Hidden Components of Expected Returns: State-Space Estimation with Prices and Earnings

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Abstract

We propose new state-space models with stock and accounting variables to estimate the expected market returns. These approaches uncover the information existing in unobserved state variables through the predictive updating system based on the Kalman filter technique. The one-step-forward in-sample prediction for the state-space model with stock variables has R^2 as 13%, where the expected market return has a persistent component. We further improve the performance of forecasting market returns by incorporating accounting variables with a state-space model, having a higher R^2, 18%. Both expected market returns and expected returns on equity have persistent components, but expected returns on equity are more persistent than expected market returns. Results from out-of-sample predictions further reinforce the forecastability of market returns based on proposed models, especially for short-range predictions.

Keywords: *State-space models, Kalman Filter, Accounting Earnings, Discount Rate News, Cash Flow News.*
JEL Codes: *C32, C53, C58, E32, G12.*

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1 Introduction

Starting from the 1920s, economists have been forecasting stock market returns or equity premiums with different methods or data. Though Lettau and Ludvigson (2001) [34] mention that excess returns are predictable based on dividends or earnings, there are contradictory findings regarding predicting stock returns. Researchers further test return predictions on different variables based on various algorithms.

In this paper, we utilize both aggregate stock and accounting variables to examine the annual stock return forecastability of various models, such as vector autoregressive model (VAR) and state-space model (SSM). By forming latent variables' processes, we estimate the annual expected market returns using the conditional expected stock and accounting variables to create unobserved time-series processes. Then, we combine the latent system with log linearized market returns to derive the predictive regressions estimated through the Kalman filter based on likelihoods. We find that more information is captured from expected state variables through the latent processes, which can improve the performance of models. These models give better predictions of market returns, with R^2 ranging from 13% to 18%.

We also compare the latent processes with other common methods. We first test basic linear regressions based on different variables and extend the simple model to an VAR model to simulate the process by including relations over time. The R^2 ranges from 6% to 10% for these models.

For state-space models, we start with defining latent variables, expected dividends, and expected returns. Following Cochrane (2007 & 2008) [16] [17] and Campbell and Shiller's decomposition (Campbell & Shiller, 1988a) [11], we generate a time-varying process for latent stock or accounting variables. The state-space model with time-varying latent stock variables improves the forecastability of aggregate stock returns. The stability of the state-space model is further tested based on the training and testing datasets.

Following Vuolteenaho (2000) [54], we make three assumptions for the analysis in this paper. First, book and market value are assumed to be positive, which is fundamental for the clean surplus identity (Equation (20)). Second, the dividend-price ratio and the book-to-market ratio are assumed to be stationary. This assumption helps to specify a value around which the price changes. The third assumption is the clean surplus identity, which links the current book value with earnings, dividends, and the last period's book value.

Our main contributions are the following. First, we examine the traditional market return forecasting models on accounting datasets. We find that the results from VAR are reasonable, but they cannot capture the potential movements in the latent variables. By examining the residuals from VAR models, we believe there is some critical information in residuals that the model does not capture. Thus, it is crucial to introduce the state-space model to uncover the dynamics behind observed variables further.

Second, we improve the forecastability of aggregate stock market returns by using two new state-space models. The first state-space model is based on dividend yield and includes the market return in the measurement equation directly rather than calculates the returns based on predicted dividend growth rates and dividend yields (Binsbergen & Koijen, 2010, [53]). Then, we can bring more information existing in unobserved variables to the predictive system, which highly improves the predictability. And, expected market returns are more persistent than expected dividend growth rates.

Our second state-space model is based on aggregate accounting data and generates better forecasting results than others. Due to the instability in dividend policies over time, we use the book-to-market ratio as a substitute for the dividend-price ratio and create the structure based on the clean surplus identity (Equation (20)) and first-order Taylor expansions. With accounting variables, we find that market returns are consistently predicted with relatively significant improvements, where expected returns on equity are more persistent than expected market returns.

By decomposing the variance of unexpected market returns, we further reinforce the value of accounting earnings, relative to unstable dividend policies, for predicting market returns. We find that cash-flow news dominates the variance when we use accounting earnings (147.9%), whereas when we use dividend growth, it explains only 2.8% of the variance. Discount-rate news accounts for 135.8% (dividend growth) and 9.5% (accounting earnings) of the variance, and the covariance between these two news components accounts for the remainder. This decomposition suggests that accounting data can significantly explain variation in unexpected market returns, and that stable quarterly earnings outperform dividend growth in capturing cash-flow news.

Further, both state-space models show great forecastability of short-range market returns because of the persistent state variables over time. Considering the unstable dividend policy, the accounting-based state-space model performs better than the other due to the persistent of cash-flow state variable, expected returns on equity.

The organization of this paper is as follows. In Section 2, we review the background of return forecastability, including historical forecasting approaches and previous state-space models. In Section 3, we describe the stock and accounting data for this paper. In Section 4, we discuss two state-space models with different information sets, and in Section 5 we outline the results from different methods. Section 6 contains our conclusions.

2 Literature Review

There is a long research history of forecasting stock returns. Starting from the 1920s, Dow and Selden (1920) [20] try to optimize the trading algorithms based on forecasting stock returns using dividend ratios. Various papers use different algorithms or variables to forecast the excess returns, considering various measurements of stock risks.

2.1 Background of Forecasting

Fore forecasting stock returns, the dividend-price ratio is used by Campbell (1987) [8], Campbell and Shiller (1988a) [11], Fama and French(1988) [21], Hodrick (1992) [27], Wolf (2000) [56], Lewellen (2004) [35], Campbell and Yogo (2006) [10], Ang and Bekaert(2006) [1], Cochrane(2007) [16], Goyal and Welch (2008) [23], and Van Binsbergen and Koijen (2010) [53]. Most of these authors argue that the dividend yield can be used to forecast stock returns, although the strength of predictions varies considerably across studies.

The research studying the dividend-price ratio contains contradictory findings regarding the forecastability of excess stock returns. For example, Goyal and Welch (2008) [23] investigate the forecasting power of a diverse group of stock and accounting variables based on out-of-sample observations. They find that the prediction models change significantly over time and that most of the predictors perform worse than predicting returns using the historical means. They also point out that available information cannot help investors make additional profits, Cochrane (2007) [16], however, argues that Goyal and Welch's results only show difficulties in using predictions to form trading strategies. In other words, the out-of-sample R-square is a statistic that evaluates the performance or usefulness of making market decisions based on the prediction, which does not indicate the rejection of the forecastability of stock returns. Cochrane (2007) [16] tests the hypothesis that "if returns are not predictable, then dividend growth must be predictable." Based on the absence of dividend growth predictability, it can indirectly defend the forecastability of stock returns.

Other researchers also estimate stock returns based on accounting variables. They find that forecasting using earning price ratio performs better than forecasting using dividend-price ratios, which is initially tested in a VAR model by Campbell and Shiller (1988b) [12] and Lamont (1998) [33]. Currently, researchers proved a positive relationship between firm-level earnings and stock prices or returns (Choi et al. (2016) [14], Bonsall et al. (2013) [5]). However, for the aggregate-level market, there is a negative relationship between earnings and stock market returns (Kothari, Lewellen, and Warner (2006) [32], Sadka (2007) [48], Sadka and Sadka (2009) [49], Hirshleifer, Hou, and Teoh (2009) [26], Patatoukas (2013) [42]). They also examine the relation between earnings news and stock returns and conclude that aggregate returns are forecastable.

2.2 Background of Variance Decomposition

Campbell and Shiller (1988b) [12] decompose unexpected stock returns into two parts, changes in expected future cash flow (dividends) news and changes in expected future discount rate (stock returns) news. Later, Campbell (1991) [9] shows that the variance of future cash flow news accounts for one third of the variance of unexpected aggregate stock returns.

Motivated by Modigliani-Miller's dividend irrelevant theory, the instability in aggregate dividend policy, and the weak results regarding forecasting long-horizon stock returns using dividend price ratios, Vuolteenaho (2000) [54] develops a return model which builds upon the relationships among book-to-market ratio, return on equity (ROE), interest rates, and returns. Using this model, Vuolteenaho (2002) [55] finds that cash-flow news significantly drives the firm-level stock returns, while expected-return information is significantly driven by aggregate-level components.

Then, based on the Campbell-Shiller present-value formula, the current period stock return can be separated into three parts: changes in conditional expected returns, changes in expected cash-flow news, and changes in expected-return news. While people estimate the firm-level relations between returns and earnings, they conclude that $Cov(N_{cf,i,t}, \Delta X_{i,t}) > 0$ and $Cov(R_{i,t}, \Delta X_{i,t}) > 0$, where $\Delta X_{i,t}$ denotes the changes in earnings for firm i. Hecht and Vuolteenaho (2006) [25] further extend the relations to the remaining variables in the decomposition, stating that the change in earnings is correlated with lagged and leading information. Moreover, the aggregate earning is negatively correlated with the return news.

