameters estimated by Mierau and Turnovsky are $\mu = 0.0566$ and an oldest survival age of 95.1 years, implying $\omega = 75.1$ for a household entering the economy at age 20.[22] This calibration of the BCL function implies a life expectancy 19.1 years for a household of age 65, which is equal to the life expectancy of 19.1 years for women of age 65 reported in the US Life Tables for the Social Security Area (Bell and Miller, 2005).[23] We set the parameters of the BCL labor supply function so that the expected retirement (and claiming) age of an entry household, conditional on survival, is 63, the average age of a new claimant in the United States, and so that the labor force participation rate at age 63 is 60% of that of prime age workers, in accordance with observed values (Table 2, Toossi, 2012). This is achieved by setting the oldest working age at 78 ($\updownarrow = 58$) and $\nu = .059$. We assume the population growth rate is 1%. These demographic parameter values imply a Social Security program dependency rate of 34.1%, which closely approximates the actual value of 34% in the United States in 2010.

5. Computing the effects of social security on growth rates and welfare

We compute equilibrium economic growth rates using a *Mathematica* coded program that is available from the authors on request. In an earlier paper (Bruce and Turnovsky, in press), we found that the computed equilibrium growth rate of the model is unique. We set up a benchmark case by introducing what we view as a realistic calibration, and compare the effect of the Social Security program on steady-state economies with, and without, annuities. To examine the robustness of our results we also conduct sensitivity analysis, by considering the effects of varying the demographic assumptions, as well as introducing Social Security "reforms" necessary to maintain PAYGO solvency. Space limitations necessitate us to focus on only the case of an annuities economy.

Table 1
The effect of social security on growth rates of steady state-economies with fair annuities (benchmark specification).

	Growth rate	PAYGO Tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	1.19%	10.24%	−40.3
No Social Security	1.91%	0	−35.8

Assumptions: Population growth rate = 1%, average claiming age = 63, life expectancy at age 65 = 19.1 years (oldest survival age = 95.1 and μ = .0566). Expected retirement/claiming age at age 20 is 63, (oldest working age = 78 ν = .059), dependency rate = 34.1%. fertility rate = 2.29%.

A key aspect is the welfare implications, and in order to carry out welfare analysis, we assume that the wage rate for the steady-state economies being compared are the same at time $t - \omega$, which is the birth year of the oldest person in the economy. Thus, for an economy that is growing faster than another, but where u is no greater, we can conclude that all generations alive as well as future generations enjoy higher utility.

5.1. The effects of social security—the benchmark case

Line 1 of Table 1 describes the simulated values for a benchmark steady-state economy with fair annuities and a PAYGO Social Security program with a replacement rate of 30% of covered earnings ($\beta = .3$), similar to the US program.[24] As seen in the table, the growth rate in the economy without Social Security is higher by .72 pps, 1.91% rather than 1.19%, than for a steady-state economy with Social Security. The payroll tax rate required for PAYGO solvency is 10.24%, which is approximately the FICA tax rate needed to finance current Old-Age, Survivors, and Disabilities Insurance (OASDI) benefits at full employment in the U.S. program. The implied dependency rate (dependents per covered worker) is 34.1%. In the table, we also report the utility multiplier as given by Eq. (12). The utility multiplier is more than 11.2% higher in the economy without Social Security, (−35.8 rather than −40.3).[25] Because the wage growth rate is also higher in the economy without Social Security, the expected lifetime utilities of all existing and future households are increased by 11.2% or more.

As mentioned, the "perpetual youth" specification in Saint-Paul (1992) is a special case of the demographic structure adopted in this paper. Because there is no finite maximum length of life in this model, to calibrate it and preserve comparability, we set ω and ℓ to an arbitrarily large number (1000) and set $\mu = -1/58.5$, implying a life expectancy at age 20 of 58.5 which is the same life expectancy at age 20 implied by our parameterized BCL survival function. In order to compare the effects of Social Security, we set $\nu = -1/108.1$ which gives a dependency rate of 34.1% as in the benchmark model.[26] All other parameters are the same as in the benchmark specification.[27]

