Permanent and Transitory Components of Recessions

Chang-Jin Kim
Korea University

and

Christian J. Murray
University of Houston

May 3, 1999

Abstract

We propose a generalization of existing business cycle models which allows us to decompose recessions into permanent and transitory components. We find that the transitory component of recessions accounts for between 77% and 96% of the observed variance of monthly indicator series. Our results suggest the following three phase characterization of the business cycle: recession, high-growth recovery during which output *partially* reverts to its previous peak, and normal growth following the recovery. In addition, we find significant timing differences between the permanent and transitory components of recessions; most notably the lack of the usual high-growth recovery phase following the 1990-91 recession.

JEL Codes: C32, E32

Kim: Department of Economics, Korea University, Seoul, 136-701, Korea (cjkim@kuccnx.korea.ac.kr); **Murray:** (Corresponding author) Department of Economics, University of Houston, Houston, TX 77204-5882 (cjmurray@uh.edu), Tel: 713-743-3835, Fax: 713-743-3798

1. Introduction

The importance of the comovement of economic time series and business cycle asymmetry was recognized by early scholars of the business cycle. In their landmark study, Burns and Mitchell (1946) highlighted comovement as one of the two empirical regularities of the business cycle:

> "...a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle."

The other regularity of the business cycle, asymmetry, is the idea that expansions are fundamentally different than recessions. This goes back at least as far as Mitchell (1927):

> ".... the most violent declines exceed the most considerable advances. The abrupt declines usually occur in crises; the greatest gains occur in periods of revival... Business contraction seems to be a briefer and more violent process than business expansion."

Recently, researchers have used the tools of modern time series analysis to explicitly model comovement and asymmetry. Stock and Watson (1989, 1991, 1993) estimate a linear dynamic factor model which captures the comovement across economic time series through an unobserved permanent component common to each series. Hamilton (1989) incorporates business cycle asymmetry in a univariate nonlinear model which allows the growth rate of output to be dependent on the 'state' of the economy. The results from his regime-switching model suggest that the business cycle is characterized by two states: positive growth (expansion) or negative growth (recession).

While comovement and asymmetry have traditionally been analyzed in isolation, in a recent paper, Diebold and Rudebusch (1996) provide empirical and theoretical support for comovement and asymmetry as important features of the business cycle and suggest that they should be analyzed simultaneously. Accordingly, M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a) estimate a unified model in which the common growth component in Stock and Watson's (1989, 1991, 1993) dynamic factor model is subject to the type of regime switching advocated by Hamilton (1989).

Meanwhile, recent literature has provided ample evidence supporting the notion that recessions are transitory in nature, *i.e.* they only temporarily lower the level of output. Within a univariate framework, Beaudry and Koop (1993), Sichel (1994), and Kim and Nelson (1999a) provide evidence of 'peak-reverting' behavior in real output; a tendency for output to revert to its previous peak following a recession. A direct implication of 'peak-reversion' is that shocks during recessions are transitory. In light of this finding, Sichel (1994) proposes a three phase characterization of the business cycle: recession, high-growth recovery during which output reverts to its previous peak, and moderate growth following the recovery.

The literature also provides evidence that all recessions are not alike. Sichel (1994) and Boldin (1994) are among those who suggest that the 1990-91 recession was unique. In particular, Sichel notes the lack of a high-growth recovery phase following the 1990-91 recession.

The regime-switching dynamic factor models estimated by M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a) are unable to capture peak-reversion in output, since they restrict attention to a two phase business cycle, as in Hamilton's (1989) univariate model. In addition, recessions only arise from one source, a switch in the common growth component.

In this paper, we present a more general regime-switching dynamic factor model of the business cycle which allows for peak-reversion, as well as the possibility that recessions arise from more than one source. Our results suggest that peak-reversion is important in explaining business cycle dynamics within a multivariate framework. Specifically, the transitory component of recessions accounts for between 77% and 96% of the observed variance of monthly indicator series. This suggests that following a recession, there is a high-growth recovery phase during which monthly indicator series *partially* revert to their previous peaks. In addition, we find significant timing differences between the permanent and transitory components of recessions; most notably the lack of the usual high-growth recovery phase following the 1990-91 recession.

This paper is organized as follows. Section 2 provides a review of comovement and asymmetry in the empirical business cycle literature. Section 3 presents a generalization of previous regime-switching dynamic factor models, which allows for a common peak-reverting component that switches independently of the common growth component. Section 4 presents our empirical results. Finally, Section 5 summarizes and offers concluding remarks.

2. Business Cycle Asymmetry and Comovement in the Empirical Literature

2.1 Asymmetry within a Univariate Framework: Asymmetry in Growth Rates vs. Peak-Reversion in Levels

Since the seminal paper by Neftçi (1984) on the first formal statistical test of asymmetry in economic time series, the literature has modeled business cycle asymmetry in at least two ways:[1] i) asymmetry in the growth of real output and ii) asymmetry in the transitory component of real output. Hamilton (1989) is an example of the former while Beaudry and Koop (1993), Sichel (1994), and Kim and Nelson (1999a) are examples of the latter.

Hamilton (1989) models business cycle asymmetry by allowing the growth rate of real output to be governed by an unobserved Markov switching state variable. His results characterize the economy as being in one of two states: positive growth (expansion) or negative growth (recession).

While the two-state model of Hamilton (1989) has been successful at identifying the NBER business cycle dates, it is unable to capture the peak-reverting behavior of real output following a recession, or asymmetry in the persistence of shocks, as reported in the more recent literature. For example, Beaudry and Koop (1993) and Sichel (1994), using data on postwar U.S. real GDP, show that a variable measuring the current depth of a recession contains information useful for predicting the subsequent growth of real GDP, suggesting the existence of a third, high-growth recovery phase. Furthermore, Beaudry and Koop (1993) report that innovations during recessions are much less persistent than those during booms, suggesting asymmetry in the persistence of shocks between booms and recessions. A direct implication of this peak-reverting behavior is that declines in economic activity contain an important transitory component.[2] At first glance, extending Hamilton's (1989) two-state model the business cycle into a three-state Markov-switching model may seem fruitful in capturing a third, high-growth recovery phase. However,

[1] Diebold and Rudebusch (1990), Diebold, Rudebusch, and Sichel (1993), Durland and McCurdy (1994), and Kim and Nelson (1998a) also discuss asymmetry in the duration dependence of booms and recessions. However, we do not explicitly deal with this issue in the current paper.

[2] Such empirical evidence is be consistent with Friedman's (1964, 1993) 'plucking' model of the business cycle, in which output cannot exceed a ceiling level, but will sometimes be plucked downward by a recession. DeLong and Summer's (1988) 'output-gaps' view of the business cycle would also predict such behavior.

Sichel (1994) reports that the three-phase Markov model is not especially informative about the particular pattern of the three phases in his sample.