2.3 Background of Estimated Models

When forecasting stock returns using dividend price ratios and other accounting data, the type of forecasting model is important. In empirical work, the Vector Autoregressive model (VAR) is the most common model used for predicting stock returns, see examples such as Cochrane (2007) [16] and Brandt and Kang (2004) [6]. In contrast to the VAR model, Rytchkov (2007) [47], Pastor and Stambaugh (2009) [41], Van Binsbergen and Koijen (2010)

[53], and Monache et al. (2021) [38] focus on predicting stock returns based on the state-space model and treating the expected stock return as a latent variable.

State-space models allow time variation in parameters and automatically apply restrictions in the updating process of latent components in the model. For modeling returns based on present-value representations, state-space models can handle complex relations and efficiently utilize all the information. After considering the movements in market-wide data, all the papers above show evidence regarding improvement in stock return forecastability.

Besides the dividends and accounting variables, improvements in computational technologies induce lots of research based on event-driven data or different markets. These research focus more on interesting predictors, feature selections, and data mining. For example, Bijl et al. (2016) [4] find that data from Google traffic can predict the stock returns. Salisu et al. (2019) [50] proposed an alternative approach to forecast market returns based on Bitcoin prices. In other words, the additional available public or private information further improves predictability.

There are also other interesting and useful factors, including investor sentiments (Huang et al. (2015) [29], Li et al. (2015) [36]), Ren et al. (2018) [45], Audrino et al. (2020) [3]), financial news (Atkins et al. (2018) [2], Nam and Seong (2019) [39]), technical indicators (Neely et al. (2014) [40], Lin (2018) [37], Dai et al. (2020) [18]), and others. Also, more advanced and maturer machine learning techniques are widely used to improve the predictability of market returns, including big data with principal component analysis (De Mol et al. (2008) [19], Brodie et al. (2009) [7], Carrasco and Rossi (2016) [13], Reichlin et al. (2017) [44]), regression trees (Rossi (2018) [46], Rasekhschaffe (2019) [43]), deep learning (Chong et al. (2017) [15], Fischer and Krauss (2018) [22], Hu et al. (2018) [28]), etc. In this paper, we focus on building state-space models based on stock and accounting variables, and the modern methods are revisited in the future.

3 Data

Following Sadka and Sadka (2009) [49] and Vuolteenaho (2002) [55], for the analysis in this paper, we generate aggregate-level dividend growth rates, dividend-price ratios, returns, returns on equity, book equity, market value, and book-to-market ratio.

3.1 Basic Data

The basic data contain all firm data in the Center for Research in Security Prices (CRSP) and COMPUSTAT databases, obtained from Wharton Research Data Services (WRDS) [51]. The CRSP data contain monthly stock market returns (with and without dividend) based on the value-weighted portfolios of NYSE, AMEX, and NASDAQ stocks for the period 1950 - 2019. The COMPUSTAT annual research file contains relevant accounting information for most publicly traded firms, including book values, market values, and returns on equity. In addition, we use rolled-over 90-day Treasury bills (the risk-free rate) and the Consumer Price Index (CPI) from CRSP, and we construct the corresponding series for dividend growth rates, dividend-price ratios, returns, returns on equity, and risk-free rates. All variables are measured at an annual frequency.

3.2 Data Manipulations

By utilizing the value-weighted stock market returns from CRSP, we construct the price-dividend ratio and the dividend growth ratio. For firm-level data from COMPUSTAT, firms must have December as the fiscal-year end to align accounting variables across firms. To filter out data errors, we exclude firms with market values below $10 million and book-to-market ratios greater than 100 or less than 0.01.

Firm-level variables are calculated as follows. The market value of equity is the product of common shares outstanding and the closing price in the corresponding fiscal years. For book equity, we use the total common equity (data item 60). If the data is not available, we use the liquidity value (data item 235) as a substitute. To account for taxes, we add short-term and/or long-term deferred taxes (data item 35 and 71) to book equity when available. If neither total common equity nor liquidity value is available, we use the clean surplus identity (Equation (20)) to approximate book value. All firm-level book equity must be non-negative to be included in the analysis.

We treat firms' net incomes (data item 172) as earnings; if this item is missing, we approximate earnings using the clean surplus identity (Equation (20)). Return on equity (ROE), or profitability, is defined as earnings divided by the previous period's book equity. Intuitively, firms cannot lose more than their book equity. Thus, when earnings are negative, we require the absolute value of earnings to be smaller than book equity.

To convert firm-level data into aggregate-level data, we utilize market capitalization to compute value-weighted variables. Market-level series are calculated as the value-weighted mean of the corresponding firm-level variables, scaled by the price level in the fiscal year.

3.3 Descriptive Statistics

Table 1 reports descriptive statistics for the aggregate-level variables, including means, standard deviations, and quantiles for log excess returns, log excess returns on equity, and other variables from 1950 to 2019. These statistics are similar to those in Vuolteenaho (2002) [55] and Hecht and Vuolteenaho (2006) [25], although the sample length differs. Comparing log returns with log excess returns on equity, stock market returns are more volatile than accounting-based ROE (standard deviation of 0.16 vs. 0.05). In addition, the log dividend growth rate exhibits substantial fluctuations over time, with a standard deviation about five times its mean.

Comparing the log dividend-price ratio with the log book-to-market ratio, we observe similar volatility, as reflected in comparable standard deviations. This provides support for the state-space model below, in which the book-to-market ratio can serve as a substitute for the dividend-price ratio. Similarly, log excess earnings can serve as a substitute for dividend growth rates, given the similar quantiles in the sample (except at the extremes).

Table 1: Descriptive Statistics: Summary

	Mean	SD	Min	25%	Median	75%	Max
r - r_f	0.0576	0.1628	-0.4959	-0.0358	0.0944	0.1638	0.3970
Δd	0.0208	0.1172	-0.2747	-0.0614	0.0106	0.0798	0.3268
dp	-3.4429	0.3417	-4.1785	-3.6771	-3.4735	-3.1795	-2.8394
e - r_f	0.1024	0.0524	0.0011	0.0654	0.0935	0.1405	0.2794
θ	-0.4452	0.3151	-1.1664	-0.6735	-0.4788	-0.2110	0.1760

This table reports means, standard deviations, and quantiles of log excess return, r - r_f, log dividend growth rate, Δd, log dividend-price ratio, dp, log excess return on equity, e - r_f, and log book-to-market ratio, θ.
These statistics are estimated based on the data from CRSP and COMPUSTAT from 1950 - 2019.

Table 2 reports the correlations among the logged variables. Consistent with the literature (e.g., Cochrane, 2007 [16]), the correlation between the log dividend growth rate and log excess returns is about 0.7, indicating that log dividend growth shares common variation with log returns. We also observe a similar relationship between the log dividend yield and the log book-to-market ratio, which further supports the use of the clean surplus identity.

The log dividend-price ratio has a small correlation with market returns but relatively large correlations with accounting returns. This suggests that the clean surplus identity can support a model with better forecasting performance for market returns, as the variables are more closely related to aggregate firm-level states.

The negative correlation (-0.6) between log excess return on equity and log dividend yield suggests that firms' returns on equity decline as they distribute more dividends. A similar relationship holds between the log book-to-market ratio and log excess return on equity, with a correlation of about -0.5.

A notable feature of the data is that the correlation between log excess return on equity and log returns is about 0.11, indicating a weak relationship between market and accounting returns. Intuitively, in an open economy, when aggregating across firms, aggregate returns on equity may not be strongly linked to domestic aggregate-level market returns.

Table 2: Descriptive Statistics: Correlations

	r - r_f	Δd	dp	$e - r_f$	θ
r - r_f	1				
Δd	0.6942	1			
dp	-0.0028	0.0005	1		
$e - r_f$	0.1127	0.1891	-0.6413	1	
θ	-0.1637	-0.1154	0.8965	-0.4871	1

This table reports correlations among log excess return, r - r_f, log dividend growth rate, dg, log dividend-price ratio, dp, log excess return on equity, e - r_f, and log book-to-market ratio, θ. Sample from 1950 - 2019, CRSP and COMPUSTAT.

3.4 OLS Forecasting Performance

This section summarizes the in-sample forecasting ability of stock and accounting variables for stock returns. Table 3 presents regressions of real returns and profitability on lagged dividend-price and book-to-market ratios, as specified below.

$$\begin{aligned} r_{t+1} &= a + bdp_t + \varepsilon_{t+1} \\ r_{t+1} - r_{f,t+1} &= a + bdp_t + \varepsilon_{t+1} \\ \Delta d_{t+1} &= a + bdp_t + \varepsilon_{t+1} \\ r_{t+1} &= a + b\theta_t + \varepsilon_{t+1} \\ r_{t+1} - r_{f,t+1} &= a + b\theta_t + \varepsilon_{t+1} \\ e_{t+1} - r_{f,t+1} &= a + b\theta_t + \varepsilon_{t+1} \end{aligned} \tag{1}$$

where a denotes the intercept, b denotes the slope coefficient, r_{t+1} denotes the real log market return at time $t+1$, $r_{f,t+1}$ denotes the log risk-free rate at time $t+1$, Δd_{t+1} denotes the log dividend growth rate at time $t+1$, dp_t denotes the log dividend-price ratio at time t, θ_t denotes the log book-to-market ratio at time t, and $e_{t+1} - r_{f,t+1}$ denotes the log excess return on equity at time $t+1$.

