Use It or Lose It#

Fahad Khalil,* Doyoung Kim,** Jacques Lawarree***

July 18, 2019

Abstract

We examine the optimality of budget policies imposed by a funding authority on a bureaucrat who operates under a fixed budget. In particular, we study a 'use-it-or-lose-it' policy under which the bureaucrat has to return any unspent budget without being able to 'roll over' any part to the next period. Instead of returning the unspent budget, the bureaucrat can go on a spending spree and engage in policy drift, which is inversely related to his motivation. The bureaucrat's motivation represents how well matched he is with the bureaucracy's mission. We show that a use-it-or-lose-it policy is complementary to motivation as it has stronger ex ante positive incentive effects on more motivated bureaucrats. Such ex ante positive effects can overcome the ex post inefficiency of the policy and make a use-it-or-lose-it policy optimal when the bureaucrat is well matched with the bureaucracy's mission or when its budget is large.

Keywords: bureaucracy, budget, policy drift, intrinsic motivation, optimal contract, incentives.

We thank the associate editor and the referees for very helpful comments. Data sharing is not applicable to this article as no new data were created or analyzed in this study.

* Department of Economics, University of Washington, Seattle; khalil@uw.edu
** Department of Economics, Sogang University, Seoul; dkim@sogang.ac.kr
*** Department of Economics, University of Washington, Seattle; and ECARES, Brussels; lawarree@uw.edu

1. Introduction

Many bureaucracies operate with fixed budgets under a "use-it-or-lose-it" (UILI) policy. If their budget is not spent by the end of the fiscal year, they must return any unspent budget to the funding authority.[1] This creates incentives for bureaucrats to spend any remaining budget in year-end spending sprees. For instance, bureaucrats offer generous contracts, procure unnecessary equipment, and travel to exotic places for conferences at the end of fiscal year. Colorful examples of wasted expenditures abound. The Senate Homeland Security and Governmental Affairs Subcommittee Chairman Sen. Rand Paul (R-Ky.) said he found old cartridges stacked to the ceiling for an obsolete printer that a previous subcommittee chairman ordered in an end-of-year spending binge.[2] A recent report from the Veterans Affairs Office of the Inspector General (2016) identified $311,000 spent on television sets that were put into storage and never used. This phenomenon is not limited to the U.S. In Canada, where the fiscal year ends in March, this period is known as "March Madness".[3]

Congress has long been concerned with the implications of UILI, which are summarized in numerous General Accounting Office (GAO) reports.[4] For example, the 1980 GAO report notes that "Waste occurs through funding of low-priority projects, stimulating demand for unplanned products or services and shortcutting the procurement process." To discourage wasteful spending sprees at the end of the fiscal year, Congress introduced a bipartisan bill in 2017 with financial bonuses for public employees who identify and return unused funds.[5]

Nonetheless, there has been very little research by economists on the topic as noted by Liebman and Mahoney (2017). Analyzing U.S. federal procurement spending, they show that expenditure in the last week of the fiscal year is 4.9 times higher than the weekly average for the

---

[1] This phenomenon is observed in many countries as shown in a 2009 IMF report (Lienert and Ljungman (2009)). It is also the case for a majority of federal and state agencies in the United States, and for many private organizations.
[2] Subcommittee on Federal Spending Oversight and Emergency Management (2017).
[3] For instance, see the CBC report, "Rush order for 31,000 smartphones signals return of 'March Madness' budget rush," (2018), https://www.cbc.ca/news/politics/march-madness-fiscal-smartphones-1.4634779.
[4] GAO reports 1980a, 1980b, 1985, 1998.
[5] Bonuses for Cost-Cutters Act of 2017 https://www.congress.gov/bill/115th-congress/senate-bill/1830 .

rest of the year, and that year-end projects are associated with a significant drop in quality scores. Stressing the inefficiency of such budget policies, they calculate that allowing bureaucracies to roll over unspent budgets into the subsequent fiscal year could lead to welfare gains of up to 13%.[6]

Proponents of UILI cite loss of policy control and oversight if agencies are allowed to roll over unspent budgets.[7] Although UILI may lead to inefficient year-end spending, by restricting expenditure to a well-defined time period, UILI allows for regular monitoring of spending obligations. Accordingly, a recent International Monetary Fund report cautions against hasty moves to allow rolling over of unspent budgets, arguing for strict restrictions if it is to be allowed (Lienert and Ljungman (2009)).

In this paper we suggest another benefit of UILI. We present an economic rationale for UILI by highlighting a typically ignored benefit of this policy and the role of motivation of bureaucrats. We show that UILI provides a bureaucrat with ex ante incentives to exert effort to make the bureaucracy more efficient. The key observation is that a spending spree also represents a penalty for a bureaucrat who cares more about the mission of the bureaucracy than those last minute spending sprees and will therefore exert effort to avoid such last minute spending. Thus, we argue that being able to roll over unspent budgets may help reduce ex post inefficient spending, but it would hurt ex ante incentives relative to UILI.

Our model of bureaucracy has two key elements: a fixed budget and a motivated bureaucrat. First, most bureaucracies operate under fixed budgets that are largely unrelated to their performance. There is a large literature in political science arguing that funding authorities should have little control over bureaucratic agencies other than being able to fix their budgets.[8] The economics literature (e.g., Tirole (1994)) has also pointed out the difficulty of measuring the performance of bureaucracies. This inability to observe the output leaves no choice to the

---

[6] Similarly, surveying practitioners in Department of Defense financial management and contracting communities, McPherson (2007) found that 75 percent favor the ability to roll over unspent budgets.

[7] When the funding authority is Congress and its members run for re-election, they will want their constituencies to benefit from the allocated budget before the next election.

[8] For instance, Brehm and Gates (1997) argue that bureaucrats should be protected from political influence. See also Aberbach et al. (1981) and Moe (1997).

funding authority besides providing fixed budgets to the bureaucrat.

A second key element of our model is the motivation of the bureaucrat. The economics and political science literatures have stressed that most bureaucrats do not operate under explicit incentives but are intrinsically motivated to fulfil the mission of the bureaucracy. As noted by Rose-Ackerman (1986), bureaucrats are trained in "professions which emphasize not only technical competence but also conscientious devotion to duty".[9] Besley and Ghatak (2005) and Prendergast (2007) argue that bureaucrats are often intrinsically motivated to deliver goods or services they are engaged to produce (see also Benabou and Tirole (2003) on intrinsic motivation).[10] Following Besley and Ghatak (2005), we define motivation by the degree to which the preference of a bureaucrat is aligned with the mission of the bureaucracy. For example, an environmentalist working for the EPA would have high motivation.

However, even motivated bureaucrats can go on spending sprees referred to as policy drift (Migué and Bélanger (1975), Antle and Eppen (1985)). This policy drift often includes expenses less related to the mission of the bureaucracy and is sometimes seen as the "bureaucratic equivalent of personal income" (Moe (1997)).[11] Therefore, the bureaucrat in our model has twin objectives of output and policy drift, but a more motivated bureaucrat is more interested in the mission of the agency and less tempted by policy drift. We show that a UILI budget policy is complementary to motivation.

We present a two-period model of a bureaucracy. A principal (funding authority) provides a bureaucrat with a fixed budget and cannot contract on the bureaucrat's output. The bureaucrat carries out the agency's mission in an environment with uncertain cost. In period 1, the bureaucrat exerts an effort that increases the probability that the cost of production will be low.

---

[9] Brehm and Gates (1997), discussing the role of professional standard norms and self-selection, write in the preface to their book, "the police officer, the social worker, the NASA engineer, the health inspector chose their jobs not for the possibility of maximizing leisure, or even for the material rewards of the job, but for the intrinsic character of the job itself."

[10] For instance, Dhillon et al. (2017) show that the perception of the level of intrinsic motivation for bureaucrats can determine their incentive to accept bribe.

[11] Biglaiser and Ma (2007) study another example of policy drift when public employees are allowed to moonlight in the private sector.

This effort captures the bureaucrat's attempts to identify opportunities for savings and better leverage existing resources. As U.S. Senator Peters explained recently during Congressional hearings, "many Federal purchases require more than a simple wave of a wand to complete. Depending on the purchase, agencies must consider the required specs for the service or the product, request and evaluate bids from vendors, negotiate prices, obtain managerial approval on purchase orders, and draft contracts detailing the terms of the agreements".[12] If the cost is low, the bureaucrat can utilize the budget more effectively to promote the agency's mission, but if the cost is high, it may not be efficient to utilize the full budget on the core mission. If the budget is not fully utilized, the bureaucrat can either engage in policy drift, or, if the principal allows it, the bureaucrat can roll over part or all of the budget hoping for a low cost of production in period 2.

While the benefit from avoiding wasteful spending is obvious, the incentive effect of allowing the bureaucrat to roll over unspent budget is subtle. On the one hand, being able to affect the outcome over two periods increases the incentive to work hard. On the other hand, the second period payoff provides for a safety net for the bureaucrat, which undermines effort incentives as he now has a second chance to attain low cost. In contrast, under UILI, a motivated bureaucrat will work hard to increase the likelihood of low cost of production since he has only one chance at utilizing the budget. We show that this incentive effect can not only lead to higher effort under UILI but it may be strong enough to overcome the negative impact of wasteful spending associated with UILI. This is an example of the tension that often exists between ex post efficiency and ex ante incentives.[13] We show that UILI is an optimal budget policy if the bureaucrat is highly motivated or if the agency budget is large.

As already mentioned, there is a large literature in economics and political science studying the incentives (or lack thereof) for bureaucrats. Other reasons for low-powered incentives for bureaucrats are lack of time consistency (Tirole (1994)), career concerns (Dewatripont et al. (1999) and Alesina and Tabellini (2007)), multitasking (Holmstrom and Milgrom (1991)), and multiple

[12] Subcommittee on Federal Spending Oversight and Emergency Management (2017).
[13] There is a large related literature in incentives studying the trade-off between commitment and flexibility (see Halac and Yared (2017, 2018) for recent contributions and references therein). Another example is the literature on rules versus discretion in monetary and fiscal policies (see Taylor (2017) for a recent survey).

principals (Martimort (1996, 2007) and Dixit (2002)).[14] None of these papers, however, study the incentive effects of budget policies.

