Is There a Structural Break in the Equity Premium?

by

Chang-Jin Kim†
Korea University

James C. Morley†
Washington University

and

Charles R. Nelson†
University of Washington

February 22, 2000

† Kim: Department of Economics, Korea University, Seoul, 136-701, Korea; Morley: Department of Economics, Washington University, St. Louis, MO 63130-4899, U.S.A.; Nelson: Department of Economics, University of Washington, Seattle, WA 98195, U.S.A.

† We appreciate helpful comments from the participants in the CSSS seminar at the University of Washington for helpful comments, but the responsibility for errors is entirely the authors'. Support from the National Science Foundation under grants SES-9818789 and the Ford and Louisa Van Voorhis endowment at the University of Washington is gratefully acknowledged. This paper was written while Kim was visiting the Department Economics, University of Washington. Kim greatly appreciates support from the Department.

Note: Kim's current address (until Aug., 2000): Dept. of Econ, Univ. of Washington, Seattle, WA 98195 (changjin@u.washington.edu)

Abstract

In this paper, we apply a Bayesian approach to test for a structural break with unknown breakpoint in an empirical model of excess returns that allows the equity premium to change in response to recurrent changes in the level of volatility. The main questions we seek to answer with our approach are the following: Is there evidence of changes in the equity premium over time? If so, can these changes be explained as the consequence of recurrent changes in the level of volatility? Or, alternatively, does the equity premium undergo a one-time permanent structural break?

For monthly excess returns on a value-weighted portfolio of NYSE stocks between 1926-1991, we find strong evidence for a structural break in the Markov-switching variance process around 1941. However, the data provide little evidence of a concurrent structural break in the equity premium. Instead, the data suggest that changes in the equity premium are mainly a consequence of recurrent changes in the level of volatility.

Key Words: Equity Premium, Bayes Factor, Marginal Likelihood, Markov Switching, Risk, Structural Break, Volatility Feedback Effect

"The Bayesian moral is simple: Never make anything more than relative probability statements about the models explicitly entertained. Be suspicious of those who promise more!" [Poirier (1995), p. 614]

1. Introduction

In a recent paper, Pastor and Stambaugh (1998) find evidence of structural breaks in the equity premium and the level of stock market volatility. However, they assume monthly excess stock returns are i.i.d. Normal within each structural regime. In this paper, we apply a Bayesian approach to test for a structural break with unknown breakpoint for an empirical model of excess returns that relaxes the strict i.i.d. assumption. The main questions we seek to answer with our approach are the following. Is there evidence of large changes in the equity premium over time? If so, can these changes be explained as the consequence of recurrent changes in the level of volatility? Or, alternatively, does the equity premium undergo a one-time permanent structural break?

The model we employ to answer these questions allows for recurrent changes in the equity premium corresponding to changes in a Markov-switching level of market volatility. The model, originally developed by Turner, Startz, and Nelson (1989) and discussed in detail in Kim, Morley, and Nelson (2000), is able to account for volatility feedback in realized returns. In this paper, we extend the model to allow for a one-time permanent structural break with unknown breakpoint in the equity premium and/or the variance process.

The Bayesian framework used in the empirical analysis allows us to answer the main questions posed above through a comparison of various restricted versions of our basic model. In particular, we employ Chib's (1995) procedure for calculating the Bayes factor for competing models through direct calculation of marginal likelihood functions for each model. This procedure makes formal evaluation of Markov-switching volatility and/or a structural breakpoint with unknown breakpoint much easier than it would be in a classical framework due to the presence of nuisance parameters.

Our main results can be summarized as follows. First, for monthly excess returns on a value-weighted portfolio of NYSE stocks between 1926-1991, there is strong evidence of recurrent changes in the equity premium over time corresponding to changes in a Markov-switching level of volatility. In particular, models with a Markov-switching variance process and a volatility feedback effect are strongly preferred to models with a constant variance and no volatility feedback, with a negative volatility feedback effect implying a positive tradeoff between volatility and the equity premium. Furthermore, these findings hold whether or not there is a structural break. Second, there is strong evidence for a structural break in the Markov-switching variance process around 1941, the same year in our sample that Pastor and Stambaugh (1998) found a structural break in their paper. However, the data provide little evidence of a concurrent structural break in the parameters related to the equity premium given the level of volatility. Instead, the data suggest that changes in the equity premium are a consequence of recurrent changes in the level of volatility. Finally, we find that, unlike the i.i.d. model of stock returns, the model with volatility feedback and a structural break in the Markov-switching variance process is able to eliminate residual autocorrelation and capture the negative skewness and excess kurtosis observed in the historical data.

The rest of the paper is organized as follows. In Section 2, we motivate and present details of the model of stock returns employed in the paper. In Section 3, we discuss issues related to the incorporation of a structural break with unknown breakpoint in the model and outline the restricted versions of the general model to be considered in the empirical analysis. Section 4 provides an overview of the Bayesian approach employed in the paper and presents the empirical results. Section 5 concludes. Tables and figures follow the appendix and a list of references.

2. An Empirical Model of Stock Returns with Volatility Feedback

The literature on stock market volatility provides two alternative hypotheses for how volatility is related to the stock returns: the leverage hypothesis and the volatility feedback hypothesis. The line of research which focuses on the leverage hypothesis assumes that the volatility process is endogenous. According to this hypothesis, a drop in the value of stock or a negative return increases financial leverage, which makes the stock riskier and increases its volatility [Cox and Ross (1976), Black (1976), Christie (1982), and Schwert (1989)] As Bekaert and Wu (1997) note, the 'leverage effect' has become almost synonymous with asymmetry in stock returns volatility. The line of research which focuses on the volatility feedback hypothesis, on the other hand, assumes that the volatility process is exogenous. According to this hypothesis, stock returns respond to changes in volatility [French, Schwert, and Stambaugh (1987), Turner, Startz, and Nelson (1989), Campbell and Hentschel (1992), and Kim, Morley, and Nelson (2000)]. For example, given that volatility is a good measure of risk and the conditional volatility process is persistent, an increase in volatility implies higher future expected returns as a compensation for the increase in non-diversifiable risk, which, in turn, decreases current stock price. Meanwhile, the different responses of returns to positive and negative changes in the level of volatility generates asymmetry in the stock return behavior even when the volatility process is symmetric. Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2000) use the log-linear present-value model of stock prices developed by Campbell and Shiller (1988a,b) to provide a theoretical background on the volatility feedback effect. Their empirical findings confirm the existence of a negative volatility feedback effect, which has also been found in French, Schwert, and Stambaugh (1987) and Turner, Startz, and Nelson (1989). [1]

We employ the following version of the model with volatility feedback and Markov-switching variance proposed by Turner, Startz, and Nelson (1989) and Kim, Morley, and Nelson (2000):

$$y_t = \mu + \gamma E[\sigma^2_{S_t}|\psi_{t-1}] + \delta(\sigma^2_{S_t} - E[\sigma^2_{S_t}|\psi_{t-1}]) + e_t, \quad S_t = \{0, 1\} \tag{1}$$

$$e_t|S_t \sim i.i.d.N(0, \sigma^2_{S_t}), \tag{2}$$

$$\sigma^2_{S_t} = \sigma^2_0 + (\sigma^2_1 - \sigma^2_0)S_t, \quad \sigma^2_0 < \sigma^2_1, \tag{3}$$

[1] While Campbell and Hentschel (1992) and French, Schwert, and Stambaugh (1987) investigate volatility feedback within GARCH-type variance process, Turner, Startz, and Nelson (1989)and Kim, Morley, and Nelson (2000) investigate it for a Markov-switching variance process.