As seen in Table 2, the qualitative results in the perpetual youth and benchmark specifications are similar. Growth rates are higher in the perpetual youth specification, but the impact of Social Security on growth rates and welfare are significantly smaller. In particular, the growth rate in the economy without Social Security is just .29 pps greater, about 40% of the benchmark increase. We find this result to hold consistently across varying parameters, which implies that the Blanchard model seriously understates the negative impact of Social

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[19] Estimates of the elasticity of output with respect to public capital are extraordinarily wide ranging. In a comprehensive analysis, summarizing 538 estimates drawn from 68 different studies, primarily using data from OECD countries, Bom and Ligthart (2010) report estimates ranging between the extremes of −1.7 and 2.0, although the majority are clustered over a far narrower range. Our chosen value of 0.25 is somewhat above their reported average of 0.18, but well within the consensus range. Moreover, taking account of the fact that because of depreciation of government capital, the elasticity of output with respect to government expenditure flows (as in our model) is likely to exceed the elasticities with respect to public capital, reported in this study, thereby further justifying setting $(1 - \alpha)\xi = 0.25$.

[20] The value of ξ is relevant only for determining the size of the capital externality.

[21] The only point to note is that ρ, being a pure rate of time preference, is somewhat smaller than the conventional value for the representative agent model ($\rho = 0.04$). This is because the latter implicitly discount for mortality factors, which we are explicitly incorporating in our analysis.

[22] Mierau and Turnovsky show that the BCL function with estimated parameters tracks the data very tightly, except at the extreme old-age tail.

[23] The longer life expectancy of women is most appropriate for our analysis because the Social Security benefit in the United States is a dual survival annuity, and the majority of beneficiaries are married.

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[24] The Social Security replacement rate for a worker with average income who does not claim until the age of full entitlement is nearly 40%. However, a majority of workers claim a reduced benefit at an earlier age, which reduces the average replacement rate.

[25] Recall that we equalized the wage rate of the oldest person in the two steady-state economies.

[26] This value implies that the number of expected working years conditional on survival at age 20 is over 108, which of course is unreasonably high. This reflects the non-realism of the perpetual youth assumption.

[27] Saint-Paul assumes a logarithmic utility function ($\varepsilon = 0$), but we maintain $\varepsilon = -2/3$ for comparability.

Table 2
The effect of social security on growth rates of steady-state economies with fair annuities (perpetual youth specification).

	Growth rate (g)	PAYGO Tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	1.72%	10.24%	−30.9
No social Security	2.01%	0	−28.7

Assumptions: Population growth rate = 1%, life expectancy at age 20 = 58.5 years (oldest survival age = 1000 and $\mu = -1/58.5$). Expected retirement/claiming age conditional on survival at age 20 is 108.1, (oldest working age = 1000 and $\nu = -1/108.1$), dependency rate = 34.1%.

Table 3
The effect of Social Security on growth rates of steady-state economies without annuities (benchmark specification).

	Growth rate (g)	PAYGO Tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	0.64%	10.24%	−42.6
No Social Security	1.35%	0	−37.8

Assumptions: Population growth rate = 1%, average claiming age = 63, life expectancy at age 65 = 19.1 years (oldest survival age = 95.1 and $\mu = .0566$). Expected retirement/claiming age at age 20 is 63, (oldest working age = 78, $\nu = .059$), dependency rate = 34.1%.

Security on growth, as compared to that implied by the more realistic BCL mortality function. Intuitively, this is because mortality is independent of age in the Blanchard economy, agents have a greater incentive to maintain their savings as they age, and less incentive for Social Security to reduce it.

Although the negative impact of PAYGO Social Security on the growth rate is a standard result, the mechanism in our model is novel and provides new insights. The key relationship to understanding this is the equilibrium equation, Eq. (21′), where it is seen that the net impact of the Social Security program on the growth rate operates entirely through its effect on the consumption aggregator, Σ^C. From the definition of Σ^C given in Eq. (14b), we see that the presence of the Social Security program impinges both directly and indirectly on this term. Its direct impact is to reduce the "net income flow" term, $(1 - \tau - \beta)L(z) + \beta$, in the wealth multiplier $h(0)$ of the newborn cohort, defined in Eq. (10). But it also has a positive indirect effect on Σ^C through the growth rate, as the impact is aggregated over time by individuals, and across cohorts. In all of our simulations the latter effect dominates, Σ^C therefore increases, and the equilibrium growth rate declines.