Our paper is also related to the literature studying how much discretion to allocate to bureaucrats. Allowing the bureaucrat to roll over his budget is a mild form of discretion while delegation is a more extreme form. For instance, Hiriart and Martimort (2012) argue that delegation may be optimal in a hierarchy of congress-regulator-firm. In their model, the regulator has private information about some potential damage (e.g., pollution) by the firm. They find that delegation allows the regulator to tailor the contract to the potential damage. Shin and Strausz (2014) also study the benefit of delegation when there is private information transmission over time. There is a well-developed literature on delegation started by Laffont and Martimort (1998), Faure-Grimaud et al. (2003), Mookherjee and Tsumagari (2004), Celik (2009) and surveyed in Mookherjee (2013).[15]

The rest of the paper is organized as follows. In section 2, we present the model. In section 3, we derive our main results. In section 4, we present extensions of our model using alternative assumptions. We conclude the paper in section 5.

2. Model

We consider the relationship between a funding authority (principal; she) and a bureaucrat (agent; he). The funding authority could be the legislature. She cares about output for the public but does not have the time or the ability to manage the agent who runs the production process. In particular, we assume the funding authority cannot observe the output and therefore delegates the task of producing the output to the bureaucrat, and gives him a fixed, exogenously given, budget $B > 0$. The principal chooses the budget policy (whether to allow the bureaucrat to roll over unspent budgets or not) to maximize the expected output net of the budget given to the bureaucrat. As an extension, we later let the principal to choose the size of the budget and show its relation

[14] For empirical evidence of low-powered incentives for bureaucrats, see Borcherding and Besocke (2003).
[15] For the literature focusing on delegation under incomplete contracts, see Dessein (2014).

with the budget policies.

With this budget B, the bureaucrat produces an output $X \geq 0$ at cost $C(X) = \frac{c_i}{2}X^2$, where the cost parameter c_i can take two values, for $i \in \{L, H\}$: c_L with probability q and c_H with probability $1 - q$, where $c_H > c_L > 0$, implying that c_L represents a state with low cost whereas c_H a state with high cost. The bureaucrat can exert an effort to improve the cost realization, e.g., by initiating production tasks and organizing the bureaucracy in a way that improves the chances of a lower cost environment as we indicated in the introduction. This is consistent with recurrent concerns in GAO reports that, "poor planning causes a bunching up of procurement at the end of the year".[16]

A higher effort is more likely to produce a low cost realization. Formally, the bureaucrat chooses an effort $q \in [0,1]$ at cost $\frac{1}{2\psi}q^2$, where $\psi > 0$.[17] The bureaucrat's effort and the cost realization are privately known to the bureaucrat. Given the cost c_i, the bureaucrat produces output X_i, where $i \in \{L, H\}$. The budget constraint requires that the cost must be covered by the budget, $\frac{c_i}{2}X_i^2 \leq B$, and we define the unspent budget in each state as $B - \frac{c_i}{2}X_i^2$.

The principal sets the policy regarding budget rollover at the beginning of the relationship. We define $\rho \in \{0,1\}$ as an indicator for rollover. If the principal chooses $\rho = 1$, it allows for rollover of unspent budgets, $B - \frac{c_i}{2}X_i^2$, and we call this the roll-it-over (RIO) policy choice. The bureaucrat can postpone production hoping for a lower cost environment next period as he obtains a second independent draw for the cost parameter c_i based on the same q chosen in the previous period.[18] We call period 1 the period when the original draw takes place and period 2 when the second draw occurs if rollover was exercised. The bureaucrat can choose between producing in period 1 or rolling over the budget to produce in period 2. To be clear, under RIO, we assume that the bureaucrat can only produce once. As will become apparent, our modeling makes it easy

[16] See for instance GAO (1980a).
[17] Instead of directly choosing the probability, we can also model that the bureaucrat chooses effort e that affects the probability q such that $q'(e) > 0$. The results of our paper are robust to this alternative modeling. Furthermore, with a low enough choice of ψ in the cost function of effort, we can ensure that the solution for q is a probability.
[18] In the other extreme, when the draws in the two periods are perfectly correlated, there is no reason to delay the production and the principal's budget policies are irrelevant.

to see the key trade-off associated with budget policies. In the extension section, we study the cases in which a separate budget is given in each of the two periods and production takes place in both periods. In addition, the bureaucrat can choose a different q in period 2. In that richer two-period setting, we can also study the implications of the bureaucrat rolling over only a fraction of the budget. We assume that the discount rate is zero.

If the principal does not allow for rollover, the bureaucrat has to use all of the budget in period 1 or lose it as he has to return any unspent budget, $B - \frac{c_i}{2} X_i^2$, to the principal. This is the use-it-or-lose-it (UILI) policy.

As argued in the introduction, the bureaucrat cares about the output net of production cost, $X_i - \frac{c_i}{2} X_i^2$, but he also values unspent budget after production, $B - \frac{c_i}{2} X_i^2$, as it allows him to go on a spending spree or engage in policy drift. We use the term policy drift for the rest of our paper as it is a broader concept than a spending spree. As in Khalil et al. (2013), we model the bureaucrat's value of policy drift by the expression $k\left(B - \frac{c_i}{2} X_i^2\right)$, where the parameter $k \in (0,1)$ represents the bureaucrat's relative preference for unspent budget.

A smaller value of k represents a more motivated bureaucrat, i.e., one who mainly cares about the output. For example, a k close to zero represents the case of an environmentalist in charge of the EPA or a school teacher in charge of the department of education. A very high k would mean that the value of the unspent budget is close to being personal income for the bureaucrat. For an extreme example, as reported in the New York Times (2018), sheriffs in Alabama were, until recently, legally allowed to keep for themselves unspent money for prisoners' meals. Two area sheriffs bought a truckload of sausages at a bargain and fed corn dogs to prisoners at every meal for about three months. Another sheriff invested \$150,000 of unspent jail food money into a used car dealership, while yet another one bought a beach house for \$740,000 with leftover jail food money.

We assume that the bureaucrat's outside option is normalized to zero. Then the participation constraint of the bureaucrat is always satisfied since he has the option to do zero effort and produce zero output. If the bureaucrat's outside option is strictly positive and above the bureaucrat's expected utility from the offered contract, the principal must give a direct transfer to the bureaucrat to satisfy the outside option.[19]

The timing of the model can be summarized as follows. In period 1, (i) the principal sets the budget policy ρ; (ii) the principal gives a budget B to the bureaucrat; (iii) the bureaucrat exerts effort q; (iv) the bureaucrat learns the first-period cost realization c_i; (v) the bureaucrat determines whether to roll over the budget (under RIO) or produces output X_i. If rollover occurs, the bureaucrat learns the second-period cost realization c_i and then produces output X_i.

3. Optimality of UILI

We start by characterizing the bureaucrat's output (and rollover decisions under RIO), and then his effort choice, before studying the principal's budget policy decision.

The bureaucrat's output and rollover decisions

Under UILI, the bureaucrat only determines output given the realized cost. Given B, k, and c_i, the bureaucrat's problem is to choose X_i in order to

$$\max \ X_i - \frac{c_i}{2}X_i^2 + k\left[B - \frac{c_i}{2}X_i^2\right]$$
$$\text{s.t.} \ (BG_i) \quad B \geq \frac{c_i}{2}X_i^2.$$

Ignoring the budget constraint (BG_i), the output produced by the bureaucrat for cost realization c_i is given by $X_i^* = \frac{1}{(1+k)c_i}$, independent of B, and the cost is $C_i(X_i^*) = \frac{1}{2(1+k)^2 c_i}$. It implies that if $B \geq \frac{1}{2(1+k)^2 c_i}$, the budget is not binding, and the bureaucrat produces his desired (i.e., unconstrained) output and uses any remaining unspent budget for policy drift $B - C_i(X_i^*) \geq 0$. Otherwise, the budget constraint is binding, and the bureaucrat produces the constrained

[19] If the budget is chosen optimally (see section 4.1), a direct transfer is more efficient than increasing B to satisfy the outside option.

output $X_i^* = \sqrt{\frac{2B}{c_i}}$ and has no chance for policy drift as $B - C_i(X_i^*) = 0$.

Defining by $B_i \stackrel{\text{def}}{=} \frac{1}{2(1+k)^2 c_i}$, the cutoff of B below which the budget is binding for cost realization $i \in \{L, H\}$, there are three different regimes depending on the size of the budget: the budget is binding for both cost realizations ($B < B_H$), only for the low cost realization ($B \in [B_H, B_L)$), or is not binding for either cost realization ($B \geq B_L$).

We define the bureaucrat's equilibrium payoff by $U_i(B, k, c_i) \stackrel{\text{def}}{=} X_i^* - C_i(X_i^*) + k[B - C_i(X_i^*)]$ and the difference in the bureaucrat's equilibrium payoffs between the two possible cost realizations by $\Delta U(B, k, c_L, c_H) \stackrel{\text{def}}{=} U_L - U_H > 0$. Intuitively, when the cost is low, the bureaucrat can produce the same output as in the high cost environment but at a lower cost and thereby benefit from a larger policy drift. Thus he has a higher payoff under a low cost realization.[20] For the rest of the paper, we will suppress the arguments and write U_i and ΔU to simplify the notation.

Under RIO, the bureaucrat chooses either to produce output in period 1 or to roll over the budget and wait for a second draw of the cost parameter to produce in period 2. Thus, the bureaucrat produces output only once either in period 1 or in period 2. Hence, when he produces, his output choice is the same as under UILI.[21] He compares his payoff when producing in period 1 versus his expected payoff when rolling over the budget and producing in period 2.

If the first-period costs are low, the bureaucrat has no reason to delay production to the second period as he prefers low-cost production ($U_L > U_H$). However, if the first-period costs are high, instead of producing output in period 1, he will exercise his option by rolling over the entire budget hoping for a lower cost realization in period 2.[22] This is the main benefit of RIO: allowing the bureaucrat to delay production hoping that the cost realization will be low in the next

[20] A formal proof is given in the proof of lemma 1 in the appendix.
[21] The result of identical output under the two policies is due to the assumption of identical budget in each policy and the assumption that the bureaucrat only produces once in the base model. In section 4, the agent can produce in multiple periods and the output is no longer identical under the two policies.
[22] We will extend the model to discuss partial rollover in section 4.

period. Denoting with r_i the amount of rollover budget under cost realization c_i, we obtain the following lemma.

LEMMA 1: *If he is allowed to roll over the budget, the bureaucrat will do so when the cost is high, i.e.,* $r_L^* = 0$ *and* $r_H^* = B$.

PROOF: In Appendix A.