In an effort to capture peak-reverting behavior and asymmetry in the persistence of shocks, Kim and Nelson (1999a) propose a model of the business cycle in which they allow for asymmetric behavior in the transitory component of real output. They allow the transitory component of output to be 'plucked' down during a recession. Their results suggest that during expansions output fluctuations are mainly permanent, and that during recessions they are mainly transitory. This is in line with Friedman's (1964, 1993) 'plucking' model of economic fluctuations. Evidence in favor of Friedman's plucking model, or asymmetry in the transitory component of output, has also been reported by Wynne and Balke (1992), and Goodwin and Sweeney (1993).

2.2 Comovement within a Linear Multivariate Framework

The comovement of economic time series over the business cycle has been extensively exploited in the construction of composite indexes of coincident and leading economic indicators. These indexes, initially developed by Mitchell and Burns (1938), have played an important role in summarizing and forecasting aggregate macroeconomic performance. However, only recently has the comovement of economic time series been investigated, by Stock and Watson (1989, 1991, 1993), within the context of explicit probability models.

The essence of the linear dynamic factor model proposed by Stock and Watson is that the comovement across economic time series can be captured by a single unobserved factor common to all the series. Utilizing the Kalman filter, Stock and Watson extract an estimate of the common component, which is then interpreted as a new experimental composite index of economic activity. By employing the four monthly coincident indicator series used to construct the Department of Commerce (DOC) composite index, they show that the new experimental index implied by the model corresponds closely to the DOC index.

Indeed, Stock and Watson's probability model has provided a unified statistical framework for analyzing comovement across economic time series. Gregory et al.'s (1997) measure of world business cycle, for example, is one of the interesting recent applications of Stock and Watson's linear dynamic factor model.

2.3 A Synthesis: Asymmetry in the Common Growth Component within a Multivariate Framework

Filardo (1994) and Diebold and Rudebusch (1996) note that when Hamilton's (1989) Markov-switching model is applied to monthly coincident variables, the correlation between inferences on the state of the business cycle and the NBER reference cycle is much weaker than originally documented by Hamilton (1989) for quarterly real GNP. One potential reason for this failure is that monthly data are noisier than quarterly data, as outliers in monthly data are averaged out in quarterly data. Making inferences on the state of the economy from noisier monthly data would be more difficult. However, employing additional information has helped alleviate this problem. Filardo (1994), for example, exploits the time varying nature of the transition probabilities as functions of leading indicators within a univariate framework. Alternatively, Diebold and Rudebusch (1996) suggest taking advantage of the 'comovement' feature of economic time series over the business cycle, and thus, propose a regime-switching dynamic factor model which embodies the two defining features of the business cycle established by Burns and Mitchell (1946): business cycle asymmetry and comovement.[3]

In order to combine these two features of the business cycle, Diebold and Rudebusch (1996) propose a dynamic factor model in which the common growth component of Stock and Watson (1989, 1991, 1993) is subject to a regime switching Markov state variable as in Hamilton (1989). Accordingly, M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a) estimate a dynamic factor model with regime switching. All three papers construct experimental indexes of coincident indicators which encompass both comovement across economic time series and asymmetry.

3. A Generalization: Asymmetry in the Common Growth Component and Peak-Reversion with a Multivariate Framework

3.1. Model Specification

A potential drawback to the regime-switching dynamic factor model proposed by Diebold and Rudebusch (1996) and estimated by M.-J. Kim and Yoo (1995), Chauvet

[3] Kim and Nelson (1998b), in their Bayesian tests of Markov switching in the business cycle, also argue that the evidence of Markov-switching, or asymmetry, is much more compelling within a multivariate framework.

(1998), and Kim and Nelson (1998a), is that it is unable to capture the potential transitory nature of recessions discussed in Section 2.1. As in Hamilton's (1989) univariate model, since the growth rate of the common component is assumed to be regime-switching, their model lacks in a mechanism through which peak-reverting behavior may be incorporated. In addition, they only allow recessions to arise from one source, a switch in the common growth component.

Even though the literature discussed in Section 2.1, such as Wynne and Balke (1992), Beaudry and Koop (1993), Sichel (1994), and Kim and Nelson (1999a) provide copious evidence of the transitory nature of recessions, their results are entirely univariate. The purpose of this section is to provide a model with which one can analyze the potential transitory nature of recessions within a multivariate framework, and assess the relative importance of permanent and transitory shocks during recessions. This is done by generalizing previous regime-switching dynamic factor models to include a regime-switching common transitory (or peak-reverting) component, as well as a regime-switching common permanent component.

Each individual time series Y_{it} (in logs), for i=1,.,.N, consists of a deterministic time trend DT_{it}, a stochastic permanent component with a unit root P_{it}, and a transitory component T_{it}. We write each series as:

(3.1) $$Y_{it} = DT_{it} + P_{it} + T_{it}$$

(3.2) $$DT_{it} = a_i + D_i t$$

(3.3) $$P_{it} = g_i C_t + z_{it}$$

(3.4) $$T_{it} = l_i x_t + w_{it}$$

where C_t and x_t are the common permanent and common transitory components, respectively; z_{it} and w_{it} are the idiosyncratic permanent and transitory components, respectively. The g_i terms are permanent factor loadings, and indicate the extent to which each series is affected by the common permanent component, C_t. Similarly, the transitory factor loadings, l_i, indicate the extent to which each series is affected by the common transitory component, x_t.

Taking first differences, writing the model in deviations from means, and in the absence of cointegration, we have:

(3.5) $$Dy_{it} = g_i Dc_t + l_i Dx_t + z_{it}$$

where $Dy_{it} = DY_{it} - D\overline{Y}_i$, $Dc_t = DC_t - f(1)^{-1}d$, and $z_{it} = Dz_{it} + Dw_{it}$.

The common permanent component is subject to the type of regime-switching proposed by Hamilton (1989):

(3.6) $$f(L)Dc_t = m_{S_{1t}} + v_t \;, v_t \sim iid\ N(0,1)$$

(3.7) $$m_{S_{1t}} = m_0 + m_1 S_{1t} \;,\; S_{1t} = \{0,1\}$$

(3.8) $$\Pr[S_{1t} = 0 \mid S_{1,t-1} = 0] = q_1 ,\quad \Pr[S_{1t} = 1 \mid S_{1,t-1} = 1] = p_1 .$$

S_{1t} is a latent Markov-switching state variable that switches between 0 and 1 with transition probabilities given by equation (3.8). The common permanent component, c_t, grows at rate $f(1)^{-1}(m_0)$ when $S_{1t} = 0$, and at rate $f(1)^{-1}(m_0 + m_1)$ when $S_{1t} = 1$.