where y_t is excess returns on market portfolio; ψ_{t-1} is information set up to time t-1; and S_t is a discrete-valued, latent first-order Markov-switching process with transition probabilities given by:

$$Pr[S_t = 1|S_{t-1} = 1] = p_{11} \quad and \quad Pr[S_t = 0|S_{t-1} = 0] = p_{00}. \tag{4}$$

We first discuss the nature of the variance dynamics in the above specification. The variance process is modeled as recurrent and endogenous switches between high variance (σ_1^2) and low variance (σ_0^2) states. The dynamics of the latent variable S_t, given by the transition probabilities in (4), determines the nature of the variance process. Thus, the following alternative specification for the dynamics of S_t would provide us with insights into the nature of the variance process:

$$S_t = \lambda_0 + \lambda_1 S_{t-1} + v_t, \tag{5}$$

where v_t is an appropriately defined, discretely valued shocks with mean zero; $\lambda_0 = 1 - p_{00}$ and $\lambda_1 = p_{00} + p_{11} - 1$ (see Hamilton, 1989). Notice that equations (3) and (5) imply the steady-state or the long-run variance is given by:

$$E[\sigma_{S_t}^2] = \sigma_0^2 + (\sigma_1^2 - \sigma_0^2) \times \frac{1 - p_{00}}{2 - p_{00} - p_{11}} \tag{6}$$

and the persistent of the variance process is given by:

$$\lambda_1 = p_{00} + p_{11} - 1 \tag{7}$$

Furthermore, expected durations of high variance state and low variance states are given by:

$$DR_0 = \frac{1}{1 - p_{00}} \quad and \quad DR_1 = \frac{1}{1 - p_{11}}, \tag{8}$$

respectively. Thus, a structural break in the parameters that describe the dynamics of the volatility process in equations (3) and (4) would imply a structural break in the steady-state volatility in equation (6), the persistence of volatility in equation (7), and the expected durations of high and low volatility states in equation (8).

We now turn our attention to how the volatility feedback effect works in the above model. At the beginning of time t, the latent variable S_t that governs the variance $\sigma_{S_t}^2$

is not realized. However, agents' expectation of $\sigma^2_{S_t}$ conditional on available information ($E[\sigma^2_{S_t}|\psi_{t-1}]$) affects stock price at the beginning of time t. The coefficient γ measures the effect of an expected change in variance on returns at time t. Suppose that, at the end of time t, S_t and $\sigma^2_{S_t}$ is realized and that it is different from its expectation at the beginning of time t. Such difference would affect returns at the end of time t through the volatility feedback effect. The coefficient δ measures the significance of the volatility feedback effect. If the realized variance state at the end of time t is higher than the expected variance state at the beginning of time t, for example, in the presence of a persistent volatility process agents would revise their expectations about future volatility in the upward direction. Furthermore, if there is a positive tradeoff between expected returns and risk as measured by volatility, this will increase future expected returns, driving the current stock price down. Thus, a positive tradeoff between expected returns and risk is consistent with an existence of negative volatility feedback effect ($\delta < 0$). Note that, in reality, S_t and $\sigma^2_{S_t}$ may not be fully observed by the agents at the end of time t. However, while $Pr[S_t = 1|\psi_{t-1}]$, available from estimation, may be termed as a prior distribution that agents form about the state at the beginning of time t, S_t itself may be considered a reasonable approximation to a posterior distribution that agents form at the end of time t. For further discussions and theoretical background on the volatility feedback effect in the presence of Markov-switching variance, readers are referred to Turner, Startz, and Nelson (1989) and Kim, Morley, and Nelson (2000).

3. Incorporating a One-Time Permanent Structural Break in the Model

Past literature provides evidence of a structural break in U.S. stock return behavior since 1926. For example, Viceria (1997), based on a rigorous stability test, provides statistical support to the presumption that there was a structural break in the behavior of multi-period holding returns in the 1950's. More recently, Pastor and Stambaugh (1998) identify three breaks in the one-month excess returns in their sample [1834:1 - 1996:12], with the locations of breaks being 1928, 1941 and 1991. Focusing on the sample that covers 1926:1 - 1991:12, we thus allow for the possibility of a one-time permanent structural break with an unknown break point in the parameters of the model introduced in Section 2. [2]

In particular, the model in Section 2 is extended as follows: [3]

$$y_t = \mu_{D_t} + \gamma_{D_t} E[\sigma^2_{S_t,D_t}|\psi_{t-1}] + \delta_{D_t}(\sigma^2_{S_t,D_t} - E[\sigma^2_{S_t,D_t}|\psi_{t-1}]) + e_t, \tag{9}$$

$$e_t|S_t, D_t \sim i.i.d.N(0, \sigma^2_{S_t,D_t}), \quad D_t = \{0,1\},\ S_t = \{0,1\} \tag{10}$$

$$D_t = 0,\ for\ 1 \leq t \leq \tau, \quad D_t = 1,\ for\ \tau + 1 \leq t \leq T, \tag{11}$$

where τ $(1 \leq \tau < T)$ is an unknown break point, and where the parameters and the variance dynamics before and after the structural break point (τ) are given by:

Before Structural Break ($D_t = 0$)

$$\mu_{D_t} = \mu_0; \quad \gamma_{D_t} = \gamma_0; \quad \delta_{D_t} = \delta_0, \tag{12}$$

$$\sigma^2_{S_t,D_t} = \sigma^2_{0,0}(1 - S_t) + \sigma^2_{1,0}S_t, \quad \sigma^2_{0,0} < \sigma^2_{1,0}, \tag{13}$$

$$Pr[S_t = 1|S_{t-1} = 1, D_t] = p_{11,0}; \quad Pr[S_t = 0|S_{t-1} = 0, D_t] = p_{00,0} \tag{14}$$

$$E[\sigma^2_{S_t,D_t}] = \sigma^2_{0,0} + (\sigma^2_{1,0} - \sigma^2_{0,0}) \times \frac{1 - p_{00,0}}{2 - p_{00,0} - p_{11,0}}, \tag{15}$$

$$\lambda_{1,D_t} = p_{00,0} + p_{11,0} - 1, \tag{16}$$

After Structural Break ($D_t = 1$)

$$\mu_{D_t} = \mu_1; \quad \gamma_{D_t} = \gamma_1; \quad \delta_{D_t} = \delta_1, \tag*{(12)$'$}$$

$$\sigma^2_{S_t,D_t} = \sigma^2_{0,1}(1 - S_t) + \sigma^2_{1,1}S_t, \quad \sigma^2_{0,1} < \sigma^2_{1,1}, \tag*{(13)$'$}$$

$$Pr[S_t = 1|S_{t-1} = 1, D_t] = p_{11,1}; \quad Pr[S_t = 0|S_{t-1} = 0, D_t = 1] = p_{00,1}, \tag*{(14)$'$}$$

$$E[\sigma^2_{S_t,D_t}] = \sigma^2_{0,1} + (\sigma^2_{1,1} - \sigma^2_{0,1}) \times \frac{1 - p_{00,1}}{2 - p_{00,1} - p_{11,1}}, \tag*{(15)$'$}$$

---

[2] Exclusion of the post-1991 data is justified in the *data description* in Section 4.2.

[3] Kim and Nelson (1999a) employ a similar framework in analyzing the nature of structural break in the U.S. business cycle with Markov-switching mean growth rate of real GDP.

$$\lambda_{1,D_t} = p_{00,0} + p_{11,0} - 1, \tag{16$'$}$$

where the parameters in equations (12) and (12)' describe the conditional mean of excess return; equations (13)-(14) and (13)'-(14)' describe the dynamics of short-run variances before and after the structural break; equations (15) and (15)' are the long-run variances before and after structural break; and equation (16) and (16)' are the persistence measures of the variance dynamics before and after the structural break. The parameters μ, γ, and δ in equations (12) and (12)' determine expected excess return given the level of volatility, and we will refer to a break in these parameters as a break in mean.