The intuition is as follows. With Social Security, agents are aware that there is a group of individuals – the retirees – that will be supported by the program. While they are working, households know that they will have to support the retirees, and thus over that portion of their lifespan they will have fewer resources for their own consumption. They also know that if they live to retirement, they, themselves, will be beneficiaries. However, they discount these future benefits at a rate greater than the biological rate offered by the PAYGO system, and on balance the initial wealth multiplier, $h(0)$, is reduced. The overall effect is to reduce their current consumption in accordance with Eq. (9a).

Recalling the agent's budget constraint, Eq. (1b), we see that while Social Security reduces the agents' current consumption, it also reduces his rate of asset accumulation, doing so by a greater amount. As a result, the growth rate of capital declines more than does consumption. Aggregating overall agents, this leads to an increase in the C/K ratio, sustaining a decline in the equilibrium growth rate in accordance with Eq. (21′).

Moreover, this reduction in $h(0)$ due to Social Security implies that households in the economy with Social Security have lower consumption levels at every age, and therefore lower expected lifetime utility. The decrease in the utility caused by Social Security is not surprising. Households have access to actuarially-fair life annuities with or without Social Security, so the policy cannot improve welfare due to annuity market failure. Furthermore, saving which is an activity with a positive externality is reduced by Social Security. To address these issues, in the next two sections we consider an economy without annuities and with a "corrective" capital subsidy.

5.2. The effect of social security in economies without annuities

Table 3 reports the computed effects in an economy without annuities, using the benchmark parameter specification. Although growth rates are reduced overall in such an economy, the negative impact of Social Security is of comparable magnitude to that of an economy with annuities. The growth rate is .71 pps higher in the steady-state economy without Social Security, which, because the base growth rate is lower in economies lacking annuities, is a larger *proportionate* change than for the annuities economy. Since we find that the utility multiplier and growth rate are decreased by the presence of PAYGO Social Security in economies without annuities, present and future generations are made less well off by the program. The utility multiplier is increased 11.3% without Social Security, an increase that is comparable to the increase in the multiplier in economies with annuities. Note also that with, or without, Social Security, households are less well off in the economy without annuities than they would be in a comparable economy with life annuities.

Table 4 reports the computed effects for the perpetual youth specification of the economy without annuities. Growth rates are lower than in the annuities economies, but the magnitude of the effect of Social Security is the same as in the annuities economies at about .27 pps, about 38% as large as the impact of Social Security in economies with realistic mortality.

5.3. The effect of corrective capital subsidies

Our assumed production technology implies that the aggregate capital stock imposes a (positive) production externality χ_k as given by Eq. (18c). The benchmark calibration implies an aggregate capital externality rate of 10.24%, which combined with a private return to capital of 6.7% implies a social rate of return to capital of 16.94%.[28] Given the significant production externality on capital, it is perhaps not surprising that PAYGO Social Security, which reduces private capital accumulation, should reduce utility even when life annuities are absent.

The production externality on capital implies that a capital subsidy can be Pareto improving. Saint-Paul (1992, p. 1254) derives a formal condition that is necessary and sufficient for a capital subsidy to increase the well-being of current and future generations. In this section, we compute the effects of such a capital subsidy, which we denote σ, for our general model. Table 5 reports the economic growth rates and utility multipliers in steady-state economies for different levels of σ. We find that the growth rate increases monotonically with the subsidy rate up to the "Pigovian" rate of 10.24%. The utility multiplier reaches a maximum at a subsidy rate of 3.4% and declines thereafter.

At a subsidy rate of 6.5%, the utility multiplier equals the same value as in the case of a zero capital subsidy. Thus, for all positive subsidy rates less than 6.5% both the utility multiplier and the growth rate are higher, so we can conclude that all existing and future generations younger than the oldest households are unambiguously better

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[28] The importance of the capital externality is characteristic of the Romer production function and the fact that to maintain endogenous growth requires that the technology simultaneously exhibit constant returns to scale in (i) the private factors of production, and (ii) the endogenously accumulated factors, in this case private and aggregate capital, together with the stylized fact that the labor share of output exceeds that of capital. One justification for this is that aggregate capital proxies for the general level of technology in the economy, which is not explicitly introduced into the analysis, but which nevertheless is a critical determinant of growth.

Table 4
The effect of Social Security on growth rates of steady-state economies without annuities (perpetual youth specification).