The bureaucrat's effort decision

Under UILI ($\rho = 0$), the bureaucrat chooses q to maximize

$$qU_L + (1-q)U_H - \frac{q^2}{2\psi}.$$

Denoting by q_ρ the effort chosen by the bureaucrat given $\rho \in \{0,1\}$, the first-order condition for the bureaucrat's effort choice problem is

$$(U_L - U_H) = \frac{q_0}{\psi} \Leftrightarrow q_0 = \psi\Delta U. \tag{1}$$

The marginal benefit of effort under UILI comes from increasing the likelihood of the low cost realization relative to the high cost realization, and it is given by the utility gap, i.e., ΔU in (1). A higher utility gap gives stronger incentives to exert effort. We call this effect on incentives the 'utility gap effect.'

Under RIO ($\rho = 1$), the bureaucrat's effort determines the likelihood of cost realizations for both periods, and he chooses q to maximize

$$qU_L + (1-q)EU - \frac{q^2}{2\psi},$$

where $EU \equiv [qU_L + (1-q)U_H]$ represents the expected utility of the second draw. The first-order condition is

$$[U_L - EU] + (1 - q_1)\frac{\partial EU}{\partial q} = \frac{q_1}{\psi} \Leftrightarrow q_1 = \frac{2\psi\Delta U}{1 + 2\psi\Delta U}, \quad (2)$$

since $\frac{\partial EU}{\partial q} = \Delta U$. The marginal benefit of effort has two terms under RIO. There is an obviously positive effect of rollover on the bureaucrat's incentive: being able to affect the outcome over two periods. This is captured by the last term of the marginal benefit of effort under RIO in condition (2): $(1 - q_1)\frac{\partial EU}{\partial q} = (1 - q_1)\Delta U$. We simply call it the 'second period effect' of RIO. However, having a second chance also provides a 'safety net,' which can dampen incentives under RIO. As in the case for UILI, there is a utility gap effect on incentives under RIO, but the utility gap is smaller since the 'bad outcome' is now the second draw instead of the high cost realization – the bureaucrat receives the expected utility from a second draw instead of receiving U_H. The first term in (2), $[U_L - EU]$, is the utility gap, which can be rewritten as $(1 - q_1)\Delta U$. Thus, the utility gap is smaller under RIO than under UILI.

Comparing the two policies, the RIO policy has an additional incentive effect: a positive second period effect from a safety net – if the bureaucrat fails in period 1 (i.e., when costs are high), he has another chance of spending the budget efficiently in period 2. But this safety net reduces the cost of failure in period 1 and reduces the incentives from the utility gap effect. The key intuition is that the RIO allows the bureaucrat to allocate his budget more efficiently between two periods, but this discretion can diminish the marginal value of effort. By contrast, under the UILI, the bureaucrat has no such discretion and hence can have a stronger incentive to make cost-reducing effort. We capture this trade-off in the next proposition.[23]

PROPOSITION 1: *UILI can provide stronger incentives for effort than RIO. More specifically, there exists a cost threshold* $\widehat{c_L} \in (0, c_H)$, *where* $\Delta U(\widehat{c_L}) \stackrel{\text{def}}{=} \frac{1}{2\psi}$, *such that* $q_0 > q_1$ *if and only if* $c_L < \widehat{c_L}$.

---

[23] From (1) and (2), we can see that both q_0 and q_1 are a function of ψ and ΔU, which itself is a function of k, B, c_H and c_L. The comparison between q_0 and q_1 immediately gives Proposition 1.

PROOF: In Appendix A.

The proposition says that UILI can lead to a higher effort than RIO if c_L is small enough. A smaller c_L increases ΔU as it increases U_L, and ΔU is the key determinant of q from conditions (1) and (2). Under both budget policies, an increase in ΔU increases q but at different rates as explained below.

Under UILI, the bureaucrat's effort incentives stem only from the utility gap effect, which is ΔU. RIO has an apparent advantage in effort incentives since there are two effects: the utility gap effect and the second period effect. However, the two effects sum up to: $2(1-q_1)\Delta U$. Thus, when effort is high ($q_1 > \frac{1}{2}$), UILI provides stronger incentives than RIO. Intuitively, a higher effort (an increase in q) makes both effects smaller under RIO. The utility gap effect ($U_L - EU$) decreases with q as the expected utility from a second draw (EU) increases. This is due to the safety net provided by RIO. In addition, the second period effect is also less relevant as q increases because the probability of reaching the second period is low (a good outcome is more likely in period 1).

We now study comparative static effects on the threshold $\widehat{c_L}$ in proposition 2, where we examine how the bureaucrat's motivation, the size of the budget and his cost of effort affect his incentives.

PROPOSITION 2: *The cost threshold* $\widehat{c_L}$ *below which UILI elicits higher effort than RIO weakly increases:*

(i) *with the bureaucrat's motivation:* $\frac{\partial \widehat{c_L}}{\partial k} < 0$ *for* $B > B_H$ *and* $\frac{\partial \widehat{c_L}}{\partial k} = 0$ *for* $B \leq B_H$

(ii) *with the size of the budget:* $\frac{\partial \widehat{c_L}}{\partial B} > 0$ *for* $B < B_L$, $\frac{\partial \widehat{c_L}}{\partial B} = 0$ *for* $B \geq B_L$

(iii) *with a lower cost of effort:* $\frac{\partial \widehat{c_L}}{\partial \psi} > 0$.

PROOF: In Appendix A.

Consider first the impact of the bureaucrat's motivation. When the budget is large enough to generate a policy drift ($B > B_H$), the cost threshold is inversely related to k. In other words, a more motivated bureaucrat has stronger incentives under UILI. This may appear to be counterintuitive since a more motivated bureaucrat benefits less from policy drift. Under UILI, when cost is high, the bureaucrat has to engage in policy drift as he does not have a second chance to produce output. Since a more motivated bureaucrat benefits less from policy drift, he will work harder to avoid having to engage in policy drift.[24] When the budget is so low that the entire budget is spent on output even when cost is high, the bureaucrat's motivation plays no role in the choice of budget policy.

Next, consider the impact of the size of budget on the strength of the incentive effect of UILI relative to RIO. When the budget is binding for some cost realization, the proposition shows that the cost threshold increases with B. In other words, UILI provides stronger incentives when the bureaucrat is given a larger budget. Intuitively, an increase in B relaxes the budget constraint and will lead to higher production as the bureaucrat's output choice better reflects the underlying efficiency. This implies an increase in ΔU and it makes UILI relatively more effective. When the budget is not binding for either cost realization, a change in B has no impact on incentives as it has no impact on output or ΔU.

The proposition also shows that UILI is more effective when the cost of effort is lower (higher ψ) as it leads to a higher ΔU.

Finally, we note that the bureaucrat prefers to work under RIO than UILI. Intuitively, under RIO, while the bureaucrat cannot commit not to roll over, he can still choose the same effort as under UILI but has a higher payoff because he has an option to delay production. Of course, he would not choose the same effort because it would not maximize his payoff.[25]

---

[24] Technically, when the budget is not binding, a decrease in k increases ΔU, making the cost threshold decreases with k. When the budget is not binding, the policy drift is larger after a high cost realization. It implies that a decrease in k has a larger negative impact on U_H relative to U_L.

[25] To see this formally, note that $q_0 U_L + (1-q_0)U_H - \frac{q_0^2}{2\psi} < q_0 U_L + (1-q_0)EU(q_0) - \frac{q_0^2}{2\psi} < q_1 U_L + (1-q_1)EU(q_1) - \frac{q_1^2}{2\psi}$. The first inequality shows that the bureaucrat's payoff when he chooses q_0 under UILI is smaller than his payoff when he chooses q_0 under RIO because $EU(q_0) > U_H$. The second inequality comes from the fact that q_1 maximizes the bureaucrat's payoff under RIO.

The principal's budget policy decision

Having characterized the bureaucrat's decisions on output, rollover, and effort, we now study the principal's decision on the budget policy: choose UILI ($\rho = 0$) or RIO ($\rho = 1$)? The principal chooses $\rho \in \{0,1\}$ to maximize her expected payoff. We define V_ρ as the principal's expected payoff. Then

$$V_0 = q_0 X_L^* + (1 - q_0) X_H^* - B,$$

$$V_1 = q_1 X_L^* + (1 - q_1)[q_1 X_L^* + (1 - q_1) X_H^*] - B.$$

The difference of the principal's payoffs under the two policies is

$$\begin{aligned} V_0 - V_1 &= (q_0 - q_1)(X_L^* - X_H^*) - (1 - q_1) q_1 (X_L^* - X_H^*) \\ &= (q_0 - \hat{q}_1)(X_L^* - X_H^*), \end{aligned} \tag{3}$$

where $\hat{q}_1 = q_1 + (1 - q_1) q_1$ is the aggregate probability over two periods of obtaining c_L under $\rho = 1$. Recall that the bureaucrat's choice of output X_i^* is independent of the budget policy. With (1) and (2), condition (3) immediately gives the following proposition.

PROPOSITION 3: *UILI can be preferred to RIO by the principal. More specifically, there exists a cost threshold $\bar{c_L} \in (0, \hat{c_L})$, where $\Delta U(\bar{c_L}) \stackrel{\text{def}}{=} \frac{\sqrt{3}}{2\psi}$, such that $V_0 > V_1$ if and only if $c_L < \bar{c_L}$.*

PROOF: In Appendix A.

The proposition says that UILI can be preferred to RIO, and it is an example of the tension that often exists between ex post efficiency and ex ante incentives. In propositions 1 and 2, we discussed the effect of budget policies on the bureaucrat's incentives, but the optimal policy from the principal's perspective depends not only on the bureaucrat's incentive effect but also on the ex-post inefficiency associated with lower output and policy drift under UILI. It is only if the incentive effect of UILI overcomes the inefficiency associated with it that the principal prefers UILI. Accordingly, in proposition 3, the threshold c_L below which UILI is optimal is smaller than the threshold for which UILI provides stronger effort incentives in proposition 1.

In our model, the inefficiency associated with UILI occurs when cost is high. The bureaucrat does not get a second chance to produce output, and there is a low output given the high cost and ensuing policy drift. Compared to RIO, UILI lowers the principal's payoff by $(1-q_1)q_1(X_L^*-X_H^*)$, which is the second term in (3). Thus, for UILI to be optimal, it has to provide relatively stronger incentives for effort, which requires c_L to be smaller than the threshold identified in proposition 2. Given that ΔU is decreasing in c_L, ΔU must be larger to provide the bureaucrat with much stronger incentives.