Table 3: OLS Forecasting Performance

Regressions	a		b		t	R^2	$\sigma(bx)$
	est	se	est	se			
$r_{t+1} = a + bdp_t + \varepsilon_{t+1}$	0.4808	0.1343	0.1200	0.0400	2.9988	0.0618	0.0412
$r_{t+1} - r_{f,t+1} = a + bdp_t + \varepsilon_{t+1}$	0.4041	0.1343	0.1013	0.0392	2.5816	0.0452	0.0347
$\Delta d_{t+1} = a + bdp_t + \varepsilon_{t+1}$	0.0605	0.0722	0.0109	0.0218	0.5012	0.0010	0.0037
$r_{t+1} = a + b\theta_t + \varepsilon_{t+1}$	0.1234	0.0252	0.1255	0.0576	2.1776	0.0572	0.0396
$r_{t+1} - r_{f,t+1} = a + b\theta_t + \varepsilon_{t+1}$	0.1086	0.0268	0.1195	0.0545	2.1931	0.0534	0.0377
$e_{t+1} - r_{f,t+1} = a + b\theta_t + \varepsilon_{t+1}$	0.0676	0.0176	-0.0802	0.0294	-2.7228	0.2321	0.0253

This table reports linear regressions based on real variables. *est* represents estimated values of coefficients and *se* denotes the HAC standard errors of estimations. r is the real log market return, r_f is the log risk-free rate, r - r_f is the log excess return, Δd is the log dividend growth rate, e - r_f is the log excess return on equity, dp is the log dividend-price ratio, and θ is the log book-to-market ratio. Annual data, 1950 - 2019, from CRSP & COMPUSTAT. t reports the t-values for the coefficient, b, in each regression. $\sigma(bx)$ shows the standard deviations of the fitted value of the regression.

The coefficients show the forecastability of log excess returns based on dividend-price and book-to-market ratios, which is mentioned by Cochrane (2007) [16], Hecht and Vuolteenaho (2006) [25], etc. Moreover, based on the coefficients and R^2 values from the third and last regressions, variation in the market dividend yield does not forecast future dividend growth, whereas variation in the market book-to-market ratio forecasts future growth opportunities. This provides evidence that incorporating accounting variables can improve the performance of forecasting market returns.

The estimated standard deviations of expected returns from the first and fourth regressions are similar, at around 4%, which is much smaller than the sample standard deviation of market returns (16%). For the fitted expected dividend growth rate, the standard deviation is about 0.3%, which is much smaller than the sample standard deviation of 12%. These results are similar to those in Cochrane (2007) [16]. The standard deviation of fitted log excess ROE captures about half of the volatility observed in the sample, which provides further evidence that accounting earnings are a better predictor of market returns than the dividend growth rate.

4 Models

In this section, we define and evaluate different models based on both stock and accounting variables, including vector autoregressive and state-sapce models.

4.1 Vector Autoregressive Model

The vector autoregressive (VAR) model is the most common framework for multivariate prediction using returns and accounting variables. In this section, we present two models to capture the dynamic relationships among log returns, log dividend growth, and the log dividend-price ratio, as well as among log returns, log returns on equity, and the log book-to-market ratio. These models are used to characterize the sample dynamics and to test whether they are useful for predicting expected returns. The first VAR model, based on stock variables, is given in Equation (2).

$$\begin{bmatrix} r_t \\ \Delta d_t \\ dp_t \end{bmatrix} = \begin{bmatrix} a_r \\ a_d \\ a_{dp} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ \Delta d_{t-1} \\ dp_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t,r} \\ \varepsilon_{t,d} \\ \varepsilon_{t,dp} \end{bmatrix} \tag{2}$$

which can be represented as:

$$z_t = A_t + Bz_{t-1} + \varepsilon_t \tag{3}$$

The VAR for accounting variables is shown in Equation (4), which is similar to the above VAR model.

$$\begin{bmatrix} r_t \\ e_t - r_{f,t} \\ \theta_t \end{bmatrix} = \begin{bmatrix} a_r \\ a_e \\ a_\theta \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} r_{t-1} \\ e_{t-1} - r_{f,t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t,r} \\ \varepsilon_{t,e} \\ \varepsilon_{t,\theta} \end{bmatrix} \tag{4}$$

In general, with one lag, there are twelve coefficients and six covariance terms to be estimated from 210 annual observations. To select the VAR order consistently, we use Akaike's Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Hannan-Quinn Information Criterion (HQC). The optimal lag order for the accounting-variable VAR model (Equation 2) is one. Although the optimal lag order for the stock-variable VAR model (Equation 4) is two, the higher-order VAR yields a lower adjusted R^2 because it uses more degrees of freedom. Therefore, this section reports the VAR(1) specification.

Table 4 reports the results for each regression in the estimated VAR models (Equations (2) and (4)), along with the results of serial correlation tests. Similar to Vuolteenaho (2002) [55], expected returns on equity are higher when past returns and past ROE are higher and when the log book-to-market ratio is lower. The results also show that the dividend-price ratio and the book-to-market ratio depend strongly on their own lagged values.

The serial correlation test yields a p-value of 0.0001 for the stock-variable VAR model, indicating strong autocorrelation in the residuals. A two-lag stock-variable VAR still exhibits residual serial correlation, with a p-value of 0.03. In contrast, the p-value for the accounting-variable VAR model is about 0.2, so we fail to reject the null hypothesis of no residual serial correlation. This suggests that the accounting-variable VAR provides a better fit to the dynamics.

Overall, log returns are poorly explained in both models, whereas the dividend-price and

Table 4: Short VAR for Aggregate Stock Market

	r_t	Δd_t	dp_t	r_t	$e_t - r_f$	θ_t
const	0.495	0.094	-0.269	0.104	0.016	-0.040
	(0.197)	(0.131)	(0.129)	(0.046)	(0.009)	(0.041)
r_{t-1}	-0.272	-0.315	-0.043	-0.036	0.002	0.011
	(-0.162)	(0.107)	(0.106)	(0.123)	(0.024)	(0.110)
Δd_{t-1}	0.389	-0.024	-0.429	-	-	-
	(0.230)	(0.152)	(0.150)	-	-	-
dp_{t-1}	0.121	0.014	0.922	-	-	-
	(0.057)	(0.037)	(0.037)	-	-	-
$e_{t-1} - r_f$	-	-	-	0.309	0.770	-0.341
	-	-	-	(0.448)	(0.085)	(0.401)
θ_{t-1}	-	-	-	0.146	-0.020	0.855
	-	-	-	(0.073)	(0.014)	(0.066)
R^2	0.109	0.214	0.908	0.066	0.662	0.791
Adjusted R^2	0.067	0.177	0.903	0.022	0.646	0.781
Serial Test (p-value)		0.0001			0.2	

This table reports the parameter estimates for the short VAR. This table includes the log stock market return (r), log dividend growth rate (Δd), log dividend-price ratio (dp), log excess return on equity ($e - r_f$), and log book-to-market ratio (θ). The parameters in the table are estimated based on the following system:

$$z_t = A + Bz_{t-1} + \varepsilon_t \tag{5}$$

For each regression, the estimates and standard errors (in parenthesis) are reported, along with the R^2 and adjusted R^2. Also, for each VAR model, the p-value from the serial correlation test (Breusch-Godfrey LM test) is reported. The Null hypothesis for the serial correlation test is: *no serial correlation exists.* Sample from 1950 - 2019, CRSP and COMPUSTAT.

book-to-market ratios are estimated precisely and are highly persistent. Motivated by these findings, we introduce a state-space model with a latent AR(1) process for expected returns on equity and expected dividend yield.

4.2 State-Space Model (Market Returns and Dividend Growth)

Following Campbell and Shiller (1988) [11], this section uses a log-linear approximation of prices, dividends, and returns to present a state-space model for log excess returns and dividend growth. Increasing the order of the latent process may not significantly improve performance, because it mainly adds additional correlation terms among lagged variables and reinforces relationships among the latent states (Cochrane, 2008 [17]). Therefore, an AR(1) latent process is sufficient for forecasting stock market returns.