	Growth rate	PAYGO Tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	0.60%	10.24%	−34.5
No Social Security	0.87%	0	−32.1

Assumptions: Population growth rate = 1%, life expectancy at age 20 = 58.5 years (oldest survival age = 1000 and $\mu = -1/58.5$). Expected retirement/claiming age conditional on survival at age 20 is 108.1, (oldest working age = 1000 and $\nu = -1/108.1$), dependency rate = 34.1%.

Table 5
The effects of a capital subsidy on growth in economies without annuities and without Social Security ($\beta = 0$).

Capital subsidy rate (σ)	Economic growth rate (g)	Utility multiplier (u)
0	1.35%	−37.8
1%	1.76%	−37.0
2%	2.15%	−36.5
3%	2.53%	−36.31
3.4%	2.68%	−36.29
4%	2.90%	−36.34
5%	3.24%	−36.7
6%	3.56%	−37.3
6.5%	3.71%	−37.8
10.24% (χ_k)	4.52%	−45.9

Assumptions: Population growth rate = 1%, average claiming age = 63, life expectancy at age 65 = 19.1 years (oldest survival age = 95.1 and $\mu = .0566$). Expected retirement/claiming age at age 20 is 63, (oldest working age = 78, $\nu = .059$), dependency rate = 34.1%.

off with the subsidy than without.[29] Subsidy rates above 6.5%, including the "Pigovian" rate of 10.24%, have ambiguous welfare effects with older workers made worse off. This is because the utility multiplier is reduced while the growth rate, and thus the wage of younger households, is higher. As pointed out by Saint-Paul (1992), for high subsidy rates there is generational conflict, with younger and future generations more likely to benefit from a high subsidy and older generations more likely to lose.

We can also conclude that all generations benefit from an increase in the subsidy rate whenever it is less than 3.4%. Increasing the subsidy rate further has ambiguous incremental effects on welfare because it increases the growth rate but decreases the utility multiplier. At a subsidy rate between 3.4% and 6.5%, an increase in the subsidy rate may decrease the welfare of some extant generations, particularly older ones, but increases the welfare of future generations.

In Table 6 we report the impact of Social Security in steady-state economies with a capital subsidy. We set the subsidy rate at 6.5% which, as we have just described, unambiguously increases the welfare of all existing and future generations except the oldest, who are held harmless. The steady-state economy grows at rate that is .87 pps lower than the economy without Social Security, and the utility multiplier is 15.8% lower. Thus, Social Security reduces growth and welfare in economies without annuities by an even greater amount when the capital market externality is "corrected" with a Pareto improving subsidy.[30]

5.4. Social security and the age profile of consumption

Consumption over the life-cycle is known to exhibit a characteristic hump or inverse-U profile (see Gourinchas and Parker, 2002; Bullard and Feigenbaum, 2003, and, more recently, Fernandez-Villaverde and Krueger, 2007). This profile is not obtained in the annuities model,

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[29] The oldest workers are equally well off in both steady state economies.
[30] This result holds also for higher subsidy rates and for economies with annuities. We do not report those results here.

Table 6
The effects of Social Security with a corrective subsidy (6.5%).

	Growth rate	PAYGO Tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	2.84%	10.24%	−44.9
No Social Security	3.71%	0	−37.8

Assumptions: Population growth rate = 1%, average claiming age = 63, life expectancy at age 65 = 19.1 years (oldest survival age = 95.1 and $\mu = .0566$). Expected retirement/claiming age at age 20 is 63, (oldest working age = 78, $\nu = .059$), dependency rate = 34.1%.

because consumption rises or falls monotonically with age depending on whether $r - \rho$ is positive or negative. However it is present in the economy without annuities. This follows directly from Eqs. (3″) and (6′) and is a consequence of a rising mortality hazard rate.

In Fig. 1 we simulate the implied age profile for an individual household (to simplify the comparison of the profiles, we assume a unit wage rate at the time the household enters the economy in all cases) with and without Social Security using the benchmark calibration for the economy without annuities. Given the parameters of the benchmark case, utility rises steadily with age, but drops sharply when the household attains very old age (about 58 years after it enters the economy, or 78 years old.) Consumption then declines sharply to zero.[31] This profile is similar, with or without, Social Security, although the profile is flattened slightly in the Social Security case, when consumption is reduced at every age. This is because the initial wealth of the household is less in the Social Security economy.