Since the optimality of UILI depends on whether ΔU is large enough, the comparative statics of the optimality of UILI with respect to k, B, and ψ are the same as the ones in proposition 2 as stated in the following proposition.

PROPOSITION 4: *The cost threshold* $\overline{c_L}$ *below which the principal prefers UILI to RIO weakly increases with the bureaucrat's motivation, with the size of the budget, and with a lower cost of effort.*

The proposition shows that the range of parameters over which UILI is optimal increases with the bureaucrat's motivation and the size of the budget. The popular press emphasizes the ex post inefficiency due to UILI and ignores its ex ante incentive effects on the bureaucrat's effort. Bureaucrats work to increase the chance of good opportunities to pursue agency objectives. We find that the value of these opportunities are relatively higher when budgets are larger, and when bureaucrats are more motivated.

We can interpret a lower value of *k* as tighter control of the funding authority over the policy drift, for instance by increasing the bureaucrat's accountability. This would make it more difficult for a bureaucrat to divert funds from the agency's main mission. An example is the Bonuses for Cost-Cutters Act of 2016, which was an effort to encourage any employee of the agency to report wasteful spending by rewarding them with cash bonuses up to 10% of the returned funds. In our model, a high k means a less strict control (the extra budget can be spent to attend a conference in a nice and sunny place). An example is when the Alabama Attorney General ruled in a 2008 opinion that a "sheriff may retain any surplus from the food service allowance as personal income."[26] The principal prefers a stricter control since it induces the bureaucrat to exert a higher effort and produce more output.[27] Our result in proposition 4 then shows that UILI is more likely to be optimal when such accountability is high.

4. Extensions

4.1. Endogenous budget

In this section, we discuss the case where the principal chooses not only the budget policy ρ but also the size of the budget B. The principal's payoff depends on the effort and output chosen by the bureaucrat, which do not necessarily imply a concave objective function in B for the principal's problem. This familiar technical problem[28] makes it difficult to characterize precisely the optimal budget chosen by the principal. Nevertheless, we can provide the lower and upper bounds of the optimal budget that a principal would choose.

In Appendix C, we show that, regardless of the budget policy, the principal would not choose a budget larger than an amount such that there is unspent budget in the good state, or smaller than an amount such that the budget is binding in both states. In other words, the optimal budget $B \in [B_H, B_L]$, and the budget can be binding only for the low cost realization.

To understand the intuition behind this result, recall that, in addition to a physical cost of production, the bureaucrat has an opportunity cost of production as he can engage in policy drift. Since leaving unspent budget in the hands of the bureaucrat induces costly policy drift in both states, there is no reason to give budget B above B_L.[29] When $B < B_H$, there is no policy drift, and the principal has no incentive to lower B below B_H.

[26] The New York Times (2018).
[27] We formally prove in appendix B that a decrease in k increases effort (since there is an increase in ΔU) and output.
[28] See Khalil et al. (2013).
[29] One may wonder if the budget is strictly binding in the low cost realization: $B < B_L$. However, we cannot eliminate the possibility that $B = B_L$ at the optimum. That is, we can show that a decrease in B from B_L can decrease the principal's payoff. It is because the loss in low-cost output due to a decrease in B from B_L can be too high as the bureaucrat produces less output than what the principal would have produced.

Recall from proposition 2 that UILI provides stronger incentives over a larger range of parameters for a more motivated bureaucrat only if $B > B_H$, and for a larger budget only if $B < B_L$. It is indeed the case as the principal chooses the budget between B_H and B_L. Also, both values of B_H and B_L decrease with k, therefore, both B_H and B_L are higher with more motivated bureaucrats.

With $B \in [B_H, B_L]$, there would be budget left after producing the desired (i.e., unconstrained) output if cost is high. It is then of interest to study the benefit of partial rollover in period 1 under RIO that may relax the budget constraint in period 2. We do that next by allowing production in each period.

4.2. Period-by-period production: *partial rollover under RIO*

The only option in the base model was an extreme roll-over: either the entire budget or nothing. In this section, we show that our main result, the optimality of UILI, continues to hold in the more realistic case when the bureaucrat produces and receives a new budget in every period. The trade-off between ex ante incentives and ex post efficiency remains. We find that UILI can again be optimal for the principal, particularly when the budget is large. The bureaucrat can increase ex post efficiency by rolling over part of the budget when cost is high in period 1, but the benefit of this partial rollover gets weaker for relatively large budgets. Intuitively, a larger budget makes production less constrained, and therefore the benefit from rolling over the budget is smaller.

As in the previous section, the bureaucrat exerts effort only at the beginning of period 1. Based on this effort, he has two independent draws of the cost: one in period 1 and another one in period 2. Given the cost realization, he can produce output in each of the two periods. The bureaucrat receives the same fixed budget B at the beginning of each period for a total budget of $2B$. Under UILI, he must spend his entire budget in each period. Under RIO, he can choose to spend part of the budget producing in period 1 and rolling over part of it. That is, we can now study the possibility of 'partial' rollover, where $r_i \in [0, B]$, with $i = L, H$.

First, we characterize when partial rollover takes place. Based on the optimal budget studied in the previous subsection, we assume that the principal offers $B \in (B_H, B_L)$, and we further assume here that $2B \geq B_H + B_L$.[30] The condition implies that under RIO, X_H will not be reduced in period 1 to increase output in period 2. The reason is that, after a high cost realization in period 1, the bureaucrat can rollover $(B - B_H)$ while producing the unconstrained amount of $X_H^* = \frac{1}{(1+k)c_H}$ in period 1. This rolled-over amount makes the total available budget in period 2 larger than B_L, the amount needed to produce the unconstrained amount of output after a low cost realization. Thus partial rollover allows the bureaucrat to produce the unconstrained X_L if cost in period 2 is low.

We can show that $r_L^* = 0$ and $r_H^* = B - B_H$ (see Lemma 2 in Appendix A). The budget is binding for the low cost realization in period 1. As the bureaucrat does not have enough budget to produce his unconstrained output, he decides to use the entire budget for production without rolling over any amount ($r_L^* = 0$). In contrast, the budget is not binding for the high cost realization. The bureaucrat has money to roll over after producing his unconstrained amount $X_H = \frac{1}{(1+k)c_H}$ at a cost of B_H, implying that $r_H^* = B - B_H$.[31]

Second, comparing the principal's payoffs under the two policies, we can see that UILI can be optimal if the incentive effect on effort is strong enough and if the efficiency gain from rolled-over budget is small, e.g., when B is large. The principal's payoff under UILI is

$$V_0 = 2q_0 X_L^*(B) + 2[1 - q_0]X_H^* - 2B \tag{4}$$

The principal's payoff under RIO is

$$V_1 = 2q_1 X_L^*(B) + 2(1 - q_1)X_H^* + (1 - q_1)q_1[X_L^*(B + r_H^*) - X_L^*(B)] - 2B \tag{5}$$

[30] This restriction biases the analysis towards RIO, and we now briefly discuss the opposite case where $2B < B_H + B_L$. As $B - B_H < B_L - B$ in this case, if the bureaucrat produces the desired output $X_H = \frac{1}{(1+k)c_H}$ at a cost of B_H in period 1 and rolls over the unspent budget $r_H = B - B_H$, he cannot produce the desired output $X_L = \frac{1}{(1+k)c_L}$ in period 2, which costs him B_L. Thus, to improve the outcome of the second-period production, the bureaucrat wants to choose $r_H > B - B_H$, sacrificing the first-period production as the bureaucrat produces $X_H < \frac{1}{(1+k)c_H}$. RIO then results in the lower outcome of production in period 1, and thereby UILI can be optimal even for a milder condition.

[31] We show in the appendix that setting $r_H^* = B - B_H$ is without loss of generality since he is indifferent between rolling over any $r_H \in [B - B_L, B - B_H]$. This is because a dollar as policy drift is equally valuable in period 1 or 2.

From (4) and (5), the difference of the principal's payoffs under the two policies is then

$$V_0 - V_1 = 2(q_0 - q_1)[X_L^*(B) - X_H^*] - (1 - q_1)q_1[X_L^*(B + r_H^*) - X_L^*(B)]. \quad (6)$$

The first term represents the incentive effect, the principal's gain from increasing the chance of the low cost output under UILI relative to RIO. This term is similar to what we found in the base model, and we show in Appendix A (Proposition 5) that there again exists a threshold $\widetilde{c_L}$ below which the effort under UILI is higher than under RIO: $q_0 > q_1$. The second term represents the ex post efficiency gain, the principal's gain under RIO from the bureaucrat's ability to use rolled-over budgets to produce more efficiently in period 2.

Thus, we can see that UILI can be optimal if the incentive effect is large enough to cover the loss associated with inefficient use of resources ex post (so the first term in RHS of (6) dominates the second term).[32] This would be the case for example if the budget B is close to B_L. In this case, RIO provides little scope to increase output when cost is low in period 2. Since production based on B is already close to production based on B_L, the rolled-over budget allows only minor increase in output. Then, if the incentive effect of UILI is large enough (when c_L is sufficiently smaller than $\widetilde{c_L}$), we find UILI to be optimal, as seen in the following example.

Consider the following parameters: $c_L = .1$, $c_H = 1$, $k = .5$, $\psi = .165$. Given B, we can compute that $q_0 > q_1$, i.e., UILI has a stronger incentive effect than RIO. To evaluate the benefit of efficient production in period 2 allowed by RIO, we first compute $B_L = 2.22$ and $B_H = .22$. If the principal chooses a budget B close to B_L, say $B = 2$, the amount rolled-over after c_H in period 1 is large ($r_H = 2 - .22 = 1.78$) but only a small amount ($B_L - B = .22$) is used to produce the unconstrained output X_L in period 2. The benefit of RIO is relatively small. We find that indeed UILI brings a higher payoff to the principal.[33]

---

[32] Notice the similarity between (6) and (3), which states $V_0 - V_1$ in the base model. Both shows the tradeoff between the ex-ante incentives and ex-post inefficiency of UILI. A subtle difference is on the ex-post inefficiency of UILI (or the ex-post efficiency of RIO). In the base model, RIO improves the second-period efficiency by replacing high-cost production with a chance for low-cost production. Here, RIO improves the second-period efficiency by making low-cost production larger.

[33] In this example, we find that $q_0 = .9871 > .9857 = q_1$ and $V_0 - V_1 = .011 > 0$.