The latent variable D_t, which determines a one-time permanent structural break in the sample, can be modeled as a two-state Markov process, as suggested by Chib (1998) and employed by Kim and Nelson (1999a). This is done by appropriately constraining the transition probabilities so that we have one-time permanent shift from $D_\tau = 0$ to $D_{\tau+1} = 1$ at an unknown break point τ. For example, the transition probabilities should be constrained such that, conditional on $D_t = 0$ there always exists non-zero probability that D_{t+1} may be 1, but conditional on $D_{\tau+1} = 1$ the probability that $D_{\tau+2} = 0$ should always be 0, so that we have $D_t = 1$ for $t \geq \tau + 1$. The following specification for the transition probabilities achieves this goal:

$$Pr[D_t = 0|D_t = 0] = q_{00};\ \Pr[D_t = 1|D_t = 0] = 1 - q_{00}, \tag{17}$$

$$Pr[D_t = 1|D_t = 1] = 1;\ \Pr[D_t = 0|D_t = 1] = 0, \tag{18}$$

$$0 < q_{00} < 1, \tag{19}$$

where the expected duration of $D_t = 0$, or the expected duration of a regime before a structural break occurs, is given by $E(\tau) = 1/(1 - q_{00})$.

Restricted versions of the above general model would then allow us to test various hypotheses. Among these are i) the significance of Markov-switching variance; ii) the significance of the volatility feedback effect; iii) the significance of a structural break in the variance process; iv) the significance of a structural break in the mean in the presence of a structural break in the variance process, etc. For this purpose, we consider various alternative models with different underlying assumptions or restrictions:

MODEL 1: Homoscedastic within Subsamples;
No Volatility Feedback

$$\sigma^2_{0,D_t} = \sigma^2_{1,D_t} = \sigma^{*2}_{D_t}, \quad D_t = 0, 1 \tag{20}$$

$$\gamma_{D_t} = 0, \quad \delta_{D_t} = 0, \quad D_t = 0, 1, \tag{21}$$

i) **Model 1-A:** No structural break in mean; No structural break in variance

$$\mu_0 = \mu_1; \quad \sigma^{*2}_0 = \sigma^{*2}_1$$

ii) **Model 1-B:** No structural break in mean; Structural break in variance

$$\mu_0 = \mu_1; \quad \sigma^{*2}_0 > \sigma^{*2}_1$$

iii) **Model 1-C:** Structural break in both mean and variance

$$\mu_0 \neq \mu_1; \quad \sigma^{*2}_0 > \sigma^{*2}_1$$

MODEL 2: Markov-switching Variance within subsamples;
No Volatility Feedback

$$\sigma^2_{0,D_t} < \sigma^2_{1,D_t} \quad D_t = 0, 1 \tag{22}$$

$$\gamma_{D_t} = 0, \quad \delta_{D_t} = 0, \quad D_t = 0, 1, \tag{23}$$

i) **Model 2-A:** No structural break in mean; no structural break in the variance dynamics

$$\mu_0 = \mu_1$$

$$\sigma^2_{0,0} = \sigma^2_{0,1}; \quad \sigma^2_{1,0} = \sigma^2_{1,1}$$

$$p_{00,0} = p_{00,1}; \quad p_{11,0} = p_{11,1}$$

ii) **Model 2-B:** No structural break in mean; structural break in the variance dynamics

$$\mu_0 = \mu_1$$

$$\sigma^2_{0,0} > \sigma^2_{0,1}; \quad \sigma^2_{1,0} > \sigma^2_{1,1}$$

$$p_{00,0} \neq p_{00,1}; \quad p_{11,0} \neq p_{11,1}$$

iii) **Model 2-C:** Structural break in both mean and the variance dynamics

$$\mu_0 \neq \mu_1$$

$$\sigma^2_{0,0} > \sigma^2_{0,1}; \quad \sigma^2_{1,0} > \sigma^2_{1,1}$$

$$p_{00,0} \neq p_{00,1}; \quad p_{11,0} \neq p_{11,1}$$

MODEL 3: Markov-Switching Variance within Subsamples; With Volatility Feedback

$$\sigma^2_{0,D_t} < \sigma^2_{1,D_t} \quad D_t = 0, 1 \tag{24}$$

$$\gamma_{D_t} \neq 0, \quad \delta_{D_t} \neq 0, \quad D_t = 0, 1, \tag{25}$$

i) **Model 3-A:** No structural break in conditional mean; no structural break in the variance dynamics

$$\mu_0 = \mu_1; \quad \gamma_0 = \gamma_1; \quad \delta_0 = \delta_1$$

$$\sigma^2_{0,0} = \sigma^2_{0,1}; \quad \sigma^2_{1,0} = \sigma^2_{1,1}$$

$$p_{00,0} = p_{00,1}; \quad p_{11,0} = p_{11,1}$$

ii) **Model 3-B:** No structural break in conditional mean; structural break in the variance dynamics

$$\mu_0 = \mu_1; \quad \gamma_0 = \gamma_1; \quad \delta_0 = \delta_1$$

$$\sigma^2_{0,0} > \sigma^2_{0,1}; \quad \sigma^2_{1,0} > \sigma^2_{1,1}$$

$$p_{00,0} \neq p_{00,1}; \quad p_{11,0} \neq p_{11,1}$$

iii) **Model 3-C:** Structural break in both conditional mean and the variance dynamics

$$\mu_0 \neq \mu_1; \quad \gamma_0 \neq \gamma_1; \quad \delta_0 \neq \delta_1$$

$$\sigma^2_{0,0} > \sigma^2_{0,1}; \quad \sigma^2_{1,0} > \sigma^2_{1,1}$$

$$p_{00,0} \neq p_{00,1}; \quad p_{11,0} \neq p_{11,1}$$

4. Empirical Results

4.1. The Bayesian Approach: Inferences and Model Selection

Testing for Markov-switching and/or structural break with unknown break point within the classical framework would be extremely difficult due to the nuisance parameters that exist only under the alternative hypothesis. In the case of testing for Markov-switching, the transition probabilities are the nuisance parameters. [4] In the case of testing for a structural break, the unknown break point is the nuisance parameter. Within the Bayesian framework, however, nuisance parameters that exist only under the alternative hypothesis do not pose any special problem. We thus cast the problem of making inference for each model and the problem of model selection into the Bayesian framework. The hierarchical nature of the model allows us to easily employ Gibbs sampling in obtaining the marginal posterior distributions of the variates of each model. Appendix 1 describes a brief description of the Gibbs sampling approach within the context of a general model (Model 3-C) in Section 3. [5]

Concerning the Bayesian model selection procedure, we assume that data $\tilde{Y}_T = \{y_1,\ y_2, \ldots, y_T\}$ have arisen from one of the models defined in Section 3, according to a probability function (marginal likelihood) $m(\tilde{Y}_T|\omega)$, where ω is the model indicator parameter. Within the Bayesian framework, the Bayes factor has been widely used for model comparison. It is defined as the ratio of marginal likelihoods for models under consideration:

$$B_{ij} = \frac{m(\tilde{Y}_T|\omega = i)}{m(\tilde{Y}_T|\omega = j)}, \quad i \neq j, \tag{26}$$

[4] Refer to Hansen (1992) and Garcia (1998) for tests of Markov switching within the classical framework.