4.2.1 Log Linearization

The model starts with the return definition, where R_{t+1} denotes the simple return at time $t+1$, P_{t+1} denotes the price of the aggregate stock market at time $t+1$, and D_{t+1} denotes the dividend at time $t+1$:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \tag{6}$$

Using a first-order Taylor expansion, the continuously compounded return can be approximated as follows:

$$r_{t+1} = \kappa - \rho dp_{t+1} + \Delta d_{t+1} + dp_t \tag{7}$$

where $dp_{t+1} \equiv \ln\left(P_{t+1}/D_{t+1}\right)$, $\Delta d_{t+1} \equiv \ln\left(D_{t+1}/D_t\right)$, $\rho \equiv \frac{\bar{P}/\bar{D}}{1+\bar{P}/\bar{D}}$ and $\kappa \equiv \ln(1 + \frac{\bar{P}}{\bar{D}}) + \rho \bar{dp}$. $\bar{D}$ and $\bar{P}$ are the sample average of aggregate dividends and prices, and $\bar{dp}$ is the sample average of linearized dividend-price ratio. Then, the log-linearized dp_t can be written as

$$dp_t = -\frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \tag{8}$$

4.2.2 Assumptions for Latent Processes

Similar to Van Binsbergen and Koijen (2008) [53] and Cochrane (2008) [17], we assume that expected returns and dividend growth follow the AR(1) processes below. Here, $\mu_t \equiv E(r_{t+1} \mid I_t)$ denotes the conditional expectation of the log return at time $t+1$, $g_t \equiv E(\Delta d_{t+1} \mid I_t)$ denotes the conditional expectation of log dividend growth at time $t+1$, $\varepsilon_{\mu,t+1}$ represents shocks to expected returns at time $t+1$, $\varepsilon_{g,t+1}$ represents shocks to expected dividend growth at time $t+1$, and $\varepsilon_{d,t+1}$ denotes the residual in dividend growth at time $t+1$.

$$\begin{aligned} \mu_{t+1} &= \phi_\mu \mu_t + \varepsilon_{\mu,t+1} \\ g_{t+1} &= \phi_g g_t + \varepsilon_{g,t+1} \end{aligned} \tag{9}$$

We use E_t to denote expectations conditional on the information set (I_t) up to time period t. By taking the conditional expectation, Equation (8) becomes:

$$dp_t = \frac{\mu_t}{1-\phi_\mu \rho} - \frac{g_t}{1-\phi_g \rho} - \frac{\kappa}{1-\rho} \tag{10}$$

The log dividend growth follows the equation below:

$$\Delta d_{t+1} = g_t + \varepsilon_{d,t+1} \tag{11}$$

Then, by substituting the equations back to Equation (7), the return can be written as:

$$r_{t+1} = \mu_t + \varepsilon_{d,t+1} - \rho(k_\mu \varepsilon_{\mu,t+1} - k_g \varepsilon_{g,t+1}) \tag{12}$$

where $k_\mu \equiv \frac{1}{1-\rho\phi_\mu}$ and $k_g \equiv \frac{1}{1-\rho\phi_g}$.

4.2.3 Measurement & Transition Equations

Given the processes above, we have the following state-space model, where the observable variables are $y_{t+1} = (r_{t+1}, \Delta d_{t+1})'$ and the unobserved variables are $\beta_t = (\mu_t, g_t, \varepsilon_{\mu,t+1}, \varepsilon_{g,t+1}, \varepsilon_{d,t+1})'$.
Measurement Equation:

$$\underbrace{\begin{bmatrix} r_{t+1} \\ \Delta d_{t+1} \end{bmatrix}}_{\equiv y_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 & -\rho k_\mu & \rho k_g & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\equiv H} \underbrace{\begin{bmatrix} \mu_t \\ g_t \\ \varepsilon_{\mu,t+1} \\ \varepsilon_{g,t+1} \\ \varepsilon_{d,t+1} \end{bmatrix}}_{\equiv \beta_t} \tag{13}$$

Transition Equation:

$$\underbrace{\begin{bmatrix} \mu_t \\ g_t \\ \varepsilon_{\mu,t+1} \\ \varepsilon_{g,t+1} \\ \varepsilon_{d,t+1} \end{bmatrix}}_{\equiv \beta_t} = \underbrace{\begin{bmatrix} \phi_\mu & 0 & 1 & 0 & 0 \\ 0 & \phi_g & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\equiv F} \underbrace{\begin{bmatrix} \mu_{t-1} \\ g_{t-1} \\ \varepsilon_{\mu,t} \\ \varepsilon_{g,t} \\ \varepsilon_{d,t} \end{bmatrix}}_{\equiv \beta_{t-1}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{g,t+1} \\ \varepsilon_{d,t+1} \end{bmatrix}}_{\equiv v_t} \tag{14}$$

For the error terms, to simplify the estimating process, the variances are decomposed as:

$$\sigma_{ij} = \sigma_i \times \sigma_j \times \rho_{ij} \qquad \forall i, j \in \{\mu, g, d\} \text{ and } i \neq j \tag{15}$$

4.2.4 Estimations for Stock-variable State-Space Model

Given the measurement and transition equations above, we have the following state-space model.

$$\text{Measurement Equation: } y_{t+1} = H\beta_t \tag{16}$$

$$\text{Transition Equation: } \beta_t = F\beta_{t-1} + v_t \qquad v_t \overset{i.i.d}{\sim} N(0, Q) \tag{17}$$

Following the above models, we utilize the Kalman filter (Hamilton, 1994 [24]) and conditional maximum likelihood estimation to estimate the vector of parameters[‡]:

$$(\phi_\mu, \phi_g, \sigma_\mu, \sigma_g, \sigma_d, \rho_{\mu g}, \rho_{\mu d}, \rho_{gd}) \tag{18}$$

The optimization problem is solved by using the R package *astsa* [52]. Also, when we consider the reduced form of the state-space model, there are more parameters than can be identified with the observed data. Following Cochrane(2008) [17], Van Binsbergen and Koijen (2010) [53], and Rytchkov (2012) [47], we restrict the correlation between expected dividend growth shocks and realized dividend growth shocks to be 0, $\rho_{gd} = 0$.

To examine the performance of state-space models, following Van Binsbergen and Koijen (2010) [53], we compute the R^2 values for returns as:

$$R^2_{\text{Ret}} = 1 - \frac{v\hat{a}r\left(r_{t+1} - \mu_t^F\right)}{v\hat{a}r\left(r_t\right)} \tag{19}$$

where $v\hat{a}r$ denotes the sample variance, and μ_t^F is the predictions of expected returns (μ_t).

[‡]See Appendix A.1 for the details of the Kalman Filter

Table 5 shows the conditional maximum likelihood estimates of the parameters for the previous model (Equations (13) to (18)), including log returns and log dividend yields. Also, this table reports the R^2 (Equation (19)) for in-sample one-step-ahead predictions of market returns. The one-step-forward in-sample predictions of market returns are further estimated in Table 7 and plotted in Figure 1. The out-of-sample predictions are generated in Table 9 and Figure 2.

From Table 5, we find that the conditional expected return is highly persistent over time, with a significant coefficient ($\phi_\mu = 0.9621$) at 5% level. The coefficient of conditional expected dividend growth (ϕ_g) is not significant at 5% level, with a value equal to 0.1788. The R^2 for in-sample predictions of market returns is 12.60%, and we find an R^2 value of 14.04% for dividend growth rates.

Shocks for expected returns and dividend growth rates are smaller than the shock for expected dividend growth rates. It shows that most of the information existing in expected market returns is carried over time, but the expected dividend policy is not significantly predicted due to high volatility.

These results are consistent with Cochrane (2007) [16], Pastor and Stambaugh (2009)[41], and Binsbergen and Koijen (2010) [53], showing that expected returns are more persistent than conditional expected dividend growth. But, the R^2 estimated base on Equation (19) is slightly higher than the one from Binsbergen and Koijen (2010) [53], which may be caused by different observable variables and the length of data.

4.3 State-Space Model (Market Returns and Returns on Equity)

Following Campbell and Shiller (1998a) [11], Vuolteenaho (2000) [54], and Vuolteenaho (2002) [55], by assuming the clean-surplus identity, this section presents a State-Space-Model based on returns, returns on equity, and book-to-market ratios.

Table 5: SSM Estimation Results based on Returns & Dividend Yield

	Estimates	S.E.
ϕ_μ	0.9621	0.0207
ϕ_g	0.1788	0.1235
σ_μ	0.0308	0.0120
σ_g	0.0824	0.1357
σ_d	0.0089	0.0073
$\rho_{\mu g}$	0.8181	0.1091
$\rho_{\mu d}$	-0.0604	1.1004
Market Return R^2	0.1260	
Dividend Growth R^2	0.1404	

This table reports the estimations of parameters for the state-space model based on the log returns (μ_t) and log dividend growth rate (g_t) from Equations (13) to (18). The restriction, $\rho_{gd} = 0$, is implemented. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2019, CRSP and COMPUSTAT. This table also reports the R^2 (Equation (19)) for one-step-ahead predictions (in-sample) of market returns and dividend growth rate. The one-step-forward predictions of market returns are plotted in Figure 1.