4.3. Period-by-period effort

In the previous subsection, the bureaucrat chooses his effort once at the beginning of period 1. Therefore, even if he wanted to adjust his effort in period 2 to take advantage of a higher budget due to a rollover, he could not by assumption. We now study this effort adjustment possibility by briefly considering the case where the bureaucrat can choose his effort in each period 1 and 2. Effort adjustment should make RIO more effective at taking advantage of rolled-over budget, but we verify that UILI can still be optimal for the principal as the key tradeoffs remain largely unaltered. Relative to UILI, we find that RIO will induce lower effort in period 1 but a higher effort in period 2 due to the larger rolled-over budget. This is similar to the base model where RIO has a weaker utility gap effect but adds a second period effect. With separate efforts in each period, we need to compare the 'total effort' over two periods under UILI and RIO, and we find that the total effort can indeed be higher under UILI. Moreover, as in the previous subsection, UILI is preferred to RIO if the 'total effort' is higher under UILI and the ex post efficiency gain under RIO is small, e.g., when the budget is high.

We continue to assume that the bureaucrat receives a budget $B \in (B_H, B_L)$ in each period, with $2B \geq B_H + B_L$, and produces output in each period. The bureaucrat again rolls over the unspent budget only if the cost realization in period 1 is high: $r_L^* = 0$ and $r_H^* = B - B_H$.

We modify the model such that the effort in a period determines the distribution of costs only in that period. Let q_ρ^t define the effort chosen by the bureaucrat under UILI $(\rho = 0)$ and RIO $(\rho = 1)$ at period $t = 1, 2$. Note that q_1^2 depends on cost realization $i \in \{H, L\}$ in period 1: $q_1^2 \in \{q_{1H}^2, q_{1L}^2\}$, and for notational simplicity, let $q_1 \equiv q_1^1$, $q_H \equiv q_{1H}^2$, and $q_L \equiv q_{1L}^2$.

We first compare efforts under UILI and RIO for each period. Since period 2 is just a repetition of period 1 under UILI, we find that $q_0^1 = q_0^2 = q_0$. In contrast, under RIO, $q_1^1 \neq q_1^2$ as the bureaucrat has more budget (from rollover) in period 2. Since the bureaucrat's effort determines the distribution of costs only in one period, the second period effect on the bureaucrat's incentives (the second period effort) is separated from the utility gap effect (the first period effort). The second period effort is higher under RIO because the bureaucrat has more budget in period 2 after a rollover. However, the first period effort is smaller under RIO because a higher effort in period 1 reduces the chance of partial rollover. To summarize: $q_0 > q_1$ and $q_0 = q_L \leq q_H$. We provide a detailed analysis and a proof in the appendix (Proposition 6).

As UILI provides stronger incentives in one period and weaker incentives in the other period, we need to calculate the aggregate probability of having a low cost over two periods, denoted by $\tilde{q}_\rho$. The aggregate probabilities under $\rho = 0$ and $\rho = 1$ are $\tilde{q}_0 = 2q_0$, and $\tilde{q}_1 = q_1 + q_1 q_L + (1 - q_1) q_H$.

We now compare the principal's payoffs under the two policies to see that UILI can be optimal if the incentive effect on effort is strong enough. The principal's payoff over two periods under UILI is $V_0 = 2q_0 X_L^*(B) + 2[1 - q_0] X_H^* - 2B$, and under RIO is $V_1 = \tilde{q}_1 X_L^*(B) + (2 - \tilde{q}_1) X_H^* + (1 - q_1) q_H [X_L^*(B + r_H^*) - X_L^*(B)] - 2B$. Therefore, the difference between the principal's overall payoffs under two policies is

$$V_0 - V_1 = (\tilde{q}_0 - \tilde{q}_1)[X_L^*(B) - X_H^*] - (1 - q_1) q_H [X_L^*(B + r_H^*) - X_L^*(B)]. \qquad (7)$$

Again, as in the previous section, we can see that UILI can be optimal if the incentive effect is large enough to cover the loss associated with inefficient use of resources ex post (so the first term in RHS of (7) dominates the second term). The main difference is that the first term of equation (7) refers to the aggregate probabilities and it says that for UILI to dominate, the aggregate probability under UILI $\tilde{q}_0$ must necessarily be greater than that under RIO $\tilde{q}_1$. To show that the set of parameters such that UILI is preferred to RIO is non empty, we present the following example.

Consider the following parameters: $c_L = .1$, $c_H = 1$, $k = .5$, $\psi = .33$. Given B, one can compute that $q_0 > q_1$, i.e., UILI has a stronger incentive effect than RIO, but also that the aggregate effort is higher under UILI: $\tilde{q}_0 > \tilde{q}_1$. As in the previous example, we can compute that $B_L = 2.22$ and $B_H = .22$. With $B = 2$, the amount of rolled-over budget is identical to the previous example. Finally, we find that indeed UILI brings a higher payoff to the principal.[34]

[34] In this example, we find that $q_0 = .997 > .994 = q_1$, $\tilde{q}_0 = 1.993 > 1.991 = \tilde{q}_1$ and $V_0 - V_1 = .014 > 0$.

5. Conclusion

Bureaucracies operating under use-it-or-lose-it fiscal rules must return any unspent budget at the end of the fiscal period. This policy has been criticized because it leads to year-end spending sprees when bureaucrats rush to spend unused budgets without much planning or care. Proponents of UILI have argued that allowing the bureaucrat to roll over unspent budget (RIO policy) weakens control and oversight of allocated budgets. In this paper, we uncover a previously ignored effect of UILI: it gives the bureaucrat incentives to work harder to avoid such wasteful spending. In other words, the prospect that unspent funds will be wasted can lead the bureaucrat to work harder to spend them productively in the current year.

Thus, our paper is yet another example of the tension between ex post efficiency and ex ante incentives. UILI provides stronger ex ante incentives but at the cost of ex post inefficiencies. We explain how UILI is complementary to the bureaucrat's motivation and show that UILI can be optimal in well-matched bureaucracies or when bureaucrats face high accountability. We also highlight the role of budget size in determining the optimality of UILI.

Our model suggests that the principal could benefit from complementing UILI with financial incentives to return unspent budget. Suppose that the principal allocates a bonus "β" to the bureaucrat for any returned dollar. The bureaucrat would return unspent budget only if $\beta \geq k$. The principal would still recover part of the unspent budget while the bureaucrat would be at least as well off. However, this policy raises ethical questions about a bureaucrat benefiting directly from a bonus resulting from unspent budget as it could lead to abuse similar to the case of sheriffs and jail food money in Alabama mentioned above.

Finally, as noted in a 2009 IMF report, rollover provisions can induce cost saving effort.[35] Then, RIO will have a positive *ex post* incentive effect that we have ignored in our analysis. Instead, we have highlighted the importance of *ex ante* incentives to create a higher chance (for low cost) to be able to use resources efficiently.

[35] See Lienert and Ljungman (2009).

Appendix A

Proof of Lemma 1

(i) For $B < B_H$,

$$\Delta U = \left[\left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - B\right] - \left[\left(\frac{2B}{c_H}\right)^{\frac{1}{2}} - B\right] = \left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - \left(\frac{2B}{c_H}\right)^{\frac{1}{2}} > 0.$$

(ii) For $B \in [B_H, B_L)$,

$$\Delta U = \left[\left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - B\right] - \left[\frac{1}{(1+k)c_H} - \frac{1}{2(1+k)^2 c_H} + k\left(B - \frac{1}{2(1+k)^2 c_H}\right)\right]$$

$$= \left[\left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - B\right] - \left[\frac{1}{2(1+k)c_H} + kB\right].$$

Here,

$$\frac{\partial \Delta U}{\partial B} = (2c_L B)^{-\frac{1}{2}} - 1 - k > 0.$$

The inequality holds for $B \in [B_H, B_L)$ because $\frac{\partial^2 \Delta U}{\partial B^2} < 0$ and $\left.\frac{\partial \Delta U}{\partial B}\right|_{B=B_L} = 0.$ We evaluate ΔU at $B = B_H$:

$$\Delta U|_{B=B_H} = \left(\frac{2}{c_L}\frac{1}{2(1+k)^2 c_H}\right)^{\frac{1}{2}} - \frac{1}{2(1+k)^2 c_H} - \left[\frac{1}{2(1+k)c_H} + \frac{k}{2(1+k)^2 c_H}\right]$$

$$= \left(\frac{1}{(1+k)^2 c_L c_H}\right)^{\frac{1}{2}} - \frac{1}{(1+k)c_H} = (1+k)^{-1} c_H^{-\frac{1}{2}}\left[c_L^{-\frac{1}{2}} - c_H^{-\frac{1}{2}}\right] > 0.$$

As $\frac{\partial \Delta U}{\partial B} > 0$, this inequality implies that $\Delta U > 0$ for $B \in [B_H, B_L)$.

(iii) For $B \geq B_L$,

$$\Delta U = \left[\frac{1}{(1+k)c_L} - \frac{1}{2(1+k)^2 c_L} + k\left(B - \frac{1}{2(1+k)^2 c_L}\right)\right] - \left[\frac{1}{(1+k)c_H} - \frac{1}{2(1+k)^2 c_H} + k\left(B - \frac{1}{2(1+k)^2 c_H}\right)\right]$$

$$= \frac{1}{2(1+k)c_L} - \frac{1}{2(1+k)c_H} > 0.$$

Since $\Delta U = U_L - U_H > 0$ for all B,

$$U_L > qU_L + (1-q)U_H \Rightarrow r_L^* = 0,$$

$$U_H < qU_L + (1-q)U_H \Rightarrow r_H^* = B. \quad \blacksquare$$

Proof of Proposition 1

From the proof of Lemma 1,

$$\Delta U = \begin{cases} \left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - \left(\frac{2B}{c_H}\right)^{\frac{1}{2}} & for\ B < B_H \\ \left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - B - \left[\frac{1}{2(1+k)c_H} + kB\right] & for\ B \in [B_H, B_L) \\ \frac{1}{2(1+k)c_L} - \frac{1}{2(1+k)c_H} & for\ B \geq B_L. \end{cases}$$

From this, it can be readily checked that $\frac{\partial \Delta U}{\partial c_L} < 0$, $\lim_{c_L \to 0} \Delta U = \infty$, and $\lim_{c_L \to c_H} \Delta U = 0$. Thus there exists $\widehat{c_L} \in (0, c_H)$, where $\Delta U(\widehat{c_L}) \stackrel{\text{def}}{=} \frac{1}{2\psi}$, such that $\Delta U > \frac{1}{2\psi}$ for $c_L < \widehat{c_L}$. $\blacksquare$