A more interesting profile is the cross section of consumption across age at a point in time, illustrated in Fig. 2. Again, two economies are shown, one with Social Security and one without, at a given time. The current wage is normalized at unity. In the economy without Social Security, consumption rises more slowly across younger households of different ages, before dropping for households of very old age. Also, in the cross section, young households have lower consumption in the Social Security economy than in the economy without Social Security.[32] However, after age 45 (25 years after the household enters the economy), consumption in the Social Security economy surpasses that of the economy without Social Security, and remains higher for the rest of the age distribution. One cannot draw welfare conclusions from this cross section because the wage grows faster in the economy without Social Security. Households who are currently younger will have higher consumption levels than currently older households when they reach similar ages. The profile does suggest that a PAYGO Social Security program can have a marked effect on the observed age cross section of consumption at a point in time, notably lower consumption by younger households and higher consumption by older households.

6. The impact of aging and compensating social security program changes

One of the more important Social Security issues facing policy makers is the problem of maintaining the solvency of a PAYGO Social Security system in the presence of increasing life expectancy. Between 1950 and 2010 in the United States life expectancy at 65 increased by 4.1 years for women and 3.74 years for men. In recent decades, life expectancy at 65 has been increasing at about one year per decade. Maintaining the solvency of a PAYGO program with increasing life expectancy requires program adjustments that may take the form of tax rate increases, benefit rate reductions, or demographic or policy

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[31] Consumption falls to zero as the household approaches the maximum survival age because survival falls to zero. If we introduce a subsistence level of consumption, the pattern remains the same with consumption falling to the subsistence level, rather than zero, as the household approaches the oldest possible survival age.
[32] Again, we normalize the wage of the oldest living household for the comparison economies.
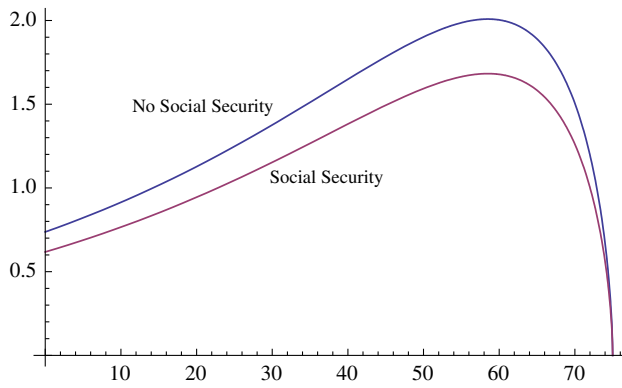
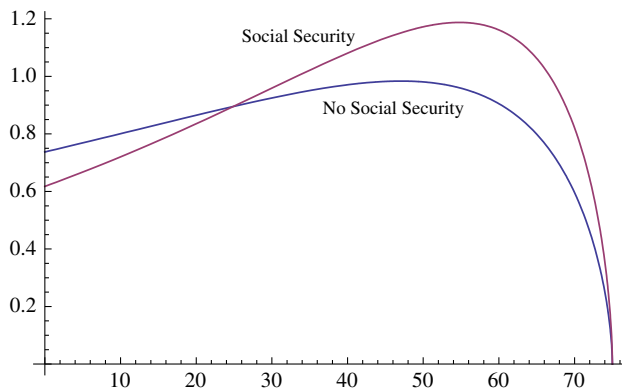**Fig. 1.** Lifetime consumption profile (no annuities).

Fig. 2. Cross section consumption profile (no annuities).

changes that prevent the program dependency rate from rising. In this section we compute the effect of increasing life expectancy on Social Security and growth, as well as the effects of the compensating policy and demographic changes.

6.1. The effect of longer life expectancy on benefit and tax rates

In Table 7, we simulate the sensitivity of the impact of Social Security to life expectancy at age 65 for an economy with annuities. Using the BCL function, life expectancy at age x is $\int_{z=x}^{z=\omega} \frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - e^{\mu \cdot x}} \cdot dz = \frac{1}{\mu} \cdot \left(\frac{\mu \cdot (\omega - x)}{1 - e^{\mu \cdot (x - \omega)}} - 1 \right)$. Life expectancy may increase from either reduced mortality hazard over remaining life (an increase in the parameter μ) or an increase in the oldest survival age (parameter ω). Since the oldest survival age has increased little over time, in Table 7 we augment life expectancy at age 65 by one year, from 19.1 to 20.1, by increasing the parameter μ from .0566 to .07224. We do this both in the economies with, and without, Social Security.