4.3.1 Clean Surplus Identity

Vuolteenaho (2000) [54] constructs the clean-surplus identity based on book equity (B_t), earnings (X_t) and dividends (D_t).

$$B_t = B_{t-1} + X_t - D_t \tag{20}$$

Based on the clean surplus accounting identity, Equation (20), and return on equity (ROE):

$$E_t = X_t / B_{t-1} \tag{21}$$

we can define log returns for market and accounting as:

$$r_t + f_t \equiv \ln\left(\frac{M_t + D_t}{M_{t-1}}\right) = \ln\left(1 + \frac{\Delta M_t + D_t}{M_{t-1}}\right) = \ln\left(1 + R_t + F_t\right) \tag{22}$$

$$e_t \equiv \ln\left(\frac{B_t + D_t}{B_{t-1}}\right) = \ln\left(1 + \frac{\Delta B_t + D_t}{B_{t-1}}\right) = \ln\left(1 + E_t\right) \tag{23}$$

where M_t denotes the market value at time t, F_t denotes the interest rate at time t, f_t denotes $\ln(1 + F_t)$ at time t, and D_t denotes the dividend at time t [54].

4.3.2 Log Linearization

By defining $\delta_t = d_t - b_t$, where d_t is the log dividend at time t and b_t is the log book equity at time t, the log market returns can be written as:

$$r_t + f_t = \ln\left(\exp\left(-\delta_t\right) + 1\right) + \Delta d_t + \delta_{t-1} \tag{24}$$

By defining $\gamma_t = d_t - m_t$, where m_t is the log market value at time t, the log accounting returns can be written as:

$$e_t = \log\left(\exp\left(-\gamma_t\right) + 1\right) + \Delta d_t + \gamma_{t-1} \tag{25}$$

Then, by using Equation (25) to subtract Equation (24), the identity can be approximated based on the first-order Taylor Series as:

$$\begin{aligned} e_t - r_t - f_t &= \ln(1 + \bar{B}/\bar{D}) + \frac{-\bar{B}/\bar{D}}{1 + \bar{B}/\bar{D}}\left(\gamma_t - \bar{\gamma}\right) - \ln(1 + \bar{M}/\bar{D}) - \frac{-\bar{M}/\bar{D}}{1 + \bar{M}/\bar{D}}\left(\delta_t - \bar{\delta}\right) - \theta_{t-1} \\ &\approx \alpha + \rho\theta_t - \theta_{t-1} + \kappa_t \end{aligned} \tag{26}$$

where θ_t denotes the log book to market ratio, α denotes a constant parameter, ρ denotes the discount ratio (a constant parameter smaller than 1), and κ_t denotes the approximation errors [§].

Similar to the log linearization for dividend yield, the log-linearized book-to-market ratio can be written as:

$$\theta_{t-1} = \frac{\alpha}{1-p} + \sum_{j=0}^{\infty} \rho^j r_{t+j} - \sum_{j=0}^{\infty} \rho^j e^*_{t+j} \tag{27}$$

where $e^*_{t+j} \equiv e_t - \kappa_t - f_t$, which denotes the log excess return on equity.

4.3.3 Assumptions for Latent Processes

Similar to the state-space model above, we assume that expected returns and excess return on equity follow the AR(1) process below, where $\mu_t \equiv E(r_{t+1}|I_t)$, $h_t \equiv E(e^*_{t+1}|I_t)$, $\varepsilon_{\mu,t+1}$ represents the shocks for expected return at time $t+1$, $\varepsilon_{h,t+1}$ represents the shocks for expected excess return on equity at time $t+1$, $\varepsilon_{e,t+1}$ represents the approximation errors for excess return on equity at time $t+1$.

$$\begin{aligned} \mu_{t+1} &= \phi_\mu \mu_t + \varepsilon_{\mu,t+1} \\ h_{t+1} &= \phi_h h_t + \varepsilon_{h,t+1} \end{aligned} \tag{28}$$

By taking the conditional expectation based on the information set, I_t, the log-linearized book-to-market ratio is:

$$\theta_t = \frac{\alpha}{1-\rho} + \frac{\mu_t}{1-\phi_\mu \rho} - \frac{h_t}{1-\phi_h \rho} \tag{29}$$

The log excess return on equity follows the equation below:

$$e^*_{t+1} = h_t + \varepsilon_{e,t+1} \tag{30}$$

[§] See Appendix A.2 for approximating κ_t in detail.

After plugging the result back into Equation (26), the identity becomes:

$$r_{t+1} = \mu_t + \varepsilon_{e,t+1} + \rho \left(k_h \varepsilon_{h,t+1} - k_\mu \varepsilon_{\mu,t+1} \right) \tag{31}$$

where $k_h \equiv \frac{1}{1-\rho\phi_h}$ and $k_\mu \equiv \frac{1}{1-\rho\phi_\mu}$. Then, we no longer have the dividends existing in the model.

4.3.4 Measurement & Transition Equations

Given the processes above, we have the following state-space model, where the observable variables are $y_{t+1} = (r_{t+1}, e^*_{t+1})'$ and the unobserved variables are $\beta_t = (\mu_t, h_t, \varepsilon_{\mu,t+1}, \varepsilon_{h,t+1}, \varepsilon_{e,t+1})'$.

Measurement Equation:

$$\underbrace{\begin{bmatrix} r_{t+1} \\ e^*_{t+1} \end{bmatrix}}_{\equiv y_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 & -\rho k_\mu & \rho k_h & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\equiv H} \underbrace{\begin{bmatrix} \mu_t \\ h_t \\ \varepsilon_{\mu,t+1} \\ \varepsilon_{h,t+1} \\ \varepsilon_{e^*,t+1} \end{bmatrix}}_{\equiv \beta_t} \tag{32}$$

Transition Equation:

$$\underbrace{\begin{bmatrix} \mu_t \\ h_t \\ \varepsilon_{\mu,t+1} \\ \varepsilon_{h,t+1} \\ \varepsilon_{e^*,t+1} \end{bmatrix}}_{\equiv \beta_t} = \underbrace{\begin{bmatrix} \phi_\mu & 0 & 1 & 0 & 0 \\ 0 & \phi_h & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\equiv F} \underbrace{\begin{bmatrix} \mu_{t-1} \\ h_{t-1} \\ \varepsilon_{\mu,t} \\ \varepsilon_{h,t} \\ \varepsilon_{e^*,t} \end{bmatrix}}_{\equiv \beta_{t-1}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{\mu,t+1} \\ \varepsilon_{h,t+1} \\ \varepsilon_{e^*,t+1} \end{bmatrix}}_{\equiv v_t} \tag{33}$$

For the error terms, to simplify the estimating process, similar to the previous state-space model, covariances are decomposed as:

$$\sigma_{ij} = \sigma_i \times \sigma_j \times \rho_{ij} \qquad \forall i, j \in \{\mu, h, e^*\} \text{ and } i \neq j \tag{34}$$

4.3.5 Estimations for Accounting-variable State-Space Model

Given the measurement and transition equations above, we have the following state-space model.

$$\text{Measurement Equation: } y_{t+1} = H\beta_t \tag{35}$$

$$\text{Transition Equation: } \beta_t = F\beta_{t-1} + v_t \qquad v_t \overset{i.i.d}{\sim} N(0, Q) \tag{36}$$

To simplify the estimating process, we utilize the Kalman filter (Hamilton, 1994 [24]) and conditional maximum likelihood estimation to estimate the vector of parameters:

$$(\phi_\mu, \phi_h, \sigma_\mu, \sigma_h, \sigma_{e^*}, \rho_{\mu h}, \rho_{\mu e^*}, \rho_{he^*}) \tag{37}$$

The optimization problem is solved by using the R package *astsa* [52]. Similar to the previous state-space model, we restrict the correlation between expected ROE shocks and realized ROE shocks to be 0, $\rho_{he} = 0$.

Table 6 shows the conditional maximum likelihood estimates of the parameters for the previous model (Equations (32) to (37)), including log returns and log returns on equity. Also, this table reports the in-sample one-step-forward predictions R^2 for market returns and dividend growth. The one-step-forward in-sample predictions are further estimated in Table 7 and plotted in Figure 1. The out-of-sample predictions are generated in Table 10 and Figure 2.

From Table 6, we find that the conditional expected return and return on equity are highly persistent over time, with statistically significant coefficients ($\phi_\mu = 0.8886$ and $\phi_h = 0.9619$) at the 5% level. Unlike the expected dividend growth rate, expected return on equity is more persistent than expected market returns, which supports the view that aggregate earnings are more stable than dividend policies. Intuitively, if firms have a high log return on equity in the previous period, they tend to have higher returns on equity in the next period.

In addition, the correlation between expected returns and returns on equity is 0.8647, which is larger (and more statistically significant) than the correlation between expected returns and dividend growth, 0.8181. This suggests that, when aggregating across firms, higher expected ROE is associated with higher expected market returns, providing more informative signals for forecasting market returns.

In Table 6, the R^2 for in-sample predictions of market returns is 17.85%, and the R^2 for returns on equity is 23.63%. Compared with Table 5, the one-step-ahead R^2 for market returns is higher. Although the dividend yield (or dividend-price ratio) contains information about future stock market returns, unstable dividend policies lead to weaker predictive performance than models based on accounting variables. Moreover, the R^2 for returns on equity exceeds that for dividend growth, further indicating that accounting data are more persistent than dividends. Section 5 provides a more detailed analysis of these findings.