[5] For a direct comparison of the Bayesian inferences and the classical inferences of the Markov-switching models or state-space models, readers are referred to Kim and Nelson (1999b).

where B_{ij} refers to the Bayes factor in favor of Model i over Model j. Various ways of Bayesian model comparison or calculating the Bayes factor have been proposed in the literature. For example, Carlin and Polson (1991), George and McCulloch (1993), Geweke (1996), and Carlin and Chib (1995) provide a procedure for model comparison based on the sensitivity of the posterior probability of the model indicator parameter ω to the prior probability. Kim and Nelson (2000) extend Carlin and Chib's (1995) procedure to deal with tests of Markov-switching in univariate and dynamic factor models. Verdinelli and Wasserman (1995) and Koop and Potter (1999) suggest a way to indirectly calculating the Bayes factor using the 'Savage-Dickey' density ratio for the nested models. Alternatively, Chib (1995) suggests a procedure for directly calculating the marginal likelihoods based on the Gibbs output. [6] Kim and Nelson (1999a) apply Chib's (1995) procedure to test for a structural break in a Markov-switching model of the business cycle. In this paper, we employ Chib's (1995) procedure as implemented by Kim and Nelson (1999). Appendix 2 provides a brief description of the procedure within the context of the general model (Model 3-C) with volatility feedback and a structural break. The prior distributions employed are also shown in Appendix 2.

For Bayesian model selection, we adopt the following criteria suggested by Jeffreys (1961) and Kass and Raftery (1993):

1) $BF_{ij} > 1$: Evidence supports Model i;

2) $10^{-\frac{1}{2}} < BF_{ij} < 1$: very slight evidence against model i;

3) $10^{-1} < BF_{ij} < 10^{-\frac{1}{2}}$: slight evidence against model i;

4) $10^{-2} < BF_{ij} < 10^{-1}$: strong to very strong evidence against Model i,

where, for given log marginal likelihoods ($ln[m(\tilde{Y}_T|\omega = i)]$) calculated, the Bayes factor BF_{ij} in favor of Model i over Model j is calculated as: $BF_{ij} = exp(ln[m(\tilde{Y}_T|\omega = i)] - ln[m(\tilde{Y}_T|\omega = j)])$.

4.2. Empirical Results: Volatility Feedback and the Nature of Structural Break in the U.S. Stock Market [1921:1 - 1991:12]

[6] For a general discussion of Bayesian model comparison and the issues related to the calculation of the Bayes factors, readers are referred to Kass and Raftery (1995).

Data Description

The data are excess stock returns on a value-weighted portfolio of all NYSE stocks over the yield on one-month U.S. Treasury bills from the CRSP files. We use continuously compounded total monthly excess returns. Continuously compounded returns are calculated by taking natural logarithms of simple gross returns and annualized by multiplying by 12.

Our sample covers the period of 1926:1 - 1991:12. Evidence that the returns process may have changed in recent years motivates the ending date. Table 1 shows that average annualized excess return for the recent period 1992:1 - 1998:12 is almost twice as high as the historical average that covers the period 1926:1 - 1991:12. In explaining such an unusual stock price run-up since 1992, Heaton and Lucas (1999) examine a number of potential fundamentals-based explanations including changes in market participation patterns or changes in portfolio diversification. Balke and Wohar (1999), based on a dynamic common factor model, attribute the recent high price/dividend ratio to the market's expectations of future dividend growth, supporting the 'New Economy' explanation for the recent stock market behavior.

Evidence of a Markov-switching Variance and the Volatility Feedback Effect:

In Table 2, we report the log marginal likelihoods for all the models considered. In Table 3, we report posterior moments of the parameters for the models with Markov-switching variance and the volatility feedback effect: Models 3-A, 3-B and 3-C. [7]

We first consider the significance of Markov-switching variance in stock returns. Under the assumption of no structural break in the sample, a comparison of the log marginal likelihoods for Models 1-A and 2-A (-812.86 and -672.79, respectively) shows that there is decisive evidence in favor of Markov-switching variance over homoscedastic returns. The same conclusion holds under the assumption of a structural break in the variance process from a comparison of Models 1-B and 2-B. That is, the evidence of Markov-

[7] Posterior moments of the parameters for the other models are available from the authors upon request.

switching variance is robust with respect to our assumption about the structural break in the sample.

Given very decisive evidence of a Markov-switching variance process, we turn to the significance of a negative volatility feedback effect within a framework with no structural break in the mean. For models with volatility feedback effect (Models 3-A and 3-B), Table 3 shows that the posterior means for the volatility feedback parameters (δ_0) are negative with relatively small standard deviations. Under the assumption of no structural break in the variance process, a comparison of the log marginal likelihoods for Models 2-A and 3-A (-672.69 and -671.10, respectively) provides slight evidence in favor of a negative volatility feedback effect. However, when a structural break is allowed in the variance process, a comparison of the log marginal likelihoods for Models 2-B and 3-B (-666.14 and -661.86, respectively) provide very strong evidence of a negative volatility feedback effect. Thus, the volatility feedback effect is supported by the data, especially when a structural break is allowed in the variance process. Again, as discussed in Kim, Morley, and Nelson (2000), a negative volatility feedback effect implies a positive tradeoff between volatility and the equity premium.

Evidence of a Structural Break in the Variance Process

Given the empirical support of Markov-switching variance and the volatility feedback effect from the previous section, we evaluate the significance of a structural break in the variance process in this section, within a framework with no structural break in the mean. This is done by comparing the log marginal likelihoods for Models 3-A and 3-B (-671.10 and -611.86, respectively). Model 3-B, which allows for a structural break in the variance process, is decisively preferred over model 3-A, which allows for no structural break. Even for the cases in which we ignore Markov-switching variance or the volatility feedback effect, there is strong evidence of a structural break in the variance. That is, Model 1-B is decisively preferred to Model 1-A and Model 2-A is decisively preferred to Model 2-B. Thus, evidence of a structural break in the variance process is robust with respect to assumptions about the variance process and the volatility feedback effect. In

Figure 1, posterior probability of the break point from Model 3-B is depicted against the historical excess returns. The distribution of the break point is centered around 1941, the same year in our sample that Pastor and Stambaugh (1998) found a structural break in their paper.

As discussed in Section 2, a structural break in the transition probabilities that govern the variance process, among other things, implies a structural break in the persistence of the volatility process (λ_1) and the expected durations of high and low variance states. Table 4 compares the posterior distributions of the variates that describe the nature of the structural break in the variance process from Model 3-B: the persistence (λ_1) of the variance process and the expected durations of high and low variance states before and after the structural break. Notice that the posterior mean of the persistence parameter is considerably larger for the pre-break sample than for the post-break sample. While the 90% posterior band for the persistence of the volatility is (0.7719, 0.9553) for the pre-break sample, it is (0.5370, 0.8899) for the post break sample. Table 4 also shows that the expected duration of the high variance state is considerably longer for the pre-break data with approximately the same expected duration of the low variance state for the pre- and post-break data.

Evidence of a Structural Break in the Equity Premium

Given the empirical evidence of a structural break in the variance process, inference on a structural break in the parameters related to the equity premium (μ, γ, and δ) seems to be robust with respect to different assumptions. In all cases, models with no structural break in the equity premium are preferred to those with a structural break in both the volatility and the equity premium. Under the assumption of homoscedasticity, Model 1-B is preferred to Model 2-B with the Bayes factor is 1.49. Under the assumption of a Markov-switching variance without volatility feedback, Model 2-B is preferred to Model 2-B the Bayes factor of 2.51. Under the assumption of a Markov-switching variance and a volatility feedback effect, Model 3-B is preferred to Model 3-C with the Bayes factor of 20.70. However, as the Bayes factors suggest, while the evidence against a structural break in the equity premium is only marginal for the first two cases in which volatility feedback is ignored, the evidence is much stronger in the third case with volatility feedback. That is, even though we find strong evidence of a structural break in the variance process, the data provide little evidence of a concurrent structural break in the parameters related to the equity premium. Instead, the data suggest that changes in the equity premium are a consequence of recurrent changes in the level of volatility. The results in this section suggest that, without appropriately taking into account the volatility feedback effect, there is a higher probability of making an inference about a spurious structural break in the equity premium.