Assuming the birth rate remains constant, an increase in life expectancy at age 65 of one year would increase the population growth rate from 1% to 1.15% according to the demographic constraint. We maintain the expected claiming/retirement age at 63. Comparing row 1 in Tables 7 and 1, we see that the steady-state economy with longer life expectancy grows slightly faster than in the benchmark case (+.07 pps with Social Security) and the difference between the growth rates in steady-state economies with and without Social Security is slightly greater (.75 pps rather than .72 pps). The dependency rate is increased from 34.1% to 35.2% by higher life expectancy, so the PAYGO solvent tax rate is increased accordingly by .33 pps from 10.24% to 10.57%.

Table 7
The effect of higher life expectancy and increasing Social Security taxes or decreasing benefit rates (annuities specification).

	Growth rate	PAYGO tax rate	Utility multiplier (u)
Social Security ($\beta = .3$)	1.26%	10.57%	−41.2
Social Security ($\beta = .2907$)	1.28%	10.24%	−41.1
No Social Security	2.01%	0	−36.4

Assumptions: Life expectancy at age 65 equals 20.1 years ($\mu = .07244$ and $n = .0115$). Dependency rate = 35.2%.

Alternatively, we can maintain Social Security PAYGO solvency without changing the payroll tax rate by reducing the benefit rate. In Table 7, we assume the increase in life expectancy is compensated by reducing the benefit rate .92 pps from 30% to 29.08%. The lower replacement rate is chosen to maintain the tax rate at 10.24%. Comparing line 2 and 1 in Table 7 we see that the steady-state economy with a compensatory lower benefit rate has a marginally higher growth rate and utility multiplier than the economy with a compensatory higher tax rate.

6.2. The effect maintaining the dependency rate with increased life expectancy

The increase in life expectancy by one year at age 65 alters the solvency of the PAYGO Social Security program by increasing the dependency rate from 34.1% to 35.2%. In this section we consider two changes that maintain the dependency rate as life expectancy increases. In the first, and most policy relevant case, we maintain the dependency rate by increasing the average claiming age for Social Security benefits.[33] Specifically, we consider a steady-state economy with augmented life expectancy plus an average retirement/claiming age that is augmented by 0.43 years, which maintains the dependency rate at 34.1% (and required tax rate at 10.24%). To increase the expected claiming age, conditional on survival, from 63 to 63.47 (as of age 20), the labor supply shape parameter ν is increased from .059 to 0.06155 both in the economies with and without Social Security. Comparing column 1 in Tables 7 and 8 we see that a later claiming (and retirement) age decreases the growth rate slightly with or without Social Security, although it raises the utility multiplier.[34] By comparison to a steady-state economy without Social Security, the growth rate is reduced by .73 pps and the utility multiplier is reduced by 11.4%.

Historically, the solvency of PAYGO Social Security has been maintained, in part, by increasing the inflow of covered workers. Since, in the United States, most workers are now covered by Social Security, the opportunity for extending this alternative is now limited. However some analysts claim that PAYGO Social Security solvency problems can be similarly eliminated, or at least mitigated, by policies that "promote growth." In our model, and we expect in practice as well, a higher economic growth rate has no impact on solvency because wage growth increases both revenues and the cost of benefits.[35] However, a higher fertility rate does reduce the program dependency rate.[36] The government may increase the fertility rate using pronatalist policies, such as baby bonuses, or equivalently by relaxing immigration restrictions on younger immigrant workers.

In Table 9 we consider steady-state economies with augmented life expectancy, but with the fertility rate increased by .07 pps from 2.29% to 2.36%. This increase is sufficient to maintain the program

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[33] We assume this is matched with an increase in the average retirement age.
[34] This latter fact is of dubious interest since our utility function specification does not allow for utility from increased leisure.
[35] We assume Social Security benefits are indexed to wage growth, which is the case for U.S. Social Security up to the age at which the household claims benefits. However, even if benefits are not indexed to wages after claiming, in the steady state the cost of benefits would still grow at the rate of wage growth.
[36] We have already seen, a higher population growth rate brought about by longer life expectancy reduces the dependency rate if the fertility rate is held constant.