In order to capture peak-reversion, the common transitory component is subject to the type of regime switching advocated by Kim and Nelson (1999a):

(3.9) $$f^*(L)x_t = t_{S_{2t}} + u_t \;, u_t \sim iid\ N(0,1)$$

(3.10) $$t_{S_{2t}} = tS_{2t} \;, S_{2t} = \{0,1\}$$

(3.11) $$\Pr[S_{2t} = 0 \mid S_{2,t-1} = 0] = q_2 ,\quad \Pr[S_{2t} = 1 \mid S_{2,t-1} = 1] = p_2 .$$

S_{2t} is a latent Markov-switching state variable, independent of S_{1t}, whose transitions are governed by the probabilities in equation (3.11). The term, t, is the size of the 'pluck'. If $t < 0$, then the transitory component is plucked down during a recession. Following the pluck then there is a tendency for output to revert to its previous peak.

We assume that the idiosyncratic components have the following autoregressive structure:

(3.12) $$y_i(L)z_{it} = e_{it} \;, e_{it} \sim iid\ N(0, s_i^2) .$$

The innovation variances of the common components have been normalized to unity to identify the model; all innovations are assumed to be mutually and serially uncorrelated at all leads and lags; and the roots of $f(L) = 0$, $f^*(L) = 0$, and $y_i(L) = 0$ lie outside the unit circle.

Our model reduces to models previously estimated in the literature with the appropriate restrictions. With only one series (N=1), $t = 0$, and one root of $f^*(L) = 0$ on the unit circle, we have Hamilton's (1989) univariate model. With N=1, and $m_1 = 0$ we have Kim and Nelson's (1999a) univariate model. In the multivariate framework, when $l_i = 0$ and $m_1 = 0$, the linear dynamic factor model of Stock and Watson (1989, 1991, 1993) emerges. When $l_i = 0$, we have Diebold and Rudebusch's (1996) nonlinear dynamic factor model estimated by M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a).

Note that in our model, the common growth component, Dc_t, and the common transitory component, x_t, are governed by two different state variables, S_{1t} and S_{2t}. This allows a recession to arise from one of two sources; a switch in the common growth component, or a 'pluck' in the common cycle. In addition, the timing and duration of S_{1t} and S_{2t} is allowed to vary across recessions.

In estimating dynamic factor models, model identification is an important issue. Within a linear dynamic factor model, (*i.e.* no regime-switching) the two common factors Dc_t and x_t in equation (3.5) are not separately identified if the permanent and transitory factor loadings are equal $(g_i = l_i)$, as is the case with the idiosyncratic components z_{it} and w_{it}. Similarly, within the nonlinear dynamic factor model above, the two common components will not be identified if they are governed by the same state variable and have equal factor loadings (*i.e.* $S_{1t} = S_{2t}$ and $g_i = l_i$). However, the empirical literature suggests to us that these restrictions, in particular $S_{1t} = S_{2t}$, do not hold. Imposing them would require that all recessions have both permanent and transitory components, and would preclude the possibility that the 1990-91 recession was not followed by a high-growth recovery phase. Thus, our model allows us to assess the extent to which recessions differ in terms of the contributions of the permanent and transitory components.

3.2 Estimation of the Model

Since the state variables, S_{1t} and S_{2t}, are unobserved, our model is nonlinear, and calculation of the exact Gaussian likelihood function is not possible. To estimate the parameters, as well as the unobserved components, we cast our model into its state space representation and use Kim's (1994) approximate maximum likelihood estimation algorithm. Section 1 of the Appendix presents the state space representation of our model. Section 2 a presents a detailed description of the estimation algorithm. Section 3 demonstrates how we construct C_t from Dc_t.

4. Permanent and Transitory Components of Recessions: Empirical Results

4.1 Data

Our data consist of four monthly series on the index of industrial production (IP), personal income less transfer payments (GMYXPQ), manufacturing and trade sales (MTQ), and civilian labor force employed in nonagricultural industries (LHNAG).[4] The first three series are from the Department of Commerce (DOC) list of coincident indicators. Even though the DOC lists employees on nonagricultural payrolls (LPNAG) as a coincident indicator, Stock and Watson (1989, 1991, 1993) report the variable as somewhat lagging. In order to avoid the additional complexity that arises from including a lagging variable, we follow Chauvet (1998) in considering LHNAG as a replacement for LPNAG. Chauvet (1998) has shown that the LHNAG series is a coincident variable, unlike the LPNAG series. The four series are available monthly from 1959.01 through 1998.10. Personal income less transfer payments and manufacturing and trade sales are expressed in chained 1992 dollars, and the index of industrial production is equal to 100 between 1992.06 and 1992.07.

All four series appear to be individually integrated, but not cointegrated. Specifically, the Augmented Dickey-Fuller test cannot be rejected at the 10% level for any of the series.[5] Using Johansen's (1991) tests for cointegration, we are unable to reject the null hypothesis that there are no cointegrating vectors at the 10% level.

[4] DRI codes are in parentheses.
[5] We use the general to specific lag selection procedure studied by Hall (1994) and Ng and Perron (1995). Results are invariant to a maximum lag of 12 and 24.

4.2. Empirical Results

Concerning the dynamic specification of the common and idiosyncratic components in equations (3.6), (3.9), and (3.12) we consider 1st and 2nd order autoregressions to describe their dynamics. We consider four cases in which both of the common components are either an AR(1) or an AR(2), and all idiosyncratic components are either an AR(1) or an AR(2). Based on various diagnostic checks we settle on a parsimonious AR(1) specification for all components, as in Chauvet (1998). Empirical results are robust to alternative specifications. For all autoregressive structures considered, the implied factors, as well as filtered and smoothed probabilities, are virtually indistinguishable. The parameter estimates of the model and their standard errors are reported in Table 1.

If the factor loadings for the transitory component, l_i, $i = 1, 2, 3, 4$ are all zero, our model collapses to a dynamic factor model with a regime-switching common growth component. As we cannot test the joint hypothesis that these transitory factor loadings are all zero due to the non-standard nature of the problem,[6] we test whether the factor loadings for the transitory component are individually significant. The asymptotic t-ratios for these parameters indicate that they are all individually significant at the 1% level. This suggests that the common transitory factor may not be ignored in explaining the data.

We are now in a position to calculate the relative importance of permanent and transitory shocks during recessions. In order to do this, we set the symmetric innovation variances of the idiosyncratic and common components to zero. This first restriction is harmless, since the explicit idea in the work of Burns and Mitchell (1946) was that a recession only occurred when a number of economic variables simultaneously contracted. We also eliminate the common symmetric shocks since the ability of these models to predict recessions has been judged entirely on the estimated behavior of the unobserved Markov-switching variables. We then write the variance of each observed series as:

[6] Under the null hypothesis that $l_i = 0$ for all i, the parameters associated with common transitory component x_t in equation (3.9) are not identified. While such problem has received attention from Hansen (1992, 1996) and Garcia (1998), the distribution of the test statistic in the presence of nuisance parameters that exist only under the alternative hypothesis is unknown for the state space model we are dealing with. However, the individual hypothesis that $\lambda_i = 0$ for *one* i does not render any parameters unidentified under the null hypothesis, and standard distribution theory is valid.