Negative Skewness and Excess Kurtosis in the Data and Residual Autocorrelation

The historical excess returns are characterized by a strong negative skewness and an excess kurtosis [skewness = -0.4425; kurtosis = 9.8741]. We first show that a Markov-switching variance explains most of the excess kurtosis in the historical data, and that the volatility feedback effect explains most of the negative skewness. We then provide an additional diagnostic check for our model selection based on residual autocorrelation. For these purposes, we examine the empirical distribution of the standardized residuals for each model considered. [8]

Table 5 reports the kurtosis and the skewness for the standardized residuals from each model, as well as the p-values for a test of Normality. We notice that models with the i.i.d. assumption (Models 1-A, 1-B, and 1-C) cannot completely explain either the excess kurtosis or the negative skewness in the data. When the Markov-switching variance

[8] For Model 3-B, for example, the residual we use is equivalent to a measure of news about future dividend in the terminology of Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2000). It is calculated in the following way:

$$\hat{e}_t = y_t - E[y_t|\psi_T],$$

where $E[y_t|\psi_T]$ measures the dynamics of the mean with volatility feedback and it is estimated by:

$$E[y_t|\psi_T] = \mu + \gamma E[\sigma^2_{S_t,D_t}|\psi_{t-1}] + \delta(E[\sigma^2_{S_t,D_t}|\psi_T] - E[\sigma^2_{S_t,D_t}|\psi_{t-1}]),$$

where ψ_{t-1} is information up to time t-1 and ψ_T is information up to time T. The term $E[\sigma^2_{S_t,D_t}|\psi_T]$ is used as a proxy for the unobserved true variance $\sigma^2_{S_t,D_t}$.

process is considered for Models 2-A, 2-B, and 2-C, the measures of kurtosis decrease to close to 3, suggesting these models do fine jobs in explaining the excess kurtosis. However, in the absence of the volatility feedback effect in these models, much of the negative skewness still remains unexplained. When the volatility feedback effect as well as Markov-switching variance are taken into account for Models 3-A, 3-B and 3-C, the joint hypothesis of Normality is not rejected. In fact, out of all the alternative models considered for empirical analysis, Model 3-B is by far the most preferred one based on a comparison of the log marginal likelihoods. The distribution of the standardized residuals from Model 3-B, in particular, suggests that the volatility feedback effect with Markov-switching variance process, along with a structural break in the volatility process but not in the equity premium, reasonably explain the negative skewness as well as the excess kurtosis in the historical data.

Note that the existence of the volatility feedback effect with a persistent volatility process implies non-zero autocorrelation in the mean of the data. Thus, the misspecified models that do not account for the volatility feedback effect would reveal some low-order autocorrelation in the residual. Table 6 reports tests of autocorrelation for the standardized residuals from various models. For models with the i.i.d. assumption, we reject the null hypothesis that the standardized residuals are white noise at a 5% significance level. For models with Markov-switching variance but without volatility feedback, even though the p-values are in general larger than in cases with the i.i.d. assumption we reject the null hypothesis. However, for Model 3-B with structural break in the variance process and volatility feedback, our most preferred model, we cannot reject the null hypothesis of no autocorrelation at a 5% significance level.

5. Summary and Conclusion

Pastor and Stambaugh (1998) develop and apply a Bayesian framework for estimating the equity premium in the presence of structural breaks in long historical time series data. However, like Merton (1980), they consider a positive relation between risk and return as a reasonable prior belief, rather than a regularity that should be verified empirically.

They also ignore equity premium changes with higher-frequency fluctuations in volatility by assuming i.i.d. returns within each subsample separated by structural break points. In this paper, we extend Pastor and Stambaugh's (1998) work in two important ways: First, within our extended Bayesian framework, a relation between risk and return is a regularity that can be verified empirically. Second, we allow for the possibility that the equity premium may change in response to recurrent changes in the level of volatility within subsamples separated by a structural break. The question of whether there is a structural break in the behavior of stock returns is directly relevant to the appropriate use of long historical data in empirical finance. For example, it has been noted in numerous studies, including Fama and French (1988), Poterba and Summers (1988), M. Kim, Nelson, and Startz (1991), that the reported evidence of mean reversion in stock prices depends in large part upon the inclusion of pre-WWII data in estimation.

Studying the period 1926:1 - 1991:12 we find strong evidence for a structural break in the variance process for returns around 1941. However, we find little evidence of a concurrent structural break in the parameters related to the equity premium. Instead, changes in the equity premium are shown to be a consequence of recurrent changes in the level of volatility. These findings suggest that, while it is important to account for heteroscedasticity in estimating the equity premium, it would also be reasonable to include the pre-WWII data. In fact, using the data that includes pre-WWII observations, M. Kim, Nelson, and Startz (1991), McQueen (1992), Kim, Nelson, and Startz (1998), and Kim and Nelson (1998) find much weaker evidence of mean reversion when accounting for heteroscedasticity in estimation, and Kim, Morley, and Nelson (1999) show that evidence of mean reversion may be a consequence of the volatility feedback effect.

Other relevant findings in this paper are strong evidence of a Markov-switching variance process in returns, strong evidence of a negative volatility feedback effect, which implies a positive tradeoff between volatility and the equity premium, and an ability of the model with volatility feedback and a structural break in the variance process to capture the negative skewness and excess kurtosis observed in the historical data.

Appendix 1: Bayesian Inference of Model 3-C via Gibbs Sampling

For Bayesian inference of the model, given appropriate priors we need the marginal posterior distributions for the followings: $\tilde{\sigma^2} = [\,\sigma^2_{0,0} \quad \sigma^2_{0,1} \quad \sigma^2_{1,0} \quad \sigma^2_{1,1}\,]'$; $\tilde{p} = [\,p_{00} \quad p_{11}\,]'$; q_{00}; $\tilde{\mu} = [\,\mu_0 \quad \mu_1 \quad \gamma_0 \quad \gamma_1 \quad \delta_0 \quad \delta_1\,]'$; $\tilde{D}_T = [\,D_1 \quad \ldots \quad D_T\,]'$; $\tilde{S}_T = [\,S_1 \quad \ldots \quad S_T\,]'$; $\tilde{S}^{\dagger}_T = [\,S^{\dagger}_1 \quad \ldots \quad S^{\dagger}_T\,]'$, where $S^{\dagger}_t = E[S_t|\tilde{Y}_{t-1}]$ and where $\tilde{Y}_t = [\,y_1 \quad \ldots \quad y_t\,]'$.