Table 8
The effect of higher life expectancy and maintaining program dependency rate by increasing the average retirement/claiming age (annuities specification).

	Growth rate	PAYGO tax rate	Utility multiplier
Social Security ($\beta = .3$)	1.25%	10.24%	−41.1
No Social Security	1.97%	0	−36.4

Assumptions: Average claiming/retirement age = 63.43 ($\nu = .06155$) and life expectancy at age 65 equals 20.1 years ($\mu = .07244$ and n = .0115), dependency rate = 34.1%.

Table 9
The impact of higher life expectancy and maintaining the program dependency rate by increasing the fertility rate (annuities specification).

	Growth Rate	PAYGO tax rate	Utility multiplier
Social Security ($\beta = .3$)	1.24%	10.24%	−41.2
No Social Security	1.97%	0	−36.5

Assumptions: Life expectancy at age 65 equals 20.1 years ($\mu = .07244$), average claiming/retirement age = 63 ($\nu = .059$), n = 1.26%, dependency rate = 34.1%, fertility rate = 2.36%

dependency rate at 34.1%. Comparing Table 9 and rows 1 and 3 of Table 7, we see that increased fertility reduces the growth rate in steady-state economies with and without Social Security and decreases the utility multiplier marginally. Of the four policies considered for maintaining program solvency with increasing life expectancy, the fertility policy has the most detrimental impact on economic growth.[37] Reducing the program benefit rate is the best response for both growth and utility, which is not surprising since Social Security reduces growth and welfare in this model.

7. Conclusions

This paper has addressed the effect of a PAYGO Social Security in a balanced growth model of overlapping generations with a realistic demographic structure. We have compared the effect of Social Security in economies with and without life annuities. Steady-state economies with PAYGO Social Security programs grow more slowly than those without Social Security and, normalizing the wage of the eldest workers, lifetime expected utility is lower for both existing and future households. Also, we find that while the qualitative effects of Social Security in our model are similar to those derived analytically by Saint-Paul (1992), the negative impact of Social Security on the growth rate is significantly greater with a realistic survival function than with the exponential survival function.

Another interesting finding is that PAYGO Social Security reduces welfare (the utilities of existing and future households) even in the absence of annuities markets. This is important because annuities market failure is often cited as the welfare justification for Social Security. One possible reason for this is the fact that aggregate capital has a significant production externality under the Romer technology assumed in this paper. To explore this issue, we imposed a capital ownership subsidy to "correct" for such an externality. Although a capital subsidy, below a certain rate, increases both growth and welfare, we find that the presence of a Social Security program still reduces growth and welfare in steady-state economies.

Another possible reason why Social Security does not improve welfare despite the absence of annuities markets is the fact that households receive a form of annuity in our model in the form of the transfer payments they receive over their lives from the recycled decedent wealth. Although not reported here, we also computed the effects on growth and welfare when decedent wealth is recycled in the form of lump-sum wealth to new-born households. This eliminates any annuity effect of recycled wealth. Although this alternative recycling scheme increased the growth rate and utility multipliers relative to the recycling scheme used here, it remains true that Social Security reduces growth and welfare. Thus, in the model we have

considered, we must conclude there is no clear welfare justification for PAYGO Social Security programs.[38]

Our focus on balanced growth paths, originating back in the infinite past, while instructive, prevents us from addressing other important policy issues, where the transitional dynamics become integral. One such issue pertains to changes in the financing of the Social Security system, such as moving from a PAYGO system, of the type addressed here, to a fully funded system. Such a restructuring of the Social Security system inevitably requires introducing a specific starting date at which the structural changes begin to take effect. Different cohorts will be affected differently, depending upon where they are in their respective life cycles at that time, and this will introduce transitional dynamics, at least until the structural change has worked through all the current cohorts. This in turn raises interesting questions relating to the time horizon over which changes should be implemented. The longer they take, the more cohorts, including currently unborn, are potentially affected. These are important directions in which this paper could be usefully extended.