Table 6: SSM Estimation Results based on Returns & Returns on Equity

	Estimates	S.E.
ϕ_μ	0.8886	0.1043
ϕ_h	0.9619	0.0359
σ_μ	0.0237	0.0187
σ_h	0.0202	0.1049
σ_e	0.0762	0.0070
$\rho_{\mu h}$	0.8647	4.2596
$\rho_{\mu e}$	-0.1598	0.1735
Market Return R^2	0.1785	
Return on Equity R^2	0.2363	

This table reports the estimations of parameters for the state-space model based on the log returns (μ_t) and log returns on equity (e_t^*) from Equations (32) to (37). The restriction, $\rho_{he} = 0$, is implemented. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2019, CRSP and COMPUSTAT. This table also reports the R^2 (Equation (19)) for one-step-ahead predictions (in-sample) of market returns and dividend growths. The one-step-forward predictions of market returns are plotted in Figure 1.

5 Results

In this section, we compare and discuss the estimated results from the previous models. We then decompose the variances of the price-dividend ratio, the book-to-market ratio, and unexpected returns in both state-space models. Finally, we split the sample into training and testing sets to evaluate the stability of the state-space models' out-of-sample predictive performance.

5.1 In-sample Predictions

Table 7 reports the in-sample one-step-ahead prediction R^2 for all methods described above. The linear regressions of returns on the dividend-price ratio or the book-to-market ratio deliver the lowest R^2, although the coefficients are statistically significant. These results are consistent with prior findings in Van Binsbergen and Koijen (2010) [53], Goyal and Welch (2008) [23], Cochrane (2007) [16], and Vuolteenaho (2002) [54]. Predictions from VAR models suggest that the stock market dynamics can be represented with lag orders selected by information criteria. However, when estimating VARs, the presence of unit-root behavior in the dividend-price and book-to-market ratios weakens predictive performance, which is also reflected in the serial correlation tests in Table 4. The state-space models deliver better in-sample predictions than the preceding approaches and can roughly capture the main movements in market returns.

Based on Tables 5 and 6, the unexpected components capture important variation relevant for return prediction. The latent process in the state-space model with market returns and dividend growth is similar to Van Binsbergen and Koijen (2010), exhibiting greater persistence in expected market returns than in expected dividend growth. The state-space model based on market returns and returns on equity yields the best in-sample predictions. It also indicates persistence in both expected market returns and expected ROE, which improves the forecastability of the predictive system.

Figure 1 plots the in-sample fitted values from the different models. The OLS and VAR models exhibit similar trends in their market return predictions. The stock-variable state-space model fluctuates around the initial expected return level and does not fully capture the time variation in returns. In contrast, the accounting-based state-space model tracks the movement of market returns over the 70-year sample, reinforcing the conclusion that accounting data can improve the performance of market return prediction.

Table 7: In-Sample Predictions R^2

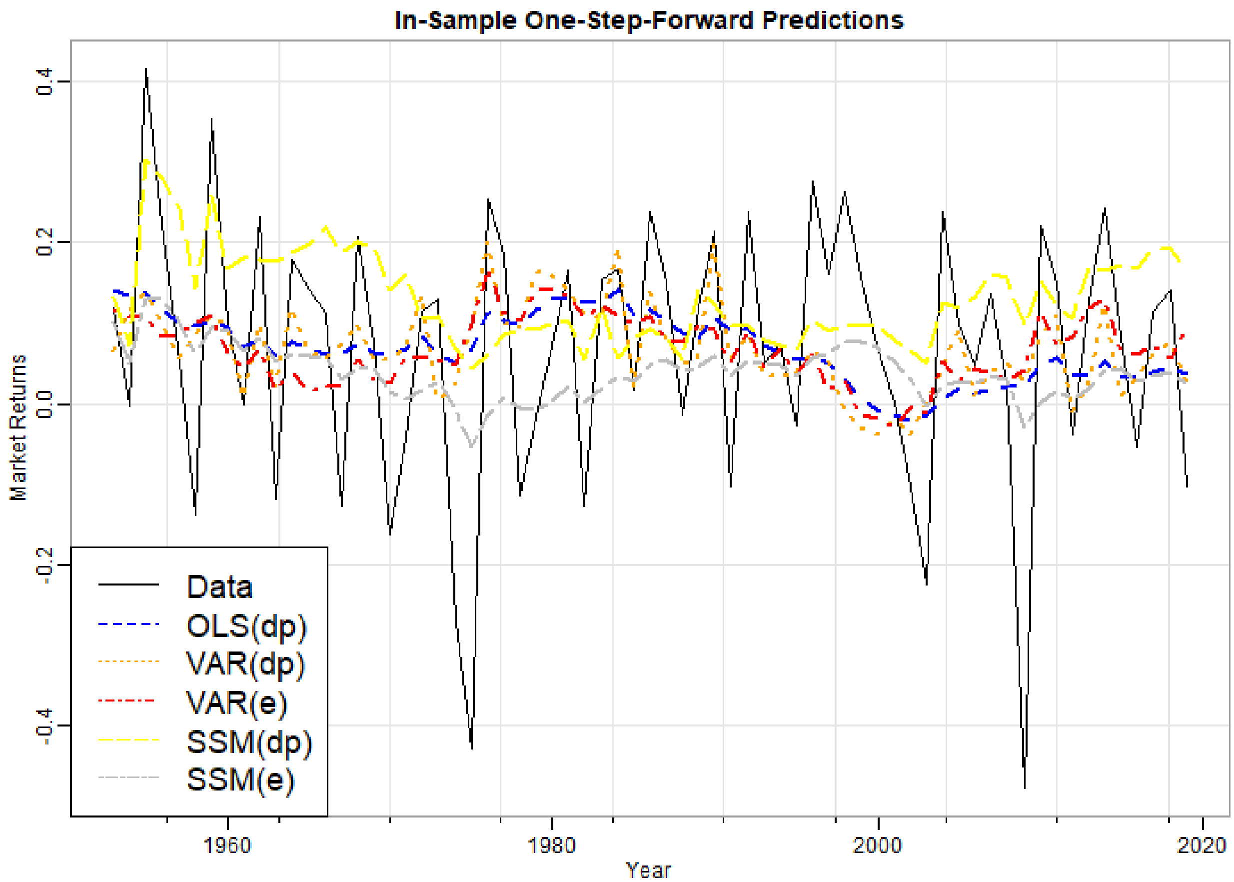
	R^2
OLS(dp)	0.0618
OLS(bm)	0.0572
VAR(dp, dg)	0.1090
VAR(e, theta)	0.0659
SSM(dp)	0.1260
SSM(e)	0.1785

This table reports the R^2 (Equation (19)) for one-step-ahead predictions based on different models. OLS(dp) and OLS(bm) denotes the predictions based on the first and fourth regressions in table 3, VAR(dp, dp) denotes the predictions based on the short VAR model (including returns, dividend yield, and dividend growth rate), VAR(e, theta) denotes the short VAR model (including returns, returns on equity, and book-to-market ratio), SSM(dp) is the state-space model based on returns and dividend-price ratio, and SSM(e) denotes the state-space model based on returns and return on equity. The predictions are plotted in Figure 1. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2019, CRSP and COMPUSTAT.

5.2 Variance Decomposition

Following Campbell and Shiller (1988a, 1988b) [11] [12], Vuolteenaho (2002) [55] and Binsbergen and Koijen (2010) [53], we derive variance decompositions of price-dividend ratio, book-to-market ratio, and unexpected returns in both state-space models. The variance

Figure 1: In-sample One-step-forward Predictions



This figure plots the one-step-forward predictions based on different models. OLS(dp) denotes the predictions based on table 3, VAR(dp, dp) denotes the predictions based on the short VAR model (including returns, dividend yield, and dividend growth rate), VAR(ROE, b/m ratio) denotes the short VAR model (including returns, returns on equity, and book-to-market ratio), SSM(dp) is the state-space model based on returns and dividend-price ratio, and SSM(e) denotes the state-space model based on returns and return on equity. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2019, CRSP and COMPUSTAT decomposition for the price-dividend ratio is given as:

$$\begin{aligned}\operatorname{var}(dp_t) = &\left(\frac{1}{1-\phi_\mu\rho}\right)^2 \operatorname{var}(\mu_t) + \left(\frac{1}{1-\phi_g\rho}\right)^2 \operatorname{var}(g_t) \\ &- \frac{2}{(1-\phi_\mu\rho)(1-\phi_g\rho)} \operatorname{cov}(\mu_t, g_t)\end{aligned} \tag{38}$$

The first term denotes the variation due to expected returns (discount rate news); the second term measures the variation due to expected dividend growth rates (cash-flow news); and the last term represents the covariation between these variation.