Proof of Proposition 2

Recall from the proof of proposition 1 that

$$\begin{cases} \left(\frac{2B}{\widehat{c_L}}\right)^{\frac{1}{2}} - \left(\frac{2B}{c_H}\right)^{\frac{1}{2}} \stackrel{\text{def}}{=} \frac{1}{2\psi} & for\ B < B_H \\ \left(\frac{2B}{\widehat{c_L}}\right)^{\frac{1}{2}} - B - \left[\frac{1}{2(1+k)c_H} + kB\right] \stackrel{\text{def}}{=} \frac{1}{2\psi} & for\ B \in [B_H, B_L) \\ \frac{1}{2(1+k)\widehat{c_L}} - \frac{1}{2(1+k)c_H} \stackrel{\text{def}}{=} \frac{1}{2\psi} & for\ B \geq B_L. \end{cases}$$

(i) For $B < B_H$, taking differentiation with respect to k, B, ψ gives

$$\frac{\partial \widehat{c_L}}{\partial k} = 0.$$

$$\frac{1}{2}\left(\frac{2B}{\widehat{c_L}}\right)^{-\frac{1}{2}}\frac{1}{\widehat{c_L}^2}\left[2\widehat{c_L}-2B\frac{\partial\widehat{c_L}}{\partial B}\right]-\frac{1}{2}\left(\frac{2B}{c_H}\right)^{-\frac{1}{2}}\frac{2}{c_H}=0\Rightarrow\frac{\partial\widehat{c_L}}{\partial B}=\left(\frac{2\widehat{c_L}^3}{B}\right)^{\frac{1}{2}}\left[(2\widehat{c_L}B)^{-\frac{1}{2}}-(2c_HB)^{-\frac{1}{2}}\right]>0.$$

$$-\frac{1}{2}\left(\frac{2B}{\widehat{c_L}}\right)^{-\frac{1}{2}}2B\frac{\partial\widehat{c_L}}{\partial\psi}=-\frac{1}{2\psi^2}\ \Rightarrow\ \frac{\partial\widehat{c_L}}{\partial\psi}=(2\widehat{c_L}B)^{-\frac{1}{2}}\psi^{-2}>0.$$

(ii) For $B\in[B_H,B_L)$, taking differentiation with respect to k,B,ψ gives

$$-\frac{1}{2}\left(\frac{2B}{\widehat{c_L}}\right)^{-\frac{1}{2}}\frac{2B}{\widehat{c_L}^2}\frac{\partial\widehat{c_L}}{\partial k}+\frac{2c_H}{[2(1+k)c_H]^2}-B=0$$

$$\Rightarrow\frac{\partial\widehat{c_L}}{\partial k}=-\left(\frac{2\widehat{c_L}^3}{B}\right)^{\frac{1}{2}}\left(B-\frac{1}{2(1+k)^2c_H}\right)=-\left(\frac{2\widehat{c_L}^3}{B}\right)^{\frac{1}{2}}(B-B_H)\le 0.$$

$$\frac{1}{2}\left(\frac{2B}{\widehat{c_L}}\right)^{-\frac{1}{2}}\frac{1}{\widehat{c_L}^2}\left[2\widehat{c_L}-2B\frac{\partial\widehat{c_L}}{\partial B}\right]-1-k=0\Rightarrow\frac{\partial\widehat{c_L}}{\partial B}=\left(\frac{2\widehat{c_L}^3}{B}\right)^{\frac{1}{2}}\left[(2\widehat{c_L}B)^{-\frac{1}{2}}-1-k\right]>0,$$

where the inequality holds because $(2c_LB)^{-\frac{1}{2}}-1-k>0$ for $B\in[B_H,B_L)$ as shown in the proof of Lemma 1.

$$-\frac{1}{2}\left(\frac{2B}{\widehat{c_L}}\right)^{-\frac{1}{2}}2B\frac{\partial\widehat{c_L}}{\partial\psi}=-\frac{1}{2\psi^2}\ \Rightarrow\ \frac{\partial\widehat{c_L}}{\partial\psi}=(2\widehat{c_L}B)^{-\frac{1}{2}}\psi^{-2}>0.$$

(iii) For $B\ge B_L$, taking differentiation with respect to k,B,ψ gives

$$-\frac{1}{[2(1+k)\widehat{c_L}]^2}\left[2\widehat{c_L}+2(1+k)\frac{\partial\widehat{c_L}}{\partial k}\right]+\frac{2c_H}{[2(1+k)c_H]^2}=0$$

$$\Rightarrow\frac{\partial\widehat{c_L}}{\partial k}=-\frac{1}{2(1+k)}\left[\frac{1}{2(1+k)^2\widehat{c_L}}-\frac{1}{2(1+k)^2\widehat{c_L}}\right]=-\frac{1}{2(1+k)}(B_L-B_H)<0.$$

$$\frac{\partial\widehat{c_L}}{\partial B}=0.$$

$$-\frac{1}{[2(1+k)\widehat{c_L}]^2}\left[2(1+k)\frac{\partial\widehat{c_L}}{\partial\psi}\right]=-\frac{1}{2\psi^2}\ \Rightarrow\ \frac{\partial\widehat{c_L}}{\partial\psi}=(1+k)\widehat{c_L}^2\psi^{-2}>0.\quad\blacksquare$$

Proof of Proposition 3

Given that $X_L^*>X_H^*$, we can follow the same steps as in the proof of proposition 1 to compare V_0 and V_1: there exists $\overline{c_L}\in(0,c_H)$, where $\Delta U(\overline{c_L})\overset{\text{def}}{=}\frac{\sqrt{3}}{2\psi}$, such that $\Delta U>\frac{\sqrt{3}}{2\psi}$ for $c_L<\overline{c_L}$.

Since $\frac{\partial \Delta U}{\partial c_L} < 0$ and $\Delta U(\widehat{c_L}) \stackrel{\text{def}}{=} \frac{1}{2\psi} < \frac{\sqrt{3}}{2\psi}$, $\widetilde{c_L} < \widehat{c_L}$. ■

Period-by-period production

PROPOSITION 5: *With period-by-period production, UILI can provide stronger incentives for effort than RIO. More specifically, there exists* $\widetilde{c_L} \in (0, c_H)$, *where* $\Delta U(\widetilde{c_L}) \stackrel{\text{def}}{=} \frac{1}{4\psi}$, *such that* $q_0 > q_1$ *if and only if* $c_L < \widetilde{c_L}$.

PROOF: Consider first the case of UILI. Given $B \in (B_H, B_L)$, the bureaucrat produces output in each period such that $X_L^* = \sqrt{\frac{2B}{c_L}}$, $X_H^* = \frac{1}{(1+k)c_H}$. The bureaucrat chooses q to

$$\max q[U_L(B) + qU_L(B) + (1-q)U_H(B)] + (1-q)[U_H(B) + qU_L(B) + (1-q)U_H(B)] - \frac{q^2}{2\psi}.$$

From this, the bureaucrat's choice of effort q_0 satisfies the following condition:

$$2\Delta U(B) = \frac{q_0}{\psi}. \tag{A1}$$

Next, consider the case of RIO. In period 2, the bureaucrat has $B + r_i$, where r_i is the budget that is rolled over when the first-period cost realization is $i \in \{L, H\}$. With this budget, the bureaucrat produces output such that:

$$X_L^*(B + r_i) = \begin{cases} \sqrt{\frac{2(B+r_i)}{c_L}} & if\ r_i < B_L - B \\ \frac{1}{(1+k)c_L} & otherwise \end{cases}, \quad X_H^* = \frac{1}{(1+k)c_H}.$$

In period 1, given c_i, the bureaucrat produces output such that:

$$X_L^*(B - r_L) = \sqrt{\frac{2(B-r_L)}{c_L}}, \quad X_H^*(B - r_H) = \begin{cases} \frac{1}{(1+k)c_H} & if\ r_H \le B - B_H \\ \sqrt{\frac{2(B-r_H)}{c_H}} & otherwise \end{cases}.$$

The bureaucrat chooses r_i to

$$\max \; U_i(B - r_i) + qU_L(B + r_i) + (1 - q)U_H(B + r_i).$$

From this, we can show that the bureaucrat chooses partial rollover, which is characterized in Lemma 2 next.

LEMMA 2: *With period-by-period production,* $r_L^* = 0$ *and* $r_H^* = B - B_H$.

PROOF: Given that $c = c_L$, the first-order condition is:

$$-U_L'(B - r_L) + qU_L'(B + r_L) + (1 - q)U_H'(B + r_L) < 0.$$

We show why it holds as inequality. When $B < B_L$, as shown in the proof of Lemma 1,

$$U_L'(B) = (2Bc_L)^{-\frac{1}{2}} - 1 > k = U_H'(B).$$

In addition, note that $U_L''(B) < 0$. Thus $U_L'(B - r_L) > U_L'(B + r_L)$ and $U_L'(B - r_L) > U_H'(B + r_L) = k$ for any r_L. Thus the first-order condition holds as inequality as shown above, implying that $r_L^* = 0$.

Next, given that $c = c_H$, the first-order condition is:

$$-U_H'(B - r_H) + qU_L'(B + r_H) + (1 - q)U_H'(B + r_H) \gtreqless 0.$$

Note that $X_L(B + r_H)$, which is the second-period output in the low cost realization, and $X_H(B - r_H)$, which is the first-period output in the high cost realization, change qualitatively as r_H changes, so does the first-order condition. There are two cutoffs of r_H above and below which the first-order condition changes: $B_L - B$ and $B - B_H$. As $B_L - B \leq B - B_H$ (because we assume that $2B \geq B_H + B_L$), we evaluate the first-order condition in three different regimes: (i) $r_H < B_L - B$, (ii) $r_H \in [B_L - B, B - B_H]$, and (iii) $r_H > B - B_H$.

(i) For $r_H < B - B_L$, the budget is binding in the second-period low cost realization. The first-order condition becomes:

$$-k + q\left\{[2(B + r_H)c_L]^{-\frac{1}{2}} - 1\right\} + (1 - q)k > 0.$$

The inequality holds because $[2(B+r_H)c_L]^{-\frac{1}{2}}-1>k$ as long as the budget is binding. It implies that $r_H \geq B-B_L$, violating the initial condition that $r_H < B-B_L$. (ii) For $r_H \in [B_L-B, B-B_H]$, the budget is not binding either in the first-period high cost realization or in the second-period low cost realization. The first-order condition becomes:

$$-k+qk+(1-q)k=0.$$

(iii) For $r_H > B-B_H$, the budget is binding in the first-period high cost realization. The first-order condition becomes:

$$-\left\{[2(B-r_H)c_H]^{-\frac{1}{2}}-1\right\}+qk+(1-q)k<0.$$

The inequality holds because we have shown that $[2(B-r_H)c_H]^{-\frac{1}{2}}-1>k$ as long as the budget is binding. It implies that $r_H \leq B-B_H$, violating the initial condition that $r_H > B-B_H$.