(4.1) $$\text{var}(Dy_{it}) = g_i^2 \text{ var}(Dc_t) + l_i^2 \text{ var}(Dx_t).$$

This calculation requires the variance of the unobserved state variables. Hamilton (1989) demonstrates that the variance of S_{it}, i =1, 2, is calculated by:[7]

(4.2) $$\text{var}(S_{it}) = p_i(1-p_i)p_{i0} + q_i(1-q_i)(1-p_{i0})$$

where $p_{i0} = (1-q_i)/(2-q_i-p_i)$ is the steady state probability that $S_{it} = 1$.

We decompose the variance of each individual series into that due to the common permanent component and that due to the common transitory component. The second column of Table 2 reports the fraction of the observed variance which can be attributed to the common transitory component, x_t. Our parameter estimates indicate that between 77% and 96% of the observed variance of monthly indicator series during recessionary periods is temporary. This suggests that the high-growth recovery phase exhibits 'partial peak-reversion.' Accordingly, we would characterize the business cycle as having the following three phases: recession, high-growth recovery during which output *partially* reverts to its previous peak, and normal growth following the recovery.

We now focus on the timing and duration of the two common components. While we cannot reject the joint null hypothesis that $q_1 = q_2$ and $p_1 = p_2$, this does not imply that $S_{1t} = S_{2t}$. The filtered and smoothed probabilities[8] of contraction for the common permanent component depicted in Figures 1 and 2 and those for the common transitory component depicted in Figures 3 and 4 confirm this. The probability that the common permanent component is contracting accords quite well with the shaded NBER recessionary dates as in M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998a). However, the probability that x_t is contracting is in general different from that of C_t. Table 3 reports the contractionary periods for C_t, x_t, as well as the NBER reference cycle dates. Both common components contract during the first five recessions in the sample, although their timing and duration are somewhat different. With the exception of the 1960-61 recession, x_t has a shorter contractionary duration than C_t.

---

[7] See Hamilton (1989, p.362).

[8] The filtered probability at time *t* uses information available at time *t*, whereas the smoothed probability uses information available at time *T*.

This is consistent with an expected contractionary duration of 6.49 months and 6.13 months for C_t and x_t respectively.[9]

The most notable difference however between the two components is that the common transitory component completely misses the 1990-91 recession. Comparing the estimates of the common permanent and transitory factors in Figures 5 and 6 also confirms this. This corroborates Sichel's (1994) finding that there was no high-growth recovery phase following the 1990-91 recession.

Our results thus suggest that each recession differs in terms of the contribution of the common permanent and common transitory factors. While the first five recessions contain both permanent and transitory variation, the timing and duration of the common components is different. In addition, the 1990-91 recession does not contain a transitory component.

In Section 3.1, we discussed that our general model may not be identified if there was a common state variable and the permanent and transitory factor loadings were equal ($S_{1t} = S_{2t}$ for all t and $g_i = l_i$ for all i). In order to enhance the credibility of our inferences above, we further performed two more diagnostic checks.

First, even though we cannot test both of these restrictions jointly, we can test the joint null hypothesis that $g_i = l_i$ for all i, without imposing the restriction that $S_{1t} = S_{2t}$. The p-value for the resulting test turned out to be close to zero, rejecting the null very strongly.

The fact that the common components switch at different times may cast doubt on our calculations of the relative importance of transitory shocks reported in Table 2. For instance, it is obviously untrue that 96% of the variance of industrial production during the 1990-91 recession was transitory. As a second diagnostic check, and to assess the robustness of our results in Table 2, we estimated our model with the restriction that $S_{1t} = S_{2t}$, *i.e.* both common components switch together. For this restricted model, the fraction of the variance of the indicator series which is due to the common transitory component is reported in the third column of Table 2. The results now range from 93% to

[9] With constant transition probabilities, the expected duration of a contraction is $(1-p_i)^{-1}$ for i=1,2.

98%, bolstering our earlier finding that the transitory component accounts for most of the observed recessionary variance. We should note that this restricted model completely misses the 1990-91 recession. Note that our earlier discussion implied that the transitory factor was a dominating source of business cycle asymmetry. Thus, by forcing $S_{1t} = S_{2t}$, the regime probabilities which result are dominated by those of the common transitory component in our general model. This provides indirect evidence that $S_{1t} \neq S_{2t}$.

5. Summary and Conclusions

While existing business cycle models which incorporate both comovement and asymmetry have been successful at identifying recessionary periods and constructing indexes of economic activity, they have two possible shortcomings. First, since they only model asymmetry in the common growth component of economic time series, they are unable to capture potential peak-reverting behavior. Second, they only allow recessions to arise from only one source. This prevents certain qualitative differences to exist between recessions, such as the absence of a high-growth recovery phase following the 1990-91 recession.

We propose a generalization of existing business cycle models which allows us to decompose recessions into permanent and transitory components. Specifically, we extend the regime-switching dynamic factor model proposed by Diebold and Rudebusch (1996) to allow for a common transitory, as well as a common permanent, component. Our results indicate that between 77% and 96% of the observed recessionary variance of monthly indicator series is due to the common transitory component. This suggests that most negative shocks over the business cycle are temporary. We call this 'partial peak-reversion.' Accordingly, we view the business cycle as having three phases: recession, partial recovery, and normal growth.

In addition, we find that each recession differs in terms of the contribution of the common permanent and common transitory factors. Five of the six recessions from 1959-1998 contain both a permanent and transitory component, although they vary both in timing and duration. The most notable recessionary difference is the absence of the usual high-growth recovery phase following the 1990-91 recession.

References

Albert, J.H. and S. Chib, “Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts,” *Journal of Business and Economic Statistics* 11 (1993), 1-15.

Beaudry, P. and G. Koop, “Do Recessions Permanently Change Output?,” *Journal of Monetary Economics* 31 (1993), 149-163.

Boldin, M.D., “Dating Turning Points in the Business Cycle,” *Journal of Business* 67 (1994), 93-131.

Burns, A.F. and W.A. Mitchell, *Measuring Business Cycles* (New York: NBER, 1946).

Chauvet, M., “An Econometric Characterization of Business Cycle Dynamics with Factor Structure and Regime Switching”, *International Economic Review* 39 (1998), 969-996.

Carter, C.K. and P. Kohn, “On Gibbs Sampling for State Space Models,” *Biometrica* 81 (1994), 541-553.

Cover, J.P., “Asymmetric Effects of Positive and Negative Money Supply Shocks,” *Quarterly Journal of Economics* 107 (1992), 1261-1282.