For the book-to-market ratio, the variance decomposition is given as:

$$\begin{aligned}\operatorname{var}(\theta_t) = &\left(\frac{1}{1-\phi_\mu\rho}\right)^2 \operatorname{var}(\mu_t) + \left(\frac{1}{1-\phi_h\rho}\right)^2 \operatorname{var}(h_t) \\ &- \frac{2}{(1-\phi_\mu\rho)(1-\phi_h\rho)} \operatorname{cov}(\mu_t, h_t)\end{aligned} \tag{39}$$

Similar to the price-dividend ratio, we decompose the book-to-market ratio into three components: variation due to expected returns (discount-rate news), variation due to expected returns on equity (cash-flow news), and the covariance between these two news components.

Panel A of Table 8 reports the variance decompositions for the price-dividend and book-to-market ratios. Following Van Binsbergen and Koijen (2010) [53], we standardize the right-hand side of Equations (38) and (39) so that the three components sum to 100%. Consistent with Van Binsbergen and Koijen (2010) [53], we find that most of the variation in the price-dividend ratio is driven by variation in expected returns. In contrast, both expected market returns and expected returns on equity play important roles in explaining variation in the book-to-market ratio.

Finally, we decompose the variation in unexpected aggregate stock returns into dividend growth news and discount-rate news as follows:

$$\begin{aligned}\operatorname{var}\left(r_{t+1}-\mu_t\right)=&\left(\rho k_\mu\right)^2 \operatorname{var}\left(\varepsilon_{\mu,t+1}\right)+\operatorname{var}\left(\varepsilon_{d,t+1}+\rho k_g \varepsilon_{g,t+1}\right)\\&-2\rho k_\mu \operatorname{cov}\left(\varepsilon_{\mu,t+1},\varepsilon_{d,t+1}+\rho k_g \varepsilon_{g,t+1}\right)\end{aligned} \tag{40}$$

This equation follows similar algorithms as above, where the variance of unexpected returns is decomposed into three parts: variation in discount rate news, variation in cash-flow news, and co covariation between these two components. The second term groups the news from real and expected dividend growth rates together.

For the decomposition of variance of unexpected aggregate stock returns with returns on equity and market returns, it is given as:

$$\begin{aligned}\operatorname{var}\left(r_{t+1}-\mu_t\right)=&\left(\rho k_\mu\right)^2 \operatorname{var}\left(\varepsilon_{\mu,t+1}\right)+\operatorname{var}\left(\varepsilon_{e,t+1}+\rho k_h \varepsilon_{h,t+1}\right)\\&-2\rho k_\mu \operatorname{cov}\left(\varepsilon_{\mu,t+1},\varepsilon_{e,t+1}+\rho k_h \varepsilon_{h,t+1}\right)\end{aligned} \tag{41}$$

As before, we group the news from real and expected returns on equity together to form the cash-flow news (second term). Then, we compute the influence of discount news, cash-flow news, and the covariance between these two terms.

In Panel B from Table 8, we show the results of variance decompositions for unexpected returns with two different variable sets, dividend growth rates and returns on equity. Similar to Panel A, we standardize all terms on the right-hand size of Equations (40) & (41), so the sum of these terms is 100%.

In Panel B, in the variance decomposition based on returns on equity, cash-flow shocks play a more prominent role in explaining the variance of unexpected market returns. In addition, the correlation between discount-rate news and cash-flow news is higher than in the decomposition based on dividend growth. This difference reflects the persistence of expected returns on equity (ϕ_h), which is larger and more statistically significant than the persistence of expected dividend growth, as shown in Tables 5 and 6. Overall, using accounting data not only improves the forecastability of market returns but also indicates that cash-flow news plays a central role in explaining variation in unexpected market returns.

Table 8: Variance Decompositions of the Price-Dividend/Book-to-Market Ratio and Unexpected Market Returns

	Discount Rates	Cash Flows	Covariance
Panel A: Decomposition of Price-Dividend/Book-to-Market Ratio			
Price-Dividend Ratio	115.6%	2.57%	-18.2%
Book-to-Market Ratio	76.5%	154.2%	-130.6%
Panel B: Decomposition of Unexpected Market Returns			
Returns & Dividends	135.8%	2.8%	-38.6%
Returns & Returns on Equity	9.5%	147.9%	-57.43%

This table reports variance decompositions of price-dividend ratio, book-to-market ratio, and unexpected returns. "Discount Rates" refers to variation due to expected return variation, "Cash Flows" refers to the variation due to expected dividend growth rates or expected returns on equity variation, and "covariance" refers to the covariation between these two terms. In Panel A, we present variance decompositions of price-dividend ratio and book-to-market ratio based on Equation (38) & (39). In Panel B, we present variance decompositions of unexpected returns with different variables based on Equation (40) & (41). The models are estimated by conditional maximum likelihood using sample data from 1950 - 2019, CRSP and COMPUSTAT.

5.3 Out-of-sample Predictions

To generate out-of-sample predictions, we split the sample into training and testing sets. The training set covers 1950-2012, and the testing set covers 2013-2019.

Table 9 reports parameter estimates for the state-space model based on market returns and dividend growth, using the training (in-sample) and testing (out-of-sample) samples. The results are similar to those obtained using the full sample (Table 5) in Section 4.2. Expected returns remain more persistent than expected dividend growth over time. Because the training sample is smaller, the in-sample R^2 is lower than the value reported in Table 5.

However, the out-of-sample R^2 is high, at 31.42%. When forecasting market returns, the state-space model performs well at short horizons. Because expected returns are persistent over time, with a statistically significant estimate of ϕ_μ, the model explains a substantial portion of out-of-sample variation in market returns.

Table 9: Out-of-Sample SSM Estimated Results based on Dividend Yield & Returns

	Estimates	S.E.
ϕ_μ	0.9574	0.0230
ϕ_g	0.1896	0.3089
σ_μ	0.0313	0.0146
σ_g	0.0443	0.4993
σ_d	0.1081	0.2045
$\rho_{\mu g}$	0.8749	3.613
$\rho_{\mu d}$	0.07518	2.3398
In-sample Market Returns R^2	0.0611	
Out-of-sample Market Returns R^2	0.3142	

This table reports the estimations of parameters for the state-space model based on the log returns and log dividend growth rate from equations (13) to (18). The restriction, $\rho_{gd} = 0$, is implemented. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2012 and tested on sample data from 2013 - 2019, CRSP and COMPUSTAT. This table also reports the R^2 (Equation (19)) for the one-step-forward predictions (in-sample) and one-step-forward predictions. The one-step-forward predictions are plotted in Figure 2.

Table 10 reports parameter estimates and R^2 values for the state-space model based on market returns and return on equity, using the training (in-sample) and testing (out-of-sample) samples. The results are similar to those in Table 6, which uses the full sample. Expected returns on equity remain more persistent than expected market returns over time. As with the stock-variable state-space model, the smaller training sample leads to a lower in-sample R^2.

In addition, the out-of-sample R^2 is 35.62%, which is higher than the corresponding value for the state-space model with returns and dividends. This suggests that, with significant and persistent state variables (ϕ_μ and ϕ_h), the latent predictive process in the accounting-based state-space model captures more information relevant for forecasting market returns. This provides further evidence that accounting earnings are more stable than dividend growth for predicting market returns.

Table 10: Out-of-Sample SSM Estimated Results based on Returns on Equity & Returns