Thus the bureaucrat chooses $r_H \in [B_L-B, B-B_H]$, and he is indifferent among any of them because they leads the bureaucrat to produce the same output. Then as we can assume without loss of generality that the bureaucrat rolls over all of the unspent budget, $r_H^* = B-B_H$. This concludes the proof of lemma 2. ■

Continuing with the proof of proposition 5, the bureaucrat chooses his effort q to

$$\max \; q\{U_L(B)+qU_L(B)+(1-q)U_H(B)\} \\ +(1-q)\{U_H(B-r_H^*)+qU_L(B+r_H^*)+(1-q)U_H(B+r_H^*)\}-\frac{q^2}{2\psi}.$$

From this, the bureaucrat's choice of effort q_1 satisfies the following condition:[36]

$$2\Delta U(B)-q_1[U_L(B+r_H^*)-(U_L(B)+kr_H^*)] \\ +(1-q_1)[U_L(B+r_H^*)-(U_L(B)+kr_H^*)] = \frac{q_1}{\psi}. \qquad \text{(A2)}$$

[36] Note that the output for the high cost realization is independent of the amount of rollover. So we use the fact that $U_H(B-r_H^*)=U_H(B)-kr_H^*$ and $U_H(B+r_H^*)=U_H(B)+kr_H^*$ to derive (A2).

Let $Z \stackrel{\text{def}}{=} U_L(B + r_H^*) - U_L(B) - kr_H^* > 0$, where the inequality holds because $U_L'(B) = (2c_L B)^{-\frac{1}{2}} - 1 > k$ (as shown in the proof of Lemma 1) when $B < B_L$. With this, from (A2)

$$q_1 = \frac{2\psi\Delta U(B) + \psi Z}{1 + 2\psi Z}.$$

With $q_0 = 2\psi\Delta U(B)$ from (A1), as $Z > 0$, it is immediate that $q_0 > q_1$ if and only if $\psi\Delta U(B) > \frac{1}{4}$. Then following the same procedure as in the proof of proposition 1, there exists $\widetilde{c_L} \in (0, c_H)$, where $\Delta U(\widetilde{c_L}) \stackrel{\text{def}}{=} \frac{1}{4\psi}$, such that $\Delta U > \frac{1}{4\psi}$ for $c_L < \widetilde{c_L}$. This concludes the proof of proposition 5. ■

Period-by-period effort

PROPOSITION 6: *With period-by-period effort, UILI provides stronger incentives in period 1 and weaker incentives in period 2 than RIO. More specifically*, $q_0 > q_1$ and $q_0 = q_L \leq q_H$.

PROOF: Consider first the case of UILI. In period 2, the bureaucrat chooses the same effort as in the base model since it is the last period: $q_0 = \psi\Delta U(B)$. In period 1, he chooses q to

$$\max \; q\left[U_L(B) + q_0 U_L(B) + (1 - q_0)U_H(B) - \frac{q_0^2}{2\psi}\right] + (1 - q)\left[U_H(B) + q_0 U_L(B) + (1 - q_0)U_H(B) - \frac{q_0^2}{2\psi}\right] - \frac{q^2}{2\psi}.$$

From this, it is easy to see that the bureaucrat again chooses the same effort. Thus, in both periods

$$q_0 = \psi\Delta U(B). \tag{A3}$$

Next, consider the case of RIO. In period 2, the bureaucrat's effort decision depends on how much budget rolled over from period 1. The amount of rollover is contingent on the first-period cost realization $i \in \{H, L\}$, so does the bureaucrat's effort in period 2. With r_i from period 1, the bureaucrat chooses q_i to

$$\max\ q_i U_L(B + r_i) + (1 - q_i)U_H(B + r_i) - \frac{q_i^2}{2\psi}.$$

From this,

$$q_i = \psi \Delta U(B + r_i). \tag{A4}$$

In period 1, the bureaucrat chooses r_i to

$$\max\ U_i(B - r_i) + q_i(B + r_i)U_L(B + r_i) + \big(1 - q_i(B + r_i)\big)U_H(B + r_i) - \frac{q_i(B + r_i)^2}{2\psi}.$$

From this, we can show the following lemma.

LEMMA 3: *With period-by-period effort,* $r_L^* = 0$ *and* $r_H^* = B - B_H$.

PROOF: The first-order condition for the choice of r_i is

$$\begin{aligned}
&-U_i'(B - r_i) + q_i(B + r_i)U_L'(B + r_i) + \big(1 - q_i(B + r_i)\big)U_H'(B + r_i) \\
&\qquad + q_L'(B + r_i)\left[\big(U_L(B + r_i) - U_H(B + r_i)\big) - \frac{q_i(B + r_i)}{\psi}\right] \\
&\qquad = -U_i'(B - r_i) + q_i(B + r_i)U_L'(B + r_i) + \big(1 - q_i(B + r_i)\big)U_H'(B + r_i),
\end{aligned}$$

where the equality holds because $q_i = \psi \Delta U(B + r_i)$ from the bureaucrat's effort choice problem. Since this first-order condition is the same as the one in proof of Lemma 2, the rest of the proof is the same as well. This concludes the proof of lemma 3. ■

With this rollover amount, from (A4) we can recover the bureaucrat's effort chosen in period 2:

$$q_L = \psi \Delta U(B) = q_0, \tag{A5}$$

$$q_H = \psi \Delta U(B + r_H^*) \geq q_0. \tag{A6}$$

The bureaucrat chooses q in period 1 to

$$\max\ q\left[U_L(B)+q_LU_L(B)+(1-q_L)U_H(B)-\frac{q_L^2}{2\psi}\right]$$
$$+(1-q)\left[U_H(B-r_H^*)+q_HU_L(B+r_H^*)+(1-q_H)U_H(B+r_H^*)-\frac{q_H^2}{2\psi}\right]-\frac{q^2}{2\psi}.$$

From this,[37]

$$q_1=\psi(1+q_L)[U_L(B)-U_H(B)]-\psi q_H[U_L(B+r_H^*)-U_H(B+r_H^*)]+\frac{q_H^2-q_L^2}{2}$$
$$=(1+q_0)q_0-q_H^2+\frac{q_H^2-q_0^2}{2}=q_0-\frac{1}{2}(q_H^2-q_0^2). \qquad \text{(A7)}$$

From (A3), (A5) ~ (A7), $q_0>q_1$ and $q_0=q_L\leq q_H$. This concludes the proof of proposition 6. ■

Appendix B

We prove here that the principal weakly prefers a lower k under either budget policy by showing that both outputs and the effort increase as k decreases.

Consider first the case of UILI. The principal's payoff is $V_0=q_0X_L^*+(1-q_0)X_H^*-B$, where $q_0=\psi\Delta U$. Differentiating V_0 with respect to k gives

$$\frac{\partial V_0}{\partial k}=q_0\frac{\partial X_L^*}{\partial k}+(1-q_0)\frac{\partial X_H^*}{\partial k}+\frac{\partial q_0}{\partial k}(X_L^*-X_H^*).$$

Since $q_0=\psi\Delta U$, the sign of $\frac{\partial q_0}{\partial k}$ is the same as that of $\frac{\partial \Delta U}{\partial k}$. From the definition of ΔU,

$$\frac{\partial \Delta U}{\partial k}=\left(B-\frac{c_L}{2}(X_L^*)^2\right)-\left(B-\frac{c_H}{2}(X_H^*)^2\right).$$

For $B<B_H$, as the budget is binding for both cost realizations, we have $X_i^*=\sqrt{\frac{2B}{c_i}}$ and $B=\frac{c_L}{2}(X_L^*)^2=\frac{c_H}{2}(X_H^*)^2$, implying that $\frac{\partial X_L^*}{\partial k}=\frac{\partial X_H^*}{\partial k}=0$ and $\frac{\partial \Delta U}{\partial k}=0$. Thus, $\frac{\partial V_0}{\partial k}=0$. For

[37] Note that as $B-r_H^*=B_H$, we have $U_H(B-r_H^*)=U_H(B)-kr_H^*$, $U_H(B+r_H^*)=U_H(B)+kr_H^*$, and therefore $U_H(B-r_H^*)=2U_H(B)-U_H(B+r_H^*)$. We use this to derive (A7).

$B \in [B_H, B_L]$, as the budget is binding only for low cost, we have $X_L^* = \sqrt{\frac{2B}{c_L}}$, $X_H^* = \frac{1}{(1+k)c_H}$, and $B = \frac{c_L}{2}(X_L^*)^2 > \frac{c_H}{2}(X_H^*)^2$, implying that $\frac{\partial X_L^*}{\partial k} = 0$, $\frac{\partial X_H^*}{\partial k} < 0$, and $\frac{\partial \Delta U}{\partial k} < 0$. Thus, $\frac{\partial V_0}{\partial k} < 0$. For $B > B_L$, as the budget is not binding for either cost realization, we have $X_i^* = \frac{1}{(1+k)c_i}$ and $B > \frac{c_L}{2}(X_L^*)^2 > \frac{c_H}{2}(X_H^*)^2$, implying that $\frac{\partial X_L^*}{\partial k} < 0$, $\frac{\partial X_H^*}{\partial k} < 0$, and $\frac{\partial \Delta U}{\partial k} < 0$. Thus, $\frac{\partial V_0}{\partial k} < 0$.

Consider next the case of RIO. The principal's payoff is $V_1 = \hat{q}_1 X_L^* + (1-\hat{q}_1)X_H^* - B$, where $\hat{q}_1 = q_1 + (1-q_1)q_1$ and $q_1 = \frac{2\psi\Delta U}{1+2\psi\Delta U}$. Differentiating V_1 with respect to k gives

$$\frac{\partial V_1}{\partial k} = \hat{q}_1 \frac{\partial X_L^*}{\partial k} + (1-\hat{q}_1)\frac{\partial X_H^*}{\partial k} + \frac{\partial \hat{q}_1}{\partial k}(X_L^* - X_H^*).$$

From $\hat{q}_1$ defined above, we can check that the sign of $\frac{\partial \hat{q}_1}{\partial k}$ is the same as that of $\frac{\partial q_1}{\partial k}$, which is again the same of that of $\frac{\partial \Delta U}{\partial k}$.