These marginal posterior distributions may be obtained from the joint posterior distribution,

$$p(\tilde{\mu}, \tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{S}^{\dagger}_T, \tilde{p}, q_{00}|\tilde{Y}_T). \tag{A.1}$$

However, the hierarchical nature of the model allows us to easily employ Gibbs sampling in obtaining the marginal posterior distributions of interest. This is done by successively sampling from the full conditional densities. The following describes the Gibbs sampling procedure:

i) Generate $\tilde{S}_T$ and $\tilde{S}^{\dagger}_T$ from $p(\tilde{S}_T, \tilde{S}^{\dagger}_T|\tilde{\mu}, \tilde{\sigma}^2, \tilde{D}_T, \tilde{p}, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$, $\tilde{S}_T$ is independent of q_{00};

ii) Generate $\tilde{D}_T$ from $p(\tilde{D}_T|\tilde{\mu}, \tilde{\sigma}^2, \tilde{S}_T, \tilde{S}^{\dagger}_T, q_{00}, \tilde{Y}_T)$, where, conditional on $\tilde{S}_T$ and $\tilde{S}^{\dagger}_T$, $\tilde{D}_T$ is independent of $\tilde{p}$;

iii) Generate $\tilde{p}$ from $p(\tilde{p}|\tilde{S}_T)$, where, conditional on $\tilde{S}_T$, $\tilde{p}$ is independent of the other variates;

iv) Generate q_{00} from $p(q_{00}|\tilde{D}_T)$, where, conditional on $\tilde{D}_T$, q_{00} is independent of the other variates.

v) Generate $\tilde{\mu}$ from $p(\tilde{\mu}|\tilde{\sigma}^2, \tilde{D}_T, \tilde{S}_T, \tilde{S}^{\dagger}_T, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$, $\tilde{S}_T$, and $\tilde{S}^{\dagger}_T$, $\tilde{\mu}$ is independent of $\tilde{p}$ and q_{00};

vi) Generate $\tilde{\sigma}^2$ from $p(\tilde{\sigma}^2|\tilde{\mu}, \tilde{D}_T, \tilde{S}_T, \tilde{S}^{\dagger}_T, \tilde{Y}_T)$, where, conditional on $\tilde{D}_T$, $\tilde{S}_T$, and $\tilde{S}^{\dagger}_T$, $\tilde{\sigma^2}$ is independent of $\tilde{p}$ and q_{00};

The above procedure is a straightforward extension of Albert and Chib's (1993) Bayes inference via Gibbs sampling of autoregressive time series subject to Markov mean and variance shifts. Kim and Nelson (1998) extended Chib's (1993) procedure to incorporate a one-time permanent structural break in the parameters of the Markov-switching. Notice that as a byproduct of generating $\tilde{D}_T$ in ii), we can get the marginal posterior distribution of the break point, τ, such that $D_1 = \ldots = D_\tau = 0$ and $D_{\tau+1} = \ldots = D_T = 0$.

Prior distributions employed are described as follows:

Parameters Before Structural Break ($D_t = 0$)

$$[\,\mu_0 \quad \gamma_0 \quad \delta_0\,]' \sim N(0, 0.2^2\sigma^2_{S_t,D_t} I_3);$$

$$\frac{1}{\sigma_0^{*2}} \sim \Gamma(\frac{1}{2}, \frac{1}{2});$$

$$\frac{1}{\sigma^2_{0,0}} \sim \Gamma(\frac{1}{2}, \frac{2}{2}); \quad \frac{1}{\sigma^2_{0,1}} \sim \Gamma(\frac{1}{2}, \frac{1}{2});$$

$$p_{00,0} \sim Beta(4, 1); \quad p_{11,1} \sim Beta(4, 1);$$

Parameters After Structural Break ($S_t = 1$)

$$[\,\mu_1 \quad \gamma_1 \quad \delta_1\,]' \sim N(0, 0.2^2\sigma^2_{S_t,D_t} I_3);$$

$$\frac{1}{\sigma_1^{*2}} \sim \Gamma(\frac{1}{2}, \frac{2}{2});$$

$$\frac{1}{\sigma^2_{1,0}} \sim \Gamma(\frac{1}{2}, \frac{4}{2}); \quad \frac{1}{\sigma^2_{1,1}} \sim \Gamma(\frac{1}{2}, \frac{2}{2});$$

$$p_{11,0} \sim Beta(4, 1); \quad p_{11,1} \sim Beta(4, 1);$$

Parameter for Structural Break

$$q_{00} \sim Beta(20, 0.1),$$

where $\Gamma(\frac{a}{2}, \frac{b}{2})$ refers to the Gamma distribution and $Beta(\alpha, \beta)$ refers to the Beta distribution. Table 3 describes the moments of these priors. All the inferences in the next section are based on 10,000 out of 11,000 Gibbs simulations, after discarding the first 1,000 simulations. Sensitivity analysis has been performed, but the qualitative results were robust with respect to a wide range of moments employed for these priors.

Appendix 2. Calculating the Marginal Likelihood

In this section, we present a procedure for directly calculating the marginal likelihoods for models under our consideration, by extending Chib's (1995) as applied by Kim and Nelson (1999). The procedure is described within the context of Model 3-C, a general model.

Define $\tilde{\theta} = [\,\tilde{\mu}' \quad \tilde{p}' \quad q_{00} \quad \tilde{\sigma^2}'\,]'$ to be a vector of the parameters of the model. Then, as in Chib (1995) the marginal density of $\tilde{Y}_T = [\,y_1 \quad \ldots \quad y_T\,]'$, by virtue of being the normalizing constant of the posterior density, can be written as:

$$m(\tilde{Y}_T) = \frac{f(\tilde{Y}_T|\tilde{\theta})\pi(\tilde{\theta})}{\pi(\tilde{\theta}|\tilde{Y}_T)}, \qquad (A.2)$$

where the numerator is the product of the sampling density and the prior, with all integrating constants included, and the denominator is the posterior density of $\tilde{\theta}$. As the above identity holds for any $\tilde{\theta}$, we may evaluate $m(\tilde{Y}_T)$ at the posterior mean $\tilde{\theta}^*$. Taking the logarithm of the above equation for computational convenience, we have:

$$ln\ m(\tilde{Y}_T) = ln\ f(\tilde{Y}_T|\tilde{\theta}^*) + ln\ \pi(\tilde{\theta}^*) - ln\ \pi(\tilde{\theta}^*|\tilde{Y}_T) \qquad (A.3)$$

The log likelihood function and the log of the prior density at $\tilde{\theta} = \tilde{\theta}^*$ can be evaluated relatively easily. First, the log likelihood function is given by:

$$ln\ f(\tilde{Y}_T|\tilde{\theta}^*) = \sum_{t=1}^{T} ln(\sum_{S_t=0}^{1}\sum_{D_t=0}^{1} p(S_t, D_t|\tilde{Y}_{t-1}, \tilde{\theta}^*) f(y_t|\tilde{Y}_{t-1}, S_t, D_t, \tilde{\theta}^*)), \qquad (A.4)$$

Second, the log of prior density is given by:

$$ln\ \pi(\tilde{\theta}^*) = ln\ \pi(\tilde{\mu}^*) + ln\ \pi(\tilde{\sigma^2}^*) + ln\ \pi(\tilde{p}^*, q_{00}^*), \tag{A.5}$$

where it is a priori assumed that $\tilde{\mu}$, $\tilde{\phi}$, $\tilde{\sigma^2}$, $\tilde{p}$, and q_{00} are independent of one another.

For an evaluation of the posterior density at $\tilde{\theta} = \tilde{\theta}^*$ we consider the following decomposition of the posterior density:

$$\pi(\tilde{\theta}^*|\tilde{Y}_T) = \pi(\tilde{\mu}^*|\tilde{Y}_T)\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{Y}_T)\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T), \tag{A.6}$$

where

$$\begin{aligned}&\pi(\tilde{\mu}^*|\tilde{Y}_T)\\ &= \int \pi(\tilde{\mu}^*, |\tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{p}, q_{00}, \tilde{Y}_T)\pi(\tilde{\sigma^2}, \tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{p}, q_{00}|\tilde{Y}_T)d\tilde{\sigma^2}d\tilde{D}_T d\tilde{S}_T d\tilde{S}_T^{\dagger} d\tilde{p} dq_{00},\end{aligned} \tag{A.7}$$

$$\begin{aligned}&\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{Y}_T)\\ &= \int \pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{p}, q_{00}, \tilde{Y}_T)\pi(\tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}|\tilde{\mu}^*, \tilde{p}, q_{00}, \tilde{Y}_T)d\tilde{D}_T d\tilde{S}_T d\tilde{S}_T^{\dagger} d\tilde{p} dq_{00},\end{aligned} \tag{A.8}$$

and

$$\begin{aligned}&\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)\\ &\qquad = \int \pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^* \tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{Y}_T)\pi(\tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)d\tilde{D}_T d\tilde{S}_T d\tilde{S}_T^{\dagger}\end{aligned} \tag{A.9}$$