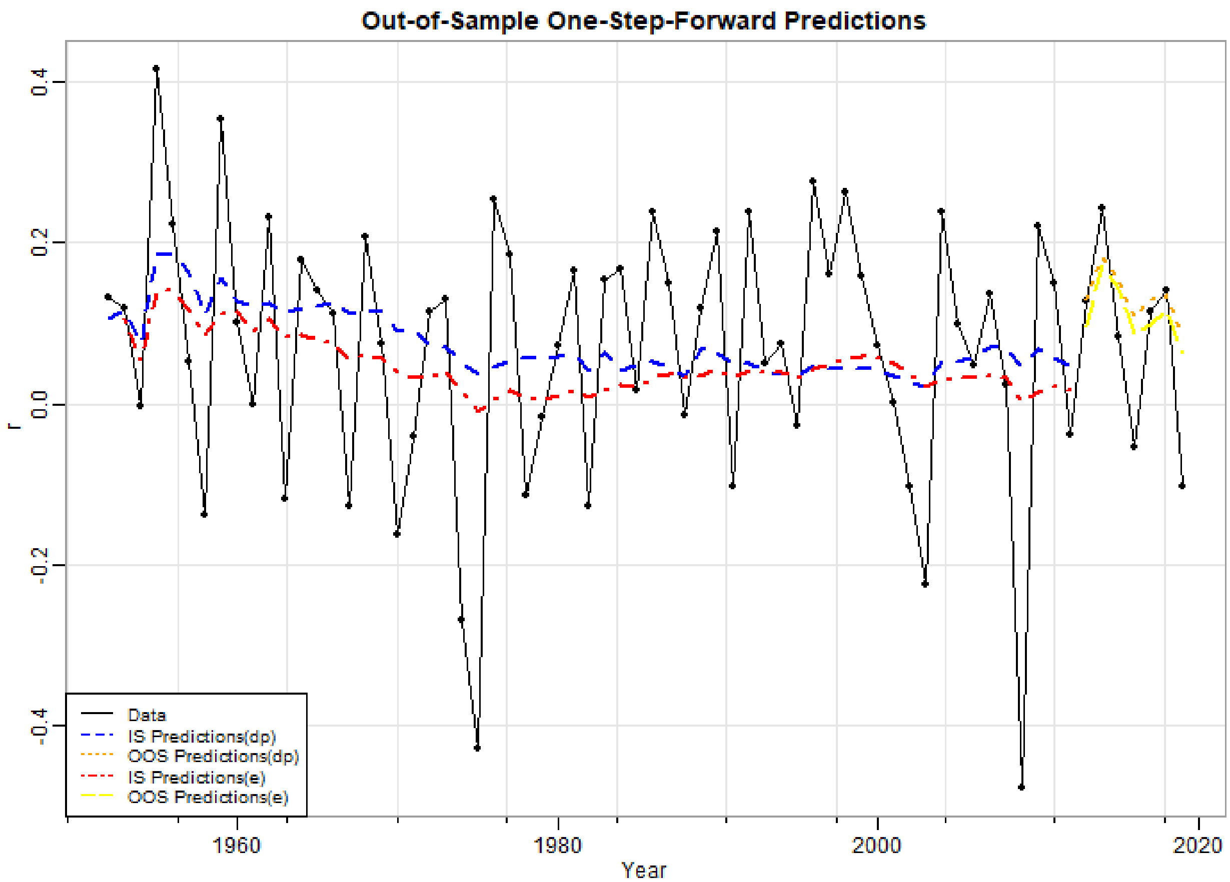
	Estimates	S.E.
ϕ_μ	0.9353	0.1177
ϕ_h	0.9815	0.0365
σ_μ	0.0141	0.296
σ_h	0.0131	0.0085
σ_e	0.0688	0.0066
$\rho_{\mu h}$	0.8108	3.4930
$\rho_{\mu e}$	-0.3318	0.5706
In-sample Market Returns R^2	0.0920	
Out-of-sample Market Returns R^2	0.3562	

This table reports the estimations of parameters for the state-space model based on the log returns and log dividend growth rate from equations (32) to (37). The restriction, $\rho_{he} = 0$, is implemented. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2012 and tested on sample data from 2013 - 2019, CRSP and COMPUSTAT. This table also reports the R^2 (Equation (19)) for the one-step-forward predictions (in-sample) and one-step-forward predictions. The one-step-forward predictions are plotted in Figure 2.

Figure 2 further supports these findings. The in-sample predictions from the state-space model based on returns and dividends fluctuate around the mean of market returns, whereas the in-sample predictions from the model based on market returns and returns on equity capture much more of the time-series variation. The out-of-sample predictions are broadly similar across the two state-space models, but the accounting-based model tracks market returns more closely by exhibiting more similar movements over time.

As a result, this section shows that the state-space model performs well and is stable within the sample. Accounting variables outperform dividend yields because dividend policies are unstable over time. Although the out-of-sample predictions are similar across models,

Figure 2: Out-of-sample One-step-forward Predictions



This plot shows the in-sample and out-of-sample one-step-forward predictions based on dividend-price ratio or log returns on equity. IS Predictions (dp) & OOS Predictions (dp) are estimated based on the state-space model of returns and dividend-price ratio. IS Predictions (e) & OOS Predictions (e) are estimated based on the state-space model of returns and returns on equity. The models are estimated by conditional maximum likelihood using sample data from 1950 - 2012 and tested on sample data from 2013 - 2019, CRSP and COMPUSTAT.

the predictability of expected returns on equity can improve the long-horizon performance of the accounting-based state-space model. In addition, when the sample is truncated, the estimated coefficients may vary across time ranges, but predictive performance appears to remain broadly consistent.

6 Conclusion

In this paper, we evaluate the performance of several models based on identities linking stock market and accounting variables. We also estimate a present-value approach to predict annual market returns by introducing latent variables, including expected market returns and expected dividend growth. The model combines a log-linearized identity to characterize the dynamics underlying the observed variables and uses the Kalman filter to estimate the latent states. Relative to VAR models, state-space models can better accommodate data instability and changes in dividend policy by allowing the latent processes to evolve over time, thereby reducing unexplained variation in the residuals.

We then propose an approach based on accounting variables, motivated by the clean surplus identity. Accounting variables are more stable than stock market variables, particularly in the presence of changes in dividend policy. As a result, latent processes based on accounting variables further improve the forecastability of future market returns. The one-step-ahead in-sample prediction R^2 for the state-space model with stock variables is 13%, whereas the state-space model with accounting variables has a higher R^2 of 18%. In addition, cash-flow news based on expected returns on equity is more persistent than the ones based on expected market returns. Results from both in-sample and out-of-sample exercises indicate that the proposed approaches improve the predictability of future returns.

These approaches generate latent processes for expected market returns and incorporate return predictions into the measurement equation through the log-linearization of market returns. In this case, information contained in the latent state variables is incorporated through the recursive updating of the predictive system. Incorporating additional variables into the latent dynamics may be useful, but doing so would require specifying more complex identities among the variables. Over long horizons, predictions tend to converge toward sample averages due to the mitigation of state variables. The filtering framework could also be extended using alternative algorithms as computational tools improve. More advanced techniques, such as machine learning and deep learning, may further improve feature selection and state updating, especially with architectures such as long short-term memory (LSTM) networks. Finally, the growing availability of data may enable the construction of alternative predictive systems with different linear constraints.

A Appendix

A.1 Kalman Filter

This section shows details on the Kalman Filter for our state-space models. The discussion follows a general case and uses the state-space model with returns and dividends in Section 4.2 as an example. The state-space model with market returns and returns on equity in Section 4.3 should follow a similar setup. In this model, unobserved state variables are $\beta_t \equiv (\mu_t, g_t, \varepsilon_{\mu,t+1}, \varepsilon_{g,t+1}, \varepsilon_{d,t+1})'$, and observed variables are $y_{t+1} \equiv (r_{t+1}, \Delta d_{t+1})'$. First, we have the measurement and transition equations as follows:

$$\text{Measurement Equation: } y_{t+1} = H\beta_t$$
$$\text{Transition Equation: } \beta_t = F\beta_{t-1} + v_t \qquad v_t \overset{i.i.d}{\sim} N(0, Q)$$

where H, F, and v_t follows Equations (13) and (14).

The Kalman Filter is given by [31] [30]:

$$\beta_{0|0} = E[\beta_0], P_{0|0} = E[\beta_0\beta_0']$$
$$\beta_{t|t-1} = F\beta_{t-1|t-1}$$
$$P_{t|t-1} = FP_{t-1|t-1}F' + Q$$
$$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - H\beta_{t|t-1}$$
$$f_{t|t-1} = HP_{t|t-1}H'$$
$$\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1}H'f_{t|t-1}^{-1}\eta_{t|t-1}$$
$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H'f_{t|t-1}^{-1}HP_{t|t-1}$$

The likelihood is based on prediction errors ($\eta_{t|t-1}$) and the covariance matrix ($f_{t|t-1}$) through each iteration from $t = 0$ to $t = T$.

$$l(\theta) = -\frac{1}{2}\sum_t \ln\left((2\pi)^n \left|f_{t|t-1}\right|\right) - \frac{1}{2}\sum_t \eta'_{t|t-1} f_{t|t-1}^{-1} \eta_{t|t-1}$$

Then, we can maximize the conditional likelihood to estimate the vector of parameters:

$$(\phi_\mu, \phi_g, \sigma_\mu, \sigma_g, \sigma_d, \rho_{\mu g}, \rho_{\mu d}, \rho_{gd})$$

A.2 Estimation of Linearized Earning Identity

This part estimate the linearized earning identity (Equation (26)) by regressing the sum of the return on equity at time period t, book-to-market ratio at time period $t-1$, and negative market return and risk-free rate at time period t on the book-to-market ratio at time period t. Thus, the equation could be re-written as:

$$e_t - r_t - f_t + \theta_{t-1} \approx \alpha + \rho\theta_t + \kappa_t \tag{42}$$

where α and ρ are constants and estimated from the regression.

From Table 11, results of the regression show the estimated ρ and α for the earning model. The lagged model explains 92.2% of the variance in the one-step ahead book-to-market ratio, while the discount rate is about 0.91. Though the estimation of α is not significant, the constant is eliminated through the substitution back to the identity. And the state-space-model is not affected.

Table 11: Estimated Identity by OLS

	Estimates	S.E.	t-stats
α	0.0045	0.0179	0.2500
ρ	0.9105	0.0327	27.880
Residual SE		0.0837	
R^2		0.9220	

This table reports estimations of the regression for the above earning identity (Equation (42)). The dependent variable is the returns on equity minus market return and risk-free rate at time period t, then minus the lagged book-to-market ratio. The independent variable is the book-to-market ratio at time period t. Sample data from 1950 - 2019, CRSP and COMPUSTAT.

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