Appendix A. The parameterized growth model

$$g = A \cdot (1-\gamma) - \delta - (1-\alpha) \cdot A \cdot \frac{\Sigma^C}{\Sigma^L} - n \tag{A1}$$

$$\Sigma^C = m(0) \cdot h(0) \cdot \int_{x=0}^{x=\omega} e^{\left(\frac{r-\rho}{1-\varepsilon} - g - n\right) \cdot x} \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot x}}{e^{\mu \cdot \omega} - 1}\right) \cdot dx \tag{A2}$$

$$\Sigma^C = m(0) \cdot h(0) \cdot \int_{x=0}^{x=\omega} e^{\left(\frac{r-\rho}{1-\varepsilon} - g - n\right) \cdot x} \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot x}}{e^{\mu \cdot \omega} - 1}\right)^{\frac{2-\varepsilon}{1-\varepsilon}} \cdot dx \tag{NA2}$$

$$\Sigma^L = \int_{x=0}^{x=\ell} e^{-n \cdot x} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot x}}{e^{\mu \cdot \omega} - 1}\right) \cdot \left(\frac{e^{\nu \cdot \ell} - e^{\nu \cdot z}}{e^{\nu \cdot \ell} - 1}\right) \cdot dx \tag{A3}$$

$$h(0) = (1-\tau-\beta) \int_{z=0}^{z=\ell} e^{-(r-g) \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right) \cdot \left(\frac{e^{\nu \cdot \ell} - e^{\nu \cdot z}}{e^{\nu \cdot \ell} - 1}\right) \cdot dz$$
$$+ \beta \cdot \int_{z=0}^{z=\omega} e^{-(r-g) \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right) \cdot dz \tag{A4}$$

$$m(0) = \left[\int_{z=0}^{z=\omega} e^{\frac{\varepsilon \cdot r - \rho}{1-\varepsilon} \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right) \cdot dz\right]^{-1} \tag{A5}$$

$$m(0) = \left[\int_{z=0}^{z=\omega} e^{\frac{\varepsilon \cdot r - \rho}{1-\varepsilon} \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right)^{\frac{2-\varepsilon}{1-\varepsilon}} \cdot dz\right]^{-1} \tag{NA5}$$

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[37] Because fertility is exogenous in our model, note that the reduction in the growth rate caused by higher fertility as shown by comparing Tables 7 and 9 is consistent with the finding of Zhang (1995) that a Social Security induced reduction in fertility could cause an increase in the growth rate. Also note when interpreting this policy in terms of increased immigration of younger workers, we are assuming that such workers do not bring either financial or human capital with them. Thus the results should not be interpreted as extending to immigration policies that encourage the inflow of young workers with capital.

[38] Our analysis assumes that all agents within a cohort are identical and have identical earnings. By extending this framework to introduce heterogeneity across workers in terms of their abilities and earnings, one would expect to find a justification for Social Security in terms of objectives involving income distribution and inequality.

$$\{\tau_s, \tau_i, \tau_k\} = \left\{ \beta \cdot \frac{\int_{x=0}^{x=\omega} e^{-n \cdot x} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot x}}{e^{\mu \cdot \omega} - 1}\right) \cdot dx - \Sigma^L}{\Sigma^L}, \frac{\gamma}{1-\alpha}, \frac{\sigma}{(1-\alpha) \cdot A} \right\} \tag{A6}$$

$$\int_{x=0}^{x=\omega} \pi \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot x}}{e^{\mu \cdot \omega} - 1}\right) \cdot e^{-n \cdot x} dx = 1 \tag{A7}$$

$$u = \frac{[m(0) \cdot h(0)]^\varepsilon}{\varepsilon} \cdot \int_{z=0}^{x=\omega} e^{\left(\frac{\varepsilon \cdot r - \rho}{1 - \varepsilon}\right) \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right) \cdot dz \tag{A8}$$

$$u = \frac{[m(0) \cdot h(0)]^\varepsilon}{\varepsilon} \cdot \int_{z=0}^{z=\omega} e^{\left(\frac{\varepsilon \cdot r - \rho}{1 - \varepsilon}\right) \cdot z} \cdot \left(\frac{e^{\mu \cdot \omega} - e^{\mu \cdot z}}{e^{\mu \cdot \omega} - 1}\right)^{\frac{1}{1 - \varepsilon}} \cdot dz \tag{NA8}$$

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