DeLong, J.B. and L.H. Summers, “How Does Macroeconomic Policy Affect Output?,” in W. C. Brainard and G. L. Perry, eds., *Brookings Papers on Economic Activity* (Washington, DC: The Brookings Institution, 1988), 433-494.

Diebold, F.X. and G.D. Rudebusch, “A Nonparametric Investigation of Duration Dependence in the American Business Cycle,” *Journal of Political Economy* 98 (1990), 596-616.

Diebold, F.X. and G.D. Rudebusch, “Measuring Business Cycles: A Modern Perspective,” *The Review of Economics and Statistics* 78 (1996), 67-77.

Diebold, F.X., G.D. Rudebusch, and D. E. Sichel, 1993, “Further Evidence on Business Cycle Duration Dependence,” in: J. H. Stock and M. W. Watson, eds., *Business Cycles, Indicators, and Forecasting* (Chicago: University of Chicago Press, 1993), 255-280.

Durland, J.M. and T.H. McCurdy, “Duration-Dependent Transitions in a Markov Model of U. S. GNP Growth,” *Journal of Business and Economic Statistics* 12 (1994), 279-288.

Filardo, A.J., "Business Cycle Phases and Their Transitional Dynamics," *Journal of Business and Economic Statistics* 12 (1994), 299-308.

Friedman, M., 1964, "Monetary Studies of the National Bureau, the National Bureau Enters its 45th year," 44th annual report, 7-25; Reprinted in M. Friedman, *The Optimum Quantity of Money and Other Essays*, (Chicago: Aldine, 1969), 261-284.

Friedman, M., "The 'Plucking Model' of Business Fluctuations Revisited," *Economic Inquiry* 31 (1993), 171-177.

Garcia, R., "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models," *International Economics Review* 39 (1998), 763-788.

Goodwin, T.H. and R.J. Sweeney, "International Evidence on Friedman's Theory of the Business Cycle" *Economic Inquiry* 31 (1993), 178-193.

Gordon, K. and A.F.M. Smith, "Modeling and Monitoring Discontinuous Changes in Time Series," in J.C. Spall, ed., *Bayesian Analysis of Time Series and Dynamic Linear Models* (New York: Marcel Dekker, 1988), 359-392.

Gregory, A.W., A.C. Head, and J. Raynauld, "Measuring World Business Cycles," *International Economic Review,* 38 (1997), 677-701.

Hamilton, J.D., "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica* 57 (1989), 357-384.

Hansen, B.E., "The Likelihood Ratio Test under Non-Standard Conditions: Testing the Markov Switching Model of GNP," *Journal of Applied Econometrics* 7 (1992), S61-S82.

Hansen, B.E., "Inference When a Nuisance Parameter is Not Identified Under the Null Hypothesis," *Econometrica* 64 (1996), 413-430.

Harrison, P.J. and C. F. Stevens, "Bayesian Forecasting," *Journal of the Royal Statistical Society*, B 38 (1976), 205-247.

Johansen, S., "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models," *Econometrica* 59 (1991), 1551-1580.

Keynes, J.M., *The General Theory of Employment, Interest and Money* (London: Macmillan, 1936).

Kim, C.-J., "Unobserved-Component Time Series Models with Markov-Switching Heteroskedasticity: Changes in Regime and the Link Between Inflation Rates and Inflation Uncertainty," *Journal of Business and Economic Statistics* 11 (1993), 341-

349.

Kim, C.-J., "Dynamic Linear Models with Markov Switching," *Journal of Econometrics* 60 (1994), 1-22.

Kim, C.-J. and C.R. Nelson, "Business Cycle Turning Points, a New Coincident Index, and Tests for Duration Dependence Based on a Dynamic Factor Model with Markov Switching," *The Review of Economics and Statistics* 80 (1998a), 188-201.

Kim, C.-J. and C.R. Nelson, "A Bayesian Approach to Testing for Markov Switching in Univariate and Dynamic Factor Models," University of Washington, 1998b.

Kim, C.-J. and C.R. Nelson, "Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components," forthcoming, *Journal of Money, Credit, and Banking* (1999a).

Kim, C.-J. and C.R. Nelson, *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, (Cambridge: MIT Press, 1999b).

Kim, M.-J. and J.-S. Yoo, "New Index of Coincident Indicators: A Multivariate Markov Switching Factor Model Approach," *Journal of Monetary Economics* 36 (1995), 607-630.

Lam, P., "The Hamilton Model with a General Autoregressive Component: Estimation and Comparison with Other Models of Economic Time Series," *Journal of Monetary Economics* 26 (1990), 409-432.

Mitchell, W.A., *Business Cycles: The Problem and its Setting* (New York: NBER, 1927).

Mitchell, W.A., and A.F. Burns, *Statistical Indicators of Cyclical Revi*vals (New York: NBER Bulletin 69, 1938)

Neftçi, S.N., "Are Economic Time Series Asymmetric over the Business Cycle?," *Journal of Political Economy* 92 (1984), 307-328.

Sichel, D.E., "Are Business Cycles Asymmetric? A Correction," *Journal of Political Economy* 97 (1989), 1055-1060.

Sichel, D.E., "Business Cycle Duration Dependence: A Parametric Approach," *The Review of Economics and Statistics* 73 (1991), 254-260.

Sichel, D.E., "Inventories and the Three Phases of the Business Cycle," *Journal of*

Business and Economic Statistics 12 (1994), 269-277.

Stock, J.H. and M.W. Watson, "New Indexes of Coincident and Leading Indicators," in O.J. Blanchard and S. Fischer, eds., *NBER Macroeconomics Annual* (Cambridge: MIT Press, 1989), 351-393.

Stock, J.H. and M.W. Watson, "A Probability Model of the Coincident Economic Indicators," in K. Lahiri and G.H. Moore, eds., *Leading Economic Indicators: New Approaches and Forecasting Records* (New York: Cambridge University Press, 1991), 63-85.

Stock, J.H. and M.W. Watson, "A Procedure for Predicting Recessions with Leading Indicators: Econometric Issues and Recent Experiences," in J.H. Stock and M.W. Watson, eds., *Business Cycles, Indicators, and Forecasting* (Chicago: University of Chicago Press, 1993) 95-156.

Wynne, M. A. and N. S. Balke, "Are Deep Recessions Followed by Strong Recoveries?," *Economics Letters* 39 (1992), 183-189.