The above decomposition of the posterior density suggests that $\pi(\tilde{\mu}^*|\tilde{Y}_T)$ can be calculated based on draws from the full Gibbs run, and $\pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{Y}_T)$, and $\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)$ can be calculated based on draws from the reduced Gibbs runs. The following explains how each of these can be calculated based on output from appropriate Gibbs runs:

$$\hat{\pi}(\tilde{\mu}^*|\tilde{Y}_T) = \frac{1}{G}\sum_{g=1}^{G} \pi(\tilde{\mu}^*, |\tilde{\sigma^2}^g, \tilde{D}_T^{g_1}, \tilde{S}_T^{g_1}, \tilde{S}_T^{\dagger g_1}, \tilde{p}^{g_1}, q_{00}^{g_1}, \tilde{Y}_T), \tag{A.10}$$

$$\hat{\pi}(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{Y}_T) = \frac{1}{G}\sum_{g_2=1}^{G} \pi(\tilde{\sigma^2}^*|\tilde{\mu}^*, \tilde{D}_T^{g_2}, \tilde{S}_T^{g_2}, \tilde{S}_T^{\dagger g_2}, \tilde{p}^{g_2}, q_{00}^{g_2}, \tilde{Y}_T), \tag{A.11}$$

$$\hat{\pi}(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)$$
$$= \frac{1}{G}\sum_{g_3=1}^{G} \pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{D}_T^{g_3}, \tilde{S}_T^{g_3}, \tilde{S}_T^{\dagger g_3}, \tilde{Y}_T), \qquad (A.12)$$

where the superscript g refers to the $g-th$ draw of the full Gibbs run and the superscript g_i, $i = 1, 2, 3$, refers to the $g_i - th$ draw from the appropriate reduced Gibbs runs. Thus, apart from the usual G iterations for the full Gibbs run, we need additional $2 \times G$ iterations for the appropriate reduced Gibbs run. In order to calculate $\pi(\tilde{p}^*, q_{00}^*|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{Y}_T)$, for example, we need output from an additional G iterations for the following reduced Gibbs run: First, we generate $\tilde{p}$ and q_{00} from $p(\tilde{p}, q_{00}|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{D}_T, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{Y}_T)$; Second, we generate $\tilde{D}_T$ from $p(\tilde{D}_T|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{S}_T, \tilde{S}_T^{\dagger}, \tilde{p}, q_{00}, \tilde{Y}_T)$; Third, we generate $\tilde{S}_T$ and $\tilde{S}_T^{\dagger}$ from $p(\tilde{S}_T, \tilde{S}_T^{\dagger}|\tilde{\mu}^*, \tilde{\sigma^2}^*, \tilde{D}_T, \tilde{p}, q_{00}, \tilde{Y}_T)$. Notice that throughout the reduced Gibbs run, $\tilde{\mu}$, and $\tilde{\sigma^2}$ are not generated and they are set equal to their posterior means $\tilde{\mu}^*$ and $\tilde{\sigma^2}^*$, respectively.

References

[1] Bekaert, Geert and Guojun Wu, 1999, "Asymmetric Volatility and Risk in Equity Markets," *Review of Financial Studies,* forthcoming.

[2] Balke, Nathan S. and Mark E. Wohar, "Why Are Stock Prices So High? Dividend Growth or Discount Factors?" Working paper, Southern Methodist University and University of Nebraska at Omaha.

[3] Black, Fischer, 1976, "Studies of Stock Price Volatility Changes," Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economical Statistics Section, 177-181.

[4] Campbell, John Y., 1987, "Stock returns and the term structure," *Journal of Financial Economics,* 18, 373-399.

[5] Campbell, John Y. and Ludger Hentschel, 1992, "No News Is Good News: An Asymmetric Model of Changing Volatility in Stock Returns," *Journal of Financial Economics,* 31, 281-318.

[6] Campbell, John Y. and Robert Shiller, 1988a, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies,* 1, 195-227.

[7] Campbell, John Y. and Robert Shiller, 1988b, "Stock prices, Earnings, and Expected Dividends," *Journal of Finance,* 43, 661-676.

[8] Carlin, B. P., and Chib, S, 1995, "Bayesian Model Choice via Markov Chain Monte Carlo Methods," *Journal of the Royal Statistical Society,* Series B, 57, No. 3, 473-484.

[9] Carlin, B. P., and Polson, N. G., 1991, "Inference for Nonconjugate Bayesian Models Using the Gibbs Sampler," *Canadian Journal of Statistics,* 19, 399-405.

[10] Cecchetti, Stephen G., Pok-Sang Lam, and Nelson C. Mark, 1990, "Mean Reversion in Equilibrium Asset Prices," *American Economic Review,* 80 (3), 398-418.

[11] Chib, Siddhartha, 1998, "Estimation and Comparison of Multiple Change-point Models," *Journal of Econometrics,* 86, 221-241.

[12] Christie, Andrew A., 1982, "The Stochastic Behavior of Common Stock Variances - Value, Leverage and Interest rate effects," *Journal of Financial Economics,* 10, 407-

432.

[13] Cox, John C. and Stephen A. Ross, 1976, "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics,* 3, 145-166.

[14] French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, "Expected stock returns and volatility," *Journal of Financial Economics* 19, 3-29.

[15] Garcia, Rene, 1998, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models," *International Economic Review*, Vol. 39, No. 3, 763-788.

[16] Geweke, John, 1996, "Variable Selection and Model Comparison in Regression," *Bayesian Statistics 5,* ed. J.O. Berger, J. M. Bernardo, A. P. Dawid, and A. F. M. Smith), Oxford: Oxford University Press.

[17] George, E. I. and McCulloch, R. E., 1993, "Variable Selection via Gibbs Sampling," *Journal of American Statistical Association,* 88, 881-889.

[18] Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle, 1993, "On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *Journal of Finance,* 48, 1779-1801.

[19] Hamilton, James D., 1989, "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica,* 57, 357-384.

[20] Hansen, B. E., 1992, "The Likelihood Ratio Test Under Nonstandard Conditions: Testing the Markov Switching Model of GNP," *Journal of Applied Econometrics,* Vol. 7, S61-S82.

[21] Harvey, Campbell R., 1989, "Time-varying conditional covariances in tests of asset pricing models," *Journal of Financial Economics,* 24, 289-317.

[22] Heaton, John and Deborah Lucas, 1999, "Stock Prices and Fundamentals," Working paper, Northwestern University.

[23] Jeffreys, H., 1961, Theory of Probability, 3rd ed., Oxford: Clarendon Press.

[24] Kass, Robert E. and Raftery, Adrian E., 1995, "Bayes Factors," *Journal of the American Statistical Association,* Vol. 90, No. 430, 773-795.

[25] Kim, Chang-Jin, James Morley, and Charles R. Nelson, 1999, "Does an Intertemporal Tradeoff between Risk and Return Explain Mean Reversion in Stock Prices?" Working paper, University of Washington and Washington University.

[26] Kim, Chang-Jin, James Morley, and Charles R. Nelson, 2000, "Is There a Intertemporal Tradeoff Between Stock Market Risk and Return?," Working paper, University of Washington and Washington University.

[27] Kim, Chang-Jin and Charles R. Nelson, 1999a, "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *Review of Economics and Statistics,* 81(4): 608-616.

[28] Kim, Chang-Jin and Charles R. Nelson, 1999b, *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications,* MIT Press: Cambridge.