Since X_i^* and ΔU are under RIO are the same as under UILI, $\frac{\partial X_i^*}{\partial k}$ and $\frac{\partial \Delta U}{\partial k}$ under RIO are the same as under UILI. Thus, $\frac{\partial V_1}{\partial k} = 0$ for $B < B_H$ and $\frac{\partial V_1}{\partial k} < 0$ for $B \geq B_H$. ■

Appendix C

We prove here that the principal chooses $B^* \in [B_H, B_L]$ regardless of ρ.

(i) Suppose that $B > B_L$. The budget is not binding for either cost realization, so $X_i = \frac{1}{(1+k)c_i}$, which is independent of B. The principal's payoff is then $V = q(B)X_L + \big(1 - q(B)\big)X_H - B$. When B changes by dB, V changes by:

$$dV = [q'(B)(X_L - X_H) - 1]dB.$$

We evaluate it at $B > B_L$. As q is an increasing function of ΔU regardless of ρ, so let $q(B) = f\big(\Delta U(B)\big)$. Since $\Delta U = \frac{1}{2(1+k)(c_H - \Delta c)} - \frac{1}{2(1+k)c_H}$ for $B \geq B_L$, $\Delta U'(B) = 0$. Then $q'(B) = f'(\Delta U)\Delta U'(B) = 0$. Thus

$$dV = -dB < 0,$$

which implies that the principal's payoff increases by decreasing B to B_L, implying $B^* \leq B_L$.

(ii) Suppose that $B < B_H$. The budget is binding for both cost realizations, so $X_i = \left(\frac{2B}{c_i}\right)^{\frac{1}{2}}$. The principal's payoff is then $V = q(B)\left(\frac{2B}{c_L}\right)^{\frac{1}{2}} + (1 - q(B))\left(\frac{2B}{c_H}\right)^{\frac{1}{2}} - B$. When B changes by dB, V changes by:

$$dV = \left\{q'(B)\left[\left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - \left(\frac{2B}{c_H}\right)^{\frac{1}{2}}\right] + q(B)(2c_L B)^{-\frac{1}{2}} + \left(1 - q(B)\right)(2c_H B)^{-\frac{1}{2}} - 1\right\} dB.$$

Since $\Delta U = \left(\frac{2B}{c_L}\right)^{\frac{1}{2}} - \left(\frac{2B}{c_H}\right)^{\frac{1}{2}}$ for $B < B_H$, we have $\Delta U'(B) > 0$, implying that $q'(B) = f'(\Delta U)\Delta U'(B) > 0$ regardless of ρ. With this and $c_L < c_H$,

$$dV > \left[(2c_H B)^{-\frac{1}{2}} - 1\right] dB.$$

Since $(2c_H B)^{-\frac{1}{2}}$ decreases with B, with the upper bound of $B = B_H$,

$$dV > \left[(2c_H B)^{-\frac{1}{2}} - 1\right] dB > \left[\left(2c_H \frac{1}{2(1+k)^2 c_H}\right)^{-\frac{1}{2}} - 1\right] dB = kdB > 0.$$

This implies that the principal becomes better off by increasing B to B_H, indicating that $B^* \geq B_H$. ■

References

Aberbach, J., Putnam, R., and Rockman, B. *Bureaucrats and Politicians in Western Democracies*. Harvard University Press, Cambridge, Massachusetts, 1981.

Alesina, A. and Tabellini, G. "Bureaucrats or Politicians? Part I: A Single Policy Task." *American Economic Review*, Vol. 97 (2007), pp. 169-179.

Antle, R. and Eppen, G. "Capital Rationing and Organizational Slack in Capital Budgeting." *Management Science*, Vol. 31 (1985), pp. 163-174.

Benabou, R. and Tirole, J. "Intrinsic and Extrinsic Motivation." *Review of Economic Studies*, Vol. 70 (2003), pp. 489-520.

Besley, T. and Ghatak, M. "Competition and Incentives with Motivated Agents." *American Economic Review*, Vol. 95 (2005), pp. 616-636.

Biglaiser, G. and Ma, C. A. "Moonlighting: Public Service and Private Practice." *RAND Journal of Economics*, Vol. 38 (2007), pp. 1113-1133.

Borcherding, T. and Besocke, P. "The Contemporary Political Economy Approach to Bureaucracy." in C. Rowley and F. Schneider, eds., *The Encyclopedia of Public Choice*, 2003.

Brehm, J. and Gates, S. *Working, Shirking, and Sabotage*. University of Michigan Press, Ann Arbor, Michigan, 1997.

Celik, G. "Mechanism Design with Collusive Supervision." *Journal of Economic Theory*, Vol. 144 (2009), pp. 69-95.

Dessein, W. "Incomplete Contracts and Firm Boundaries: New Directions." *Journal of Law, Economics, and Organization*, Vol. 30 (2014), pp. i13-i36.

Dewatripont, M., Jewitt, I., and Tirole, J. "The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies." *Review of Economic Studies*, Vol. 66 (1999), pp. 199–217.

Dhillon A, Nicolò A, and Xu F. "Corruption, Intrinsic Motivation, and The Love of Praise." *Journal of Public Economic Theory*, Vol. 19 (2017), pp.1077–1098.

Faure-Grimaud, A., Laffont, J.-J., and Martimort, D. "Collusion, Delegation and Supervision with Soft Information." *Review of Economic Studies*, Vol. 70 (2003), pp. 253-279.

Fichtner, J. and Greene, R. "Curbing the Surge in Year-End Federal Government Spending: Reforming 'Use It or Lose It' Rules" George Mason University working paper, 2014.

General Accounting Office, "Federal year-end spending: Symptom of a larger problem," 1980a.

General Accounting Office, "Government agencies need effective planning to curb unnecessary year-end planning," 1980b.

General Accounting Office, "Federal year-end spending patterns for fiscal years 1982, 1983, and 1984," 1985.

General Accounting Office, "Year-end spending; reforms underway but better reporting and oversight needed," 1998.

Halac, M. and Yared, P. "Commitment vs. Flexibility with Costly Verification," CEPR Discussion Paper, No. DP12572, 2017.

Halac, M. and Yared, P. "Fiscal Rules and Discretion in a World Economy," *American Economic Review*, 108, (2018), pp. 2305-2334.

Hiriart, Y. and Martimort, D. "How Much Discretion for Risk Regulators?" *Rand Journal of Economics*, Vol. 43 (2012), pp. 283-314.

Holmstrom, B. and Milgrom, P. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design." *Journal of Law, Economics, and Organization,* Vol. 7 (1991), pp. 24-52.

Khalil, F., Kim, D. and Lawarree, J. "Contracts Offered by Bureaucrats," *RAND Journal of Economics*, Vol. 44 (2013), pp. 686-711.

Laffont, J.-J. and Martimort, D. “Collusion and Delegation.” *RAND Journal of Economics*, Vol. 29 (1998), pp. 280-305.

Liebman, J. and Mahoney, N. “Do Expiring Budgets Lead to Wasteful Year-End Spending? Evidence from Federal Procurement” *American Economic Review*, Vol. 107 (2017), pp. 3510-3549.

Lienert, I., and Ljungman G. “Carry-Over of Budget Authority.” Public Financial Management Technical Guidance Note, Fiscal Affairs Department, International Monetary Fund, 2009.

Martimort, D. “The Multiprincipal Nature of Government.” *European Economic Review*, Vol. 40 (1996), pp. 673-685.

Martimort, D. “Multi-Contracting Mechanism Design.” in R. Blundell, W. Newey and T. Persson, eds., *Advances in Economic Theory: Proceedings of the World Congress of the Econometric Society*. Cambridge University Press, 2007.

McPherson, M. F. “An Analysis of Year-End Spending and the Feasibility of a Carryover Incentive for Federal Agencies.” MBA Professional Report, Naval Postgraduate School, Monterey, CA, 2007.

Migué, J.-L. and Bélanger, G. “Toward a General Theory of Managerial Discretion.” *Public Choice*, Vol. 17 (1974), pp. 27–43.

Moe, T. “The Positive Theory of Public Bureaucracy.” in D. Mueller, eds., *Perspectives on Public Choice*: *A Handbook*. Cambridge University Press, Cambridge, 1997.

Mookherjee, D. “Incentives in Hierarchy” in R. Gibbons and J. Roberts, eds., *Handbook of Organizational Economics*, Princeton University Press, 2013.

Mookherjee, D. and Tsumagari, M. “The Organization of Supplier Networks: Effects of Delegation and Intermediation.” *Econometrica*, Vol. 72 (2004), pp. 1179-1219.

New York Times, “*Alabama Moves to Limit Sheriffs from Pocketing Jail Food Money*,” Alan Blinder, July 11, 2018.

Ogrysko N. “Endless CRs, appropriations problems drive end of year spending surge” at https://federalnewsradio.com/contractsawards/2015/09/endless-crs-appropriations-problems-drive-end-year-spending-surge/, Federal News Radio, 2015.

Prendergast, C. “Limits of Bureaucratic Efficiency.” *Journal of Political Economy*, Vol. 111 (2003), pp. 929-958.

Prendergast, C. “The Motivation and Bias of Bureaucrats,” *American Economic Review*, Vol. 97 (2007), pp. 180-196.

Rose-Ackerman, S. “Reforming Public Bureaucracy through Economic Incentives?" *Journal of Law, Economics and Organization*, Vol. 2 (1986), pp. 131-161.

Shin, D. and Strausz, R. “Delegation and Dynamic Incentives,” *Rand Journal of Economics*, Vol. 45 (2014), pp. 495-520.

Subcommittee on Federal Spending Oversight and Emergency Management, “Prudent Planning or Wasteful Binge? Another Look at End of the Year Spending,” 115th Congress , September 20, 2017. https://www.gpo.gov/fdsys/pkg/CHRG-115shrg29444/pdf/CHRG-115shrg29444.pdf

Taylor, J. “Rules Versus Discretion: Assessing the Debate Over the Conduct of Monetary Policy,” NBER Working Paper, No. 24149, 2017.

Tirole, J. “The Internal Organization of Government.” *Oxford Economic Papers*, Vol. 46 (1994), pp. 1-29.

Veterans Affair Office of Inspector General “Review of Alleged Waste of Funds at the VA Medical Center in Detroit, Michigan”, https://www.va.gov/oig/pubs/VAOIG-16-02729-350.pdf, 2016.