Appendix

1. Representation

In this section, we discuss representation of the model presented in Section 3. We employ the following state space representation for equations (3.5) – (3.12) assuming AR(1) dynamics for the common growth, common transitory, and idiosyncratic components. Even though our model involves two unobserved Markov-switching variables, S_{1t} and S_{2t} the dynamics can be represented by a single Markov-switching variable, S_t, in the following manner:

$$S_t = 1 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 0$$

$$S_t = 2 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 1$$

$$S_t = 3 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 0$$

$$S_t = 4 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 1$$

with

$$\Pr[S_t = j \mid S_{t-1} = i] = p_{ij}$$

and

$$\sum_{j=1}^{4} p_{ij} = 1.$$

Independence between S_{1t} and S_{2t} amounts to restrictions in the transition probabilities which describe the dynamics of the newly defined S_t. In our case, p_{ij} are functions of q_1, p_1, q_2, and p_2. For example,

$p_{11} = \Pr[S_t = 1 \mid S_{t-1} = 1] = \Pr[S_{1t} = 0 \mid S_{1,t-1} = 0] \Pr[S_{2t} = 0 \mid S_{2,t-1} = 0] = q_1 q_2$.

We employ the following state space representation:

Measurement Equation: $$Dy_t = Hx_t$$

Transition Equation: $$x_t = a_{S_t} + Fx_{t-1} + V_t$$

$$E(V_t V_t') = Q\,,$$

where

$$H = \begin{bmatrix} g_1 & l_1 & -l_1 & 1 & 0 & 0 & 0 \\ g_2 & l_2 & -l_2 & 0 & 1 & 0 & 0 \\ g_3 & l_3 & -l_3 & 0 & 0 & 1 & 0 \\ g_4 & l_4 & -l_4 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$x_t = \begin{bmatrix} Dc_t \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \end{bmatrix}, a_{S_t} = a_{S_{1t},S_{2t}} = \begin{bmatrix} m_{S_{1t}} \\ t_{S_{2t}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, V_t = \begin{bmatrix} v_t \\ u_t \\ 0 \\ e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix}, F = \begin{bmatrix} f & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & y_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & y_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & y_4 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & s_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & s_3^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & s_4^2 \end{bmatrix}.$$

2. Estimation

Defining S_t and its transitional dynamics as in equations (3.14) - (3.19), the above state-space model is a special case of that considered by Kim (1994). The following describes Kim's approximate maximum likelihood estimation algorithm. For details of the nature of the approximation and the Bayesian alternative to the estimation procedure, readers are referred to Kim and Nelson (1999b).

If the Markov-switching state variables were observed, the state space model presented in Section 3 would be linear and Gaussian, and calculation of the exact likelihood function with the Kalman filter would be possible. The unobservability of the state variables, however, induces nonlinearity in the transition equation of the state space representation, and calculation of the exact likelihood function via the Kalman filter is computationally intractable. As noted by Harrison and Stevens (1976) and Gordon and Smith (1988), if there are *M* possible states at each time period (4 in our case), each iteration of the filter produces an *M*-fold increase in the number of states to consider. With a sample size of *T*, there would be us 4^T cases to consider; an impractical computational burden. Kim (1994) proposes a method to approximate the likelihood function for state space models with Markov switching in both the measurement and transition equations. The algorithm is computationally efficient, and experience suggests that the degree of approximation is small; see Kim (1994) and Kim and Nelson (1999b).

Conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter equations can be written as:

$$x_{t|t-1}^{(i,j)} = a_{S_j} + Fx_{t-1|t-1}^{i}$$

$$P_{t|t-1}^{(i,j)} = FP_{t-1|t-1}^{i}F' + Q$$

$$h_{t|t-1}^{(i,j)} = Dy_t - Hx_{t|t-1}^{(i,j)}$$

$$f_{t|t-1}^{(i,j)} = HP_{t|t-1}^{(i,j)}H'$$

$$x_{t|t}^{(i,j)} = x_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1}h_{t|t-1}^{(i,j)}$$

$$P_{t|t}^{(i,j)} = (I - P_{t|t-1}^{(i,j)}H'[f_{t|t-1}^{(i,j)}]^{-1})HP_{t|t-1}^{(i,j)}$$

where $x_{t|t-1}^{(i,j)}$ is an inference on x_t based on information up to time *t*-1, conditional on

$S_t = j$ and $S_{t-1} = i$; $x_{t|t}^{(i,j)}$ is an inference on x_t based on information up to time *t*, conditional on $S_t = j$ and $S_{t-1} = i$; $P_{t|t-1}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ are the MSE matrices of $x_{t|t-1}^{(i,j)}$ and $x_{t|t}^{(i,j)}$ respectively; $h_{t|t-1}^{(i,j)}$ is the conditional forecast error of Δy_t based on information up to time *t*-1; $f_{t|t-1}^{(i,j)}$ is the conditional variance of $h_{t|t-1}^{(i,j)}$.

As noted by Harrison and Stevens (1976) and Gordon and Smith (1988) each iteration of the Kalman filter produces a 4-fold increase in the number of cases to consider. To render the Kalman filter operational, we need to collapse the 4^2 posteriors ($x_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$) into 4 at each iteration. Collapsing requires the following approximations suggested by Harrison and Stevens (1976):

$$x_{t|t}^{j} = \frac{\sum_{i=1}^{4} \Pr[S_{t-1} = i, S_t = j \mid W_t] x_{t|t}^{(i,j)}}{\Pr[S_t = j \mid W_t]},$$

and

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{4} \Pr[S_{t-1} = i, S_t = j \mid W_t] \left\{ P_{t|t}^{(i,j)} + (x_{t|t}^{j} - x_{t|t}^{(i,j)})(x_{t|t}^{j} - x_{t|t}^{(i,j)})' \right\}}{\Pr[S_t = j \mid W_t]},$$

where W_t refers to information available at time *t*.

In order to obtain the probability terms necessary for collapsing, we perform the following procedure due to Hamilton (1989):

<u>Step 1:</u>

At the beginning of the t^{th} iteration, given $\Pr[S_{t-1} = i \mid W_{t-1}]$, we calculate

$$\Pr[S_t = j, S_{t-1} = i \mid W_{t-1}] = \Pr[S_t = j \mid S_{t-1} = i] \Pr[S_{t-1} = i \mid W_t],$$

<u>Step 2:</u>

Consider the joint density of Δy_t, S_t, and S_{t-1}:

$$f(Dy_t, S_t = j, S_{t-1} = i \mid W_{t-1}) = f(Dy_t \mid S_t = j, S_{t-1} = i, W_{t-1}) \Pr[S_t = j, S_{t-1} = i \mid W_{t-1}]$$

from which the marginal density of Δy_t is obtained by:

$$f(Dy_t \mid W_{t-1}) = \sum_{i=1}^{4} \sum_{j=1}^{4} f(Dy_t, S_t = j, S_{t-1} = i \mid W_{t-1})$$

$$= \sum_{i=1}^{4} \sum_{j=1}^{4} f(Dy_t \mid S_t = j, S_{t-1} = i, W_{t-1}) \Pr[S_t = j, S_{t-1} = i \mid W_{t-1}]$$

where the conditional density $f(Dy_t \mid S_t = j, S_{t-1} = i, W_{t-1})$ is obtained via the prediction-error decomposition:

$$f(Dy_t \mid S_t = j, S_{t-1} = i, W_{t-1}) = (2p)^{-\frac{T}{2}} \left| f_{t|t-1}^{(i,j)} \right|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} h_{t|t-1}^{(i,j)\prime} f_{t|t-1}^{(i,j)-1} h_{t|t-1}^{(i,j)} \right\}.$$