[29] Kim, Chang-Jin and Charles R. Nelson, 1998, "Testing for Mean Reversion in Heteroskedatic Data II: Autoregression Tests Based on Gibbs-Sampling-Augmented Randomization," *Journal of Empirical Finance,* 5, 385-396.

[30] Kim, Chang-Jin and Charles R. Nelson, 2000, "A Bayesian Approach to Testing for Markov Switching in Univariate and Dynamic Factor Models," Forthcoming, *International Economic Review.*

[31] Kim, Chang-Jin, Charles R. Nelson, and Richard Startz, 1998, "Testing for Mean Reversion in Heteroskedastic Data Based on Gibbs-Sampling-Augmented Randomization," *Journal of Empirical Finance,* 5, 131-154.

[32] Kim, Myung-Jig, Charles R. Nelson, and Richard Startz, 1991, "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," *Review of Economic Studies,* 48, 515-528.

[33] Koop, Gary and Simon M. Potter, 1999, "Bayes Factors and Nonlinearity: Evidence from Economic Time Series," *Journal of Econometrics*, 88, 251-281.

[34] Lucas, Robert E. Jr., 1978, "Asset Prices in an Exchange Economy," *Econometrica,* 66, 1429-45.

[35] McQueen, G., 1992, "Long-Horizon Mean-Reverting Stock Prices Revisited," *Journal of Financial and Quantitative Analysis,* 27, 1-18.

[36] Merton, Robert C., 1980, "On Estimating the Expected Return on the Market: An Explanatory Investigation," *Journal of Financial Economics,* 8, 323-361.

[37] Pastor, Lubos and Robert F. Stambaugh, 1998, "The Equity Premium and Structural

Breaks," Working paper, The Wharton School, University of Pennsylvania.

[38] Pindyck, Robert S., 1984, "Risk, Inflation, and the Stock Market," *American Economic Review,* 45, 267-290.

[39] Poirier, Dale J., 1995, *Intermediate Statistics and Econometrics,* Cambridge: The MIT Press.

[40] Scruggs, John T., 1998, "Resolving the Puzzling Intertemporal Relation Between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach," *Journal of Finance,* 53, 575-603.

[41] Turner, Christopher M., Richard Startz, and Charles R. Nelson, 1989, "A Markov model of heteroscedasticity, risk, and learning in the stock market," *Journal of Financial Economics,* 25, 3-22.

[42] Verdinelli, I. and L. Wasserman, 1995, "Computing Bayes Factors Using a Generalization of the Savage-Dickey Density Ratio," *Journal of the American Statistical Association*, 90, 614-618.

[43] Viceira, Luis M., "Testing for Structural Change in the Predictability of Asset Returns," Working paper, Dept. of Economics, Harvard University.

[44] Whitelaw, Robert F., 1994, "Time variations and covariations in the expectation and volatility of stock market returns," *Journal of Finance,* 49, 515-541.

Table 1. Descriptive Statistics: Historical Returns

Sample Period	Mean	Standard Deviation
1926:1 - 1991:12	0.0590	0.6764
1926:1 - 1998:12	0.0647	0.6557
1926:1 - 1940:12	0.0165	1.0780
1941:1 - 1991:12	0.0715	0.5012
1992:1 - 1998:12	0.1184	0.4112

Table 2. Log Marginal Likelihoods for Various Models under Consideration

Assumptions	**Cases Considered**		$ln[m(\tilde{Y}_T)]$
homoscedasticity within subsamples; no volatility feedback	Model 1-A:	no structural break	-812.87
	Model 1-B:	structural break in variance	-713.35
	Model 1-C:	structural break in mean and variance	-713.75
Markov-switching variance within subsamples; no volatility feedback	Model 2-A:	no structural break	-672.79
	Model 2-B:	structural break in variance	-666.14
	Model 2-C:	structural break in mean and variance	-667.06
Markov-switching variance within subsamples; with volatility feedback	Model 3-A:	no structural break	-671.10
	Model 3-B:	structural break in variance	-661.86
	Model 3-C:	structural break in mean and variance	-664.89

Table 3. Posterior Moments from Models 3-B and 3-C: Markov-Switching Variance with Volatility Feedback Effect [1926:1 - 1991:12]

Parameters	**Model 3-B**		**Model 3-C**	
	Mean	SD	Mean	SD
μ_0	0.1045	0.0320	0.1473	0.0830
μ_1	–	–	0.0675	0.0479
γ_0	-0.0719	0.0977	-0.0830	0.1019
γ_1	–	–	0.0563	0.1775
δ_0	-0.3204	0.1114	-0.1945	0.1127
δ_1	–	–	-0.3930	0.1719
$\sigma^2_{0,0}$	0.2944	0.1028	0.2651	0.0529
$\sigma^2_{0,1}$	0.1738	0.0190	0.1703	0.0189
$\sigma^2_{1,0}$	2.2554	0.4845	2.2635	0.4740
$\sigma^2_{1,1}$	0.6582	0.2224	0.6077	0.1867
$p_{00,0}$	0.9469	0.0319	0.9519	0.0275
$p_{00,1}$	0.9516	0.0288	0.9488	0.0269
$p_{11,0}$	0.9332	0.0427	0.9265	0.0422
$p_{11,1}$	0.7987	0.0990	0.7968	0.0976
q_{00}	0.9939	0.0074	0.9952	0.0048
$ln[m(\tilde{Y}_T)]$	-661.86		-664.89	
Bayes Factor in favor of Model 3-B over Model 3-C: 20.70				

1. SD refers to standard deviation;

Table 4. The Nature of Structural Break in the Variance Process: Posterior Moments from Model 3-B

	Moments		90% Posterior Bands	
	Mean	SD	Lower Bound	Upper Bound
	Expected Duration of Low Variance State (Months)			
Before break	27.15	20.08	9.44	64.13
After break	28.85	20.08	10.10	67.13
	Expected Duration of High Variance State (Months)			
Before break	24.26	23.51	6.61	71.02
After break	6.46	4.79	2.56	13.78
	Persistence of Volatility (λ_1)			
Before break	0.8799	0.0603	0.7719	0.9553
After break	0.7458	0.1125	0.5370	0.8999

Table 5. Kurtosis and Skewness for Standardized Residuals from Various Models

Model	Skewness	Kurtosis	p-value (Normality Test)
Model 1-A	-0.4425	9.8741	0.0000
Model 1-B	-0.5087	5.4514	0.0000
Model 1-C	-0.5091	5.4725	0.0000
Model 2-A	-0.1940	3.0005	0.0836
Model 2-B	-0.2885	3.2246	0.0018
Model 2-C	-0.1642	2.6889	0.0342
Model 3-A	-0.0611	3.0422	0.7588
Model 3-B	-0.0579	2.8100	0.4417
Model 3-C	-0.0566	2.7847	0.3767

Table 6. Tests of Autoocorrelations for Standardized Residuals from Various Models (P-Values)

Model	Lag 1	Lag 3	Lag 5	Lag 12	Lag 24	Lag 36
Model 1-A	0.004	0.000	0.000	0.002	0.000	0.000
Model 1-B	0.012	0.059	0.009	0.146	0.035	0.171
Model 1-C	0.012	0.061	0.010	0.149	0.033	0.166
Model 2-A	0.079	0.286	0.019	0.180	0.064	0.216
Model 2-B	0.088	0.393	0.050	0.323	0.091	0.244
Model 2-C	0.142	0.467	0.046	0.278	0.066	0.235
Model 3-A	0.179	0.433	0.037	0.233	0.071	0.162
Model 3-B	0.289	0.670	0.075	0.330	0.076	0.237
Model 3-C	0.338	0.706	0.092	0.369	0.083	0.248

Figure 1. Excess Returns and Posterior Probability of the Break Point from Model 3-B