<u>Step 3:</u>

Once Δy_t is observed at the end of time t, we update the probability terms:

$$\Pr[S_t = j, S_{t-1} = i \mid W_t]$$

$$= \Pr[S_t = j, S_{t-1} = i \mid W_{t-1}, Dy_t]$$

$$= \frac{f(S_t = j, S_{t-1} = i, Dy_t \mid W_{t-1})}{f(Dy_t \mid W_{t-1})}$$

$$= \frac{f(Dy_t \mid S_t = j, S_{t-1} = i, W_{t-1}) \Pr[S_t = j, S_{t-1} = i \mid W_{t-1}]}{f(Dy_t \mid W_{t-1})}$$

with

$$\Pr[S_t = j \mid W_t] \sum_{i=1}^{4} \Pr[S_t = j, S_{t-1} = i \mid W_t].$$

To initialize the above filter, we use the steady-state probabilities.

As a by product of the above filter in Step 2, we obtain the log likelihood function:

$$\ln L = \sum_{t=1}^{T} ln(f(\Delta y_t \mid y_{t-1})$$

which can be maximized with respect to the parameters of the model.

3. Constructing C_t from Dc_t

Since the data are in deviations from their means, d and $D = [D_1 \quad D_2 \quad D_3 \quad D_4]'$ are concentrated out of the likelihood function. As in Stock and Watson (1991), we can use the steady state Kalman gain retrieve these terms in the following manner:

$$d = E'(I_r - (I_r - K^* H)F)^{-1} K^* D\bar{y},$$

$$\tilde{D} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = D\bar{y} - \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} d,$$

where K^* is the steady state Kalman gain, $E_1' = [1\,0\,0 \ldots 0]'$, and r is the dimension of the state vector. Once d is retrieved, given $\Delta\tilde{c}_T = [\Delta c_1\ \Delta c_2\ \ldots \Delta c_T]'$, and arbitrary initial value C_0, we obtain $C_t = d + \Delta c_t + C_{t-1}$, $t = 1,2,\ldots,T$.

Table 1. Maximum Likelihood Estimates
Monthly Data, 1959.01-1998.10
(i = IP, GMYXPQ, MTQ, LHNAG)

	Transition Probabilities		
q_1	0.977	p_1	0.846
	(0.012)		(0.074)
q_2	0.984	p_2	0.837
	(0.006)		(0.053)
	Regime Dependent Parameters		
μ_0	0.248	μ_1	-2.136
	(0.083)		(0.434)
τ	-6.361		
	(0.935)		
	Permanent Factor Loadings		
γ_1	0.243	γ_2	0.272
	(0.065)		(0.075)
γ_3	0.164	γ_4	0.196
	(0.048)		(0.050)
	Transitory Factor Loadings		
λ_1	0.435	λ_2	0.197
	(0.054)		(0.040)
λ_3	0.279	λ_4	0.127
	(0.038)		(0.031)
	Autogressive Parameters for the Common Components		
ϕ	0.663	ϕ^*	0.693
	(0.114)		(0.046)
	Autoregressive Parameters for the Idiosyncratic Components		
ψ_1	0.124	ψ_2	-0.081
	(0.115)		(0.054)
ψ_3	-0.340	ψ_4	-0.242
	(0.046)		(0.048)
	Idiosyncratic Innovation Standard Deviations		
σ_1	0.466	σ_2	0.754
	(0.046)		(0.031)
σ_3	0.762	σ_4	0.861
	(0.28)		(0.030)
	(0.29)		
	$\ln L$ = -1056.583		

Standard errors are in parentheses.

Table 2. The Relative Importance of the Transitory Component During Recessions

Series	$S_{1t} \neq S_{2t}$	$S_{1t} = S_{2t}$
Industrial Production	96.29%	98.20%
Personal Income	80.83%	98.04%
Manufacturing and Trade Sales	95.94%	98.83%
Employment	77.21%	93.17%

Table 3. Contractionary Periods for both Common Components

C_t	x_t	NBER Chronology
1960.06 - 1960.12 (07)	1960.02 - 1961.03 (14)	1960.04 - 1961.02 (11)
1970.08 - 1970.12 (05)	1970.09 - 1970.11 (03)	1969.12 - 1970.11 (12)
1973.12 - 1975.04 (17)	1973.12 - 1975.03 (04)	1973.11 - 1975.03 (17)
1980.01 - 1980.05 (05)	1980.03 - 1980.06 (04)	1980.01 - 1980.07 (07)
1981.08 - 1983.02 (19)	1981.10 - 1982.01 (04)	1981.07 - 1982.11 (17)
1990.05 - 1991.02 (10)		1990.07 - 1991.03 (09)

Note: C_t is said to contract when $Pr[S_{1t}=1|\psi_T]>0.5$ and x_t is said to contract when $Pr[S_{2t}=1|\psi_T]>0.5$, where ψ_T denotes information available at time T. Durations of the contractionary periods are in parentheses. Ommited from the table is a one period contraction that occurred for x_t in 1970.01.

Figure 1. Filtered Probability that C_t is Contracting

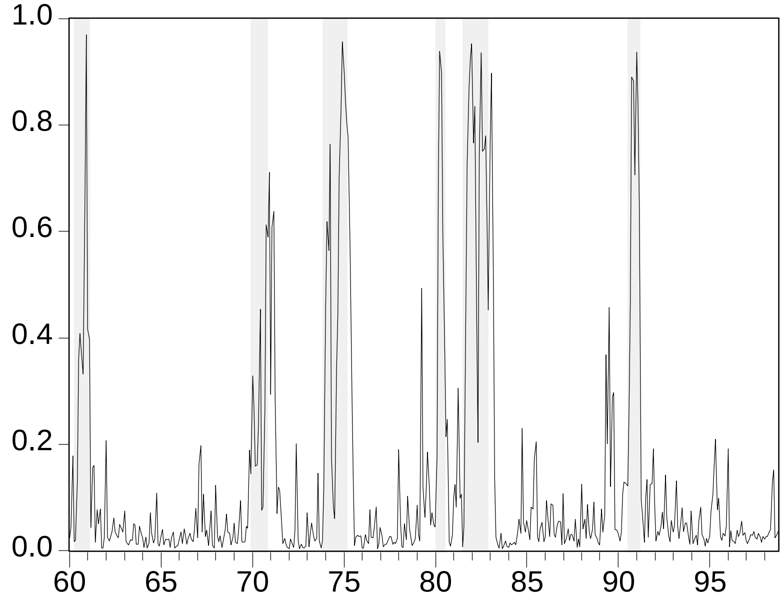

Figure 2. Smoothed Probability C_t that is Contracting

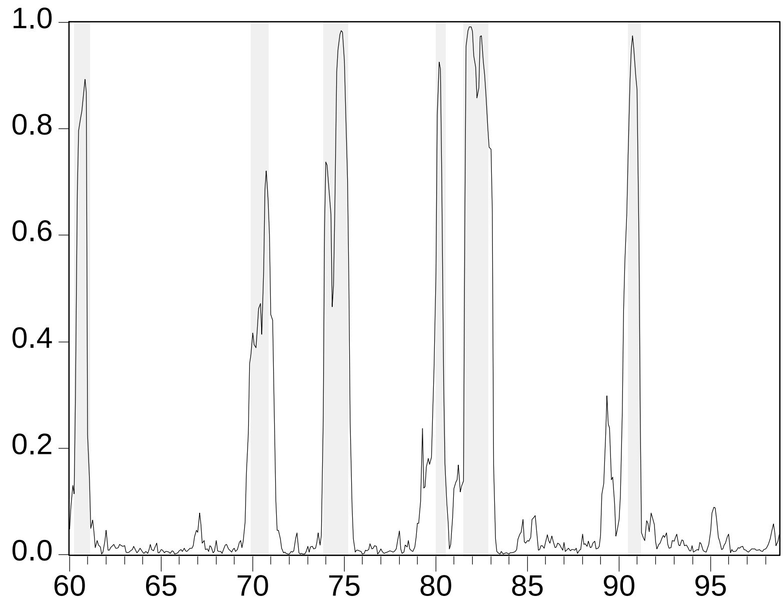

Figure 3. Filtered Probability that x_t is Contracting

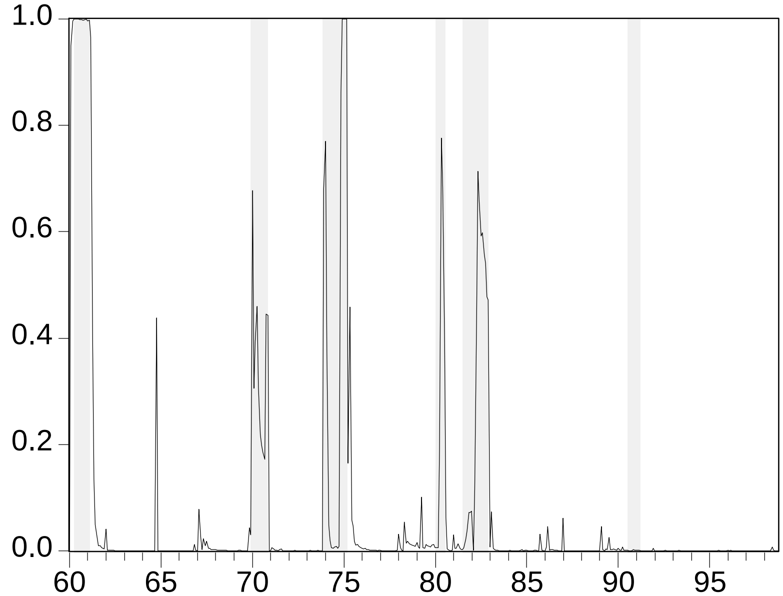

Figure 4. Smoothed Probability that x_t is Contracting

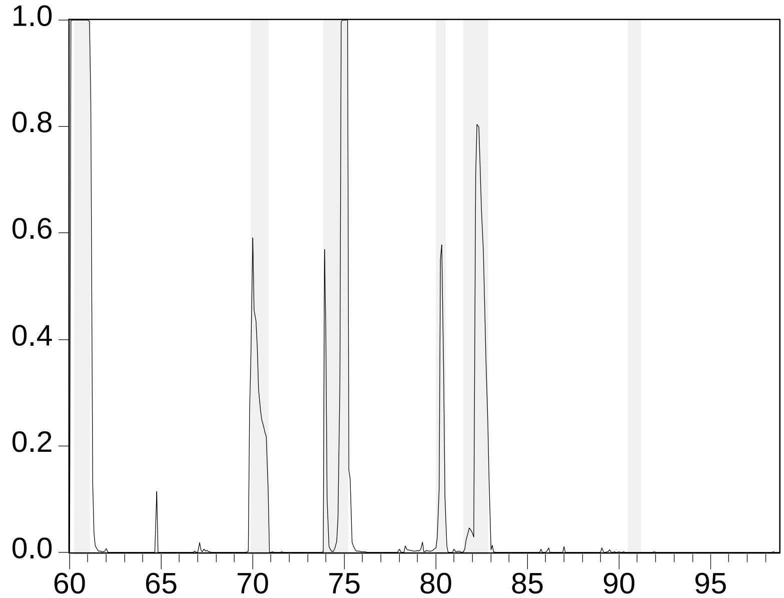

Figure 5. Common Permanent Component

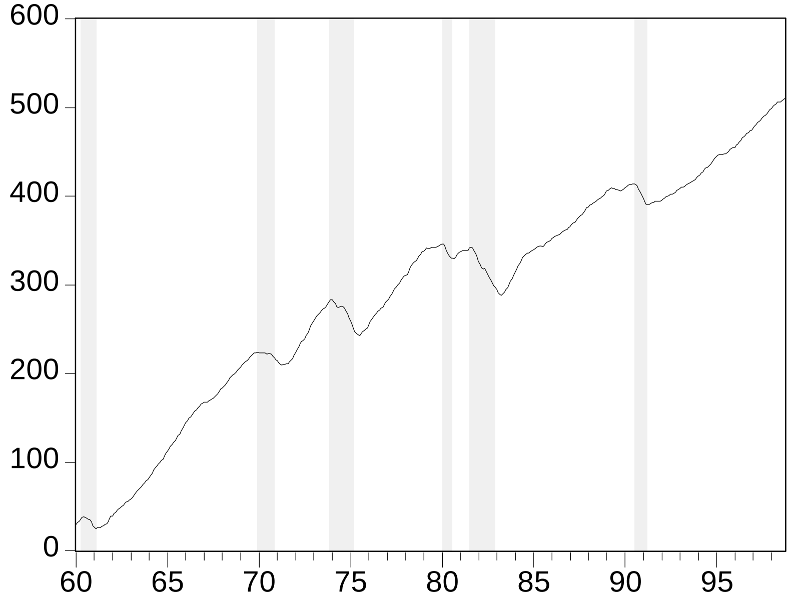

Figure 6. Common Transitory Component